Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

1-Algebraic-functions/1.2-Trinomial-products/1.2.2-Quartic/41-1.2.2.4-f-x-^m-d+e-x^2-^q-a+b-x^2+c-x^4-^p

Nasser M. Abbasi

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [413]. This is test number [41].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.27 (410)	0.73 (3)
Mathematica	98.31 (406)	1.69 (7)
Maple	91.04 (376)	8.96 (37)
Fricas	79.66 (329)	20.34 (84)
Giac	62.71 (259)	37.29 (154)
Mupad	52.78 (218)	47.22 (195)
Maxima	34.87 (144)	65.13 (269)
Sympy	31.72 (131)	68.28 (282)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

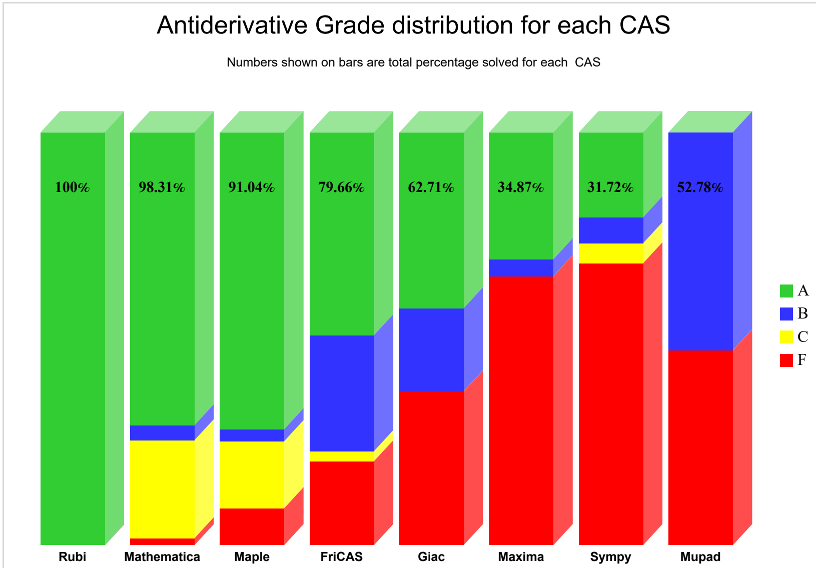
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

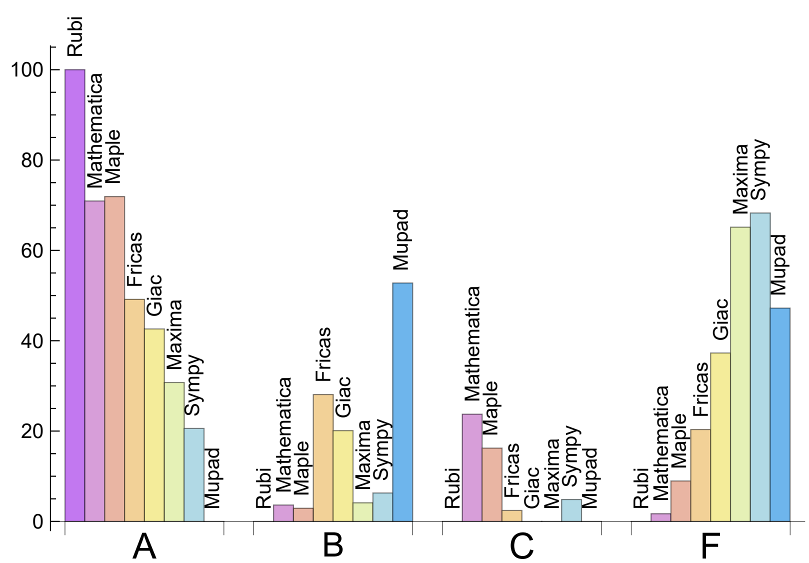
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.274	0.000	0.000	0.726
Maple	71.913	2.906	16.223	8.959
Mathematica	70.944	3.632	23.729	1.695
Fricas	49.153	28.087	2.421	20.339
Giac	42.615	20.097	0.000	37.288
Maxima	30.751	4.116	0.000	65.133
Sympy	20.581	6.295	4.843	68.281
Mupad	0.000	52.785	0.000	47.215

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	3	100.00	0.00	0.00
Mathematica	7	100.00	0.00	0.00
Maple	37	100.00	0.00	0.00
Fricas	84	80.95	19.05	0.00
Giac	154	73.38	11.04	15.58
Mupad	195	0.00	100.00	0.00
Maxima	269	71.75	0.00	28.25
Sympy	282	67.73	32.27	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.24
Giac	0.51
Rubi	0.52
Maple	0.79
Mathematica	3.01
Sympy	3.14
Mupad	8.36
Fricas	13.06

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	109.45	1.14	94.00	1.00
Maple	197.24	0.94	146.50	0.84
Rubi	224.45	1.01	186.50	1.01
Mathematica	349.61	1.36	160.50	0.96
Sympy	504.31	3.06	121.00	1.09
Giac	856.24	3.32	145.00	1.12
Fricas	2055.09	6.49	205.00	1.80
Mupad	4535.03	13.49	169.00	1.95

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

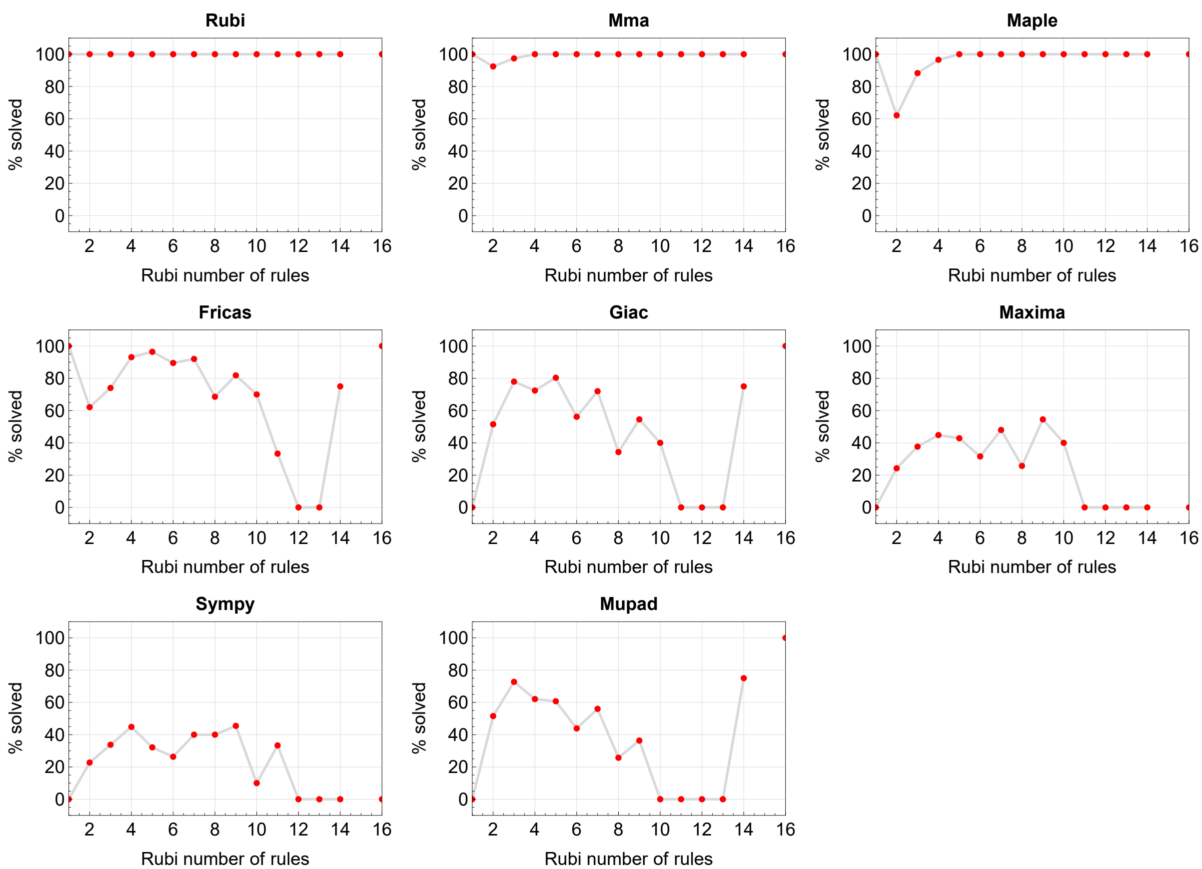


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

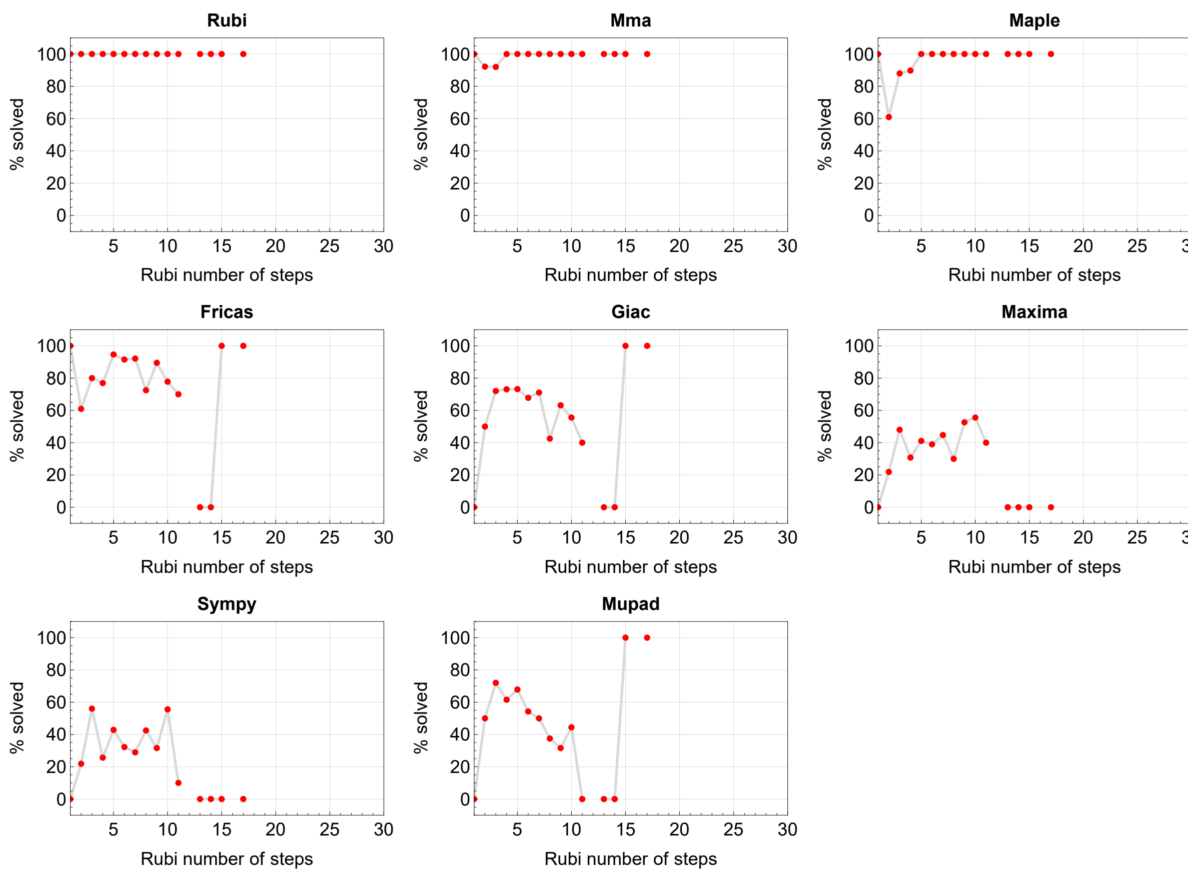


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

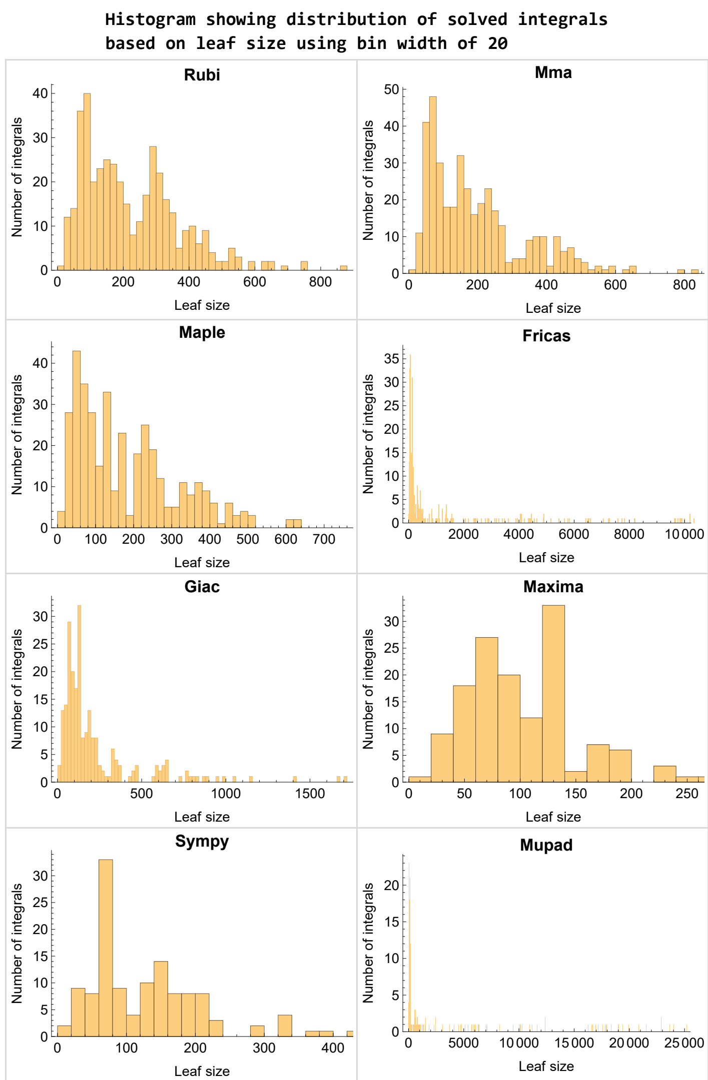


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

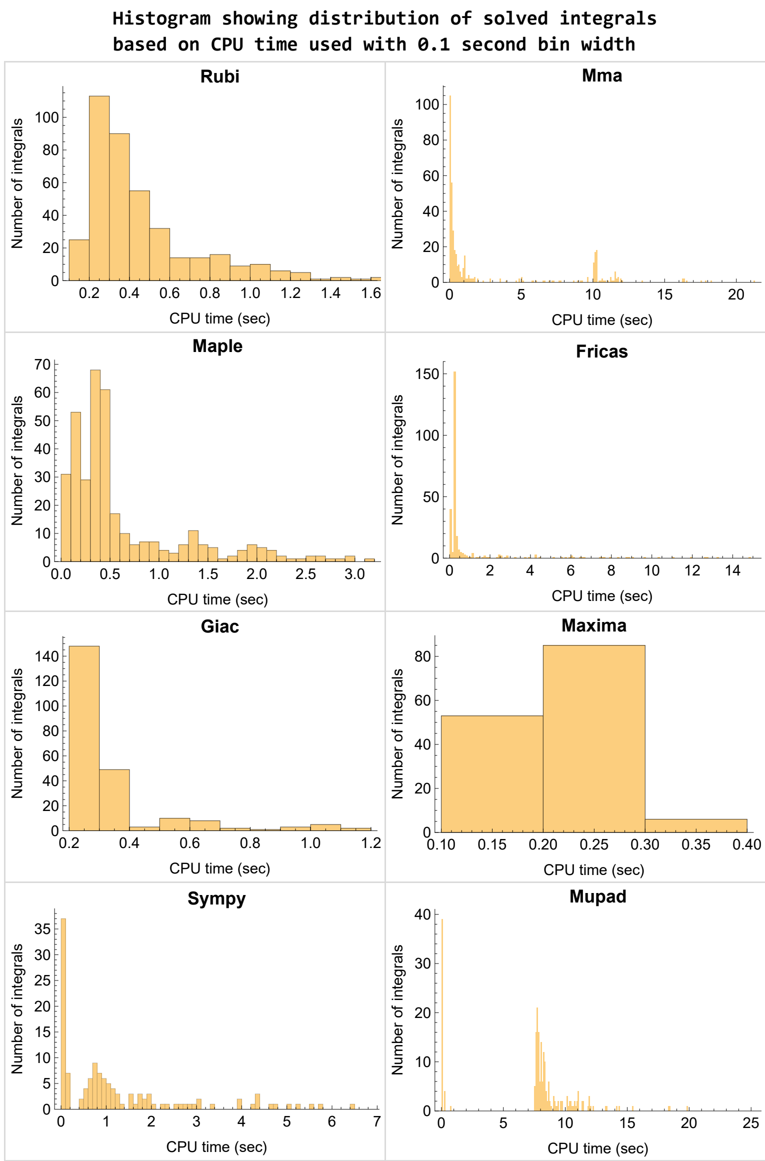


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

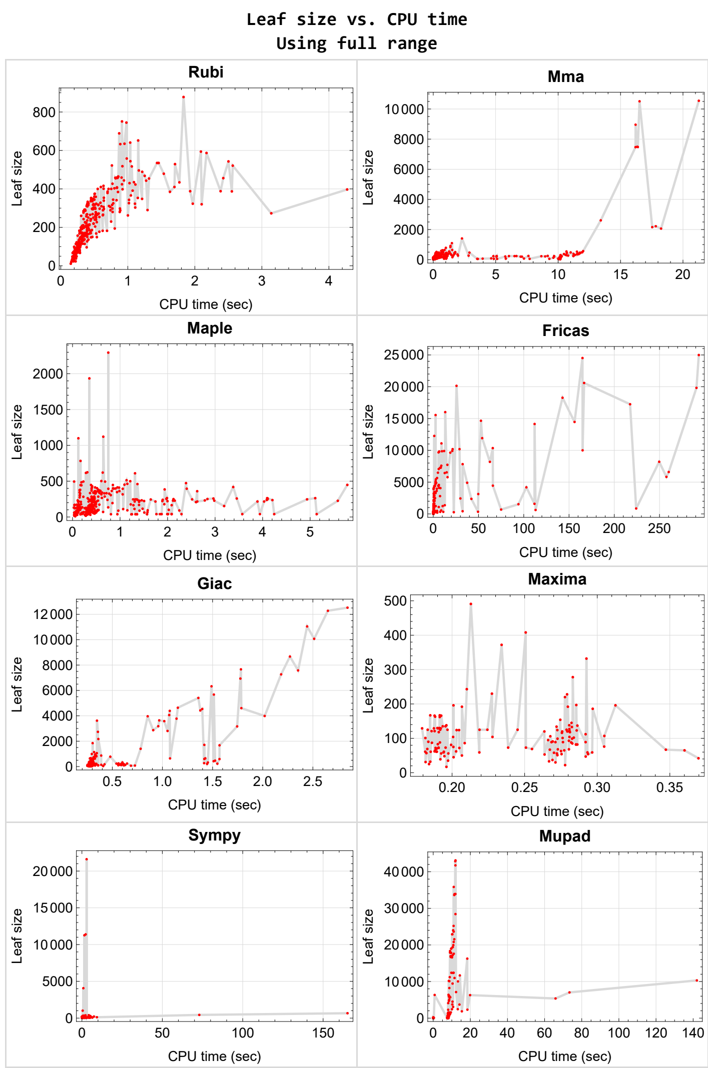


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{14, 354, 355, 359, 360, 370, 376\}$

Mathematica $\{151, 152, 153, 154, 155, 163, 164, 165, 166, 167, 168, 189, 190, 192, 193, 199, 200, 201, 202, 203, 225, 397, 398, 399\}$

Maple $\{76, 78, 80, 82, 84, 86\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

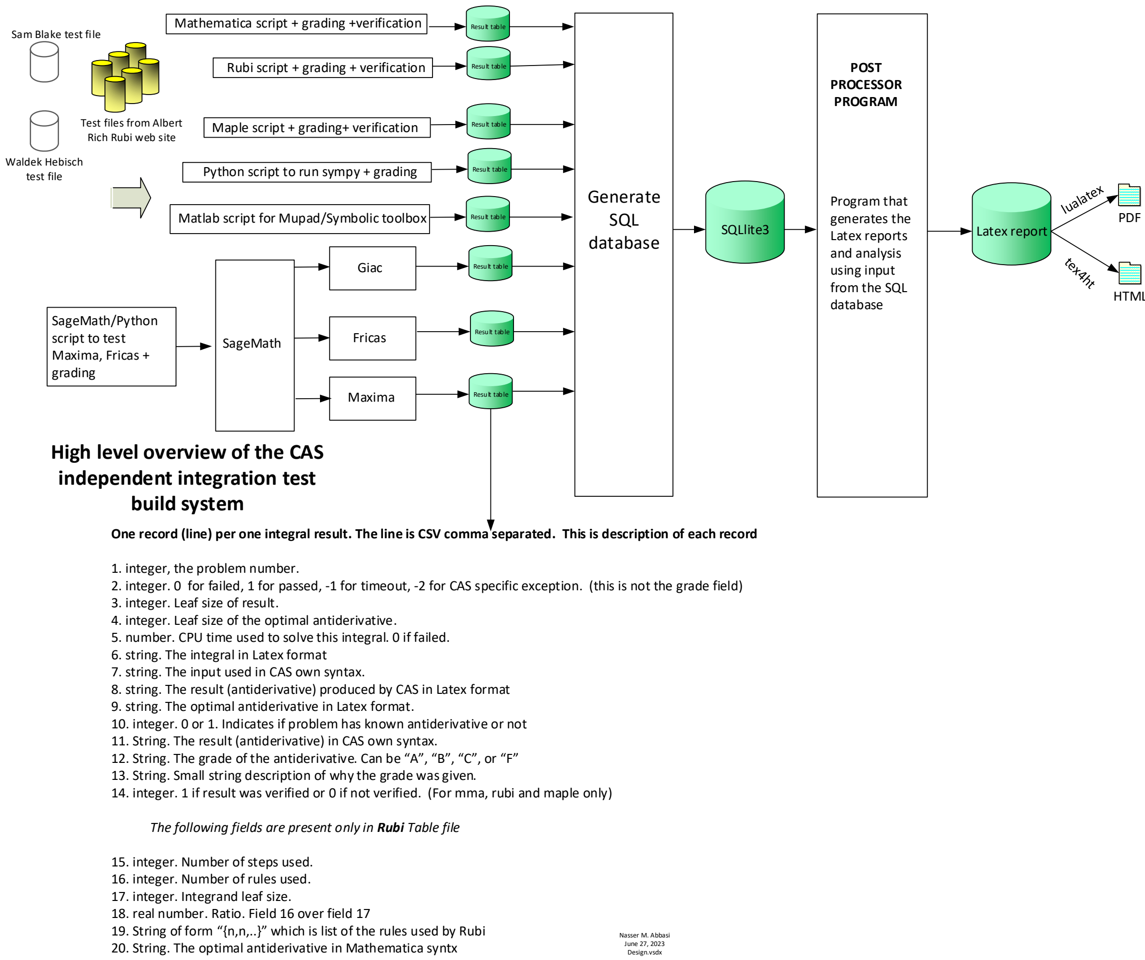
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413 }

B grade { }

C grade { }

F normal fail { 111, 117, 224 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 20, 21, 22, 23, 24, 25, 26, 32, 33, 34, 35, 36, 37, 38, 44, 45, 46, 47, 48, 49, 55, 57, 59, 61, 62, 63, 64, 65, 67, 69, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 156, 157, 158, 159, 160, 161, 162, 169, 170, 171, 172, 173, 174, 175, 181, 182, 183, 184, 185, 186, 187, 188, 194, 195, 196, 197, 198, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 261, 262, 263, 264, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 311, 312, 313, 314, 315, 322, 323, 324, 325, 326, 332, 333, 334, 335, 336, 342, 343, 344, 345, 346, 347, 354, 355, 358, 359, 360, 376, 377, 378, 379, 380, 388, 391, 392, 393, 401, 402, 403, 404, 405, 406, 413 }

B grade { 56, 58, 60, 66, 68, 76, 82, 362, 363, 365, 366, 373, 374, 385, 394 }

C grade { 15, 16, 17, 18, 19, 27, 28, 29, 30, 31, 39, 40, 41, 42, 43, 50, 51, 52, 53, 54, 151, 152, 153, 154, 155, 163, 164, 165, 166, 167, 168, 176, 177, 178, 179, 180, 189, 190, 191, 192, 193, 199, 200, 201, 202, 203, 224, 259, 260, 265, 266, 310, 316, 317, 318, 319, 320, 321, 327, 328, 329, 330, 331, 337, 338, 339, 340, 341, 348, 349, 350, 351, 352, 353, 356, 357, 361, 364, 367, 368, 369, 370, 371, 372, 375, 381, 382, 383, 384, 386, 387, 389, 390, 395, 396, 397, 398, 399 }

F normal fail { 400, 407, 408, 409, 410, 411, 412 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 20, 21, 22, 23, 24, 25, 26, 32, 33, 34, 35, 36, 37, 38, 44, 45, 46, 47, 48, 49, 57, 59, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 79, 81, 83, 89, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 110, 111, 112, 113, 114, 115, 116, 117, 122, 123, 124, 127, 128, 129, 130, 131, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 222, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 261, 262, 263, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 311, 312, 313, 314, 315, 322, 323, 324, 325, 326, 332, 333, 334, 335, 336, 342, 343, 344, 345, 346, 347, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399 }

B grade { 55, 56, 58, 60, 65, 87, 88, 125, 126, 220, 221, 264 }

C grade { 15, 16, 17, 18, 19, 27, 28, 29, 30, 31, 39, 40, 41, 42, 43, 50, 51, 52, 53, 54, 76, 78, 80, 82, 84, 85, 86, 107, 108, 109, 118, 119, 120, 121, 132, 133, 134, 135, 136, 259, 260, 265, 266, 310, 316, 317, 318, 319, 320, 321, 327, 328, 329, 330, 331, 337, 338, 339, 340, 341, 348, 349, 350, 351, 352, 353, 413 }

F normal fail { 90, 91, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 44, 45, 46, 47, 48, 49, 50, 57, 59, 61, 62, 63, 64, 67, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 198, 199, 200, 201, 202, 203, 229, 230, 231, 232, 233, 234, 235, 236, 244, 245, 246, 247, 248, 249, 250, 261, 262, 263, 264, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 296, 297, 298, 299, 311, 312, 313, 314, 315, 324, 336 }

B grade { 55, 56, 58, 60, 65, 66, 68, 70, 87, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 196, 197, 220, 221, 222, 237, 238, 239, 240, 241, 242, 243, 252, 253, 254, 255, 256, 257, 258, 303, 304, 305, 306, 307, 308, 309, 332, 333, 334, 335, 342, 343, 344, 345, 346, 347, 354, 355, 356, 357, 358, 359, 361, 362, 363, 364, 365, 366, 367, 368, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 395, 396, 397, 413 }

C grade { 42, 43, 51, 52, 53, 54, 259, 260, 265, 266 }

F normal fail { 18, 19, 30, 31, 90, 91, 154, 155, 166, 167, 168, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 316, 317, 318, 319, 320, 321, 327, 328, 329, 330, 331, 337, 338, 339, 340, 341, 348, 349, 350, 351, 352, 353, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412 }

F(-1) timedout fail { 251, 295, 300, 301, 302, 310, 322, 323, 325, 326, 360, 369, 370, 394, 398, 399 }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 12, 13, 14, 20, 24, 25, 26, 32, 33, 37, 38, 44, 45, 46, 47, 48, 49, 55, 57, 59, 61, 62, 63, 64, 65, 67, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 93, 94, 95, 96, 97, 98, 99, 100, 101, 137, 138, 139, 140, 142, 143, 144, 145, 146, 147, 148, 149, 150, 156, 157, 158, 159, 160, 161, 162, 181, 182, 183, 184, 185, 186, 187, 188, 194, 195, 196, 197, 198, 220, 221, 222, 229, 230, 231, 232, 233, 234, 235, 236, 244, 245, 246, 247, 248, 249, 250, 251, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279 }

B grade { 8, 9, 10, 11, 21, 22, 23, 34, 35, 36, 56, 58, 60, 66, 68, 70, 92 }

C grade { }

F normal fail { 15, 16, 17, 18, 19, 27, 28, 29, 30, 31, 39, 40, 41, 42, 43, 50, 51, 52, 53, 54, 90, 91, 107, 108, 109, 110, 111, 118, 119, 120, 121, 122, 123, 132, 133, 134, 135, 136, 141, 151, 152, 153, 154, 155, 163, 164, 165, 166, 167, 168, 176, 177, 178, 179, 180, 189, 190, 191, 192, 193, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 259, 260, 261, 262, 263, 264, 265, 266, 310, 314, 315, 316, 317, 318, 319, 320, 321, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413 }

F(-1) timedout fail { }

F(-2) exception fail { 102, 103, 104, 105, 106, 112, 113, 114, 115, 116, 117, 124, 125, 126, 127, 128, 129, 130, 131, 169, 170, 171, 172, 173, 174, 175, 237, 238, 239, 240, 241, 242, 243, 252, 253, 254, 255, 256, 257, 258, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 311, 312, 313, 322, 323, 324 }

2.1.6 Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 20, 21, 22, 23, 24, 32, 33, 34, 35, 44, 45, 46, 47, 48, 49, 57, 59, 61, 62, 63, 64, 67, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 92, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 112, 113, 114, 115, 116, 117, 124, 125, 127, 128, 129, 130, 131, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 156, 158, 159, 160, 161, 169, 170, 171, 173, 181, 182, 183, 184, 185, 194, 195, 196, 197, 198, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 315, 334, 336, 413 }

B grade { 13, 14, 25, 26, 36, 37, 38, 55, 56, 58, 60, 65, 66, 68, 70, 87, 88, 89, 93, 94, 107, 108, 109, 110, 111, 118, 119, 120, 121, 122, 123, 126, 132, 133, 134, 135, 136, 147, 148, 149, 150, 157, 162, 174, 175, 186, 187, 188, 220, 221, 222, 303, 304, 305, 306, 307, 308, 309, 343, 344, 345, 347, 354, 355, 356, 357, 358, 359, 360, 367, 368, 369, 370, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385 }

C grade { }

F normal fail { 15, 16, 17, 18, 19, 27, 28, 29, 30, 31, 39, 40, 41, 42, 43, 50, 51, 52, 53, 54, 90, 91, 151, 152, 153, 154, 155, 163, 164, 165, 166, 167, 168, 176, 177, 178, 179, 180, 189, 190, 191, 192, 193, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 259, 260, 261, 262, 263, 264, 265, 266, 316, 317, 318, 319, 320, 321, 327, 328, 329, 330, 331, 337, 338, 339, 340, 341, 348, 349, 350, 351, 352, 353, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412 }

F(-1) timedout fail { 310, 363, 364, 365, 366, 374, 375, 389, 390, 391, 392, 393, 395, 396, 397, 398, 399 }

F(-2) exception fail { 172, 311, 312, 313, 314, 322, 323, 324, 325, 326, 332, 333, 335, 342, 346, 361, 362, 371, 372, 373, 386, 387, 388, 394 }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 18, 20, 21, 22, 23, 24, 25, 26, 30, 32, 33, 34, 35, 36, 37, 38, 42, 44, 45, 46, 47, 48, 49, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 78, 80, 82, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 158, 171, 172, 173, 184, 185, 186, 195, 196, 220, 221, 222, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 354, 355, 356, 357, 358, 359, 360, 367, 368, 369, 370, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385 }

C grade { }

F normal fail { }

F(-1) timedout fail { 15, 16, 17, 19, 27, 28, 29, 31, 39, 40, 41, 43, 50, 51, 52, 54, 75, 77, 79, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 187, 188, 189, 190, 191, 192, 193, 194, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 361, 362, 363, 364, 365, 366, 371, 372, 373, 374, 375, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 20, 21, 22, 23, 24, 25, 26, 32, 33, 34, 35, 36, 37, 38, 44, 45, 46, 57, 59, 61, 62, 63, 64, 67, 69, 71, 72, 73, 74, 76, 95, 96, 97, 98, 99, 100, 101, 137, 138, 139, 140, 141, 142, 143, 144, 156, 158, 169, 170, 171, 181, 182, 183, 184, 273, 274, 275, 276, 277, 278, 279, 281, 282, 284, 285, 288, 289, 290, 291, 292, 293 }

B grade { 47, 48, 49, 55, 56, 58, 60, 65, 66, 68, 70, 92, 103, 104, 114, 115, 129, 157, 220, 221, 222, 280, 283, 286, 287, 294 }

C grade { 15, 16, 17, 18, 19, 27, 28, 29, 30, 31, 39, 40, 41, 42, 43, 50, 51, 52, 53, 54 }

F normal fail { 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 172, 173, 174, 175, 176, 177, 178, 179, 180, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 223, 225, 226, 227, 228, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 368, 369, 370, 371, 372, 373, 374, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 405, 413 }

F(-1) timedout fail { 102, 105, 106, 107, 108, 109, 110, 111, 112, 113, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 130, 131, 132, 133, 134, 135, 136, 219, 224, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 367, 375, 400, 401, 402, 403, 404, 406, 407, 408, 409, 410, 411, 412 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	150	149	126	125	125	151	125	125
N.S.	1	1.01	1.00	0.85	0.84	0.84	1.01	0.84	0.84
time (sec)	N/A	0.351	0.006	0.282	0.192	0.248	0.024	0.265	0.063

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	149	126	125	125	155	125	125
N.S.	1	1.00	1.00	0.85	0.84	0.84	1.04	0.84	0.84
time (sec)	N/A	0.296	0.005	0.288	0.197	0.262	0.024	0.266	0.063

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	88	146	125	124	124	150	124	124
N.S.	1	0.99	1.64	1.40	1.39	1.39	1.69	1.39	1.39
time (sec)	N/A	0.243	0.005	0.283	0.203	0.245	0.027	0.268	0.062

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	141	122	121	121	148	121	121
N.S.	1	1.00	1.00	0.87	0.86	0.86	1.05	0.86	0.86
time (sec)	N/A	0.285	0.004	0.313	0.186	0.255	0.024	0.267	0.060

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	146	142	123	125	122	150	125	122
N.S.	1	1.03	1.00	0.87	0.88	0.86	1.06	0.88	0.86
time (sec)	N/A	0.284	0.008	0.254	0.224	0.254	0.079	0.267	0.063

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	139	122	121	127	143	121	121
N.S.	1	1.00	1.00	0.88	0.87	0.91	1.03	0.87	0.87
time (sec)	N/A	0.282	0.007	0.273	0.188	0.252	0.078	0.286	0.059

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	145	142	123	125	129	150	135	122
N.S.	1	1.02	1.00	0.87	0.88	0.91	1.06	0.95	0.86
time (sec)	N/A	0.300	0.007	0.288	0.245	0.237	0.096	0.262	0.060

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	89	56	44	102	48	60	54	42
N.S.	1	1.33	0.84	0.66	1.52	0.72	0.90	0.81	0.63
time (sec)	N/A	0.217	0.084	0.355	0.284	0.238	0.606	0.268	7.729

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	72	51	39	93	43	54	45	37
N.S.	1	1.41	1.00	0.76	1.82	0.84	1.06	0.88	0.73
time (sec)	N/A	0.194	0.070	0.278	0.280	0.238	0.576	0.262	7.817

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	47	42	30	67	34	48	38	32
N.S.	1	1.07	0.95	0.68	1.52	0.77	1.09	0.86	0.73
time (sec)	N/A	0.173	0.070	0.258	0.267	0.237	0.515	0.278	7.564

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	65	69	49	99	56	75	76	45
N.S.	1	1.12	1.19	0.84	1.71	0.97	1.29	1.31	0.78
time (sec)	N/A	0.209	0.125	0.464	0.283	0.252	4.779	0.294	7.607

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	61	71	49	88	72	83	91	51
N.S.	1	1.03	1.20	0.83	1.49	1.22	1.41	1.54	0.86
time (sec)	N/A	0.210	0.125	0.467	0.282	0.245	3.307	0.283	7.887

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	61	72	54	91	72	76	129	56
N.S.	1	0.97	1.14	0.86	1.44	1.14	1.21	2.05	0.89
time (sec)	N/A	0.209	0.198	0.438	0.269	0.247	2.850	0.276	7.739

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	63	60	48	59	59	63	116	43
N.S.	1	1.09	1.03	0.83	1.02	1.02	1.09	2.00	0.74
time (sec)	N/A	0.197	0.175	0.356	0.293	0.243	2.569	0.270	7.866

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	223	82	40	0	69	78	0	0
N.S.	1	1.07	0.39	0.19	0.00	0.33	0.38	0.00	0.00
time (sec)	N/A	0.379	4.641	5.138	0.000	0.084	1.029	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	197	68	40	0	63	78	0	0
N.S.	1	1.03	0.35	0.21	0.00	0.33	0.41	0.00	0.00
time (sec)	N/A	0.310	3.957	1.841	0.000	0.080	0.912	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	183	48	37	0	58	76	0	0
N.S.	1	1.04	0.27	0.21	0.00	0.33	0.43	0.00	0.00
time (sec)	N/A	0.272	3.569	1.237	0.000	0.082	0.820	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	181	53	38	0	0	78	0	61
N.S.	1	1.06	0.31	0.22	0.00	0.00	0.46	0.00	0.36
time (sec)	N/A	0.285	3.520	1.356	0.000	0.000	0.994	0.000	7.773

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	199	54	40	0	0	83	0	0
N.S.	1	1.04	0.28	0.21	0.00	0.00	0.43	0.00	0.00
time (sec)	N/A	0.316	7.719	2.010	0.000	0.000	1.104	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	114	66	54	127	58	121	80	52
N.S.	1	1.37	0.80	0.65	1.53	0.70	1.46	0.96	0.63
time (sec)	N/A	0.224	0.103	0.327	0.281	0.242	0.938	0.270	7.642

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	93	61	49	118	53	109	71	47
N.S.	1	1.39	0.91	0.73	1.76	0.79	1.63	1.06	0.70
time (sec)	N/A	0.204	0.097	0.296	0.281	0.237	0.848	0.275	7.694

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	72	56	44	95	48	95	57	42
N.S.	1	1.20	0.93	0.73	1.58	0.80	1.58	0.95	0.70
time (sec)	N/A	0.179	0.088	0.282	0.266	0.238	0.794	0.268	7.539

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	90	79	56	138	67	117	90	55
N.S.	1	1.15	1.01	0.72	1.77	0.86	1.50	1.15	0.71
time (sec)	N/A	0.224	0.142	0.441	0.281	0.256	9.318	0.286	7.579

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	78	78	62	122	78	114	102	64
N.S.	1	0.96	0.96	0.77	1.51	0.96	1.41	1.26	0.79
time (sec)	N/A	0.224	0.161	0.497	0.281	0.248	4.354	0.286	8.020

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	89	83	65	123	82	133	146	71
N.S.	1	1.03	0.97	0.76	1.43	0.95	1.55	1.70	0.83
time (sec)	N/A	0.226	0.209	0.493	0.286	0.260	5.507	0.283	7.929

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	89	83	65	112	82	148	158	82
N.S.	1	1.09	1.01	0.79	1.37	1.00	1.80	1.93	1.00
time (sec)	N/A	0.232	0.229	0.479	0.292	0.262	5.793	0.310	8.079

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	251	74	40	0	79	160	0	0
N.S.	1	1.07	0.31	0.17	0.00	0.34	0.68	0.00	0.00
time (sec)	N/A	0.405	7.264	4.244	0.000	0.083	1.800	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	225	68	40	0	73	160	0	0
N.S.	1	1.03	0.31	0.18	0.00	0.33	0.73	0.00	0.00
time (sec)	N/A	0.358	5.812	1.875	0.000	0.082	1.645	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	207	49	38	0	68	158	0	0
N.S.	1	1.05	0.25	0.19	0.00	0.35	0.80	0.00	0.00
time (sec)	N/A	0.319	5.115	1.250	0.000	0.085	1.510	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	209	53	38	0	0	160	0	48
N.S.	1	1.05	0.27	0.19	0.00	0.00	0.80	0.00	0.24
time (sec)	N/A	0.328	4.853	1.355	0.000	0.000	1.840	0.000	7.765

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	211	54	40	0	0	163	0	0
N.S.	1	1.05	0.27	0.20	0.00	0.00	0.81	0.00	0.00
time (sec)	N/A	0.330	9.297	2.087	0.000	0.000	1.728	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	91	51	39	104	43	48	46	38
N.S.	1	1.36	0.76	0.58	1.55	0.64	0.72	0.69	0.57
time (sec)	N/A	0.220	0.073	0.309	0.281	0.243	0.742	0.293	7.874

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	63	42	30	76	34	32	37	32
N.S.	1	1.24	0.82	0.59	1.49	0.67	0.63	0.73	0.63
time (sec)	N/A	0.200	0.094	0.285	0.305	0.244	0.679	0.279	7.835

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	49	41	29	65	33	32	33	27
N.S.	1	1.40	1.17	0.83	1.86	0.94	0.91	0.94	0.77
time (sec)	N/A	0.172	0.076	0.279	0.360	0.244	0.634	0.266	7.852

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	28	32	20	42	26	22	26	19
N.S.	1	1.17	1.33	0.83	1.75	1.08	0.92	1.08	0.79
time (sec)	N/A	0.157	0.073	0.274	0.370	0.243	0.614	0.272	7.858

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	40	50	30	67	41	31	61	30
N.S.	1	1.05	1.32	0.79	1.76	1.08	0.82	1.61	0.79
time (sec)	N/A	0.180	0.090	0.375	0.347	0.243	2.769	0.285	0.298

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	44	46	31	47	47	31	66	31
N.S.	1	1.05	1.10	0.74	1.12	1.12	0.74	1.57	0.74
time (sec)	N/A	0.183	0.101	0.297	0.292	0.249	1.599	0.280	7.675

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	62	55	43	59	50	88	114	43
N.S.	1	1.07	0.95	0.74	1.02	0.86	1.52	1.97	0.74
time (sec)	N/A	0.209	0.158	0.326	0.297	0.255	3.948	0.280	7.783

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	192	74	40	0	59	75	0	0
N.S.	1	1.04	0.40	0.22	0.00	0.32	0.41	0.00	0.00
time (sec)	N/A	0.307	10.026	3.943	0.000	0.082	1.149	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	168	66	40	0	50	75	0	0
N.S.	1	1.01	0.40	0.24	0.00	0.30	0.45	0.00	0.00
time (sec)	N/A	0.272	10.038	1.765	0.000	0.079	1.050	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	157	48	38	0	47	73	0	0
N.S.	1	1.01	0.31	0.25	0.00	0.30	0.47	0.00	0.00
time (sec)	N/A	0.246	10.026	0.808	0.000	0.075	0.778	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	178	53	38	0	73	75	0	48
N.S.	1	1.03	0.31	0.22	0.00	0.42	0.43	0.00	0.28
time (sec)	N/A	0.280	10.038	1.323	0.000	0.080	0.813	0.000	8.229

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	192	54	40	0	84	80	0	0
N.S.	1	1.02	0.29	0.21	0.00	0.44	0.42	0.00	0.00
time (sec)	N/A	0.310	10.026	1.993	0.000	0.081	0.993	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	67	51	39	89	62	66	45	97
N.S.	1	1.16	0.88	0.67	1.53	1.07	1.14	0.78	1.67
time (sec)	N/A	0.227	0.111	0.304	0.271	0.240	6.472	0.264	8.231

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	44	32	63	58	48	39	89
N.S.	1	1.00	0.98	0.71	1.40	1.29	1.07	0.87	1.98
time (sec)	N/A	0.185	0.124	0.296	0.275	0.240	5.013	0.271	8.116

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	41	29	54	52	39	33	82
N.S.	1	1.00	1.17	0.83	1.54	1.49	1.11	0.94	2.34
time (sec)	N/A	0.169	0.111	0.290	0.294	0.250	4.341	0.263	8.066

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	22	31	31	16	16
N.S.	1	1.00	1.00	0.85	1.10	1.55	1.55	0.80	0.80
time (sec)	N/A	0.150	0.103	0.280	0.278	0.257	3.018	0.271	0.032

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	50	52	35	56	61	212	61	40
N.S.	1	1.09	1.13	0.76	1.22	1.33	4.61	1.33	0.87
time (sec)	N/A	0.186	0.143	0.320	0.272	0.257	7.463	0.279	7.974

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	69	59	43	68	77	228	82	47
N.S.	1	1.06	0.91	0.66	1.05	1.18	3.51	1.26	0.72
time (sec)	N/A	0.205	0.148	0.326	0.280	0.258	4.622	0.290	7.980

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	204	70	40	0	77	75	0	0
N.S.	1	1.04	0.36	0.20	0.00	0.39	0.38	0.00	0.00
time (sec)	N/A	0.316	10.040	3.572	0.000	0.080	2.097	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	183	68	40	0	98	75	0	0
N.S.	1	1.03	0.38	0.23	0.00	0.55	0.42	0.00	0.00
time (sec)	N/A	0.279	10.031	2.293	0.000	0.083	1.988	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	183	66	38	0	98	73	0	0
N.S.	1	1.02	0.37	0.21	0.00	0.54	0.41	0.00	0.00
time (sec)	N/A	0.276	10.032	1.568	0.000	0.083	1.974	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	206	71	38	0	107	75	0	48
N.S.	1	1.05	0.36	0.19	0.00	0.55	0.38	0.00	0.24
time (sec)	N/A	0.321	10.049	0.944	0.000	0.085	2.688	0.000	7.951

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	227	54	40	0	118	80	0	0
N.S.	1	1.06	0.25	0.19	0.00	0.55	0.37	0.00	0.00
time (sec)	N/A	0.368	10.025	2.083	0.000	0.087	3.003	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	189	2295	372	1571	21612	3620	1539
N.S.	1	1.00	0.70	8.53	1.38	5.84	80.34	13.46	5.72
time (sec)	N/A	0.453	0.429	0.752	0.234	0.264	2.913	0.347	8.616

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	67	153	122	129	129	134	132	121
N.S.	1	1.06	2.43	1.94	2.05	2.05	2.13	2.10	1.92
time (sec)	N/A	0.278	0.019	0.140	0.179	0.231	0.035	0.275	0.071

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	153	124	129	129	141	133	123
N.S.	1	1.00	1.00	0.81	0.84	0.84	0.92	0.87	0.80
time (sec)	N/A	0.329	0.018	0.132	0.192	0.231	0.025	0.261	7.625

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	49	151	124	129	129	136	133	123
N.S.	1	1.09	3.36	2.76	2.87	2.87	3.02	2.96	2.73
time (sec)	N/A	0.234	0.016	0.139	0.190	0.229	0.044	0.257	0.063

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	153	124	129	129	139	133	123
N.S.	1	1.00	1.00	0.81	0.84	0.84	0.91	0.87	0.80
time (sec)	N/A	0.314	0.015	0.128	0.187	0.239	0.028	0.283	0.064

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	33	149	124	129	129	133	133	123
N.S.	1	1.14	5.14	4.28	4.45	4.45	4.59	4.59	4.24
time (sec)	N/A	0.191	0.011	0.124	0.192	0.256	0.036	0.256	0.063

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	143	121	125	125	134	130	120
N.S.	1	1.00	1.00	0.85	0.87	0.87	0.94	0.91	0.84
time (sec)	N/A	0.293	0.013	0.145	0.183	0.238	0.032	0.253	0.064

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	88	149	122	130	127	131	134	121
N.S.	1	0.95	1.60	1.31	1.40	1.37	1.41	1.44	1.30
time (sec)	N/A	0.217	0.020	0.050	0.191	0.235	0.129	0.263	7.722

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	141	123	125	131	124	128	119
N.S.	1	1.00	1.00	0.87	0.89	0.93	0.88	0.91	0.84
time (sec)	N/A	0.300	0.023	0.051	0.219	0.244	0.125	0.254	0.064

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	147	123	130	133	131	144	120
N.S.	1	1.00	1.00	0.84	0.88	0.90	0.89	0.98	0.82
time (sec)	N/A	0.315	0.027	0.051	0.190	0.245	0.151	0.261	7.722

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	122	1121	192	759	11387	1848	1483
N.S.	1	1.00	0.60	5.52	0.95	3.74	56.09	9.10	7.31
time (sec)	N/A	0.344	0.090	0.647	0.207	0.256	2.325	0.304	8.341

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	38	85	29	61	61	76	61	61
N.S.	1	1.12	2.50	0.85	1.79	1.79	2.24	1.79	1.79
time (sec)	N/A	0.189	0.003	0.098	0.190	0.240	0.023	0.251	0.054

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	83	62	61	61	75	61	61
N.S.	1	1.00	1.00	0.75	0.73	0.73	0.90	0.73	0.73
time (sec)	N/A	0.221	0.007	0.107	0.194	0.229	0.022	0.252	0.053

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	27	83	20	61	61	75	61	61
N.S.	1	1.17	3.61	0.87	2.65	2.65	3.26	2.65	2.65
time (sec)	N/A	0.175	0.002	0.063	0.189	0.251	0.022	0.255	0.052

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	83	62	61	61	75	61	61
N.S.	1	1.00	1.00	0.75	0.73	0.73	0.90	0.73	0.73
time (sec)	N/A	0.210	0.003	0.102	0.185	0.247	0.020	0.263	0.051

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	61	61	71	76	61
N.S.	1	1.00	1.00	0.91	5.55	5.55	6.45	6.91	5.55
time (sec)	N/A	0.146	0.002	0.057	0.182	0.253	0.023	0.273	0.052

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	58	57	57	68	57	57
N.S.	1	1.00	1.00	0.79	0.78	0.78	0.93	0.78	0.78
time (sec)	N/A	0.202	0.003	0.106	0.182	0.238	0.028	0.271	0.051

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	80	59	62	58	75	62	58
N.S.	1	1.00	1.00	0.74	0.78	0.72	0.94	0.78	0.72
time (sec)	N/A	0.209	0.004	0.038	0.203	0.232	0.041	0.283	0.053

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	60	59	62	66	59	59
N.S.	1	1.00	1.00	0.82	0.81	0.85	0.90	0.81	0.81
time (sec)	N/A	0.205	0.003	0.034	0.219	0.235	0.038	0.268	0.052

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	80	61	62	64	75	69	60
N.S.	1	1.00	1.00	0.76	0.78	0.80	0.94	0.86	0.75
time (sec)	N/A	0.218	0.003	0.041	0.188	0.265	0.048	0.257	0.053

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	85	80	91	54	129	0	98	0
N.S.	1	0.59	0.55	0.63	0.37	0.89	0.00	0.68	0.00
time (sec)	N/A	0.230	1.049	2.247	0.275	0.257	0.000	0.270	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	A	A	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	83	62	276	45	31	29	150	40	103
N.S.	1	0.75	3.33	0.54	0.37	0.35	1.81	0.48	1.24
time (sec)	N/A	0.224	0.547	0.312	0.182	0.259	3.907	0.281	8.083

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	69	69	62	33	98	0	57	0
N.S.	1	0.71	0.71	0.64	0.34	1.01	0.00	0.59	0.00
time (sec)	N/A	0.193	1.027	1.510	0.267	0.273	0.000	0.270	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	92	67	125	48	35	33	0	60	83
N.S.	1	0.73	1.36	0.52	0.38	0.36	0.00	0.65	0.90
time (sec)	N/A	0.236	0.378	0.345	0.197	0.252	0.000	0.268	8.171

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	73	72	67	37	105	0	61	0
N.S.	1	0.72	0.71	0.66	0.37	1.04	0.00	0.60	0.00
time (sec)	N/A	0.206	1.029	1.577	0.269	0.260	0.000	0.273	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	137	80	165	58	48	48	0	128	125
N.S.	1	0.58	1.20	0.42	0.35	0.35	0.00	0.93	0.91
time (sec)	N/A	0.252	0.357	0.345	0.195	0.254	0.000	0.259	8.320

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	124	108	165	125	300	0	98	0
N.S.	1	0.81	0.71	1.08	0.82	1.96	0.00	0.64	0.00
time (sec)	N/A	0.260	1.048	2.086	0.279	0.276	0.000	0.264	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	A	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	77	54	181	37	65	42	0	38	48
N.S.	1	0.70	2.35	0.48	0.84	0.55	0.00	0.49	0.62
time (sec)	N/A	0.198	0.653	0.067	0.188	0.242	0.000	0.275	7.728

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	122	108	164	124	301	0	98	0
N.S.	1	0.78	0.69	1.05	0.79	1.93	0.00	0.63	0.00
time (sec)	N/A	0.233	1.041	1.281	0.273	0.275	0.000	0.264	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	161	98	92	87	88	119	0	106	0
N.S.	1	0.61	0.57	0.54	0.55	0.74	0.00	0.66	0.00
time (sec)	N/A	0.267	1.031	0.104	0.192	0.246	0.000	0.272	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	138	124	176	134	334	0	113	0
N.S.	1	0.73	0.65	0.93	0.71	1.76	0.00	0.59	0.00
time (sec)	N/A	0.309	1.051	1.224	0.272	0.268	0.000	0.271	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	223	129	130	120	138	205	0	182	0
N.S.	1	0.58	0.58	0.54	0.62	0.92	0.00	0.82	0.00
time (sec)	N/A	0.308	1.056	0.136	0.196	0.250	0.000	0.270	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	400	215	160	1099	491	853	0	2171	0
N.S.	1	0.54	0.40	2.75	1.23	2.13	0.00	5.43	0.00
time (sec)	N/A	0.414	0.458	0.123	0.213	0.277	0.000	0.361	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	153	112	495	243	381	0	993	0
N.S.	1	0.55	0.41	1.79	0.88	1.38	0.00	3.60	0.00
time (sec)	N/A	0.328	0.143	0.033	0.210	0.256	0.000	0.296	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	92	86	131	75	94	0	263	0
N.S.	1	0.60	0.56	0.86	0.49	0.61	0.00	1.72	0.00
time (sec)	N/A	0.265	0.064	0.027	0.201	0.249	0.000	0.267	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	134	106	78	0	0	0	0	0	0
N.S.	1	0.79	0.58	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.246	0.098	0.000	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	154	133	101	0	0	0	0	0	0
N.S.	1	0.86	0.66	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.265	0.109	0.000	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	49	25	40	86	47	155	32	59
N.S.	1	1.44	0.74	1.18	2.53	1.38	4.56	0.94	1.74
time (sec)	N/A	0.184	0.008	0.083	0.209	0.265	4.387	0.269	7.813

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	45	62	135	92	0	196	108
N.S.	1	1.00	0.52	0.72	1.57	1.07	0.00	2.28	1.26
time (sec)	N/A	0.236	0.059	0.121	0.195	0.267	0.000	0.301	7.926

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	115	68	99	196	140	0	331	169
N.S.	1	0.90	0.53	0.77	1.53	1.09	0.00	2.59	1.32
time (sec)	N/A	0.261	0.073	0.161	0.201	0.265	0.000	0.280	7.973

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	170	166	170	166	166	202	193	169
N.S.	1	1.02	1.00	1.02	1.00	1.00	1.22	1.16	1.02
time (sec)	N/A	0.448	0.039	0.147	0.188	0.271	0.038	0.273	0.045

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	166	170	166	166	204	193	169
N.S.	1	1.00	1.00	1.02	1.00	1.00	1.23	1.16	1.02
time (sec)	N/A	0.355	0.037	0.138	0.192	0.247	0.032	0.285	7.899

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	164	154	170	166	166	199	193	169
N.S.	1	0.99	0.93	1.02	1.00	1.00	1.20	1.16	1.02
time (sec)	N/A	0.406	0.047	0.128	0.193	0.259	0.036	0.320	0.028

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	161	166	163	163	199	189	165
N.S.	1	1.00	1.00	1.03	1.01	1.01	1.24	1.17	1.02
time (sec)	N/A	0.321	0.040	0.146	0.188	0.239	0.035	0.325	0.027

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	165	162	167	167	164	199	193	166
N.S.	1	1.02	1.00	1.03	1.03	1.01	1.23	1.19	1.02
time (sec)	N/A	0.337	0.077	0.040	0.192	0.244	0.138	0.271	7.774

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	156	167	162	168	185	185	163
N.S.	1	1.00	1.00	1.07	1.04	1.08	1.19	1.19	1.04
time (sec)	N/A	0.308	0.070	0.043	0.192	0.261	0.142	0.279	0.029

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	164	162	169	167	170	197	212	166
N.S.	1	1.01	1.00	1.04	1.03	1.05	1.22	1.31	1.02
time (sec)	N/A	0.345	0.054	0.049	0.185	0.276	0.196	0.296	0.032

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	132	126	136	0	421	0	126	1343
N.S.	1	0.99	0.95	1.02	0.00	3.17	0.00	0.95	10.10
time (sec)	N/A	0.335	0.045	0.203	0.000	0.290	0.000	0.616	7.875

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	96	93	98	0	312	434	91	979
N.S.	1	0.99	0.96	1.01	0.00	3.22	4.47	0.94	10.09
time (sec)	N/A	0.283	0.046	0.142	0.000	0.284	72.994	0.616	8.008

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	72	71	65	0	219	287	67	606
N.S.	1	1.01	1.00	0.92	0.00	3.08	4.04	0.94	8.54
time (sec)	N/A	0.245	0.033	0.096	0.000	0.276	5.220	0.686	7.711

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	81	128	76	0	249	0	78	2424
N.S.	1	1.04	1.64	0.97	0.00	3.19	0.00	1.00	31.08
time (sec)	N/A	0.279	0.074	0.067	0.000	0.279	0.000	0.726	10.104

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	114	186	126	0	385	0	124	3729
N.S.	1	1.02	1.66	1.12	0.00	3.44	0.00	1.11	33.29
time (sec)	N/A	0.358	0.146	0.090	0.000	0.392	0.000	0.629	10.265

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	272	327	95	0	5140	0	4391	10177
N.S.	1	1.04	1.25	0.36	0.00	19.69	0.00	16.82	38.99
time (sec)	N/A	0.563	0.255	0.101	0.000	2.570	0.000	1.073	8.400

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	212	251	65	0	2632	0	3179	6366
N.S.	1	1.02	1.21	0.31	0.00	12.65	0.00	15.28	30.61
time (sec)	N/A	0.416	0.105	0.070	0.000	0.618	0.000	0.958	8.233

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	173	45	0	1569	0	1402	4109
N.S.	1	1.00	1.01	0.26	0.00	9.12	0.00	8.15	23.89
time (sec)	N/A	0.305	0.062	0.059	0.000	0.438	0.000	0.783	8.071

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	187	206	173	0	2914	0	2805	6335
N.S.	1	0.99	1.09	0.92	0.00	15.42	0.00	14.84	33.52
time (sec)	N/A	0.392	0.187	0.093	0.000	0.786	0.000	1.051	0.779

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	F	B	F(-1)	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	0	267	232	0	5442	0	2870	10101
N.S.	1	0.00	0.99	0.86	0.00	20.08	0.00	10.59	37.27
time (sec)	N/A	0.000	0.206	0.108	0.000	2.886	0.000	0.908	8.694

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	220	208	282	0	1323	0	239	2282
N.S.	1	1.04	0.98	1.33	0.00	6.24	0.00	1.13	10.76
time (sec)	N/A	0.488	0.193	0.227	0.000	0.347	0.000	0.647	8.065

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	169	160	211	0	849	0	194	1527
N.S.	1	1.15	1.09	1.44	0.00	5.78	0.00	1.32	10.39
time (sec)	N/A	0.362	0.122	0.175	0.000	0.296	0.000	0.614	8.163

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	109	111	126	0	538	394	120	283
N.S.	1	1.02	1.04	1.18	0.00	5.03	3.68	1.12	2.64
time (sec)	N/A	0.268	0.057	0.118	0.000	0.258	4.227	0.586	7.612

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	96	101	95	0	474	374	102	264
N.S.	1	1.02	1.07	1.01	0.00	5.04	3.98	1.09	2.81
time (sec)	N/A	0.241	0.056	0.102	0.000	0.258	2.279	0.560	7.675

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	180	243	212	0	1014	0	201	7119
N.S.	1	1.20	1.62	1.41	0.00	6.76	0.00	1.34	47.46
time (sec)	N/A	0.444	0.226	0.118	0.000	0.596	0.000	0.613	12.299

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	0	379	300	0	1635	0	250	10034
N.S.	1	0.00	1.70	1.35	0.00	7.33	0.00	1.12	45.00
time (sec)	N/A	0.000	0.340	0.155	0.000	1.149	0.000	0.562	13.332

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	425	400	455	217	0	7252	0	5675	16604
N.S.	1	0.94	1.07	0.51	0.00	17.06	0.00	13.35	39.07
time (sec)	N/A	0.759	0.761	0.131	0.000	7.531	0.000	1.513	10.139

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	318	362	178	0	4658	0	4538	12396
N.S.	1	0.95	1.08	0.53	0.00	13.86	0.00	13.51	36.89
time (sec)	N/A	0.585	0.586	0.130	0.000	2.308	0.000	1.397	10.982

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	265	298	152	0	3467	0	3776	9444
N.S.	1	0.96	1.08	0.55	0.00	12.56	0.00	13.68	34.22
time (sec)	N/A	0.441	0.422	0.106	0.000	1.158	0.000	1.140	10.412

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	279	304	178	0	4885	0	4426	12349
N.S.	1	0.95	1.04	0.61	0.00	16.67	0.00	15.11	42.15
time (sec)	N/A	0.544	0.510	0.179	0.000	3.281	0.000	1.378	10.668

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	389	379	382	379	0	7583	0	5408	17591
N.S.	1	0.97	0.98	0.97	0.00	19.49	0.00	13.90	45.22
time (sec)	N/A	0.703	0.698	0.160	0.000	8.891	0.000	1.358	11.087

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	522	518	487	484	0	10190	0	6327	21554
N.S.	1	0.99	0.93	0.93	0.00	19.52	0.00	12.12	41.29
time (sec)	N/A	0.916	0.795	0.187	0.000	21.405	0.000	1.489	11.357

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	365	391	435	624	0	3196	0	598	4501
N.S.	1	1.07	1.19	1.71	0.00	8.76	0.00	1.64	12.33
time (sec)	N/A	1.087	0.428	0.311	0.000	0.535	0.000	1.418	10.379

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	297	354	495	0	2167	0	466	3062
N.S.	1	1.17	1.39	1.95	0.00	8.53	0.00	1.83	12.06
time (sec)	N/A	0.574	0.301	0.228	0.000	0.395	0.000	1.525	10.737

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	154	261	343	0	1378	0	318	593
N.S.	1	1.05	1.79	2.35	0.00	9.44	0.00	2.18	4.06
time (sec)	N/A	0.316	0.200	0.146	0.000	0.304	0.000	1.453	7.846

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	203	233	303	0	1369	0	268	625
N.S.	1	1.10	1.26	1.64	0.00	7.40	0.00	1.45	3.38
time (sec)	N/A	0.408	0.165	0.150	0.000	0.334	0.000	1.416	7.852

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	172	172	273	0	1226	0	228	587
N.S.	1	1.01	1.01	1.61	0.00	7.21	0.00	1.34	3.45
time (sec)	N/A	0.328	0.148	0.149	0.000	0.314	0.000	1.548	7.818

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	148	142	147	0	1109	661	208	517
N.S.	1	1.06	1.02	1.06	0.00	7.98	4.76	1.50	3.72
time (sec)	N/A	0.294	0.089	0.138	0.000	0.296	165.164	1.445	7.741

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	306	396	442	0	2494	0	421	11674
N.S.	1	1.21	1.57	1.75	0.00	9.90	0.00	1.67	46.33
time (sec)	N/A	0.635	0.417	0.187	0.000	1.950	0.000	1.510	14.337

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	363	405	642	616	0	3956	0	648	16265
N.S.	1	1.12	1.77	1.70	0.00	10.90	0.00	1.79	44.81
time (sec)	N/A	0.837	0.903	0.282	0.000	3.981	0.000	1.427	18.347

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	554	544	644	445	0	9636	0	3987	22911
N.S.	1	0.98	1.16	0.80	0.00	17.39	0.00	7.20	41.36
time (sec)	N/A	1.052	1.520	0.169	0.000	18.804	0.000	2.019	10.325

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	461	458	543	375	0	7060	0	7578	19041
N.S.	1	0.99	1.18	0.81	0.00	15.31	0.00	16.44	41.30
time (sec)	N/A	0.817	1.269	0.155	0.000	6.011	0.000	2.353	9.742

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	380	386	447	328	0	5650	0	3164	16688
N.S.	1	1.02	1.18	0.86	0.00	14.87	0.00	8.33	43.92
time (sec)	N/A	0.710	1.080	0.160	0.000	4.207	0.000	1.745	9.432

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	438	403	436	374	0	7270	0	7267	18992
N.S.	1	0.92	1.00	0.85	0.00	16.60	0.00	16.59	43.36
time (sec)	N/A	0.809	1.013	0.147	0.000	8.330	0.000	2.183	9.694

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	460	464	516	447	0	9909	0	4605	22914
N.S.	1	1.01	1.12	0.97	0.00	21.54	0.00	10.01	49.81
time (sec)	N/A	0.872	1.297	0.389	0.000	21.044	0.000	1.787	10.184

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	23	25	18	17	17	17	19	17
N.S.	1	0.92	1.00	0.72	0.68	0.68	0.68	0.76	0.68
time (sec)	N/A	0.190	0.008	0.042	0.196	0.256	0.052	0.273	0.033

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	23	25	18	25	17	17	19	17
N.S.	1	0.92	1.00	0.72	1.00	0.68	0.68	0.76	0.68
time (sec)	N/A	0.213	0.004	0.035	0.184	0.254	0.049	0.277	0.017

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	38	37	31	30	30	37	30	32
N.S.	1	1.03	1.00	0.84	0.81	0.81	1.00	0.81	0.86
time (sec)	N/A	0.200	0.010	0.047	0.271	0.262	0.056	0.263	7.558

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	38	37	31	53	30	37	30	32
N.S.	1	1.03	1.00	0.84	1.43	0.81	1.00	0.81	0.86
time (sec)	N/A	0.217	0.005	0.034	0.264	0.257	0.051	0.261	0.020

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	54	45	38	0	47	44	38	41
N.S.	1	1.20	1.00	0.84	0.00	1.04	0.98	0.84	0.91
time (sec)	N/A	0.219	0.023	0.053	0.000	0.255	0.062	0.409	0.031

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	116	69	58	104	61	105	102	102
N.S.	1	1.14	0.68	0.57	1.02	0.60	1.03	1.00	1.00
time (sec)	N/A	0.265	0.107	0.334	0.201	0.252	0.776	0.307	0.272

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	90	64	53	87	56	94	88	85
N.S.	1	1.11	0.79	0.65	1.07	0.69	1.16	1.09	1.05
time (sec)	N/A	0.231	0.083	0.203	0.186	0.254	0.735	0.287	7.680

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	79	59	48	70	51	80	74	67
N.S.	1	1.07	0.80	0.65	0.95	0.69	1.08	1.00	0.91
time (sec)	N/A	0.213	0.081	0.179	0.189	0.263	0.703	0.271	7.666

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	99	88	83	89	95	0	98	86
N.S.	1	1.05	0.94	0.88	0.95	1.01	0.00	1.04	0.91
time (sec)	N/A	0.269	0.132	0.521	0.276	0.272	0.000	0.294	0.245

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	102	91	83	89	112	0	138	84
N.S.	1	1.05	0.94	0.86	0.92	1.15	0.00	1.42	0.87
time (sec)	N/A	0.286	0.153	0.503	0.268	0.268	0.000	0.305	8.006

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	104	91	86	106	112	0	169	0
N.S.	1	1.05	0.92	0.87	1.07	1.13	0.00	1.71	0.00
time (sec)	N/A	0.283	0.182	0.480	0.268	0.260	0.000	0.322	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	99	70	66	99	90	0	189	0
N.S.	1	1.10	0.78	0.73	1.10	1.00	0.00	2.10	0.00
time (sec)	N/A	0.242	0.219	0.409	0.273	0.249	0.000	0.309	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	125	80	71	116	95	0	233	0
N.S.	1	1.13	0.72	0.64	1.05	0.86	0.00	2.10	0.00
time (sec)	N/A	0.275	0.245	0.398	0.276	0.266	0.000	0.290	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	151	80	76	133	100	0	255	0
N.S.	1	1.14	0.61	0.58	1.01	0.76	0.00	1.93	0.00
time (sec)	N/A	0.305	0.307	0.435	0.283	0.260	0.000	0.314	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	322	344	237	226	0	139	0	0	0
N.S.	1	1.07	0.74	0.70	0.00	0.43	0.00	0.00	0.00
time (sec)	N/A	0.494	6.063	5.582	0.000	0.085	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	305	316	234	221	0	134	0	0	0
N.S.	1	1.04	0.77	0.72	0.00	0.44	0.00	0.00	0.00
time (sec)	N/A	0.398	5.090	1.998	0.000	0.097	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	279	292	229	216	0	129	0	0	0
N.S.	1	1.05	0.82	0.77	0.00	0.46	0.00	0.00	0.00
time (sec)	N/A	0.347	5.058	1.745	0.000	0.093	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	284	287	231	225	0	0	0	0	0
N.S.	1	1.01	0.81	0.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.349	4.815	1.407	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	305	320	237	228	0	0	0	0	0
N.S.	1	1.05	0.78	0.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.406	9.679	2.151	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	146	79	68	135	71	218	207	0
N.S.	1	1.15	0.62	0.54	1.06	0.56	1.72	1.63	0.00
time (sec)	N/A	0.289	0.184	0.237	0.194	0.267	1.206	0.298	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	120	74	63	118	66	190	179	0
N.S.	1	1.13	0.70	0.59	1.11	0.62	1.79	1.69	0.00
time (sec)	N/A	0.268	0.143	0.214	0.191	0.267	1.160	0.324	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	113	69	58	101	61	165	151	127
N.S.	1	1.14	0.70	0.59	1.02	0.62	1.67	1.53	1.28
time (sec)	N/A	0.247	0.132	0.192	0.182	0.275	1.100	0.293	7.785

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	131	99	87	120	106	0	113	0
N.S.	1	1.10	0.83	0.73	1.01	0.89	0.00	0.95	0.00
time (sec)	N/A	0.321	0.201	0.475	0.276	0.270	0.000	0.303	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	132	102	96	120	122	0	153	0
N.S.	1	1.08	0.84	0.79	0.98	1.00	0.00	1.25	0.00
time (sec)	N/A	0.322	0.227	0.527	0.264	0.292	0.000	0.313	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	135	101	96	137	122	0	190	0
N.S.	1	1.06	0.80	0.76	1.08	0.96	0.00	1.50	0.00
time (sec)	N/A	0.324	0.255	0.532	0.286	0.280	0.000	0.309	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	135	101	96	154	122	0	227	0
N.S.	1	1.06	0.80	0.76	1.21	0.96	0.00	1.79	0.00
time (sec)	N/A	0.321	0.344	0.548	0.275	0.268	0.000	0.319	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	356	375	249	236	0	149	0	0	0
N.S.	1	1.05	0.70	0.66	0.00	0.42	0.00	0.00	0.00
time (sec)	N/A	0.513	9.168	4.206	0.000	0.083	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	331	351	244	231	0	144	0	0	0
N.S.	1	1.06	0.74	0.70	0.00	0.44	0.00	0.00	0.00
time (sec)	N/A	0.454	7.622	2.783	0.000	0.085	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	308	314	239	226	0	139	0	0	0
N.S.	1	1.02	0.78	0.73	0.00	0.45	0.00	0.00	0.00
time (sec)	N/A	0.395	6.662	1.405	0.000	0.104	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	312	325	235	233	0	0	0	0	0
N.S.	1	1.04	0.75	0.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.410	6.508	1.456	0.000	0.000	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	314	325	247	238	0	0	0	0	0
N.S.	1	1.04	0.79	0.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.407	10.248	2.127	0.000	0.000	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	331	353	244	238	0	0	0	0	0
N.S.	1	1.07	0.74	0.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.462	10.270	4.088	0.000	0.000	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	165	135	125	0	315	325	136	0
N.S.	1	1.08	0.88	0.82	0.00	2.06	2.12	0.89	0.00
time (sec)	N/A	0.350	0.524	0.186	0.000	0.273	1.052	0.334	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	104	104	88	0	233	233	96	0
N.S.	1	1.04	1.04	0.88	0.00	2.33	2.33	0.96	0.00
time (sec)	N/A	0.247	0.329	0.121	0.000	0.274	0.920	0.309	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	77	77	65	0	178	165	67	92
N.S.	1	1.01	1.01	0.86	0.00	2.34	2.17	0.88	1.21
time (sec)	N/A	0.215	0.290	0.087	0.000	0.264	0.834	0.294	8.168

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	89	85	76	0	517	0	0	81
N.S.	1	0.99	0.94	0.84	0.00	5.74	0.00	0.00	0.90
time (sec)	N/A	0.265	0.245	0.097	0.000	0.319	0.000	0.000	8.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	82	81	71	0	197	0	124	103
N.S.	1	1.02	1.01	0.89	0.00	2.46	0.00	1.55	1.29
time (sec)	N/A	0.234	0.334	0.123	0.000	0.305	0.000	0.300	8.149

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	134	145	98	0	255	0	339	0
N.S.	1	1.08	1.17	0.79	0.00	2.06	0.00	2.73	0.00
time (sec)	N/A	0.311	0.572	0.135	0.000	0.348	0.000	0.306	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	195	186	138	0	339	0	571	0
N.S.	1	1.10	1.05	0.78	0.00	1.92	0.00	3.23	0.00
time (sec)	N/A	0.416	0.808	0.163	0.000	0.407	0.000	0.310	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	403	400	532	448	0	431	0	0	0
N.S.	1	0.99	1.32	1.11	0.00	1.07	0.00	0.00	0.00
time (sec)	N/A	0.562	11.508	5.782	0.000	0.094	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	333	479	395	0	346	0	0	0
N.S.	1	0.99	1.43	1.18	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	0.411	11.006	2.403	0.000	0.097	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	285	302	362	0	300	0	0	0
N.S.	1	1.01	1.07	1.28	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.332	10.184	0.987	0.000	0.097	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	312	316	448	385	0	294	0	0	0
N.S.	1	1.01	1.44	1.23	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.396	10.707	1.938	0.000	0.086	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	376	374	373	418	0	347	0	0	0
N.S.	1	0.99	0.99	1.11	0.00	0.92	0.00	0.00	0.00
time (sec)	N/A	0.511	10.467	3.381	0.000	0.094	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	112	64	53	90	56	65	60	0
N.S.	1	1.14	0.65	0.54	0.92	0.57	0.66	0.61	0.00
time (sec)	N/A	0.271	0.115	0.206	0.184	0.248	0.912	0.291	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	83	59	48	73	51	54	53	0
N.S.	1	1.08	0.77	0.62	0.95	0.66	0.70	0.69	0.00
time (sec)	N/A	0.234	0.102	0.195	0.239	0.241	0.871	0.270	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	60	54	43	56	46	51	46	0
N.S.	1	1.07	0.96	0.77	1.00	0.82	0.91	0.82	0.00
time (sec)	N/A	0.200	0.097	0.181	0.193	0.240	0.816	0.273	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	51	47	36	39	39	42	39	35
N.S.	1	1.04	0.96	0.73	0.80	0.80	0.86	0.80	0.71
time (sec)	N/A	0.192	0.091	0.165	0.192	0.252	0.796	0.275	8.098

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	71	63	52	58	75	0	78	56
N.S.	1	1.03	0.91	0.75	0.84	1.09	0.00	1.13	0.81
time (sec)	N/A	0.235	0.121	0.448	0.270	0.264	0.000	0.283	8.203

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	66	58	49	51	78	0	101	83
N.S.	1	1.06	0.94	0.79	0.82	1.26	0.00	1.63	1.34
time (sec)	N/A	0.206	0.141	0.332	0.273	0.257	0.000	0.292	0.277

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	90	63	64	68	83	0	145	0
N.S.	1	1.08	0.76	0.77	0.82	1.00	0.00	1.75	0.00
time (sec)	N/A	0.242	0.177	0.355	0.273	0.269	0.000	0.294	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	116	75	66	85	90	0	167	0
N.S.	1	1.12	0.72	0.63	0.82	0.87	0.00	1.61	0.00
time (sec)	N/A	0.274	0.240	0.375	0.283	0.264	0.000	0.301	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	298	309	229	216	0	129	0	0	0
N.S.	1	1.04	0.77	0.72	0.00	0.43	0.00	0.00	0.00
time (sec)	N/A	0.390	10.226	3.891	0.000	0.099	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	270	277	222	208	0	122	0	0	0
N.S.	1	1.03	0.82	0.77	0.00	0.45	0.00	0.00	0.00
time (sec)	N/A	0.341	10.228	2.597	0.000	0.093	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	260	159	194	0	117	0	0	0
N.S.	1	1.01	0.62	0.75	0.00	0.46	0.00	0.00	0.00
time (sec)	N/A	0.306	10.125	0.840	0.000	0.086	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	278	287	224	211	0	124	0	0	0
N.S.	1	1.03	0.81	0.76	0.00	0.45	0.00	0.00	0.00
time (sec)	N/A	0.356	10.233	1.400	0.000	0.082	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	302	311	237	228	0	140	0	0	0
N.S.	1	1.03	0.78	0.75	0.00	0.46	0.00	0.00	0.00
time (sec)	N/A	0.397	10.241	2.114	0.000	0.081	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	84	59	48	73	86	0	52	0
N.S.	1	1.09	0.77	0.62	0.95	1.12	0.00	0.68	0.00
time (sec)	N/A	0.232	0.179	0.216	0.196	0.246	0.000	0.273	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	58	54	43	56	81	0	46	52
N.S.	1	1.04	0.96	0.77	1.00	1.45	0.00	0.82	0.93
time (sec)	N/A	0.206	0.144	0.199	0.197	0.259	0.000	0.292	7.849

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	32	46	0	21	21
N.S.	1	1.00	1.00	0.88	1.28	1.84	0.00	0.84	0.84
time (sec)	N/A	0.166	0.144	0.065	0.185	0.260	0.000	0.292	7.578

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	70	62	53	65	107	0	78	0
N.S.	1	1.06	0.94	0.80	0.98	1.62	0.00	1.18	0.00
time (sec)	N/A	0.212	0.206	0.339	0.277	0.266	0.000	0.295	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	97	70	61	82	124	0	122	0
N.S.	1	1.08	0.78	0.68	0.91	1.38	0.00	1.36	0.00
time (sec)	N/A	0.252	0.229	0.370	0.286	0.247	0.000	0.311	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	307	318	219	216	0	184	0	0	0
N.S.	1	1.04	0.71	0.70	0.00	0.60	0.00	0.00	0.00
time (sec)	N/A	0.404	10.230	4.031	0.000	0.082	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	286	292	219	216	0	179	0	0	0
N.S.	1	1.02	0.77	0.76	0.00	0.63	0.00	0.00	0.00
time (sec)	N/A	0.374	10.251	2.625	0.000	0.080	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	282	292	219	216	0	179	0	0	0
N.S.	1	1.04	0.78	0.77	0.00	0.63	0.00	0.00	0.00
time (sec)	N/A	0.355	10.252	1.308	0.000	0.081	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	309	318	228	223	0	192	0	0	0
N.S.	1	1.03	0.74	0.72	0.00	0.62	0.00	0.00	0.00
time (sec)	N/A	0.402	10.258	1.506	0.000	0.082	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	326	346	234	228	0	207	0	0	0
N.S.	1	1.06	0.72	0.70	0.00	0.63	0.00	0.00	0.00
time (sec)	N/A	0.448	10.261	2.983	0.000	0.082	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	297	297	430	0	0	0	0	0	0
N.S.	1	1.00	1.45	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.527	11.763	0.000	0.000	0.000	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	297	297	386	0	0	0	0	0	0
N.S.	1	1.00	1.30	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.472	11.613	0.000	0.000	0.000	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	295	295	386	0	0	0	0	0	0
N.S.	1	1.00	1.31	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.484	11.561	0.000	0.000	0.000	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	295	295	370	0	0	0	0	0	0
N.S.	1	1.00	1.25	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.473	11.654	0.000	0.000	0.000	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	299	299	567	0	0	0	0	0	0
N.S.	1	1.00	1.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.505	12.005	0.000	0.000	0.000	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	299	299	490	0	0	0	0	0	0
N.S.	1	1.00	1.64	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.482	11.936	0.000	0.000	0.000	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	297	297	487	0	0	0	0	0	0
N.S.	1	1.00	1.64	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.495	11.910	0.000	0.000	0.000	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	297	297	447	0	0	0	0	0	0
N.S.	1	1.00	1.51	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.470	11.745	0.000	0.000	0.000	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	297	297	354	0	0	0	0	0	0
N.S.	1	1.00	1.19	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.463	11.500	0.000	0.000	0.000	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	297	297	242	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.477	11.226	0.000	0.000	0.000	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	295	295	241	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.473	11.221	0.000	0.000	0.000	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	295	295	356	0	0	0	0	0	0
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.488	11.537	0.000	0.000	0.000	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	303	303	375	0	0	0	0	0	0
N.S.	1	1.00	1.24	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.502	11.600	0.000	0.000	0.000	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	303	303	397	0	0	0	0	0	0
N.S.	1	1.00	1.31	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.469	11.597	0.000	0.000	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	301	301	395	0	0	0	0	0	0
N.S.	1	1.00	1.31	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.477	11.573	0.000	0.000	0.000	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	301	301	460	0	0	0	0	0	0
N.S.	1	1.00	1.53	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.486	11.741	0.000	0.000	0.000	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	191	1935	408	1357	11266	2736	769
N.S.	1	1.00	0.79	7.96	1.68	5.58	46.36	11.26	3.16
time (sec)	N/A	0.433	0.815	0.354	0.251	0.261	1.529	0.357	8.067

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	117	783	230	573	4068	1142	429
N.S.	1	1.00	0.75	5.05	1.48	3.70	26.25	7.37	2.77
time (sec)	N/A	0.321	0.449	0.168	0.227	0.255	0.876	0.318	7.867

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	59	82	104	171	1015	338	171
N.S.	1	1.00	0.71	0.99	1.25	2.06	12.23	4.07	2.06
time (sec)	N/A	0.236	0.080	0.079	0.228	0.255	0.467	0.300	7.677

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	194	194	156	0	0	0	0	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.389	0.346	0.000	0.000	0.000	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	392	0	160	0	0	0	0	0	0
N.S.	1	0.00	0.41	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	1.534	0.000	0.000	0.000	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	319	319	466	0	0	0	0	0	0
N.S.	1	1.00	1.46	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.521	2.900	0.000	0.000	0.000	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	317	317	267	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.486	1.983	0.000	0.000	0.000	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	317	317	267	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.493	2.831	0.000	0.000	0.000	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	323	323	307	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.525	11.343	0.000	0.000	0.000	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	148	134	103	120	277	0	123	181
N.S.	1	1.10	1.00	0.77	0.90	2.07	0.00	0.92	1.35
time (sec)	N/A	0.454	0.070	0.412	0.281	2.891	0.000	0.292	8.010

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	127	99	92	107	212	0	108	166
N.S.	1	1.08	0.84	0.78	0.91	1.80	0.00	0.92	1.41
time (sec)	N/A	0.384	0.074	0.410	0.305	1.185	0.000	0.281	7.882

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	104	77	80	89	170	0	90	138
N.S.	1	0.99	0.73	0.76	0.85	1.62	0.00	0.86	1.31
time (sec)	N/A	0.268	0.031	0.398	0.292	0.730	0.000	0.280	8.061

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	84	66	72	82	145	0	85	944
N.S.	1	0.88	0.69	0.75	0.85	1.51	0.00	0.89	9.83
time (sec)	N/A	0.224	0.033	0.389	0.285	0.365	0.000	0.274	8.743

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	87	67	75	82	146	0	86	328
N.S.	1	0.91	0.70	0.78	0.85	1.52	0.00	0.90	3.42
time (sec)	N/A	0.221	0.031	0.381	0.285	0.428	0.000	0.284	8.309

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	116	134	90	101	201	0	103	527
N.S.	1	1.02	1.18	0.79	0.89	1.76	0.00	0.90	4.62
time (sec)	N/A	0.289	0.061	0.399	0.276	4.431	0.000	0.279	8.043

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	128	169	107	120	265	0	131	820
N.S.	1	0.99	1.31	0.83	0.93	2.05	0.00	1.02	6.36
time (sec)	N/A	0.306	0.075	0.386	0.279	22.700	0.000	0.280	8.290

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	154	209	127	145	338	0	170	1017
N.S.	1	0.99	1.34	0.81	0.93	2.17	0.00	1.09	6.52
time (sec)	N/A	0.342	0.069	0.425	0.282	49.460	0.000	0.269	8.597

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	359	359	344	279	0	4414	0	370	6097
N.S.	1	1.00	0.96	0.78	0.00	12.30	0.00	1.03	16.98
time (sec)	N/A	0.516	0.261	0.408	0.000	5.530	0.000	0.288	8.484

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	373	263	0	4354	0	338	5908
N.S.	1	1.00	1.08	0.76	0.00	12.62	0.00	0.98	17.12
time (sec)	N/A	0.469	0.166	0.409	0.000	1.084	0.000	0.282	8.439

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	233	254	0	4040	0	332	5111
N.S.	1	1.00	0.69	0.76	0.00	12.02	0.00	0.99	15.21
time (sec)	N/A	0.453	0.108	0.406	0.000	0.474	0.000	0.290	8.647

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	337	232	246	0	3892	0	340	4720
N.S.	1	1.00	0.69	0.73	0.00	11.55	0.00	1.01	14.01
time (sec)	N/A	0.445	0.097	0.390	0.000	0.364	0.000	0.283	8.459

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	234	253	0	4084	0	344	4802
N.S.	1	1.00	0.70	0.75	0.00	12.15	0.00	1.02	14.29
time (sec)	N/A	0.451	0.102	0.332	0.000	0.683	0.000	0.313	8.516

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	348	348	389	266	0	4362	0	353	5761
N.S.	1	1.00	1.12	0.76	0.00	12.53	0.00	1.01	16.55
time (sec)	N/A	0.482	0.176	0.404	0.000	1.807	0.000	0.284	8.682

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	360	367	284	0	4442	0	368	5972
N.S.	1	1.00	1.02	0.79	0.00	12.34	0.00	1.02	16.59
time (sec)	N/A	0.494	0.304	0.398	0.000	5.978	0.000	0.299	8.707

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	177	159	160	220	555	0	258	305
N.S.	1	1.05	0.94	0.95	1.30	3.28	0.00	1.53	1.80
time (sec)	N/A	0.575	0.088	0.431	0.278	8.772	0.000	0.275	8.247

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	169	142	146	197	457	0	229	647
N.S.	1	1.13	0.95	0.97	1.31	3.05	0.00	1.53	4.31
time (sec)	N/A	0.381	0.076	0.495	0.286	4.234	0.000	0.286	8.263

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	164	149	136	192	487	0	228	528
N.S.	1	1.06	0.96	0.88	1.24	3.14	0.00	1.47	3.41
time (sec)	N/A	0.354	0.072	0.415	0.280	1.787	0.000	0.279	8.276

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	159	114	132	186	492	0	196	527
N.S.	1	1.07	0.77	0.89	1.25	3.30	0.00	1.32	3.54
time (sec)	N/A	0.317	0.100	0.423	0.297	1.745	0.000	0.274	8.241

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	171	144	146	196	458	0	208	649
N.S.	1	1.13	0.95	0.97	1.30	3.03	0.00	1.38	4.30
time (sec)	N/A	0.350	0.068	0.421	0.313	4.215	0.000	0.291	8.410

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	211	241	171	228	686	0	289	1082
N.S.	1	1.01	1.15	0.82	1.09	3.28	0.00	1.38	5.18
time (sec)	N/A	0.397	0.128	0.457	0.279	74.982	0.000	0.283	9.055

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	235	248	181	278	852	0	357	1337
N.S.	1	1.00	1.05	0.77	1.18	3.61	0.00	1.51	5.67
time (sec)	N/A	0.430	0.312	0.494	0.283	224.195	0.000	0.268	9.143

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	263	278	209	332	0	0	361	1545
N.S.	1	0.99	1.05	0.79	1.25	0.00	0.00	1.36	5.83
time (sec)	N/A	0.489	0.303	0.481	0.293	0.000	0.000	0.295	9.492

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	712	652	431	334	0	9856	0	599	18343
N.S.	1	0.92	0.61	0.47	0.00	13.84	0.00	0.84	25.76
time (sec)	N/A	1.141	0.212	0.457	0.000	9.624	0.000	0.293	9.057

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	687	641	428	339	0	9822	0	614	17909
N.S.	1	0.93	0.62	0.49	0.00	14.30	0.00	0.89	26.07
time (sec)	N/A	1.029	0.257	0.460	0.000	6.610	0.000	0.294	9.022

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	685	635	423	327	0	9678	0	607	17180
N.S.	1	0.93	0.62	0.48	0.00	14.13	0.00	0.89	25.08
time (sec)	N/A	0.925	0.183	0.477	0.000	5.778	0.000	0.292	10.458

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	685	633	428	339	0	9774	0	622	17812
N.S.	1	0.92	0.62	0.49	0.00	14.27	0.00	0.91	26.00
time (sec)	N/A	0.916	0.182	0.438	0.000	6.449	0.000	0.297	9.657

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	689	689	429	334	0	9892	0	621	17945
N.S.	1	1.00	0.62	0.48	0.00	14.36	0.00	0.90	26.04
time (sec)	N/A	0.814	0.198	0.368	0.000	11.957	0.000	0.299	9.257

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	745	745	499	355	0	10188	0	659	24015
N.S.	1	1.00	0.67	0.48	0.00	13.68	0.00	0.88	32.23
time (sec)	N/A	0.949	0.242	0.502	0.000	29.014	0.000	0.285	10.732

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	751	751	513	355	0	10352	0	645	20828
N.S.	1	1.00	0.68	0.47	0.00	13.78	0.00	0.86	27.73
time (sec)	N/A	0.913	0.266	0.488	0.000	65.773	0.000	0.277	11.043

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	40	110	0	32	0	0	0
N.S.	1	1.00	0.57	1.57	0.00	0.46	0.00	0.00	0.00
time (sec)	N/A	0.261	10.186	1.065	0.000	0.120	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	36	112	0	56	0	0	0
N.S.	1	1.00	0.51	1.60	0.00	0.80	0.00	0.00	0.00
time (sec)	N/A	0.258	10.173	1.303	0.000	0.113	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	43	46	88	0	42	0	0	0
N.S.	1	0.43	0.46	0.89	0.00	0.42	0.00	0.00	0.00
time (sec)	N/A	0.209	10.125	1.408	0.000	0.082	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	34	37	88	0	31	0	0	0
N.S.	1	0.56	0.61	1.44	0.00	0.51	0.00	0.00	0.00
time (sec)	N/A	0.188	10.152	0.963	0.000	0.085	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	67	54	93	0	44	0	0	0
N.S.	1	0.59	0.48	0.82	0.00	0.39	0.00	0.00	0.00
time (sec)	N/A	0.174	10.100	1.581	0.000	0.086	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	99	35	93	0	31	0	0	0
N.S.	1	1.74	0.61	1.63	0.00	0.54	0.00	0.00	0.00
time (sec)	N/A	0.192	10.142	1.154	0.000	0.084	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	60	168	0	75	0	0	0
N.S.	1	1.00	0.81	2.27	0.00	1.01	0.00	0.00	0.00
time (sec)	N/A	0.277	10.118	1.399	0.000	0.103	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	56	115	0	74	0	0	0
N.S.	1	1.00	0.76	1.55	0.00	1.00	0.00	0.00	0.00
time (sec)	N/A	0.270	10.166	1.474	0.000	0.116	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	143	128	128	124	206	0	156	0
N.S.	1	0.59	0.53	0.53	0.51	0.85	0.00	0.64	0.00
time (sec)	N/A	0.293	0.477	0.345	0.205	0.318	0.000	0.276	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	81	56	51	50	50	0	68	0
N.S.	1	0.75	0.52	0.47	0.46	0.46	0.00	0.63	0.00
time (sec)	N/A	0.247	0.037	0.305	0.207	0.280	0.000	0.272	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	114	96	103	81	155	0	109	0
N.S.	1	0.64	0.54	0.58	0.46	0.87	0.00	0.61	0.00
time (sec)	N/A	0.238	0.123	0.324	0.199	0.271	0.000	0.277	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	96	83	80	45	123	0	84	0
N.S.	1	0.63	0.55	0.53	0.30	0.81	0.00	0.55	0.00
time (sec)	N/A	0.247	0.089	0.327	0.201	0.293	0.000	0.272	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	111	101	94	59	134	0	111	0
N.S.	1	0.63	0.57	0.53	0.33	0.76	0.00	0.63	0.00
time (sec)	N/A	0.255	0.173	0.330	0.194	0.269	0.000	0.282	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	110	90	106	83	141	0	100	0
N.S.	1	0.62	0.51	0.60	0.47	0.80	0.00	0.56	0.00
time (sec)	N/A	0.264	0.129	0.340	0.190	0.277	0.000	0.280	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	82	72	73	72	72	76	79	73
N.S.	1	1.05	0.92	0.94	0.92	0.92	0.97	1.01	0.94
time (sec)	N/A	0.277	0.024	0.362	0.205	0.263	0.024	0.259	0.023

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	78	73	72	72	82	79	73
N.S.	1	1.00	1.00	0.94	0.92	0.92	1.05	1.01	0.94
time (sec)	N/A	0.242	0.016	0.351	0.191	0.258	0.024	0.260	0.017

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	79	72	73	72	72	76	79	73
N.S.	1	1.05	0.96	0.97	0.96	0.96	1.01	1.05	0.97
time (sec)	N/A	0.256	0.019	0.350	0.189	0.251	0.025	0.288	0.016

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	70	69	69	78	76	70
N.S.	1	1.00	1.00	0.96	0.95	0.95	1.07	1.04	0.96
time (sec)	N/A	0.230	0.014	0.289	0.183	0.269	0.023	0.285	0.016

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	77	74	71	73	70	73	79	70
N.S.	1	1.04	1.00	0.96	0.99	0.95	0.99	1.07	0.95
time (sec)	N/A	0.245	0.018	0.311	0.251	0.242	0.085	0.266	0.019

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	71	74	69	74	73	74	70
N.S.	1	1.00	1.00	1.04	0.97	1.04	1.03	1.04	0.99
time (sec)	N/A	0.226	0.026	0.302	0.255	0.250	0.080	0.269	0.019

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	75	71	73	73	76	71	96	70
N.S.	1	1.01	0.96	0.99	0.99	1.03	0.96	1.30	0.95
time (sec)	N/A	0.249	0.032	0.284	0.191	0.247	0.130	0.269	0.021

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	165	159	0	426	320	174	251
N.S.	1	1.00	0.98	0.95	0.00	2.54	1.90	1.04	1.49
time (sec)	N/A	0.504	0.105	0.346	0.000	0.335	0.650	0.422	7.700

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	136	133	122	0	350	189	136	179
N.S.	1	1.01	0.99	0.90	0.00	2.59	1.40	1.01	1.33
time (sec)	N/A	0.409	0.056	0.346	0.000	0.269	0.621	0.390	7.680

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	102	90	0	302	162	99	95
N.S.	1	1.00	0.96	0.85	0.00	2.85	1.53	0.93	0.90
time (sec)	N/A	0.301	0.056	0.371	0.000	0.272	0.526	0.359	7.788

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	94	88	79	0	268	153	83	77
N.S.	1	1.13	1.06	0.95	0.00	3.23	1.84	1.00	0.93
time (sec)	N/A	0.253	0.048	0.304	0.000	0.267	0.418	0.329	7.720

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	92	89	85	0	267	155	89	81
N.S.	1	1.03	1.00	0.96	0.00	3.00	1.74	1.00	0.91
time (sec)	N/A	0.274	0.048	0.329	0.000	0.271	0.595	0.359	7.630

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	115	105	94	0	316	167	96	98
N.S.	1	1.08	0.99	0.89	0.00	2.98	1.58	0.91	0.92
time (sec)	N/A	0.323	0.048	0.343	0.000	0.354	0.771	0.325	7.638

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	149	135	122	0	360	284	135	128
N.S.	1	1.10	0.99	0.90	0.00	2.65	2.09	0.99	0.94
time (sec)	N/A	0.528	0.066	0.340	0.000	0.266	1.045	0.294	7.696

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	182	166	149	0	436	328	173	156
N.S.	1	1.09	0.99	0.89	0.00	2.61	1.96	1.04	0.93
time (sec)	N/A	0.596	0.078	0.347	0.000	0.340	1.248	0.287	7.797

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	182	170	151	0	504	235	172	223
N.S.	1	1.05	0.98	0.87	0.00	2.91	1.36	0.99	1.29
time (sec)	N/A	0.676	0.083	0.333	0.000	0.276	1.919	0.307	7.751

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	150	141	120	0	462	212	134	137
N.S.	1	1.05	0.99	0.84	0.00	3.23	1.48	0.94	0.96
time (sec)	N/A	0.466	0.063	0.327	0.000	0.256	1.719	0.283	7.710

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	133	122	106	0	421	201	116	118
N.S.	1	1.07	0.98	0.85	0.00	3.40	1.62	0.94	0.95
time (sec)	N/A	0.320	0.073	0.319	0.000	0.279	1.294	0.285	7.759

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	125	110	107	0	391	196	109	112
N.S.	1	1.09	0.96	0.93	0.00	3.40	1.70	0.95	0.97
time (sec)	N/A	0.270	0.068	0.290	0.000	0.319	0.770	0.286	7.713

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	135	124	111	0	421	202	118	118
N.S.	1	1.06	0.98	0.87	0.00	3.31	1.59	0.93	0.93
time (sec)	N/A	0.347	0.099	0.365	0.000	0.268	1.026	0.276	7.700

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	170	141	124	0	476	214	131	138
N.S.	1	1.20	0.99	0.87	0.00	3.35	1.51	0.92	0.97
time (sec)	N/A	0.438	0.062	0.365	0.000	0.266	1.351	0.266	7.740

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	194	173	151	0	514	330	171	168
N.S.	1	1.13	1.01	0.88	0.00	3.01	1.93	1.00	0.98
time (sec)	N/A	0.791	0.080	0.378	0.000	0.260	1.788	0.272	7.728

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	227	228	216	0	0	0	233	7024
N.S.	1	0.99	0.99	0.94	0.00	0.00	0.00	1.01	30.54
time (sec)	N/A	0.560	0.180	0.507	0.000	0.000	0.000	0.596	73.398

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	185	186	172	0	616	0	194	2304
N.S.	1	0.98	0.98	0.91	0.00	3.26	0.00	1.03	12.19
time (sec)	N/A	0.434	0.121	0.565	0.000	113.048	0.000	0.587	18.413

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	157	139	137	0	421	0	155	1853
N.S.	1	0.99	0.88	0.87	0.00	2.66	0.00	0.98	11.73
time (sec)	N/A	0.380	0.068	0.439	0.000	32.438	0.000	0.577	15.421

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	131	114	112	0	321	0	131	3704
N.S.	1	0.99	0.86	0.85	0.00	2.43	0.00	0.99	28.06
time (sec)	N/A	0.330	0.049	0.459	0.000	11.072	0.000	0.563	14.133

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	116	112	113	0	321	0	133	2434
N.S.	1	0.87	0.84	0.85	0.00	2.41	0.00	1.00	18.30
time (sec)	N/A	0.311	0.048	0.362	0.000	7.675	0.000	0.593	13.230

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	168	242	166	0	0	0	170	6285
N.S.	1	1.01	1.45	0.99	0.00	0.00	0.00	1.02	37.63
time (sec)	N/A	0.441	0.214	0.419	0.000	0.000	0.000	0.559	19.843

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	204	331	215	0	0	0	232	5368
N.S.	1	1.00	1.61	1.05	0.00	0.00	0.00	1.13	26.19
time (sec)	N/A	0.530	0.215	0.497	0.000	0.000	0.000	0.541	65.887

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	266	426	284	0	0	0	327	10300
N.S.	1	0.99	1.59	1.06	0.00	0.00	0.00	1.22	38.43
time (sec)	N/A	0.651	0.277	0.541	0.000	0.000	0.000	0.601	141.998

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	387	387	463	413	0	24520	0	12525	41755
N.S.	1	1.00	1.20	1.07	0.00	63.36	0.00	32.36	107.89
time (sec)	N/A	2.600	0.407	0.474	0.000	164.883	0.000	2.845	11.970

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	323	385	331	0	20147	0	11046	33892
N.S.	1	1.00	1.19	1.02	0.00	62.37	0.00	34.20	104.93
time (sec)	N/A	1.138	0.337	0.487	0.000	25.695	0.000	2.443	11.481

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	323	264	0	15553	0	8670	25202
N.S.	1	1.00	1.15	0.94	0.00	55.55	0.00	30.96	90.01
time (sec)	N/A	0.886	0.222	0.431	0.000	2.574	0.000	2.271	10.978

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	277	213	0	12269	0	6936	19401
N.S.	1	1.00	1.10	0.85	0.00	48.88	0.00	27.63	77.29
time (sec)	N/A	0.591	0.320	0.436	0.000	1.116	0.000	1.777	10.548

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	274	215	0	16013	0	7664	23640
N.S.	1	1.00	1.08	0.85	0.00	63.04	0.00	30.17	93.07
time (sec)	N/A	0.627	0.173	0.337	0.000	13.244	0.000	1.783	11.030

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	340	276	0	20595	0	10072	33644
N.S.	1	1.00	1.14	0.93	0.00	69.11	0.00	33.80	112.90
time (sec)	N/A	0.930	0.262	0.462	0.000	166.674	0.000	2.513	11.326

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	348	348	410	349	0	24988	0	12281	42882
N.S.	1	1.00	1.18	1.00	0.00	71.80	0.00	35.29	123.22
time (sec)	N/A	1.284	0.357	0.488	0.000	293.422	0.000	2.650	11.881

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	866	878	215	264	0	0	0	0	43112
N.S.	1	1.01	0.25	0.30	0.00	0.00	0.00	0.00	49.78
time (sec)	N/A	1.833	0.304	1.988	0.000	0.000	0.000	0.000	12.008

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	296	259	369	0	1525	0	0	0
N.S.	1	1.09	0.95	1.36	0.00	5.61	0.00	0.00	0.00
time (sec)	N/A	0.659	0.970	0.571	0.000	94.121	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	219	199	286	0	1231	0	0	0
N.S.	1	1.05	0.96	1.38	0.00	5.92	0.00	0.00	0.00
time (sec)	N/A	0.486	0.646	0.456	0.000	10.363	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	172	160	190	0	1050	0	0	0
N.S.	1	1.02	0.95	1.13	0.00	6.25	0.00	0.00	0.00
time (sec)	N/A	0.387	0.941	0.404	0.000	0.871	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	174	240	0	2367	0	0	0
N.S.	1	1.00	0.94	1.29	0.00	12.73	0.00	0.00	0.00
time (sec)	N/A	0.470	0.457	0.446	0.000	42.124	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	361	354	158	225	0	1094	0	214	0
N.S.	1	0.98	0.44	0.62	0.00	3.03	0.00	0.59	0.00
time (sec)	N/A	0.644	0.638	0.453	0.000	0.476	0.000	0.341	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	424	435	209	246	0	0	0	0	0
N.S.	1	1.03	0.49	0.58	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.106	5.769	4.103	0.000	0.000	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	417	408	204	239	0	0	0	0	0
N.S.	1	0.98	0.49	0.57	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.855	5.058	1.366	0.000	0.000	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	401	127	341	0	0	0	0	0
N.S.	1	1.05	0.33	0.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.641	9.622	0.621	0.000	0.000	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	399	400	208	240	0	0	0	0	0
N.S.	1	1.00	0.52	0.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.713	10.182	1.897	0.000	0.000	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	311	154	153	0	0	0	0	0
N.S.	1	0.86	0.43	0.42	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.724	10.173	3.190	0.000	0.000	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	546	535	224	263	0	0	0	0	0
N.S.	1	0.98	0.41	0.48	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.418	10.222	4.095	0.000	0.000	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	482	517	545	620	0	0	0	0	0
N.S.	1	1.07	1.13	1.29	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.049	10.668	0.635	0.000	0.000	0.000	0.000	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	384	345	494	0	0	0	0	0
N.S.	1	1.07	0.96	1.37	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.845	1.942	0.685	0.000	0.000	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	289	266	361	0	1589	0	0	0
N.S.	1	1.07	0.99	1.34	0.00	5.91	0.00	0.00	0.00
time (sec)	N/A	0.621	1.602	0.535	0.000	111.846	0.000	0.000	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	350	344	258	360	0	0	0	0	0
N.S.	1	0.98	0.74	1.03	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.792	0.955	0.474	0.000	0.000	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	562	558	230	344	0	0	0	0	0
N.S.	1	0.99	0.41	0.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.988	1.099	0.559	0.000	0.000	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	463	454	214	251	0	0	0	0	0
N.S.	1	0.98	0.46	0.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.328	8.963	2.839	0.000	0.000	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	428	430	209	246	0	0	0	0	0
N.S.	1	1.00	0.49	0.57	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.038	10.196	1.351	0.000	0.000	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	722	423	213	258	0	0	0	0	0
N.S.	1	0.59	0.30	0.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.131	10.236	2.964	0.000	0.000	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	625	468	219	258	0	0	0	0	0
N.S.	1	0.75	0.35	0.41	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.242	10.246	4.107	0.000	0.000	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	553	535	224	263	0	0	0	0	0
N.S.	1	0.97	0.41	0.48	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.448	10.272	5.107	0.000	0.000	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	181	166	211	0	1364	0	0	0
N.S.	1	1.05	0.96	1.22	0.00	7.88	0.00	0.00	0.00
time (sec)	N/A	0.401	0.781	0.442	0.000	12.718	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	136	146	171	0	1084	0	0	0
N.S.	1	0.99	1.07	1.25	0.00	7.91	0.00	0.00	0.00
time (sec)	N/A	0.309	0.416	0.362	0.000	0.964	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	96	101	0	357	0	74	0
N.S.	1	1.00	1.12	1.17	0.00	4.15	0.00	0.86	0.00
time (sec)	N/A	0.226	0.280	0.349	0.000	0.337	0.000	0.276	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	144	166	0	1097	0	0	0
N.S.	1	1.00	1.04	1.20	0.00	7.95	0.00	0.00	0.00
time (sec)	N/A	0.351	0.476	0.402	0.000	0.458	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	214	209	206	0	1414	0	207	0
N.S.	1	0.98	0.96	0.94	0.00	6.49	0.00	0.95	0.00
time (sec)	N/A	0.422	1.005	0.456	0.000	0.679	0.000	0.296	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	418	410	127	222	0	0	0	0	0
N.S.	1	0.98	0.30	0.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.605	10.208	1.890	0.000	0.000	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	268	99	134	0	0	0	0	0
N.S.	1	1.09	0.40	0.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.402	10.122	0.750	0.000	0.000	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	266	80	70	0	0	0	0	0
N.S.	1	1.09	0.33	0.29	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.388	10.072	0.643	0.000	0.000	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	399	427	147	178	0	0	0	0	0
N.S.	1	1.07	0.37	0.45	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.796	10.185	1.931	0.000	0.000	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	422	444	219	258	0	0	0	0	0
N.S.	1	1.05	0.52	0.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.981	10.181	3.456	0.000	0.000	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	269	251	365	0	4901	0	0	0
N.S.	1	1.14	1.06	1.55	0.00	20.77	0.00	0.00	0.00
time (sec)	N/A	0.563	0.938	0.586	0.000	37.451	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	168	180	229	0	1381	0	468	0
N.S.	1	1.01	1.08	1.37	0.00	8.27	0.00	2.80	0.00
time (sec)	N/A	0.362	0.643	0.437	0.000	0.543	0.000	0.306	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	162	178	216	0	1349	0	451	0
N.S.	1	1.02	1.12	1.36	0.00	8.48	0.00	2.84	0.00
time (sec)	N/A	0.339	0.699	0.432	0.000	0.582	0.000	0.293	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	167	182	223	0	1379	0	468	0
N.S.	1	1.01	1.10	1.34	0.00	8.31	0.00	2.82	0.00
time (sec)	N/A	0.331	0.904	0.395	0.000	0.558	0.000	0.282	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	268	247	285	0	4909	0	0	0
N.S.	1	1.01	0.93	1.07	0.00	18.45	0.00	0.00	0.00
time (sec)	N/A	0.539	1.139	0.677	0.000	2.444	0.000	0.000	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	419	416	363	413	0	6486	0	777	0
N.S.	1	0.99	0.87	0.99	0.00	15.48	0.00	1.85	0.00
time (sec)	N/A	0.660	1.681	0.945	0.000	5.173	0.000	0.480	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	459	199	246	0	0	0	0	0
N.S.	1	1.02	0.44	0.55	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.941	10.268	4.935	0.000	0.000	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	423	464	199	246	0	0	0	0	0
N.S.	1	1.10	0.47	0.58	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.847	9.675	2.516	0.000	0.000	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	422	455	199	244	0	0	0	0	0
N.S.	1	1.08	0.47	0.58	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.821	7.104	1.646	0.000	0.000	0.000	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	423	457	199	246	0	0	0	0	0
N.S.	1	1.08	0.47	0.58	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.849	6.996	1.274	0.000	0.000	0.000	0.000	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	422	459	199	246	0	0	0	0	0
N.S.	1	1.09	0.47	0.58	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.875	10.150	1.319	0.000	0.000	0.000	0.000	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	468	489	211	253	0	0	0	0	0
N.S.	1	1.04	0.45	0.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.222	10.208	1.975	0.000	0.000	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	406	397	475	473	0	5829	0	959	11195
N.S.	1	0.98	1.17	1.17	0.00	14.36	0.00	2.36	27.57
time (sec)	N/A	4.298	1.304	2.388	0.000	257.626	0.000	0.329	8.899

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	324	320	383	396	0	4182	0	776	8222
N.S.	1	0.99	1.18	1.22	0.00	12.91	0.00	2.40	25.38
time (sec)	N/A	2.095	0.979	0.870	0.000	102.891	0.000	0.322	8.681

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	303	350	316	0	2435	0	626	5705
N.S.	1	1.04	1.20	1.08	0.00	8.34	0.00	2.14	19.54
time (sec)	N/A	1.124	0.954	0.607	0.000	29.968	0.000	0.327	9.191

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	236	256	227	0	1085	0	449	717
N.S.	1	1.17	1.27	1.12	0.00	5.37	0.00	2.22	3.55
time (sec)	N/A	0.394	0.586	0.387	0.000	6.081	0.000	0.306	8.603

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	288	274	358	0	3126	0	724	10964
N.S.	1	1.02	0.98	1.27	0.00	11.12	0.00	2.58	39.02
time (sec)	N/A	0.920	0.636	0.465	0.000	49.740	0.000	0.313	11.487

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	382	388	348	364	0	6592	0	784	19959
N.S.	1	1.02	0.91	0.95	0.00	17.26	0.00	2.05	52.25
time (sec)	N/A	2.385	1.054	0.825	0.000	260.181	0.000	0.325	10.828

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	552	587	445	463	0	0	0	1057	33925
N.S.	1	1.06	0.81	0.84	0.00	0.00	0.00	1.91	61.46
time (sec)	N/A	2.154	1.700	1.153	0.000	0.000	0.000	0.346	11.971

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	390	388	786	360	0	6534	0	0	0
N.S.	1	0.99	2.02	0.92	0.00	16.75	0.00	0.00	0.00
time (sec)	N/A	1.840	1.010	2.623	0.000	14.868	0.000	0.000	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	324	326	7468	368	0	3260	0	0	0
N.S.	1	1.01	23.05	1.14	0.00	10.06	0.00	0.00	0.00
time (sec)	N/A	0.963	16.200	0.400	0.000	2.498	0.000	0.000	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	321	2607	233	0	985	0	0	0
N.S.	1	1.34	10.86	0.97	0.00	4.10	0.00	0.00	0.00
time (sec)	N/A	0.468	13.416	0.365	0.000	0.832	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	289	626	326	0	2402	0	0	0
N.S.	1	0.99	2.15	1.12	0.00	8.25	0.00	0.00	0.00
time (sec)	N/A	0.731	0.734	0.771	0.000	1.634	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	373	385	7477	398	0	4095	0	0	0
N.S.	1	1.03	20.05	1.07	0.00	10.98	0.00	0.00	0.00
time (sec)	N/A	1.594	16.384	0.906	0.000	6.508	0.000	0.000	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	512	521	10511	500	0	5773	0	0	0
N.S.	1	1.02	20.53	0.98	0.00	11.28	0.00	0.00	0.00
time (sec)	N/A	2.513	16.520	1.214	0.000	15.781	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	460	398	501	447	0	8200	0	867	16951
N.S.	1	0.87	1.09	0.97	0.00	17.83	0.00	1.88	36.85
time (sec)	N/A	0.932	1.763	0.849	0.000	249.643	0.000	0.389	9.376

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	327	310	373	335	0	4444	0	802	12392
N.S.	1	0.95	1.14	1.02	0.00	13.59	0.00	2.45	37.90
time (sec)	N/A	0.611	1.199	0.569	0.000	65.872	0.000	0.341	9.762

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	353	379	402	0	0	0	838	28434
N.S.	1	1.02	1.10	1.16	0.00	0.00	0.00	2.42	82.18
time (sec)	N/A	1.141	1.393	0.515	0.000	0.000	0.000	0.341	11.986

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	417	434	427	399	0	0	0	898	35855
N.S.	1	1.04	1.02	0.96	0.00	0.00	0.00	2.15	85.98
time (sec)	N/A	1.804	1.735	0.851	0.000	0.000	0.000	0.331	11.061

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	595	594	1407	460	0	19825	0	0	0
N.S.	1	1.00	2.36	0.77	0.00	33.32	0.00	0.00	0.00
time (sec)	N/A	2.088	2.301	1.366	0.000	290.822	0.000	0.000	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	491	496	916	394	0	11917	0	0	0
N.S.	1	1.01	1.87	0.80	0.00	24.27	0.00	0.00	0.00
time (sec)	N/A	1.180	1.380	0.753	0.000	54.072	0.000	0.000	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	487	522	8958	411	0	7721	0	0	0
N.S.	1	1.07	18.39	0.84	0.00	15.85	0.00	0.00	0.00
time (sec)	N/A	0.756	16.204	0.406	0.000	15.604	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	424	7491	347	0	4059	0	0	0
N.S.	1	1.63	28.81	1.33	0.00	15.61	0.00	0.00	0.00
time (sec)	N/A	0.892	16.330	0.936	0.000	7.918	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	523	529	1095	451	0	7830	0	0	0
N.S.	1	1.01	2.09	0.86	0.00	14.97	0.00	0.00	0.00
time (sec)	N/A	1.742	1.494	1.093	0.000	32.456	0.000	0.000	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	273	346	316	0	3615	0	4637	917
N.S.	1	0.97	1.23	1.12	0.00	12.86	0.00	16.50	3.26
time (sec)	N/A	3.175	0.793	1.055	0.000	3.733	0.000	1.153	8.337

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	240	285	228	0	2053	0	4060	776
N.S.	1	1.05	1.24	1.00	0.00	8.97	0.00	17.73	3.39
time (sec)	N/A	0.506	0.543	0.536	0.000	1.470	0.000	1.067	8.350

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	198	169	162	0	871	0	591	649
N.S.	1	1.09	0.93	0.89	0.00	4.79	0.00	3.25	3.57
time (sec)	N/A	0.346	0.233	0.452	0.000	0.755	0.000	1.569	8.300

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	230	264	271	0	1232	0	3639	669
N.S.	1	0.95	1.10	1.12	0.00	5.11	0.00	15.10	2.78
time (sec)	N/A	0.776	0.523	0.461	0.000	2.718	0.000	0.968	8.398

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	290	307	330	0	2799	0	1675	825
N.S.	1	1.00	1.06	1.14	0.00	9.65	0.00	5.78	2.84
time (sec)	N/A	1.331	1.454	0.669	0.000	7.499	0.000	1.569	8.241

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	323	588	250	0	2860	0	1710	1024
N.S.	1	0.99	1.81	0.77	0.00	8.80	0.00	5.26	3.15
time (sec)	N/A	2.031	0.639	2.124	0.000	1.303	0.000	1.418	8.238

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	262	412	262	0	1430	0	3580	870
N.S.	1	1.00	1.57	1.00	0.00	5.44	0.00	13.61	3.31
time (sec)	N/A	1.002	0.411	0.319	0.000	0.672	0.000	1.018	8.340

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	257	237	168	0	759	0	641	989
N.S.	1	1.17	1.08	0.76	0.00	3.45	0.00	2.91	4.50
time (sec)	N/A	0.391	0.299	0.392	0.000	0.352	0.000	1.076	8.272

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	263	471	239	0	1998	0	3965	1234
N.S.	1	0.99	1.78	0.90	0.00	7.54	0.00	14.96	4.66
time (sec)	N/A	0.658	0.451	0.735	0.000	0.492	0.000	0.853	8.169

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	199	100	0	318	0	209	383
N.S.	1	1.00	2.07	1.04	0.00	3.31	0.00	2.18	3.99
time (sec)	N/A	0.395	0.439	2.025	0.000	0.286	0.000	0.330	8.390

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	479	479	838	394	0	18271	0	0	0
N.S.	1	1.00	1.75	0.82	0.00	38.14	0.00	0.00	0.00
time (sec)	N/A	1.465	1.372	1.056	0.000	142.828	0.000	0.000	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	366	366	594	305	0	14654	0	0	0
N.S.	1	1.00	1.62	0.83	0.00	40.04	0.00	0.00	0.00
time (sec)	N/A	1.015	0.775	0.805	0.000	52.679	0.000	0.000	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	292	252	0	11094	0	0	0
N.S.	1	1.00	0.98	0.85	0.00	37.23	0.00	0.00	0.00
time (sec)	N/A	0.729	10.674	0.465	0.000	9.052	0.000	0.000	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	227	233	0	3395	0	0	0
N.S.	1	1.00	0.95	0.97	0.00	14.15	0.00	0.00	0.00
time (sec)	N/A	0.487	8.657	0.404	0.000	2.462	0.000	0.000	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	179	224	0	4557	0	0	0
N.S.	1	1.00	0.74	0.92	0.00	18.75	0.00	0.00	0.00
time (sec)	N/A	0.330	0.137	0.379	0.000	6.168	0.000	0.000	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	271	266	0	6431	0	0	0
N.S.	1	1.00	0.97	0.95	0.00	22.97	0.00	0.00	0.00
time (sec)	N/A	0.719	10.804	0.617	0.000	12.662	0.000	0.000	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	341	320	332	0	8187	0	0	0
N.S.	1	1.00	0.94	0.97	0.00	24.01	0.00	0.00	0.00
time (sec)	N/A	0.767	10.590	0.714	0.000	62.573	0.000	0.000	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	443	443	383	410	0	9998	0	0	0
N.S.	1	1.00	0.86	0.93	0.00	22.57	0.00	0.00	0.00
time (sec)	N/A	1.251	11.235	0.979	0.000	164.999	0.000	0.000	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	350	456	10546	502	0	0	0	0	0
N.S.	1	1.30	30.13	1.43	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.326	21.270	0.606	0.000	0.000	0.000	0.000	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	343	449	382	0	14462	0	0	0
N.S.	1	0.95	1.25	1.06	0.00	40.17	0.00	0.00	0.00
time (sec)	N/A	0.978	0.686	0.540	0.000	156.125	0.000	0.000	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	333	317	463	393	0	14146	0	0	0
N.S.	1	0.95	1.39	1.18	0.00	42.48	0.00	0.00	0.00
time (sec)	N/A	0.731	0.636	0.400	0.000	111.971	0.000	0.000	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	341	325	2061	429	0	17249	0	0	0
N.S.	1	0.95	6.04	1.26	0.00	50.58	0.00	0.00	0.00
time (sec)	N/A	0.746	18.246	0.413	0.000	217.475	0.000	0.000	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	339	410	2158	513	0	0	0	0	0
N.S.	1	1.21	6.37	1.51	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.602	17.538	1.137	0.000	0.000	0.000	0.000	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	419	543	2218	610	0	0	0	0	0
N.S.	1	1.30	5.29	1.46	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.452	17.822	1.315	0.000	0.000	0.000	0.000	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	243	243	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.462	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	313	306	272	0	0	0	0	0	0
N.S.	1	0.98	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.714	1.096	0.000	0.000	0.000	0.000	0.000	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	256	251	211	0	0	0	0	0	0
N.S.	1	0.98	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.491	0.534	0.000	0.000	0.000	0.000	0.000	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	210	210	183	0	0	0	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.380	0.355	0.000	0.000	0.000	0.000	0.000	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	198	203	168	0	0	0	0	0	0
N.S.	1	1.03	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.376	0.319	0.000	0.000	0.000	0.000	0.000	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	262	258	218	0	0	0	0	0	0
N.S.	1	0.98	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.498	0.423	0.000	0.000	0.000	0.000	0.000	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	322	316	259	0	0	0	0	0	0
N.S.	1	0.98	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.583	0.454	0.000	0.000	0.000	0.000	0.000	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	339	339	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.656	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	273	273	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.535	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	162	162	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.337	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	190	199	0	0	0	0	0	0	0
N.S.	1	1.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.298	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	264	264	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.549	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	328	328	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.605	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	44	58	101	0	120	0	42	0
N.S.	1	1.10	1.45	2.52	0.00	3.00	0.00	1.05	0.00
time (sec)	N/A	0.226	0.869	0.382	0.000	0.247	0.000	0.301	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [252] had the largest ratio of [.7272729999999999947]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.01	20	0.150
2	A	2	2	1.00	20	0.100
3	A	5	4	0.99	18	0.222
4	A	2	2	1.00	17	0.118
5	A	4	3	1.03	20	0.150
6	A	2	2	1.00	20	0.100
7	A	4	3	1.02	20	0.150
8	A	9	8	1.33	20	0.400
9	A	6	5	1.41	20	0.250
10	A	5	4	1.07	18	0.222
11	A	8	7	1.12	20	0.350
12	A	8	7	1.03	20	0.350
13	A	9	8	0.97	20	0.400
14	A	7	6	1.09	20	0.300
15	A	10	10	1.07	20	0.500
16	A	7	7	1.03	20	0.350
17	A	6	6	1.04	17	0.353
18	A	6	6	1.06	20	0.300
19	A	8	8	1.04	20	0.400
20	A	10	9	1.37	20	0.450
21	A	8	7	1.39	20	0.350
22	A	6	5	1.20	18	0.278

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	9	8	1.15	20	0.400
24	A	10	9	0.96	20	0.450
25	A	10	9	1.03	20	0.450
26	A	10	9	1.09	20	0.450
27	A	11	11	1.07	20	0.550
28	A	8	8	1.03	20	0.400
29	A	8	8	1.05	17	0.471
30	A	8	8	1.05	20	0.400
31	A	8	8	1.05	20	0.400
32	A	8	7	1.36	20	0.350
33	A	8	7	1.24	20	0.350
34	A	5	4	1.40	20	0.200
35	A	4	3	1.17	18	0.167
36	A	7	6	1.05	20	0.300
37	A	6	5	1.05	20	0.250
38	A	8	7	1.07	20	0.350
39	A	8	8	1.04	20	0.400
40	A	6	6	1.01	20	0.300
41	A	4	4	1.01	17	0.235
42	A	6	6	1.03	20	0.300
43	A	8	8	1.02	20	0.400
44	A	7	6	1.16	20	0.300
45	A	6	5	1.00	20	0.250
46	A	5	4	1.00	20	0.200
47	A	3	2	1.00	18	0.111
48	A	7	6	1.09	20	0.300
49	A	8	7	1.06	20	0.350
50	A	8	8	1.04	20	0.400
51	A	5	5	1.03	20	0.250
52	A	6	6	1.02	17	0.353
53	A	8	8	1.05	20	0.400
54	A	9	9	1.06	20	0.450
55	A	3	3	1.00	25	0.120
56	A	5	4	1.06	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	3	3	1.00	23	0.130
58	A	5	4	1.09	23	0.174
59	A	3	3	1.00	23	0.130
60	A	5	4	1.14	21	0.190
61	A	3	3	1.00	20	0.150
62	A	6	5	0.95	23	0.217
63	A	3	3	1.00	23	0.130
64	A	5	4	1.00	23	0.174
65	A	3	3	1.00	23	0.130
66	A	5	4	1.12	21	0.190
67	A	3	3	1.00	21	0.143
68	A	5	4	1.17	21	0.190
69	A	3	3	1.00	21	0.143
70	A	2	2	1.00	19	0.105
71	A	3	3	1.00	18	0.167
72	A	5	4	1.00	21	0.190
73	A	3	3	1.00	21	0.143
74	A	5	4	1.00	21	0.190
75	A	5	5	0.59	33	0.152
76	A	6	5	0.75	31	0.161
77	A	4	4	0.71	30	0.133
78	A	6	5	0.73	33	0.152
79	A	4	4	0.72	33	0.121
80	A	6	5	0.58	33	0.152
81	A	6	6	0.81	33	0.182
82	A	5	4	0.70	31	0.129
83	A	5	5	0.78	30	0.167
84	A	6	5	0.61	33	0.152
85	A	9	9	0.73	33	0.273
86	A	6	5	0.58	33	0.152
87	A	4	4	0.54	35	0.114
88	A	4	4	0.55	35	0.114
89	A	4	4	0.60	35	0.114
90	A	4	4	0.79	35	0.114

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	A	4	4	0.86	35	0.114
92	A	2	2	1.44	29	0.069
93	A	5	4	1.00	31	0.129
94	A	5	4	0.90	31	0.129
95	A	4	3	1.02	25	0.120
96	A	2	2	1.00	25	0.080
97	A	4	3	0.99	23	0.130
98	A	2	2	1.00	22	0.091
99	A	4	3	1.02	25	0.120
100	A	2	2	1.00	25	0.080
101	A	4	3	1.01	25	0.120
102	A	4	3	0.99	25	0.120
103	A	4	3	0.99	25	0.120
104	A	6	5	1.01	23	0.217
105	A	4	3	1.04	25	0.120
106	A	4	3	1.02	25	0.120
107	A	5	5	1.04	25	0.200
108	A	3	3	1.02	25	0.120
109	A	2	2	1.00	22	0.091
110	A	3	3	0.99	25	0.120
111	F	0	0	N/A	0.000	N/A
112	A	5	4	1.04	25	0.160
113	A	7	6	1.15	25	0.240
114	A	5	4	1.02	25	0.160
115	A	5	4	1.02	23	0.174
116	A	6	5	1.20	25	0.200
117	F	0	0	N/A	0.000	N/A
118	A	6	6	0.94	25	0.240
119	A	5	5	0.95	25	0.200
120	A	3	3	0.96	25	0.120
121	A	4	4	0.95	22	0.182
122	A	6	6	0.97	25	0.240
123	A	8	8	0.99	25	0.320

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
124	A	7	6	1.07	25	0.240
125	A	9	8	1.17	25	0.320
126	A	6	5	1.05	25	0.200
127	A	7	6	1.10	25	0.240
128	A	6	5	1.01	25	0.200
129	A	6	5	1.06	23	0.217
130	A	8	7	1.21	25	0.280
131	A	8	7	1.12	25	0.280
132	A	7	7	0.98	25	0.280
133	A	6	6	0.99	25	0.240
134	A	6	6	1.02	25	0.240
135	A	5	5	0.92	25	0.200
136	A	6	6	1.01	22	0.273
137	A	5	4	0.92	21	0.190
138	A	6	5	0.92	22	0.227
139	A	6	5	1.03	17	0.294
140	A	7	6	1.03	18	0.333
141	A	6	5	1.20	22	0.227
142	A	8	7	1.14	25	0.280
143	A	6	5	1.11	25	0.200
144	A	6	5	1.07	23	0.217
145	A	9	8	1.05	25	0.320
146	A	9	8	1.05	25	0.320
147	A	9	8	1.05	25	0.320
148	A	6	5	1.10	25	0.200
149	A	8	7	1.13	25	0.280
150	A	10	9	1.14	25	0.360
151	A	7	7	1.07	25	0.280
152	A	6	6	1.04	25	0.240
153	A	4	4	1.05	22	0.182
154	A	5	5	1.01	25	0.200
155	A	7	7	1.05	25	0.280
156	A	9	8	1.15	25	0.320
157	A	7	6	1.13	25	0.240

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
158	A	7	6	1.14	23	0.261
159	A	11	10	1.10	25	0.400
160	A	11	10	1.08	25	0.400
161	A	11	10	1.06	25	0.400
162	A	11	10	1.06	25	0.400
163	A	9	9	1.05	25	0.360
164	A	8	8	1.06	25	0.320
165	A	6	6	1.02	22	0.273
166	A	6	6	1.04	25	0.240
167	A	7	7	1.04	25	0.280
168	A	9	9	1.07	25	0.360
169	A	7	6	1.08	27	0.222
170	A	5	4	1.04	27	0.148
171	A	5	4	1.01	25	0.160
172	A	7	6	0.99	27	0.222
173	A	5	4	1.02	27	0.148
174	A	7	6	1.08	27	0.222
175	A	9	8	1.10	27	0.296
176	A	6	6	0.99	27	0.222
177	A	5	5	0.99	27	0.185
178	A	4	4	1.01	24	0.167
179	A	6	6	1.01	27	0.222
180	A	8	8	0.99	27	0.296
181	A	9	8	1.14	25	0.320
182	A	7	6	1.08	25	0.240
183	A	5	4	1.07	25	0.160
184	A	5	4	1.04	23	0.174
185	A	7	6	1.03	25	0.240
186	A	5	4	1.06	25	0.160
187	A	7	6	1.08	25	0.240
188	A	9	8	1.12	25	0.320
189	A	5	5	1.04	25	0.200
190	A	5	5	1.03	25	0.200
191	A	3	3	1.01	22	0.136

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
192	A	5	5	1.03	25	0.200
193	A	6	6	1.03	25	0.240
194	A	7	6	1.09	25	0.240
195	A	5	4	1.04	25	0.160
196	A	3	2	1.00	23	0.087
197	A	6	5	1.06	25	0.200
198	A	7	6	1.08	25	0.240
199	A	6	6	1.04	25	0.240
200	A	5	5	1.02	25	0.200
201	A	5	5	1.04	22	0.227
202	A	6	6	1.03	25	0.240
203	A	8	8	1.06	25	0.320
204	A	2	2	1.00	31	0.065
205	A	2	2	1.00	31	0.065
206	A	2	2	1.00	31	0.065
207	A	2	2	1.00	31	0.065
208	A	2	2	1.00	31	0.065
209	A	2	2	1.00	31	0.065
210	A	2	2	1.00	31	0.065
211	A	2	2	1.00	31	0.065
212	A	2	2	1.00	31	0.065
213	A	2	2	1.00	31	0.065
214	A	2	2	1.00	31	0.065
215	A	2	2	1.00	31	0.065
216	A	2	2	1.00	31	0.065
217	A	2	2	1.00	31	0.065
218	A	2	2	1.00	31	0.065
219	A	2	2	1.00	31	0.065
220	A	2	2	1.00	27	0.074
221	A	2	2	1.00	27	0.074
222	A	2	2	1.00	25	0.080
223	A	3	3	1.00	27	0.111
224	F	0	0	N/A	0.000	N/A

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
225	A	2	2	1.00	29	0.069
226	A	2	2	1.00	29	0.069
227	A	2	2	1.00	29	0.069
228	A	2	2	1.00	29	0.069
229	A	6	5	1.10	22	0.227
230	A	6	5	1.08	22	0.227
231	A	4	3	0.99	22	0.136
232	A	7	6	0.88	22	0.273
233	A	6	5	0.91	20	0.250
234	A	4	3	1.02	22	0.136
235	A	4	3	0.99	22	0.136
236	A	4	3	0.99	22	0.136
237	A	2	2	1.00	22	0.091
238	A	2	2	1.00	22	0.091
239	A	2	2	1.00	22	0.091
240	A	2	2	1.00	22	0.091
241	A	2	2	1.00	19	0.105
242	A	2	2	1.00	22	0.091
243	A	2	2	1.00	22	0.091
244	A	5	4	1.05	22	0.182
245	A	7	6	1.13	22	0.273
246	A	7	6	1.06	22	0.273
247	A	6	5	1.07	22	0.227
248	A	6	5	1.13	20	0.250
249	A	4	3	1.01	22	0.136
250	A	4	3	1.00	22	0.136
251	A	4	3	0.99	22	0.136
252	A	17	16	0.92	22	0.727
253	A	15	14	0.93	22	0.636
254	A	15	14	0.93	22	0.636
255	A	15	14	0.92	22	0.636
256	A	2	2	1.00	19	0.105
257	A	2	2	1.00	22	0.091
258	A	2	2	1.00	22	0.091

Continued on next page

2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
259	A	5	4	1.00	20	0.200
260	A	5	4	1.00	22	0.182
261	A	6	6	0.43	22	0.273
262	A	3	3	0.56	24	0.125
263	A	1	1	0.59	20	0.050
264	A	1	1	1.74	22	0.045
265	A	5	4	1.00	22	0.182
266	A	5	4	1.00	24	0.167
267	A	8	7	0.59	37	0.189
268	A	6	5	0.75	35	0.143
269	A	7	6	0.64	34	0.176
270	A	8	7	0.63	37	0.189
271	A	7	6	0.63	37	0.162
272	A	8	7	0.62	37	0.189
273	A	4	3	1.05	25	0.120
274	A	2	2	1.00	25	0.080
275	A	4	3	1.05	23	0.130
276	A	2	2	1.00	22	0.091
277	A	4	3	1.04	25	0.120
278	A	2	2	1.00	25	0.080
279	A	4	3	1.01	25	0.120
280	A	4	4	1.00	25	0.160
281	A	3	3	1.01	25	0.120
282	A	4	4	1.00	25	0.160
283	A	4	4	1.13	22	0.182
284	A	5	5	1.03	25	0.200
285	A	3	3	1.08	25	0.120
286	A	4	4	1.10	25	0.160
287	A	3	3	1.09	25	0.120
288	A	5	5	1.05	25	0.200
289	A	4	4	1.05	25	0.160
290	A	5	5	1.07	25	0.200
291	A	5	5	1.09	22	0.227
292	A	7	7	1.06	25	0.280

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
293	A	4	4	1.20	25	0.160
294	A	6	6	1.13	25	0.240
295	A	4	3	0.99	27	0.111
296	A	4	3	0.98	27	0.111
297	A	4	3	0.99	27	0.111
298	A	4	3	0.99	27	0.111
299	A	7	6	0.87	25	0.240
300	A	4	3	1.01	27	0.111
301	A	4	3	1.00	27	0.111
302	A	4	3	0.99	27	0.111
303	A	2	2	1.00	27	0.074
304	A	2	2	1.00	27	0.074
305	A	2	2	1.00	27	0.074
306	A	2	2	1.00	27	0.074
307	A	2	2	1.00	24	0.083
308	A	2	2	1.00	27	0.074
309	A	2	2	1.00	27	0.074
310	A	5	4	1.01	31	0.129
311	A	11	10	1.09	29	0.345
312	A	9	8	1.05	29	0.276
313	A	8	7	1.02	27	0.259
314	A	11	10	1.00	29	0.345
315	A	4	3	0.98	29	0.103
316	A	10	10	1.03	29	0.345
317	A	8	8	0.98	29	0.276
318	A	6	6	1.05	26	0.231
319	A	8	8	1.00	29	0.276
320	A	6	6	0.86	29	0.207
321	A	13	13	0.98	29	0.448
322	A	13	12	1.07	29	0.414
323	A	11	10	1.07	29	0.345
324	A	10	9	1.07	27	0.333
325	A	13	12	0.98	29	0.414
326	A	4	3	0.99	29	0.103

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
327	A	13	13	0.98	29	0.448
328	A	11	11	1.00	26	0.423
329	A	11	11	0.59	29	0.379
330	A	13	13	0.75	29	0.448
331	A	14	14	0.97	29	0.483
332	A	9	8	1.05	29	0.276
333	A	7	6	0.99	29	0.207
334	A	4	3	1.00	27	0.111
335	A	4	3	1.00	29	0.103
336	A	4	3	0.98	29	0.103
337	A	4	4	0.98	29	0.138
338	A	3	3	1.09	29	0.103
339	A	3	3	1.09	26	0.115
340	A	8	8	1.07	29	0.276
341	A	10	10	1.05	29	0.345
342	A	9	8	1.14	29	0.276
343	A	6	5	1.01	29	0.172
344	A	6	5	1.02	29	0.172
345	A	6	5	1.01	27	0.185
346	A	4	3	1.01	29	0.103
347	A	4	3	0.99	29	0.103
348	A	8	8	1.02	29	0.276
349	A	8	8	1.10	29	0.276
350	A	8	8	1.08	29	0.276
351	A	8	8	1.08	29	0.276
352	A	8	8	1.09	26	0.308
353	A	9	9	1.04	29	0.310
354	A	4	3	0.98	29	0.103
355	A	4	3	0.99	29	0.103
356	A	7	6	1.04	29	0.207
357	A	5	4	1.17	27	0.148
358	A	4	3	1.02	29	0.103
359	A	4	3	1.02	29	0.103
360	A	4	3	1.06	29	0.103

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
361	A	7	6	0.99	29	0.207
362	A	6	5	1.01	29	0.172
363	A	7	6	1.34	26	0.231
364	A	4	4	0.99	29	0.138
365	A	5	5	1.03	29	0.172
366	A	6	6	1.02	29	0.207
367	A	8	7	0.87	29	0.241
368	A	8	7	0.95	27	0.259
369	A	4	3	1.02	29	0.103
370	A	4	3	1.04	29	0.103
371	A	8	7	1.00	29	0.241
372	A	7	6	1.01	29	0.207
373	A	8	7	1.07	26	0.269
374	A	7	6	1.63	29	0.207
375	A	4	4	1.01	29	0.138
376	A	4	3	0.97	29	0.103
377	A	7	6	1.05	29	0.207
378	A	5	4	1.09	27	0.148
379	A	4	3	0.95	29	0.103
380	A	4	3	1.00	29	0.103
381	A	5	5	0.99	29	0.172
382	A	5	5	1.00	29	0.172
383	A	6	5	1.17	26	0.192
384	A	4	4	0.99	29	0.138
385	A	5	5	1.00	25	0.200
386	A	2	2	1.00	29	0.069
387	A	2	2	1.00	29	0.069
388	A	2	2	1.00	29	0.069
389	A	2	2	1.00	29	0.069
390	A	4	3	1.00	26	0.115
391	A	2	2	1.00	29	0.069
392	A	2	2	1.00	29	0.069
393	A	2	2	1.00	29	0.069
394	A	7	6	1.30	29	0.207

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
395	A	4	4	0.95	29	0.138
396	A	4	4	0.95	29	0.138
397	A	4	4	0.95	26	0.154
398	A	5	5	1.21	29	0.172
399	A	6	6	1.30	29	0.207
400	A	3	3	1.00	29	0.103
401	A	4	3	0.98	27	0.111
402	A	4	3	0.98	27	0.111
403	A	4	3	1.00	27	0.111
404	A	4	3	1.03	25	0.120
405	A	4	3	0.98	27	0.111
406	A	4	3	0.98	27	0.111
407	A	2	2	1.00	27	0.074
408	A	2	2	1.00	27	0.074
409	A	2	2	1.00	27	0.074
410	A	3	3	1.05	24	0.125
411	A	2	2	1.00	27	0.074
412	A	2	2	1.00	27	0.074
413	A	6	5	1.10	28	0.179

CHAPTER 3

Listing of integrals

3.1 $\int x^3(d+ex^2)(a+cx^4)^5\,dx$

3.1.1 Optimal result

Integrand size = 20, antiderivative size = 149

$$\int x^3(d+ex^2)\left(a+cx^4\right)^5\,dx = \frac{1}{4}a^5dx^4+\frac{1}{6}a^5ex^6+\frac{5}{8}a^4cdx^8+\frac{1}{2}a^4cex^{10}+\frac{5}{6}a^3c^2dx^{12}+\frac{5}{7}a^3c^2ex^{14}+\frac{5}{8}a^2c^3dx^{16}+\frac{5}{9}a^2c^3ex^{18}+\frac{1}{4}ac^4dx^{20}+\frac{5}{22}ac^4ex^{22}+\frac{1}{24}c^5dx^{24}+\frac{1}{26}c^5ex^{26}$$

output
```
1/4*a^5*d*x^4+1/6*a^5*e*x^6+5/8*a^4*c*d*x^8+1/2*a^4*c*e*x^10+5/6*a^3*c^2*d
*x^12+5/7*a^3*c^2*e*x^14+5/8*a^2*c^3*d*x^16+5/9*a^2*c^3*e*x^18+1/4*a*c^4*d
*x^20+5/22*a*c^4*e*x^22+1/24*c^5*d*x^24+1/26*c^5*e*x^26
```

3.1.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00

$$\int x^3(d+ex^2)\left(a+cx^4\right)^5\,dx = \frac{1}{4}a^5dx^4+\frac{1}{6}a^5ex^6+\frac{5}{8}a^4cdx^8+\frac{1}{2}a^4cex^{10}+\frac{5}{6}a^3c^2dx^{12}+\frac{5}{7}a^3c^2ex^{14}+\frac{5}{8}a^2c^3dx^{16}+\frac{5}{9}a^2c^3ex^{18}+\frac{1}{4}ac^4dx^{20}+\frac{5}{22}ac^4ex^{22}+\frac{1}{24}c^5dx^{24}+\frac{1}{26}c^5ex^{26}$$

input
```
Integrate[x^3*(d + e*x^2)*(a + c*x^4)^5,x]
```

output
```
(a^5*d*x^4)/4 + (a^5*e*x^6)/6 + (5*a^4*c*d*x^8)/8 + (a^4*c*e*x^10)/2 + (5*
a^3*c^2*d*x^12)/6 + (5*a^3*c^2*e*x^14)/7 + (5*a^2*c^3*d*x^16)/8 + (5*a^2*c
^3*e*x^18)/9 + (a*c^4*d*x^20)/4 + (5*a*c^4*e*x^22)/22 + (c^5*d*x^24)/24 +
(c^5*e*x^26)/26
```

3.1.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1579, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+cx^4\right)^5 \left(d+ex^2\right) \, dx$$

$$\Big\downarrow \text{ 1579}$$

$$\frac{1}{2} \int x^2 \left(ex^2+d\right) \left(cx^4+a\right)^5 dx^2$$

$$\Big\downarrow \text{ 522}$$

$$\frac{1}{2} \int \left(c^5ex^{24}+c^5dx^{22}+5ac^4ex^{20}+5ac^4dx^{18}+10a^2c^3ex^{16}+10a^2c^3dx^{14}+10a^3c^2ex^{12}+10a^3c^2dx^{10}+5a^4cex^8+\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{1}{2}a^5dx^4+\frac{1}{3}a^5ex^6+\frac{5}{4}a^4cdx^8+a^4cex^{10}+\frac{5}{3}a^3c^2dx^{12}+\frac{10}{7}a^3c^2ex^{14}+\frac{5}{4}a^2c^3dx^{16}+\frac{10}{9}a^2c^3ex^{18}+\frac{1}{2}ac^4dx^{20}+\right.$$

input
```
Int[x^3*(d + e*x^2)*(a + c*x^4)^5,x]
```

output
```
((a^5*d*x^4)/2 + (a^5*e*x^6)/3 + (5*a^4*c*d*x^8)/4 + a^4*c*e*x^10 + (5*a^3
*c^2*d*x^12)/3 + (10*a^3*c^2*e*x^14)/7 + (5*a^2*c^3*d*x^16)/4 + (10*a^2*c^
3*e*x^18)/9 + (a*c^4*d*x^20)/2 + (5*a*c^4*e*x^22)/11 + (c^5*d*x^24)/12 + (
c^5*e*x^26)/13)/2
```

3.1.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.85

method	result
gosper	$\frac{1}{4}a^5d\,x^4+\frac{1}{6}a^5e\,x^6+\frac{5}{8}a^4cd\,x^8+\frac{1}{2}a^4ce\,x^{10}+\frac{5}{6}a^3c^2d\,x^{12}+\frac{5}{7}a^3c^2e\,x^{14}+\frac{5}{8}a^2c^3d\,x^{16}+\frac{5}{9}a^2c^3e\,x$
default	$\frac{1}{4}a^5d\,x^4+\frac{1}{6}a^5e\,x^6+\frac{5}{8}a^4cd\,x^8+\frac{1}{2}a^4ce\,x^{10}+\frac{5}{6}a^3c^2d\,x^{12}+\frac{5}{7}a^3c^2e\,x^{14}+\frac{5}{8}a^2c^3d\,x^{16}+\frac{5}{9}a^2c^3e\,x$
norman	$\frac{1}{4}a^5d\,x^4+\frac{1}{6}a^5e\,x^6+\frac{5}{8}a^4cd\,x^8+\frac{1}{2}a^4ce\,x^{10}+\frac{5}{6}a^3c^2d\,x^{12}+\frac{5}{7}a^3c^2e\,x^{14}+\frac{5}{8}a^2c^3d\,x^{16}+\frac{5}{9}a^2c^3e\,x$
risch	$\frac{1}{4}a^5d\,x^4+\frac{1}{6}a^5e\,x^6+\frac{5}{8}a^4cd\,x^8+\frac{1}{2}a^4ce\,x^{10}+\frac{5}{6}a^3c^2d\,x^{12}+\frac{5}{7}a^3c^2e\,x^{14}+\frac{5}{8}a^2c^3d\,x^{16}+\frac{5}{9}a^2c^3e\,x$
parallelrisch	$\frac{1}{4}a^5d\,x^4+\frac{1}{6}a^5e\,x^6+\frac{5}{8}a^4cd\,x^8+\frac{1}{2}a^4ce\,x^{10}+\frac{5}{6}a^3c^2d\,x^{12}+\frac{5}{7}a^3c^2e\,x^{14}+\frac{5}{8}a^2c^3d\,x^{16}+\frac{5}{9}a^2c^3e\,x$

input
```
int(x^3*(e*x^2+d)*(c*x^4+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^5*d*x^4+1/6*a^5*e*x^6+5/8*a^4*c*d*x^8+1/2*a^4*c*e*x^10+5/6*a^3*c^2*d
*x^12+5/7*a^3*c^2*e*x^14+5/8*a^2*c^3*d*x^16+5/9*a^2*c^3*e*x^18+1/4*a*c^4*d
*x^20+5/22*a*c^4*e*x^22+1/24*c^5*d*x^24+1/26*c^5*e*x^26
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.84

$$\int x^3(d+ex^2)\left(a+cx^4\right)^5\,dx = \frac{1}{26}c^5ex^{26}+\frac{1}{24}c^5dx^{24}+\frac{5}{22}ac^4ex^{22}+\frac{1}{4}ac^4dx^{20}+\frac{5}{9}a^2c^3ex^{18}+\frac{5}{8}a^2c^3dx^{16}+\frac{5}{7}a^3c^2ex^{14}+\frac{5}{6}a^3c^2dx^{12}+\frac{1}{2}a^4cex^{10}+\frac{5}{8}a^4cdx^8+\frac{1}{6}a^5ex^6+\frac{1}{4}a^5dx^4$$

input
```
integrate(x^3*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="fricas")
```

output
```
1/26*c^5*e*x^26 + 1/24*c^5*d*x^24 + 5/22*a*c^4*e*x^22 + 1/4*a*c^4*d*x^20 +
 5/9*a^2*c^3*e*x^18 + 5/8*a^2*c^3*d*x^16 + 5/7*a^3*c^2*e*x^14 + 5/6*a^3*c^
2*d*x^12 + 1/2*a^4*c*e*x^10 + 5/8*a^4*c*d*x^8 + 1/6*a^5*e*x^6 + 1/4*a^5*d*
x^4
```

3.1.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.01

$$\int x^3(d+ex^2)\left(a+cx^4\right)^5\,dx = \frac{a^5dx^4}{4}+\frac{a^5ex^6}{6}+\frac{5a^4cdx^8}{8}+\frac{a^4cex^{10}}{2}+\frac{5a^3c^2dx^{12}}{6}+\frac{5a^3c^2ex^{14}}{7}+\frac{5a^2c^3dx^{16}}{8}+\frac{5a^2c^3ex^{18}}{9}+\frac{ac^4dx^{20}}{4}+\frac{5ac^4ex^{22}}{22}+\frac{c^5dx^{24}}{24}+\frac{c^5ex^{26}}{26}$$

input
```
integrate(x**3*(e*x**2+d)*(c*x**4+a)**5,x)
```

output
```
a**5*d*x**4/4 + a**5*e*x**6/6 + 5*a**4*c*d*x**8/8 + a**4*c*e*x**10/2 + 5*a
**3*c**2*d*x**12/6 + 5*a**3*c**2*e*x**14/7 + 5*a**2*c**3*d*x**16/8 + 5*a**
2*c**3*e*x**18/9 + a*c**4*d*x**20/4 + 5*a*c**4*e*x**22/22 + c**5*d*x**24/2
4 + c**5*e*x**26/26
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.84

$$\int x^3 (d+ex^2) (a+cx^4)^5 \, dx = \frac{1}{26} c^5 e x^{26} + \frac{1}{24} c^5 d x^{24} + \frac{5}{22} a c^4 e x^{22} + \frac{1}{4} a c^4 d x^{20} + \frac{5}{9} a^2 c^3 e x^{18} + \frac{5}{8} a^2 c^3 d x^{16} + \frac{5}{7} a^3 c^2 e x^{14} + \frac{5}{6} a^3 c^2 d x^{12} + \frac{1}{2} a^4 c e x^{10} + \frac{5}{8} a^4 c d x^8 + \frac{1}{6} a^5 e x^6 + \frac{1}{4} a^5 d x^4$$

input
```
integrate(x^3*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="maxima")
```

output
```
1/26*c^5*e*x^26 + 1/24*c^5*d*x^24 + 5/22*a*c^4*e*x^22 + 1/4*a*c^4*d*x^20 +
 5/9*a^2*c^3*e*x^18 + 5/8*a^2*c^3*d*x^16 + 5/7*a^3*c^2*e*x^14 + 5/6*a^3*c^
2*d*x^12 + 1/2*a^4*c*e*x^10 + 5/8*a^4*c*d*x^8 + 1/6*a^5*e*x^6 + 1/4*a^5*d*
x^4
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.84

$$\int x^3 (d+ex^2) (a+cx^4)^5 \, dx = \frac{1}{26} c^5 e x^{26} + \frac{1}{24} c^5 d x^{24} + \frac{5}{22} a c^4 e x^{22} + \frac{1}{4} a c^4 d x^{20} + \frac{5}{9} a^2 c^3 e x^{18} + \frac{5}{8} a^2 c^3 d x^{16} + \frac{5}{7} a^3 c^2 e x^{14} + \frac{5}{6} a^3 c^2 d x^{12} + \frac{1}{2} a^4 c e x^{10} + \frac{5}{8} a^4 c d x^8 + \frac{1}{6} a^5 e x^6 + \frac{1}{4} a^5 d x^4$$

input
```
integrate(x^3*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="giac")
```

output
```
1/26*c^5*e*x^26 + 1/24*c^5*d*x^24 + 5/22*a*c^4*e*x^22 + 1/4*a*c^4*d*x^20 +
 5/9*a^2*c^3*e*x^18 + 5/8*a^2*c^3*d*x^16 + 5/7*a^3*c^2*e*x^14 + 5/6*a^3*c^
2*d*x^12 + 1/2*a^4*c*e*x^10 + 5/8*a^4*c*d*x^8 + 1/6*a^5*e*x^6 + 1/4*a^5*d*
x^4
```

3.1.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.84

$$\int x^3(d+ex^2)\left(a+cx^4\right)^5\,dx = \frac{e\,a^5\,x^6}{6}+\frac{d\,a^5\,x^4}{4}+\frac{e\,a^4\,c\,x^{10}}{2}+\frac{5\,d\,a^4\,c\,x^8}{8}+\frac{5\,e\,a^3\,c^2\,x^{14}}{7}+\frac{5\,d\,a^3\,c^2\,x^{12}}{6}+\frac{5\,e\,a^2\,c^3\,x^{18}}{9}+\frac{5\,d\,a^2\,c^3\,x^{16}}{8}+\frac{5\,e\,a\,c^4\,x^{22}}{22}+\frac{d\,a\,c^4\,x^{20}}{4}+\frac{e\,c^5\,x^{26}}{26}+\frac{d\,c^5\,x^{24}}{24}$$

input
```
int(x^3*(a + c*x^4)^5*(d + e*x^2),x)
```

output
```
(a^5*d*x^4)/4 + (a^5*e*x^6)/6 + (c^5*d*x^24)/24 + (c^5*e*x^26)/26 + (5*a^3
*c^2*d*x^12)/6 + (5*a^2*c^3*d*x^16)/8 + (5*a^3*c^2*e*x^14)/7 + (5*a^2*c^3*
e*x^18)/9 + (5*a^4*c*d*x^8)/8 + (a*c^4*d*x^20)/4 + (a^4*c*e*x^10)/2 + (5*a
*c^4*e*x^22)/22
```

3.2 $\int x^2(d+ex^2)(a+cx^4)^5\,dx$

3.2.1 Optimal result

Integrand size = 20, antiderivative size = 149

$$\int x^2(d+ex^2)\left(a+cx^4\right)^5\,dx=\frac{1}{3}a^5dx^3+\frac{1}{5}a^5ex^5+\frac{5}{7}a^4cdx^7+\frac{5}{9}a^4cex^9+\frac{10}{11}a^3c^2dx^{11}+\frac{10}{13}a^3c^2ex^{13}+\frac{2}{3}a^2c^3dx^{15}+\frac{10}{17}a^2c^3ex^{17}+\frac{5}{19}ac^4dx^{19}+\frac{5}{21}ac^4ex^{21}+\frac{1}{23}c^5dx^{23}+\frac{1}{25}c^5ex^{25}$$

output
```
1/3*a^5*d*x^3+1/5*a^5*e*x^5+5/7*a^4*c*d*x^7+5/9*a^4*c*e*x^9+10/11*a^3*c^2*
d*x^11+10/13*a^3*c^2*e*x^13+2/3*a^2*c^3*d*x^15+10/17*a^2*c^3*e*x^17+5/19*a
*c^4*d*x^19+5/21*a*c^4*e*x^21+1/23*c^5*d*x^23+1/25*c^5*e*x^25
```

3.2.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00

$$\int x^2(d+ex^2)\left(a+cx^4\right)^5\,dx=\frac{1}{3}a^5dx^3+\frac{1}{5}a^5ex^5+\frac{5}{7}a^4cdx^7+\frac{5}{9}a^4cex^9+\frac{10}{11}a^3c^2dx^{11}+\frac{10}{13}a^3c^2ex^{13}+\frac{2}{3}a^2c^3dx^{15}+\frac{10}{17}a^2c^3ex^{17}+\frac{5}{19}ac^4dx^{19}+\frac{5}{21}ac^4ex^{21}+\frac{1}{23}c^5dx^{23}+\frac{1}{25}c^5ex^{25}$$

input
```
Integrate[x^2*(d + e*x^2)*(a + c*x^4)^5,x]
```

output
```
(a^5*d*x^3)/3 + (a^5*e*x^5)/5 + (5*a^4*c*d*x^7)/7 + (5*a^4*c*e*x^9)/9 + (1
0*a^3*c^2*d*x^11)/11 + (10*a^3*c^2*e*x^13)/13 + (2*a^2*c^3*d*x^15)/3 + (10
*a^2*c^3*e*x^17)/17 + (5*a*c^4*d*x^19)/19 + (5*a*c^4*e*x^21)/21 + (c^5*d*x
^23)/23 + (c^5*e*x^25)/25
```

3.2.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1585, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+cx^4\right)^5 \left(d+ex^2\right) dx$$

$$\Big\downarrow \text{1585}$$

$$\int \left(a^5dx^2+a^5ex^4+5a^4cdx^6+5a^4cex^8+10a^3c^2dx^{10}+10a^3c^2ex^{12}+10a^2c^3dx^{14}+10a^2c^3ex^{16}+5ac^4dx^{18}+5ac\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}a^5dx^3+\frac{1}{5}a^5ex^5+\frac{5}{7}a^4cdx^7+\frac{5}{9}a^4cex^9+\frac{10}{11}a^3c^2dx^{11}+\frac{10}{13}a^3c^2ex^{13}+\frac{2}{3}a^2c^3dx^{15}+\frac{10}{17}a^2c^3ex^{17}+\frac{5}{19}ac^4dx^{19}+\frac{5}{21}ac^4ex^{21}+\frac{1}{23}c^5dx^{23}+\frac{1}{25}c^5ex^{25}$$

input
```
Int[x^2*(d + e*x^2)*(a + c*x^4)^5,x]
```

output
```
(a^5*d*x^3)/3 + (a^5*e*x^5)/5 + (5*a^4*c*d*x^7)/7 + (5*a^4*c*e*x^9)/9 + (1
0*a^3*c^2*d*x^11)/11 + (10*a^3*c^2*e*x^13)/13 + (2*a^2*c^3*d*x^15)/3 + (10
*a^2*c^3*e*x^17)/17 + (5*a*c^4*d*x^19)/19 + (5*a*c^4*e*x^21)/21 + (c^5*d*x
^23)/23 + (c^5*e*x^25)/25
```

3.2.3.1 Defintions of rubi rules used

rule 1585
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.2.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.85

method	result
gosper	$\frac{1}{3}a^5d\,x^3+\frac{1}{5}a^5e\,x^5+\frac{5}{7}a^4cd\,x^7+\frac{5}{9}a^4ce\,x^9+\frac{10}{11}a^3c^2d\,x^{11}+\frac{10}{13}a^3c^2e\,x^{13}+\frac{2}{3}a^2c^3d\,x^{15}+\frac{10}{17}a^2c^3e$
default	$\frac{1}{3}a^5d\,x^3+\frac{1}{5}a^5e\,x^5+\frac{5}{7}a^4cd\,x^7+\frac{5}{9}a^4ce\,x^9+\frac{10}{11}a^3c^2d\,x^{11}+\frac{10}{13}a^3c^2e\,x^{13}+\frac{2}{3}a^2c^3d\,x^{15}+\frac{10}{17}a^2c^3e$
norman	$\frac{1}{3}a^5d\,x^3+\frac{1}{5}a^5e\,x^5+\frac{5}{7}a^4cd\,x^7+\frac{5}{9}a^4ce\,x^9+\frac{10}{11}a^3c^2d\,x^{11}+\frac{10}{13}a^3c^2e\,x^{13}+\frac{2}{3}a^2c^3d\,x^{15}+\frac{10}{17}a^2c^3e$
risch	$\frac{1}{3}a^5d\,x^3+\frac{1}{5}a^5e\,x^5+\frac{5}{7}a^4cd\,x^7+\frac{5}{9}a^4ce\,x^9+\frac{10}{11}a^3c^2d\,x^{11}+\frac{10}{13}a^3c^2e\,x^{13}+\frac{2}{3}a^2c^3d\,x^{15}+\frac{10}{17}a^2c^3e$
parallelrisch	$\frac{1}{3}a^5d\,x^3+\frac{1}{5}a^5e\,x^5+\frac{5}{7}a^4cd\,x^7+\frac{5}{9}a^4ce\,x^9+\frac{10}{11}a^3c^2d\,x^{11}+\frac{10}{13}a^3c^2e\,x^{13}+\frac{2}{3}a^2c^3d\,x^{15}+\frac{10}{17}a^2c^3e$

input
```
int(x^2*(e*x^2+d)*(c*x^4+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^5*d*x^3+1/5*a^5*e*x^5+5/7*a^4*c*d*x^7+5/9*a^4*c*e*x^9+10/11*a^3*c^2*
d*x^11+10/13*a^3*c^2*e*x^13+2/3*a^2*c^3*d*x^15+10/17*a^2*c^3*e*x^17+5/19*a
*c^4*d*x^19+5/21*a*c^4*e*x^21+1/23*c^5*d*x^23+1/25*c^5*e*x^25
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.84

$$\int x^2(d+ex^2)\left(a+cx^4\right)^5\,dx=\frac{1}{25}\,c^5ex^{25}+\frac{1}{23}\,c^5dx^{23}+\frac{5}{21}\,ac^4ex^{21}+\frac{5}{19}\,ac^4dx^{19}+\frac{10}{17}\,a^2c^3ex^{17}+\frac{2}{3}\,a^2c^3dx^{15}+\frac{10}{13}\,a^3c^2ex^{13}+\frac{10}{11}\,a^3c^2dx^{11}+\frac{5}{9}\,a^4cex^9+\frac{5}{7}\,a^4cdx^7+\frac{1}{5}\,a^5ex^5+\frac{1}{3}\,a^5dx^3$$

input
```
integrate(x^2*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="fricas")
```

output
```
1/25*c^5*e*x^25 + 1/23*c^5*d*x^23 + 5/21*a*c^4*e*x^21 + 5/19*a*c^4*d*x^19
+ 10/17*a^2*c^3*e*x^17 + 2/3*a^2*c^3*d*x^15 + 10/13*a^3*c^2*e*x^13 + 10/11
*a^3*c^2*d*x^11 + 5/9*a^4*c*e*x^9 + 5/7*a^4*c*d*x^7 + 1/5*a^5*e*x^5 + 1/3*
a^5*d*x^3
```

3.2.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.04

$$\int x^2(d+ex^2)\left(a+cx^4\right)^5\,dx = \frac{a^5dx^3}{3}+\frac{a^5ex^5}{5}+\frac{5a^4cdx^7}{7}+\frac{5a^4cex^9}{9}+\frac{10a^3c^2dx^{11}}{11}+\frac{10a^3c^2ex^{13}}{13}+\frac{2a^2c^3dx^{15}}{3}+\frac{10a^2c^3ex^{17}}{17}+\frac{5ac^4dx^{19}}{19}+\frac{5ac^4ex^{21}}{21}+\frac{c^5dx^{23}}{23}+\frac{c^5ex^{25}}{25}$$

input
```
integrate(x**2*(e*x**2+d)*(c*x**4+a)**5,x)
```

output
```
a**5*d*x**3/3 + a**5*e*x**5/5 + 5*a**4*c*d*x**7/7 + 5*a**4*c*e*x**9/9 + 10
*a**3*c**2*d*x**11/11 + 10*a**3*c**2*e*x**13/13 + 2*a**2*c**3*d*x**15/3 +
10*a**2*c**3*e*x**17/17 + 5*a*c**4*d*x**19/19 + 5*a*c**4*e*x**21/21 + c**5
*d*x**23/23 + c**5*e*x**25/25
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.84

$$\int x^2(d+ex^2)\left(a+cx^4\right)^5\,dx = \frac{1}{25}\,c^5ex^{25}+\frac{1}{23}\,c^5dx^{23}+\frac{5}{21}\,ac^4ex^{21}+\frac{5}{19}\,ac^4dx^{19}+\frac{10}{17}\,a^2c^3ex^{17}+\frac{2}{3}\,a^2c^3dx^{15}+\frac{10}{13}\,a^3c^2ex^{13}+\frac{10}{11}\,a^3c^2dx^{11}+\frac{5}{9}\,a^4cex^9+\frac{5}{7}\,a^4cdx^7+\frac{1}{5}\,a^5ex^5+\frac{1}{3}\,a^5dx^3$$

input
```
integrate(x^2*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="maxima")
```

output
```
1/25*c^5*e*x^25 + 1/23*c^5*d*x^23 + 5/21*a*c^4*e*x^21 + 5/19*a*c^4*d*x^19
+ 10/17*a^2*c^3*e*x^17 + 2/3*a^2*c^3*d*x^15 + 10/13*a^3*c^2*e*x^13 + 10/11
*a^3*c^2*d*x^11 + 5/9*a^4*c*e*x^9 + 5/7*a^4*c*d*x^7 + 1/5*a^5*e*x^5 + 1/3*
a^5*d*x^3
```

3.2.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.84

$$\int x^2(d+ex^2)\left(a+cx^4\right)^5\,dx = \frac{1}{25}c^5ex^{25}+\frac{1}{23}c^5dx^{23}+\frac{5}{21}ac^4ex^{21}+\frac{5}{19}ac^4dx^{19}+\frac{10}{17}a^2c^3ex^{17}+\frac{2}{3}a^2c^3dx^{15}+\frac{10}{13}a^3c^2ex^{13}+\frac{10}{11}a^3c^2dx^{11}+\frac{5}{9}a^4cex^9+\frac{5}{7}a^4cdx^7+\frac{1}{5}a^5ex^5+\frac{1}{3}a^5dx^3$$

input
```
integrate(x^2*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="giac")
```

output
```
1/25*c^5*e*x^25 + 1/23*c^5*d*x^23 + 5/21*a*c^4*e*x^21 + 5/19*a*c^4*d*x^19
+ 10/17*a^2*c^3*e*x^17 + 2/3*a^2*c^3*d*x^15 + 10/13*a^3*c^2*e*x^13 + 10/11
*a^3*c^2*d*x^11 + 5/9*a^4*c*e*x^9 + 5/7*a^4*c*d*x^7 + 1/5*a^5*e*x^5 + 1/3*
a^5*d*x^3
```

3.2.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.84

$$\int x^2(d+ex^2)\left(a+cx^4\right)^5\,dx = \frac{e\,a^5\,x^5}{5}+\frac{d\,a^5\,x^3}{3}+\frac{5\,e\,a^4\,c\,x^9}{9}+\frac{5\,d\,a^4\,c\,x^7}{7}+\frac{10\,e\,a^3\,c^2\,x^{13}}{13}+\frac{10\,d\,a^3\,c^2\,x^{11}}{11}+\frac{10\,e\,a^2\,c^3\,x^{17}}{17}+\frac{2\,d\,a^2\,c^3\,x^{15}}{3}+\frac{5\,e\,a\,c^4\,x^{21}}{21}+\frac{5\,d\,a\,c^4\,x^{19}}{19}+\frac{e\,c^5\,x^{25}}{25}+\frac{d\,c^5\,x^{23}}{23}$$

input
```
int(x^2*(a + c*x^4)^5*(d + e*x^2),x)
```

output
```
(a^5*d*x^3)/3 + (a^5*e*x^5)/5 + (c^5*d*x^23)/23 + (c^5*e*x^25)/25 + (10*a^
3*c^2*d*x^11)/11 + (2*a^2*c^3*d*x^15)/3 + (10*a^3*c^2*e*x^13)/13 + (10*a^2
*c^3*e*x^17)/17 + (5*a^4*c*d*x^7)/7 + (5*a*c^4*d*x^19)/19 + (5*a^4*c*e*x^9
)/9 + (5*a*c^4*e*x^21)/21
```

3.3 $\int x(d+ex^2)\left(a+cx^4\right)^5\,dx$

3.3.1 Optimal result

Integrand size = 18, antiderivative size = 89

$$\int x\left(d+ex^2\right)\left(a+cx^4\right)^5\,dx=\frac{1}{2}a^5dx^2+\frac{5}{6}a^4cdx^6+a^3c^2dx^{10}+\frac{5}{7}a^2c^3dx^{14}+\frac{5}{18}ac^4dx^{18}+\frac{1}{22}c^5dx^{22}+\frac{e\left(a+cx^4\right)^6}{24c}$$

output
```
1/2*a^5*d*x^2+5/6*a^4*c*d*x^6+a^3*c^2*d*x^10+5/7*a^2*c^3*d*x^14+5/18*a*c^4
*d*x^18+1/22*c^5*d*x^22+1/24*e*(c*x^4+a)^6/c
```

3.3.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.64

$$\int x\left(d+ex^2\right)\left(a+cx^4\right)^5\,dx=\frac{1}{2}a^5dx^2+\frac{1}{4}a^5ex^4+\frac{5}{6}a^4cdx^6+\frac{5}{8}a^4cex^8+a^3c^2dx^{10}+\frac{5}{6}a^3c^2ex^{12}+\frac{5}{7}a^2c^3dx^{14}+\frac{5}{8}a^2c^3ex^{16}+\frac{5}{18}ac^4dx^{18}+\frac{1}{4}ac^4ex^{20}+\frac{1}{22}c^5dx^{22}+\frac{1}{24}c^5ex^{24}$$

input
```
Integrate[x*(d + e*x^2)*(a + c*x^4)^5,x]
```

output
```
(a^5*d*x^2)/2 + (a^5*e*x^4)/4 + (5*a^4*c*d*x^6)/6 + (5*a^4*c*e*x^8)/8 + a^
3*c^2*d*x^10 + (5*a^3*c^2*e*x^12)/6 + (5*a^2*c^3*d*x^14)/7 + (5*a^2*c^3*e*
x^16)/8 + (5*a*c^4*d*x^18)/18 + (a*c^4*e*x^20)/4 + (c^5*d*x^22)/22 + (c^5*
e*x^24)/24
```

3.3.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1577, 455, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+cx^4\right)^5\left(d+ex^2\right)\,dx$$

$$\Big\downarrow \text{1577}$$

$$\frac{1}{2}\int \left(ex^2+d\right)\left(cx^4+a\right)^5dx^2$$

$$\Big\downarrow \text{455}$$

$$\frac{1}{2}\left(d\int \left(cx^4+a\right)^5dx^2+\frac{e\left(a+cx^4\right)^6}{12c}\right)$$

$$\Big\downarrow \text{210}$$

$$\frac{1}{2}\left(d\int \left(c^5x^{20}+5ac^4x^{16}+10a^2c^3x^{12}+10a^3c^2x^8+5a^4cx^4+a^5\right)dx^2+\frac{e\left(a+cx^4\right)^6}{12c}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(d\left(a^5x^2+\frac{5}{3}a^4cx^6+2a^3c^2x^{10}+\frac{10}{7}a^2c^3x^{14}+\frac{5}{9}ac^4x^{18}+\frac{c^5x^{22}}{11}\right)+\frac{e\left(a+cx^4\right)^6}{12c}\right)$$

input
```
Int[x*(d + e*x^2)*(a + c*x^4)^5,x]
```

output
```
((e*(a + c*x^4)^6)/(12*c) + d*(a^5*x^2 + (5*a^4*c*x^6)/3 + 2*a^3*c^2*x^10
+ (10*a^2*c^3*x^14)/7 + (5*a*c^4*x^18)/9 + (c^5*x^22)/11))/2
```

3.3.3.1 Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.3.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.40

method	result
gosper	$\frac{5}{8}e\,a^2c^3x^{16}+\frac{5}{18}a\,c^4d\,x^{18}+\frac{1}{4}ea\,c^4x^{20}+\frac{1}{22}c^5d\,x^{22}+\frac{1}{24}e\,c^5x^{24}+\frac{1}{2}a^5d\,x^2+\frac{1}{4}e\,a^5x^4+\frac{5}{6}a^4cd\,x^6$
default	$\frac{5}{8}e\,a^2c^3x^{16}+\frac{5}{18}a\,c^4d\,x^{18}+\frac{1}{4}ea\,c^4x^{20}+\frac{1}{22}c^5d\,x^{22}+\frac{1}{24}e\,c^5x^{24}+\frac{1}{2}a^5d\,x^2+\frac{1}{4}e\,a^5x^4+\frac{5}{6}a^4cd\,x^6$
norman	$\frac{5}{8}e\,a^2c^3x^{16}+\frac{5}{18}a\,c^4d\,x^{18}+\frac{1}{4}ea\,c^4x^{20}+\frac{1}{22}c^5d\,x^{22}+\frac{1}{24}e\,c^5x^{24}+\frac{1}{2}a^5d\,x^2+\frac{1}{4}e\,a^5x^4+\frac{5}{6}a^4cd\,x^6$
risch	$\frac{5}{8}e\,a^2c^3x^{16}+\frac{5}{18}a\,c^4d\,x^{18}+\frac{1}{4}ea\,c^4x^{20}+\frac{1}{22}c^5d\,x^{22}+\frac{1}{24}e\,c^5x^{24}+\frac{1}{2}a^5d\,x^2+\frac{1}{4}e\,a^5x^4+\frac{5}{6}a^4cd\,x^6$
parallelrisch	$\frac{5}{8}e\,a^2c^3x^{16}+\frac{5}{18}a\,c^4d\,x^{18}+\frac{1}{4}ea\,c^4x^{20}+\frac{1}{22}c^5d\,x^{22}+\frac{1}{24}e\,c^5x^{24}+\frac{1}{2}a^5d\,x^2+\frac{1}{4}e\,a^5x^4+\frac{5}{6}a^4cd\,x^6$

input
```
int(x*(e*x^2+d)*(c*x^4+a)^5,x,method=_RETURNVERBOSE)
```

output
```
5/8*e*a^2*c^3*x^16+5/18*a*c^4*d*x^18+1/4*e*a*c^4*x^20+1/22*c^5*d*x^22+1/24
*e*c^5*x^24+1/2*a^5*d*x^2+1/4*e*a^5*x^4+5/6*a^4*c*d*x^6+5/8*e*c*a^4*x^8+a^
3*c^2*d*x^10+5/6*e*a^3*c^2*x^12+5/7*a^2*c^3*d*x^14
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.39

$$\int x(d+ex^2)\left(a+cx^4\right)^5\,dx = \frac{1}{24}\,c^5ex^{24}+\frac{1}{22}\,c^5dx^{22}+\frac{1}{4}\,ac^4ex^{20}+\frac{5}{18}\,ac^4dx^{18}+\frac{5}{8}\,a^2c^3ex^{16}+\frac{5}{7}\,a^2c^3dx^{14}+\frac{5}{6}\,a^3c^2ex^{12}+a^3c^2dx^{10}+\frac{5}{8}\,a^4cex^8+\frac{5}{6}\,a^4cdx^6+\frac{1}{4}\,a^5ex^4+\frac{1}{2}\,a^5dx^2$$

input
```
integrate(x*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="fricas")
```

output
```
1/24*c^5*e*x^24 + 1/22*c^5*d*x^22 + 1/4*a*c^4*e*x^20 + 5/18*a*c^4*d*x^18 +
 5/8*a^2*c^3*e*x^16 + 5/7*a^2*c^3*d*x^14 + 5/6*a^3*c^2*e*x^12 + a^3*c^2*d*
x^10 + 5/8*a^4*c*e*x^8 + 5/6*a^4*c*d*x^6 + 1/4*a^5*e*x^4 + 1/2*a^5*d*x^2
```

3.3.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.69

$$\int x(d+ex^2)\left(a+cx^4\right)^5\,dx = \frac{a^5dx^2}{2}+\frac{a^5ex^4}{4}+\frac{5a^4cdx^6}{6}+\frac{5a^4cex^8}{8}+a^3c^2dx^{10}+\frac{5a^3c^2ex^{12}}{6}+\frac{5a^2c^3dx^{14}}{7}+\frac{5a^2c^3ex^{16}}{8}+\frac{5ac^4dx^{18}}{18}+\frac{ac^4ex^{20}}{4}+\frac{c^5dx^{22}}{22}+\frac{c^5ex^{24}}{24}$$

input
```
integrate(x*(e*x**2+d)*(c*x**4+a)**5,x)
```

output
```
a**5*d*x**2/2 + a**5*e*x**4/4 + 5*a**4*c*d*x**6/6 + 5*a**4*c*e*x**8/8 + a*
*3*c**2*d*x**10 + 5*a**3*c**2*e*x**12/6 + 5*a**2*c**3*d*x**14/7 + 5*a**2*c
**3*e*x**16/8 + 5*a*c**4*d*x**18/18 + a*c**4*e*x**20/4 + c**5*d*x**22/22 +
 c**5*e*x**24/24
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.39

$$\int x(d+ex^2)\left(a+cx^4\right)^5\,dx = \frac{1}{24}c^5ex^{24}+\frac{1}{22}c^5dx^{22}+\frac{1}{4}ac^4ex^{20}+\frac{5}{18}ac^4dx^{18}+\frac{5}{8}a^2c^3ex^{16}+\frac{5}{7}a^2c^3dx^{14}+\frac{5}{6}a^3c^2ex^{12}+a^3c^2dx^{10}+\frac{5}{8}a^4cex^8+\frac{5}{6}a^4cdx^6+\frac{1}{4}a^5ex^4+\frac{1}{2}a^5dx^2$$

input
```
integrate(x*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="maxima")
```

output
```
1/24*c^5*e*x^24 + 1/22*c^5*d*x^22 + 1/4*a*c^4*e*x^20 + 5/18*a*c^4*d*x^18 +
 5/8*a^2*c^3*e*x^16 + 5/7*a^2*c^3*d*x^14 + 5/6*a^3*c^2*e*x^12 + a^3*c^2*d*
x^10 + 5/8*a^4*c*e*x^8 + 5/6*a^4*c*d*x^6 + 1/4*a^5*e*x^4 + 1/2*a^5*d*x^2
```

3.3.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.39

$$\int x(d+ex^2)\left(a+cx^4\right)^5\,dx = \frac{1}{24}c^5ex^{24}+\frac{1}{22}c^5dx^{22}+\frac{1}{4}ac^4ex^{20}+\frac{5}{18}ac^4dx^{18}+\frac{5}{8}a^2c^3ex^{16}+\frac{5}{7}a^2c^3dx^{14}+\frac{5}{6}a^3c^2ex^{12}+a^3c^2dx^{10}+\frac{5}{8}a^4cex^8+\frac{5}{6}a^4cdx^6+\frac{1}{4}a^5ex^4+\frac{1}{2}a^5dx^2$$

input
```
integrate(x*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="giac")
```

output
```
1/24*c^5*e*x^24 + 1/22*c^5*d*x^22 + 1/4*a*c^4*e*x^20 + 5/18*a*c^4*d*x^18 +
 5/8*a^2*c^3*e*x^16 + 5/7*a^2*c^3*d*x^14 + 5/6*a^3*c^2*e*x^12 + a^3*c^2*d*
x^10 + 5/8*a^4*c*e*x^8 + 5/6*a^4*c*d*x^6 + 1/4*a^5*e*x^4 + 1/2*a^5*d*x^2
```

3.3.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.39

$$\int x(d+ex^2)\left(a+cx^4\right)^5\,dx=\frac{e\,a^5\,x^4}{4}+\frac{d\,a^5\,x^2}{2}+\frac{5\,e\,a^4\,c\,x^8}{8}+\frac{5\,d\,a^4\,c\,x^6}{6}+\frac{5\,e\,a^3\,c^2\,x^{12}}{6}+d\,a^3\,c^2\,x^{10}+\frac{5\,e\,a^2\,c^3\,x^{16}}{8}+\frac{5\,d\,a^2\,c^3\,x^{14}}{7}+\frac{e\,a\,c^4\,x^{20}}{4}+\frac{5\,d\,a\,c^4\,x^{18}}{18}+\frac{e\,c^5\,x^{24}}{24}+\frac{d\,c^5\,x^{22}}{22}$$

input
```
int(x*(a + c*x^4)^5*(d + e*x^2),x)
```

output
```
(a^5*d*x^2)/2 + (a^5*e*x^4)/4 + (c^5*d*x^22)/22 + (c^5*e*x^24)/24 + a^3*c^
2*d*x^10 + (5*a^2*c^3*d*x^14)/7 + (5*a^3*c^2*e*x^12)/6 + (5*a^2*c^3*e*x^16
)/8 + (5*a^4*c*d*x^6)/6 + (5*a*c^4*d*x^18)/18 + (5*a^4*c*e*x^8)/8 + (a*c^4
*e*x^20)/4
```

3.4 $\int (d+ex^2) (a+cx^4)^5 \, dx$

3.4.1 Optimal result

Integrand size = 17, antiderivative size = 141

$$\int (d+ex^2) (a+cx^4)^5 \, dx = a^5dx + \frac{1}{3}a^5ex^3 + a^4cdx^5 + \frac{5}{7}a^4cex^7 + \frac{10}{9}a^3c^2dx^9 + \frac{10}{11}a^3c^2ex^{11} + \frac{10}{13}a^2c^3dx^{13} + \frac{2}{3}a^2c^3ex^{15} + \frac{5}{17}ac^4dx^{17} + \frac{5}{19}ac^4ex^{19} + \frac{1}{21}c^5dx^{21} + \frac{1}{23}c^5ex^{23}$$

output
```
a^5*d*x+1/3*a^5*e*x^3+a^4*c*d*x^5+5/7*a^4*c*e*x^7+10/9*a^3*c^2*d*x^9+10/11
*a^3*c^2*e*x^11+10/13*a^2*c^3*d*x^13+2/3*a^2*c^3*e*x^15+5/17*a*c^4*d*x^17+
5/19*a*c^4*e*x^19+1/21*c^5*d*x^21+1/23*c^5*e*x^23
```

3.4.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00

$$\int (d+ex^2) (a+cx^4)^5 \, dx = a^5dx + \frac{1}{3}a^5ex^3 + a^4cdx^5 + \frac{5}{7}a^4cex^7 + \frac{10}{9}a^3c^2dx^9 + \frac{10}{11}a^3c^2ex^{11} + \frac{10}{13}a^2c^3dx^{13} + \frac{2}{3}a^2c^3ex^{15} + \frac{5}{17}ac^4dx^{17} + \frac{5}{19}ac^4ex^{19} + \frac{1}{21}c^5dx^{21} + \frac{1}{23}c^5ex^{23}$$

input
```
Integrate[(d + e*x^2)*(a + c*x^4)^5,x]
```

output
```
a^5*d*x + (a^5*e*x^3)/3 + a^4*c*d*x^5 + (5*a^4*c*e*x^7)/7 + (10*a^3*c^2*d*
x^9)/9 + (10*a^3*c^2*e*x^11)/11 + (10*a^2*c^3*d*x^13)/13 + (2*a^2*c^3*e*x^
15)/3 + (5*a*c^4*d*x^17)/17 + (5*a*c^4*e*x^19)/19 + (c^5*d*x^21)/21 + (c^5
*e*x^23)/23
```

3.4.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1468, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^4\right)^5 \left(d+ex^2\right) \, dx$$

$$\Big\downarrow \text{1468}$$

$$\int \left(a^5d+a^5ex^2+5a^4cdx^4+5a^4cex^6+10a^3c^2dx^8+10a^3c^2ex^{10}+10a^2c^3dx^{12}+10a^2c^3ex^{14}+5ac^4dx^{16}+5ac^4ex$$

$$\Big\downarrow \text{2009}$$

$$a^5dx+\frac{1}{3}a^5ex^3+a^4cdx^5+\frac{5}{7}a^4cex^7+\frac{10}{9}a^3c^2dx^9+\frac{10}{11}a^3c^2ex^{11}+\frac{10}{13}a^2c^3dx^{13}+\frac{2}{3}a^2c^3ex^{15}+\frac{5}{17}ac^4dx^{17}+\frac{5}{19}ac^4ex^{19}+\frac{1}{21}c^5dx^{21}+\frac{1}{23}c^5ex^{23}$$

input
```
Int[(d + e*x^2)*(a + c*x^4)^5,x]
```

output
```
a^5*d*x + (a^5*e*x^3)/3 + a^4*c*d*x^5 + (5*a^4*c*e*x^7)/7 + (10*a^3*c^2*d*
x^9)/9 + (10*a^3*c^2*e*x^11)/11 + (10*a^2*c^3*d*x^13)/13 + (2*a^2*c^3*e*x^
15)/3 + (5*a*c^4*d*x^17)/17 + (5*a*c^4*e*x^19)/19 + (c^5*d*x^21)/21 + (c^5
*e*x^23)/23
```

3.4.3.1 Defintions of rubi rules used

rule 1468
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.4.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.87

method	result
gosper	$a^5dx+\frac{1}{3}a^5e\,x^3+a^4cd\,x^5+\frac{5}{7}a^4ce\,x^7+\frac{10}{9}a^3c^2d\,x^9+\frac{10}{11}a^3c^2e\,x^{11}+\frac{10}{13}a^2c^3d\,x^{13}+\frac{2}{3}a^2c^3e\,x^{15}$
default	$a^5dx+\frac{1}{3}a^5e\,x^3+a^4cd\,x^5+\frac{5}{7}a^4ce\,x^7+\frac{10}{9}a^3c^2d\,x^9+\frac{10}{11}a^3c^2e\,x^{11}+\frac{10}{13}a^2c^3d\,x^{13}+\frac{2}{3}a^2c^3e\,x^{15}$
norman	$a^5dx+\frac{1}{3}a^5e\,x^3+a^4cd\,x^5+\frac{5}{7}a^4ce\,x^7+\frac{10}{9}a^3c^2d\,x^9+\frac{10}{11}a^3c^2e\,x^{11}+\frac{10}{13}a^2c^3d\,x^{13}+\frac{2}{3}a^2c^3e\,x^{15}$
risch	$a^5dx+\frac{1}{3}a^5e\,x^3+a^4cd\,x^5+\frac{5}{7}a^4ce\,x^7+\frac{10}{9}a^3c^2d\,x^9+\frac{10}{11}a^3c^2e\,x^{11}+\frac{10}{13}a^2c^3d\,x^{13}+\frac{2}{3}a^2c^3e\,x^{15}$
parallelrisch	$a^5dx+\frac{1}{3}a^5e\,x^3+a^4cd\,x^5+\frac{5}{7}a^4ce\,x^7+\frac{10}{9}a^3c^2d\,x^9+\frac{10}{11}a^3c^2e\,x^{11}+\frac{10}{13}a^2c^3d\,x^{13}+\frac{2}{3}a^2c^3e\,x^{15}$

input
```
int((e*x^2+d)*(c*x^4+a)^5,x,method=_RETURNVERBOSE)
```

output
```
a^5*d*x+1/3*a^5*e*x^3+a^4*c*d*x^5+5/7*a^4*c*e*x^7+10/9*a^3*c^2*d*x^9+10/11
*a^3*c^2*e*x^11+10/13*a^2*c^3*d*x^13+2/3*a^2*c^3*e*x^15+5/17*a*c^4*d*x^17+
5/19*a*c^4*e*x^19+1/21*c^5*d*x^21+1/23*c^5*e*x^23
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.86

$$\int (d+ex^2)\,(a+cx^4)^5\,dx=\frac{1}{23}\,c^5ex^{23}+\frac{1}{21}\,c^5dx^{21}+\frac{5}{19}\,ac^4ex^{19}+\frac{5}{17}\,ac^4dx^{17}+\frac{2}{3}\,a^2c^3ex^{15}+\frac{10}{13}\,a^2c^3dx^{13}+\frac{10}{11}\,a^3c^2ex^{11}+\frac{10}{9}\,a^3c^2dx^9+\frac{5}{7}\,a^4cex^7+a^4cdx^5+\frac{1}{3}\,a^5ex^3+a^5dx$$

input
```
integrate((e*x^2+d)*(c*x^4+a)^5,x, algorithm="fricas")
```

output
```
1/23*c^5*e*x^23 + 1/21*c^5*d*x^21 + 5/19*a*c^4*e*x^19 + 5/17*a*c^4*d*x^17
+ 2/3*a^2*c^3*e*x^15 + 10/13*a^2*c^3*d*x^13 + 10/11*a^3*c^2*e*x^11 + 10/9*
a^3*c^2*d*x^9 + 5/7*a^4*c*e*x^7 + a^4*c*d*x^5 + 1/3*a^5*e*x^3 + a^5*d*x
```

3.4.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.05

$$\int \left(d+ex^2\right)\left(a+cx^4\right)^5\,dx = a^5dx + \frac{a^5ex^3}{3} + a^4cdx^5 + \frac{5a^4cex^7}{7} + \frac{10a^3c^2dx^9}{9} + \frac{10a^3c^2ex^{11}}{11} + \frac{10a^2c^3dx^{13}}{13} + \frac{2a^2c^3ex^{15}}{3} + \frac{5ac^4dx^{17}}{17} + \frac{5ac^4ex^{19}}{19} + \frac{c^5dx^{21}}{21} + \frac{c^5ex^{23}}{23}$$

input
```
integrate((e*x**2+d)*(c*x**4+a)**5,x)
```

output
```
a**5*d*x + a**5*e*x**3/3 + a**4*c*d*x**5 + 5*a**4*c*e*x**7/7 + 10*a**3*c**
2*d*x**9/9 + 10*a**3*c**2*e*x**11/11 + 10*a**2*c**3*d*x**13/13 + 2*a**2*c*
*3*e*x**15/3 + 5*a*c**4*d*x**17/17 + 5*a*c**4*e*x**19/19 + c**5*d*x**21/21
 + c**5*e*x**23/23
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.86

$$\int \left(d+ex^2\right)\left(a+cx^4\right)^5\,dx = \frac{1}{23}\,c^5ex^{23} + \frac{1}{21}\,c^5dx^{21} + \frac{5}{19}\,ac^4ex^{19} + \frac{5}{17}\,ac^4dx^{17} + \frac{2}{3}\,a^2c^3ex^{15} + \frac{10}{13}\,a^2c^3dx^{13} + \frac{10}{11}\,a^3c^2ex^{11} + \frac{10}{9}\,a^3c^2dx^9 + \frac{5}{7}\,a^4cex^7 + a^4cdx^5 + \frac{1}{3}\,a^5ex^3 + a^5dx$$

input
```
integrate((e*x^2+d)*(c*x^4+a)^5,x, algorithm="maxima")
```

output
```
1/23*c^5*e*x^23 + 1/21*c^5*d*x^21 + 5/19*a*c^4*e*x^19 + 5/17*a*c^4*d*x^17
+ 2/3*a^2*c^3*e*x^15 + 10/13*a^2*c^3*d*x^13 + 10/11*a^3*c^2*e*x^11 + 10/9*
a^3*c^2*d*x^9 + 5/7*a^4*c*e*x^7 + a^4*c*d*x^5 + 1/3*a^5*e*x^3 + a^5*d*x
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.86

$$\int (d+ex^2)(a+cx^4)^5 \,dx = \frac{1}{23}c^5ex^{23} + \frac{1}{21}c^5dx^{21} + \frac{5}{19}ac^4ex^{19} + \frac{5}{17}ac^4dx^{17} + \frac{2}{3}a^2c^3ex^{15} + \frac{10}{13}a^2c^3dx^{13} + \frac{10}{11}a^3c^2ex^{11} + \frac{10}{9}a^3c^2dx^9 + \frac{5}{7}a^4cex^7 + a^4cdx^5 + \frac{1}{3}a^5ex^3 + a^5dx$$

input
```
integrate((e*x^2+d)*(c*x^4+a)^5,x, algorithm="giac")
```

output
```
1/23*c^5*e*x^23 + 1/21*c^5*d*x^21 + 5/19*a*c^4*e*x^19 + 5/17*a*c^4*d*x^17
+ 2/3*a^2*c^3*e*x^15 + 10/13*a^2*c^3*d*x^13 + 10/11*a^3*c^2*e*x^11 + 10/9*
a^3*c^2*d*x^9 + 5/7*a^4*c*e*x^7 + a^4*c*d*x^5 + 1/3*a^5*e*x^3 + a^5*d*x
```

3.4.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.86

$$\int (d+ex^2)(a+cx^4)^5 \,dx = \frac{e\,a^5\,x^3}{3} + d\,a^5\,x + \frac{5\,e\,a^4\,c\,x^7}{7} + d\,a^4\,c\,x^5 + \frac{10\,e\,a^3\,c^2\,x^{11}}{11} + \frac{10\,d\,a^3\,c^2\,x^9}{9} + \frac{2\,e\,a^2\,c^3\,x^{15}}{3} + \frac{10\,d\,a^2\,c^3\,x^{13}}{13} + \frac{5\,e\,a\,c^4\,x^{19}}{19} + \frac{5\,d\,a\,c^4\,x^{17}}{17} + \frac{e\,c^5\,x^{23}}{23} + \frac{d\,c^5\,x^{21}}{21}$$

input
```
int((a + c*x^4)^5*(d + e*x^2),x)
```

output
```
(a^5*e*x^3)/3 + (c^5*d*x^21)/21 + (c^5*e*x^23)/23 + a^5*d*x + (10*a^3*c^2*
d*x^9)/9 + (10*a^2*c^3*d*x^13)/13 + (10*a^3*c^2*e*x^11)/11 + (2*a^2*c^3*e*
x^15)/3 + a^4*c*d*x^5 + (5*a*c^4*d*x^17)/17 + (5*a^4*c*e*x^7)/7 + (5*a*c^4
*e*x^19)/19
```

3.5 $\int \frac{(d+ex^2)(a+cx^4)^5}{x}\,dx$

3.5.1 Optimal result

Integrand size = 20, antiderivative size = 142

$$\int \frac{(d+ex^2)(a+cx^4)^5}{x}\,dx = \frac{1}{2}a^5ex^2+\frac{5}{4}a^4cdx^4+\frac{5}{6}a^4cex^6+\frac{5}{4}a^3c^2dx^8 + a^3c^2ex^{10}+\frac{5}{6}a^2c^3dx^{12}+\frac{5}{7}a^2c^3ex^{14}+\frac{5}{16}ac^4dx^{16} + \frac{5}{18}ac^4ex^{18}+\frac{1}{20}c^5dx^{20}+\frac{1}{22}c^5ex^{22}+a^5d\log(x)$$

output
```
1/2*a^5*e*x^2+5/4*a^4*c*d*x^4+5/6*a^4*c*e*x^6+5/4*a^3*c^2*d*x^8+a^3*c^2*e*
x^10+5/6*a^2*c^3*d*x^12+5/7*a^2*c^3*e*x^14+5/16*a*c^4*d*x^16+5/18*a*c^4*e*
x^18+1/20*c^5*d*x^20+1/22*c^5*e*x^22+a^5*d*ln(x)
```

3.5.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)(a+cx^4)^5}{x}\,dx = \frac{1}{2}a^5ex^2+\frac{5}{4}a^4cdx^4+\frac{5}{6}a^4cex^6+\frac{5}{4}a^3c^2dx^8 + a^3c^2ex^{10}+\frac{5}{6}a^2c^3dx^{12}+\frac{5}{7}a^2c^3ex^{14}+\frac{5}{16}ac^4dx^{16} + \frac{5}{18}ac^4ex^{18}+\frac{1}{20}c^5dx^{20}+\frac{1}{22}c^5ex^{22}+a^5d\log(x)$$

input
```
Integrate[((d + e*x^2)*(a + c*x^4)^5)/x,x]
```

output
```
(a^5*e*x^2)/2 + (5*a^4*c*d*x^4)/4 + (5*a^4*c*e*x^6)/6 + (5*a^3*c^2*d*x^8)/
4 + a^3*c^2*e*x^10 + (5*a^2*c^3*d*x^12)/6 + (5*a^2*c^3*e*x^14)/7 + (5*a*c^
4*d*x^16)/16 + (5*a*c^4*e*x^18)/18 + (c^5*d*x^20)/20 + (c^5*e*x^22)/22 + a
^5*d*Log[x]
```

3.5.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1579, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^4)^5(d+ex^2)}{x}dx$$

$$\Big\downarrow \text{1579}$$

$$\frac{1}{2}\int \frac{(ex^2+d)(cx^4+a)^5}{x^2}dx^2$$

$$\Big\downarrow \text{522}$$

$$\frac{1}{2}\int \Big(c^5ex^{20}+c^5dx^{18}+5ac^4ex^{16}+5ac^4dx^{14}+10a^2c^3ex^{12}+10a^2c^3dx^{10}+10a^3c^2ex^8+10a^3c^2dx^6+5a^4cex^4+$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\Big(a^5d\log(x^2)+a^5ex^2+\frac{5}{2}a^4cdx^4+\frac{5}{3}a^4cex^6+\frac{5}{2}a^3c^2dx^8+2a^3c^2ex^{10}+\frac{5}{3}a^2c^3dx^{12}+\frac{10}{7}a^2c^3ex^{14}+\frac{5}{8}ac^4dx^{16}+$$

input
```
Int[((d + e*x^2)*(a + c*x^4)^5)/x,x]
```

output
```
(a^5*e*x^2 + (5*a^4*c*d*x^4)/2 + (5*a^4*c*e*x^6)/3 + (5*a^3*c^2*d*x^8)/2 +
 2*a^3*c^2*e*x^10 + (5*a^2*c^3*d*x^12)/3 + (10*a^2*c^3*e*x^14)/7 + (5*a*c^
4*d*x^16)/8 + (5*a*c^4*e*x^18)/9 + (c^5*d*x^20)/10 + (c^5*e*x^22)/11 + a^5
*d*Log[x^2])/2
```

3.5.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.5.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.87

method	result
default	$\frac{a^5 e x^2}{2}+\frac{5a^4 cd x^4}{4}+\frac{5a^4 ce x^6}{6}+\frac{5a^3c^2d x^8}{4}+a^3c^2e x^{10}+\frac{5a^2c^3d x^{12}}{6}+\frac{5a^2c^3e x^{14}}{7}+\frac{5a c^4d x^{16}}{16}+\frac{5a c^4e x^{18}}{18}$
norman	$\frac{a^5 e x^2}{2}+\frac{5a^4 cd x^4}{4}+\frac{5a^4 ce x^6}{6}+\frac{5a^3c^2d x^8}{4}+a^3c^2e x^{10}+\frac{5a^2c^3d x^{12}}{6}+\frac{5a^2c^3e x^{14}}{7}+\frac{5a c^4d x^{16}}{16}+\frac{5a c^4e x^{18}}{18}$
risch	$\frac{a^5 e x^2}{2}+\frac{5a^4 cd x^4}{4}+\frac{5a^4 ce x^6}{6}+\frac{5a^3c^2d x^8}{4}+a^3c^2e x^{10}+\frac{5a^2c^3d x^{12}}{6}+\frac{5a^2c^3e x^{14}}{7}+\frac{5a c^4d x^{16}}{16}+\frac{5a c^4e x^{18}}{18}$
parallelrisch	$\frac{a^5 e x^2}{2}+\frac{5a^4 cd x^4}{4}+\frac{5a^4 ce x^6}{6}+\frac{5a^3c^2d x^8}{4}+a^3c^2e x^{10}+\frac{5a^2c^3d x^{12}}{6}+\frac{5a^2c^3e x^{14}}{7}+\frac{5a c^4d x^{16}}{16}+\frac{5a c^4e x^{18}}{18}$

input
```
int((e*x^2+d)*(c*x^4+a)^5/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^5*e*x^2+5/4*a^4*c*d*x^4+5/6*a^4*c*e*x^6+5/4*a^3*c^2*d*x^8+a^3*c^2*e*
x^10+5/6*a^2*c^3*d*x^12+5/7*a^2*c^3*e*x^14+5/16*a*c^4*d*x^16+5/18*a*c^4*e*
x^18+1/20*c^5*d*x^20+1/22*c^5*e*x^22+a^5*d*ln(x)
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex^2)(a+cx^4)^5}{x}\,dx = \frac{1}{22}c^5ex^{22} + \frac{1}{20}c^5dx^{20} + \frac{5}{18}ac^4ex^{18} + \frac{5}{16}ac^4dx^{16} + \frac{5}{7}a^2c^3ex^{14} + \frac{5}{6}a^2c^3dx^{12} + a^3c^2ex^{10} + \frac{5}{4}a^3c^2dx^8 + \frac{5}{6}a^4cex^6 + \frac{5}{4}a^4cdx^4 + \frac{1}{2}a^5ex^2 + a^5d\log(x)$$

input
```
integrate((e*x^2+d)*(c*x^4+a)^5/x,x, algorithm="fricas")
```

output
```
1/22*c^5*e*x^22 + 1/20*c^5*d*x^20 + 5/18*a*c^4*e*x^18 + 5/16*a*c^4*d*x^16
+ 5/7*a^2*c^3*e*x^14 + 5/6*a^2*c^3*d*x^12 + a^3*c^2*e*x^10 + 5/4*a^3*c^2*d
*x^8 + 5/6*a^4*c*e*x^6 + 5/4*a^4*c*d*x^4 + 1/2*a^5*e*x^2 + a^5*d*log(x)
```

3.5.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex^2)(a+cx^4)^5}{x}\,dx = a^5d\log(x) + \frac{a^5ex^2}{2} + \frac{5a^4cdx^4}{4} + \frac{5a^4cex^6}{6} + \frac{5a^3c^2dx^8}{4} + a^3c^2ex^{10} + \frac{5a^2c^3dx^{12}}{6} + \frac{5a^2c^3ex^{14}}{7} + \frac{5ac^4dx^{16}}{16} + \frac{5ac^4ex^{18}}{18} + \frac{c^5dx^{20}}{20} + \frac{c^5ex^{22}}{22}$$

input
```
integrate((e*x**2+d)*(c*x**4+a)**5/x,x)
```

output
```
a**5*d*log(x) + a**5*e*x**2/2 + 5*a**4*c*d*x**4/4 + 5*a**4*c*e*x**6/6 + 5*
a**3*c**2*d*x**8/4 + a**3*c**2*e*x**10 + 5*a**2*c**3*d*x**12/6 + 5*a**2*c*
*3*e*x**14/7 + 5*a*c**4*d*x**16/16 + 5*a*c**4*e*x**18/18 + c**5*d*x**20/20
 + c**5*e*x**22/22
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex^2)(a+cx^4)^5}{x}\,dx = \frac{1}{22}c^5ex^{22} + \frac{1}{20}c^5dx^{20} + \frac{5}{18}ac^4ex^{18} + \frac{5}{16}ac^4dx^{16} + \frac{5}{7}a^2c^3ex^{14} + \frac{5}{6}a^2c^3dx^{12} + a^3c^2ex^{10} + \frac{5}{4}a^3c^2dx^8 + \frac{5}{6}a^4cex^6 + \frac{5}{4}a^4cdx^4 + \frac{1}{2}a^5ex^2 + \frac{1}{2}a^5d\log\left(x^2\right)$$

input
```
integrate((e*x^2+d)*(c*x^4+a)^5/x,x, algorithm="maxima")
```

output
```
1/22*c^5*e*x^22 + 1/20*c^5*d*x^20 + 5/18*a*c^4*e*x^18 + 5/16*a*c^4*d*x^16
+ 5/7*a^2*c^3*e*x^14 + 5/6*a^2*c^3*d*x^12 + a^3*c^2*e*x^10 + 5/4*a^3*c^2*d
*x^8 + 5/6*a^4*c*e*x^6 + 5/4*a^4*c*d*x^4 + 1/2*a^5*e*x^2 + 1/2*a^5*d*log(x
^2)
```

3.5.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex^2)(a+cx^4)^5}{x}\,dx = \frac{1}{22}c^5ex^{22} + \frac{1}{20}c^5dx^{20} + \frac{5}{18}ac^4ex^{18} + \frac{5}{16}ac^4dx^{16} + \frac{5}{7}a^2c^3ex^{14} + \frac{5}{6}a^2c^3dx^{12} + a^3c^2ex^{10} + \frac{5}{4}a^3c^2dx^8 + \frac{5}{6}a^4cex^6 + \frac{5}{4}a^4cdx^4 + \frac{1}{2}a^5ex^2 + \frac{1}{2}a^5d\log\left(x^2\right)$$

input
```
integrate((e*x^2+d)*(c*x^4+a)^5/x,x, algorithm="giac")
```

output
```
1/22*c^5*e*x^22 + 1/20*c^5*d*x^20 + 5/18*a*c^4*e*x^18 + 5/16*a*c^4*d*x^16
+ 5/7*a^2*c^3*e*x^14 + 5/6*a^2*c^3*d*x^12 + a^3*c^2*e*x^10 + 5/4*a^3*c^2*d
*x^8 + 5/6*a^4*c*e*x^6 + 5/4*a^4*c*d*x^4 + 1/2*a^5*e*x^2 + 1/2*a^5*d*log(x
^2)
```

3.5.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex^2)(a+cx^4)^5}{x}\,dx = \frac{a^5\,e\,x^2}{2} + \frac{c^5\,d\,x^{20}}{20} + \frac{c^5\,e\,x^{22}}{22} + a^5\,d\,\ln(x) + \frac{5\,a^3\,c^2\,d\,x^8}{4} + \frac{5\,a^2\,c^3\,d\,x^{12}}{6} + a^3\,c^2\,e\,x^{10} + \frac{5\,a^2\,c^3\,e\,x^{14}}{7} + \frac{5\,a^4\,c\,d\,x^4}{4} + \frac{5\,a\,c^4\,d\,x^{16}}{16} + \frac{5\,a^4\,c\,e\,x^6}{6} + \frac{5\,a\,c^4\,e\,x^{18}}{18}$$

input
```
int(((a + c*x^4)^5*(d + e*x^2))/x,x)
```

output
```
(a^5*e*x^2)/2 + (c^5*d*x^20)/20 + (c^5*e*x^22)/22 + a^5*d*log(x) + (5*a^3*
c^2*d*x^8)/4 + (5*a^2*c^3*d*x^12)/6 + a^3*c^2*e*x^10 + (5*a^2*c^3*e*x^14)/
7 + (5*a^4*c*d*x^4)/4 + (5*a*c^4*d*x^16)/16 + (5*a^4*c*e*x^6)/6 + (5*a*c^4
*e*x^18)/18
```

3.6 $\int \frac{(d+ex^2)(a+cx^4)^5}{x^2} dx$

3.6.1 Optimal result

Integrand size = 20, antiderivative size = 139

$$\int \frac{(d+ex^2)(a+cx^4)^5}{x^2} dx = -\frac{a^5 d}{x} + a^5 ex + \frac{5}{3}a^4 cdx^3 + a^4 cex^5 + \frac{10}{7}a^3c^2dx^7 + \frac{10}{9}a^3c^2ex^9 + \frac{10}{11}a^2c^3dx^{11} + \frac{10}{13}a^2c^3ex^{13} + \frac{1}{3}ac^4dx^{15} + \frac{5}{17}ac^4ex^{17} + \frac{1}{19}c^5dx^{19} + \frac{1}{21}c^5ex^{21}$$

output
```
-a^5*d/x+a^5*e*x+5/3*a^4*c*d*x^3+a^4*c*e*x^5+10/7*a^3*c^2*d*x^7+10/9*a^3*c
^2*e*x^9+10/11*a^2*c^3*d*x^11+10/13*a^2*c^3*e*x^13+1/3*a*c^4*d*x^15+5/17*a
*c^4*e*x^17+1/19*c^5*d*x^19+1/21*c^5*e*x^21
```

3.6.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)(a+cx^4)^5}{x^2} dx = -\frac{a^5 d}{x} + a^5 ex + \frac{5}{3}a^4 cdx^3 + a^4 cex^5 + \frac{10}{7}a^3c^2dx^7 + \frac{10}{9}a^3c^2ex^9 + \frac{10}{11}a^2c^3dx^{11} + \frac{10}{13}a^2c^3ex^{13} + \frac{1}{3}ac^4dx^{15} + \frac{5}{17}ac^4ex^{17} + \frac{1}{19}c^5dx^{19} + \frac{1}{21}c^5ex^{21}$$

input
```
Integrate[((d + e*x^2)*(a + c*x^4)^5)/x^2,x]
```

output
```
-((a^5*d)/x) + a^5*e*x + (5*a^4*c*d*x^3)/3 + a^4*c*e*x^5 + (10*a^3*c^2*d*x
^7)/7 + (10*a^3*c^2*e*x^9)/9 + (10*a^2*c^3*d*x^11)/11 + (10*a^2*c^3*e*x^13
)/13 + (a*c^4*d*x^15)/3 + (5*a*c^4*e*x^17)/17 + (c^5*d*x^19)/19 + (c^5*e*x
^21)/21
```

3.6.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1585, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^4)^5(d+ex^2)}{x^2}\,dx$$

$$\Big\downarrow \text{1585}$$

$$\int \left(\frac{a^5d}{x^2}+a^5e+5a^4cdx^2+5a^4cex^4+10a^3c^2dx^6+10a^3c^2ex^8+10a^2c^3dx^{10}+10a^2c^3ex^{12}+5ac^4dx^{14}+5ac^4ex^{16}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5d}{x}+a^5ex+\frac{5}{3}a^4cdx^3+a^4cex^5+\frac{10}{7}a^3c^2dx^7+\frac{10}{9}a^3c^2ex^9+\frac{10}{11}a^2c^3dx^{11}+\frac{10}{13}a^2c^3ex^{13}+\frac{1}{3}ac^4dx^{15}+\frac{5}{17}ac^4ex^{17}+\frac{1}{19}c^5dx^{19}+\frac{1}{21}c^5ex^{21}$$

input
```
Int[((d + e*x^2)*(a + c*x^4)^5)/x^2,x]
```

output
```
-((a^5*d)/x) + a^5*e*x + (5*a^4*c*d*x^3)/3 + a^4*c*e*x^5 + (10*a^3*c^2*d*x
^7)/7 + (10*a^3*c^2*e*x^9)/9 + (10*a^2*c^3*d*x^11)/11 + (10*a^2*c^3*e*x^13
)/13 + (a*c^4*d*x^15)/3 + (5*a*c^4*e*x^17)/17 + (c^5*d*x^19)/19 + (c^5*e*x
^21)/21
```

3.6.3.1 Defintions of rubi rules used

rule 1585
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.6.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.88

method	result
default	$-\frac{a^5 d}{x}+a^5 e x+\frac{5a^4 c d\,x^3}{3}+a^4 c e\,x^5+\frac{10a^3 c^2 d\,x^7}{7}+\frac{10a^3 c^2 e\,x^9}{9}+\frac{10a^2 c^3 d\,x^{11}}{11}+\frac{10a^2 c^3 e\,x^{13}}{13}+\frac{a\,c^4 d\,x^{15}}{3}+$
risch	$-\frac{a^5 d}{x}+a^5 e x+\frac{5a^4 c d\,x^3}{3}+a^4 c e\,x^5+\frac{10a^3 c^2 d\,x^7}{7}+\frac{10a^3 c^2 e\,x^9}{9}+\frac{10a^2 c^3 d\,x^{11}}{11}+\frac{10a^2 c^3 e\,x^{13}}{13}+\frac{a\,c^4 d\,x^{15}}{3}+$
norman	$\frac{-d\,a^5+a^5 e\,x^2+\frac{5}{3}a^4 c d\,x^4+a^4 c e\,x^6+\frac{10}{7}a^3 c^2 d\,x^8+\frac{10}{9}a^3 c^2 e\,x^{10}+\frac{10}{11}a^2 c^3 d\,x^{12}+\frac{10}{13}a^2 c^3 e\,x^{14}+\frac{1}{3}a\,c^4 d\,x^{16}+\frac{5}{17}a\,c^4 e\,x^{18}+\frac{1}{19}c^5 d\,x}{x}$
gosper	$-\frac{-138567c^5 e\,x^{22}-153153c^5 d\,x^{20}-855855a\,c^4 e\,x^{18}-969969a\,c^4 d\,x^{16}-2238390a^2 c^3 e\,x^{14}-2645370a^2 c^3 d\,x^{12}-3233230a^3 c^2 e}{2909907x}$
parallelrisch	$\frac{138567c^5 e\,x^{22}+153153c^5 d\,x^{20}+855855a\,c^4 e\,x^{18}+969969a\,c^4 d\,x^{16}+2238390a^2 c^3 e\,x^{14}+2645370a^2 c^3 d\,x^{12}+3233230a^3 c^2 e\,x^{10}}{2909907x}$

input
```
int((e*x^2+d)*(c*x^4+a)^5/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^5*d/x+a^5*e*x+5/3*a^4*c*d*x^3+a^4*c*e*x^5+10/7*a^3*c^2*d*x^7+10/9*a^3*c
^2*e*x^9+10/11*a^2*c^3*d*x^11+10/13*a^2*c^3*e*x^13+1/3*a*c^4*d*x^15+5/17*a
*c^4*e*x^17+1/19*c^5*d*x^19+1/21*c^5*e*x^21
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex^2)\,(a+cx^4)^5}{x^2}\,dx = \frac{138567\,c^5ex^{22}+153153\,c^5dx^{20}+855855\,ac^4ex^{18}+969969\,ac^4dx^{16}+2238390\,a^2c^3ex^{14}+2645370\,a^2c^3dx^{1}}{290}$$

input
```
integrate((e*x^2+d)*(c*x^4+a)^5/x^2,x, algorithm="fricas")
```

output
```
1/2909907*(138567*c^5*e*x^22 + 153153*c^5*d*x^20 + 855855*a*c^4*e*x^18 + 9
69969*a*c^4*d*x^16 + 2238390*a^2*c^3*e*x^14 + 2645370*a^2*c^3*d*x^12 + 323
3230*a^3*c^2*e*x^10 + 4157010*a^3*c^2*d*x^8 + 2909907*a^4*c*e*x^6 + 484984
5*a^4*c*d*x^4 + 2909907*a^5*e*x^2 - 2909907*a^5*d)/x
```

3.6.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.03

$$\int \frac{(d+ex^2)(a+cx^4)^5}{x^2}\,dx = -\frac{a^5d}{x}+a^5ex+\frac{5a^4cdx^3}{3}+a^4cex^5+\frac{10a^3c^2dx^7}{7}+\frac{10a^3c^2ex^9}{9}+\frac{10a^2c^3dx^{11}}{11}+\frac{10a^2c^3ex^{13}}{13}+\frac{ac^4dx^{15}}{3}+\frac{5ac^4ex^{17}}{17}+\frac{c^5dx^{19}}{19}+\frac{c^5ex^{21}}{21}$$

input
```
integrate((e*x**2+d)*(c*x**4+a)**5/x**2,x)
```

output
```
-a**5*d/x + a**5*e*x + 5*a**4*c*d*x**3/3 + a**4*c*e*x**5 + 10*a**3*c**2*d*
x**7/7 + 10*a**3*c**2*e*x**9/9 + 10*a**2*c**3*d*x**11/11 + 10*a**2*c**3*e*
x**13/13 + a*c**4*d*x**15/3 + 5*a*c**4*e*x**17/17 + c**5*d*x**19/19 + c**5
*e*x**21/21
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex^2)(a+cx^4)^5}{x^2}\,dx = \frac{1}{21}\,c^5ex^{21}+\frac{1}{19}\,c^5dx^{19}+\frac{5}{17}\,ac^4ex^{17}+\frac{1}{3}\,ac^4dx^{15}+\frac{10}{13}\,a^2c^3ex^{13}+\frac{10}{11}\,a^2c^3dx^{11}+\frac{10}{9}\,a^3c^2ex^9+\frac{10}{7}\,a^3c^2dx^7+a^4cex^5+\frac{5}{3}\,a^4cdx^3+a^5ex-\frac{a^5d}{x}$$

input
```
integrate((e*x^2+d)*(c*x^4+a)^5/x^2,x, algorithm="maxima")
```

output
```
1/21*c^5*e*x^21 + 1/19*c^5*d*x^19 + 5/17*a*c^4*e*x^17 + 1/3*a*c^4*d*x^15 +
 10/13*a^2*c^3*e*x^13 + 10/11*a^2*c^3*d*x^11 + 10/9*a^3*c^2*e*x^9 + 10/7*a
^3*c^2*d*x^7 + a^4*c*e*x^5 + 5/3*a^4*c*d*x^3 + a^5*e*x - a^5*d/x
```

3.6.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex^2)(a+cx^4)^5}{x^2}\,dx = \frac{1}{21}c^5ex^{21} + \frac{1}{19}c^5dx^{19} + \frac{5}{17}ac^4ex^{17} + \frac{1}{3}ac^4dx^{15} + \frac{10}{13}a^2c^3ex^{13} + \frac{10}{11}a^2c^3dx^{11} + \frac{10}{9}a^3c^2ex^9 + \frac{10}{7}a^3c^2dx^7 + a^4cex^5 + \frac{5}{3}a^4cdx^3 + a^5ex - \frac{a^5d}{x}$$

input
```
integrate((e*x^2+d)*(c*x^4+a)^5/x^2,x, algorithm="giac")
```

output
```
1/21*c^5*e*x^21 + 1/19*c^5*d*x^19 + 5/17*a*c^4*e*x^17 + 1/3*a*c^4*d*x^15 +
 10/13*a^2*c^3*e*x^13 + 10/11*a^2*c^3*d*x^11 + 10/9*a^3*c^2*e*x^9 + 10/7*a
^3*c^2*d*x^7 + a^4*c*e*x^5 + 5/3*a^4*c*d*x^3 + a^5*e*x - a^5*d/x
```

3.6.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex^2)(a+cx^4)^5}{x^2}\,dx = \frac{c^5\,d\,x^{19}}{19} - \frac{a^5\,d}{x} + \frac{c^5\,e\,x^{21}}{21} + a^5\,e\,x + \frac{10\,a^3\,c^2\,d\,x^7}{7} + \frac{10\,a^2\,c^3\,d\,x^{11}}{11} + \frac{10\,a^3\,c^2\,e\,x^9}{9} + \frac{10\,a^2\,c^3\,e\,x^{13}}{13} + \frac{5\,a^4\,c\,d\,x^3}{3} + \frac{a\,c^4\,d\,x^{15}}{3} + a^4\,c\,e\,x^5 + \frac{5\,a\,c^4\,e\,x^{17}}{17}$$

input
```
int(((a + c*x^4)^5*(d + e*x^2))/x^2,x)
```

output
```
(c^5*d*x^19)/19 - (a^5*d)/x + (c^5*e*x^21)/21 + a^5*e*x + (10*a^3*c^2*d*x^
7)/7 + (10*a^2*c^3*d*x^11)/11 + (10*a^3*c^2*e*x^9)/9 + (10*a^2*c^3*e*x^13)
/13 + (5*a^4*c*d*x^3)/3 + (a*c^4*d*x^15)/3 + a^4*c*e*x^5 + (5*a*c^4*e*x^17
)/17
```

3.7 $\int \frac{(d+ex^2)(a+cx^4)^5}{x^3}\,dx$

3.7.1 Optimal result

Integrand size = 20, antiderivative size = 142

$$\int \frac{(d+ex^2)(a+cx^4)^5}{x^3}\,dx = -\frac{a^5d}{2x^2}+\frac{5}{2}a^4cdx^2+\frac{5}{4}a^4cex^4+\frac{5}{3}a^3c^2dx^6+\frac{5}{4}a^3c^2ex^8+a^2c^3dx^{10}+\frac{5}{6}a^2c^3ex^{12}+\frac{5}{14}ac^4dx^{14}+\frac{5}{16}ac^4ex^{16}+\frac{1}{18}c^5dx^{18}+\frac{1}{20}c^5ex^{20}+a^5e\log(x)$$

output
```
-1/2*a^5*d/x^2+5/2*a^4*c*d*x^2+5/4*a^4*c*e*x^4+5/3*a^3*c^2*d*x^6+5/4*a^3*c
^2*e*x^8+a^2*c^3*d*x^10+5/6*a^2*c^3*e*x^12+5/14*a*c^4*d*x^14+5/16*a*c^4*e*
x^16+1/18*c^5*d*x^18+1/20*c^5*e*x^20+a^5*e*ln(x)
```

3.7.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)(a+cx^4)^5}{x^3}\,dx = -\frac{a^5d}{2x^2}+\frac{5}{2}a^4cdx^2+\frac{5}{4}a^4cex^4+\frac{5}{3}a^3c^2dx^6+\frac{5}{4}a^3c^2ex^8+a^2c^3dx^{10}+\frac{5}{6}a^2c^3ex^{12}+\frac{5}{14}ac^4dx^{14}+\frac{5}{16}ac^4ex^{16}+\frac{1}{18}c^5dx^{18}+\frac{1}{20}c^5ex^{20}+a^5e\log(x)$$

input
```
Integrate[((d + e*x^2)*(a + c*x^4)^5)/x^3,x]
```

output
```
-1/2*(a^5*d)/x^2 + (5*a^4*c*d*x^2)/2 + (5*a^4*c*e*x^4)/4 + (5*a^3*c^2*d*x^
6)/3 + (5*a^3*c^2*e*x^8)/4 + a^2*c^3*d*x^10 + (5*a^2*c^3*e*x^12)/6 + (5*a*
c^4*d*x^14)/14 + (5*a*c^4*e*x^16)/16 + (c^5*d*x^18)/18 + (c^5*e*x^20)/20 +
 a^5*e*Log[x]
```

3.7.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1579, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+cx^4\right)^5 \left(d+ex^2\right)}{x^3} \, dx$$

$$\Big\downarrow \; 1579$$

$$\frac{1}{2}\int \frac{\left(ex^2+d\right)\left(cx^4+a\right)^5}{x^4}dx^2$$

$$\Big\downarrow \; 522$$

$$\frac{1}{2}\int \Big(c^5ex^{18}+c^5dx^{16}+5ac^4ex^{14}+5ac^4dx^{12}+10a^2c^3ex^{10}+10a^2c^3dx^8+10a^3c^2ex^6+10a^3c^2dx^4+5a^4cex^2+5$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\Big(-\frac{a^5d}{x^2}+a^5e\log\left(x^2\right)+5a^4cdx^2+\frac{5}{2}a^4cex^4+\frac{10}{3}a^3c^2dx^6+\frac{5}{2}a^3c^2ex^8+2a^2c^3dx^{10}+\frac{5}{3}a^2c^3ex^{12}+\frac{5}{7}ac^4dx^{14}+$$

input
```
Int[((d + e*x^2)*(a + c*x^4)^5)/x^3,x]
```

output
```
(-((a^5*d)/x^2) + 5*a^4*c*d*x^2 + (5*a^4*c*e*x^4)/2 + (10*a^3*c^2*d*x^6)/3
 + (5*a^3*c^2*e*x^8)/2 + 2*a^2*c^3*d*x^10 + (5*a^2*c^3*e*x^12)/3 + (5*a*c^
4*d*x^14)/7 + (5*a*c^4*e*x^16)/8 + (c^5*d*x^18)/9 + (c^5*e*x^20)/10 + a^5*
e*Log[x^2])/2
```

3.7.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.7.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.87

method	result
default	$-\frac{a^5 d}{2x^2}+\frac{5a^4cd\,x^2}{2}+\frac{5a^4ce\,x^4}{4}+\frac{5a^3c^2d\,x^6}{3}+\frac{5a^3c^2e\,x^8}{4}+a^2c^3d\,x^{10}+\frac{5a^2c^3e\,x^{12}}{6}+\frac{5a\,c^4d\,x^{14}}{14}+\frac{5a\,c^4e\,x^{16}}{16}+$
risch	$-\frac{a^5 d}{2x^2}+\frac{5a^4cd\,x^2}{2}+\frac{5a^4ce\,x^4}{4}+\frac{5a^3c^2d\,x^6}{3}+\frac{5a^3c^2e\,x^8}{4}+a^2c^3d\,x^{10}+\frac{5a^2c^3e\,x^{12}}{6}+\frac{5a\,c^4d\,x^{14}}{14}+\frac{5a\,c^4e\,x^{16}}{16}+$
norman	$\frac{a^2c^3d\,x^{12}-\frac{1}{2}d\,a^5+\frac{1}{18}c^5d\,x^{20}+\frac{1}{20}c^5e\,x^{22}+\frac{5}{14}a\,c^4d\,x^{16}+\frac{5}{16}a\,c^4e\,x^{18}+\frac{5}{6}a^2c^3e\,x^{14}+\frac{5}{3}a^3c^2d\,x^8+\frac{5}{4}a^3c^2e\,x^{10}+\frac{5}{2}a^4cd\,x^4+\frac{5}{4}a^4ce}{x^2}$
parallelrisch	$\frac{252c^5e\,x^{22}+280c^5d\,x^{20}+1575a\,c^4e\,x^{18}+1800a\,c^4d\,x^{16}+4200a^2c^3e\,x^{14}+5040a^2c^3d\,x^{12}+6300a^3c^2e\,x^{10}+8400a^3c^2d\,x^8+6300a}{5040x^2}$

input
```
int((e*x^2+d)*(c*x^4+a)^5/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^5*d/x^2+5/2*a^4*c*d*x^2+5/4*a^4*c*e*x^4+5/3*a^3*c^2*d*x^6+5/4*a^3*c
^2*e*x^8+a^2*c^3*d*x^10+5/6*a^2*c^3*e*x^12+5/14*a*c^4*d*x^14+5/16*a*c^4*e*
x^16+1/18*c^5*d*x^18+1/20*c^5*e*x^20+a^5*e*ln(x)
```

3.7.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex^2)(a+cx^4)^5}{x^3}\,dx$$
$$= \frac{252\,c^5ex^{22} + 280\,c^5dx^{20} + 1575\,ac^4ex^{18} + 1800\,ac^4dx^{16} + 4200\,a^2c^3ex^{14} + 5040\,a^2c^3dx^{12} + 6300\,a^3c^2ex^{10} \cdots}{5040\,x^2}$$

input
```
integrate((e*x^2+d)*(c*x^4+a)^5/x^3,x, algorithm="fricas")
```

output
```
1/5040*(252*c^5*e*x^22 + 280*c^5*d*x^20 + 1575*a*c^4*e*x^18 + 1800*a*c^4*d
*x^16 + 4200*a^2*c^3*e*x^14 + 5040*a^2*c^3*d*x^12 + 6300*a^3*c^2*e*x^10 +
8400*a^3*c^2*d*x^8 + 6300*a^4*c*e*x^6 + 12600*a^4*c*d*x^4 + 5040*a^5*e*x^2
*log(x) - 2520*a^5*d)/x^2
```

3.7.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex^2)(a+cx^4)^5}{x^3}\,dx = -\frac{a^5d}{2x^2} + a^5e\log(x) + \frac{5a^4cdx^2}{2} + \frac{5a^4cex^4}{4} + \frac{5a^3c^2dx^6}{3} + \frac{5a^3c^2ex^8}{4} + a^2c^3dx^{10} + \frac{5a^2c^3ex^{12}}{6} + \frac{5ac^4dx^{14}}{14} + \frac{5ac^4ex^{16}}{16} + \frac{c^5dx^{18}}{18} + \frac{c^5ex^{20}}{20}$$

input
```
integrate((e*x**2+d)*(c*x**4+a)**5/x**3,x)
```

output
```
-a**5*d/(2*x**2) + a**5*e*log(x) + 5*a**4*c*d*x**2/2 + 5*a**4*c*e*x**4/4 +
 5*a**3*c**2*d*x**6/3 + 5*a**3*c**2*e*x**8/4 + a**2*c**3*d*x**10 + 5*a**2*
c**3*e*x**12/6 + 5*a*c**4*d*x**14/14 + 5*a*c**4*e*x**16/16 + c**5*d*x**18/
18 + c**5*e*x**20/20
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex^2)(a+cx^4)^5}{x^3}\,dx = \frac{1}{20}\,c^5ex^{20} + \frac{1}{18}\,c^5dx^{18} + \frac{5}{16}\,ac^4ex^{16} + \frac{5}{14}\,ac^4dx^{14} + \frac{5}{6}\,a^2c^3ex^{12} + a^2c^3dx^{10} + \frac{5}{4}\,a^3c^2ex^8 + \frac{5}{3}\,a^3c^2dx^6 + \frac{5}{4}\,a^4cex^4 + \frac{5}{2}\,a^4cdx^2 + \frac{1}{2}\,a^5e\log\left(x^2\right) - \frac{a^5d}{2\,x^2}$$

input
```
integrate((e*x^2+d)*(c*x^4+a)^5/x^3,x, algorithm="maxima")
```

output
```
1/20*c^5*e*x^20 + 1/18*c^5*d*x^18 + 5/16*a*c^4*e*x^16 + 5/14*a*c^4*d*x^14
+ 5/6*a^2*c^3*e*x^12 + a^2*c^3*d*x^10 + 5/4*a^3*c^2*e*x^8 + 5/3*a^3*c^2*d*
x^6 + 5/4*a^4*c*e*x^4 + 5/2*a^4*c*d*x^2 + 1/2*a^5*e*log(x^2) - 1/2*a^5*d/x
^2
```

3.7.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex^2)(a+cx^4)^5}{x^3}\,dx = \frac{1}{20}\,c^5ex^{20} + \frac{1}{18}\,c^5dx^{18} + \frac{5}{16}\,ac^4ex^{16} + \frac{5}{14}\,ac^4dx^{14} + \frac{5}{6}\,a^2c^3ex^{12} + a^2c^3dx^{10} + \frac{5}{4}\,a^3c^2ex^8 + \frac{5}{3}\,a^3c^2dx^6 + \frac{5}{4}\,a^4cex^4 + \frac{5}{2}\,a^4cdx^2 + \frac{1}{2}\,a^5e\log\left(x^2\right) - \frac{a^5ex^2 + a^5d}{2\,x^2}$$

input
```
integrate((e*x^2+d)*(c*x^4+a)^5/x^3,x, algorithm="giac")
```

output
```
1/20*c^5*e*x^20 + 1/18*c^5*d*x^18 + 5/16*a*c^4*e*x^16 + 5/14*a*c^4*d*x^14
+ 5/6*a^2*c^3*e*x^12 + a^2*c^3*d*x^10 + 5/4*a^3*c^2*e*x^8 + 5/3*a^3*c^2*d*
x^6 + 5/4*a^4*c*e*x^4 + 5/2*a^4*c*d*x^2 + 1/2*a^5*e*log(x^2) - 1/2*(a^5*e*
x^2 + a^5*d)/x^2
```

3.7.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex^2)(a+cx^4)^5}{x^3}\,dx = \frac{c^5\,d\,x^{18}}{18} - \frac{a^5\,d}{2\,x^2} + \frac{c^5\,e\,x^{20}}{20} + a^5\,e\,\ln(x) + \frac{5\,a^3\,c^2\,d\,x^6}{3} + a^2\,c^3\,d\,x^{10} + \frac{5\,a^3\,c^2\,e\,x^8}{4} + \frac{5\,a^2\,c^3\,e\,x^{12}}{6} + \frac{5\,a^4\,c\,d\,x^2}{2} + \frac{5\,a\,c^4\,d\,x^{14}}{14} + \frac{5\,a^4\,c\,e\,x^4}{4} + \frac{5\,a\,c^4\,e\,x^{16}}{16}$$

input
```
int(((a + c*x^4)^5*(d + e*x^2))/x^3,x)
```

output
```
(c^5*d*x^18)/18 - (a^5*d)/(2*x^2) + (c^5*e*x^20)/20 + a^5*e*log(x) + (5*a^
3*c^2*d*x^6)/3 + a^2*c^3*d*x^10 + (5*a^3*c^2*e*x^8)/4 + (5*a^2*c^3*e*x^12)
/6 + (5*a^4*c*d*x^2)/2 + (5*a*c^4*d*x^14)/14 + (5*a^4*c*e*x^4)/4 + (5*a*c^
4*e*x^16)/16
```

3.8 $\int x^5(2+3x^2)\sqrt{5+x^4}\,dx$

3.8.1 Optimal result

Integrand size = 20, antiderivative size = 67

$$\int x^5(2+3x^2)\sqrt{5+x^4}\,dx = -\frac{5}{8}x^2\sqrt{5+x^4}+\frac{3}{10}x^4(5+x^4)^{3/2} - \frac{1}{4}(4-x^2)(5+x^4)^{3/2}-\frac{25}{8}\operatorname{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)$$

output
```
3/10*x^4*(x^4+5)^(3/2)-1/4*(-x^2+4)*(x^4+5)^(3/2)-25/8*arcsinh(1/5*x^2*5^(
1/2))-5/8*x^2*(x^4+5)^(1/2)
```

3.8.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.84

$$\int x^5(2+3x^2)\sqrt{5+x^4}\,dx = \frac{1}{40}\sqrt{5+x^4}(-200+25x^2+20x^4+10x^6+12x^8) + \frac{25}{8}\log\left(-x^2+\sqrt{5+x^4}\right)$$

input
```
Integrate[x^5*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

output
```
(Sqrt[5 + x^4]*(-200 + 25*x^2 + 20*x^4 + 10*x^6 + 12*x^8))/40 + (25*Log[-x
^2 + Sqrt[5 + x^4]])/8
```

3.8.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.33, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1579, 533, 27, 533, 25, 455, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 (3x^2+2) \sqrt{x^4+5}\, dx$$

$$\Big\downarrow \text{ 1579}$$

$$\frac{1}{2} \int x^4 (3x^2+2) \sqrt{x^4+5}dx^2$$

$$\Big\downarrow \text{ 533}$$

$$\frac{1}{2} \left(\frac{3}{5}x^4 (x^4+5)^{3/2} - \frac{1}{5} \int 10x^2 (3-x^2) \sqrt{x^4+5}dx^2\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2} \left(\frac{3}{5}x^4 (x^4+5)^{3/2} - 2 \int x^2 (3-x^2) \sqrt{x^4+5}dx^2\right)$$

$$\Big\downarrow \text{ 533}$$

$$\frac{1}{2} \left(\frac{3}{5}x^4 (x^4+5)^{3/2} - 2\left(-\frac{1}{4} \int -\left((12x^2+5) \sqrt{x^4+5}\right) dx^2 - \frac{1}{4}(x^4+5)^{3/2} x^2\right)\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2} \left(\frac{3}{5}x^4 (x^4+5)^{3/2} - 2\left(\frac{1}{4} \int (12x^2+5) \sqrt{x^4+5}dx^2 - \frac{1}{4}x^2 (x^4+5)^{3/2}\right)\right)$$

$$\Big\downarrow \text{ 455}$$

$$\frac{1}{2} \left(\frac{3}{5}x^4 (x^4+5)^{3/2} - 2\left(\frac{1}{4}\left(5 \int \sqrt{x^4+5}dx^2 + 4(x^4+5)^{3/2}\right) - \frac{1}{4}x^2 (x^4+5)^{3/2}\right)\right)$$

$$\Big\downarrow \text{ 211}$$

$$\frac{1}{2} \left(\frac{3}{5}x^4 (x^4+5)^{3/2} - 2\left(\frac{1}{4}\left(5\left(\frac{5}{2} \int \frac{1}{\sqrt{x^4+5}}dx^2 + \frac{1}{2}\sqrt{x^4+5}x^2\right) + 4(x^4+5)^{3/2}\right) - \frac{1}{4}x^2 (x^4+5)^{3/2}\right)\right)$$

$$\Big\downarrow \text{ 222}$$

$$\frac{1}{2} \left(\frac{3}{5}x^4 (x^4+5)^{3/2} - 2\left(\frac{1}{4}\left(5\left(\frac{5}{2}\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{1}{2}\sqrt{x^4+5}x^2\right) + 4(x^4+5)^{3/2}\right) - \frac{1}{4}x^2 (x^4+5)^{3/2}\right)\right)$$

input
```
Int[x^5*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

output
```
((3*x^4*(5 + x^4)^(3/2))/5 - 2*(-1/4*(x^2*(5 + x^4)^(3/2)) + (4*(5 + x^4)^
(3/2) + 5*((x^2*Sqrt[5 + x^4])/2 + (5*ArcSinh[x^2/Sqrt[5]])/2))/4))/2
```

3.8.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

3.8.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.66

method	result	size
risch	$\frac{(12x^8+10x^6+20x^4+25x^2-200)\sqrt{x^4+5}}{40}-\frac{25\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{8}$	44
pseudoelliptic	$\frac{(12x^8+10x^6+20x^4+25x^2-200)\sqrt{x^4+5}}{40}-\frac{25\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{8}$	44
trager	$\left(\frac{3}{10}x^8+\frac{1}{4}x^6+\frac{1}{2}x^4+\frac{5}{8}x^2-5\right)\sqrt{x^4+5}-\frac{25\ln\left(x^2+\sqrt{x^4+5}\right)}{8}$	46
default	$\frac{(x^4+5)^{\frac{3}{2}}(3x^4-10)}{10}+\frac{x^2(x^4+5)^{\frac{3}{2}}}{4}-\frac{5x^2\sqrt{x^4+5}}{8}-\frac{25\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{8}$	53
elliptic	$\frac{3x^8\sqrt{x^4+5}}{10}+\frac{x^4\sqrt{x^4+5}}{2}-5\sqrt{x^4+5}+\frac{x^6\sqrt{x^4+5}}{4}+\frac{5x^2\sqrt{x^4+5}}{8}-\frac{25\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{8}$	70
meijerg	$-\frac{75\sqrt{5}\left(-\frac{8\sqrt{\pi}}{15}+\frac{4\sqrt{\pi}\left(1+\frac{x^4}{5}\right)^{\frac{3}{2}}\left(-\frac{3x^4}{5}+2\right)}{15}\right)}{8\sqrt{\pi}}-\frac{25\left(-\frac{\sqrt{\pi}\,x^2\sqrt{5}\left(\frac{6x^4}{5}+3\right)\sqrt{1+\frac{x^4}{5}}}{30}+\frac{\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}\right)}{4\sqrt{\pi}}$	84

input
```
int(x^5*(3*x^2+2)*(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/40*(12*x^8+10*x^6+20*x^4+25*x^2-200)*(x^4+5)^(1/2)-25/8*arcsinh(1/5*x^2*
5^(1/2))
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.72

$$\int x^5(2+3x^2)\sqrt{5+x^4}\,dx = \frac{1}{40}\left(12\,x^8+10\,x^6+20\,x^4+25\,x^2-200\right)\sqrt{x^4+5} + \frac{25}{8}\log\left(-x^2+\sqrt{x^4+5}\right)$$

input
```
integrate(x^5*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="fricas")
```

output
```
1/40*(12*x^8 + 10*x^6 + 20*x^4 + 25*x^2 - 200)*sqrt(x^4 + 5) + 25/8*log(-x
^2 + sqrt(x^4 + 5))
```

3.8.6 Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.90

$$\int x^5 (2+3x^2)\sqrt{5+x^4}\,dx = \sqrt{x^4+5}\left(\frac{x^6}{4}+\frac{5x^2}{8}\right) + \frac{3\sqrt{x^4+5}\left(\frac{x^8}{5}+\frac{x^4}{3}-\frac{10}{3}\right)}{2} - \frac{25\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{8}$$

input
```
integrate(x**5*(3*x**2+2)*(x**4+5)**(1/2),x)
```

output
```
sqrt(x**4 + 5)*(x**6/4 + 5*x**2/8) + 3*sqrt(x**4 + 5)*(x**8/5 + x**4/3 - 1
0/3)/2 - 25*asinh(sqrt(5)*x**2/5)/8
```

3.8.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(50) = 100.

Time = 0.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.52

$$\int x^5 (2+3x^2)\sqrt{5+x^4}\,dx = \frac{3}{10}\left(x^4+5\right)^{\frac{5}{2}} - \frac{5}{2}\left(x^4+5\right)^{\frac{3}{2}} - \frac{25\left(\frac{\sqrt{x^4+5}}{x^2}+\frac{(x^4+5)^{\frac{3}{2}}}{x^6}\right)}{8\left(\frac{2(x^4+5)}{x^4}-\frac{(x^4+5)^2}{x^8}-1\right)} - \frac{25}{16}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right) + \frac{25}{16}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

input
```
integrate(x^5*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="maxima")
```

output
```
3/10*(x^4 + 5)^(5/2) - 5/2*(x^4 + 5)^(3/2) - 25/8*(sqrt(x^4 + 5)/x^2 + (x^
4 + 5)^(3/2)/x^6)/(2*(x^4 + 5)/x^4 - (x^4 + 5)^2/x^8 - 1) - 25/16*log(sqrt
(x^4 + 5)/x^2 + 1) + 25/16*log(sqrt(x^4 + 5)/x^2 - 1)
```

3.8.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

$$\int x^5(2+3x^2)\sqrt{5+x^4}\,dx = \frac{1}{8}\left(2\,x^4+5\right)\sqrt{x^4+5}x^2 + \frac{3}{10}\left(x^4+5\right)^{\frac{5}{2}} - \frac{5}{2}\left(x^4+5\right)^{\frac{3}{2}} + \frac{25}{8}\log\left(-x^2+\sqrt{x^4+5}\right)$$

input
```
integrate(x^5*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="giac")
```

output
```
1/8*(2*x^4 + 5)*sqrt(x^4 + 5)*x^2 + 3/10*(x^4 + 5)^(5/2) - 5/2*(x^4 + 5)^(
3/2) + 25/8*log(-x^2 + sqrt(x^4 + 5))
```

3.8.9 Mupad [B] (verification not implemented)

Time = 7.73 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.63

$$\int x^5(2+3x^2)\sqrt{5+x^4}\,dx = \sqrt{x^4+5}\left(\frac{3\,x^8}{10}+\frac{x^6}{4}+\frac{x^4}{2}+\frac{5\,x^2}{8}-5\right) - \frac{25\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{8}$$

input
```
int(x^5*(x^4 + 5)^(1/2)*(3*x^2 + 2),x)
```

output
```
(x^4 + 5)^(1/2)*((5*x^2)/8 + x^4/2 + x^6/4 + (3*x^8)/10 - 5) - (25*asinh((
5^(1/2)*x^2)/5))/8
```

3.9 $\int x^3(2+3x^2)\sqrt{5+x^4}\,dx$

3.9.1 Optimal result

Integrand size = 20, antiderivative size = 51

$$\int x^3(2+3x^2)\sqrt{5+x^4}\,dx=-\frac{15}{16}x^2\sqrt{5+x^4}+\frac{1}{24}(8+9x^2)\left(5+x^4\right)^{3/2}-\frac{75}{16}\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)$$

output
```
1/24*(9*x^2+8)*(x^4+5)^(3/2)-75/16*arcsinh(1/5*x^2*5^(1/2))-15/16*x^2*(x^4
+5)^(1/2)
```

3.9.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int x^3(2+3x^2)\sqrt{5+x^4}\,dx=\frac{1}{48}\sqrt{5+x^4}(80+45x^2+16x^4+18x^6)+\frac{75}{16}\log\left(-x^2+\sqrt{5+x^4}\right)$$

input
```
Integrate[x^3*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

output
```
(Sqrt[5 + x^4]*(80 + 45*x^2 + 16*x^4 + 18*x^6))/48 + (75*Log[-x^2 + Sqrt[5
 + x^4]])/16
```

3.9.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.41, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1579, 533, 455, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(3x^2+2\right)\sqrt{x^4+5}\,dx$$

$$\Big\downarrow \text{1579}$$

$$\frac{1}{2}\int x^2\left(3x^2+2\right)\sqrt{x^4+5}dx^2$$

$$\Big\downarrow \text{533}$$

$$\frac{1}{2}\left(\frac{3}{4}x^2\left(x^4+5\right)^{3/2}-\frac{1}{4}\int\left(15-8x^2\right)\sqrt{x^4+5}dx^2\right)$$

$$\Big\downarrow \text{455}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{8}{3}\left(x^4+5\right)^{3/2}-15\int\sqrt{x^4+5}dx^2\right)+\frac{3}{4}\left(x^4+5\right)^{3/2}x^2\right)$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{8}{3}\left(x^4+5\right)^{3/2}-15\left(\frac{5}{2}\int\frac{1}{\sqrt{x^4+5}}dx^2+\frac{1}{2}\sqrt{x^4+5}x^2\right)\right)+\frac{3}{4}\left(x^4+5\right)^{3/2}x^2\right)$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{8}{3}\left(x^4+5\right)^{3/2}-15\left(\frac{5}{2}\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{2}\sqrt{x^4+5}x^2\right)\right)+\frac{3}{4}\left(x^4+5\right)^{3/2}x^2\right)$$

input
```
Int[x^3*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

output
```
((3*x^2*(5 + x^4)^(3/2))/4 + ((8*(5 + x^4)^(3/2))/3 - 15*((x^2*Sqrt[5 + x^
4])/2 + (5*ArcSinh[x^2/Sqrt[5]])/2))/4)/2
```

3.9.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

3.9.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{(18x^6+16x^4+45x^2+80)\sqrt{x^4+5}}{48}-\frac{75\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{16}$	39
pseudoelliptic	$\frac{(18x^6+16x^4+45x^2+80)\sqrt{x^4+5}}{48}-\frac{75\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{16}$	39
trager	$\left(\frac{3}{8}x^6+\frac{1}{3}x^4+\frac{15}{16}x^2+\frac{5}{3}\right)\sqrt{x^4+5}+\frac{75\ln\left(x^2-\sqrt{x^4+5}\right)}{16}$	43
default	$\frac{3x^2(x^4+5)^{\frac{3}{2}}}{8}-\frac{15x^2\sqrt{x^4+5}}{16}-\frac{75\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{16}+\frac{(x^4+5)^{\frac{3}{2}}}{3}$	46
elliptic	$\frac{5\sqrt{x^4+5}}{3}+\frac{3x^6\sqrt{x^4+5}}{8}+\frac{15x^2\sqrt{x^4+5}}{16}-\frac{75\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{16}+\frac{x^4\sqrt{x^4+5}}{3}$	58
meijerg	$-\frac{75\left(-\frac{\sqrt{\pi}\,x^2\sqrt{5}\left(\frac{6x^4}{5}+3\right)\sqrt{1+\frac{x^4}{5}}}{30}+\frac{\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}\right)}{8\sqrt{\pi}}-\frac{5\sqrt{5}\left(\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\left(2+\frac{2x^4}{5}\right)\sqrt{1+\frac{x^4}{5}}}{3}\right)}{4\sqrt{\pi}}$	84

input
```
int(x^3*(3*x^2+2)*(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/48*(18*x^6+16*x^4+45*x^2+80)*(x^4+5)^(1/2)-75/16*arcsinh(1/5*x^2*5^(1/2)
)
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int x^3(2+3x^2)\sqrt{5+x^4}\,dx=\frac{1}{48}\left(18\,x^6+16\,x^4+45\,x^2+80\right)\sqrt{x^4+5}+\frac{75}{16}\,\log\left(-x^2+\sqrt{x^4+5}\right)$$

input
```
integrate(x^3*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="fricas")
```

output
```
1/48*(18*x^6 + 16*x^4 + 45*x^2 + 80)*sqrt(x^4 + 5) + 75/16*log(-x^2 + sqrt
(x^4 + 5))
```

3.9.6 Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int x^3(2+3x^2)\sqrt{5+x^4}\,dx = \left(\frac{x^4}{3}+\frac{5}{3}\right)\sqrt{x^4+5}+\frac{3\sqrt{x^4+5}\left(\frac{x^6}{4}+\frac{5x^2}{8}\right)}{2}-\frac{75\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{16}$$

input
```
integrate(x**3*(3*x**2+2)*(x**4+5)**(1/2),x)
```

output
```
(x**4/3 + 5/3)*sqrt(x**4 + 5) + 3*sqrt(x**4 + 5)*(x**6/4 + 5*x**2/8)/2 - 7
5*asinh(sqrt(5)*x**2/5)/16
```

3.9.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(40) = 80.

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.82

$$\int x^3(2+3x^2)\sqrt{5+x^4}\,dx = \frac{1}{3}\left(x^4+5\right)^{\frac{3}{2}}-\frac{75\left(\frac{\sqrt{x^4+5}}{x^2}+\frac{(x^4+5)^{\frac{3}{2}}}{x^6}\right)}{16\left(\frac{2(x^4+5)}{x^4}-\frac{(x^4+5)^2}{x^8}-1\right)} - \frac{75}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)+\frac{75}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

input
```
integrate(x^3*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="maxima")
```

output
```
1/3*(x^4 + 5)^(3/2) - 75/16*(sqrt(x^4 + 5)/x^2 + (x^4 + 5)^(3/2)/x^6)/(2*(
x^4 + 5)/x^4 - (x^4 + 5)^2/x^8 - 1) - 75/32*log(sqrt(x^4 + 5)/x^2 + 1) + 7
5/32*log(sqrt(x^4 + 5)/x^2 - 1)
```

3.9.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.88

$$\int x^3(2+3x^2)\sqrt{5+x^4}\,dx = \frac{3}{16}\left(2\,x^4+5\right)\sqrt{x^4+5}x^2+\frac{1}{3}\left(x^4+5\right)^{\frac{3}{2}}+\frac{75}{16}\,\log\left(-x^2+\sqrt{x^4+5}\right)$$

input
```
integrate(x^3*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="giac")
```

output
```
3/16*(2*x^4 + 5)*sqrt(x^4 + 5)*x^2 + 1/3*(x^4 + 5)^(3/2) + 75/16*log(-x^2
+ sqrt(x^4 + 5))
```

3.9.9 Mupad [B] (verification not implemented)

Time = 7.82 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

$$\int x^3(2+3x^2)\sqrt{5+x^4}\,dx = \sqrt{x^4+5}\left(\frac{3\,x^6}{8}+\frac{x^4}{3}+\frac{15\,x^2}{16}+\frac{5}{3}\right)-\frac{75\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{16}$$

input
```
int(x^3*(x^4 + 5)^(1/2)*(3*x^2 + 2),x)
```

output
```
(x^4 + 5)^(1/2)*((15*x^2)/16 + x^4/3 + (3*x^6)/8 + 5/3) - (75*asinh((5^(1/
2)*x^2)/5))/16
```

3.10 $\int x(2+3x^2)\sqrt{5+x^4}\,dx$

3.10.1 Optimal result

Integrand size = 18, antiderivative size = 44

$$\int x(2+3x^2)\sqrt{5+x^4}\,dx = \frac{1}{2}x^2\sqrt{5+x^4}+\frac{1}{2}(5+x^4)^{3/2}+\frac{5}{2}\operatorname{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)$$

output
```
1/2*(x^4+5)^(3/2)+5/2*arcsinh(1/5*x^2*5^(1/2))+1/2*x^2*(x^4+5)^(1/2)
```

3.10.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int x(2+3x^2)\sqrt{5+x^4}\,dx = \frac{1}{2}\sqrt{5+x^4}(5+x^2+x^4)-\frac{5}{2}\log\left(-x^2+\sqrt{5+x^4}\right)$$

input
```
Integrate[x*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

output
```
(Sqrt[5 + x^4]*(5 + x^2 + x^4))/2 - (5*Log[-x^2 + Sqrt[5 + x^4]])/2
```

3.10.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1577, 455, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(3x^2+2\right)\sqrt{x^4+5}\,dx$$

$$\Big\downarrow \text{1577}$$

$$\frac{1}{2}\int \left(3x^2+2\right)\sqrt{x^4+5}dx^2$$

$$\Big\downarrow \text{455}$$

$$\frac{1}{2}\left(2\int \sqrt{x^4+5}dx^2+\left(x^4+5\right)^{3/2}\right)$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}\left(2\left(\frac{5}{2}\int \frac{1}{\sqrt{x^4+5}}dx^2+\frac{1}{2}\sqrt{x^4+5}x^2\right)+\left(x^4+5\right)^{3/2}\right)$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{2}\left(2\left(\frac{5}{2}\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{2}\sqrt{x^4+5}x^2\right)+\left(x^4+5\right)^{3/2}\right)$$

input
```
Int[x*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

output
```
((5 + x^4)^(3/2) + 2*((x^2*Sqrt[5 + x^4])/2 + (5*ArcSinh[x^2/Sqrt[5]])/2))
/2
```

3.10.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

3.10.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.68

method	result	size
risch	$\frac{(x^4+x^2+5)\sqrt{x^4+5}}{2}+\frac{5\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}$	30
pseudoelliptic	$\frac{(x^4+x^2+5)\sqrt{x^4+5}}{2}+\frac{5\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}$	30
default	$\frac{(x^4+5)^{\frac{3}{2}}}{2}+\frac{5\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}+\frac{x^2\sqrt{x^4+5}}{2}$	34
trager	$\left(\frac{1}{2}x^4+\frac{1}{2}x^2+\frac{5}{2}\right)\sqrt{x^4+5}+\frac{5\ln\left(x^2+\sqrt{x^4+5}\right)}{2}$	36
elliptic	$\frac{5\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}+\frac{5\sqrt{x^4+5}}{2}+\frac{x^4\sqrt{x^4+5}}{2}+\frac{x^2\sqrt{x^4+5}}{2}$	46
meijerg	$-\frac{15\sqrt{5}\left(\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\left(2+\frac{2x^4}{5}\right)\sqrt{1+\frac{x^4}{5}}}{3}\right)}{8\sqrt{\pi}}-\frac{5\left(-\frac{2\sqrt{\pi}\,x^2\sqrt{5}\,\sqrt{1+\frac{x^4}{5}}}{5}-2\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)\right)}{4\sqrt{\pi}}$	77

input
```
int(x*(3*x^2+2)*(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(x^4+x^2+5)*(x^4+5)^(1/2)+5/2*arcsinh(1/5*x^2*5^(1/2))
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int x(2+3x^2)\sqrt{5+x^4}\,dx = \frac{1}{2}\left(x^4+x^2+5\right)\sqrt{x^4+5} - \frac{5}{2}\log\left(-x^2+\sqrt{x^4+5}\right)$$

input
```
integrate(x*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(x^4 + x^2 + 5)*sqrt(x^4 + 5) - 5/2*log(-x^2 + sqrt(x^4 + 5))
```

3.10.6 Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09

$$\int x(2+3x^2)\sqrt{5+x^4}\,dx = \frac{x^2\sqrt{x^4+5}}{2} + \frac{3\left(\frac{x^4}{3}+\frac{5}{3}\right)\sqrt{x^4+5}}{2} + \frac{5\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{2}$$

input
```
integrate(x*(3*x**2+2)*(x**4+5)**(1/2),x)
```

output
```
x**2*sqrt(x**4 + 5)/2 + 3*(x**4/3 + 5/3)*sqrt(x**4 + 5)/2 + 5*asinh(sqrt(5
)*x**2/5)/2
```

3.10.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(33) = 66$.

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.52

$$\int x(2+3x^2)\sqrt{5+x^4}\,dx = \frac{1}{2}\left(x^4+5\right)^{\frac{3}{2}} + \frac{5\sqrt{x^4+5}}{2x^2\left(\frac{x^4+5}{x^4}-1\right)} + \frac{5}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right) - \frac{5}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

input
```
integrate(x*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(x^4 + 5)^(3/2) + 5/2*sqrt(x^4 + 5)/(x^2*((x^4 + 5)/x^4 - 1)) + 5/4*lo
g(sqrt(x^4 + 5)/x^2 + 1) - 5/4*log(sqrt(x^4 + 5)/x^2 - 1)
```

3.10.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int x(2+3x^2)\sqrt{5+x^4}\,dx = \frac{1}{2}\sqrt{x^4+5}x^2 + \frac{1}{2}\left(x^4+5\right)^{\frac{3}{2}} - \frac{5}{2}\log\left(-x^2+\sqrt{x^4+5}\right)$$

input
```
integrate(x*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(x^4 + 5)*x^2 + 1/2*(x^4 + 5)^(3/2) - 5/2*log(-x^2 + sqrt(x^4 + 5)
)
```

3.10.9 Mupad [B] (verification not implemented)

Time = 7.56 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.73

$$\int x(2+3x^2)\sqrt{5+x^4}\,dx = \frac{5\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{2} + \sqrt{x^4+5}\left(\frac{x^4}{2}+\frac{x^2}{2}+\frac{5}{2}\right)$$

input
```
int(x*(x^4 + 5)^(1/2)*(3*x^2 + 2),x)
```

output
```
(5*asinh((5^(1/2)*x^2)/5))/2 + (x^4 + 5)^(1/2)*(x^2/2 + x^4/2 + 5/2)
```

3.11 $\int \frac{(2+3x^2)\sqrt{5+x^4}}{x}\,dx$

3.11.1 Optimal result

Integrand size = 20, antiderivative size = 58

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x}\,dx = \frac{1}{4}(4+3x^2)\sqrt{5+x^4}+\frac{15}{4}\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)-\sqrt{5}\text{arctanh}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)$$

output
```
15/4*arcsinh(1/5*x^2*5^(1/2))-arctanh(1/5*(x^4+5)^(1/2)*5^(1/2))*5^(1/2)+1
/4*(3*x^2+4)*(x^4+5)^(1/2)
```

3.11.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.19

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x}\,dx = \frac{1}{4}\left((4+3x^2)\sqrt{5+x^4}+8\sqrt{5}\text{arctanh}\left(\frac{x^2-\sqrt{5+x^4}}{\sqrt{5}}\right) - 15\log\left(-x^2+\sqrt{5+x^4}\right)\right)$$

input
```
Integrate[((2 + 3*x^2)*Sqrt[5 + x^4])/x,x]
```

output
```
((4 + 3*x^2)*Sqrt[5 + x^4] + 8*Sqrt[5]*ArcTanh[(x^2 - Sqrt[5 + x^4])/Sqrt[
5]] - 15*Log[-x^2 + Sqrt[5 + x^4]])/4
```

3.11.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1579, 535, 538, 222, 243, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x^2+2)\sqrt{x^4+5}}{x}dx$$

$$\Big\downarrow \text{1579}$$

$$\frac{1}{2}\int \frac{(3x^2+2)\sqrt{x^4+5}}{x^2}dx^2$$

$$\Big\downarrow \text{535}$$

$$\frac{1}{2}\left(\frac{5}{2}\int \frac{3x^2+4}{x^2\sqrt{x^4+5}}dx^2+\frac{1}{2}\sqrt{x^4+5}(3x^2+4)\right)$$

$$\Big\downarrow \text{538}$$

$$\frac{1}{2}\left(\frac{5}{2}\left(3\int \frac{1}{\sqrt{x^4+5}}dx^2+4\int \frac{1}{x^2\sqrt{x^4+5}}dx^2\right)+\frac{1}{2}\sqrt{x^4+5}(3x^2+4)\right)$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{2}\left(\frac{5}{2}\left(4\int \frac{1}{x^2\sqrt{x^4+5}}dx^2+3\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)\right)+\frac{1}{2}\sqrt{x^4+5}(3x^2+4)\right)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\left(\frac{5}{2}\left(2\int \frac{1}{x^2\sqrt{x^4+5}}dx^4+3\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)\right)+\frac{1}{2}\sqrt{x^4+5}(3x^2+4)\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{5}{2}\left(4\int \frac{1}{\sqrt{x^4+5}-5}d\sqrt{x^4+5}+3\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)\right)+\frac{1}{2}\sqrt{x^4+5}(3x^2+4)\right)$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{2}\left(\frac{5}{2}\left(3\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{4\text{arctanh}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{\sqrt{5}}\right)+\frac{1}{2}\sqrt{x^4+5}(3x^2+4)\right)$$

input
```
Int[((2 + 3*x^2)*Sqrt[5 + x^4])/x,x]
```

output
```
(((4 + 3*x^2)*Sqrt[5 + x^4])/2 + (5*(3*ArcSinh[x^2/Sqrt[5]] - (4*ArcTanh[S
qrt[5 + x^4]/Sqrt[5]])/Sqrt[5]))/2)/2
```

3.11.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

3.11.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.84

method	result
default	$\frac{3x^2\sqrt{x^4+5}}{4}+\frac{15\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{4}+\sqrt{x^4+5}-\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)$
elliptic	$\frac{3x^2\sqrt{x^4+5}}{4}+\frac{15\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{4}+\sqrt{x^4+5}-\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)$
pseudoelliptic	$\frac{3x^2\sqrt{x^4+5}}{4}+\frac{15\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{4}+\sqrt{x^4+5}-\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)$
trager	$\left(\frac{3x^2}{4}+1\right)\sqrt{x^4+5}-\frac{15\ln\left(x^2-\sqrt{x^4+5}\right)}{4}+\operatorname{RootOf}\left(\_Z^2-5\right)\ln\left(\frac{\sqrt{x^4+5}-\operatorname{RootOf}\left(\_Z^2-5\right)}{x^2}\right)$
meijerg	$-\frac{\sqrt{5}\left(4\sqrt{\pi}-4\sqrt{\pi}\,\sqrt{1+\frac{x^4}{5}}+4\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{x^4}{5}}}{2}\right)-2(2-2\ln(2)+4\ln(x)-\ln(5))\sqrt{\pi}\right)}{4\sqrt{\pi}}-\frac{15\left(-\frac{2\sqrt{\pi}\,x^2\sqrt{5}\,\sqrt{1+\frac{x^4}{5}}}{5}-2\sqrt{\pi}\right.}{8\sqrt{\pi}}$

input
```
int((3*x^2+2)*(x^4+5)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
3/4*x^2*(x^4+5)^(1/2)+15/4*arcsinh(1/5*x^2*5^(1/2))+(x^4+5)^(1/2)-5^(1/2)*
arctanh(5^(1/2)/(x^4+5)^(1/2))
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x}\,dx = \frac{1}{4}\sqrt{x^4+5}(3\,x^2+4)+\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right) - \frac{15}{4}\log\left(-x^2+\sqrt{x^4+5}\right)$$

input
```
integrate((3*x^2+2)*(x^4+5)^(1/2)/x,x, algorithm="fricas")
```

output
```
1/4*sqrt(x^4 + 5)*(3*x^2 + 4) + sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2
) - 15/4*log(-x^2 + sqrt(x^4 + 5))
```

3.11.6 Sympy [A] (verification not implemented)

Time = 4.78 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.29

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x}\,dx = \frac{3x^2\sqrt{x^4+5}}{4} + \sqrt{5}\left(\sqrt{\frac{x^4}{5}+1} + \frac{\log\left(\sqrt{\frac{x^4}{5}+1}-1\right)}{2} - \frac{\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{2}\right) + \frac{15\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{4}$$

input
```
integrate((3*x**2+2)*(x**4+5)**(1/2)/x,x)
```

output
```
3*x**2*sqrt(x**4 + 5)/4 + sqrt(5)*(sqrt(x**4/5 + 1) + log(sqrt(x**4/5 + 1)
 - 1)/2 - log(sqrt(x**4/5 + 1) + 1)/2) + 15*asinh(sqrt(5)*x**2/5)/4
```

3.11.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(46) = 92.

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.71

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x}\,dx = \frac{1}{2}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right) + \sqrt{x^4+5} + \frac{15\sqrt{x^4+5}}{4\,x^2\left(\frac{x^4+5}{x^4}-1\right)} + \frac{15}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right) - \frac{15}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

input
```
integrate((3*x^2+2)*(x^4+5)^(1/2)/x,x, algorithm="maxima")
```

output
```
1/2*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) + sq
rt(x^4 + 5) + 15/4*sqrt(x^4 + 5)/(x^2*((x^4 + 5)/x^4 - 1)) + 15/8*log(sqrt
(x^4 + 5)/x^2 + 1) - 15/8*log(sqrt(x^4 + 5)/x^2 - 1)
```

3.11.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.31

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x}\,dx = \frac{1}{4}\sqrt{x^4+5}\left(3x^2+4\right)+\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right) - \frac{15}{4}\log\left(-x^2+\sqrt{x^4+5}\right)$$

input
```
integrate((3*x^2+2)*(x^4+5)^(1/2)/x,x, algorithm="giac")
```

output
```
1/4*sqrt(x^4 + 5)*(3*x^2 + 4) + sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5
))/(x^2 - sqrt(5) - sqrt(x^4 + 5))) - 15/4*log(-x^2 + sqrt(x^4 + 5))
```

3.11.9 Mupad [B] (verification not implemented)

Time = 7.61 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.78

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x}\,dx = \frac{15\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{4} - \sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}}{5}\right) + \sqrt{x^4+5}\left(\frac{3\,x^2}{4}+1\right)$$

input
```
int(((x^4 + 5)^(1/2)*(3*x^2 + 2))/x,x)
```

output
```
(15*asinh((5^(1/2)*x^2)/5))/4 - 5^(1/2)*atanh((5^(1/2)*(x^4 + 5)^(1/2))/5)
 + (x^4 + 5)^(1/2)*((3*x^2)/4 + 1)
```

3.12 $\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^3}\,dx$

3.12.1 Optimal result

Integrand size = 20, antiderivative size = 59

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^3}\,dx = -\frac{(2-3x^2)\sqrt{5+x^4}}{2x^2} + \operatorname{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{3}{2}\sqrt{5}\operatorname{arctanh}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)$$

output
```
arcsinh(1/5*x^2*5^(1/2))-3/2*arctanh(1/5*(x^4+5)^(1/2)*5^(1/2))*5^(1/2)-1/
2*(-3*x^2+2)*(x^4+5)^(1/2)/x^2
```

3.12.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.20

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^3}\,dx = \frac{(-2+3x^2)\sqrt{5+x^4}}{2x^2} + 3\sqrt{5}\operatorname{arctanh}\left(\frac{x^2-\sqrt{5+x^4}}{\sqrt{5}}\right) - \log\left(-x^2+\sqrt{5+x^4}\right)$$

input
```
Integrate[((2 + 3*x^2)*Sqrt[5 + x^4])/x^3,x]
```

output
```
((-2 + 3*x^2)*Sqrt[5 + x^4])/(2*x^2) + 3*Sqrt[5]*ArcTanh[(x^2 - Sqrt[5 + x
^4])/Sqrt[5]] - Log[-x^2 + Sqrt[5 + x^4]]
```

3.12.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1579, 536, 538, 222, 243, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x^2+2)\sqrt{x^4+5}}{x^3}dx$$

$$\Big\downarrow \text{ 1579}$$

$$\frac{1}{2}\int \frac{(3x^2+2)\sqrt{x^4+5}}{x^4}dx^2$$

$$\Big\downarrow \text{ 536}$$

$$\frac{1}{2}\left(\int \frac{2x^2+15}{x^2\sqrt{x^4+5}}dx^2-\frac{(2-3x^2)\sqrt{x^4+5}}{x^2}\right)$$

$$\Big\downarrow \text{ 538}$$

$$\frac{1}{2}\left(2\int \frac{1}{\sqrt{x^4+5}}dx^2+15\int \frac{1}{x^2\sqrt{x^4+5}}dx^2-\frac{\sqrt{x^4+5}(2-3x^2)}{x^2}\right)$$

$$\Big\downarrow \text{ 222}$$

$$\frac{1}{2}\left(15\int \frac{1}{x^2\sqrt{x^4+5}}dx^2+2\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\sqrt{x^4+5}(2-3x^2)}{x^2}\right)$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\left(\frac{15}{2}\int \frac{1}{x^2\sqrt{x^4+5}}dx^4+2\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\sqrt{x^4+5}(2-3x^2)}{x^2}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2}\left(15\int \frac{1}{\sqrt{x^4+5}-5}d\sqrt{x^4+5}+2\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\sqrt{x^4+5}(2-3x^2)}{x^2}\right)$$

$$\Big\downarrow \text{ 220}$$

$$\frac{1}{2}\left(2\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)-3\sqrt{5}\text{arctanh}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)-\frac{\sqrt{x^4+5}(2-3x^2)}{x^2}\right)$$

input
```
Int[((2 + 3*x^2)*Sqrt[5 + x^4])/x^3,x]
```

output
```
(-(((2 - 3*x^2)*Sqrt[5 + x^4])/x^2) + 2*ArcSinh[x^2/Sqrt[5]] - 3*Sqrt[5]*A
rcTanh[Sqrt[5 + x^4]/Sqrt[5]])/2
```

3.12.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 536
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_)^2, x_Symbol] :> S
imp[(-(2*c*p - d*x))*((a + b*x^2)^p/(2*p*x)), x] + Int[(a*d + 2*b*c*p*x)*((
a + b*x^2)^(p - 1)/x), x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && Integer
Q[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

3.12.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

method	result
risch	$-\frac{\sqrt{x^4+5}}{x^2}+\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)+\frac{3\sqrt{x^4+5}}{2}-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{2}$
elliptic	$-\frac{\sqrt{x^4+5}}{x^2}+\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)+\frac{3\sqrt{x^4+5}}{2}-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{2}$
default	$\frac{3\sqrt{x^4+5}}{2}-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{2}-\frac{(x^4+5)^{\frac{3}{2}}}{5x^2}+\frac{x^2\sqrt{x^4+5}}{5}+\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)$
pseudoelliptic	$\frac{-3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)x^2+3x^2\sqrt{x^4+5}+2\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)x^2-2\sqrt{x^4+5}}{2x^2}$
trager	$\frac{(3x^2-2)\sqrt{x^4+5}}{2x^2}+\ln\left(-x^2-\sqrt{x^4+5}\right)+\frac{3\operatorname{RootOf}\left(\_Z^2-5\right)\ln\left(\frac{\sqrt{x^4+5}-\operatorname{RootOf}\left(\_Z^2-5\right)}{x^2}\right)}{2}$
meijerg	$-\frac{\frac{4\sqrt{\pi}\sqrt{5}\sqrt{1+\frac{x^4}{5}}}{x^2}-4\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{4\sqrt{\pi}}-\frac{3\sqrt{5}\left(4\sqrt{\pi}-4\sqrt{\pi}\sqrt{1+\frac{x^4}{5}}+4\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{x^4}{5}}}{2}\right)-2(2-2\ln(2)+4\ln(x)-\ln(\right.}{8\sqrt{\pi}}$

input
```
int((3*x^2+2)*(x^4+5)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-(x^4+5)^(1/2)/x^2+arcsinh(1/5*x^2*5^(1/2))+3/2*(x^4+5)^(1/2)-3/2*5^(1/2)*
arctanh(5^(1/2)/(x^4+5)^(1/2))
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.22

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^3}\,dx = \frac{3\sqrt{5}x^2\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right) - 2\,x^2\log\left(-x^2+\sqrt{x^4+5}\right) - 2\,x^2 + \sqrt{x^4+5}(3\,x^2-2)}{2\,x^2}$$

input
```
integrate((3*x^2+2)*(x^4+5)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
1/2*(3*sqrt(5)*x^2*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 2*x^2*log(-x^2 +
sqrt(x^4 + 5)) - 2*x^2 + sqrt(x^4 + 5)*(3*x^2 - 2))/x^2
```

3.12.6 Sympy [A] (verification not implemented)

Time = 3.31 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.41

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^3}\,dx = -\frac{x^2}{\sqrt{x^4+5}} + \frac{3\sqrt{x^4+5}}{2} + \frac{3\sqrt{5}\log\left(x^4\right)}{4} - \frac{3\sqrt{5}\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{2} + \operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right) - \frac{5}{x^2\sqrt{x^4+5}}$$

input
```
integrate((3*x**2+2)*(x**4+5)**(1/2)/x**3,x)
```

output
```
-x**2/sqrt(x**4 + 5) + 3*sqrt(x**4 + 5)/2 + 3*sqrt(5)*log(x**4)/4 - 3*sqrt
(5)*log(sqrt(x**4/5 + 1) + 1)/2 + asinh(sqrt(5)*x**2/5) - 5/(x**2*sqrt(x**
4 + 5))
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.49

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^3}\,dx = \frac{3}{4}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right) + \frac{3}{2}\sqrt{x^4+5} - \frac{\sqrt{x^4+5}}{x^2} + \frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right) - \frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

input
```
integrate((3*x^2+2)*(x^4+5)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
3/4*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) + 3/
2*sqrt(x^4 + 5) - sqrt(x^4 + 5)/x^2 + 1/2*log(sqrt(x^4 + 5)/x^2 + 1) - 1/2
*log(sqrt(x^4 + 5)/x^2 - 1)
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.54

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^3}\,dx = \frac{3}{2}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right) + \frac{3}{2}\sqrt{x^4+5} + \frac{10}{\left(x^2-\sqrt{x^4+5}\right)^2-5} - \log\left(-x^2+\sqrt{x^4+5}\right)$$

input
```
integrate((3*x^2+2)*(x^4+5)^(1/2)/x^3,x, algorithm="giac")
```

output
```
3/2*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4
 + 5))) + 3/2*sqrt(x^4 + 5) + 10/((x^2 - sqrt(x^4 + 5))^2 - 5) - log(-x^2
+ sqrt(x^4 + 5))
```

3.12.9 Mupad [B] (verification not implemented)

Time = 7.89 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^3}\,dx = \operatorname{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right) + \frac{3\sqrt{x^4+5}}{2} - \frac{\sqrt{x^4+5}}{x^2} + \frac{\sqrt{5}\operatorname{atan}\left(\frac{\sqrt{5}\sqrt{x^4+5}\,1\mathrm{i}}{5}\right)\,3\mathrm{i}}{2}$$

input
```
int(((x^4 + 5)^(1/2)*(3*x^2 + 2))/x^3,x)
```

output
```
asinh((5^(1/2)*x^2)/5) + (5^(1/2)*atan((5^(1/2)*(x^4 + 5)^(1/2)*1i)/5)*3i)
/2 + (3*(x^4 + 5)^(1/2))/2 - (x^4 + 5)^(1/2)/x^2
```

3.13 $\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^5}\,dx$

3.13.1 Optimal result

Integrand size = 20, antiderivative size = 63

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^5}\,dx = -\frac{(1+3x^2)\sqrt{5+x^4}}{2x^4}+\frac{3}{2}\operatorname{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\operatorname{arctanh}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{2\sqrt{5}}$$

output
```
3/2*arcsinh(1/5*x^2*5^(1/2))-1/10*arctanh(1/5*(x^4+5)^(1/2)*5^(1/2))*5^(1/
2)-1/2*(3*x^2+1)*(x^4+5)^(1/2)/x^4
```

3.13.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.14

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^5}\,dx = -\frac{(1+3x^2)\sqrt{5+x^4}}{2x^4}+\frac{\operatorname{arctanh}\left(\frac{x^2-\sqrt{5+x^4}}{\sqrt{5}}\right)}{\sqrt{5}} - \frac{3}{2}\log\left(-x^2+\sqrt{5+x^4}\right)$$

input
```
Integrate[((2 + 3*x^2)*Sqrt[5 + x^4])/x^5,x]
```

output
```
-1/2*((1 + 3*x^2)*Sqrt[5 + x^4])/x^4 + ArcTanh[(x^2 - Sqrt[5 + x^4])/Sqrt[
5]]/Sqrt[5] - (3*Log[-x^2 + Sqrt[5 + x^4]])/2
```

3.13.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1579, 537, 27, 538, 222, 243, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x^2+2)\sqrt{x^4+5}}{x^5}dx$$

$$\Big\downarrow \text{ 1579}$$

$$\frac{1}{2}\int \frac{(3x^2+2)\sqrt{x^4+5}}{x^6}dx^2$$

$$\Big\downarrow \text{ 537}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int -\frac{2(3x^2+1)}{x^2\sqrt{x^4+5}}dx^2-\frac{\sqrt{x^4+5}(3x^2+1)}{x^4}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(\int \frac{3x^2+1}{x^2\sqrt{x^4+5}}dx^2-\frac{(3x^2+1)\sqrt{x^4+5}}{x^4}\right)$$

$$\Big\downarrow \text{ 538}$$

$$\frac{1}{2}\left(3\int \frac{1}{\sqrt{x^4+5}}dx^2+\int \frac{1}{x^2\sqrt{x^4+5}}dx^2-\frac{\sqrt{x^4+5}(3x^2+1)}{x^4}\right)$$

$$\Big\downarrow \text{ 222}$$

$$\frac{1}{2}\left(\int \frac{1}{x^2\sqrt{x^4+5}}dx^2+3\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\sqrt{x^4+5}(3x^2+1)}{x^4}\right)$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2\sqrt{x^4+5}}dx^4+3\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\sqrt{x^4+5}(3x^2+1)}{x^4}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2}\left(\int \frac{1}{\sqrt{x^4+5}-5}d\sqrt{x^4+5}+3\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\sqrt{x^4+5}(3x^2+1)}{x^4}\right)$$

$$\Big\downarrow \text{ 220}$$

$$\frac{1}{2}\left(3\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\text{arctanh}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{\sqrt{5}}-\frac{\sqrt{x^4+5}(3x^2+1)}{x^4}\right)$$

input
```
Int[((2 + 3*x^2)*Sqrt[5 + x^4])/x^5,x]
```

output
```
(-(((1 + 3*x^2)*Sqrt[5 + x^4])/x^4) + 3*ArcSinh[x^2/Sqrt[5]] - ArcTanh[Sqr
t[5 + x^4]/Sqrt[5]]/Sqrt[5])/2
```

3.13.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

3.13.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

method	result
elliptic	$\frac{3\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{10}-\frac{3\sqrt{x^4+5}}{2x^2}-\frac{\sqrt{x^4+5}}{2x^4}$
risch	$-\frac{3x^6+x^4+15x^2+5}{2x^4\sqrt{x^4+5}}+\frac{3\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{10}$
pseudoelliptic	$\frac{-\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)x^4+15\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)x^4-15x^2\sqrt{x^4+5}-5\sqrt{x^4+5}}{10x^4}$
trager	$-\frac{(3x^2+1)\sqrt{x^4+5}}{2x^4}+\frac{3\ln\left(-x^2-\sqrt{x^4+5}\right)}{2}+\frac{\operatorname{RootOf}\left(\_Z^2-5\right)\ln\left(\frac{\sqrt{x^4+5}-\operatorname{RootOf}\left(\_Z^2-5\right)}{x^2}\right)}{10}$
default	$-\frac{3(x^4+5)^{\frac{3}{2}}}{10x^2}+\frac{3x^2\sqrt{x^4+5}}{10}+\frac{3\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}-\frac{(x^4+5)^{\frac{3}{2}}}{10x^4}+\frac{\sqrt{x^4+5}}{10}-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{10}$
meijerg	$-\frac{\sqrt{5}\left(-\frac{5\sqrt{\pi}\left(8+\frac{4x^4}{5}\right)}{4x^4}+\frac{10\sqrt{\pi}\sqrt{1+\frac{x^4}{5}}}{x^4}+2\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{x^4}{5}}}{2}\right)-(-2\ln(2)-1+4\ln(x)-\ln(5))\sqrt{\pi}+\frac{10\sqrt{\pi}}{x^4}\right)}{20\sqrt{\pi}}-3\left(\frac{4\sqrt{\pi}\sqrt{5}}{x}\right.$

input
```
int((3*x^2+2)*(x^4+5)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
3/2*arcsinh(1/5*x^2*5^(1/2))-1/10*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))-3
/2*(x^4+5)^(1/2)/x^2-1/2*(x^4+5)^(1/2)/x^4
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.14

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^5}\,dx = \frac{\sqrt{5}x^4\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right) - 15\,x^4\log\left(-x^2+\sqrt{x^4+5}\right) - 15\,x^4 - 5\,\sqrt{x^4+5}(3\,x^2+1)}{10\,x^4}$$

input
```
integrate((3*x^2+2)*(x^4+5)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
1/10*(sqrt(5)*x^4*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 15*x^4*log(-x^2 +
sqrt(x^4 + 5)) - 15*x^4 - 5*sqrt(x^4 + 5)*(3*x^2 + 1))/x^4
```

3.13.6 Sympy [A] (verification not implemented)

Time = 2.85 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.21

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^5}\,dx = -\frac{3x^2}{2\sqrt{x^4+5}} - \frac{\sqrt{5}\operatorname{asinh}\left(\frac{\sqrt{5}}{x^2}\right)}{10} + \frac{3\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{2} - \frac{\sqrt{1+\frac{5}{x^4}}}{2x^2} - \frac{15}{2x^2\sqrt{x^4+5}}$$

input
```
integrate((3*x**2+2)*(x**4+5)**(1/2)/x**5,x)
```

output
```
-3*x**2/(2*sqrt(x**4 + 5)) - sqrt(5)*asinh(sqrt(5)/x**2)/10 + 3*asinh(sqrt
(5)*x**2/5)/2 - sqrt(1 + 5/x**4)/(2*x**2) - 15/(2*x**2*sqrt(x**4 + 5))
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.44

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^5}\,dx = \frac{1}{20}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right) - \frac{3\sqrt{x^4+5}}{2\,x^2} - \frac{\sqrt{x^4+5}}{2\,x^4} + \frac{3}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right) - \frac{3}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

input
```
integrate((3*x^2+2)*(x^4+5)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
1/20*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) - 3
/2*sqrt(x^4 + 5)/x^2 - 1/2*sqrt(x^4 + 5)/x^4 + 3/4*log(sqrt(x^4 + 5)/x^2 +
 1) - 3/4*log(sqrt(x^4 + 5)/x^2 - 1)
```

3.13.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(49) = 98.

Time = 0.28 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.05

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^5}\,dx$$
$$= \frac{1}{10}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right) + \frac{\left(x^2-\sqrt{x^4+5}\right)^3 + 15\left(x^2-\sqrt{x^4+5}\right)^2 + 5\,x^2 - 5\sqrt{x^4+5} - 75}{\left(\left(x^2-\sqrt{x^4+5}\right)^2-5\right)^2} - \frac{3}{2}\log\left(-x^2+\sqrt{x^4+5}\right)$$

input
```
integrate((3*x^2+2)*(x^4+5)^(1/2)/x^5,x, algorithm="giac")
```

output
```
1/10*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^
4 + 5))) + ((x^2 - sqrt(x^4 + 5))^3 + 15*(x^2 - sqrt(x^4 + 5))^2 + 5*x^2 -
 5*sqrt(x^4 + 5) - 75)/((x^2 - sqrt(x^4 + 5))^2 - 5)^2 - 3/2*log(-x^2 + sq
rt(x^4 + 5))
```

3.13.9 Mupad [B] (verification not implemented)

Time = 7.74 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^5}\,dx = \frac{3\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{2} - \frac{3\,\sqrt{x^4+5}}{2\,x^2} - \frac{\sqrt{x^4+5}}{2\,x^4} + \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}\,1\mathrm{i}}{5}\right)\,1\mathrm{i}}{10}$$

input
```
int(((x^4 + 5)^(1/2)*(3*x^2 + 2))/x^5,x)
```

output
```
(3*asinh((5^(1/2)*x^2)/5))/2 + (5^(1/2)*atan((5^(1/2)*(x^4 + 5)^(1/2)*1i)/
5)*1i)/10 - (3*(x^4 + 5)^(1/2))/(2*x^2) - (x^4 + 5)^(1/2)/(2*x^4)
```

3.14 $\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^7}\,dx$

3.14.1 Optimal result

Integrand size = 20, antiderivative size = 58

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^7}\,dx = -\frac{3\sqrt{5+x^4}}{4x^4}-\frac{(5+x^4)^{3/2}}{15x^6}-\frac{3\text{arctanh}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{4\sqrt{5}}$$

output
```
-1/15*(x^4+5)^(3/2)/x^6-3/20*arctanh(1/5*(x^4+5)^(1/2)*5^(1/2))*5^(1/2)-3/
4*(x^4+5)^(1/2)/x^4
```

3.14.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.03

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^7}\,dx = -\frac{\sqrt{5+x^4}(20+45x^2+4x^4)}{60x^6}+\frac{3\text{arctanh}\left(\frac{x^2-\sqrt{5+x^4}}{\sqrt{5}}\right)}{2\sqrt{5}}$$

input
```
Integrate[((2 + 3*x^2)*Sqrt[5 + x^4])/x^7,x]
```

output
```
-1/60*(Sqrt[5 + x^4]*(20 + 45*x^2 + 4*x^4))/x^6 + (3*ArcTanh[(x^2 - Sqrt[5
 + x^4])/Sqrt[5]])/(2*Sqrt[5])
```

3.14.3 Rubi [A] (warning: unable to verify)

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1579, 534, 243, 51, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x^2+2)\sqrt{x^4+5}}{x^7}dx$$

$$\Big\downarrow \text{ 1579}$$

$$\frac{1}{2}\int \frac{(3x^2+2)\sqrt{x^4+5}}{x^8}dx^2$$

$$\Big\downarrow \text{ 534}$$

$$\frac{1}{2}\left(3\int \frac{\sqrt{x^4+5}}{x^6}dx^2-\frac{2(x^4+5)^{3/2}}{15x^6}\right)$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\left(\frac{3}{2}\int \frac{\sqrt{x^4+5}}{x^4}dx^4-\frac{2(x^4+5)^{3/2}}{15x^6}\right)$$

$$\Big\downarrow \text{ 51}$$

$$\frac{1}{2}\left(\frac{3}{2}\left(\frac{1}{2}\int \frac{1}{x^2\sqrt{x^4+5}}dx^4-\frac{\sqrt{x^4+5}}{x^2}\right)-\frac{2(x^4+5)^{3/2}}{15x^6}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2}\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{x^4+5}-5}d\sqrt{x^4+5}-\frac{\sqrt{x^4+5}}{x^2}\right)-\frac{2(x^4+5)^{3/2}}{15x^6}\right)$$

$$\Big\downarrow \text{ 220}$$

$$\frac{1}{2}\left(\frac{3}{2}\left(-\frac{\text{arctanh}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{\sqrt{5}}-\frac{\sqrt{x^4+5}}{x^2}\right)-\frac{2(x^4+5)^{3/2}}{15x^6}\right)$$

input
```
Int[((2 + 3*x^2)*Sqrt[5 + x^4])/x^7,x]
```

output
```
((-2*(5 + x^4)^(3/2))/(15*x^6) + (3*(-(Sqrt[5 + x^4]/x^2) - ArcTanh[Sqrt[5
 + x^4]/Sqrt[5]]/Sqrt[5]))/2)/2
```

3.14.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

3.14. $\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^7}\,dx$

3.14.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$\frac{-9\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)x^6-\left(4x^4+45x^2+20\right)\sqrt{x^4+5}}{60x^6}$
default	$-\frac{\left(x^4+5\right)^{\frac{3}{2}}}{15x^6}-\frac{3\left(x^4+5\right)^{\frac{3}{2}}}{20x^4}+\frac{3\sqrt{x^4+5}}{20}-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{20}$
trager	$-\frac{\left(4x^4+45x^2+20\right)\sqrt{x^4+5}}{60x^6}-\frac{3\,\operatorname{RootOf}\left(\_Z^2-5\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^2-5\right)+\sqrt{x^4+5}}{x^2}\right)}{20}$
risch	$-\frac{4x^8+45x^6+40x^4+225x^2+100}{60x^6\sqrt{x^4+5}}-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{20}$
elliptic	$-\frac{\sqrt{x^4+5}}{15x^2}-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{20}-\frac{3\sqrt{x^4+5}}{4x^4}-\frac{\sqrt{x^4+5}}{3x^6}$
meijerg	$-\frac{\sqrt{5}\left(1+\frac{x^4}{5}\right)^{\frac{3}{2}}}{3x^6}-\frac{3\sqrt{5}\left(-\frac{5\sqrt{\pi}\left(8+\frac{4x^4}{5}\right)}{4x^4}+\frac{10\sqrt{\pi}\sqrt{1+\frac{x^4}{5}}}{x^4}+2\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{x^4}{5}}}{2}\right)-\left(-2\ln(2)-1+4\ln(x)-\ln(5)\right)\sqrt{\pi}+\frac{10\sqrt{}}{x^{}}\right.}{40\sqrt{\pi}}$

input
```
int((3*x^2+2)*(x^4+5)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
1/60*(-9*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))*x^6-(4*x^4+45*x^2+20)*(x^4
+5)^(1/2))/x^6
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^7}\,dx = \frac{9\sqrt{5}x^6\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)-4\,x^6-(4\,x^4+45\,x^2+20)\sqrt{x^4+5}}{60\,x^6}$$

input
```
integrate((3*x^2+2)*(x^4+5)^(1/2)/x^7,x, algorithm="fricas")
```

output
```
1/60*(9*sqrt(5)*x^6*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 4*x^6 - (4*x^4 +
 45*x^2 + 20)*sqrt(x^4 + 5))/x^6
```

3.14.6 Sympy [A] (verification not implemented)

Time = 2.57 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.09

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^7}\,dx = -\frac{\sqrt{1+\frac{5}{x^4}}}{15} - \frac{3\sqrt{5}\operatorname{asinh}\left(\frac{\sqrt{5}}{x^2}\right)}{20} - \frac{3\sqrt{1+\frac{5}{x^4}}}{4x^2} - \frac{\sqrt{1+\frac{5}{x^4}}}{3x^4}$$

input
```
integrate((3*x**2+2)*(x**4+5)**(1/2)/x**7,x)
```

output
```
-sqrt(1 + 5/x**4)/15 - 3*sqrt(5)*asinh(sqrt(5)/x**2)/20 - 3*sqrt(1 + 5/x**
4)/(4*x**2) - sqrt(1 + 5/x**4)/(3*x**4)
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^7}\,dx = \frac{3}{40}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right) - \frac{3\sqrt{x^4+5}}{4\,x^4} - \frac{(x^4+5)^{\frac{3}{2}}}{15\,x^6}$$

input
```
integrate((3*x^2+2)*(x^4+5)^(1/2)/x^7,x, algorithm="maxima")
```

output
```
3/40*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) - 3
/4*sqrt(x^4 + 5)/x^4 - 1/15*(x^4 + 5)^(3/2)/x^6
```

3.14.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(43) = 86.

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.00

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^7}\,dx$$
$$= \frac{3}{20}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right)$$
$$+ \frac{9\left(x^2-\sqrt{x^4+5}\right)^5 + 12\left(x^2-\sqrt{x^4+5}\right)^4 - 225\,x^2 + 225\sqrt{x^4+5} + 100}{6\left(\left(x^2-\sqrt{x^4+5}\right)^2-5\right)^3}$$

input
```
integrate((3*x^2+2)*(x^4+5)^(1/2)/x^7,x, algorithm="giac")
```

output
```
3/20*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^
4 + 5))) + 1/6*(9*(x^2 - sqrt(x^4 + 5))^5 + 12*(x^2 - sqrt(x^4 + 5))^4 - 2
25*x^2 + 225*sqrt(x^4 + 5) + 100)/((x^2 - sqrt(x^4 + 5))^2 - 5)^3
```

3.14.9 Mupad [B] (verification not implemented)

Time = 7.87 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.74

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^7}\,dx = -\frac{3\sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}}{5}\right)}{20} - \frac{3\sqrt{x^4+5}}{4\,x^4} - \frac{\left(x^4+5\right)^{3/2}}{15\,x^6}$$

input
```
int(((x^4 + 5)^(1/2)*(3*x^2 + 2))/x^7,x)
```

output
```
- (3*5^(1/2)*atanh((5^(1/2)*(x^4 + 5)^(1/2))/5))/20 - (3*(x^4 + 5)^(1/2))/
(4*x^4) - (x^4 + 5)^(3/2)/(15*x^6)
```

3.15 $\int x^4(2+3x^2)\sqrt{5+x^4}\,dx$

3.15.1 Optimal result

Integrand size = 20, antiderivative size = 208

$$\begin{aligned}\int x^4(2+3x^2)\sqrt{5+x^4}\,dx &= \frac{20}{21}x\sqrt{5+x^4}+\frac{2}{3}x^3\sqrt{5+x^4}-\frac{10x\sqrt{5+x^4}}{\sqrt{5}+x^2}+\frac{1}{21}x^5(6+7x^2)\sqrt{5+x^4}\\ &+\frac{10\sqrt[4]{5}(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{(\sqrt{5}+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}}\\ &-\frac{5\sqrt[4]{5}(21+2\sqrt{5})(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{(\sqrt{5}+x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{21\sqrt{5+x^4}}\end{aligned}$$

output
```
20/21*x*(x^4+5)^(1/2)+2/3*x^3*(x^4+5)^(1/2)+1/21*x^5*(7*x^2+6)*(x^4+5)^(1/
2)-10*x*(x^4+5)^(1/2)/(x^2+5^(1/2))+10*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)
))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(3
/4))),1/2*2^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(
1/2)-5/21*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*
x*5^(3/4)))*EllipticF(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/
2))*(21+2*5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)
```

3.15.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.64 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.39

$$\int x^4(2+3x^2)\sqrt{5+x^4}\,dx = \frac{1}{21}x\left(6(5+x^4)^{3/2}+7x^2(5+x^4)^{3/2} -30\sqrt{5}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-\frac{x^4}{5}\right)-35\sqrt{5}x^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{x^4}{5}\right)\right)$$

input
```
Integrate[x^4*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

output
```
(x*(6*(5 + x^4)^(3/2) + 7*x^2*(5 + x^4)^(3/2) - 30*Sqrt[5]*Hypergeometric2
F1[-1/2, 1/4, 5/4, -1/5*x^4] - 35*Sqrt[5]*x^2*Hypergeometric2F1[-1/2, 3/4,
 7/4, -1/5*x^4]))/21
```

3.15.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1597, 27, 1603, 27, 1603, 25, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(3x^2+2)\sqrt{x^4+5}\,dx$$

$$\Big\downarrow \text{1597}$$

$$\frac{10}{63}\int \frac{3x^4(7x^2+6)}{\sqrt{x^4+5}}dx+\frac{1}{21}(7x^2+6)\sqrt{x^4+5}x^5$$

$$\Big\downarrow \text{27}$$

$$\frac{10}{21}\int \frac{x^4(7x^2+6)}{\sqrt{x^4+5}}dx+\frac{1}{21}(7x^2+6)\sqrt{x^4+5}x^5$$

$$\Big\downarrow \text{1603}$$

$$\frac{10}{21}\left(\frac{7}{5}x^3\sqrt{x^4+5}-\frac{1}{5}\int \frac{15x^2(7-2x^2)}{\sqrt{x^4+5}}dx\right)+\frac{1}{21}(7x^2+6)\sqrt{x^4+5}x^5$$

$$\Big\downarrow \text{27}$$

$$\frac{10}{21}\left(\frac{7}{5}x^3\sqrt{x^4+5}-3\int\frac{x^2(7-2x^2)}{\sqrt{x^4+5}}dx\right)+\frac{1}{21}\left(7x^2+6\right)\sqrt{x^4+5}x^5$$

$\big\downarrow$ 1603

$$\frac{10}{21}\left(\frac{7}{5}x^3\sqrt{x^4+5}-3\left(-\frac{1}{3}\int-\frac{21x^2+10}{\sqrt{x^4+5}}dx-\frac{2}{3}\sqrt{x^4+5}x\right)\right)+\frac{1}{21}\left(7x^2+6\right)\sqrt{x^4+5}x^5$$

$\big\downarrow$ 25

$$\frac{10}{21}\left(\frac{7}{5}x^3\sqrt{x^4+5}-3\left(\frac{1}{3}\int\frac{21x^2+10}{\sqrt{x^4+5}}dx-\frac{2}{3}x\sqrt{x^4+5}\right)\right)+\frac{1}{21}\left(7x^2+6\right)\sqrt{x^4+5}x^5$$

$\big\downarrow$ 1512

$$\frac{10}{21}\left(\frac{7}{5}x^3\sqrt{x^4+5}-3\left(\frac{1}{3}\left(\left(10+21\sqrt{5}\right)\int\frac{1}{\sqrt{x^4+5}}dx-21\sqrt{5}\int\frac{\sqrt{5}-x^2}{\sqrt{5}\sqrt{x^4+5}}dx\right)-\frac{2}{3}x\sqrt{x^4+5}\right)\right)+\frac{1}{21}\left(7x^2+6\right)\sqrt{x^4+5}x^5$$

$\big\downarrow$ 27

$$\frac{10}{21}\left(\frac{7}{5}x^3\sqrt{x^4+5}-3\left(\frac{1}{3}\left(\left(10+21\sqrt{5}\right)\int\frac{1}{\sqrt{x^4+5}}dx-21\int\frac{\sqrt{5}-x^2}{\sqrt{x^4+5}}dx\right)-\frac{2}{3}x\sqrt{x^4+5}\right)\right)+\frac{1}{21}\left(7x^2+6\right)\sqrt{x^4+5}x^5$$

$\big\downarrow$ 761

$$\frac{10}{21}\left(\frac{7}{5}x^3\sqrt{x^4+5}-3\left(\frac{1}{3}\left(\frac{\left(10+21\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}-21\int\frac{\sqrt{5}-x^2}{\sqrt{x^4+5}}\right.\right.\right.$$
$$\frac{1}{21}\left(7x^2+6\right)\sqrt{x^4+5}x^5$$

$\big\downarrow$ 1510

$$\frac{10}{21}\left(\frac{7}{5}x^3\sqrt{x^4+5}-3\left(\frac{1}{3}\left(\frac{\left(10+21\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}-21\left(\frac{\sqrt[4]{5}(x^2+\right.\right.\right.\right.$$
$$\frac{1}{21}\left(7x^2+6\right)\sqrt{x^4+5}x^5$$

input
```
Int[x^4*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

3.15. $\int x^4(2+3x^2)\sqrt{5+x^4}\,dx$

output
```
(x^5*(6 + 7*x^2)*Sqrt[5 + x^4])/21 + (10*((7*x^3*Sqrt[5 + x^4])/5 - 3*((-2
*x*Sqrt[5 + x^4])/3 + (-21*(-((x*Sqrt[5 + x^4])/(Sqrt[5] + x^2)) + (5^(1/4
)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5
^(1/4)], 1/2])/Sqrt[5 + x^4]) + ((10 + 21*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5
 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(2*5^(1/4)
*Sqrt[5 + x^4]))/3)))/21
```

3.15.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

rule 1597
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((c*d*(m + 4*p + 3) + c*e*(4*p
 + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p + 3))), x] + Simp[4*a*(p/((4*p +
 m + 1)*(m + 4*p + 3)))   Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
 3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && Gt
Q[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (
IntegerQ[p] || IntegerQ[m])
```

rule 1603
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
 1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ
[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[
m])
```

3.15.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 5.14 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.19

method	result
meijerg	$\frac{3\sqrt{5}\,x^{7}{}_{2}F_{1}\left(-\frac{1}{2},\frac{7}{4};\frac{11}{4};-\frac{x^{4}}{5}\right)}{7}+\frac{2\sqrt{5}\,x^{5}{}_{2}F_{1}\left(-\frac{1}{2},\frac{5}{4};\frac{9}{4};-\frac{x^{4}}{5}\right)}{5}$
risch	$\frac{x(7x^{6}+6x^{4}+14x^{2}+20)\sqrt{x^{4}+5}}{21}-\frac{4\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+5i\sqrt{5}\,x^{2}}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{21\sqrt{i\sqrt{5}}\,\sqrt{x^{4}+5}}-\frac{2i\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+5i\sqrt{5}\,x^{2}}\left(F\left(\frac{x\sqrt{}}{}\right.\right.}{\sqrt{i\sqrt{5}}\,\sqrt{x^{4}}}$
default	$\frac{x^{7}\sqrt{x^{4}+5}}{3}+\frac{2x^{3}\sqrt{x^{4}+5}}{3}-\frac{2i\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+5i\sqrt{5}\,x^{2}}\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)\right)}{\sqrt{i\sqrt{5}}\,\sqrt{x^{4}+5}}+\frac{2x^{5}\sqrt{x^{4}+5}}{7}+\frac{20x\sqrt{x^{4}+}}{21}$
elliptic	$\frac{x^{7}\sqrt{x^{4}+5}}{3}+\frac{2x^{3}\sqrt{x^{4}+5}}{3}-\frac{2i\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+5i\sqrt{5}\,x^{2}}\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)\right)}{\sqrt{i\sqrt{5}}\,\sqrt{x^{4}+5}}+\frac{2x^{5}\sqrt{x^{4}+5}}{7}+\frac{20x\sqrt{x^{4}+}}{21}$

input
```
int(x^4*(3*x^2+2)*(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/7*5^(1/2)*x^7*hypergeom([-1/2,7/4],[11/4],-1/5*x^4)+2/5*5^(1/2)*x^5*hype
rgeom([-1/2,5/4],[9/4],-1/5*x^4)
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.33

$$\int x^4(2+3x^2)\sqrt{5+x^4}\,dx =$$

$$-\frac{210\,(-5)^{\frac{3}{4}}\,xE(\arcsin\left(\frac{(-5)^{\frac{1}{4}}}{x}\right)\,|-1)-190\,(-5)^{\frac{3}{4}}\,xF(\arcsin\left(\frac{(-5)^{\frac{1}{4}}}{x}\right)\,|-1)-(7\,x^{8}+6\,x^{6}+14\,x^{4}+20}{21\,x}$$

input
```
integrate(x^4*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="fricas")
```

output
```
-1/21*(210*(-5)^(3/4)*x*elliptic_e(arcsin((-5)^(1/4)/x), -1) - 190*(-5)^(3
/4)*x*elliptic_f(arcsin((-5)^(1/4)/x), -1) - (7*x^8 + 6*x^6 + 14*x^4 + 20*
x^2 - 210)*sqrt(x^4 + 5))/x
```

3.15.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.38

$$\int x^4(2+3x^2)\sqrt{5+x^4}\,dx = \frac{3\sqrt{5}x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{11}{4}\right)}+\frac{\sqrt{5}x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x**4*(3*x**2+2)*(x**4+5)**(1/2),x)
```

output
```
3*sqrt(5)*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), x**4*exp_polar(I*pi)
/5)/(4*gamma(11/4)) + sqrt(5)*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), x
**4*exp_polar(I*pi)/5)/(2*gamma(9/4))
```

3.15.7 Maxima [F]

$$\int x^4(2+3x^2)\sqrt{5+x^4}\,dx = \int \sqrt{x^4+5}(3x^2+2)x^4\,dx$$

input
```
integrate(x^4*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^4 + 5)*(3*x^2 + 2)*x^4, x)
```

3.15.8 Giac [F]

$$\int x^4(2+3x^2)\sqrt{5+x^4}\,dx=\int\sqrt{x^4+5}(3\,x^2+2)x^4\,dx$$

input
```
integrate(x^4*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^4 + 5)*(3*x^2 + 2)*x^4, x)
```

3.15.9 Mupad [F(-1)]

Timed out.

$$\int x^4(2+3x^2)\sqrt{5+x^4}\,dx=\int x^4\,\sqrt{x^4+5}\,\left(3\,x^2+2\right)\,dx$$

input
```
int(x^4*(x^4 + 5)^(1/2)*(3*x^2 + 2),x)
```

output
```
int(x^4*(x^4 + 5)^(1/2)*(3*x^2 + 2), x)
```

3.16 $\int x^2(2+3x^2)\sqrt{5+x^4}\,dx$

3.16.1 Optimal result

Integrand size = 20, antiderivative size = 192

$$\begin{aligned}\int x^2(2+3x^2)\sqrt{5+x^4}\,dx &= \frac{10}{7}x\sqrt{5+x^4}+\frac{4x\sqrt{5+x^4}}{\sqrt{5}+x^2}+\frac{1}{35}x^3(14+15x^2)\sqrt{5+x^4}\\ &\quad-\frac{4\sqrt[4]{5}(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{(\sqrt{5}+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}}\\ &\quad+\frac{\sqrt[4]{5}(14-5\sqrt{5})(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{(\sqrt{5}+x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{7\sqrt{5+x^4}}\end{aligned}$$

output
```
10/7*x*(x^4+5)^(1/2)+1/35*x^3*(15*x^2+14)*(x^4+5)^(1/2)+4*x*(x^4+5)^(1/2)/
(x^2+5^(1/2))-4*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arcta
n(1/5*x*5^(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2
+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)+1/7*5^(1/4)*(cos(2
*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticF(si
n(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(14-5*5^(1/2))*(x^2+5^(1/2))*((x^4
+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)
```

3.16.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.96 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.35

$$\int x^2(2+3x^2)\sqrt{5+x^4}\,dx = \frac{1}{21}x\left(9(5+x^4)^{3/2} - 45\sqrt{5}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-\frac{x^4}{5}\right) + 14\sqrt{5}x^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{x^4}{5}\right)\right)$$

input
```
Integrate[x^2*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

output
```
(x*(9*(5 + x^4)^(3/2) - 45*Sqrt[5]*Hypergeometric2F1[-1/2, 1/4, 5/4, -1/5*
x^4] + 14*Sqrt[5]*x^2*Hypergeometric2F1[-1/2, 3/4, 7/4, -1/5*x^4]))/21
```

3.16.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1597, 1603, 27, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(3x^2+2)\sqrt{x^4+5}\,dx$$

$$\Big\downarrow \text{1597}$$

$$\frac{2}{7}\int \frac{x^2(15x^2+14)}{\sqrt{x^4+5}}dx + \frac{1}{35}(15x^2+14)\sqrt{x^4+5}x^3$$

$$\Big\downarrow \text{1603}$$

$$\frac{2}{7}\left(5x\sqrt{x^4+5} - \frac{1}{3}\int \frac{3(25-14x^2)}{\sqrt{x^4+5}}dx\right) + \frac{1}{35}(15x^2+14)\sqrt{x^4+5}x^3$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{7}\left(5x\sqrt{x^4+5} - \int \frac{25-14x^2}{\sqrt{x^4+5}}dx\right) + \frac{1}{35}(15x^2+14)\sqrt{x^4+5}x^3$$

$$\Big\downarrow \text{1512}$$

$$\frac{2}{7}\left(-\left(25-14\sqrt{5}\right)\int \frac{1}{\sqrt{x^4+5}}dx-14\sqrt{5}\int \frac{\sqrt{5}-x^2}{\sqrt{5}\sqrt{x^4+5}}dx+5\sqrt{x^4+5}x\right)+\frac{1}{35}\left(15x^2+14\right)\sqrt{x^4+5}x^3$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{7}\left(-\left(25-14\sqrt{5}\right)\int \frac{1}{\sqrt{x^4+5}}dx-14\int \frac{\sqrt{5}-x^2}{\sqrt{x^4+5}}dx+5\sqrt{x^4+5}x\right)+\frac{1}{35}\left(15x^2+14\right)\sqrt{x^4+5}x^3$$

$$\Big\downarrow \text{761}$$

$$\frac{2}{7}\left(-14\int \frac{\sqrt{5}-x^2}{\sqrt{x^4+5}}dx-\frac{\left(25-14\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}+5\sqrt{x^4+5}x\right)+\frac{1}{35}\left(15x^2+14\right)\sqrt{x^4+5}x^3$$

$$\Big\downarrow \text{1510}$$

$$\frac{2}{7}\left(-\frac{\left(25-14\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}-14\left(\frac{\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\text{a}\right.}{\sqrt{x^4+5}}\right.\right.$$

$$\frac{1}{35}\left(15x^2+14\right)\sqrt{x^4+5}x^3$$

input
```
Int[x^2*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

output
```
(x^3*(14 + 15*x^2)*Sqrt[5 + x^4])/35 + (2*(5*x*Sqrt[5 + x^4] - 14*(-((x*Sq
rt[5 + x^4])/(Sqrt[5] + x^2)) + (5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(S
qrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4]) - ((2
5 - 14*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*Elliptic
F[2*ArcTan[x/5^(1/4)], 1/2])/(2*5^(1/4)*Sqrt[5 + x^4])))/7
```

3.16. $\int x^2(2+3x^2)\sqrt{5+x^4}\,dx$

3.16.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

rule 1597
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((c*d*(m + 4*p + 3) + c*e*(4*p
 + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p + 3))), x] + Simp[4*a*(p/((4*p +
 m + 1)*(m + 4*p + 3)))   Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
 3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && Gt
Q[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (
IntegerQ[p] || IntegerQ[m])
```

rule 1603
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
 1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ
[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[
m])
```

3.16.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.84 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.21

method	result
meijerg	$\frac{3\sqrt{5}\,x^{5}\,{}_2F_1\left(-\frac{1}{2},\frac{5}{4};\frac{9}{4};-\frac{x^4}{5}\right)}{5}+\frac{2\sqrt{5}\,x^{3}\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{x^4}{5}\right)}{3}$
risch	$\frac{x(15x^4+14x^2+50)\sqrt{x^4+5}}{35}-\frac{2\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{7\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{4i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(F\left(\frac{x\sqrt{5}\,\sqrt{i}}{5}\right.\right.}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$
default	$\frac{3x^5\sqrt{x^4+5}}{7}+\frac{10x\sqrt{x^4+5}}{7}-\frac{2\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{7\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{2x^3\sqrt{x^4+5}}{5}+\frac{4i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}}}{5}$
elliptic	$\frac{3x^5\sqrt{x^4+5}}{7}+\frac{10x\sqrt{x^4+5}}{7}-\frac{2\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{7\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{2x^3\sqrt{x^4+5}}{5}+\frac{4i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}}}{5}$

input
```
int(x^2*(3*x^2+2)*(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/5*5^(1/2)*x^5*hypergeom([-1/2,5/4],[9/4],-1/5*x^4)+2/3*5^(1/2)*x^3*hyper
geom([-1/2,3/4],[7/4],-1/5*x^4)
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.33

$$\int x^2(2+3x^2)\sqrt{5+x^4}\,dx$$
$$=\frac{140\,(-5)^{\frac{3}{4}}\,xE(\arcsin\left(\frac{(-5)^{\frac{1}{4}}}{x}\right)\,|-1)-190\,(-5)^{\frac{3}{4}}\,xF(\arcsin\left(\frac{(-5)^{\frac{1}{4}}}{x}\right)\,|-1)+(15\,x^6+14\,x^4+50\,x^2+140}{35\,x}$$

input
```
integrate(x^2*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="fricas")
```

output
```
1/35*(140*(-5)^(3/4)*x*elliptic_e(arcsin((-5)^(1/4)/x), -1) - 190*(-5)^(3/
4)*x*elliptic_f(arcsin((-5)^(1/4)/x), -1) + (15*x^6 + 14*x^4 + 50*x^2 + 14
0)*sqrt(x^4 + 5))/x
```

3.16.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.91 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.41

$$\int x^2(2+3x^2)\sqrt{5+x^4}\,dx = \frac{3\sqrt{5}x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{\sqrt{5}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x**2*(3*x**2+2)*(x**4+5)**(1/2),x)
```

output
```
3*sqrt(5)*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), x**4*exp_polar(I*pi)/
5)/(4*gamma(9/4)) + sqrt(5)*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), x**
4*exp_polar(I*pi)/5)/(2*gamma(7/4))
```

3.16.7 Maxima [F]

$$\int x^2(2+3x^2)\sqrt{5+x^4}\,dx = \int \sqrt{x^4+5}(3\,x^2+2)x^2\,dx$$

input
```
integrate(x^2*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^4 + 5)*(3*x^2 + 2)*x^2, x)
```

3.16.8 Giac [F]

$$\int x^2(2+3x^2)\sqrt{5+x^4}\,dx = \int \sqrt{x^4+5}(3\,x^2+2)x^2\,dx$$

input
```
integrate(x^2*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^4 + 5)*(3*x^2 + 2)*x^2, x)
```

3.16.9 Mupad [F(-1)]

Timed out.

$$\int x^2(2+3x^2)\sqrt{5+x^4}\,dx = \int x^2\,\sqrt{x^4+5}\,\left(3\,x^2+2\right)\,dx$$

input
```
int(x^2*(x^4 + 5)^(1/2)*(3*x^2 + 2),x)
```

output
```
int(x^2*(x^4 + 5)^(1/2)*(3*x^2 + 2), x)
```

3.17 $\int (2+3x^2)\sqrt{5+x^4}\,dx$

3.17.1 Optimal result

Integrand size = 17, antiderivative size = 176

$$\begin{aligned}&\int (2+3x^2)\sqrt{5+x^4}\,dx\\&=\frac{6x\sqrt{5+x^4}}{\sqrt{5}+x^2}+\frac{1}{15}x(10+9x^2)\sqrt{5+x^4}\\&-\frac{6\sqrt[4]{5}(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{(\sqrt{5}+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}}\\&+\frac{\sqrt[4]{5}(9+2\sqrt{5})(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{(\sqrt{5}+x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{3\sqrt{5+x^4}}\end{aligned}$$

output
```
1/15*x*(9*x^2+10)*(x^4+5)^(1/2)+6*x*(x^4+5)^(1/2)/(x^2+5^(1/2))-6*5^(1/4)*
(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*Ellipt
icE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+
5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)+1/3*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^
2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticF(sin(2*arctan(1/5*x*5^(3/4)
)),1/2*2^(1/2))*(x^2+5^(1/2))*(9+2*5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2
)/(x^4+5)^(1/2)
```

3.17.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.57 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.27

$$\int (2+3x^2)\sqrt{5+x^4}\,dx = \sqrt{5}x\left(2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-\frac{x^4}{5}\right)+x^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{x^4}{5}\right)\right)$$

input
```
Integrate[(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

output
```
Sqrt[5]*x*(2*Hypergeometric2F1[-1/2, 1/4, 5/4, -1/5*x^4] + x^2*Hypergeomet
ric2F1[-1/2, 3/4, 7/4, -1/5*x^4])
```

3.17.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {1491, 27, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3x^2+2)\sqrt{x^4+5}\,dx$$

$$\Big\downarrow \ 1491$$

$$\frac{1}{15}\int \frac{10(9x^2+10)}{\sqrt{x^4+5}}dx+\frac{1}{15}x\sqrt{x^4+5}(9x^2+10)$$

$$\Big\downarrow \ 27$$

$$\frac{2}{3}\int \frac{9x^2+10}{\sqrt{x^4+5}}dx+\frac{1}{15}x\sqrt{x^4+5}(9x^2+10)$$

$$\Big\downarrow \ 1512$$

$$\frac{2}{3}\left(\left(10+9\sqrt{5}\right)\int \frac{1}{\sqrt{x^4+5}}dx-9\sqrt{5}\int \frac{\sqrt{5}-x^2}{\sqrt{5}\sqrt{x^4+5}}dx\right)+\frac{1}{15}x\sqrt{x^4+5}(9x^2+10)$$

$$\Big\downarrow \ 27$$

$$\frac{2}{3}\left(\left(10+9\sqrt{5}\right)\int\frac{1}{\sqrt{x^4+5}}dx-9\int\frac{\sqrt{5}-x^2}{\sqrt{x^4+5}}dx\right)+\frac{1}{15}x\sqrt{x^4+5}(9x^2+10)$$

$\big\downarrow$ 761

$$\frac{2}{3}\left(\frac{\left(10+9\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}-9\int\frac{\sqrt{5}-x^2}{\sqrt{x^4+5}}dx\right)+$$
$$\frac{1}{15}x\sqrt{x^4+5}(9x^2+10)$$

$\big\downarrow$ 1510

$$\frac{2}{3}\left(\frac{\left(10+9\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}-9\left(\frac{\sqrt[4]{5}(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\arcta\right.}{\sqrt{x^4+5}}\right.\right.$$
$$\frac{1}{15}x\sqrt{x^4+5}(9x^2+10)$$

input
```
Int[(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

output
```
(x*(10 + 9*x^2)*Sqrt[5 + x^4])/15 + (2*(-9*(-((x*Sqrt[5 + x^4])/(Sqrt[5] +
 x^2)) + (5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*Ellipt
icE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4]) + ((10 + 9*Sqrt[5])*(Sqrt[5]
 + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1
/2])/(2*5^(1/4)*Sqrt[5 + x^4])))/3
```

3.17.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1491
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Simp[2*(p/((4*p + 1)*(4*p + 3)))   Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
 1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c
*d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

3.17.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.21

method	result
meijerg	$2\sqrt{5}\, x\, {}_2F_1\left(-\frac{1}{2}, \frac{1}{4}; \frac{5}{4}; -\frac{x^4}{5}\right) + \sqrt{5}\, x^3\, {}_2F_1\left(-\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -\frac{x^4}{5}\right)$
risch	$\frac{x(9x^2+10)\sqrt{x^4+5}}{15} + \frac{4\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5}, i\right)}{15\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}} + \frac{6i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5}, i\right) - \ldots\right.}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}\ldots}$
default	$\frac{2x\sqrt{x^4+5}}{3} + \frac{4\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5}, i\right)}{15\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}} + \frac{3x^3\sqrt{x^4+5}}{5} + \frac{6i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\ldots}}{5}\ldots\right.\right.}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$
elliptic	$\frac{2x\sqrt{x^4+5}}{3} + \frac{4\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5}, i\right)}{15\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}} + \frac{3x^3\sqrt{x^4+5}}{5} + \frac{6i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\ldots}}{5}\ldots\right.\right.}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$

input
```
int((3*x^2+2)*(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*5^(1/2)*x*hypergeom([-1/2,1/4],[5/4],-1/5*x^4)+5^(1/2)*x^3*hypergeom([-1
/2,3/4],[7/4],-1/5*x^4)
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.33

$$\int (2+3x^2)\sqrt{5+x^4}\,dx$$
$$=\frac{90\,(-5)^{\frac{3}{4}}\,xE(\arcsin\left(\frac{(-5)^{\frac{1}{4}}}{x}\right)\,|-1)-70\,(-5)^{\frac{3}{4}}\,xF(\arcsin\left(\frac{(-5)^{\frac{1}{4}}}{x}\right)\,|-1)+(9\,x^4+10\,x^2+90)\sqrt{x^4+5}}{15\,x}$$

input
```
integrate((3*x^2+2)*(x^4+5)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(90*(-5)^(3/4)*x*elliptic_e(arcsin((-5)^(1/4)/x), -1) - 70*(-5)^(3/4)
*x*elliptic_f(arcsin((-5)^(1/4)/x), -1) + (9*x^4 + 10*x^2 + 90)*sqrt(x^4 +
 5))/x
```

3.17.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.43

$$\int (2+3x^2)\sqrt{5+x^4}\,dx=\frac{3\sqrt{5}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{\sqrt{5}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((3*x**2+2)*(x**4+5)**(1/2),x)
```

output
```
3*sqrt(5)*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), x**4*exp_polar(I*pi)/
5)/(4*gamma(7/4)) + sqrt(5)*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), x**4*e
xp_polar(I*pi)/5)/(2*gamma(5/4))
```

3.17.7 Maxima [F]

$$\int (2+3x^2)\sqrt{5+x^4}\,dx = \int \sqrt{x^4+5}(3\,x^2+2)\,dx$$

input
```
integrate((3*x^2+2)*(x^4+5)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^4 + 5)*(3*x^2 + 2), x)
```

3.17.8 Giac [F]

$$\int (2+3x^2)\sqrt{5+x^4}\,dx = \int \sqrt{x^4+5}(3\,x^2+2)\,dx$$

input
```
integrate((3*x^2+2)*(x^4+5)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^4 + 5)*(3*x^2 + 2), x)
```

3.17.9 Mupad [F(-1)]

Timed out.

$$\int (2+3x^2)\sqrt{5+x^4}\,dx = \int \sqrt{x^4+5}\,(3\,x^2+2)\,dx$$

input
```
int((x^4 + 5)^(1/2)*(3*x^2 + 2),x)
```

output
```
int((x^4 + 5)^(1/2)*(3*x^2 + 2), x)
```

3.18 $\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^2}\,dx$

3.18.1 Optimal result

Integrand size = 20, antiderivative size = 171

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^2}\,dx = -\frac{(2-x^2)\sqrt{5+x^4}}{x}+\frac{4x\sqrt{5+x^4}}{\sqrt{5}+x^2}-\frac{4\sqrt[4]{5}(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{(\sqrt{5}+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}}+\frac{\sqrt[4]{5}(2+\sqrt{5})(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{(\sqrt{5}+x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{\sqrt{5+x^4}}$$

output
```
-(-x^2+2)*(x^4+5)^(1/2)/x+4*x*(x^4+5)^(1/2)/(x^2+5^(1/2))-4*5^(1/4)*(cos(2
*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticE(si
n(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2
))^2)^(1/2)/(x^4+5)^(1/2)+5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/c
os(2*arctan(1/5*x*5^(3/4)))*EllipticF(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(
1/2))*(2+5^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1
/2)
```

3.18.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.52 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.31

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^2}\,dx = -\frac{2\sqrt{5}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{4},\frac{3}{4},-\frac{x^4}{5}\right)}{x} + 3\sqrt{5}x\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-\frac{x^4}{5}\right)$$

input
```
Integrate[((2 + 3*x^2)*Sqrt[5 + x^4])/x^2,x]
```

output
```
(-2*Sqrt[5]*Hypergeometric2F1[-1/2, -1/4, 3/4, -1/5*x^4])/x + 3*Sqrt[5]*x*
Hypergeometric2F1[-1/2, 1/4, 5/4, -1/5*x^4]
```

3.18.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1595, 27, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x^2+2)\sqrt{x^4+5}}{x^2}dx$$

$$\Big\downarrow \; 1595$$

$$-\frac{2}{3}\int -\frac{3(2x^2+5)}{\sqrt{x^4+5}}dx - \frac{\sqrt{x^4+5}(2-x^2)}{x}$$

$$\Big\downarrow \; 27$$

$$2\int \frac{2x^2+5}{\sqrt{x^4+5}}dx - \frac{(2-x^2)\sqrt{x^4+5}}{x}$$

$$\Big\downarrow \; 1512$$

$$2\left(\left(5+2\sqrt{5}\right)\int \frac{1}{\sqrt{x^4+5}}dx - 2\sqrt{5}\int \frac{\sqrt{5}-x^2}{\sqrt{5}\sqrt{x^4+5}}dx\right) - \frac{(2-x^2)\sqrt{x^4+5}}{x}$$

$$\Big\downarrow \; 27$$

$$2\left(\left(5+2\sqrt{5}\right)\int \frac{1}{\sqrt{x^4+5}}dx-2\int \frac{\sqrt{5}-x^2}{\sqrt{x^4+5}}dx\right)-\frac{\left(2-x^2\right)\sqrt{x^4+5}}{x}$$

$$\Big\downarrow \text{761}$$

$$2\left(\frac{\left(5+2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}-2\int \frac{\sqrt{5}-x^2}{\sqrt{x^4+5}}dx\right)-\frac{\left(2-x^2\right)\sqrt{x^4+5}}{x}$$

$$\Big\downarrow \text{1510}$$

$$2\left(\frac{\left(5+2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}-2\left(\frac{\sqrt[4]{5}(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\arctan\right.}{\sqrt{x^4+5}}\right.\right.$$

$$\frac{\left(2-x^2\right)\sqrt{x^4+5}}{x}$$

input
```
Int[((2 + 3*x^2)*Sqrt[5 + x^4])/x^2,x]
```

output
```
-(((2 - x^2)*Sqrt[5 + x^4])/x) + 2*(-2*(-((x*Sqrt[5 + x^4])/(Sqrt[5] + x^2
)) + (5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[
2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4]) + ((5 + 2*Sqrt[5])*(Sqrt[5] + x^
2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/
(2*5^(1/4)*Sqrt[5 + x^4]))
```

3.18.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

rule 1595
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((d*(m + 4*p + 3) + e*(m + 1)*
x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Simp[4*(p/(f^2*(m + 1)*(m + 4*p + 3))
)   Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*
x^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.18.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.36 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.22

method	result
meijerg	$-\frac{2\sqrt{5}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};-\frac{x^4}{5}\right)}{x}+3\sqrt{5}\,x\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{x^4}{5}\right)$
default	$x\sqrt{x^4+5}+\frac{2\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}-\frac{2\sqrt{x^4+5}}{x}+\frac{4i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5}\right.\right.}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$
elliptic	$x\sqrt{x^4+5}+\frac{2\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}-\frac{2\sqrt{x^4+5}}{x}+\frac{4i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5}\right.\right.}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$
risch	$\frac{x^6-2x^4+5x^2-10}{x\sqrt{x^4+5}}+\frac{2\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{4i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\right.}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$

input
```
int((3*x^2+2)*(x^4+5)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-2*5^(1/2)/x*hypergeom([-1/2,-1/4],[3/4],-1/5*x^4)+3*5^(1/2)*x*hypergeom([
-1/2,1/4],[5/4],-1/5*x^4)
```

3.18.5 Fricas [F]

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^2}\,dx = \int \frac{\sqrt{x^4+5}(3\,x^2+2)}{x^2}\,dx$$

input
```
integrate((3*x^2+2)*(x^4+5)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
integral(sqrt(x^4 + 5)*(3*x^2 + 2)/x^2, x)
```

3.18.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.99 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.46

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^2}\,dx = \frac{3\sqrt{5}x\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{4}\\\frac{5}{4}\end{matrix}\right|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{\sqrt{5}\Gamma\left(-\frac{1}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\right|\frac{x^4e^{i\pi}}{5}\right)}{2x\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((3*x**2+2)*(x**4+5)**(1/2)/x**2,x)
```

output
```
3*sqrt(5)*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), x**4*exp_polar(I*pi)/5)/
(4*gamma(5/4)) + sqrt(5)*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), x**4*exp_
polar(I*pi)/5)/(2*x*gamma(3/4))
```

3.18.7 Maxima [F]

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^2}\,dx = \int \frac{\sqrt{x^4+5}(3\,x^2+2)}{x^2}\,dx$$

input
```
integrate((3*x^2+2)*(x^4+5)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x^2, x)
```

3.18.8 Giac [F]

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^2}\,dx = \int \frac{\sqrt{x^4+5}(3\,x^2+2)}{x^2}\,dx$$

input
```
integrate((3*x^2+2)*(x^4+5)^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x^2, x)
```

3.18.9 Mupad [B] (verification not implemented)

Time = 7.77 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.36

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^2}\,dx = \frac{3\,x\,\sqrt{x^4+5}\,{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{1}{4};\ \frac{5}{4};\ -\frac{x^4}{5}\right)}{\sqrt{\frac{x^4}{5}+1}} + \frac{2\,\sqrt{x^4+5}\,{}_2\mathrm{F}_1\left(-\frac{1}{2},-\frac{1}{4};\ \frac{3}{4};\ -\frac{5}{x^4}\right)}{x\,\sqrt{\frac{5}{x^4}+1}}$$

input
```
int(((x^4 + 5)^(1/2)*(3*x^2 + 2))/x^2,x)
```

output
```
(3*x*(x^4 + 5)^(1/2)*hypergeom([-1/2, 1/4], 5/4, -x^4/5))/(x^4/5 + 1)^(1/2
) + (2*(x^4 + 5)^(1/2)*hypergeom([-1/2, -1/4], 3/4, -5/x^4))/(x*(5/x^4 + 1
)^(1/2))
```

3.19 $\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^4}\,dx$

3.19.1 Optimal result

Integrand size = 20, antiderivative size = 192

$$\begin{aligned}\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^4}\,dx &= -\frac{6\sqrt{5+x^4}}{x} - \frac{(2-9x^2)\sqrt{5+x^4}}{3x^3} + \frac{6x\sqrt{5+x^4}}{\sqrt{5}+x^2} \\ &\quad - \frac{6\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}} \\ &\quad + \frac{\left(2+9\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{3\sqrt[4]{5}\sqrt{5+x^4}}\end{aligned}$$

output
```
-6*(x^4+5)^(1/2)/x-1/3*(-9*x^2+2)*(x^4+5)^(1/2)/x^3+6*x*(x^4+5)^(1/2)/(x^2
+5^(1/2))-6*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/
5*x*5^(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(
1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)+1/15*(cos(2*arctan(1/5
*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticF(sin(2*arctan(
1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*(2+9*5^(1/2))*((x^4+5)/(x^2+5^(
1/2))^2)^(1/2)*5^(3/4)/(x^4+5)^(1/2)
```

3.19.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.72 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.28

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^4}\,dx = -\frac{\sqrt{5}\left(2\,\text{Hypergeometric2F1}\left(-\frac{3}{4},-\frac{1}{2},\frac{1}{4},-\frac{x^4}{5}\right)+9x^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{4},\frac{3}{4},-\frac{x^4}{5}\right)\right)}{3x^3}$$

input
```
Integrate[((2 + 3*x^2)*Sqrt[5 + x^4])/x^4,x]
```

output
```
-1/3*(Sqrt[5]*(2*Hypergeometric2F1[-3/4, -1/2, 1/4, -1/5*x^4] + 9*x^2*Hype
rgeometric2F1[-1/2, -1/4, 3/4, -1/5*x^4]))/x^3
```

3.19.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1595, 25, 1605, 27, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x^2+2)\sqrt{x^4+5}}{x^4}\,dx$$

$$\Big\downarrow \text{1595}$$

$$-\frac{2}{3}\int -\frac{2x^2+45}{x^2\sqrt{x^4+5}}dx-\frac{\sqrt{x^4+5}(2-9x^2)}{3x^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{2}{3}\int \frac{2x^2+45}{x^2\sqrt{x^4+5}}dx-\frac{(2-9x^2)\sqrt{x^4+5}}{3x^3}$$

$$\Big\downarrow \text{1605}$$

$$\frac{2}{3}\left(-\frac{1}{5}\int -\frac{5(9x^2+2)}{\sqrt{x^4+5}}dx-\frac{9\sqrt{x^4+5}}{x}\right)-\frac{(2-9x^2)\sqrt{x^4+5}}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}\left(\int \frac{9x^2+2}{\sqrt{x^4+5}}dx-\frac{9\sqrt{x^4+5}}{x}\right)-\frac{(2-9x^2)\sqrt{x^4+5}}{3x^3}$$

$\Big\downarrow$ 1512

$$\frac{2}{3}\left(\left(2+9\sqrt{5}\right)\int \frac{1}{\sqrt{x^4+5}}dx-9\sqrt{5}\int \frac{\sqrt{5}-x^2}{\sqrt{5}\sqrt{x^4+5}}dx-\frac{9\sqrt{x^4+5}}{x}\right)-\frac{(2-9x^2)\sqrt{x^4+5}}{3x^3}$$

$\Big\downarrow$ 27

$$\frac{2}{3}\left(\left(2+9\sqrt{5}\right)\int \frac{1}{\sqrt{x^4+5}}dx-9\int \frac{\sqrt{5}-x^2}{\sqrt{x^4+5}}dx-\frac{9\sqrt{x^4+5}}{x}\right)-\frac{(2-9x^2)\sqrt{x^4+5}}{3x^3}$$

$\Big\downarrow$ 761

$$\frac{2}{3}\left(-9\int \frac{\sqrt{5}-x^2}{\sqrt{x^4+5}}dx+\frac{(2+9\sqrt{5})(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{(x^2+\sqrt{5})^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}-\frac{9\sqrt{x^4+5}}{x}\right)-\frac{(2-9x^2)\sqrt{x^4+5}}{3x^3}$$

$\Big\downarrow$ 1510

$$\frac{2}{3}\left(\frac{(2+9\sqrt{5})(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{(x^2+\sqrt{5})^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}-9\left(\frac{\sqrt[4]{5}(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{(x^2+\sqrt{5})^2}}E\left(2\arctan\right.}{\sqrt{x^4+5}}\right.\right.$$

$$\frac{(2-9x^2)\sqrt{x^4+5}}{3x^3}$$

input
```
Int[((2 + 3*x^2)*Sqrt[5 + x^4])/x^4,x]
```

output
```
-1/3*((2 - 9*x^2)*Sqrt[5 + x^4])/x^3 + (2*((-9*Sqrt[5 + x^4])/x - 9*(-((x*
Sqrt[5 + x^4])/(Sqrt[5] + x^2)) + (5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/
(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4]) + (
(2 + 9*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*Elliptic
F[2*ArcTan[x/5^(1/4)], 1/2])/(2*5^(1/4)*Sqrt[5 + x^4])))/3
```

3.19. $\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^4}\,dx$

3.19.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

rule 1595
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((d*(m + 4*p + 3) + e*(m + 1)*
x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Simp[4*(p/(f^2*(m + 1)*(m + 4*p + 3))
)   Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*
x^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1605
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + S
imp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*
(m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
 IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.19. $\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^4}\,dx$

3.19.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 2.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.21

method	result
meijerg	$-\frac{2\sqrt{5}\, {}_2F_1\left(-\frac{3}{4},-\frac{1}{2};\frac{1}{4};-\frac{x^4}{5}\right)}{3x^3}-\frac{3\sqrt{5}\, {}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};-\frac{x^4}{5}\right)}{x}$
default	$-\frac{3\sqrt{x^4+5}}{x}+\frac{6i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)\right)}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}-\frac{2\sqrt{x^4+5}}{3x^3}+\frac{4\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5\cdots}}{75\sqrt{i\sqrt{5}}\cdots}$
elliptic	$-\frac{3\sqrt{x^4+5}}{x}+\frac{6i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)\right)}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}-\frac{2\sqrt{x^4+5}}{3x^3}+\frac{4\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5\cdots}}{75\sqrt{i\sqrt{5}}\cdots}$
risch	$-\frac{9x^6+2x^4+45x^2+10}{3x^3\sqrt{x^4+5}}+\frac{4\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{75\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{6i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\cdots\right.\right.}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$

input
```
int((3*x^2+2)*(x^4+5)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-2/3*5^(1/2)/x^3*hypergeom([-3/4,-1/2],[1/4],-1/5*x^4)-3*5^(1/2)/x*hyperge
om([-1/2,-1/4],[3/4],-1/5*x^4)
```

3.19.5 Fricas [F]

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^4}\,dx=\int \frac{\sqrt{x^4+5}(3\,x^2+2)}{x^4}\,dx$$

input
```
integrate((3*x^2+2)*(x^4+5)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
integral(sqrt(x^4 + 5)*(3*x^2 + 2)/x^4, x)
```

3.19.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.43

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^4}\,dx = \frac{3\sqrt{5}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4x\Gamma\left(\frac{3}{4}\right)} + \frac{\sqrt{5}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2x^3\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate((3*x**2+2)*(x**4+5)**(1/2)/x**4,x)
```

output
```
3*sqrt(5)*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), x**4*exp_polar(I*pi)/5)/
(4*x*gamma(3/4)) + sqrt(5)*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), x**4*ex
p_polar(I*pi)/5)/(2*x**3*gamma(1/4))
```

3.19.7 Maxima [F]

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^4}\,dx = \int \frac{\sqrt{x^4+5}(3x^2+2)}{x^4}\,dx$$

input
```
integrate((3*x^2+2)*(x^4+5)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x^4, x)
```

3.19.8 Giac [F]

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^4}\,dx = \int \frac{\sqrt{x^4+5}(3x^2+2)}{x^4}\,dx$$

input
```
integrate((3*x^2+2)*(x^4+5)^(1/2)/x^4,x, algorithm="giac")
```

output
```
integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x^4, x)
```

3.19.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^4}\,dx = \int \frac{\sqrt{x^4+5}\,(3\,x^2+2)}{x^4}\,dx$$

input
```
int(((x^4 + 5)^(1/2)*(3*x^2 + 2))/x^4,x)
```

output
```
int(((x^4 + 5)^(1/2)*(3*x^2 + 2))/x^4, x)
```

3.20 $\int x^5(2+3x^2)\left(5+x^4\right)^{3/2}\,dx$

3.20.1 Optimal result

Integrand size = 20, antiderivative size = 83

$$\int x^5\left(2+3x^2\right)\left(5+x^4\right)^{3/2}\,dx=-\frac{25}{16}x^2\sqrt{5+x^4}-\frac{5}{24}x^2\left(5+x^4\right)^{3/2}+\frac{3}{14}x^4\left(5+x^4\right)^{5/2}-\frac{1}{42}\left(18-7x^2\right)\left(5+x^4\right)^{5/2}-\frac{125}{16}\operatorname{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)$$

output
```
-5/24*x^2*(x^4+5)^(3/2)+3/14*x^4*(x^4+5)^(5/2)-1/42*(-7*x^2+18)*(x^4+5)^(5
/2)-125/16*arcsinh(1/5*x^2*5^(1/2))-25/16*x^2*(x^4+5)^(1/2)
```

3.20.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.80

$$\int x^5\left(2+3x^2\right)\left(5+x^4\right)^{3/2}\,dx=\frac{1}{336}\sqrt{5+x^4}\left(-3600+525x^2+360x^4+490x^6+576x^8+56x^{10}+72x^{12}\right)+\frac{125}{16}\log\left(-x^2+\sqrt{5+x^4}\right)$$

input
```
Integrate[x^5*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

output
```
(Sqrt[5 + x^4]*(-3600 + 525*x^2 + 360*x^4 + 490*x^6 + 576*x^8 + 56*x^10 +
72*x^12))/336 + (125*Log[-x^2 + Sqrt[5 + x^4]])/16
```

3.20.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.37, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {1579, 533, 27, 533, 27, 455, 211, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 (3x^2+2) (x^4+5)^{3/2} dx$$

$$\Big\downarrow \text{1579}$$

$$\frac{1}{2}\int x^4 (3x^2+2) (x^4+5)^{3/2} dx^2$$

$$\Big\downarrow \text{533}$$

$$\frac{1}{2}\left(\frac{3}{7}x^4 (x^4+5)^{5/2}-\frac{1}{7}\int 2x^2 (15-7x^2) (x^4+5)^{3/2} dx^2\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{3}{7}x^4 (x^4+5)^{5/2}-\frac{2}{7}\int x^2 (15-7x^2) (x^4+5)^{3/2} dx^2\right)$$

$$\Big\downarrow \text{533}$$

$$\frac{1}{2}\left(\frac{3}{7}x^4 (x^4+5)^{5/2}-\frac{2}{7}\left(-\frac{1}{6}\int -5(18x^2+7) (x^4+5)^{3/2} dx^2-\frac{7}{6}x^2 (x^4+5)^{5/2}\right)\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{3}{7}x^4 (x^4+5)^{5/2}-\frac{2}{7}\left(\frac{5}{6}\int (18x^2+7) (x^4+5)^{3/2} dx^2-\frac{7}{6}x^2 (x^4+5)^{5/2}\right)\right)$$

$$\Big\downarrow \text{455}$$

$$\frac{1}{2}\left(\frac{3}{7}x^4 (x^4+5)^{5/2}-\frac{2}{7}\left(\frac{5}{6}\left(7\int (x^4+5)^{3/2} dx^2+\frac{18}{5}(x^4+5)^{5/2}\right)-\frac{7}{6}x^2 (x^4+5)^{5/2}\right)\right)$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}\left(\frac{3}{7}x^4 (x^4+5)^{5/2}-\frac{2}{7}\left(\frac{5}{6}\left(7\left(\frac{15}{4}\int \sqrt{x^4+5}dx^2+\frac{1}{4}(x^4+5)^{3/2} x^2\right)+\frac{18}{5}(x^4+5)^{5/2}\right)-\frac{7}{6}x^2 (x^4+5)^{5/2}\right)\right)$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}\left(\frac{3}{7}x^4 (x^4+5)^{5/2}-\frac{2}{7}\left(\frac{5}{6}\left(7\left(\frac{15}{4}\left(\frac{5}{2}\int \frac{1}{\sqrt{x^4+5}}dx^2+\frac{1}{2}\sqrt{x^4+5}x^2\right)+\frac{1}{4}(x^4+5)^{3/2} x^2\right)+\frac{18}{5}(x^4+5)^{5/2}\right)-\right.\right.$$

$$\Big\downarrow 222$$

$$\frac{1}{2}\left(\frac{3}{7}x^4\left(x^4+5\right)^{5/2}-\frac{2}{7}\left(\frac{5}{6}\left(7\left(\frac{15}{4}\left(\frac{5}{2}\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{2}\sqrt{x^4+5}x^2\right)+\frac{1}{4}\left(x^4+5\right)^{3/2}x^2\right)+\frac{18}{5}\left(x^4+5\right)^{5/2}\right)-\frac{7}{6}\right.\right.$$

input
```
Int[x^5*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

output
```
((3*x^4*(5 + x^4)^(5/2))/7 - (2*((-7*x^2*(5 + x^4)^(5/2))/6 + (5*((18*(5 +
 x^4)^(5/2))/5 + 7*((x^2*(5 + x^4)^(3/2))/4 + (15*((x^2*Sqrt[5 + x^4])/2 +
 (5*ArcSinh[x^2/Sqrt[5]])/2))/4)))/6))/7)/2
```

3.20.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

3.20.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.65

method	result
risch	$\frac{(72x^{12}+56x^{10}+576x^8+490x^6+360x^4+525x^2-3600)\sqrt{x^4+5}}{336}-\frac{125\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{16}$
pseudoelliptic	$\frac{(72x^{12}+56x^{10}+576x^8+490x^6+360x^4+525x^2-3600)\sqrt{x^4+5}}{336}-\frac{125\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{16}$
trager	$\left(\frac{3}{14}x^{12}+\frac{1}{6}x^{10}+\frac{12}{7}x^8+\frac{35}{24}x^6+\frac{15}{14}x^4+\frac{25}{16}x^2-\frac{75}{7}\right)\sqrt{x^4+5}-\frac{125\ln\left(x^2+\sqrt{x^4+5}\right)}{16}$
default	$\frac{3\sqrt{x^4+5}\,(x^4-2)\,(x^8+10x^4+25)}{14}+\frac{x^{10}\sqrt{x^4+5}}{6}+\frac{35x^6\sqrt{x^4+5}}{24}+\frac{25x^2\sqrt{x^4+5}}{16}-\frac{125\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{16}$
elliptic	$\frac{3x^{12}\sqrt{x^4+5}}{14}+\frac{12x^8\sqrt{x^4+5}}{7}+\frac{15x^4\sqrt{x^4+5}}{14}-\frac{75\sqrt{x^4+5}}{7}+\frac{x^{10}\sqrt{x^4+5}}{6}+\frac{35x^6\sqrt{x^4+5}}{24}+\frac{25x^2\sqrt{x^4+5}}{16}-$ 125 arcs
meijerg	$\frac{1125\sqrt{5}\left(\frac{16\sqrt{\pi}}{105}-\frac{2\sqrt{\pi}\left(-\frac{4}{25}x^{12}-\frac{32}{25}x^8-\frac{4}{5}x^4+8\right)\sqrt{1+\frac{x^4}{5}}}{105}\right)}{16\sqrt{\pi}}+\frac{\frac{25\sqrt{\pi}\,x^2\sqrt{5}\left(\frac{8}{25}x^8+\frac{14}{5}x^4+3\right)\sqrt{1+\frac{x^4}{5}}}{48}-\frac{125\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{16}}{\sqrt{\pi}}$

input
```
int(x^5*(3*x^2+2)*(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/336*(72*x^12+56*x^10+576*x^8+490*x^6+360*x^4+525*x^2-3600)*(x^4+5)^(1/2)
-125/16*arcsinh(1/5*x^2*5^(1/2))
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.70

$$\int x^5(2+3x^2)\,(5+x^4)^{3/2}\,dx=\frac{1}{336}\left(72\,x^{12}+56\,x^{10}+576\,x^8+490\,x^6+360\,x^4+525\,x^2-3600\right)\sqrt{x^4+5}+\frac{125}{16}\log\left(-x^2+\sqrt{x^4+5}\right)$$

input
```
integrate(x^5*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="fricas")
```

output
```
1/336*(72*x^12 + 56*x^10 + 576*x^8 + 490*x^6 + 360*x^4 + 525*x^2 - 3600)*s
qrt(x^4 + 5) + 125/16*log(-x^2 + sqrt(x^4 + 5))
```

3.20.6 Sympy [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.46

$$\int x^5(2+3x^2)\left(5+x^4\right)^{3/2}\,dx = 5\sqrt{x^4+5}\left(\frac{x^6}{4}+\frac{5x^2}{8}\right)+\frac{15\sqrt{x^4+5}\left(\frac{x^8}{5}+\frac{x^4}{3}-\frac{10}{3}\right)}{2} + \sqrt{x^4+5}\left(\frac{x^{10}}{6}+\frac{5x^6}{24}-\frac{25x^2}{16}\right)+\frac{3\sqrt{x^4+5}\left(\frac{x^{12}}{7}+\frac{x^8}{7}-\frac{20x^4}{21}+\frac{200}{21}\right)}{2}-\frac{125\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{16}$$

input
```
integrate(x**5*(3*x**2+2)*(x**4+5)**(3/2),x)
```

output
```
5*sqrt(x**4 + 5)*(x**6/4 + 5*x**2/8) + 15*sqrt(x**4 + 5)*(x**8/5 + x**4/3
- 10/3)/2 + sqrt(x**4 + 5)*(x**10/6 + 5*x**6/24 - 25*x**2/16) + 3*sqrt(x**
4 + 5)*(x**12/7 + x**8/7 - 20*x**4/21 + 200/21)/2 - 125*asinh(sqrt(5)*x**2
/5)/16
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.53

$$\int x^5(2+3x^2)\left(5+x^4\right)^{3/2}\,dx = \frac{3}{14}\left(x^4+5\right)^{\frac{7}{2}} - \frac{3}{2}\left(x^4+5\right)^{\frac{5}{2}} - \frac{125\left(\frac{3\sqrt{x^4+5}}{x^2}-\frac{8\left(x^4+5\right)^{\frac{3}{2}}}{x^6}-\frac{3\left(x^4+5\right)^{\frac{5}{2}}}{x^{10}}\right)}{48\left(\frac{3\left(x^4+5\right)}{x^4}-\frac{3\left(x^4+5\right)^2}{x^8}+\frac{\left(x^4+5\right)^3}{x^{12}}-1\right)} - \frac{125}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)+\frac{125}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

input
```
integrate(x^5*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="maxima")
```

output
```
3/14*(x^4 + 5)^(7/2) - 3/2*(x^4 + 5)^(5/2) - 125/48*(3*sqrt(x^4 + 5)/x^2 -
 8*(x^4 + 5)^(3/2)/x^6 - 3*(x^4 + 5)^(5/2)/x^10)/(3*(x^4 + 5)/x^4 - 3*(x^4
 + 5)^2/x^8 + (x^4 + 5)^3/x^12 - 1) - 125/32*log(sqrt(x^4 + 5)/x^2 + 1) +
125/32*log(sqrt(x^4 + 5)/x^2 - 1)
```

3.20.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.96

$$\int x^5(2+3x^2)\left(5+x^4\right)^{3/2}\,dx=\frac{3}{14}\left(x^4+5\right)^{\frac{7}{2}}+\frac{1}{48}\left(2\left(4x^4+5\right)x^4-75\right)\sqrt{x^4+5}x^2+\frac{5}{8}\left(2x^4+5\right)\sqrt{x^4+5}x^2-\frac{3}{2}\left(x^4+5\right)^{\frac{5}{2}}+\frac{125}{16}\log\left(-x^2+\sqrt{x^4+5}\right)$$

input
```
integrate(x^5*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="giac")
```

output
```
3/14*(x^4 + 5)^(7/2) + 1/48*(2*(4*x^4 + 5)*x^4 - 75)*sqrt(x^4 + 5)*x^2 + 5
/8*(2*x^4 + 5)*sqrt(x^4 + 5)*x^2 - 3/2*(x^4 + 5)^(5/2) + 125/16*log(-x^2 +
 sqrt(x^4 + 5))
```

3.20.9 Mupad [B] (verification not implemented)

Time = 7.64 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.63

$$\int x^5(2+3x^2)\left(5+x^4\right)^{3/2}\,dx=\sqrt{x^4+5}\left(\frac{3x^{12}}{14}+\frac{x^{10}}{6}+\frac{12x^8}{7}+\frac{35x^6}{24}+\frac{15x^4}{14}+\frac{25x^2}{16}-\frac{75}{7}\right)-\frac{125\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{16}$$

input
```
int(x^5*(x^4 + 5)^(3/2)*(3*x^2 + 2),x)
```

output
```
(x^4 + 5)^(1/2)*((25*x^2)/16 + (15*x^4)/14 + (35*x^6)/24 + (12*x^8)/7 + x^
10/6 + (3*x^12)/14 - 75/7) - (125*asinh((5^(1/2)*x^2)/5))/16
```

3.21 $\int x^3(2+3x^2)(5+x^4)^{3/2}\,dx$

3.21.1 Optimal result

Integrand size = 20, antiderivative size = 67

$$\int x^3(2+3x^2)\left(5+x^4\right)^{3/2}\,dx=-\frac{75}{32}x^2\sqrt{5+x^4}-\frac{5}{16}x^2\left(5+x^4\right)^{3/2}+\frac{1}{20}\left(4+5x^2\right)\left(5+x^4\right)^{5/2}-\frac{375}{32}\operatorname{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)$$

output
```
-5/16*x^2*(x^4+5)^(3/2)+1/20*(5*x^2+4)*(x^4+5)^(5/2)-375/32*arcsinh(1/5*x^
2*5^(1/2))-75/32*x^2*(x^4+5)^(1/2)
```

3.21.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int x^3(2+3x^2)\left(5+x^4\right)^{3/2}\,dx=\frac{1}{160}\sqrt{5+x^4}\left(800+375x^2+320x^4+350x^6+32x^8+40x^{10}\right)+\frac{375}{32}\log\left(-x^2+\sqrt{5+x^4}\right)$$

input
```
Integrate[x^3*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

output
```
(Sqrt[5 + x^4]*(800 + 375*x^2 + 320*x^4 + 350*x^6 + 32*x^8 + 40*x^10))/160
 + (375*Log[-x^2 + Sqrt[5 + x^4]])/32
```

3.21.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.39, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1579, 533, 27, 455, 211, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (3x^2+2) (x^4+5)^{3/2} \, dx$$

$$\Big\downarrow \text{1579}$$

$$\frac{1}{2} \int x^2 (3x^2+2) (x^4+5)^{3/2} \, dx^2$$

$$\Big\downarrow \text{533}$$

$$\frac{1}{2} \left(\frac{1}{2} x^2 (x^4+5)^{5/2} - \frac{1}{6} \int 3(5-4x^2) (x^4+5)^{3/2} \, dx^2\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2} \left(\frac{1}{2} x^2 (x^4+5)^{5/2} - \frac{1}{2} \int (5-4x^2) (x^4+5)^{3/2} \, dx^2\right)$$

$$\Big\downarrow \text{455}$$

$$\frac{1}{2} \left(\frac{1}{2} \left(\frac{4}{5} (x^4+5)^{5/2} - 5 \int (x^4+5)^{3/2} \, dx^2\right) + \frac{1}{2} x^2 (x^4+5)^{5/2}\right)$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2} \left(\frac{1}{2} \left(\frac{4}{5} (x^4+5)^{5/2} - 5 \left(\frac{15}{4} \int \sqrt{x^4+5} dx^2 + \frac{1}{4} (x^4+5)^{3/2} x^2\right)\right) + \frac{1}{2} x^2 (x^4+5)^{5/2}\right)$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2} \left(\frac{1}{2} \left(\frac{4}{5} (x^4+5)^{5/2} - 5 \left(\frac{15}{4} \left(\frac{5}{2} \int \frac{1}{\sqrt{x^4+5}} dx^2 + \frac{1}{2} \sqrt{x^4+5} x^2\right) + \frac{1}{4} (x^4+5)^{3/2} x^2\right)\right) + \frac{1}{2} x^2 (x^4+5)^{5/2}\right)$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{2} \left(\frac{1}{2} \left(\frac{4}{5} (x^4+5)^{5/2} - 5 \left(\frac{15}{4} \left(\frac{5}{2} \text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{1}{2} \sqrt{x^4+5} x^2\right) + \frac{1}{4} (x^4+5)^{3/2} x^2\right)\right) + \frac{1}{2} x^2 (x^4+5)^{5/2}\right)$$

input
```
Int[x^3*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

output
```
((x^2*(5 + x^4)^(5/2))/2 + ((4*(5 + x^4)^(5/2))/5 - 5*((x^2*(5 + x^4)^(3/2
))/4 + (15*((x^2*Sqrt[5 + x^4])/2 + (5*ArcSinh[x^2/Sqrt[5]])/2))/4))/2)/2
```

3.21.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

3.21.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.73

method	result
risch	$\frac{(40x^{10}+32x^8+350x^6+320x^4+375x^2+800)\sqrt{x^4+5}}{160}-\frac{375\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{32}$
pseudoelliptic	$\frac{(40x^{10}+32x^8+350x^6+320x^4+375x^2+800)\sqrt{x^4+5}}{160}-\frac{375\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{32}$
trager	$\left(\frac{1}{4}x^{10}+\frac{1}{5}x^8+\frac{35}{16}x^6+2x^4+\frac{75}{32}x^2+5\right)\sqrt{x^4+5}-\frac{375\ln\left(x^2+\sqrt{x^4+5}\right)}{32}$
default	$\frac{x^{10}\sqrt{x^4+5}}{4}+\frac{35x^6\sqrt{x^4+5}}{16}+\frac{75x^2\sqrt{x^4+5}}{32}-\frac{375\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{32}+\frac{(x^4+5)^{\frac{5}{2}}}{5}$
elliptic	$5\sqrt{x^4+5}+\frac{x^{10}\sqrt{x^4+5}}{4}+\frac{35x^6\sqrt{x^4+5}}{16}+\frac{75x^2\sqrt{x^4+5}}{32}-\frac{375\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{32}+\frac{x^8\sqrt{x^4+5}}{5}+2x^4\sqrt{x^4+5}$
meijerg	$\frac{\frac{25\sqrt{\pi}\,x^2\sqrt{5}\left(\frac{8}{25}x^8+\frac{14}{5}x^4+3\right)\sqrt{1+\frac{x^4}{5}}}{32}-\frac{375\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{32}}{\sqrt{\pi}}+\frac{75\sqrt{5}\left(-\frac{8\sqrt{\pi}}{15}+\frac{4\sqrt{\pi}\left(\frac{2}{25}x^8+\frac{4}{5}x^4+2\right)\sqrt{1+\frac{x^4}{5}}}{15}\right)}{8\sqrt{\pi}}$

input
```
int(x^3*(3*x^2+2)*(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/160*(40*x^10+32*x^8+350*x^6+320*x^4+375*x^2+800)*(x^4+5)^(1/2)-375/32*ar
csinh(1/5*x^2*5^(1/2))
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

$$\int x^3(2+3x^2)\,(5+x^4)^{3/2}\,dx=\frac{1}{160}\,(40\,x^{10}+32\,x^8+350\,x^6+320\,x^4+375\,x^2+800)\sqrt{x^4+5}+\frac{375}{32}\,\log\left(-x^2+\sqrt{x^4+5}\right)$$

input
```
integrate(x^3*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="fricas")
```

output
```
1/160*(40*x^10 + 32*x^8 + 350*x^6 + 320*x^4 + 375*x^2 + 800)*sqrt(x^4 + 5)
 + 375/32*log(-x^2 + sqrt(x^4 + 5))
```

3.21.6 Sympy [A] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.63

$$\int x^3(2+3x^2)\left(5+x^4\right)^{3/2}\,dx = 5\left(\frac{x^4}{3}+\frac{5}{3}\right)\sqrt{x^4+5}+\frac{15\sqrt{x^4+5}\left(\frac{x^6}{4}+\frac{5x^2}{8}\right)}{2} + \sqrt{x^4+5}\left(\frac{x^8}{5}+\frac{x^4}{3}-\frac{10}{3}\right)+\frac{3\sqrt{x^4+5}\left(\frac{x^{10}}{6}+\frac{5x^6}{24}-\frac{25x^2}{16}\right)}{2}-\frac{375\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{32}$$

input
```
integrate(x**3*(3*x**2+2)*(x**4+5)**(3/2),x)
```

output
```
5*(x**4/3 + 5/3)*sqrt(x**4 + 5) + 15*sqrt(x**4 + 5)*(x**6/4 + 5*x**2/8)/2
+ sqrt(x**4 + 5)*(x**8/5 + x**4/3 - 10/3) + 3*sqrt(x**4 + 5)*(x**10/6 + 5*
x**6/24 - 25*x**2/16)/2 - 375*asinh(sqrt(5)*x**2/5)/32
```

3.21.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(52) = 104.

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.76

$$\int x^3(2+3x^2)\left(5+x^4\right)^{3/2}\,dx = \frac{1}{5}\left(x^4+5\right)^{\frac{5}{2}}-\frac{125\left(\frac{3\sqrt{x^4+5}}{x^2}-\frac{8\left(x^4+5\right)^{\frac{3}{2}}}{x^6}-\frac{3\left(x^4+5\right)^{\frac{5}{2}}}{x^{10}}\right)}{32\left(\frac{3\left(x^4+5\right)}{x^4}-\frac{3\left(x^4+5\right)^2}{x^8}+\frac{\left(x^4+5\right)^3}{x^{12}}-1\right)} - \frac{375}{64}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)+\frac{375}{64}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

input
```
integrate(x^3*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="maxima")
```

output
```
1/5*(x^4 + 5)^(5/2) - 125/32*(3*sqrt(x^4 + 5)/x^2 - 8*(x^4 + 5)^(3/2)/x^6
- 3*(x^4 + 5)^(5/2)/x^10)/(3*(x^4 + 5)/x^4 - 3*(x^4 + 5)^2/x^8 + (x^4 + 5)
^3/x^12 - 1) - 375/64*log(sqrt(x^4 + 5)/x^2 + 1) + 375/64*log(sqrt(x^4 + 5
)/x^2 - 1)
```

3.21.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06

$$\int x^3(2+3x^2)\left(5+x^4\right)^{3/2}\,dx = \frac{1}{32}\left(2\left(4\,x^4+5\right)x^4-75\right)\sqrt{x^4+5}x^2 + \frac{15}{16}\left(2\,x^4+5\right)\sqrt{x^4+5}x^2 + \frac{1}{5}\left(x^4+5\right)^{\frac{5}{2}} + \frac{375}{32}\log\left(-x^2+\sqrt{x^4+5}\right)$$

input
```
integrate(x^3*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="giac")
```

output
```
1/32*(2*(4*x^4 + 5)*x^4 - 75)*sqrt(x^4 + 5)*x^2 + 15/16*(2*x^4 + 5)*sqrt(x
^4 + 5)*x^2 + 1/5*(x^4 + 5)^(5/2) + 375/32*log(-x^2 + sqrt(x^4 + 5))
```

3.21.9 Mupad [B] (verification not implemented)

Time = 7.69 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

$$\int x^3(2+3x^2)\left(5+x^4\right)^{3/2}\,dx = \sqrt{x^4+5}\left(\frac{x^{10}}{4}+\frac{x^8}{5}+\frac{35\,x^6}{16}+2\,x^4+\frac{75\,x^2}{32}+5\right) - \frac{375\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{32}$$

input
```
int(x^3*(x^4 + 5)^(3/2)*(3*x^2 + 2),x)
```

output
```
(x^4 + 5)^(1/2)*((75*x^2)/32 + 2*x^4 + (35*x^6)/16 + x^8/5 + x^10/4 + 5) -
 (375*asinh((5^(1/2)*x^2)/5))/32
```

3.22 $\int x(2+3x^2)\left(5+x^4\right)^{3/2}\,dx$

3.22.1 Optimal result

Integrand size = 18, antiderivative size = 60

$$\int x\left(2+3x^2\right)\left(5+x^4\right)^{3/2}\,dx=\frac{15}{8}x^2\sqrt{5+x^4}+\frac{1}{4}x^2\left(5+x^4\right)^{3/2}+\frac{3}{10}\left(5+x^4\right)^{5/2}+\frac{75}{8}\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)$$

output
```
1/4*x^2*(x^4+5)^(3/2)+3/10*(x^4+5)^(5/2)+75/8*arcsinh(1/5*x^2*5^(1/2))+15/
8*x^2*(x^4+5)^(1/2)
```

3.22.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int x\left(2+3x^2\right)\left(5+x^4\right)^{3/2}\,dx=\frac{1}{40}\sqrt{5+x^4}\left(300+125x^2+120x^4+10x^6+12x^8\right)-\frac{75}{8}\log\left(-x^2+\sqrt{5+x^4}\right)$$

input
```
Integrate[x*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

output
```
(Sqrt[5 + x^4]*(300 + 125*x^2 + 120*x^4 + 10*x^6 + 12*x^8))/40 - (75*Log[-
x^2 + Sqrt[5 + x^4]])/8
```

3.22.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.20, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1577, 455, 211, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(3x^2+2\right)\left(x^4+5\right)^{3/2}dx$$

$$\Big\downarrow \text{1577}$$

$$\frac{1}{2}\int \left(3x^2+2\right)\left(x^4+5\right)^{3/2}dx^2$$

$$\Big\downarrow \text{455}$$

$$\frac{1}{2}\left(2\int \left(x^4+5\right)^{3/2}dx^2+\frac{3}{5}\left(x^4+5\right)^{5/2}\right)$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}\left(2\left(\frac{15}{4}\int \sqrt{x^4+5}dx^2+\frac{1}{4}\left(x^4+5\right)^{3/2}x^2\right)+\frac{3}{5}\left(x^4+5\right)^{5/2}\right)$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}\left(2\left(\frac{15}{4}\left(\frac{5}{2}\int \frac{1}{\sqrt{x^4+5}}dx^2+\frac{1}{2}\sqrt{x^4+5}x^2\right)+\frac{1}{4}\left(x^4+5\right)^{3/2}x^2\right)+\frac{3}{5}\left(x^4+5\right)^{5/2}\right)$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{2}\left(2\left(\frac{15}{4}\left(\frac{5}{2}\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{2}\sqrt{x^4+5}x^2\right)+\frac{1}{4}\left(x^4+5\right)^{3/2}x^2\right)+\frac{3}{5}\left(x^4+5\right)^{5/2}\right)$$

input
```
Int[x*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

output
```
((3*(5 + x^4)^(5/2))/5 + 2*((x^2*(5 + x^4)^(3/2))/4 + (15*((x^2*Sqrt[5 + x
^4])/2 + (5*ArcSinh[x^2/Sqrt[5]])/2))/4))/2
```

3.22.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

3.22.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73

method	result	size
risch	$\frac{\left(12 x^{8}+10 x^{6}+120 x^{4}+125 x^{2}+300\right) \sqrt{x^{4}+5}}{40}+\frac{75 \,\operatorname{arcsinh}\left(\frac{x^{2} \sqrt{5}}{5}\right)}{8}$	44
pseudoelliptic	$\frac{\left(12 x^{8}+10 x^{6}+120 x^{4}+125 x^{2}+300\right) \sqrt{x^{4}+5}}{40}+\frac{75 \,\operatorname{arcsinh}\left(\frac{x^{2} \sqrt{5}}{5}\right)}{8}$	44
default	$\frac{75 \,\operatorname{arcsinh}\left(\frac{x^{2} \sqrt{5}}{5}\right)}{8}+\frac{x^{6} \sqrt{x^{4}+5}}{4}+\frac{25 x^{2} \sqrt{x^{4}+5}}{8}+\frac{3 \left(x^{4}+5\right)^{\frac{5}{2}}}{10}$	46
trager	$\left(\frac{3}{10} x^{8}+\frac{1}{4} x^{6}+3 x^{4}+\frac{25}{8} x^{2}+\frac{15}{2}\right) \sqrt{x^{4}+5}-\frac{75 \ln\left(x^{2}-\sqrt{x^{4}+5}\right)}{8}$	48
elliptic	$\frac{75 \,\operatorname{arcsinh}\left(\frac{x^{2} \sqrt{5}}{5}\right)}{8}+\frac{15 \sqrt{x^{4}+5}}{2}+\frac{3 x^{8} \sqrt{x^{4}+5}}{10}+3 x^{4} \sqrt{x^{4}+5}+\frac{x^{6} \sqrt{x^{4}+5}}{4}+\frac{25 x^{2} \sqrt{x^{4}+5}}{8}$	70
meijerg	$\frac{225 \sqrt{5}\, \left(-\frac{8 \sqrt{\pi}}{15}+\frac{4 \sqrt{\pi}\, \left(\frac{2}{25} x^{8}+\frac{4}{5} x^{4}+2\right) \sqrt{1+\frac{x^{4}}{5}}}{15}\right)}{16 \sqrt{\pi}}+\frac{5 \sqrt{\pi}\, x^{2} \sqrt{5}\, \left(\frac{x^{4}}{20}+\frac{5}{8}\right) \sqrt{1+\frac{x^{4}}{5}}+\frac{75 \sqrt{\pi}\, \operatorname{arcsinh}\left(\frac{x^{2} \sqrt{5}}{5}\right)}{8}}{\sqrt{\pi}}$	88

input
```
int(x*(3*x^2+2)*(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/40*(12*x^8+10*x^6+120*x^4+125*x^2+300)*(x^4+5)^(1/2)+75/8*arcsinh(1/5*x^
2*5^(1/2))
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.80

$$\int x(2+3x^2)\left(5+x^4\right)^{3/2}\,dx=\frac{1}{40}\left(12\,x^8+10\,x^6+120\,x^4+125\,x^2+300\right)\sqrt{x^4+5}-\frac{75}{8}\log\left(-x^2+\sqrt{x^4+5}\right)$$

input
```
integrate(x*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="fricas")
```

output
```
1/40*(12*x^8 + 10*x^6 + 120*x^4 + 125*x^2 + 300)*sqrt(x^4 + 5) - 75/8*log(
-x^2 + sqrt(x^4 + 5))
```

3.22.6 Sympy [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.58

$$\int x(2+3x^2)\left(5+x^4\right)^{3/2}\,dx=\frac{5x^2\sqrt{x^4+5}}{2}+\frac{15\left(\frac{x^4}{3}+\frac{5}{3}\right)\sqrt{x^4+5}}{2}+\sqrt{x^4+5}\left(\frac{x^6}{4}+\frac{5x^2}{8}\right)+\frac{3\sqrt{x^4+5}\left(\frac{x^8}{5}+\frac{x^4}{3}-\frac{10}{3}\right)}{2}+\frac{75\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{8}$$

input
```
integrate(x*(3*x**2+2)*(x**4+5)**(3/2),x)
```

output
```
5*x**2*sqrt(x**4 + 5)/2 + 15*(x**4/3 + 5/3)*sqrt(x**4 + 5)/2 + sqrt(x**4 +
 5)*(x**6/4 + 5*x**2/8) + 3*sqrt(x**4 + 5)*(x**8/5 + x**4/3 - 10/3)/2 + 75
*asinh(sqrt(5)*x**2/5)/8
```

3.22.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(45) = 90.

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.58

$$\int x(2+3x^2)\left(5+x^4\right)^{3/2}\,dx = \frac{3}{10}\left(x^4+5\right)^{\frac{5}{2}} + \frac{25\left(\frac{3\sqrt{x^4+5}}{x^2} - \frac{5\left(x^4+5\right)^{\frac{3}{2}}}{x^6}\right)}{8\left(\frac{2\left(x^4+5\right)}{x^4} - \frac{\left(x^4+5\right)^2}{x^8} - 1\right)} + \frac{75}{16}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right) - \frac{75}{16}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

input
```
integrate(x*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="maxima")
```

output
```
3/10*(x^4 + 5)^(5/2) + 25/8*(3*sqrt(x^4 + 5)/x^2 - 5*(x^4 + 5)^(3/2)/x^6)/
(2*(x^4 + 5)/x^4 - (x^4 + 5)^2/x^8 - 1) + 75/16*log(sqrt(x^4 + 5)/x^2 + 1)
 - 75/16*log(sqrt(x^4 + 5)/x^2 - 1)
```

3.22.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int x(2+3x^2)\left(5+x^4\right)^{3/2}\,dx = \frac{1}{8}\left(2\,x^4+5\right)\sqrt{x^4+5}x^2 + \frac{3}{10}\left(x^4+5\right)^{\frac{5}{2}} + \frac{5}{2}\sqrt{x^4+5}x^2 - \frac{75}{8}\log\left(-x^2+\sqrt{x^4+5}\right)$$

input
```
integrate(x*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="giac")
```

output
```
1/8*(2*x^4 + 5)*sqrt(x^4 + 5)*x^2 + 3/10*(x^4 + 5)^(5/2) + 5/2*sqrt(x^4 +
5)*x^2 - 75/8*log(-x^2 + sqrt(x^4 + 5))
```

3.22.9 Mupad [B] (verification not implemented)

Time = 7.54 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.70

$$\int x(2+3x^2)\left(5+x^4\right)^{3/2}\,dx=\frac{75\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{8}+\sqrt{x^4+5}\left(\frac{3\,x^8}{10}+\frac{x^6}{4}+3\,x^4+\frac{25\,x^2}{8}+\frac{15}{2}\right)$$

input
```
int(x*(x^4 + 5)^(3/2)*(3*x^2 + 2),x)
```

output
```
(75*asinh((5^(1/2)*x^2)/5))/8 + (x^4 + 5)^(1/2)*((25*x^2)/8 + 3*x^4 + x^6/
4 + (3*x^8)/10 + 15/2)
```

3.23 $\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x}\,dx$

3.23.1 Optimal result

Integrand size = 20, antiderivative size = 78

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x}\,dx = \frac{5}{16}(16+9x^2)\sqrt{5+x^4} + \frac{1}{24}(8+9x^2)(5+x^4)^{3/2} + \frac{225}{16}\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right) - 5\sqrt{5}\text{arctanh}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)$$

output
```
1/24*(9*x^2+8)*(x^4+5)^(3/2)+225/16*arcsinh(1/5*x^2*5^(1/2))-5*arctanh(1/5
*(x^4+5)^(1/2)*5^(1/2))*5^(1/2)+5/16*(9*x^2+16)*(x^4+5)^(1/2)
```

3.23.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x}\,dx = \frac{1}{48}\left(\sqrt{5+x^4}(320+225x^2+16x^4+18x^6) + 480\sqrt{5}\text{arctanh}\left(\frac{x^2-\sqrt{5+x^4}}{\sqrt{5}}\right) - 675\log\left(-x^2+\sqrt{5+x^4}\right)\right)$$

input
```
Integrate[((2 + 3*x^2)*(5 + x^4)^(3/2))/x,x]
```

output
```
(Sqrt[5 + x^4]*(320 + 225*x^2 + 16*x^4 + 18*x^6) + 480*Sqrt[5]*ArcTanh[(x^
2 - Sqrt[5 + x^4])/Sqrt[5]] - 675*Log[-x^2 + Sqrt[5 + x^4]])/48
```

3.23.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1579, 535, 535, 538, 222, 243, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x^2+2)(x^4+5)^{3/2}}{x}dx$$

$$\Big\downarrow \text{ 1579}$$

$$\frac{1}{2}\int \frac{(3x^2+2)(x^4+5)^{3/2}}{x^2}dx^2$$

$$\Big\downarrow \text{ 535}$$

$$\frac{1}{2}\left(\frac{5}{4}\int \frac{(9x^2+8)\sqrt{x^4+5}}{x^2}dx^2+\frac{1}{12}(9x^2+8)(x^4+5)^{3/2}\right)$$

$$\Big\downarrow \text{ 535}$$

$$\frac{1}{2}\left(\frac{5}{4}\left(\frac{5}{2}\int \frac{9x^2+16}{x^2\sqrt{x^4+5}}dx^2+\frac{1}{2}\sqrt{x^4+5}(9x^2+16)\right)+\frac{1}{12}(9x^2+8)(x^4+5)^{3/2}\right)$$

$$\Big\downarrow \text{ 538}$$

$$\frac{1}{2}\left(\frac{5}{4}\left(\frac{5}{2}\left(9\int \frac{1}{\sqrt{x^4+5}}dx^2+16\int \frac{1}{x^2\sqrt{x^4+5}}dx^2\right)+\frac{1}{2}\sqrt{x^4+5}(9x^2+16)\right)+\frac{1}{12}(9x^2+8)(x^4+5)^{3/2}\right)$$

$$\Big\downarrow \text{ 222}$$

$$\frac{1}{2}\left(\frac{5}{4}\left(\frac{5}{2}\left(16\int \frac{1}{x^2\sqrt{x^4+5}}dx^2+9\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)\right)+\frac{1}{2}\sqrt{x^4+5}(9x^2+16)\right)+\frac{1}{12}(9x^2+8)(x^4+5)^{3/2}\right)$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\left(\frac{5}{4}\left(\frac{5}{2}\left(8\int \frac{1}{x^2\sqrt{x^4+5}}dx^4+9\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)\right)+\frac{1}{2}\sqrt{x^4+5}(9x^2+16)\right)+\frac{1}{12}(9x^2+8)(x^4+5)^{3/2}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2}\left(\frac{5}{4}\left(\frac{5}{2}\left(16\int \frac{1}{\sqrt{x^4+5}-5}d\sqrt{x^4+5}+9\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)\right)+\frac{1}{2}\sqrt{x^4+5}(9x^2+16)\right)+\frac{1}{12}(9x^2+8)(x^4+5)^{3/2}\right)$$

$$\Big\downarrow 220$$

$$\frac{1}{2}\left(\frac{5}{4}\left(\frac{5}{2}\left(9\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{16\text{arctanh}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{\sqrt{5}}\right)+\frac{1}{2}\sqrt{x^4+5}(9x^2+16)\right)+\frac{1}{12}(9x^2+8)\,(x^4+5)^{3/2}\right)$$

input
```
Int[((2 + 3*x^2)*(5 + x^4)^(3/2))/x,x]
```

output
```
(((8 + 9*x^2)*(5 + x^4)^(3/2))/12 + (5*(((16 + 9*x^2)*Sqrt[5 + x^4])/2 + (
5*(9*ArcSinh[x^2/Sqrt[5]] - (16*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/Sqrt[5]))/
2))/4)/2
```

3.23.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

3.23.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.72

method	result
pseudoelliptic	$-5\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)+\frac{225\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{16}+\frac{(18x^6+16x^4+225x^2+320)\sqrt{x^4+5}}{48}$
trager	$\left(\frac{3}{8}x^6+\frac{1}{3}x^4+\frac{75}{16}x^2+\frac{20}{3}\right)\sqrt{x^4+5}+\frac{225\ln\left(-x^2-\sqrt{x^4+5}\right)}{16}-5\operatorname{RootOf}\left(\_Z^2-5\right)\ln\left(\frac{\operatorname{RootOf}\left(\_\right.}{}\right.$
default	$\frac{225\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{16}+\frac{3x^6\sqrt{x^4+5}}{8}+\frac{75x^2\sqrt{x^4+5}}{16}+\frac{x^4\sqrt{x^4+5}}{3}+\frac{20\sqrt{x^4+5}}{3}-5\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)$
elliptic	$\frac{225\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{16}+\frac{3x^6\sqrt{x^4+5}}{8}+\frac{75x^2\sqrt{x^4+5}}{16}+\frac{x^4\sqrt{x^4+5}}{3}+\frac{20\sqrt{x^4+5}}{3}-5\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)$
meijerg	$\frac{15\sqrt{5}\left(-\frac{32\sqrt{\pi}}{9}+\frac{2\sqrt{\pi}\left(\frac{4x^4}{5}+16\right)\sqrt{1+\frac{x^4}{5}}}{9}-\frac{8\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{x^4}{5}}}{2}\right)}{3}+\frac{4\left(\frac{8}{3}-2\ln(2)+4\ln(x)-\ln(5)\right)\sqrt{\pi}}{3}\right)}{8\sqrt{\pi}}+\frac{15\sqrt{\pi}\,x^2\sqrt{5}\left(\frac{x^4}{20}+\frac{5}{8}\right)\sqrt{}}{2}$

input
```
int((3*x^2+2)*(x^4+5)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-5*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))+225/16*arcsinh(1/5*x^2*5^(1/2))+
1/48*(18*x^6+16*x^4+225*x^2+320)*(x^4+5)^(1/2)
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.86

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x}\,dx = \frac{1}{48}\left(18\,x^6+16\,x^4+225\,x^2+320\right)\sqrt{x^4+5} + 5\,\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right) - \frac{225}{16}\,\log\left(-x^2+\sqrt{x^4+5}\right)$$

input
```
integrate((3*x^2+2)*(x^4+5)^(3/2)/x,x, algorithm="fricas")
```

output
```
1/48*(18*x^6 + 16*x^4 + 225*x^2 + 320)*sqrt(x^4 + 5) + 5*sqrt(5)*log(-(sqr
t(5) - sqrt(x^4 + 5))/x^2) - 225/16*log(-x^2 + sqrt(x^4 + 5))
```

3.23.6 Sympy [A] (verification not implemented)

Time = 9.32 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.50

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x}\,dx = \frac{15x^2\sqrt{x^4+5}}{4} + \left(\frac{x^4}{3}+\frac{5}{3}\right)\sqrt{x^4+5} + \frac{3\sqrt{x^4+5}\left(\frac{x^6}{4}+\frac{5x^2}{8}\right)}{2} + 5\sqrt{5}\left(\sqrt{\frac{x^4}{5}+1} + \frac{\log\left(\sqrt{\frac{x^4}{5}+1}-1\right)}{2} - \frac{\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{2}\right) + \frac{225\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{16}$$

input
```
integrate((3*x**2+2)*(x**4+5)**(3/2)/x,x)
```

output
```
15*x**2*sqrt(x**4 + 5)/4 + (x**4/3 + 5/3)*sqrt(x**4 + 5) + 3*sqrt(x**4 + 5
)*(x**6/4 + 5*x**2/8)/2 + 5*sqrt(5)*(sqrt(x**4/5 + 1) + log(sqrt(x**4/5 +
1) - 1)/2 - log(sqrt(x**4/5 + 1) + 1)/2) + 225*asinh(sqrt(5)*x**2/5)/16
```

3.23.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(62) = 124$.

Time = 0.28 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.77

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x}\,dx = \frac{1}{3}\left(x^4+5\right)^{\frac{3}{2}} + \frac{5}{2}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right) + 5\sqrt{x^4+5} + \frac{75\left(\frac{3\sqrt{x^4+5}}{x^2} - \frac{5\left(x^4+5\right)^{\frac{3}{2}}}{x^6}\right)}{16\left(\frac{2\left(x^4+5\right)}{x^4} - \frac{\left(x^4+5\right)^2}{x^8} - 1\right)} + \frac{225}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right) - \frac{225}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

input
```
integrate((3*x^2+2)*(x^4+5)^(3/2)/x,x, algorithm="maxima")
```

output
```
1/3*(x^4 + 5)^(3/2) + 5/2*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5)
+ sqrt(x^4 + 5))) + 5*sqrt(x^4 + 5) + 75/16*(3*sqrt(x^4 + 5)/x^2 - 5*(x^4
+ 5)^(3/2)/x^6)/(2*(x^4 + 5)/x^4 - (x^4 + 5)^2/x^8 - 1) + 225/32*log(sqrt(
x^4 + 5)/x^2 + 1) - 225/32*log(sqrt(x^4 + 5)/x^2 - 1)
```

3.23.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.15

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x}\,dx = \frac{1}{48}\sqrt{x^4+5}\left(\left(2\left(9x^2+8\right)x^2+225\right)x^2+320\right) + 5\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right) - \frac{225}{16}\log\left(-x^2+\sqrt{x^4+5}\right)$$

input
```
integrate((3*x^2+2)*(x^4+5)^(3/2)/x,x, algorithm="giac")
```

output
```
1/48*sqrt(x^4 + 5)*((2*(9*x^2 + 8)*x^2 + 225)*x^2 + 320) + 5*sqrt(5)*log(-
(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4 + 5))) - 225/16*
log(-x^2 + sqrt(x^4 + 5))
```

3.23.9 Mupad [B] (verification not implemented)

Time = 7.58 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.71

$$\int \frac{(2+3x^2)\,(5+x^4)^{3/2}}{x}\,dx = \frac{225\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{16} - 5\,\sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}}{5}\right) + \sqrt{x^4+5}\,\left(\frac{3\,x^6}{8}+\frac{x^4}{3}+\frac{75\,x^2}{16}+\frac{20}{3}\right)$$

input
```
int(((x^4 + 5)^(3/2)*(3*x^2 + 2))/x,x)
```

output
```
(225*asinh((5^(1/2)*x^2)/5))/16 - 5*5^(1/2)*atanh((5^(1/2)*(x^4 + 5)^(1/2)
)/5) + (x^4 + 5)^(1/2)*((75*x^2)/16 + x^4/3 + (3*x^6)/8 + 20/3)
```

3.24 $\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^3} \, dx$

3.24.1 Optimal result

Integrand size = 20, antiderivative size = 81

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^3} \, dx = \frac{3}{2}(5+x^2)\sqrt{5+x^4} - \frac{(2-x^2)(5+x^4)^{3/2}}{2x^2} + \frac{15}{2}\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{15}{2}\sqrt{5}\text{arctanh}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)$$

output
```
-1/2*(-x^2+2)*(x^4+5)^(3/2)/x^2+15/2*arcsinh(1/5*x^2*5^(1/2))-15/2*arctanh
(1/5*(x^4+5)^(1/2)*5^(1/2))*5^(1/2)+3/2*(x^2+5)*(x^4+5)^(1/2)
```

3.24.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^3} \, dx = \frac{1}{2}\left(\frac{\sqrt{5+x^4}(-10+20x^2+x^4+x^6)}{x^2} + 30\sqrt{5}\text{arctanh}\left(\frac{x^2-\sqrt{5+x^4}}{\sqrt{5}}\right) - 15\log\left(-x^2+\sqrt{5+x^4}\right)\right)$$

input
```
Integrate[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^3,x]
```

output
```
((Sqrt[5 + x^4]*(-10 + 20*x^2 + x^4 + x^6))/x^2 + 30*Sqrt[5]*ArcTanh[(x^2
- Sqrt[5 + x^4])/Sqrt[5]] - 15*Log[-x^2 + Sqrt[5 + x^4]])/2
```

3.24.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {1579, 536, 535, 27, 538, 222, 243, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x^2+2)(x^4+5)^{3/2}}{x^3}dx$$

$$\Big\downarrow \ 1579$$

$$\frac{1}{2}\int \frac{(3x^2+2)(x^4+5)^{3/2}}{x^4}dx^2$$

$$\Big\downarrow \ 536$$

$$\frac{1}{2}\left(\int \frac{(6x^2+15)\sqrt{x^4+5}}{x^2}dx^2-\frac{(2-x^2)(x^4+5)^{3/2}}{x^2}\right)$$

$$\Big\downarrow \ 535$$

$$\frac{1}{2}\left(\frac{5}{2}\int \frac{6(x^2+5)}{x^2\sqrt{x^4+5}}dx^2-\frac{(2-x^2)(x^4+5)^{3/2}}{x^2}+3(x^2+5)\sqrt{x^4+5}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{2}\left(15\int \frac{x^2+5}{x^2\sqrt{x^4+5}}dx^2-\frac{(2-x^2)(x^4+5)^{3/2}}{x^2}+3(x^2+5)\sqrt{x^4+5}\right)$$

$$\Big\downarrow \ 538$$

$$\frac{1}{2}\left(15\left(\int \frac{1}{\sqrt{x^4+5}}dx^2+5\int \frac{1}{x^2\sqrt{x^4+5}}dx^2\right)-\frac{(2-x^2)(x^4+5)^{3/2}}{x^2}+3(x^2+5)\sqrt{x^4+5}\right)$$

$$\Big\downarrow \ 222$$

$$\frac{1}{2}\left(15\left(5\int \frac{1}{x^2\sqrt{x^4+5}}dx^2+\operatorname{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)\right)-\frac{(2-x^2)(x^4+5)^{3/2}}{x^2}+3(x^2+5)\sqrt{x^4+5}\right)$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2}\left(15\left(\frac{5}{2}\int \frac{1}{x^2\sqrt{x^4+5}}dx^4+\operatorname{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)\right)-\frac{(2-x^2)(x^4+5)^{3/2}}{x^2}+3(x^2+5)\sqrt{x^4+5}\right)$$

$$\Big\downarrow \ 73$$

$$\frac{1}{2}\left(15\left(5\int \frac{1}{\sqrt{x^4+5}-5}d\sqrt{x^4+5}+\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)\right)-\frac{(2-x^2)\,(x^4+5)^{3/2}}{x^2}+3(x^2+5)\,\sqrt{x^4+5}\right)$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{2}\left(15\left(\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)-\sqrt{5}\text{arctanh}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)\right)-\frac{(2-x^2)\,(x^4+5)^{3/2}}{x^2}+3(x^2+5)\,\sqrt{x^4+5}\right)$$

input
```
Int[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^3,x]
```

output
```
(3*(5 + x^2)*Sqrt[5 + x^4] - ((2 - x^2)*(5 + x^4)^(3/2))/x^2 + 15*(ArcSinh
[x^2/Sqrt[5]] - Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]]))/2
```

3.24.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

3.24. $\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^3}\,dx$

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 536
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_)^2, x_Symbol] :> S
imp[(-(2*c*p - d*x))*((a + b*x^2)^p/(2*p*x)), x] + Int[(a*d + 2*b*c*p*x)*((
a + b*x^2)^(p - 1)/x), x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && Integer
Q[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

3.24.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.77

method	result
pseudoelliptic	$\frac{-15\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)x^2+15\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)x^2+\left(x^6+x^4+20x^2-10\right)\sqrt{x^4+5}}{2x^2}$
trager	$\frac{\left(x^6+x^4+20x^2-10\right)\sqrt{x^4+5}}{2x^2}+\frac{15\ln\left(-x^2-\sqrt{x^4+5}\right)}{2}+\frac{15\,\operatorname{RootOf}\left(\_Z^2-5\right)\ln\left(\frac{\sqrt{x^4+5}-\operatorname{RootOf}\left(\_Z^2-5\right)}{x^2}\right)}{2}$
default	$\frac{x^4\sqrt{x^4+5}}{2}+10\sqrt{x^4+5}-\frac{15\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{2}+\frac{15\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}+\frac{x^2\sqrt{x^4+5}}{2}-\frac{5\sqrt{x^4+5}}{x^2}$
elliptic	$\frac{x^4\sqrt{x^4+5}}{2}+10\sqrt{x^4+5}-\frac{15\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{2}+\frac{15\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}+\frac{x^2\sqrt{x^4+5}}{2}-\frac{5\sqrt{x^4+5}}{x^2}$
risch	$-\frac{5\sqrt{x^4+5}}{x^2}+\frac{15\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}+\frac{\sqrt{x^4+5}\left(x^4-10\right)}{2}+\frac{x^2\sqrt{x^4+5}}{2}+15\sqrt{x^4+5}-\frac{15\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{2}$
meijerg	$\frac{-\frac{5\sqrt{\pi}\sqrt{5}\left(-\frac{x^4}{10}+1\right)\sqrt{1+\frac{x^4}{5}}}{x^2}+\frac{15\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}}{\sqrt{\pi}}+\frac{45\sqrt{5}\left(-\frac{32\sqrt{\pi}}{9}+\frac{2\sqrt{\pi}\left(\frac{4x^4}{5}+16\right)\sqrt{1+\frac{x^4}{5}}}{9}-\frac{8\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{x^4}{5}}}{2}\right)}{3}+\frac{4(}{}\right.}{16\sqrt{\pi}}$

input
```
int((3*x^2+2)*(x^4+5)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-15*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))*x^2+15*arcsinh(1/5*x^2*5^(
1/2))*x^2+(x^6+x^4+20*x^2-10)*(x^4+5)^(1/2))/x^2
```

3.24.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^3}\,dx=\frac{15\sqrt{5}x^2\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)-15\,x^2\log\left(-x^2+\sqrt{x^4+5}\right)-10\,x^2+\left(x^6+x^4+\right.}{2\,x^2}$$

input
```
integrate((3*x^2+2)*(x^4+5)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
1/2*(15*sqrt(5)*x^2*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 15*x^2*log(-x^2
+ sqrt(x^4 + 5)) - 10*x^2 + (x^6 + x^4 + 20*x^2 - 10)*sqrt(x^4 + 5))/x^2
```

3.24.6 Sympy [A] (verification not implemented)

Time = 4.35 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.41

$$\int \frac{(2+3x^2)\,(5+x^4)^{3/2}}{x^3}\,dx = \frac{x^6}{2\sqrt{x^4+5}} - \frac{5x^2}{2\sqrt{x^4+5}} + \frac{(x^4+5)^{\frac{3}{2}}}{2} + \frac{15\sqrt{x^4+5}}{2} + \frac{15\sqrt{5}\log(x^4)}{4} - \frac{15\sqrt{5}\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{2} + \frac{15\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{2} - \frac{25}{x^2\sqrt{x^4+5}}$$

input
```
integrate((3*x**2+2)*(x**4+5)**(3/2)/x**3,x)
```

output
```
x**6/(2*sqrt(x**4 + 5)) - 5*x**2/(2*sqrt(x**4 + 5)) + (x**4 + 5)**(3/2)/2
+ 15*sqrt(x**4 + 5)/2 + 15*sqrt(5)*log(x**4)/4 - 15*sqrt(5)*log(sqrt(x**4/
5 + 1) + 1)/2 + 15*asinh(sqrt(5)*x**2/5)/2 - 25/(x**2*sqrt(x**4 + 5))
```

3.24.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.51

$$\int \frac{(2+3x^2)\,(5+x^4)^{3/2}}{x^3}\,dx = \frac{1}{2}\left(x^4+5\right)^{\frac{3}{2}} + \frac{15}{4}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right) + \frac{15}{2}\sqrt{x^4+5} - \frac{5\sqrt{x^4+5}}{x^2} + \frac{5\sqrt{x^4+5}}{2x^2\left(\frac{x^4+5}{x^4}-1\right)} + \frac{15}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right) - \frac{15}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

input
```
integrate((3*x^2+2)*(x^4+5)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
1/2*(x^4 + 5)^(3/2) + 15/4*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5)
 + sqrt(x^4 + 5))) + 15/2*sqrt(x^4 + 5) - 5*sqrt(x^4 + 5)/x^2 + 5/2*sqrt(x
^4 + 5)/(x^2*((x^4 + 5)/x^4 - 1)) + 15/4*log(sqrt(x^4 + 5)/x^2 + 1) - 15/4
*log(sqrt(x^4 + 5)/x^2 - 1)
```

3.24.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.26

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^3}\,dx = \frac{1}{2}\sqrt{x^4+5}\big((x^2+1)x^2+20\big) + \frac{15}{2}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right) + \frac{50}{\left(x^2-\sqrt{x^4+5}\right)^2-5} - \frac{15}{2}\log\left(-x^2+\sqrt{x^4+5}\right)$$

input
```
integrate((3*x^2+2)*(x^4+5)^(3/2)/x^3,x, algorithm="giac")
```

output
```
1/2*sqrt(x^4 + 5)*((x^2 + 1)*x^2 + 20) + 15/2*sqrt(5)*log(-(x^2 + sqrt(5)
- sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4 + 5))) + 50/((x^2 - sqrt(x^4 +
5))^2 - 5) - 15/2*log(-x^2 + sqrt(x^4 + 5))
```

3.24.9 Mupad [B] (verification not implemented)

Time = 8.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.79

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^3}\,dx = \frac{15\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{2} + \sqrt{x^4+5}\left(\frac{x^4}{2}+\frac{x^2}{2}+10\right) - \frac{5\sqrt{x^4+5}}{x^2} + \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}\sqrt{x^4+5}\,1\mathrm{i}}{5}\right)\,15\mathrm{i}}{2}$$

input
```
int(((x^4 + 5)^(3/2)*(3*x^2 + 2))/x^3,x)
```

output
```
(15*asinh((5^(1/2)*x^2)/5))/2 + (5^(1/2)*atan((5^(1/2)*(x^4 + 5)^(1/2)*1i)
/5)*15i)/2 + (x^4 + 5)^(1/2)*(x^2/2 + x^4/2 + 10) - (5*(x^4 + 5)^(1/2))/x^
2
```

3.25 $\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^5} \, dx$

3.25.1 Optimal result

Integrand size = 20, antiderivative size = 86

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^5} \, dx = -\frac{3(15-2x^2)\sqrt{5+x^4}}{4x^2} - \frac{(2-3x^2)(5+x^4)^{3/2}}{4x^4} + \frac{45}{4}\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{3}{2}\sqrt{5}\text{arctanh}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)$$

output
```
-1/4*(-3*x^2+2)*(x^4+5)^(3/2)/x^4+45/4*arcsinh(1/5*x^2*5^(1/2))-3/2*arctan
h(1/5*(x^4+5)^(1/2)*5^(1/2))*5^(1/2)-3/4*(-2*x^2+15)*(x^4+5)^(1/2)/x^2
```

3.25.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^5} \, dx = 3\sqrt{5}\text{arctanh}\left(\frac{x^2-\sqrt{5+x^4}}{\sqrt{5}}\right) + \frac{1}{4}\left(\frac{\sqrt{5+x^4}(-10-30x^2+4x^4+3x^6)}{x^4} - 45\log\left(-x^2+\sqrt{5+x^4}\right)\right)$$

input
```
Integrate[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^5,x]
```

output
```
3*Sqrt[5]*ArcTanh[(x^2 - Sqrt[5 + x^4])/Sqrt[5]] + ((Sqrt[5 + x^4]*(-10 -
30*x^2 + 4*x^4 + 3*x^6))/x^4 - 45*Log[-x^2 + Sqrt[5 + x^4]])/4
```

3.25.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {1579, 537, 27, 535, 538, 222, 243, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x^2+2)(x^4+5)^{3/2}}{x^5}dx$$

$$\Big\downarrow \text{1579}$$

$$\frac{1}{2}\int \frac{(3x^2+2)(x^4+5)^{3/2}}{x^6}dx^2$$

$$\Big\downarrow \text{537}$$

$$\frac{1}{2}\left(-\frac{3}{2}\int -\frac{2(3x^2+1)\sqrt{x^4+5}}{x^2}dx^2-\frac{(3x^2+1)(x^4+5)^{3/2}}{x^4}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(3\int \frac{(3x^2+1)\sqrt{x^4+5}}{x^2}dx^2-\frac{(3x^2+1)(x^4+5)^{3/2}}{x^4}\right)$$

$$\Big\downarrow \text{535}$$

$$\frac{1}{2}\left(3\left(\frac{5}{2}\int \frac{3x^2+2}{x^2\sqrt{x^4+5}}dx^2+\frac{1}{2}\sqrt{x^4+5}(3x^2+2)\right)-\frac{(3x^2+1)(x^4+5)^{3/2}}{x^4}\right)$$

$$\Big\downarrow \text{538}$$

$$\frac{1}{2}\left(3\left(\frac{5}{2}\left(3\int \frac{1}{\sqrt{x^4+5}}dx^2+2\int \frac{1}{x^2\sqrt{x^4+5}}dx^2\right)+\frac{1}{2}\sqrt{x^4+5}(3x^2+2)\right)-\frac{(3x^2+1)(x^4+5)^{3/2}}{x^4}\right)$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{2}\left(3\left(\frac{5}{2}\left(2\int \frac{1}{x^2\sqrt{x^4+5}}dx^2+3\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)\right)+\frac{1}{2}\sqrt{x^4+5}(3x^2+2)\right)-\frac{(3x^2+1)(x^4+5)^{3/2}}{x^4}\right)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\left(3\left(\frac{5}{2}\left(\int \frac{1}{x^2\sqrt{x^4+5}}dx^4+3\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)\right)+\frac{1}{2}\sqrt{x^4+5}(3x^2+2)\right)-\frac{(3x^2+1)(x^4+5)^{3/2}}{x^4}\right)$$

$$\Big\downarrow 73$$

$$\frac{1}{2}\left(3\left(\frac{5}{2}\left(2\int\frac{1}{\sqrt{x^4+5}-5}d\sqrt{x^4+5}+3\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)\right)+\frac{1}{2}\sqrt{x^4+5}(3x^2+2)\right)-\frac{(3x^2+1)\,(x^4+5)^{3/2}}{x^4}\right)$$

$$\Big\downarrow 220$$

$$\frac{1}{2}\left(3\left(\frac{5}{2}\left(3\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{2\text{arctanh}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{\sqrt{5}}\right)+\frac{1}{2}\sqrt{x^4+5}(3x^2+2)\right)-\frac{(3x^2+1)\,(x^4+5)^{3/2}}{x^4}\right)$$

input
```
Int[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^5,x]
```

output
```
(-(((1 + 3*x^2)*(5 + x^4)^(3/2))/x^4) + 3*(((2 + 3*x^2)*Sqrt[5 + x^4])/2 +
 (5*(3*ArcSinh[x^2/Sqrt[5]] - (2*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/Sqrt[5]))
/2))/2
```

3.25.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

3.25.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$\frac{-6\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)x^4+45\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)x^4+3\sqrt{x^4+5}\,\left(x^6+\frac{4}{3}x^4-10x^2-\frac{10}{3}\right)}{4x^4}$
default	$\frac{45\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{4}+\frac{3x^2\sqrt{x^4+5}}{4}-\frac{15\sqrt{x^4+5}}{2x^2}+\sqrt{x^4+5}-\frac{5\sqrt{x^4+5}}{2x^4}-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{2}$
elliptic	$\frac{45\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{4}+\frac{3x^2\sqrt{x^4+5}}{4}-\frac{15\sqrt{x^4+5}}{2x^2}+\sqrt{x^4+5}-\frac{5\sqrt{x^4+5}}{2x^4}-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{2}$
trager	$\frac{(3x^6+4x^4-30x^2-10)\sqrt{x^4+5}}{4x^4}+\frac{3\,\operatorname{RootOf}\left(\_Z^2-5\right)\ln\left(\frac{\sqrt{x^4+5}-\operatorname{RootOf}\left(\_Z^2-5\right)}{x^2}\right)}{2}-\frac{45\ln\left(x^2-\sqrt{x^4+5}\right)}{4}$
risch	$-\frac{5(3x^6+x^4+15x^2+5)}{2x^4\sqrt{x^4+5}}+\frac{45\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{4}+\frac{3x^2\sqrt{x^4+5}}{4}+\sqrt{x^4+5}-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{2}$
meijerg	$\frac{3\sqrt{5}\left(\frac{5\sqrt{\pi}\left(-\frac{12x^4}{5}+8\right)}{6x^4}-\frac{5\sqrt{\pi}\left(8-\frac{16x^4}{5}\right)\sqrt{1+\frac{x^4}{5}}}{6x^4}-4\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{x^4}{5}}}{2}\right)+2(1-2\ln(2)+4\ln(x)-\ln(5))\sqrt{\pi}-\frac{20\sqrt{\pi}}{3x^4}\right)}{8\sqrt{\pi}}+$

input
```
int((3*x^2+2)*(x^4+5)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/4*(-6*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))*x^4+45*arcsinh(1/5*x^2*5^(1
/2))*x^4+3*(x^4+5)^(1/2)*(x^6+4/3*x^4-10*x^2-10/3))/x^4
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.95

$$\int \frac{(2+3x^2)\,(5+x^4)^{3/2}}{x^5}\,dx=\frac{6\,\sqrt{5}x^4\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)-45\,x^4\log\left(-x^2+\sqrt{x^4+5}\right)-30\,x^4+(3\,x^6+4\,x^4}{4\,x^4}$$

input
```
integrate((3*x^2+2)*(x^4+5)^(3/2)/x^5,x, algorithm="fricas")
```

output
```
1/4*(6*sqrt(5)*x^4*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 45*x^4*log(-x^2 +
 sqrt(x^4 + 5)) - 30*x^4 + (3*x^6 + 4*x^4 - 30*x^2 - 10)*sqrt(x^4 + 5))/x^
4
```

3.25.6 Sympy [A] (verification not implemented)

Time = 5.51 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.55

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^5}\,dx = \frac{3x^6}{4\sqrt{x^4+5}} - \frac{15x^2}{4\sqrt{x^4+5}} + \sqrt{x^4+5} + \frac{\sqrt{5}\log(x^4)}{2} - \sqrt{5}\log\left(\sqrt{\frac{x^4}{5}+1}+1\right) - \frac{\sqrt{5}\,\mathrm{asinh}\left(\frac{\sqrt{5}}{x^2}\right)}{2} + \frac{45\,\mathrm{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{4} - \frac{5\sqrt{1+\frac{5}{x^4}}}{2x^2} - \frac{75}{2x^2\sqrt{x^4+5}}$$

input
```
integrate((3*x**2+2)*(x**4+5)**(3/2)/x**5,x)
```

output
```
3*x**6/(4*sqrt(x**4 + 5)) - 15*x**2/(4*sqrt(x**4 + 5)) + sqrt(x**4 + 5) +
sqrt(5)*log(x**4)/2 - sqrt(5)*log(sqrt(x**4/5 + 1) + 1) - sqrt(5)*asinh(sq
rt(5)/x**2)/2 + 45*asinh(sqrt(5)*x**2/5)/4 - 5*sqrt(1 + 5/x**4)/(2*x**2) -
 75/(2*x**2*sqrt(x**4 + 5))
```

3.25.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.43

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^5}\,dx = \frac{3}{4}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right) + \sqrt{x^4+5} - \frac{15\sqrt{x^4+5}}{2\,x^2} + \frac{15\sqrt{x^4+5}}{4\,x^2\left(\frac{x^4+5}{x^4}-1\right)} - \frac{5\sqrt{x^4+5}}{2\,x^4} + \frac{45}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right) - \frac{45}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

input
```
integrate((3*x^2+2)*(x^4+5)^(3/2)/x^5,x, algorithm="maxima")
```

output
```
3/4*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) + sq
rt(x^4 + 5) - 15/2*sqrt(x^4 + 5)/x^2 + 15/4*sqrt(x^4 + 5)/(x^2*((x^4 + 5)/
x^4 - 1)) - 5/2*sqrt(x^4 + 5)/x^4 + 45/8*log(sqrt(x^4 + 5)/x^2 + 1) - 45/8
*log(sqrt(x^4 + 5)/x^2 - 1)
```

3.25.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(68) = 136$.

Time = 0.28 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.70

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^5}\,dx = \frac{1}{4}\sqrt{x^4+5}(3x^2+4) + \frac{3}{2}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right) + \frac{5\left(\left(x^2-\sqrt{x^4+5}\right)^3+15\left(x^2-\sqrt{x^4+5}\right)^2+5x^2-5\sqrt{x^4+5}-75\right)}{\left(\left(x^2-\sqrt{x^4+5}\right)^2-5\right)^2} - \frac{45}{4}\log\left(-x^2+\sqrt{x^4+5}\right)$$

input
```
integrate((3*x^2+2)*(x^4+5)^(3/2)/x^5,x, algorithm="giac")
```

output
```
1/4*sqrt(x^4 + 5)*(3*x^2 + 4) + 3/2*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4
 + 5))/(x^2 - sqrt(5) - sqrt(x^4 + 5))) + 5*((x^2 - sqrt(x^4 + 5))^3 + 15*
(x^2 - sqrt(x^4 + 5))^2 + 5*x^2 - 5*sqrt(x^4 + 5) - 75)/((x^2 - sqrt(x^4 +
 5))^2 - 5)^2 - 45/4*log(-x^2 + sqrt(x^4 + 5))
```

3.25.9 Mupad [B] (verification not implemented)

Time = 7.93 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.83

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^5}\,dx = \frac{45\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{4} + \sqrt{x^4+5}\left(\frac{3x^2}{4}+1\right) - \frac{15\sqrt{x^4+5}}{2x^2} - \frac{5\sqrt{x^4+5}}{2x^4} + \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}\sqrt{x^4+5}\,1i}{5}\right)3i}{2}$$

input
```
int(((x^4 + 5)^(3/2)*(3*x^2 + 2))/x^5,x)
```

output
```
(45*asinh((5^(1/2)*x^2)/5))/4 + (5^(1/2)*atan((5^(1/2)*(x^4 + 5)^(1/2)*1i)
/5)*3i)/2 + (x^4 + 5)^(1/2)*((3*x^2)/4 + 1) - (15*(x^4 + 5)^(1/2))/(2*x^2)
 - (5*(x^4 + 5)^(1/2))/(2*x^4)
```

3.26 $\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^7}\,dx$

3.26.1 Optimal result

Integrand size = 20, antiderivative size = 82

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^7}\,dx = -\frac{(4-9x^2)\sqrt{5+x^4}}{4x^2} - \frac{(4+9x^2)(5+x^4)^{3/2}}{12x^6} + \operatorname{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{9}{4}\sqrt{5}\operatorname{arctanh}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)$$

output
```
-1/12*(9*x^2+4)*(x^4+5)^(3/2)/x^6+arcsinh(1/5*x^2*5^(1/2))-9/4*arctanh(1/5
*(x^4+5)^(1/2)*5^(1/2))*5^(1/2)-1/4*(-9*x^2+4)*(x^4+5)^(1/2)/x^2
```

3.26.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.01

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^7}\,dx = \frac{\sqrt{5+x^4}(-20-45x^2-16x^4+18x^6)}{12x^6} + \frac{9}{2}\sqrt{5}\operatorname{arctanh}\left(\frac{x^2-\sqrt{5+x^4}}{\sqrt{5}}\right) - \log\left(-x^2+\sqrt{5+x^4}\right)$$

input
```
Integrate[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^7,x]
```

output
```
(Sqrt[5 + x^4]*(-20 - 45*x^2 - 16*x^4 + 18*x^6))/(12*x^6) + (9*Sqrt[5]*Arc
Tanh[(x^2 - Sqrt[5 + x^4])/Sqrt[5]])/2 - Log[-x^2 + Sqrt[5 + x^4]]
```

3.26.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {1579, 537, 25, 536, 538, 222, 243, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x^2+2)(x^4+5)^{3/2}}{x^7}dx$$

$$\Big\downarrow \text{ 1579}$$

$$\frac{1}{2}\int \frac{(3x^2+2)(x^4+5)^{3/2}}{x^8}dx^2$$

$$\Big\downarrow \text{ 537}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int -\frac{(9x^2+4)\sqrt{x^4+5}}{x^4}dx^2-\frac{(9x^2+4)(x^4+5)^{3/2}}{6x^6}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{(9x^2+4)\sqrt{x^4+5}}{x^4}dx^2-\frac{(9x^2+4)(x^4+5)^{3/2}}{6x^6}\right)$$

$$\Big\downarrow \text{ 536}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\int \frac{4x^2+45}{x^2\sqrt{x^4+5}}dx^2-\frac{(4-9x^2)\sqrt{x^4+5}}{x^2}\right)-\frac{(9x^2+4)(x^4+5)^{3/2}}{6x^6}\right)$$

$$\Big\downarrow \text{ 538}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(4\int \frac{1}{\sqrt{x^4+5}}dx^2+45\int \frac{1}{x^2\sqrt{x^4+5}}dx^2-\frac{\sqrt{x^4+5}(4-9x^2)}{x^2}\right)-\frac{(9x^2+4)(x^4+5)^{3/2}}{6x^6}\right)$$

$$\Big\downarrow \text{ 222}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(45\int \frac{1}{x^2\sqrt{x^4+5}}dx^2+4\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\sqrt{x^4+5}(4-9x^2)}{x^2}\right)-\frac{(9x^2+4)(x^4+5)^{3/2}}{6x^6}\right)$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{45}{2}\int \frac{1}{x^2\sqrt{x^4+5}}dx^4+4\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\sqrt{x^4+5}(4-9x^2)}{x^2}\right)-\frac{(9x^2+4)(x^4+5)^{3/2}}{6x^6}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(45\int \frac{1}{\sqrt{x^4+5}-5}d\sqrt{x^4+5}+4\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\sqrt{x^4+5}(4-9x^2)}{x^2}\right)-\frac{(9x^2+4)\,(x^4+5)^{3/2}}{6x^6}\right)$$

$$\Big\downarrow \; 220$$

$$\frac{1}{2}\left(\frac{1}{2}\left(4\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)-9\sqrt{5}\text{arctanh}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)-\frac{\sqrt{x^4+5}(4-9x^2)}{x^2}\right)-\frac{(9x^2+4)\,(x^4+5)^{3/2}}{6x^6}\right)$$

input
```
Int[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^7,x]
```

output
```
(-1/6*((4 + 9*x^2)*(5 + x^4)^(3/2))/x^6 + (-(((4 - 9*x^2)*Sqrt[5 + x^4])/x
^2) + 4*ArcSinh[x^2/Sqrt[5]] - 9*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/2
)/2
```

3.26.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 536
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_)^2, x_Symbol] :> S
imp[(-(2*c*p - d*x))*((a + b*x^2)^p/(2*p*x)), x] + Int[(a*d + 2*b*c*p*x)*((
a + b*x^2)^(p - 1)/x), x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && Integer
Q[2*p]
```

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

3.26.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$\frac{-9\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)x^6+4\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)x^6+6\left(x^6-\frac{8}{9}x^4-\frac{5}{2}x^2-\frac{10}{9}\right)\sqrt{x^4+5}}{4x^6}$
risch	$-\frac{16x^8+45x^6+100x^4+225x^2+100}{12x^6\sqrt{x^4+5}}+\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)+\frac{3\sqrt{x^4+5}}{2}-\frac{9\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{4}$
default	$\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)-\frac{4\sqrt{x^4+5}}{3x^2}-\frac{5\sqrt{x^4+5}}{3x^6}+\frac{3\sqrt{x^4+5}}{2}-\frac{15\sqrt{x^4+5}}{4x^4}-\frac{9\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{4}$
elliptic	$\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)-\frac{4\sqrt{x^4+5}}{3x^2}-\frac{5\sqrt{x^4+5}}{3x^6}+\frac{3\sqrt{x^4+5}}{2}-\frac{15\sqrt{x^4+5}}{4x^4}-\frac{9\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{4}$
trager	$\frac{(18x^6-16x^4-45x^2-20)\sqrt{x^4+5}}{12x^6}-\ln\left(x^2-\sqrt{x^4+5}\right)+\frac{9\,\operatorname{RootOf}\left(\_Z^2-5\right)\ln\left(\frac{\sqrt{x^4+5}-\operatorname{RootOf}\left(\_Z^2-5\right)}{x^2}\right)}{4}$
meijerg	$\frac{-\frac{5\sqrt{\pi}\,\sqrt{5}\left(\frac{4x^4}{5}+1\right)\sqrt{1+\frac{x^4}{5}}}{3x^6}+\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{\sqrt{\pi}}+\frac{9\sqrt{5}\left(\frac{5\sqrt{\pi}\left(-\frac{12x^4}{5}+8\right)}{6x^4}-\frac{5\sqrt{\pi}\left(8-\frac{16x^4}{5}\right)\sqrt{1+\frac{x^4}{5}}}{6x^4}-4\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{x^4}{5}}}{2}\right.\right.}{16\sqrt{\pi}}$

input
```
int((3*x^2+2)*(x^4+5)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
1/4*(-9*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))*x^6+4*arcsinh(1/5*x^2*5^(1/
2))*x^6+6*(x^6-8/9*x^4-5/2*x^2-10/9)*(x^4+5)^(1/2))/x^6
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \frac{(2+3x^2)\left(5+x^4\right)^{3/2}}{x^7}\,dx = \frac{27\sqrt{5}x^6\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)-12\,x^6\log\left(-x^2+\sqrt{x^4+5}\right)-16\,x^6+(18\,x^6-16}{12\,x^6}$$

input
```
integrate((3*x^2+2)*(x^4+5)^(3/2)/x^7,x, algorithm="fricas")
```

output
```
1/12*(27*sqrt(5)*x^6*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 12*x^6*log(-x^2
 + sqrt(x^4 + 5)) - 16*x^6 + (18*x^6 - 16*x^4 - 45*x^2 - 20)*sqrt(x^4 + 5)
)/x^6
```

3.26.6 Sympy [A] (verification not implemented)

Time = 5.79 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.80

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^7}\,dx = -\frac{x^2}{\sqrt{x^4+5}} - \frac{\sqrt{1+\frac{5}{x^4}}}{3} + \frac{3\sqrt{x^4+5}}{2} + \frac{3\sqrt{5}\log(x^4)}{4} - \frac{3\sqrt{5}\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{2} - \frac{3\sqrt{5}\operatorname{asinh}\left(\frac{\sqrt{5}}{x^2}\right)}{4} + \operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right) - \frac{15\sqrt{1+\frac{5}{x^4}}}{4x^2} - \frac{5}{x^2\sqrt{x^4+5}} - \frac{5\sqrt{1+\frac{5}{x^4}}}{3x^4}$$

input
```
integrate((3*x**2+2)*(x**4+5)**(3/2)/x**7,x)
```

output
```
-x**2/sqrt(x**4 + 5) - sqrt(1 + 5/x**4)/3 + 3*sqrt(x**4 + 5)/2 + 3*sqrt(5)
*log(x**4)/4 - 3*sqrt(5)*log(sqrt(x**4/5 + 1) + 1)/2 - 3*sqrt(5)*asinh(sqr
t(5)/x**2)/4 + asinh(sqrt(5)*x**2/5) - 15*sqrt(1 + 5/x**4)/(4*x**2) - 5/(x
**2*sqrt(x**4 + 5)) - 5*sqrt(1 + 5/x**4)/(3*x**4)
```

3.26.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.37

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^7}\,dx = \frac{9}{8}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right) + \frac{3}{2}\sqrt{x^4+5} - \frac{\sqrt{x^4+5}}{x^2} - \frac{15\sqrt{x^4+5}}{4x^4} - \frac{(x^4+5)^{\frac{3}{2}}}{3x^6} + \frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right) - \frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

input
```
integrate((3*x^2+2)*(x^4+5)^(3/2)/x^7,x, algorithm="maxima")
```

output
```
9/8*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) + 3/
2*sqrt(x^4 + 5) - sqrt(x^4 + 5)/x^2 - 15/4*sqrt(x^4 + 5)/x^4 - 1/3*(x^4 +
5)^(3/2)/x^6 + 1/2*log(sqrt(x^4 + 5)/x^2 + 1) - 1/2*log(sqrt(x^4 + 5)/x^2
- 1)
```

3.26.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(66) = 132.

Time = 0.31 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.93

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^7}\,dx = \frac{9}{4}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right)+\frac{3}{2}\sqrt{x^4+5} + \frac{5\left(9\left(x^2-\sqrt{x^4+5}\right)^5+24\left(x^2-\sqrt{x^4+5}\right)^4-120\left(x^2-\sqrt{x^4+5}\right)^2-225\,x^2+225\sqrt{x^4+5}+400\right)}{6\left(\left(x^2-\sqrt{x^4+5}\right)^2-5\right)^3} - \log\left(-x^2+\sqrt{x^4+5}\right)$$

input
```
integrate((3*x^2+2)*(x^4+5)^(3/2)/x^7,x, algorithm="giac")
```

output
```
9/4*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4
 + 5))) + 3/2*sqrt(x^4 + 5) + 5/6*(9*(x^2 - sqrt(x^4 + 5))^5 + 24*(x^2 - s
qrt(x^4 + 5))^4 - 120*(x^2 - sqrt(x^4 + 5))^2 - 225*x^2 + 225*sqrt(x^4 + 5
) + 400)/((x^2 - sqrt(x^4 + 5))^2 - 5)^3 - log(-x^2 + sqrt(x^4 + 5))
```

3.26.9 Mupad [B] (verification not implemented)

Time = 8.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^7}\,dx = \operatorname{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)+\frac{3\sqrt{x^4+5}}{2} + \sqrt{x^4+5}\left(\frac{2}{3\,x^2}-\frac{5}{3\,x^6}\right)-\frac{2\sqrt{x^4+5}}{x^2}-\frac{15\sqrt{x^4+5}}{4\,x^4}+\frac{\sqrt{5}\,\operatorname{atan}\left(\frac{\sqrt{5}\sqrt{x^4+5}\,1\mathrm{i}}{5}\right)\,9\mathrm{i}}{4}$$

input
```
int(((x^4 + 5)^(3/2)*(3*x^2 + 2))/x^7,x)
```

output
```
asinh((5^(1/2)*x^2)/5) + (5^(1/2)*atan((5^(1/2)*(x^4 + 5)^(1/2)*1i)/5)*9i)
/4 + (3*(x^4 + 5)^(1/2))/2 + (x^4 + 5)^(1/2)*(2/(3*x^2) - 5/(3*x^6)) - (2*
(x^4 + 5)^(1/2))/x^2 - (15*(x^4 + 5)^(1/2))/(4*x^4)
```

3.27 $\int x^4(2+3x^2)\left(5+x^4\right)^{3/2}\,dx$

3.27.1 Optimal result

Integrand size = 20, antiderivative size = 235

$$\begin{aligned}\int x^4(2+3x^2)\left(5+x^4\right)^{3/2}\,dx &= \frac{200}{77}x\sqrt{5+x^4}+\frac{20}{13}x^3\sqrt{5+x^4} \\ &- \frac{300x\sqrt{5+x^4}}{13\left(\sqrt{5}+x^2\right)}+\frac{10x^5(78+77x^2)\sqrt{5+x^4}}{1001}+\frac{1}{143}x^5\left(26+33x^2\right)\left(5+x^4\right)^{3/2} \\ &+ \frac{300\sqrt[4]{5}(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{13\sqrt{5+x^4}} \\ &- \frac{50\sqrt[4]{5}(231+26\sqrt{5})\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{1001\sqrt{5+x^4}}\end{aligned}$$

output
```
1/143*x^5*(33*x^2+26)*(x^4+5)^(3/2)+200/77*x*(x^4+5)^(1/2)+20/13*x^3*(x^4+
5)^(1/2)+10/1001*x^5*(77*x^2+78)*(x^4+5)^(1/2)-300/13*x*(x^4+5)^(1/2)/(x^2
+5^(1/2))+300/13*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arct
an(1/5*x*5^(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^
2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)-50/1001*5^(1/4)*(
cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*Ellipti
cF(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*(231+26*5^(1/2)
)*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)
```

3.27.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.31

$$\int x^4(2+3x^2)\,(5+x^4)^{3/2}\,dx = \frac{1}{143}x\Bigg((26+33x^2)\,(5+x^4)^{5/2} \\ -650\sqrt{5}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},-\frac{x^4}{5}\right)-825\sqrt{5}x^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},\frac{7}{4},-\frac{x^4}{5}\right)\Bigg)$$

input
```
Integrate[x^4*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

output
```
(x*((26 + 33*x^2)*(5 + x^4)^(5/2) - 650*Sqrt[5]*Hypergeometric2F1[-3/2, 1/
4, 5/4, -1/5*x^4] - 825*Sqrt[5]*x^2*Hypergeometric2F1[-3/2, 3/4, 7/4, -1/5
*x^4]))/143
```

3.27.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules used = {1597, 1597, 27, 1603, 27, 1603, 25, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(3x^2+2)\,(x^4+5)^{3/2}\,dx$$

$$\Big\downarrow \text{1597}$$

$$\frac{30}{143}\int x^4(33x^2+26)\,\sqrt{x^4+5}dx+\frac{1}{143}(33x^2+26)\,(x^4+5)^{3/2}\,x^5$$

$$\Big\downarrow \text{1597}$$

$$\frac{30}{143}\left(\frac{10}{63}\int\frac{3x^4(77x^2+78)}{\sqrt{x^4+5}}dx+\frac{1}{21}(77x^2+78)\,\sqrt{x^4+5}x^5\right)+\frac{1}{143}(33x^2+26)\,(x^4+5)^{3/2}\,x^5$$

$$\Big\downarrow \text{27}$$

$$\frac{30}{143}\left(\frac{10}{21}\int\frac{x^4(77x^2+78)}{\sqrt{x^4+5}}dx+\frac{1}{21}(77x^2+78)\,\sqrt{x^4+5}x^5\right)+\frac{1}{143}(33x^2+26)\,(x^4+5)^{3/2}\,x^5$$

$\downarrow$ 1603

$$\frac{30}{143}\left(\frac{10}{21}\left(\frac{77}{5}x^3\sqrt{x^4+5}-\frac{1}{5}\int\frac{15x^2\left(77-26x^2\right)}{\sqrt{x^4+5}}dx\right)+\frac{1}{21}\left(77x^2+78\right)\sqrt{x^4+5}x^5\right)+$$
$$\frac{1}{143}\left(33x^2+26\right)\left(x^4+5\right)^{3/2}x^5$$

$\downarrow$ 27

$$\frac{30}{143}\left(\frac{10}{21}\left(\frac{77}{5}x^3\sqrt{x^4+5}-3\int\frac{x^2\left(77-26x^2\right)}{\sqrt{x^4+5}}dx\right)+\frac{1}{21}\left(77x^2+78\right)\sqrt{x^4+5}x^5\right)+$$
$$\frac{1}{143}\left(33x^2+26\right)\left(x^4+5\right)^{3/2}x^5$$

$\downarrow$ 1603

$$\frac{30}{143}\left(\frac{10}{21}\left(\frac{77}{5}x^3\sqrt{x^4+5}-3\left(-\frac{1}{3}\int-\frac{231x^2+130}{\sqrt{x^4+5}}dx-\frac{26}{3}\sqrt{x^4+5}x\right)\right)+\frac{1}{21}\left(77x^2+78\right)\sqrt{x^4+5}x^5\right)+$$
$$\frac{1}{143}\left(33x^2+26\right)\left(x^4+5\right)^{3/2}x^5$$

$\downarrow$ 25

$$\frac{30}{143}\left(\frac{10}{21}\left(\frac{77}{5}x^3\sqrt{x^4+5}-3\left(\frac{1}{3}\int\frac{231x^2+130}{\sqrt{x^4+5}}dx-\frac{26}{3}x\sqrt{x^4+5}\right)\right)+\frac{1}{21}\left(77x^2+78\right)\sqrt{x^4+5}x^5\right)+$$
$$\frac{1}{143}\left(33x^2+26\right)\left(x^4+5\right)^{3/2}x^5$$

$\downarrow$ 1512

$$\frac{30}{143}\left(\frac{10}{21}\left(\frac{77}{5}x^3\sqrt{x^4+5}-3\left(\frac{1}{3}\left(\left(130+231\sqrt{5}\right)\int\frac{1}{\sqrt{x^4+5}}dx-231\sqrt{5}\int\frac{\sqrt{5}-x^2}{\sqrt{5}\sqrt{x^4+5}}dx\right)-\frac{26}{3}x\sqrt{x^4+5}\right)\right)\right.$$
$$\frac{1}{143}\left(33x^2+26\right)\left(x^4+5\right)^{3/2}x^5$$

$\downarrow$ 27

$$\frac{30}{143}\left(\frac{10}{21}\left(\frac{77}{5}x^3\sqrt{x^4+5}-3\left(\frac{1}{3}\left(\left(130+231\sqrt{5}\right)\int\frac{1}{\sqrt{x^4+5}}dx-231\int\frac{\sqrt{5}-x^2}{\sqrt{x^4+5}}dx\right)-\frac{26}{3}x\sqrt{x^4+5}\right)\right)+\frac{1}{21}\left(\right.\right.$$
$$\frac{1}{143}\left(33x^2+26\right)\left(x^4+5\right)^{3/2}x^5$$

$\downarrow$ 761

3.27. $\int x^4(2+3x^2)\left(5+x^4\right)^{3/2}\,dx$

$$\frac{30}{143}\left(\frac{10}{21}\left(\frac{77}{5}x^3\sqrt{x^4+5}-3\left(\frac{1}{3}\left(\frac{\left(130+231\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}-231\int\right.\right.\right.\right.\right.$$
$$\frac{1}{143}\left(33x^2+26\right)\left(x^4+5\right)^{3/2}x^5$$

$\downarrow$ 1510

$$\frac{30}{143}\left(\frac{10}{21}\left(\frac{77}{5}x^3\sqrt{x^4+5}-3\left(\frac{1}{3}\left(\frac{\left(130+231\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}-231\left(\right.\right.\right.\right.\right.\right.$$
$$\frac{1}{143}\left(33x^2+26\right)\left(x^4+5\right)^{3/2}x^5$$

input
```
Int[x^4*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

output
```
(x^5*(26 + 33*x^2)*(5 + x^4)^(3/2))/143 + (30*((x^5*(78 + 77*x^2)*Sqrt[5 +
 x^4])/21 + (10*((77*x^3*Sqrt[5 + x^4])/5 - 3*((-26*x*Sqrt[5 + x^4])/3 + (
-231*(-((x*Sqrt[5 + x^4])/(Sqrt[5] + x^2)) + (5^(1/4)*(Sqrt[5] + x^2)*Sqrt
[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5
+ x^4]) + ((130 + 231*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x
^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(2*5^(1/4)*Sqrt[5 + x^4]))/3))
)/21))/143
```

3.27.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

rule 1597
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((c*d*(m + 4*p + 3) + c*e*(4*p
 + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p + 3))), x] + Simp[4*a*(p/((4*p +
 m + 1)*(m + 4*p + 3)))   Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
 3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && Gt
Q[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (
IntegerQ[p] || IntegerQ[m])
```

rule 1603
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
 1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ
[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[
m])
```

3.27.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 4.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.17

method	result
meijerg	$\frac{15\sqrt{5}\,x^7{}_2F_1\left(-\frac{3}{2},\frac{7}{4};\frac{11}{4};-\frac{x^4}{5}\right)}{7}+2\sqrt{5}\,x^5{}_2F_1\left(-\frac{3}{2},\frac{5}{4};\frac{9}{4};-\frac{x^4}{5}\right)$
risch	$\frac{x(231x^{10}+182x^8+1925x^6+1690x^4+1540x^2+2600)\sqrt{x^4+5}}{1001}-\frac{40\sqrt{5}\sqrt{25-5i\sqrt{5}\,x^2}\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)}{77\sqrt{i\sqrt{5}}\sqrt{x^4+5}}-\frac{60i\sqrt{25-}}{}$
default	$\frac{3x^{11}\sqrt{x^4+5}}{13}+\frac{25x^7\sqrt{x^4+5}}{13}+\frac{20x^3\sqrt{x^4+5}}{13}-\frac{60i\sqrt{25-5i\sqrt{5}\,x^2}\sqrt{25+5i\sqrt{5}\,x^2}\left(F\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\right)}{13\sqrt{i\sqrt{5}}\sqrt{x^4+5}}+\frac{2x^9}{}$
elliptic	$\frac{3x^{11}\sqrt{x^4+5}}{13}+\frac{25x^7\sqrt{x^4+5}}{13}+\frac{20x^3\sqrt{x^4+5}}{13}-\frac{60i\sqrt{25-5i\sqrt{5}\,x^2}\sqrt{25+5i\sqrt{5}\,x^2}\left(F\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\right)}{13\sqrt{i\sqrt{5}}\sqrt{x^4+5}}+\frac{2x^9}{}$

input
```
int(x^4*(3*x^2+2)*(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
15/7*5^(1/2)*x^7*hypergeom([-3/2,7/4],[11/4],-1/5*x^4)+2*5^(1/2)*x^5*hyper
geom([-3/2,5/4],[9/4],-1/5*x^4)
```

3.27.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.34

$$\int x^4(2+3x^2)\left(5+x^4\right)^{3/2}\,dx =$$
$$-\frac{23100\,(-5)^{\frac{3}{4}}\,xE(\arcsin\left(\frac{(-5)^{\frac{1}{4}}}{x}\right)\,|-1)-20500\,(-5)^{\frac{3}{4}}\,xF(\arcsin\left(\frac{(-5)^{\frac{1}{4}}}{x}\right)\,|-1)-(231\,x^{12}+182\,x^{10}+19}{1001\,x}$$

input
```
integrate(x^4*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="fricas")
```

output
```
-1/1001*(23100*(-5)^(3/4)*x*elliptic_e(arcsin((-5)^(1/4)/x), -1) - 20500*(
-5)^(3/4)*x*elliptic_f(arcsin((-5)^(1/4)/x), -1) - (231*x^12 + 182*x^10 +
1925*x^8 + 1690*x^6 + 1540*x^4 + 2600*x^2 - 23100)*sqrt(x^4 + 5))/x
```

3.27.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.80 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.68

$$\int x^4(2+3x^2)\left(5+x^4\right)^{3/2}\,dx = \frac{3\sqrt{5}x^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{15}{4}\right)} + \frac{\sqrt{5}x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{13}{4}\right)} + \frac{15\sqrt{5}x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{11}{4}\right)} + \frac{5\sqrt{5}x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x**4*(3*x**2+2)*(x**4+5)**(3/2),x)
```

output
```
3*sqrt(5)*x**11*gamma(11/4)*hyper((-1/2, 11/4), (15/4,), x**4*exp_polar(I*
pi)/5)/(4*gamma(15/4)) + sqrt(5)*x**9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,
), x**4*exp_polar(I*pi)/5)/(2*gamma(13/4)) + 15*sqrt(5)*x**7*gamma(7/4)*hy
per((-1/2, 7/4), (11/4,), x**4*exp_polar(I*pi)/5)/(4*gamma(11/4)) + 5*sqrt
(5)*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), x**4*exp_polar(I*pi)/5)/(2*
gamma(9/4))
```

3.27.7 Maxima [F]

$$\int x^4(2+3x^2)\left(5+x^4\right)^{3/2}\,dx = \int \left(x^4+5\right)^{\frac{3}{2}}\left(3\,x^2+2\right)x^4\,dx$$

input
```
integrate(x^4*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="maxima")
```

output
```
integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)*x^4, x)
```

3.27.8 Giac [F]

$$\int x^4(2+3x^2)\left(5+x^4\right)^{3/2}\,dx=\int (x^4+5)^{\frac{3}{2}}(3\,x^2+2)x^4\,dx$$

input
```
integrate(x^4*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="giac")
```

output
```
integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)*x^4, x)
```

3.27.9 Mupad [F(-1)]

Timed out.

$$\int x^4(2+3x^2)\left(5+x^4\right)^{3/2}\,dx=\int x^4\left(x^4+5\right)^{3/2}\left(3\,x^2+2\right)\,dx$$

input
```
int(x^4*(x^4 + 5)^(3/2)*(3*x^2 + 2),x)
```

output
```
int(x^4*(x^4 + 5)^(3/2)*(3*x^2 + 2), x)
```

3.28 $\int x^2(2+3x^2)\left(5+x^4\right)^{3/2}\,dx$

3.28.1 Optimal result

Integrand size = 20, antiderivative size = 219

$$\int x^2\left(2+3x^2\right)\left(5+x^4\right)^{3/2}\,dx=\frac{300}{77}x\sqrt{5+x^4}+\frac{40x\sqrt{5+x^4}}{3\left(\sqrt{5}+x^2\right)}+\frac{2}{231}x^3\left(154+135x^2\right)\sqrt{5+x^4}+\frac{1}{99}x^3\left(22+27x^2\right)\left(5+x^4\right)^{3/2}-\frac{40\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{3\sqrt{5+x^4}}+\frac{10\sqrt[4]{5}\left(154-45\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{231\sqrt{5+x^4}}$$

output
```
1/99*x^3*(27*x^2+22)*(x^4+5)^(3/2)+300/77*x*(x^4+5)^(1/2)+2/231*x^3*(135*x
^2+154)*(x^4+5)^(1/2)+40/3*x*(x^4+5)^(1/2)/(x^2+5^(1/2))-40/3*5^(1/4)*(cos
(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticE(
sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1
/2))^2)^(1/2)/(x^4+5)^(1/2)+10/231*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2
)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticF(sin(2*arctan(1/5*x*5^(3/4))
),1/2*2^(1/2))*(154-45*5^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1
/2)/(x^4+5)^(1/2)
```

3.28.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.81 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.31

$$\int x^2(2+3x^2)\,(5+x^4)^{3/2}\,dx=\frac{1}{33}x\left(9(5+x^4)^{5/2}\right.$$
$$\left.-225\sqrt{5}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},-\frac{x^4}{5}\right)+110\sqrt{5}x^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},\frac{7}{4},-\frac{x^4}{5}\right)\right)$$

input
```
Integrate[x^2*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

output
```
(x*(9*(5 + x^4)^(5/2) - 225*Sqrt[5]*Hypergeometric2F1[-3/2, 1/4, 5/4, -1/5
*x^4] + 110*Sqrt[5]*x^2*Hypergeometric2F1[-3/2, 3/4, 7/4, -1/5*x^4]))/33
```

3.28.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1597, 1597, 1603, 27, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(3x^2+2)\,(x^4+5)^{3/2}\,dx$$

$$\Big\downarrow \text{1597}$$

$$\frac{10}{33}\int x^2(27x^2+22)\sqrt{x^4+5}dx+\frac{1}{99}(27x^2+22)\,(x^4+5)^{3/2}\,x^3$$

$$\Big\downarrow \text{1597}$$

$$\frac{10}{33}\left(\frac{2}{7}\int\frac{x^2(135x^2+154)}{\sqrt{x^4+5}}dx+\frac{1}{35}(135x^2+154)\sqrt{x^4+5}x^3\right)+\frac{1}{99}(27x^2+22)\,(x^4+5)^{3/2}\,x^3$$

$$\Big\downarrow \text{1603}$$

$$\frac{10}{33}\left(\frac{2}{7}\left(45x\sqrt{x^4+5}-\frac{1}{3}\int\frac{3(225-154x^2)}{\sqrt{x^4+5}}dx\right)+\frac{1}{35}(135x^2+154)\sqrt{x^4+5}x^3\right)+$$
$$\frac{1}{99}(27x^2+22)\,(x^4+5)^{3/2}\,x^3$$

$$\Big\downarrow \text{27}$$

$$\frac{10}{33}\left(\frac{2}{7}\left(45x\sqrt{x^4+5}-\int\frac{225-154x^2}{\sqrt{x^4+5}}dx\right)+\frac{1}{35}\left(135x^2+154\right)\sqrt{x^4+5}x^3\right)+\frac{1}{99}\left(27x^2+22\right)\left(x^4+5\right)^{3/2}x^3$$

$$\Big\downarrow \text{1512}$$

$$\frac{10}{33}\left(\frac{2}{7}\left(-\left(225-154\sqrt{5}\right)\int\frac{1}{\sqrt{x^4+5}}dx-154\sqrt{5}\int\frac{\sqrt{5}-x^2}{\sqrt{5}\sqrt{x^4+5}}dx+45\sqrt{x^4+5}x\right)+\frac{1}{35}\left(135x^2+154\right)\sqrt{x^4+5}\right.$$
$$\frac{1}{99}\left(27x^2+22\right)\left(x^4+5\right)^{3/2}x^3$$

$$\Big\downarrow \text{27}$$

$$\frac{10}{33}\left(\frac{2}{7}\left(-\left(225-154\sqrt{5}\right)\int\frac{1}{\sqrt{x^4+5}}dx-154\int\frac{\sqrt{5}-x^2}{\sqrt{x^4+5}}dx+45\sqrt{x^4+5}x\right)+\frac{1}{35}\left(135x^2+154\right)\sqrt{x^4+5}x^3\right)+$$
$$\frac{1}{99}\left(27x^2+22\right)\left(x^4+5\right)^{3/2}x^3$$

$$\Big\downarrow \text{761}$$

$$\frac{10}{33}\left(\frac{2}{7}\left(-154\int\frac{\sqrt{5}-x^2}{\sqrt{x^4+5}}dx-\frac{\left(225-154\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}+45\sqrt{x^4\right.\right.$$
$$\frac{1}{99}\left(27x^2+22\right)\left(x^4+5\right)^{3/2}x^3$$

$$\Big\downarrow \text{1510}$$

$$\frac{10}{33}\left(\frac{2}{7}\left(-\frac{\left(225-154\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}-154\left(\frac{\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right.}}}{\sqrt{x}}\right.\right.\right.$$
$$\frac{1}{99}\left(27x^2+22\right)\left(x^4+5\right)^{3/2}x^3$$

input
```
Int[x^2*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

3.28. $\int x^2(2+3x^2)\left(5+x^4\right)^{3/2}\,dx$

output
```
(x^3*(22 + 27*x^2)*(5 + x^4)^(3/2))/99 + (10*((x^3*(154 + 135*x^2)*Sqrt[5
+ x^4])/35 + (2*(45*x*Sqrt[5 + x^4] - 154*(-((x*Sqrt[5 + x^4])/(Sqrt[5] +
x^2)) + (5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*Ellipti
cE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4]) - ((225 - 154*Sqrt[5])*(Sqrt[
5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)],
 1/2])/(2*5^(1/4)*Sqrt[5 + x^4]))/7))/33
```

3.28.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

rule 1597
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((c*d*(m + 4*p + 3) + c*e*(4*p
 + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p + 3))), x] + Simp[4*a*(p/((4*p +
 m + 1)*(m + 4*p + 3)))   Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
 3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && Gt
Q[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (
IntegerQ[p] || IntegerQ[m])
```

rule 1603
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
 1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ
[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[
m])
```

3.28.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.88 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.18

method	result
meijerg	$3\sqrt{5}\,x^5\,{}_2F_1\left(-\frac{3}{2},\frac{5}{4};\frac{9}{4};-\frac{x^4}{5}\right)+\frac{10\sqrt{5}\,x^3\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{x^4}{5}\right)}{3}$
risch	$\frac{x(189x^8+154x^6+1755x^4+1694x^2+2700)\sqrt{x^4+5}}{693}-\frac{60\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{77\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{8i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{2}$
default	$\frac{3x^9\sqrt{x^4+5}}{11}+\frac{195x^5\sqrt{x^4+5}}{77}+\frac{300x\sqrt{x^4+5}}{77}-\frac{60\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{77\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{2x^7\sqrt{x^4+5}}{9}+\frac{22x^3\sqrt{x}}{9}$
elliptic	$\frac{3x^9\sqrt{x^4+5}}{11}+\frac{195x^5\sqrt{x^4+5}}{77}+\frac{300x\sqrt{x^4+5}}{77}-\frac{60\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{77\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{2x^7\sqrt{x^4+5}}{9}+\frac{22x^3\sqrt{x}}{9}$

input
```
int(x^2*(3*x^2+2)*(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
3*5^(1/2)*x^5*hypergeom([-3/2,5/4],[9/4],-1/5*x^4)+10/3*5^(1/2)*x^3*hyperg
eom([-3/2,3/4],[7/4],-1/5*x^4)
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.33

$$\int x^2(2+3x^2)\left(5+x^4\right)^{3/2}\,dx=\frac{9240\,(-5)^{\frac{3}{4}}\,xE(\arcsin\left(\frac{(-5)^{\frac{1}{4}}}{x}\right)\,|-1)-11940\,(-5)^{\frac{3}{4}}\,xF(\arcsin\left(\frac{(-5)^{\frac{1}{4}}}{x}\right)\,|-1)+(189\,x^{10}+15}{693\,x}$$

input
```
integrate(x^2*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="fricas")
```

output
```
1/693*(9240*(-5)^(3/4)*x*elliptic_e(arcsin((-5)^(1/4)/x), -1) - 11940*(-5)
^(3/4)*x*elliptic_f(arcsin((-5)^(1/4)/x), -1) + (189*x^10 + 154*x^8 + 1755
*x^6 + 1694*x^4 + 2700*x^2 + 9240)*sqrt(x^4 + 5))/x
```

3.28.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.64 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.73

$$\int x^2(2+3x^2)\left(5+x^4\right)^{3/2}\,dx = \frac{3\sqrt{5}x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{13}{4}\right)} + \frac{\sqrt{5}x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{11}{4}\right)} + \frac{15\sqrt{5}x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{5\sqrt{5}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x**2*(3*x**2+2)*(x**4+5)**(3/2),x)
```

output
```
3*sqrt(5)*x**9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), x**4*exp_polar(I*pi)
/5)/(4*gamma(13/4)) + sqrt(5)*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,),
x**4*exp_polar(I*pi)/5)/(2*gamma(11/4)) + 15*sqrt(5)*x**5*gamma(5/4)*hyper
((-1/2, 5/4), (9/4,), x**4*exp_polar(I*pi)/5)/(4*gamma(9/4)) + 5*sqrt(5)*x
**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), x**4*exp_polar(I*pi)/5)/(2*gamma
(7/4))
```

3.28.7 Maxima [F]

$$\int x^2(2+3x^2)\left(5+x^4\right)^{3/2}\,dx = \int \left(x^4+5\right)^{\frac{3}{2}}\left(3\,x^2+2\right)x^2\,dx$$

input
```
integrate(x^2*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="maxima")
```

output
```
integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)*x^2, x)
```

3.28.8 Giac [F]

$$\int x^2(2+3x^2)\left(5+x^4\right)^{3/2}\,dx = \int \left(x^4+5\right)^{\frac{3}{2}}\left(3\,x^2+2\right)x^2\,dx$$

input
```
integrate(x^2*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="giac")
```

output
```
integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)*x^2, x)
```

3.28.9 Mupad [F(-1)]

Timed out.

$$\int x^2(2+3x^2)\left(5+x^4\right)^{3/2}\,dx = \int x^2\left(x^4+5\right)^{3/2}\left(3\,x^2+2\right)\,dx$$

input
```
int(x^2*(x^4 + 5)^(3/2)*(3*x^2 + 2),x)
```

output
```
int(x^2*(x^4 + 5)^(3/2)*(3*x^2 + 2), x)
```

3.29 $\int (2+3x^2)(5+x^4)^{3/2}\,dx$

3.29.1 Optimal result

Integrand size = 17, antiderivative size = 197

$$\int (2+3x^2)(5+x^4)^{3/2}\,dx = \frac{20x\sqrt{5+x^4}}{\sqrt{5}+x^2}+\frac{2}{7}x(10+7x^2)\sqrt{5+x^4}+\frac{1}{21}x(6+7x^2)(5+x^4)^{3/2}-\frac{20\sqrt[4]{5}(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{(\sqrt{5}+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}}+\frac{10\sqrt[4]{5}(7+2\sqrt{5})(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{(\sqrt{5}+x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{7\sqrt{5+x^4}}$$

output
```
1/21*x*(7*x^2+6)*(x^4+5)^(3/2)+2/7*x*(7*x^2+10)*(x^4+5)^(1/2)+20*x*(x^4+5)
^(1/2)/(x^2+5^(1/2))-20*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos
(2*arctan(1/5*x*5^(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/
2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)+10/7*5^(1/
4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*Ell
ipticF(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*(7+2*5^(1/2
))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)
```

3.29.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.25

$$\int (2+3x^2)(5+x^4)^{3/2}\,dx = 5\sqrt{5}x\left(2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},-\frac{x^4}{5}\right) + x^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},\frac{7}{4},-\frac{x^4}{5}\right)\right)$$

input
```
Integrate[(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

output
```
5*Sqrt[5]*x*(2*Hypergeometric2F1[-3/2, 1/4, 5/4, -1/5*x^4] + x^2*Hypergeom
etric2F1[-3/2, 3/4, 7/4, -1/5*x^4])
```

3.29.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {1491, 27, 1491, 27, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3x^2+2)(x^4+5)^{3/2}\,dx$$

$$\Big\downarrow \text{1491}$$

$$\frac{1}{21}\int 30(7x^2+6)\sqrt{x^4+5}dx+\frac{1}{21}x(7x^2+6)(x^4+5)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{10}{7}\int (7x^2+6)\sqrt{x^4+5}dx+\frac{1}{21}x(7x^2+6)(x^4+5)^{3/2}$$

$$\Big\downarrow \text{1491}$$

$$\frac{10}{7}\left(\frac{1}{15}\int \frac{30(7x^2+10)}{\sqrt{x^4+5}}dx+\frac{1}{5}x\sqrt{x^4+5}(7x^2+10)\right)+\frac{1}{21}x(7x^2+6)(x^4+5)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{10}{7}\left(2\int \frac{7x^2+10}{\sqrt{x^4+5}}dx+\frac{1}{5}x\sqrt{x^4+5}(7x^2+10)\right)+\frac{1}{21}x(7x^2+6)\left(x^4+5\right)^{3/2}$$

$\Big\downarrow$ 1512

$$\frac{10}{7}\left(2\left(\left(10+7\sqrt{5}\right)\int \frac{1}{\sqrt{x^4+5}}dx-7\sqrt{5}\int \frac{\sqrt{5}-x^2}{\sqrt{5}\sqrt{x^4+5}}dx\right)+\frac{1}{5}x\sqrt{x^4+5}(7x^2+10)\right)+$$
$$\frac{1}{21}x(7x^2+6)\left(x^4+5\right)^{3/2}$$

$\Big\downarrow$ 27

$$\frac{10}{7}\left(2\left(\left(10+7\sqrt{5}\right)\int \frac{1}{\sqrt{x^4+5}}dx-7\int \frac{\sqrt{5}-x^2}{\sqrt{x^4+5}}dx\right)+\frac{1}{5}x\sqrt{x^4+5}(7x^2+10)\right)+$$
$$\frac{1}{21}x(7x^2+6)\left(x^4+5\right)^{3/2}$$

$\Big\downarrow$ 761

$$\frac{10}{7}\left(2\left(\frac{\left(10+7\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}-7\int \frac{\sqrt{5}-x^2}{\sqrt{x^4+5}}dx\right)+\frac{1}{5}x\sqrt{x^4+5}(7x$$
$$\frac{1}{21}x(7x^2+6)\left(x^4+5\right)^{3/2}$$

$\Big\downarrow$ 1510

$$\frac{10}{7}\left(2\left(\frac{\left(10+7\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}-7\left(\frac{\sqrt[4]{5}(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2a\right.}{\sqrt{x^4+5}}\right.\right.\right.$$
$$\frac{1}{21}x(7x^2+6)\left(x^4+5\right)^{3/2}$$

input
```
Int[(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

output
```
(x*(6 + 7*x^2)*(5 + x^4)^(3/2))/21 + (10*((x*(10 + 7*x^2)*Sqrt[5 + x^4])/5
 + 2*(-7*(-((x*Sqrt[5 + x^4])/(Sqrt[5] + x^2)) + (5^(1/4)*(Sqrt[5] + x^2)*
Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqr
t[5 + x^4]) + ((10 + 7*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] +
x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(2*5^(1/4)*Sqrt[5 + x^4]))))/
7
```

3.29. $\int (2+3x^2)\left(5+x^4\right)^{3/2}\,dx$

3.29.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1491
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Simp[2*(p/((4*p + 1)*(4*p + 3)))   Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
 1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c
*d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

3.29. $\int (2+3x^2)(5+x^4)^{3/2}\,dx$

3.29.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.19

method	result
meijerg	$10\sqrt{5}\,x\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{x^4}{5}\right)+5\sqrt{5}\,x^3\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{x^4}{5}\right)$
risch	$\frac{x(7x^6+6x^4+77x^2+90)\sqrt{x^4+5}}{21}+\frac{8\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{7\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{4i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\left(F\left(\frac{x\sqrt{}}{}\right.\right.}{\sqrt{i\sqrt{5}}\,\sqrt{x^4}}$
default	$\frac{2x^5\sqrt{x^4+5}}{7}+\frac{30x\sqrt{x^4+5}}{7}+\frac{8\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{7\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{x^7\sqrt{x^4+5}}{3}+\frac{11x^3\sqrt{x^4+5}}{3}+\frac{4i\sqrt{25-5i}}{}$
elliptic	$\frac{2x^5\sqrt{x^4+5}}{7}+\frac{30x\sqrt{x^4+5}}{7}+\frac{8\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{7\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{x^7\sqrt{x^4+5}}{3}+\frac{11x^3\sqrt{x^4+5}}{3}+\frac{4i\sqrt{25-5i}}{}$

input
```
int((3*x^2+2)*(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
10*5^(1/2)*x*hypergeom([-3/2,1/4],[5/4],-1/5*x^4)+5*5^(1/2)*x^3*hypergeom(
[-3/2,3/4],[7/4],-1/5*x^4)
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.35

$$\int (2+3x^2)\left(5+x^4\right)^{3/2}\,dx=\frac{420\,(-5)^{\frac{3}{4}}\,xE(\arcsin\left(\frac{(-5)^{\frac{1}{4}}}{x}\right)\mid-1)-300\,(-5)^{\frac{3}{4}}\,xF(\arcsin\left(\frac{(-5)^{\frac{1}{4}}}{x}\right)\mid-1)+(7\,x^8+6\,x^6+77}{21\,x}$$

input
```
integrate((3*x^2+2)*(x^4+5)^(3/2),x, algorithm="fricas")
```

output
```
1/21*(420*(-5)^(3/4)*x*elliptic_e(arcsin((-5)^(1/4)/x), -1) - 300*(-5)^(3/
4)*x*elliptic_f(arcsin((-5)^(1/4)/x), -1) + (7*x^8 + 6*x^6 + 77*x^4 + 90*x
^2 + 420)*sqrt(x^4 + 5))/x
```

3.29.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.51 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.80

$$\int \left(2+3x^2\right)\left(5+x^4\right)^{3/2}\,dx = \frac{3\sqrt{5}x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{11}{4}\right)} + \frac{\sqrt{5}x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{9}{4}\right)} + \frac{15\sqrt{5}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{5\sqrt{5}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((3*x**2+2)*(x**4+5)**(3/2),x)
```

output
```
3*sqrt(5)*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), x**4*exp_polar(I*pi)
/5)/(4*gamma(11/4)) + sqrt(5)*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), x
**4*exp_polar(I*pi)/5)/(2*gamma(9/4)) + 15*sqrt(5)*x**3*gamma(3/4)*hyper((
-1/2, 3/4), (7/4,), x**4*exp_polar(I*pi)/5)/(4*gamma(7/4)) + 5*sqrt(5)*x*g
amma(1/4)*hyper((-1/2, 1/4), (5/4,), x**4*exp_polar(I*pi)/5)/(2*gamma(5/4)
)
```

3.29.7 Maxima [F]

$$\int \left(2+3x^2\right)\left(5+x^4\right)^{3/2}\,dx = \int \left(x^4+5\right)^{\frac{3}{2}}\left(3\,x^2+2\right)dx$$

input
```
integrate((3*x^2+2)*(x^4+5)^(3/2),x, algorithm="maxima")
```

output
```
integrate((x^4 + 5)^(3/2)*(3*x^2 + 2), x)
```

3.29.8 Giac [F]

$$\int (2+3x^2)\left(5+x^4\right)^{3/2}\,dx=\int \left(x^4+5\right)^{\frac{3}{2}}\left(3\,x^2+2\right)dx$$

input
```
integrate((3*x^2+2)*(x^4+5)^(3/2),x, algorithm="giac")
```

output
```
integrate((x^4 + 5)^(3/2)*(3*x^2 + 2), x)
```

3.29.9 Mupad [F(-1)]

Timed out.

$$\int (2+3x^2)\left(5+x^4\right)^{3/2}\,dx=\int \left(x^4+5\right)^{3/2}\left(3\,x^2+2\right)\,dx$$

input
```
int((x^4 + 5)^(3/2)*(3*x^2 + 2),x)
```

output
```
int((x^4 + 5)^(3/2)*(3*x^2 + 2), x)
```

3.30 $\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^2}\,dx$

3.30.1 Optimal result

Integrand size = 20, antiderivative size = 199

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^2}\,dx = \frac{24x\sqrt{5+x^4}}{\sqrt{5}+x^2} + \frac{6}{35}x(25+14x^2)\sqrt{5+x^4} - \frac{(14-3x^2)(5+x^4)^{3/2}}{7x} - \frac{24\sqrt[4]{5}(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{(\sqrt{5}+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}} + \frac{6\sqrt[4]{5}(14+5\sqrt{5})(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{(\sqrt{5}+x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{7\sqrt{5+x^4}}$$

output
```
-1/7*(-3*x^2+14)*(x^4+5)^(3/2)/x+6/35*x*(14*x^2+25)*(x^4+5)^(1/2)+24*x*(x^
4+5)^(1/2)/(x^2+5^(1/2))-24*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)
/cos(2*arctan(1/5*x*5^(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2
^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)+6/7*5^
(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*
EllipticF(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*(14+5*5^
(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)
```

3.30.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.85 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.27

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^2}\,dx = -\frac{10\sqrt{5}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{4},\frac{3}{4},-\frac{x^4}{5}\right)}{x} + 15\sqrt{5}x\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},-\frac{x^4}{5}\right)$$

input
```
Integrate[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^2,x]
```

output
```
(-10*Sqrt[5]*Hypergeometric2F1[-3/2, -1/4, 3/4, -1/5*x^4])/x + 15*Sqrt[5]*
x*Hypergeometric2F1[-3/2, 1/4, 5/4, -1/5*x^4]
```

3.30.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1595, 25, 1491, 27, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x^2+2)(x^4+5)^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{1595}$$

$$-\frac{6}{7}\int -\left((14x^2+15)\sqrt{x^4+5}\right)dx - \frac{(14-3x^2)(x^4+5)^{3/2}}{7x}$$

$$\Big\downarrow \text{25}$$

$$\frac{6}{7}\int (14x^2+15)\sqrt{x^4+5}dx - \frac{(14-3x^2)(x^4+5)^{3/2}}{7x}$$

$$\Big\downarrow \text{1491}$$

$$\frac{6}{7}\left(\frac{1}{15}\int \frac{30(14x^2+25)}{\sqrt{x^4+5}}dx + \frac{1}{5}x\sqrt{x^4+5}(14x^2+25)\right) - \frac{(14-3x^2)(x^4+5)^{3/2}}{7x}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{6}{7}\left(2\int \frac{14x^2+25}{\sqrt{x^4+5}}dx+\frac{1}{5}x\sqrt{x^4+5}(14x^2+25)\right)-\frac{(14-3x^2)\,(x^4+5)^{3/2}}{7x}$$

$$\Big\downarrow \text{ 1512}$$

$$\frac{6}{7}\left(2\left(\left(25+14\sqrt{5}\right)\int \frac{1}{\sqrt{x^4+5}}dx-14\sqrt{5}\int \frac{\sqrt{5}-x^2}{\sqrt{5}\sqrt{x^4+5}}dx\right)+\frac{1}{5}x\sqrt{x^4+5}(14x^2+25)\right)-\frac{(14-3x^2)\,(x^4+5)^{3/2}}{7x}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{6}{7}\left(2\left(\left(25+14\sqrt{5}\right)\int \frac{1}{\sqrt{x^4+5}}dx-14\int \frac{\sqrt{5}-x^2}{\sqrt{x^4+5}}dx\right)+\frac{1}{5}x\sqrt{x^4+5}(14x^2+25)\right)-\frac{(14-3x^2)\,(x^4+5)^{3/2}}{7x}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{6}{7}\left(2\left(\frac{(25+14\sqrt{5})\,(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{(x^2+\sqrt{5})^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}-14\int \frac{\sqrt{5}-x^2}{\sqrt{x^4+5}}dx\right)+\frac{1}{5}x\sqrt{x^4+5}(1\right.$$

$$\frac{(14-3x^2)\,(x^4+5)^{3/2}}{7x}$$

$$\Big\downarrow \text{ 1510}$$

$$\frac{6}{7}\left(2\left(\frac{(25+14\sqrt{5})\,(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{(x^2+\sqrt{5})^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}-14\left(\frac{\sqrt[4]{5}(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{(x^2+\sqrt{5})^2}}E\left(2\right.}{\sqrt{x^4+5}}\right.\right.\right.$$

$$\frac{(14-3x^2)\,(x^4+5)^{3/2}}{7x}$$

input

```
Int[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^2,x]
```

3.30. $\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^2}\,dx$

output
```
-1/7*((14 - 3*x^2)*(5 + x^4)^(3/2))/x + (6*((x*(25 + 14*x^2)*Sqrt[5 + x^4]
)/5 + 2*(-14*(-((x*Sqrt[5 + x^4])/(Sqrt[5] + x^2)) + (5^(1/4)*(Sqrt[5] + x
^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])
/Sqrt[5 + x^4]) + ((25 + 14*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[
5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(2*5^(1/4)*Sqrt[5 + x^4]
))))/7
```

3.30.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1491
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Simp[2*(p/((4*p + 1)*(4*p + 3)))   Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
 1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c
*d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

rule 1595
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((d*(m + 4*p + 3) + e*(m + 1)*
x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Simp[4*(p/(f^2*(m + 1)*(m + 4*p + 3))
)   Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*
x^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.30.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.36 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.19

method	result
meijerg	$-\frac{10\sqrt{5}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{4};\frac{3}{4};-\frac{x^4}{5}\right)}{x}+15\sqrt{5}\,x\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{x^4}{5}\right)$
risch	$\frac{15x^{10}+14x^8+300x^6-280x^4+1125x^2-1750}{35x\sqrt{x^4+5}}+\frac{12\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{7\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{24i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5}}{}$
default	$\frac{3x^5\sqrt{x^4+5}}{7}+\frac{45x\sqrt{x^4+5}}{7}+\frac{12\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{7\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}-\frac{10\sqrt{x^4+5}}{x}+\frac{2x^3\sqrt{x^4+5}}{5}+\frac{24i\sqrt{25-5i}}{}$
elliptic	$\frac{3x^5\sqrt{x^4+5}}{7}+\frac{45x\sqrt{x^4+5}}{7}+\frac{12\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{7\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}-\frac{10\sqrt{x^4+5}}{x}+\frac{2x^3\sqrt{x^4+5}}{5}+\frac{24i\sqrt{25-5i}}{}$

input
```
int((3*x^2+2)*(x^4+5)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-10*5^(1/2)/x*hypergeom([-3/2,-1/4],[3/4],-1/5*x^4)+15*5^(1/2)*x*hypergeom
([-3/2,1/4],[5/4],-1/5*x^4)
```

3.30.5 Fricas [F]

$$\int \frac{(2+3x^2)\left(5+x^4\right)^{3/2}}{x^2}\,dx=\int \frac{(x^4+5)^{\frac{3}{2}}(3\,x^2+2)}{x^2}\,dx$$

input
```
integrate((3*x^2+2)*(x^4+5)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
integral((3*x^6 + 2*x^4 + 15*x^2 + 10)*sqrt(x^4 + 5)/x^2, x)
```

3.30.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.84 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.80

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^2}\,dx = \frac{3\sqrt{5}x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\right|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{\sqrt{5}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\right|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{7}{4}\right)} + \frac{15\sqrt{5}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{4}\\\frac{5}{4}\end{matrix}\right|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{5\sqrt{5}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\right|\frac{x^4e^{i\pi}}{5}\right)}{2x\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((3*x**2+2)*(x**4+5)**(3/2)/x**2,x)
```

output
```
3*sqrt(5)*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), x**4*exp_polar(I*pi)/
5)/(4*gamma(9/4)) + sqrt(5)*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), x**
4*exp_polar(I*pi)/5)/(2*gamma(7/4)) + 15*sqrt(5)*x*gamma(1/4)*hyper((-1/2,
 1/4), (5/4,), x**4*exp_polar(I*pi)/5)/(4*gamma(5/4)) + 5*sqrt(5)*gamma(-1
/4)*hyper((-1/2, -1/4), (3/4,), x**4*exp_polar(I*pi)/5)/(2*x*gamma(3/4))
```

3.30.7 Maxima [F]

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^2}\,dx = \int \frac{(x^4+5)^{\frac{3}{2}}(3x^2+2)}{x^2}\,dx$$

input
```
integrate((3*x^2+2)*(x^4+5)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x^2, x)
```

3.30.8 Giac [F]

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^2}\,dx = \int \frac{(x^4+5)^{\frac{3}{2}}(3\,x^2+2)}{x^2}\,dx$$

input
```
integrate((3*x^2+2)*(x^4+5)^(3/2)/x^2,x, algorithm="giac")
```

output
```
integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x^2, x)
```

3.30.9 Mupad [B] (verification not implemented)

Time = 7.77 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.24

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^2}\,dx = 15\sqrt{5}\,x\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{4};\ \frac{5}{4};\ -\frac{x^4}{5}\right) + \frac{2\,(x^4+5)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{5}{4};\ -\frac{1}{4};\ -\frac{5}{x^4}\right)}{5\,x\,\left(\frac{5}{x^4}+1\right)^{3/2}}$$

input
```
int(((x^4 + 5)^(3/2)*(3*x^2 + 2))/x^2,x)
```

output
```
15*5^(1/2)*x*hypergeom([-3/2, 1/4], 5/4, -x^4/5) + (2*(x^4 + 5)^(3/2)*hype
rgeom([-3/2, -5/4], -1/4, -5/x^4))/(5*x*(5/x^4 + 1)^(3/2))
```

3.31 $\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^4} \, dx$

3.31.1 Optimal result

Integrand size = 20, antiderivative size = 201

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^4} \, dx = -\frac{2(27-2x^2)\sqrt{5+x^4}}{3x} + \frac{36x\sqrt{5+x^4}}{\sqrt{5}+x^2} - \frac{(10-9x^2)(5+x^4)^{3/2}}{15x^3} - \frac{36\sqrt[4]{5}(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{(\sqrt{5}+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}} + \frac{2\sqrt[4]{5}(27+2\sqrt{5})(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{(\sqrt{5}+x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{3\sqrt{5+x^4}}$$

output
```
-1/15*(-9*x^2+10)*(x^4+5)^(3/2)/x^3-2/3*(-2*x^2+27)*(x^4+5)^(1/2)/x+36*x*(
x^4+5)^(1/2)/(x^2+5^(1/2))-36*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/
2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2
*2^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)+2/3*
5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4))
)*EllipticF(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*(27+2*
5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)
```

3.31.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.27

$$\int \frac{(2+3x^2)\left(5+x^4\right)^{3/2}}{x^4}\,dx = -\frac{5\sqrt{5}\left(2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{4},\frac{1}{4},-\frac{x^4}{5}\right)+9x^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{4},\frac{3}{4},-\frac{x^4}{5}\right)\right)}{3x^3}$$

input
```
Integrate[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^4,x]
```

output
```
(-5*Sqrt[5]*(2*Hypergeometric2F1[-3/2, -3/4, 1/4, -1/5*x^4] + 9*x^2*Hyperg
eometric2F1[-3/2, -1/4, 3/4, -1/5*x^4]))/(3*x^3)
```

3.31.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1595, 27, 1595, 25, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x^2+2)\left(x^4+5\right)^{3/2}}{x^4}dx$$

$$\Big\downarrow \text{1595}$$

$$-\frac{2}{5}\int -\frac{5(2x^2+9)\sqrt{x^4+5}}{x^2}dx-\frac{(10-9x^2)\left(x^4+5\right)^{3/2}}{15x^3}$$

$$\Big\downarrow \text{27}$$

$$2\int \frac{(2x^2+9)\sqrt{x^4+5}}{x^2}dx-\frac{(10-9x^2)\left(x^4+5\right)^{3/2}}{15x^3}$$

$$\Big\downarrow \text{1595}$$

$$2\left(-\frac{2}{3}\int -\frac{27x^2+10}{\sqrt{x^4+5}}dx-\frac{\sqrt{x^4+5}(27-2x^2)}{3x}\right)-\frac{(10-9x^2)\left(x^4+5\right)^{3/2}}{15x^3}$$

$$\Big\downarrow \ 25$$

$$2\left(\frac{2}{3}\int \frac{27x^2+10}{\sqrt{x^4+5}}dx-\frac{(27-2x^2)\sqrt{x^4+5}}{3x}\right)-\frac{(10-9x^2)(x^4+5)^{3/2}}{15x^3}$$

$$\Big\downarrow \ 1512$$

$$2\left(\frac{2}{3}\left(\left(10+27\sqrt{5}\right)\int \frac{1}{\sqrt{x^4+5}}dx-27\sqrt{5}\int \frac{\sqrt{5}-x^2}{\sqrt{5}\sqrt{x^4+5}}dx\right)-\frac{(27-2x^2)\sqrt{x^4+5}}{3x}\right)-\frac{(10-9x^2)(x^4+5)^{3/2}}{15x^3}$$

$$\Big\downarrow \ 27$$

$$2\left(\frac{2}{3}\left(\left(10+27\sqrt{5}\right)\int \frac{1}{\sqrt{x^4+5}}dx-27\int \frac{\sqrt{5}-x^2}{\sqrt{x^4+5}}dx\right)-\frac{(27-2x^2)\sqrt{x^4+5}}{3x}\right)-\frac{(10-9x^2)(x^4+5)^{3/2}}{15x^3}$$

$$\Big\downarrow \ 761$$

$$2\left(\frac{2}{3}\left(\frac{\left(10+27\sqrt{5}\right)(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{(x^2+\sqrt{5})^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}-27\int \frac{\sqrt{5}-x^2}{\sqrt{x^4+5}}dx\right)-\frac{(27-2x^2)\sqrt{}}{3x}\right.$$

$$\frac{(10-9x^2)(x^4+5)^{3/2}}{15x^3}$$

$$\Big\downarrow \ 1510$$

$$2\left(\frac{2}{3}\left(\frac{\left(10+27\sqrt{5}\right)(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{(x^2+\sqrt{5})^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}-27\left(\frac{\sqrt[4]{5}(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{(x^2+\sqrt{5})^2}}E\Big(2}{\sqrt{x^4+5}}\right.\right.\right.$$

$$\frac{(10-9x^2)(x^4+5)^{3/2}}{15x^3}$$

input
```
Int[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^4,x]
```

3.31. $\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^4}\,dx$

output
```
-1/15*((10 - 9*x^2)*(5 + x^4)^(3/2))/x^3 + 2*(-1/3*((27 - 2*x^2)*Sqrt[5 +
x^4])/x + (2*(-27*(-((x*Sqrt[5 + x^4])/(Sqrt[5] + x^2)) + (5^(1/4)*(Sqrt[5
] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)],
1/2])/Sqrt[5 + x^4]) + ((10 + 27*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(
Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(2*5^(1/4)*Sqrt[5 +
 x^4])))/3)
```

3.31.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

rule 1595
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((d*(m + 4*p + 3) + e*(m + 1)*
x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Simp[4*(p/(f^2*(m + 1)*(m + 4*p + 3))
)   Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*
x^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.31.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 2.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.20

method	result
meijerg	$-\frac{10\sqrt{5}\, {}_2F_1\left(-\frac{3}{2},-\frac{3}{4};\frac{1}{4};-\frac{x^4}{5}\right)}{3x^3}-\frac{15\sqrt{5}\, {}_2F_1\left(-\frac{3}{2},-\frac{1}{4};\frac{3}{4};-\frac{x^4}{5}\right)}{x}$
risch	$\frac{9x^{10}+10x^8-180x^6-1125x^2-250}{15x^3\sqrt{x^4+5}}+\frac{8\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{15\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{36i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(F\left(\right.\right.}{5\sqrt{i\sqrt{5}}\,\sqrt{}}$
default	$-\frac{15\sqrt{x^4+5}}{x}+\frac{3x^3\sqrt{x^4+5}}{5}+\frac{36i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)\right)}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}-\frac{10\sqrt{x^4+5}}{3x^3}+\frac{2x\sqrt{x^4-}}{3}$
elliptic	$-\frac{15\sqrt{x^4+5}}{x}+\frac{3x^3\sqrt{x^4+5}}{5}+\frac{36i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)\right)}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}-\frac{10\sqrt{x^4+5}}{3x^3}+\frac{2x\sqrt{x^4-}}{3}$

input
```
int((3*x^2+2)*(x^4+5)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-10/3*5^(1/2)/x^3*hypergeom([-3/2,-3/4],[1/4],-1/5*x^4)-15*5^(1/2)/x*hyper
geom([-3/2,-1/4],[3/4],-1/5*x^4)
```

3.31.5 Fricas [F]

$$\int \frac{(2+3x^2)\,(5+x^4)^{3/2}}{x^4}\,dx = \int \frac{(x^4+5)^{\frac{3}{2}}\,(3\,x^2+2)}{x^4}\,dx$$

input
```
integrate((3*x^2+2)*(x^4+5)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
integral((3*x^6 + 2*x^4 + 15*x^2 + 10)*sqrt(x^4 + 5)/x^4, x)
```

3.31.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.73 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.81

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^4}\,dx = \frac{3\sqrt{5}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{\sqrt{5}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{5}{4}\right)} + \frac{15\sqrt{5}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4x\Gamma\left(\frac{3}{4}\right)} + \frac{5\sqrt{5}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2x^3\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate((3*x**2+2)*(x**4+5)**(3/2)/x**4,x)
```

output
```
3*sqrt(5)*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), x**4*exp_polar(I*pi)/
5)/(4*gamma(7/4)) + sqrt(5)*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), x**4*e
xp_polar(I*pi)/5)/(2*gamma(5/4)) + 15*sqrt(5)*gamma(-1/4)*hyper((-1/2, -1/
4), (3/4,), x**4*exp_polar(I*pi)/5)/(4*x*gamma(3/4)) + 5*sqrt(5)*gamma(-3/
4)*hyper((-3/4, -1/2), (1/4,), x**4*exp_polar(I*pi)/5)/(2*x**3*gamma(1/4))
```

3.31.7 Maxima [F]

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^4}\,dx = \int \frac{(x^4+5)^{\frac{3}{2}}(3\,x^2+2)}{x^4}\,dx$$

input
```
integrate((3*x^2+2)*(x^4+5)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x^4, x)
```

3.31.8 Giac [F]

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^4}\,dx = \int \frac{(x^4+5)^{\frac{3}{2}}(3\,x^2+2)}{x^4}\,dx$$

input
```
integrate((3*x^2+2)*(x^4+5)^(3/2)/x^4,x, algorithm="giac")
```

output
```
integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x^4, x)
```

3.31.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^4}\,dx = \int \frac{(x^4+5)^{3/2}\,(3\,x^2+2)}{x^4}\,dx$$

input
```
int(((x^4 + 5)^(3/2)*(3*x^2 + 2))/x^4,x)
```

output
```
int(((x^4 + 5)^(3/2)*(3*x^2 + 2))/x^4, x)
```

3.32 $\int \frac{x^7(2+3x^2)}{\sqrt{5+x^4}}\,dx$

3.32.1 Optimal result

Integrand size = 20, antiderivative size = 67

$$\int \frac{x^7(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{1}{3}x^4\sqrt{5+x^4}+\frac{3}{8}x^6\sqrt{5+x^4}-\frac{5}{48}\left(32+27x^2\right)\sqrt{5+x^4}+\frac{225}{16}\operatorname{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)$$

output
```
225/16*arcsinh(1/5*x^2*5^(1/2))+1/3*x^4*(x^4+5)^(1/2)+3/8*x^6*(x^4+5)^(1/2
)-5/48*(27*x^2+32)*(x^4+5)^(1/2)
```

3.32.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.76

$$\int \frac{x^7(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{1}{48}\sqrt{5+x^4}\left(-160-135x^2+16x^4+18x^6\right)-\frac{225}{16}\log\left(-x^2+\sqrt{5+x^4}\right)$$

input
```
Integrate[(x^7*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

output
```
(Sqrt[5 + x^4]*(-160 - 135*x^2 + 16*x^4 + 18*x^6))/48 - (225*Log[-x^2 + Sq
rt[5 + x^4]])/16
```

3.32.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.36, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1579, 533, 533, 27, 533, 455, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7 (3x^2+2)}{\sqrt{x^4+5}}\,dx$$

$$\Big\downarrow \text{1579}$$

$$\frac{1}{2}\int \frac{x^6 (3x^2+2)}{\sqrt{x^4+5}}dx^2$$

$$\Big\downarrow \text{533}$$

$$\frac{1}{2}\left(\frac{3}{4}x^6\sqrt{x^4+5}-\frac{1}{4}\int \frac{x^4 (45-8x^2)}{\sqrt{x^4+5}}dx^2\right)$$

$$\Big\downarrow \text{533}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{1}{3}\int -\frac{5x^2 (27x^2+16)}{\sqrt{x^4+5}}dx^2+\frac{8}{3}\sqrt{x^4+5}x^4\right)+\frac{3}{4}\sqrt{x^4+5}x^6\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{8}{3}x^4\sqrt{x^4+5}-\frac{5}{3}\int \frac{x^2 (27x^2+16)}{\sqrt{x^4+5}}dx^2\right)+\frac{3}{4}\sqrt{x^4+5}x^6\right)$$

$$\Big\downarrow \text{533}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{8}{3}x^4\sqrt{x^4+5}-\frac{5}{3}\left(\frac{27}{2}x^2\sqrt{x^4+5}-\frac{1}{2}\int \frac{135-32x^2}{\sqrt{x^4+5}}dx^2\right)\right)+\frac{3}{4}\sqrt{x^4+5}x^6\right)$$

$$\Big\downarrow \text{455}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{8}{3}x^4\sqrt{x^4+5}-\frac{5}{3}\left(\frac{1}{2}\left(32\sqrt{x^4+5}-135\int \frac{1}{\sqrt{x^4+5}}dx^2\right)+\frac{27}{2}\sqrt{x^4+5}x^2\right)\right)+\frac{3}{4}\sqrt{x^4+5}x^6\right)$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{8}{3}x^4\sqrt{x^4+5}-\frac{5}{3}\left(\frac{1}{2}\left(32\sqrt{x^4+5}-135\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)\right)+\frac{27}{2}\sqrt{x^4+5}x^2\right)\right)+\frac{3}{4}\sqrt{x^4+5}x^6\right)$$

input
```
Int[(x^7*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

output
```
((3*x^6*Sqrt[5 + x^4])/4 + ((8*x^4*Sqrt[5 + x^4])/3 - (5*((27*x^2*Sqrt[5 +
 x^4])/2 + (32*Sqrt[5 + x^4] - 135*ArcSinh[x^2/Sqrt[5]])/2))/3)/4)/2
```

3.32.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

3.32.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.58

method	result	size
risch	$\frac{(18x^6+16x^4-135x^2-160)\sqrt{x^4+5}}{48}+\frac{225\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{16}$	39
pseudoelliptic	$\frac{(18x^6+16x^4-135x^2-160)\sqrt{x^4+5}}{48}+\frac{225\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{16}$	39
trager	$\left(\frac{3}{8}x^6+\frac{1}{3}x^4-\frac{45}{16}x^2-\frac{10}{3}\right)\sqrt{x^4+5}-\frac{225\ln\left(x^2-\sqrt{x^4+5}\right)}{16}$	43
default	$\frac{3x^6\sqrt{x^4+5}}{8}-\frac{45x^2\sqrt{x^4+5}}{16}+\frac{225\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{16}+\frac{\sqrt{x^4+5}\,(x^4-10)}{3}$	51
elliptic	$\frac{3x^6\sqrt{x^4+5}}{8}-\frac{45x^2\sqrt{x^4+5}}{16}+\frac{225\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{16}+\frac{x^4\sqrt{x^4+5}}{3}-\frac{10\sqrt{x^4+5}}{3}$	58
meijerg	$\frac{-\frac{3\sqrt{\pi}\,x^2\sqrt{5}\,(-2x^4+15)\sqrt{1+\frac{x^4}{5}}}{16}+\frac{225\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{16}}{\sqrt{\pi}}+\frac{5\sqrt{5}\left(\frac{4\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\left(-\frac{4x^4}{5}+8\right)\sqrt{1+\frac{x^4}{5}}}{6}\right)}{2\sqrt{\pi}}$	84

input
```
int(x^7*(3*x^2+2)/(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/48*(18*x^6+16*x^4-135*x^2-160)*(x^4+5)^(1/2)+225/16*arcsinh(1/5*x^2*5^(1
/2))
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.64

$$\int \frac{x^7(2+3x^2)}{\sqrt{5+x^4}}\,dx=\frac{1}{48}\left(18\,x^6+16\,x^4-135\,x^2-160\right)\sqrt{x^4+5}-\frac{225}{16}\log\left(-x^2+\sqrt{x^4+5}\right)$$

input
```
integrate(x^7*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="fricas")
```

output
```
1/48*(18*x^6 + 16*x^4 - 135*x^2 - 160)*sqrt(x^4 + 5) - 225/16*log(-x^2 + s
qrt(x^4 + 5))
```

3.32.6 Sympy [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.72

$$\int \frac{x^7(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{\sqrt{x^4+5}\cdot\left(\frac{3x^6}{4}+\frac{2x^4}{3}-\frac{45x^2}{8}-\frac{20}{3}\right)}{2}+\frac{225\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{16}$$

input
```
integrate(x**7*(3*x**2+2)/(x**4+5)**(1/2),x)
```

output
```
sqrt(x**4 + 5)*(3*x**6/4 + 2*x**4/3 - 45*x**2/8 - 20/3)/2 + 225*asinh(sqrt
(5)*x**2/5)/16
```

3.32.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.55

$$\int \frac{x^7(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{1}{3}\left(x^4+5\right)^{\frac{3}{2}}-5\sqrt{x^4+5}-\frac{75\left(\frac{5\sqrt{x^4+5}}{x^2}-\frac{3\left(x^4+5\right)^{\frac{3}{2}}}{x^6}\right)}{16\left(\frac{2\left(x^4+5\right)}{x^4}-\frac{\left(x^4+5\right)^2}{x^8}-1\right)} + \frac{225}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{225}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

input
```
integrate(x^7*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="maxima")
```

output
```
1/3*(x^4 + 5)^(3/2) - 5*sqrt(x^4 + 5) - 75/16*(5*sqrt(x^4 + 5)/x^2 - 3*(x^
4 + 5)^(3/2)/x^6)/(2*(x^4 + 5)/x^4 - (x^4 + 5)^2/x^8 - 1) + 225/32*log(sqr
t(x^4 + 5)/x^2 + 1) - 225/32*log(sqrt(x^4 + 5)/x^2 - 1)
```

3.32.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.69

$$\int \frac{x^7(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{1}{48}\sqrt{x^4+5}\left(\left(2\left(9x^2+8\right)x^2-135\right)x^2-160\right)-\frac{225}{16}\log\left(-x^2+\sqrt{x^4+5}\right)$$

input
```
integrate(x^7*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="giac")
```

output
```
1/48*sqrt(x^4 + 5)*((2*(9*x^2 + 8)*x^2 - 135)*x^2 - 160) - 225/16*log(-x^2
 + sqrt(x^4 + 5))
```

3.32.9 Mupad [B] (verification not implemented)

Time = 7.87 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.57

$$\int \frac{x^7(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{225\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{16} - \sqrt{x^4+5}\left(-\frac{3\,x^6}{8} - \frac{x^4}{3} + \frac{45\,x^2}{16} + \frac{10}{3}\right)$$

input
```
int((x^7*(3*x^2 + 2))/(x^4 + 5)^(1/2),x)
```

output
```
(225*asinh((5^(1/2)*x^2)/5))/16 - (x^4 + 5)^(1/2)*((45*x^2)/16 - x^4/3 - (
3*x^6)/8 + 10/3)
```

3.33 $\int \frac{x^5(2+3x^2)}{\sqrt{5+x^4}}\,dx$

3.33.1 Optimal result

Integrand size = 20, antiderivative size = 51

$$\int \frac{x^5(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{1}{2}x^4\sqrt{5+x^4}-\frac{1}{2}(10-x^2)\sqrt{5+x^4}-\frac{5}{2}\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)$$

output
```
-5/2*arcsinh(1/5*x^2*5^(1/2))+1/2*x^4*(x^4+5)^(1/2)-1/2*(-x^2+10)*(x^4+5)^
(1/2)
```

3.33.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int \frac{x^5(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{1}{2}\sqrt{5+x^4}(-10+x^2+x^4)+\frac{5}{2}\log\left(-x^2+\sqrt{5+x^4}\right)$$

input
```
Integrate[(x^5*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

output
```
(Sqrt[5 + x^4]*(-10 + x^2 + x^4))/2 + (5*Log[-x^2 + Sqrt[5 + x^4]])/2
```

3.33.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.24, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1579, 533, 27, 533, 27, 455, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(3x^2+2)}{\sqrt{x^4+5}}\,dx$$

$$\Big\downarrow \text{ 1579}$$

$$\frac{1}{2}\int \frac{x^4(3x^2+2)}{\sqrt{x^4+5}}dx^2$$

$$\Big\downarrow \text{ 533}$$

$$\frac{1}{2}\left(x^4\sqrt{x^4+5}-\frac{1}{3}\int \frac{6x^2(5-x^2)}{\sqrt{x^4+5}}dx^2\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(x^4\sqrt{x^4+5}-2\int \frac{x^2(5-x^2)}{\sqrt{x^4+5}}dx^2\right)$$

$$\Big\downarrow \text{ 533}$$

$$\frac{1}{2}\left(x^4\sqrt{x^4+5}-2\left(-\frac{1}{2}\int -\frac{5(2x^2+1)}{\sqrt{x^4+5}}dx^2-\frac{1}{2}\sqrt{x^4+5}x^2\right)\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(x^4\sqrt{x^4+5}-2\left(\frac{5}{2}\int \frac{2x^2+1}{\sqrt{x^4+5}}dx^2-\frac{1}{2}x^2\sqrt{x^4+5}\right)\right)$$

$$\Big\downarrow \text{ 455}$$

$$\frac{1}{2}\left(x^4\sqrt{x^4+5}-2\left(\frac{5}{2}\left(\int \frac{1}{\sqrt{x^4+5}}dx^2+2\sqrt{x^4+5}\right)-\frac{1}{2}x^2\sqrt{x^4+5}\right)\right)$$

$$\Big\downarrow \text{ 222}$$

$$\frac{1}{2}\left(x^4\sqrt{x^4+5}-2\left(\frac{5}{2}\left(\operatorname{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)+2\sqrt{x^4+5}\right)-\frac{1}{2}x^2\sqrt{x^4+5}\right)\right)$$

input
```
Int[(x^5*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

output
```
(x^4*Sqrt[5 + x^4] - 2*(-1/2*(x^2*Sqrt[5 + x^4]) + (5*(2*Sqrt[5 + x^4] + A
rcSinh[x^2/Sqrt[5]]))/2))/2
```

3.33.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

3.33. $\int \frac{x^5(2+3x^2)}{\sqrt{5+x^4}}\,dx$

3.33.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.59

method	result	size
risch	$\frac{(x^4+x^2-10)\sqrt{x^4+5}}{2}-\frac{5\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}$	30
pseudoelliptic	$\frac{(x^4+x^2-10)\sqrt{x^4+5}}{2}-\frac{5\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}$	30
trager	$\left(\frac{1}{2}x^4+\frac{1}{2}x^2-5\right)\sqrt{x^4+5}-\frac{5\ln\left(x^2+\sqrt{x^4+5}\right)}{2}$	36
default	$\frac{\sqrt{x^4+5}\,(x^4-10)}{2}+\frac{x^2\sqrt{x^4+5}}{2}-\frac{5\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}$	39
elliptic	$\frac{x^4\sqrt{x^4+5}}{2}-5\sqrt{x^4+5}+\frac{x^2\sqrt{x^4+5}}{2}-\frac{5\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}$	46
meijerg	$\frac{15\sqrt{5}\left(\frac{4\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\left(-\frac{4x^4}{5}+8\right)\sqrt{1+\frac{x^4}{5}}}{6}\right)}{4\sqrt{\pi}}+\frac{\frac{\sqrt{\pi}\,x^2\sqrt{5}\,\sqrt{1+\frac{x^4}{5}}}{2}-\frac{5\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}}{\sqrt{\pi}}$	77

input
```
int(x^5*(3*x^2+2)/(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(x^4+x^2-10)*(x^4+5)^(1/2)-5/2*arcsinh(1/5*x^2*5^(1/2))
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.67

$$\int \frac{x^5(2+3x^2)}{\sqrt{5+x^4}}\,dx=\frac{1}{2}\left(x^4+x^2-10\right)\sqrt{x^4+5}+\frac{5}{2}\log\left(-x^2+\sqrt{x^4+5}\right)$$

input
```
integrate(x^5*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(x^4 + x^2 - 10)*sqrt(x^4 + 5) + 5/2*log(-x^2 + sqrt(x^4 + 5))
```

3.33.6 Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.63

$$\int \frac{x^5(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{\sqrt{x^4+5}(x^4+x^2-10)}{2} - \frac{5\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{2}$$

input
```
integrate(x**5*(3*x**2+2)/(x**4+5)**(1/2),x)
```

output
```
sqrt(x**4 + 5)*(x**4 + x**2 - 10)/2 - 5*asinh(sqrt(5)*x**2/5)/2
```

3.33.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.49

$$\int \frac{x^5(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{1}{2}\left(x^4+5\right)^{\frac{3}{2}} - \frac{15}{2}\sqrt{x^4+5} + \frac{5\sqrt{x^4+5}}{2x^2\left(\frac{x^4+5}{x^4}-1\right)} - \frac{5}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right) + \frac{5}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

input
```
integrate(x^5*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(x^4 + 5)^(3/2) - 15/2*sqrt(x^4 + 5) + 5/2*sqrt(x^4 + 5)/(x^2*((x^4 +
5)/x^4 - 1)) - 5/4*log(sqrt(x^4 + 5)/x^2 + 1) + 5/4*log(sqrt(x^4 + 5)/x^2
- 1)
```

3.33.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

$$\int \frac{x^5(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{1}{2}\sqrt{x^4+5}\left((x^2+1)x^2-10\right) + \frac{5}{2}\log\left(-x^2+\sqrt{x^4+5}\right)$$

input
```
integrate(x^5*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(x^4 + 5)*((x^2 + 1)*x^2 - 10) + 5/2*log(-x^2 + sqrt(x^4 + 5))
```

3.33.9 Mupad [B] (verification not implemented)

Time = 7.84 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.63

$$\int \frac{x^5(2+3x^2)}{\sqrt{5+x^4}}\,dx = \sqrt{x^4+5}\left(\frac{x^4}{2}+\frac{x^2}{2}-5\right)-\frac{5\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{2}$$

input
```
int((x^5*(3*x^2 + 2))/(x^4 + 5)^(1/2),x)
```

output
```
(x^4 + 5)^(1/2)*(x^2/2 + x^4/2 - 5) - (5*asinh((5^(1/2)*x^2)/5))/2
```

3.34 $\int \frac{x^3(2+3x^2)}{\sqrt{5+x^4}}\,dx$

3.34.1 Optimal result

Integrand size = 20, antiderivative size = 35

$$\int \frac{x^3(2+3x^2)}{\sqrt{5+x^4}}\,dx=\frac{1}{4}\left(4+3x^2\right)\sqrt{5+x^4}-\frac{15}{4}\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)$$

output
```
-15/4*arcsinh(1/5*x^2*5^(1/2))+1/4*(3*x^2+4)*(x^4+5)^(1/2)
```

3.34.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

$$\int \frac{x^3(2+3x^2)}{\sqrt{5+x^4}}\,dx=\frac{1}{4}\left(4+3x^2\right)\sqrt{5+x^4}+\frac{15}{4}\log\left(-x^2+\sqrt{5+x^4}\right)$$

input
```
Integrate[(x^3*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

output
```
((4 + 3*x^2)*Sqrt[5 + x^4])/4 + (15*Log[-x^2 + Sqrt[5 + x^4]])/4
```

3.34.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.40, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1579, 533, 455, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(3x^2+2)}{\sqrt{x^4+5}}\,dx$$

$$\Big\downarrow \text{1579}$$

$$\frac{1}{2}\int \frac{x^2(3x^2+2)}{\sqrt{x^4+5}}dx^2$$

$$\Big\downarrow \text{533}$$

$$\frac{1}{2}\left(\frac{3}{2}x^2\sqrt{x^4+5}-\frac{1}{2}\int \frac{15-4x^2}{\sqrt{x^4+5}}dx^2\right)$$

$$\Big\downarrow \text{455}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(4\sqrt{x^4+5}-15\int \frac{1}{\sqrt{x^4+5}}dx^2\right)+\frac{3}{2}\sqrt{x^4+5}x^2\right)$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(4\sqrt{x^4+5}-15\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)\right)+\frac{3}{2}\sqrt{x^4+5}x^2\right)$$

input
```
Int[(x^3*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

output
```
((3*x^2*Sqrt[5 + x^4])/2 + (4*Sqrt[5 + x^4] - 15*ArcSinh[x^2/Sqrt[5]])/2)/
2
```

3.34.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

3.34.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

method	result	size
risch	$-\frac{15\,\mathrm{arcsinh}\left(\frac{x^{2}\sqrt{5}}{5}\right)}{4}+\frac{\left(3x^{2}+4\right)\sqrt{x^{4}+5}}{4}$	29
default	$\frac{3x^{2}\sqrt{x^{4}+5}}{4}-\frac{15\,\mathrm{arcsinh}\left(\frac{x^{2}\sqrt{5}}{5}\right)}{4}+\sqrt{x^{4}+5}$	32
elliptic	$\frac{3x^{2}\sqrt{x^{4}+5}}{4}-\frac{15\,\mathrm{arcsinh}\left(\frac{x^{2}\sqrt{5}}{5}\right)}{4}+\sqrt{x^{4}+5}$	32
pseudoelliptic	$\frac{3x^{2}\sqrt{x^{4}+5}}{4}-\frac{15\,\mathrm{arcsinh}\left(\frac{x^{2}\sqrt{5}}{5}\right)}{4}+\sqrt{x^{4}+5}$	32
trager	$\left(\frac{3x^{2}}{4}+1\right)\sqrt{x^{4}+5}+\frac{15\ln\left(x^{2}-\sqrt{x^{4}+5}\right)}{4}$	33
meijerg	$\frac{\frac{3\sqrt{\pi}\,x^{2}\sqrt{5}\,\sqrt{1+\frac{x^{4}}{5}}}{4}-\frac{15\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{x^{2}\sqrt{5}}{5}\right)}{4}}{\sqrt{\pi}}+\frac{\sqrt{5}\left(-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{1+\frac{x^{4}}{5}}\right)}{2\sqrt{\pi}}$	70

input
```
int(x^3*(3*x^2+2)/(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-15/4*arcsinh(1/5*x^2*5^(1/2))+1/4*(3*x^2+4)*(x^4+5)^(1/2)
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{x^3(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{1}{4}\sqrt{x^4+5}\left(3\,x^2+4\right) + \frac{15}{4}\log\left(-x^2+\sqrt{x^4+5}\right)$$

input
```
integrate(x^3*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(x^4 + 5)*(3*x^2 + 4) + 15/4*log(-x^2 + sqrt(x^4 + 5))
```

3.34.6 Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{x^3(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{\left(\frac{3x^2}{2}+2\right)\sqrt{x^4+5}}{2} - \frac{15\,\mathrm{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{4}$$

input
```
integrate(x**3*(3*x**2+2)/(x**4+5)**(1/2),x)
```

output
```
(3*x**2/2 + 2)*sqrt(x**4 + 5)/2 - 15*asinh(sqrt(5)*x**2/5)/4
```

3.34.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(28) = 56.

Time = 0.36 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.86

$$\int \frac{x^3(2+3x^2)}{\sqrt{5+x^4}}\,dx = \sqrt{x^4+5} + \frac{15\sqrt{x^4+5}}{4\,x^2\left(\frac{x^4+5}{x^4}-1\right)} - \frac{15}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right) + \frac{15}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

input
```
integrate(x^3*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(x^4 + 5) + 15/4*sqrt(x^4 + 5)/(x^2*((x^4 + 5)/x^4 - 1)) - 15/8*log(sq
rt(x^4 + 5)/x^2 + 1) + 15/8*log(sqrt(x^4 + 5)/x^2 - 1)
```

3.34.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{x^3(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{1}{4}\sqrt{x^4+5}\left(3\,x^2+4\right)+\frac{15}{4}\log\left(-x^2+\sqrt{x^4+5}\right)$$

input
```
integrate(x^3*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(x^4 + 5)*(3*x^2 + 4) + 15/4*log(-x^2 + sqrt(x^4 + 5))
```

3.34.9 Mupad [B] (verification not implemented)

Time = 7.85 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{x^3(2+3x^2)}{\sqrt{5+x^4}}\,dx = \sqrt{x^4+5}\left(\frac{3\,x^2}{4}+1\right)-\frac{15\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{4}$$

input
```
int((x^3*(3*x^2 + 2))/(x^4 + 5)^(1/2),x)
```

output
```
(x^4 + 5)^(1/2)*((3*x^2)/4 + 1) - (15*asinh((5^(1/2)*x^2)/5))/4
```

3.35 $\int \frac{x(2+3x^2)}{\sqrt{5+x^4}}\,dx$

3.35.1 Optimal result

Integrand size = 18, antiderivative size = 24

$$\int \frac{x(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{3\sqrt{5+x^4}}{2} + \operatorname{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)$$

output
```
arcsinh(1/5*x^2*5^(1/2))+3/2*(x^4+5)^(1/2)
```

3.35.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \frac{x(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{3\sqrt{5+x^4}}{2} - \log\left(-x^2+\sqrt{5+x^4}\right)$$

input
```
Integrate[(x*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

output
```
(3*Sqrt[5 + x^4])/2 - Log[-x^2 + Sqrt[5 + x^4]]
```

3.35.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1577, 455, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(3x^2+2)}{\sqrt{x^4+5}}\,dx$$

$$\Big\downarrow \; 1577$$

$$\frac{1}{2}\int \frac{3x^2+2}{\sqrt{x^4+5}}dx^2$$

$$\Big\downarrow \; 455$$

$$\frac{1}{2}\left(2\int \frac{1}{\sqrt{x^4+5}}dx^2+3\sqrt{x^4+5}\right)$$

$$\Big\downarrow \; 222$$

$$\frac{1}{2}\left(2\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)+3\sqrt{x^4+5}\right)$$

input
```
Int[(x*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

output
```
(3*Sqrt[5 + x^4] + 2*ArcSinh[x^2/Sqrt[5]])/2
```

3.35.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

3.35.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

method	result	size
default	$\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)+\frac{3\sqrt{x^4+5}}{2}$	20
risch	$\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)+\frac{3\sqrt{x^4+5}}{2}$	20
elliptic	$\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)+\frac{3\sqrt{x^4+5}}{2}$	20
pseudoelliptic	$\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)+\frac{3\sqrt{x^4+5}}{2}$	20
trager	$\frac{3\sqrt{x^4+5}}{2}+\ln\left(x^2+\sqrt{x^4+5}\right)$	23
meijerg	$\frac{3\sqrt{5}\left(-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{1+\frac{x^4}{5}}\right)}{4\sqrt{\pi}}+\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)$	39

input
```
int(x*(3*x^2+2)/(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsinh(1/5*x^2*5^(1/2))+3/2*(x^4+5)^(1/2)
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{3}{2}\sqrt{x^4+5}-\log\left(-x^2+\sqrt{x^4+5}\right)$$

input
```
integrate(x*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="fricas")
```

output
```
3/2*sqrt(x^4 + 5) - log(-x^2 + sqrt(x^4 + 5))
```

3.35.6 Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{3\sqrt{x^4+5}}{2} + \operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)$$

input
```
integrate(x*(3*x**2+2)/(x**4+5)**(1/2),x)
```

output
```
3*sqrt(x**4 + 5)/2 + asinh(sqrt(5)*x**2/5)
```

3.35.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(19) = 38.

Time = 0.37 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.75

$$\int \frac{x(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{3}{2}\sqrt{x^4+5} + \frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right) - \frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

input
```
integrate(x*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="maxima")
```

output
```
3/2*sqrt(x^4 + 5) + 1/2*log(sqrt(x^4 + 5)/x^2 + 1) - 1/2*log(sqrt(x^4 + 5)
/x^2 - 1)
```

3.35.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{3}{2}\sqrt{x^4+5} - \log\left(-x^2+\sqrt{x^4+5}\right)$$

input
```
integrate(x*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="giac")
```

output
```
3/2*sqrt(x^4 + 5) - log(-x^2 + sqrt(x^4 + 5))
```

3.35.9 Mupad [B] (verification not implemented)

Time = 7.86 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{x(2+3x^2)}{\sqrt{5+x^4}}\,dx = \operatorname{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right) + \frac{3\sqrt{x^4+5}}{2}$$

input
```
int((x*(3*x^2 + 2))/(x^4 + 5)^(1/2),x)
```

output
```
asinh((5^(1/2)*x^2)/5) + (3*(x^4 + 5)^(1/2))/2
```

3.36 $\int \frac{2+3x^2}{x\sqrt{5+x^4}}\,dx$

3.36.1 Optimal result

Integrand size = 20, antiderivative size = 38

$$\int \frac{2+3x^2}{x\sqrt{5+x^4}}\,dx = \frac{3}{2}\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{\text{arctanh}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{\sqrt{5}}$$

output
```
3/2*arcsinh(1/5*x^2*5^(1/2))-1/5*arctanh(1/5*(x^4+5)^(1/2)*5^(1/2))*5^(1/2
)
```

3.36.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.32

$$\int \frac{2+3x^2}{x\sqrt{5+x^4}}\,dx = \frac{2\text{arctanh}\left(\frac{x^2-\sqrt{5+x^4}}{\sqrt{5}}\right)}{\sqrt{5}} - \frac{3}{2}\log\left(-x^2+\sqrt{5+x^4}\right)$$

input
```
Integrate[(2 + 3*x^2)/(x*Sqrt[5 + x^4]),x]
```

output
```
(2*ArcTanh[(x^2 - Sqrt[5 + x^4])/Sqrt[5]])/Sqrt[5] - (3*Log[-x^2 + Sqrt[5
+ x^4]])/2
```

3.36.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1579, 538, 222, 243, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+2}{x\sqrt{x^4+5}}dx$$

$$\Big\downarrow \text{1579}$$

$$\frac{1}{2}\int \frac{3x^2+2}{x^2\sqrt{x^4+5}}dx^2$$

$$\Big\downarrow \text{538}$$

$$\frac{1}{2}\left(3\int \frac{1}{\sqrt{x^4+5}}dx^2+2\int \frac{1}{x^2\sqrt{x^4+5}}dx^2\right)$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{2}\left(2\int \frac{1}{x^2\sqrt{x^4+5}}dx^2+3\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)\right)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\left(\int \frac{1}{x^2\sqrt{x^4+5}}dx^4+3\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(2\int \frac{1}{\sqrt{x^4+5}-5}d\sqrt{x^4+5}+3\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)\right)$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{2}\left(3\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{2\text{arctanh}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{\sqrt{5}}\right)$$

input
```
Int[(2 + 3*x^2)/(x*Sqrt[5 + x^4]),x]
```

output
```
(3*ArcSinh[x^2/Sqrt[5]] - (2*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/Sqrt[5])/2
```

3.36.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

3.36.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{3 \operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}-\frac{\sqrt{5}\, \operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{5}$	30
elliptic	$\frac{3 \operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}-\frac{\sqrt{5}\, \operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{5}$	30
pseudoelliptic	$\frac{3 \operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}-\frac{\sqrt{5}\, \operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{5}$	30
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2-5\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^2-5\right)+\sqrt{x^4+5}}{x^2}\right)}{5}-\frac{3\ln\left(x^2-\sqrt{x^4+5}\right)}{2}$	45
meijerg	$\frac{\sqrt{5}\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{x^4}{5}}}{2}\right)+\left(-2\ln(2)+4\ln(x)-\ln(5)\right)\sqrt{\pi}\right)}{10\sqrt{\pi}}+\frac{3 \operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}$	58

input
```
int((3*x^2+2)/x/(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/2*arcsinh(1/5*x^2*5^(1/2))-1/5*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))
```

3.36.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

$$\int \frac{2+3x^2}{x\sqrt{5+x^4}}\,dx = \frac{1}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)-\frac{3}{2}\log\left(-x^2+\sqrt{x^4+5}\right)$$

input
```
integrate((3*x^2+2)/x/(x^4+5)^(1/2),x, algorithm="fricas")
```

output
```
1/5*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 3/2*log(-x^2 + sqrt(x^4
+ 5))
```

3.36.6 Sympy [A] (verification not implemented)

Time = 2.77 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \frac{2+3x^2}{x\sqrt{5+x^4}}\,dx = -\frac{\sqrt{5}\operatorname{asinh}\left(\frac{\sqrt{5}}{x^2}\right)}{5} + \frac{3\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{2}$$

input
```
integrate((3*x**2+2)/x/(x**4+5)**(1/2),x)
```

output
```
-sqrt(5)*asinh(sqrt(5)/x**2)/5 + 3*asinh(sqrt(5)*x**2/5)/2
```

3.36.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(30) = 60.

Time = 0.35 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.76

$$\int \frac{2+3x^2}{x\sqrt{5+x^4}}\,dx = \frac{1}{10}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right) + \frac{3}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right) - \frac{3}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

input
```
integrate((3*x^2+2)/x/(x^4+5)^(1/2),x, algorithm="maxima")
```

output
```
1/10*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) + 3
/4*log(sqrt(x^4 + 5)/x^2 + 1) - 3/4*log(sqrt(x^4 + 5)/x^2 - 1)
```

3.36.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(30) = 60.

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.61

$$\int \frac{2+3x^2}{x\sqrt{5+x^4}}\,dx = \frac{1}{5}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right) - \frac{3}{2}\log\left(-x^2+\sqrt{x^4+5}\right)$$

input
```
integrate((3*x^2+2)/x/(x^4+5)^(1/2),x, algorithm="giac")
```

output
```
1/5*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4
 + 5))) - 3/2*log(-x^2 + sqrt(x^4 + 5))
```

3.36.9 Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{2+3x^2}{x\sqrt{5+x^4}}\,dx = \frac{3\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{2} - \frac{\sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}}{5}\right)}{5}$$

input
```
int((3*x^2 + 2)/(x*(x^4 + 5)^(1/2)),x)
```

output
```
(3*asinh((5^(1/2)*x^2)/5))/2 - (5^(1/2)*atanh((5^(1/2)*(x^4 + 5)^(1/2))/5)
)/5
```

3.37 $\int \frac{2+3x^2}{x^3\sqrt{5+x^4}}\,dx$

3.37.1 Optimal result

Integrand size = 20, antiderivative size = 42

$$\int \frac{2+3x^2}{x^3\sqrt{5+x^4}}\,dx=-\frac{\sqrt{5+x^4}}{5x^2}-\frac{3\operatorname{arctanh}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{2\sqrt{5}}$$

output
```
-3/10*arctanh(1/5*(x^4+5)^(1/2)*5^(1/2))*5^(1/2)-1/5*(x^4+5)^(1/2)/x^2
```

3.37.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

$$\int \frac{2+3x^2}{x^3\sqrt{5+x^4}}\,dx=-\frac{\sqrt{5+x^4}}{5x^2}+\frac{3\operatorname{arctanh}\left(\frac{x^2-\sqrt{5+x^4}}{\sqrt{5}}\right)}{\sqrt{5}}$$

input
```
Integrate[(2 + 3*x^2)/(x^3*Sqrt[5 + x^4]),x]
```

output
```
-1/5*Sqrt[5 + x^4]/x^2 + (3*ArcTanh[(x^2 - Sqrt[5 + x^4])/Sqrt[5]])/Sqrt[5
]
```

3.37.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1579, 534, 243, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+2}{x^3\sqrt{x^4+5}}dx$$

$$\Big\downarrow \text{1579}$$

$$\frac{1}{2}\int \frac{3x^2+2}{x^4\sqrt{x^4+5}}dx^2$$

$$\Big\downarrow \text{534}$$

$$\frac{1}{2}\left(3\int \frac{1}{x^2\sqrt{x^4+5}}dx^2-\frac{2\sqrt{x^4+5}}{5x^2}\right)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\left(\frac{3}{2}\int \frac{1}{x^2\sqrt{x^4+5}}dx^4-\frac{2\sqrt{x^4+5}}{5x^2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(3\int \frac{1}{\sqrt{x^4+5}-5}d\sqrt{x^4+5}-\frac{2\sqrt{x^4+5}}{5x^2}\right)$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{2}\left(-\frac{3\text{arctanh}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{\sqrt{5}}-\frac{2\sqrt{x^4+5}}{5x^2}\right)$$

input
```
Int[(2 + 3*x^2)/(x^3*Sqrt[5 + x^4]),x]
```

output
```
((-2*Sqrt[5 + x^4])/(5*x^2) - (3*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/Sqrt[5])/
2
```

3.37.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

3.37.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

method	result	size
default	$-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{10}-\frac{\sqrt{x^4+5}}{5x^2}$	31
risch	$-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{10}-\frac{\sqrt{x^4+5}}{5x^2}$	31
elliptic	$-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{10}-\frac{\sqrt{x^4+5}}{5x^2}$	31
pseudoelliptic	$\frac{-3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)x^2-2\sqrt{x^4+5}}{10x^2}$	36
trager	$-\frac{\sqrt{x^4+5}}{5x^2}-\frac{3\operatorname{RootOf}\left(\_Z^2-5\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^2-5\right)+\sqrt{x^4+5}}{x^2}\right)}{10}$	41
meijerg	$-\frac{\sqrt{5}\,\sqrt{1+\frac{x^4}{5}}}{5x^2}+\frac{3\sqrt{5}\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{x^4}{5}}}{2}\right)+(-2\ln(2)+4\ln(x)-\ln(5))\sqrt{\pi}\right)}{20\sqrt{\pi}}$	64

input
```
int((3*x^2+2)/x^3/(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-3/10*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))-1/5*(x^4+5)^(1/2)/x^2
```

3.37.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12

$$\int \frac{2+3x^2}{x^3\sqrt{5+x^4}}\,dx=\frac{3\,\sqrt{5}x^2\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)-2\,x^2-2\,\sqrt{x^4+5}}{10\,x^2}$$

input
```
integrate((3*x^2+2)/x^3/(x^4+5)^(1/2),x, algorithm="fricas")
```

output
```
1/10*(3*sqrt(5)*x^2*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 2*x^2 - 2*sqrt(x
^4 + 5))/x^2
```

3.37.6 Sympy [A] (verification not implemented)

Time = 1.60 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

$$\int \frac{2+3x^2}{x^3\sqrt{5+x^4}}\,dx = -\frac{\sqrt{1+\frac{5}{x^4}}}{5} - \frac{3\sqrt{5}\operatorname{asinh}\left(\frac{\sqrt{5}}{x^2}\right)}{10}$$

input
```
integrate((3*x**2+2)/x**3/(x**4+5)**(1/2),x)
```

output
```
-sqrt(1 + 5/x**4)/5 - 3*sqrt(5)*asinh(sqrt(5)/x**2)/10
```

3.37.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12

$$\int \frac{2+3x^2}{x^3\sqrt{5+x^4}}\,dx = \frac{3}{20}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right) - \frac{\sqrt{x^4+5}}{5\,x^2}$$

input
```
integrate((3*x^2+2)/x^3/(x^4+5)^(1/2),x, algorithm="maxima")
```

output
```
3/20*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) - 1
/5*sqrt(x^4 + 5)/x^2
```

3.37.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(31) = 62.

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.57

$$\int \frac{2+3x^2}{x^3\sqrt{5+x^4}}\,dx = \frac{3}{10}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right) + \frac{2}{\left(x^2-\sqrt{x^4+5}\right)^2-5}$$

input
```
integrate((3*x^2+2)/x^3/(x^4+5)^(1/2),x, algorithm="giac")
```

output
```
3/10*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^
4 + 5))) + 2/((x^2 - sqrt(x^4 + 5))^2 - 5)
```

3.37.9 Mupad [B] (verification not implemented)

Time = 7.68 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

$$\int \frac{2+3x^2}{x^3\sqrt{5+x^4}}\,dx = -\frac{3\sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}}{5}\right)}{10} - \frac{\sqrt{x^4+5}}{5\,x^2}$$

input
```
int((3*x^2 + 2)/(x^3*(x^4 + 5)^(1/2)),x)
```

output
```
- (3*5^(1/2)*atanh((5^(1/2)*(x^4 + 5)^(1/2))/5))/10 - (x^4 + 5)^(1/2)/(5*x
^2)
```

3.38 $\int \frac{2+3x^2}{x^5\sqrt{5+x^4}}\,dx$

3.38.1 Optimal result

Integrand size = 20, antiderivative size = 58

$$\int \frac{2+3x^2}{x^5\sqrt{5+x^4}}\,dx=-\frac{\sqrt{5+x^4}}{10x^4}-\frac{3\sqrt{5+x^4}}{10x^2}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{10\sqrt{5}}$$

output
```
1/50*arctanh(1/5*(x^4+5)^(1/2)*5^(1/2))*5^(1/2)-1/10*(x^4+5)^(1/2)/x^4-3/1
0*(x^4+5)^(1/2)/x^2
```

3.38.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \frac{2+3x^2}{x^5\sqrt{5+x^4}}\,dx=\frac{1}{50}\left(-\frac{5(1+3x^2)\sqrt{5+x^4}}{x^4}-2\sqrt{5}\operatorname{arctanh}\left(\frac{x^2-\sqrt{5+x^4}}{\sqrt{5}}\right)\right)$$

input
```
Integrate[(2 + 3*x^2)/(x^5*Sqrt[5 + x^4]),x]
```

output
```
((-5*(1 + 3*x^2)*Sqrt[5 + x^4])/x^4 - 2*Sqrt[5]*ArcTanh[(x^2 - Sqrt[5 + x^
4])/Sqrt[5]])/50
```

3.38.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1579, 539, 27, 534, 243, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+2}{x^5\sqrt{x^4+5}}dx$$

$$\Big\downarrow \text{1579}$$

$$\frac{1}{2}\int \frac{3x^2+2}{x^6\sqrt{x^4+5}}dx^2$$

$$\Big\downarrow \text{539}$$

$$\frac{1}{2}\left(-\frac{1}{10}\int -\frac{2(15-x^2)}{x^4\sqrt{x^4+5}}dx^2-\frac{\sqrt{x^4+5}}{5x^4}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{1}{5}\int \frac{15-x^2}{x^4\sqrt{x^4+5}}dx^2-\frac{\sqrt{x^4+5}}{5x^4}\right)$$

$$\Big\downarrow \text{534}$$

$$\frac{1}{2}\left(\frac{1}{5}\left(-\int \frac{1}{x^2\sqrt{x^4+5}}dx^2-\frac{3\sqrt{x^4+5}}{x^2}\right)-\frac{\sqrt{x^4+5}}{5x^4}\right)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\left(\frac{1}{5}\left(-\frac{1}{2}\int \frac{1}{x^2\sqrt{x^4+5}}dx^4-\frac{3\sqrt{x^4+5}}{x^2}\right)-\frac{\sqrt{x^4+5}}{5x^4}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{1}{5}\left(-\int \frac{1}{\sqrt{x^4+5}-5}d\sqrt{x^4+5}-\frac{3\sqrt{x^4+5}}{x^2}\right)-\frac{\sqrt{x^4+5}}{5x^4}\right)$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{2}\left(\frac{1}{5}\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{\sqrt{5}}-\frac{3\sqrt{x^4+5}}{x^2}\right)-\frac{\sqrt{x^4+5}}{5x^4}\right)$$

input
```
Int[(2 + 3*x^2)/(x^5*Sqrt[5 + x^4]),x]
```

output
```
(-1/5*Sqrt[5 + x^4]/x^4 + ((-3*Sqrt[5 + x^4])/x^2 + ArcTanh[Sqrt[5 + x^4]/
Sqrt[5]]/Sqrt[5])/5)/2
```

3.38.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

3.38.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.74

method	result	size
default	$-\frac{3\sqrt{x^4+5}}{10x^2}-\frac{\sqrt{x^4+5}}{10x^4}+\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{50}$	43
elliptic	$-\frac{3\sqrt{x^4+5}}{10x^2}-\frac{\sqrt{x^4+5}}{10x^4}+\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{50}$	43
risch	$-\frac{3x^6+x^4+15x^2+5}{10x^4\sqrt{x^4+5}}+\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{50}$	46
pseudoelliptic	$\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)x^4-15x^2\sqrt{x^4+5}-5\sqrt{x^4+5}}{50x^4}$	47
trager	$-\frac{(3x^2+1)\sqrt{x^4+5}}{10x^4}-\frac{\operatorname{RootOf}\left(\_Z^2-5\right)\ln\left(\frac{\sqrt{x^4+5}-\operatorname{RootOf}\left(\_Z^2-5\right)}{x^2}\right)}{50}$	50
meijerg	$\frac{\sqrt{5}\left(\frac{5\sqrt{\pi}\left(8+\frac{4x^4}{5}\right)}{8x^4}-\frac{5\sqrt{\pi}\sqrt{1+\frac{x^4}{5}}}{x^4}+\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{x^4}{5}}}{2}\right)-\frac{(1-2\ln(2)+4\ln(x)-\ln(5))\sqrt{\pi}}{2}-\frac{5\sqrt{\pi}}{x^4}\right)}{50\sqrt{\pi}}-\frac{3\sqrt{5}\sqrt{1+\frac{x^4}{5}}}{10x^2}$	105

input
```
int((3*x^2+2)/x^5/(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-3/10*(x^4+5)^(1/2)/x^2-1/10*(x^4+5)^(1/2)/x^4+1/50*5^(1/2)*arctanh(5^(1/2
)/(x^4+5)^(1/2))
```

3.38.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.86

$$\int \frac{2+3x^2}{x^5\sqrt{5+x^4}}\,dx = \frac{\sqrt{5}x^4\log\left(\frac{\sqrt{5}+\sqrt{x^4+5}}{x^2}\right)-15\,x^4-5\,\sqrt{x^4+5}(3\,x^2+1)}{50\,x^4}$$

input
```
integrate((3*x^2+2)/x^5/(x^4+5)^(1/2),x, algorithm="fricas")
```

output
```
1/50*(sqrt(5)*x^4*log((sqrt(5) + sqrt(x^4 + 5))/x^2) - 15*x^4 - 5*sqrt(x^4
 + 5)*(3*x^2 + 1))/x^4
```

3.38.6 Sympy [A] (verification not implemented)

Time = 3.95 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.52

$$\int \frac{2+3x^2}{x^5\sqrt{5+x^4}}\,dx = \frac{\sqrt{5}\left(-\frac{\log\left(\sqrt{\frac{x^4}{5}+1}-1\right)}{4}+\frac{\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{4}-\frac{1}{4\left(\sqrt{\frac{x^4}{5}+1}+1\right)}-\frac{1}{4\left(\sqrt{\frac{x^4}{5}+1}-1\right)}\right)}{25} - \frac{3\sqrt{5}\sqrt{5x^4+25}}{50x^2}$$

input
```
integrate((3*x**2+2)/x**5/(x**4+5)**(1/2),x)
```

output
```
sqrt(5)*(-log(sqrt(x**4/5 + 1) - 1)/4 + log(sqrt(x**4/5 + 1) + 1)/4 - 1/(4
*(sqrt(x**4/5 + 1) + 1)) - 1/(4*(sqrt(x**4/5 + 1) - 1)))/25 - 3*sqrt(5)*sq
rt(5*x**4 + 25)/(50*x**2)
```

3.38.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02

$$\int \frac{2+3x^2}{x^5\sqrt{5+x^4}}\,dx = -\frac{1}{100}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right) - \frac{3\sqrt{x^4+5}}{10\,x^2} - \frac{\sqrt{x^4+5}}{10\,x^4}$$

input
```
integrate((3*x^2+2)/x^5/(x^4+5)^(1/2),x, algorithm="maxima")
```

output
```
-1/100*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) -
 3/10*sqrt(x^4 + 5)/x^2 - 1/10*sqrt(x^4 + 5)/x^4
```

3.38.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(43) = 86$.

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.97

$$\int \frac{2+3x^2}{x^5\sqrt{5+x^4}}\,dx = -\frac{1}{50}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right) + \frac{\left(x^2-\sqrt{x^4+5}\right)^3+15\left(x^2-\sqrt{x^4+5}\right)^2+5\,x^2-5\sqrt{x^4+5}-75}{5\left(\left(x^2-\sqrt{x^4+5}\right)^2-5\right)^2}$$

input
```
integrate((3*x^2+2)/x^5/(x^4+5)^(1/2),x, algorithm="giac")
```

output
```
-1/50*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x
^4 + 5))) + 1/5*((x^2 - sqrt(x^4 + 5))^3 + 15*(x^2 - sqrt(x^4 + 5))^2 + 5*
x^2 - 5*sqrt(x^4 + 5) - 75)/((x^2 - sqrt(x^4 + 5))^2 - 5)^2
```

3.38.9 Mupad [B] (verification not implemented)

Time = 7.78 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.74

$$\int \frac{2+3x^2}{x^5\sqrt{5+x^4}}\,dx = \frac{\sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt{5}\sqrt{x^4+5}}{5}\right)}{50} - \frac{3\sqrt{x^4+5}}{10\,x^2} - \frac{\sqrt{x^4+5}}{10\,x^4}$$

input
```
int((3*x^2 + 2)/(x^5*(x^4 + 5)^(1/2)),x)
```

output
```
(5^(1/2)*atanh((5^(1/2)*(x^4 + 5)^(1/2))/5))/50 - (3*(x^4 + 5)^(1/2))/(10*
x^2) - (x^4 + 5)^(1/2)/(10*x^4)
```

3.39 $\int \frac{x^4(2+3x^2)}{\sqrt{5+x^4}}\,dx$

3.39.1 Optimal result

Integrand size = 20, antiderivative size = 185

$$\int \frac{x^4(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{2}{3}x\sqrt{5+x^4}+\frac{3}{5}x^3\sqrt{5+x^4}-\frac{9x\sqrt{5+x^4}}{\sqrt{5}+x^2} + \frac{9\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}} - \frac{\sqrt[4]{5}\left(27+2\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{6\sqrt{5+x^4}}$$

output
```
2/3*x*(x^4+5)^(1/2)+3/5*x^3*(x^4+5)^(1/2)-9*x*(x^4+5)^(1/2)/(x^2+5^(1/2))+
9*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4
)))*EllipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*((x^
4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)-1/6*5^(1/4)*(cos(2*arctan(1/5*x*
5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticF(sin(2*arctan(1/5
*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*(27+2*5^(1/2))*((x^4+5)/(x^2+5^(1/
2))^2)^(1/2)/(x^4+5)^(1/2)
```

3.39.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.40

$$\int \frac{x^4(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{1}{15}x\left(\left(10+9x^2\right)\sqrt{5+x^4}-10\sqrt{5}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{x^4}{5}\right)-9\sqrt{5}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{x^4}{5}\right)\right)$$

input
```
Integrate[(x^4*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

output
```
(x*((10 + 9*x^2)*Sqrt[5 + x^4] - 10*Sqrt[5]*Hypergeometric2F1[1/4, 1/2, 5/
4, -1/5*x^4] - 9*Sqrt[5]*x^2*Hypergeometric2F1[1/2, 3/4, 7/4, -1/5*x^4]))/
15
```

3.39.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1603, 27, 1603, 25, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(3x^2+2)}{\sqrt{x^4+5}}dx$$

$$\Big\downarrow \text{1603}$$

$$\frac{3}{5}x^3\sqrt{x^4+5}-\frac{1}{5}\int \frac{5x^2(9-2x^2)}{\sqrt{x^4+5}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{5}x^3\sqrt{x^4+5}-\int \frac{x^2(9-2x^2)}{\sqrt{x^4+5}}dx$$

$$\Big\downarrow \text{1603}$$

$$\frac{1}{3}\int -\frac{27x^2+10}{\sqrt{x^4+5}}dx+\frac{2}{3}\sqrt{x^4+5}x+\frac{3}{5}\sqrt{x^4+5}x^3$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{3}\int \frac{27x^2+10}{\sqrt{x^4+5}}dx+\frac{2}{3}\sqrt{x^4+5}x+\frac{3}{5}\sqrt{x^4+5}x^3$$

$$\Big\downarrow \text{ 1512}$$

$$\frac{1}{3}\left(27\sqrt{5}\int \frac{\sqrt{5}-x^2}{\sqrt{5}\sqrt{x^4+5}}dx-\left(10+27\sqrt{5}\right)\int \frac{1}{\sqrt{x^4+5}}dx\right)+\frac{2}{3}\sqrt{x^4+5}x+\frac{3}{5}\sqrt{x^4+5}x^3$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\left(27\int \frac{\sqrt{5}-x^2}{\sqrt{x^4+5}}dx-\left(10+27\sqrt{5}\right)\int \frac{1}{\sqrt{x^4+5}}dx\right)+\frac{2}{3}\sqrt{x^4+5}x+\frac{3}{5}\sqrt{x^4+5}x^3$$

$$\Big\downarrow \text{ 761}$$

$$\frac{1}{3}\left(27\int \frac{\sqrt{5}-x^2}{\sqrt{x^4+5}}dx-\frac{\left(10+27\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}\right)+$$
$$\frac{2}{3}\sqrt{x^4+5}x+\frac{3}{5}\sqrt{x^4+5}x^3$$

$$\Big\downarrow \text{ 1510}$$

$$\frac{1}{3}\left(27\left(\frac{\sqrt[4]{5}(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}}-\frac{x\sqrt{x^4+5}}{x^2+\sqrt{5}}\right)-\frac{\left(10+27\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\text{E}}{2\sqrt[4]{5}\sqrt{x^4}}\right.$$
$$\frac{2}{3}\sqrt{x^4+5}x+\frac{3}{5}\sqrt{x^4+5}x^3$$

input
```
Int[(x^4*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

output
```
(2*x*Sqrt[5 + x^4])/3 + (3*x^3*Sqrt[5 + x^4])/5 + (27*(-((x*Sqrt[5 + x^4])
/(Sqrt[5] + x^2)) + (5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2
)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4]) - ((10 + 27*Sqrt[
5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x
/5^(1/4)], 1/2])/(2*5^(1/4)*Sqrt[5 + x^4]))/3
```

3.39. $\int \frac{x^4(2+3x^2)}{\sqrt{5+x^4}}\,dx$

3.39.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

rule 1603
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
 1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ
[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[
m])
```

3.39.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 3.94 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.22

method	result
meijerg	$\frac{3\sqrt{5}\, x^{7} {}_{2}F_{1}\left(\frac{1}{2},\frac{7}{4};\frac{11}{4};-\frac{x^{4}}{5}\right)}{35}+\frac{2\sqrt{5}\, x^{5} {}_{2}F_{1}\left(\frac{1}{2},\frac{5}{4};\frac{9}{4};-\frac{x^{4}}{5}\right)}{25}$
risch	$\frac{x\left(9x^{2}+10\right)\sqrt{x^{4}+5}}{15}-\frac{2\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+5i\sqrt{5}\,x^{2}}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{15\sqrt{i\sqrt{5}}\,\sqrt{x^{4}+5}}-\frac{9i\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+5i\sqrt{5}\,x^{2}}\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-\right.}{5\sqrt{i\sqrt{5}}\,\sqrt{x^{4}+5}}$
default	$\frac{3x^{3}\sqrt{x^{4}+5}}{5}-\frac{9i\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+5i\sqrt{5}\,x^{2}}\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)\right)}{5\sqrt{i\sqrt{5}}\,\sqrt{x^{4}+5}}+\frac{2x\sqrt{x^{4}+5}}{3}-\frac{2\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+}}{15\sqrt{i\sqrt{5}}}$
elliptic	$\frac{3x^{3}\sqrt{x^{4}+5}}{5}-\frac{9i\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+5i\sqrt{5}\,x^{2}}\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)\right)}{5\sqrt{i\sqrt{5}}\,\sqrt{x^{4}+5}}+\frac{2x\sqrt{x^{4}+5}}{3}-\frac{2\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+}}{15\sqrt{i\sqrt{5}}}$

input
```
int(x^4*(3*x^2+2)/(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/35*5^(1/2)*x^7*hypergeom([1/2,7/4],[11/4],-1/5*x^4)+2/25*5^(1/2)*x^5*hyp
ergeom([1/2,5/4],[9/4],-1/5*x^4)
```

3.39.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.32

$$\int \frac{x^4(2+3x^2)}{\sqrt{5+x^4}}\,dx =$$
$$-\frac{135\,(-5)^{\frac{3}{4}}\,xE(\arcsin\left(\frac{(-5)^{\frac{1}{4}}}{x}\right)\,|-1)-125\,(-5)^{\frac{3}{4}}\,xF(\arcsin\left(\frac{(-5)^{\frac{1}{4}}}{x}\right)\,|-1)-(9\,x^4+10\,x^2-135)\sqrt{x^4}}{15\,x}$$

input
```
integrate(x^4*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="fricas")
```

output
```
-1/15*(135*(-5)^(3/4)*x*elliptic_e(arcsin((-5)^(1/4)/x), -1) - 125*(-5)^(3
/4)*x*elliptic_f(arcsin((-5)^(1/4)/x), -1) - (9*x^4 + 10*x^2 - 135)*sqrt(x
^4 + 5))/x
```

3.39.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.41

$$\int \frac{x^4(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{3\sqrt{5}x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{20\Gamma\left(\frac{11}{4}\right)} + \frac{\sqrt{5}x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{10\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x**4*(3*x**2+2)/(x**4+5)**(1/2),x)
```

output
```
3*sqrt(5)*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), x**4*exp_polar(I*pi)/
5)/(20*gamma(11/4)) + sqrt(5)*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), x*
*4*exp_polar(I*pi)/5)/(10*gamma(9/4))
```

3.39.7 Maxima [F]

$$\int \frac{x^4(2+3x^2)}{\sqrt{5+x^4}}\,dx = \int \frac{(3\,x^2+2)x^4}{\sqrt{x^4+5}}\,dx$$

input
```
integrate(x^4*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 2)*x^4/sqrt(x^4 + 5), x)
```

3.39.8 Giac [F]

$$\int \frac{x^4(2+3x^2)}{\sqrt{5+x^4}}\,dx = \int \frac{(3\,x^2+2)x^4}{\sqrt{x^4+5}}\,dx$$

input
```
integrate(x^4*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="giac")
```

output
```
integrate((3*x^2 + 2)*x^4/sqrt(x^4 + 5), x)
```

3.39.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(2+3x^2)}{\sqrt{5+x^4}}\,dx = \int \frac{x^4\,(3\,x^2+2)}{\sqrt{x^4+5}}\,dx$$

input
```
int((x^4*(3*x^2 + 2))/(x^4 + 5)^(1/2),x)
```

output
```
int((x^4*(3*x^2 + 2))/(x^4 + 5)^(1/2), x)
```

3.40 $\int \frac{x^2(2+3x^2)}{\sqrt{5+x^4}}\,dx$

3.40.1 Optimal result

Integrand size = 20, antiderivative size = 166

$$\int \frac{x^2(2+3x^2)}{\sqrt{5+x^4}}\,dx = x\sqrt{5+x^4}+\frac{2x\sqrt{5+x^4}}{\sqrt{5}+x^2} - \frac{2\sqrt[4]{5}(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}} + \frac{\sqrt[4]{5}(2-\sqrt{5})\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt{5+x^4}}$$

output
```
x*(x^4+5)^(1/2)+2*x*(x^4+5)^(1/2)/(x^2+5^(1/2))-2*5^(1/4)*(cos(2*arctan(1/
5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticE(sin(2*arctan
(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2
)/(x^4+5)^(1/2)+1/2*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*a
rctan(1/5*x*5^(3/4)))*EllipticF(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*
(2-5^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)
```

3.40.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.40

$$\int \frac{x^2(2+3x^2)}{\sqrt{5+x^4}}\,dx = x\sqrt{5+x^4}-\sqrt{5}x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{x^4}{5}\right) + \frac{2x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{x^4}{5}\right)}{3\sqrt{5}}$$

input
```
Integrate[(x^2*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

output
```
x*Sqrt[5 + x^4] - Sqrt[5]*x*Hypergeometric2F1[1/4, 1/2, 5/4, -1/5*x^4] + (
2*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, -1/5*x^4])/(3*Sqrt[5])
```

3.40.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1603, 27, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(3x^2+2)}{\sqrt{x^4+5}}\,dx$$

$$\Big\downarrow \text{ 1603}$$

$$x\sqrt{x^4+5}-\frac{1}{3}\int \frac{3(5-2x^2)}{\sqrt{x^4+5}}dx$$

$$\Big\downarrow \text{ 27}$$

$$x\sqrt{x^4+5}-\int \frac{5-2x^2}{\sqrt{x^4+5}}dx$$

$$\Big\downarrow \text{ 1512}$$

$$-\left(5-2\sqrt{5}\right)\int \frac{1}{\sqrt{x^4+5}}dx-2\sqrt{5}\int \frac{\sqrt{5}-x^2}{\sqrt{5}\sqrt{x^4+5}}dx+\sqrt{x^4+5}x$$

$$\Big\downarrow \text{ 27}$$

$$-\left(5-2\sqrt{5}\right)\int \frac{1}{\sqrt{x^4+5}}dx-2\int \frac{\sqrt{5}-x^2}{\sqrt{x^4+5}}dx+\sqrt{x^4+5}x$$

$$\Big\downarrow \text{761}$$

$$-2\int \frac{\sqrt{5}-x^2}{\sqrt{x^4+5}}dx-\frac{(5-2\sqrt{5})\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}+\sqrt{x^4+5}x$$

$$\Big\downarrow \text{1510}$$

$$-\frac{(5-2\sqrt{5})\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}-2\left(\frac{\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}}-\frac{x\sqrt{x^4+5}}{x^2+\sqrt{5}}\right)+\sqrt{x^4+5}x$$

input
```
Int[(x^2*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

output
```
x*Sqrt[5 + x^4] - 2*(-((x*Sqrt[5 + x^4])/(Sqrt[5] + x^2)) + (5^(1/4)*(Sqrt
[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)]
, 1/2])/Sqrt[5 + x^4]) - ((5 - 2*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(
Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(2*5^(1/4)*Sqrt[5 +
 x^4])
```

3.40.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

rule 1603
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
 1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ
[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[
m])
```

3.40.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.76 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.24

method	result
meijerg	$\frac{3\sqrt{5}\,x^{5}{}_{2}F_{1}\left(\frac{1}{2},\frac{5}{4};\frac{9}{4};-\frac{x^{4}}{5}\right)}{25}+\frac{2\sqrt{5}\,x^{3}{}_{2}F_{1}\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{x^{4}}{5}\right)}{15}$
default	$x\sqrt{x^{4}+5}-\frac{\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+5i\sqrt{5}\,x^{2}}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{5\sqrt{i\sqrt{5}}\,\sqrt{x^{4}+5}}+\frac{2i\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+5i\sqrt{5}\,x^{2}}\,\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}\cdots}{}\right)\right)}{5\sqrt{i\sqrt{5}}\,\sqrt{x^{4}+5}}$
risch	$x\sqrt{x^{4}+5}-\frac{\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+5i\sqrt{5}\,x^{2}}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{5\sqrt{i\sqrt{5}}\,\sqrt{x^{4}+5}}+\frac{2i\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+5i\sqrt{5}\,x^{2}}\,\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}\cdots}{}\right)\right)}{5\sqrt{i\sqrt{5}}\,\sqrt{x^{4}+5}}$
elliptic	$x\sqrt{x^{4}+5}-\frac{\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+5i\sqrt{5}\,x^{2}}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{5\sqrt{i\sqrt{5}}\,\sqrt{x^{4}+5}}+\frac{2i\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+5i\sqrt{5}\,x^{2}}\,\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}\cdots}{}\right)\right)}{5\sqrt{i\sqrt{5}}\,\sqrt{x^{4}+5}}$

input
```
int(x^2*(3*x^2+2)/(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/25*5^(1/2)*x^5*hypergeom([1/2,5/4],[9/4],-1/5*x^4)+2/15*5^(1/2)*x^3*hype
rgeom([1/2,3/4],[7/4],-1/5*x^4)
```

3.40.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.30

$$\int \frac{x^2(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{2\,(-5)^{\frac{3}{4}}\,xE(\arcsin\left(\frac{(-5)^{\frac{1}{4}}}{x}\right)\,|-1)-3\,(-5)^{\frac{3}{4}}\,xF(\arcsin\left(\frac{(-5)^{\frac{1}{4}}}{x}\right)\,|-1)+\sqrt{x^4+5}(x^2+2)}{x}$$

input
```
integrate(x^2*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="fricas")
```

output
```
(2*(-5)^(3/4)*x*elliptic_e(arcsin((-5)^(1/4)/x), -1) - 3*(-5)^(3/4)*x*elli
ptic_f(arcsin((-5)^(1/4)/x), -1) + sqrt(x^4 + 5)*(x^2 + 2))/x
```

3.40.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.45

$$\int \frac{x^2(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{3\sqrt{5}x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{20\Gamma\left(\frac{9}{4}\right)}+\frac{\sqrt{5}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{10\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x**2*(3*x**2+2)/(x**4+5)**(1/2),x)
```

output
```
3*sqrt(5)*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), x**4*exp_polar(I*pi)/5
)/(20*gamma(9/4)) + sqrt(5)*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), x**4
*exp_polar(I*pi)/5)/(10*gamma(7/4))
```

3.40.7 Maxima [F]

$$\int \frac{x^2(2+3x^2)}{\sqrt{5+x^4}}\,dx = \int \frac{(3\,x^2+2)x^2}{\sqrt{x^4+5}}\,dx$$

input
```
integrate(x^2*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 2)*x^2/sqrt(x^4 + 5), x)
```

3.40.8 Giac [F]

$$\int \frac{x^2(2+3x^2)}{\sqrt{5+x^4}}\,dx = \int \frac{(3\,x^2+2)x^2}{\sqrt{x^4+5}}\,dx$$

input
```
integrate(x^2*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="giac")
```

output
```
integrate((3*x^2 + 2)*x^2/sqrt(x^4 + 5), x)
```

3.40.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(2+3x^2)}{\sqrt{5+x^4}}\,dx = \int \frac{x^2\,(3\,x^2+2)}{\sqrt{x^4+5}}\,dx$$

input
```
int((x^2*(3*x^2 + 2))/(x^4 + 5)^(1/2),x)
```

output
```
int((x^2*(3*x^2 + 2))/(x^4 + 5)^(1/2), x)
```

3.41 $\int \frac{2+3x^2}{\sqrt{5+x^4}}\,dx$

3.41.1 Optimal result

Integrand size = 17, antiderivative size = 155

$$\int \frac{2+3x^2}{\sqrt{5+x^4}}\,dx=\frac{3x\sqrt{5+x^4}}{\sqrt{5}+x^2}-\frac{3\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}}+\frac{\left(2+3\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{5+x^4}}$$

output
```
3*x*(x^4+5)^(1/2)/(x^2+5^(1/2))-3*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)
^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(3/4)))
,1/2*2^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)+
1/10*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*E
llipticF(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*(2+3*5^(1
/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)*5^(3/4)/(x^4+5)^(1/2)
```

3.41.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.31

$$\int \frac{2+3x^2}{\sqrt{5+x^4}}\,dx = \frac{x\left(2\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{x^4}{5}\right)+x^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{x^4}{5}\right)\right)}{\sqrt{5}}$$

input
```
Integrate[(2 + 3*x^2)/Sqrt[5 + x^4],x]
```

output
```
(x*(2*Hypergeometric2F1[1/4, 1/2, 5/4, -1/5*x^4] + x^2*Hypergeometric2F1[1
/2, 3/4, 7/4, -1/5*x^4]))/Sqrt[5]
```

3.41.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+2}{\sqrt{x^4+5}}\,dx$$

$$\Big\downarrow \text{1512}$$

$$\left(2+3\sqrt{5}\right)\int \frac{1}{\sqrt{x^4+5}}dx-3\sqrt{5}\int \frac{\sqrt{5}-x^2}{\sqrt{5}\sqrt{x^4+5}}dx$$

$$\Big\downarrow \text{27}$$

$$\left(2+3\sqrt{5}\right)\int \frac{1}{\sqrt{x^4+5}}dx-3\int \frac{\sqrt{5}-x^2}{\sqrt{x^4+5}}dx$$

$$\Big\downarrow \text{761}$$

$$\frac{\left(2+3\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}-3\int \frac{\sqrt{5}-x^2}{\sqrt{x^4+5}}dx$$

$$\Big\downarrow \text{1510}$$

$$\frac{\left(2+3\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}-3\left(\frac{\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}}-\frac{x\sqrt{x^4+5}}{x^2+\sqrt{5}}\right)$$

input
```
Int[(2 + 3*x^2)/Sqrt[5 + x^4],x]
```

output
```
-3*(-((x*Sqrt[5 + x^4])/(Sqrt[5] + x^2)) + (5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(
5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 +
x^4]) + ((2 + 3*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]
*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(2*5^(1/4)*Sqrt[5 + x^4])
```

3.41.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

3.41.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.81 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.25

method	result	s
meijerg	$\frac{2\sqrt{5}\, x\, {}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{x^4}{5}\right)}{5}+\frac{\sqrt{5}\, x^3\, {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{x^4}{5}\right)}{5}$	3
default	$\frac{2\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{25\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{3i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)\right)}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$	1
elliptic	$\frac{2\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{25\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{3i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)\right)}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$	1

input
```
int((3*x^2+2)/(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*5^(1/2)*x*hypergeom([1/4,1/2],[5/4],-1/5*x^4)+1/5*5^(1/2)*x^3*hypergeo
m([1/2,3/4],[7/4],-1/5*x^4)
```

3.41.5 Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.30

$$\int \frac{2+3x^2}{\sqrt{5+x^4}}\,dx = \frac{15\,(-5)^{\frac{3}{4}}\,xE\left(\arcsin\left(\frac{(-5)^{\frac{1}{4}}}{x}\right)\,|-1\right)-13\,(-5)^{\frac{3}{4}}\,xF\left(\arcsin\left(\frac{(-5)^{\frac{1}{4}}}{x}\right)\,|-1\right)+15\sqrt{x^4+5}}{5\,x}$$

input
```
integrate((3*x^2+2)/(x^4+5)^(1/2),x, algorithm="fricas")
```

output
```
1/5*(15*(-5)^(3/4)*x*elliptic_e(arcsin((-5)^(1/4)/x), -1) - 13*(-5)^(3/4)*
x*elliptic_f(arcsin((-5)^(1/4)/x), -1) + 15*sqrt(x^4 + 5))/x
```

3.41.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.78 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.47

$$\int \frac{2+3x^2}{\sqrt{5+x^4}}\,dx = \frac{3\sqrt{5}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{20\Gamma\left(\frac{7}{4}\right)} + \frac{\sqrt{5}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{10\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((3*x**2+2)/(x**4+5)**(1/2),x)
```

output
```
3*sqrt(5)*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), x**4*exp_polar(I*pi)/5
)/(20*gamma(7/4)) + sqrt(5)*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), x**4*ex
p_polar(I*pi)/5)/(10*gamma(5/4))
```

3.41.7 Maxima [F]

$$\int \frac{2+3x^2}{\sqrt{5+x^4}}\,dx = \int \frac{3\,x^2+2}{\sqrt{x^4+5}}\,dx$$

input
```
integrate((3*x^2+2)/(x^4+5)^(1/2),x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 2)/sqrt(x^4 + 5), x)
```

3.41.8 Giac [F]

$$\int \frac{2+3x^2}{\sqrt{5+x^4}}\,dx = \int \frac{3\,x^2+2}{\sqrt{x^4+5}}\,dx$$

input
```
integrate((3*x^2+2)/(x^4+5)^(1/2),x, algorithm="giac")
```

output
```
integrate((3*x^2 + 2)/sqrt(x^4 + 5), x)
```

3.41.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x^2}{\sqrt{5+x^4}}\,dx = \int \frac{3\,x^2+2}{\sqrt{x^4+5}}\,dx$$

input
```
int((3*x^2 + 2)/(x^4 + 5)^(1/2),x)
```

output
```
int((3*x^2 + 2)/(x^4 + 5)^(1/2), x)
```

3.42 $\int \frac{2+3x^2}{x^2\sqrt{5+x^4}}\,dx$

3.42.1 Optimal result

Integrand size = 20, antiderivative size = 173

$$\int \frac{2+3x^2}{x^2\sqrt{5+x^4}}\,dx = -\frac{2\sqrt{5+x^4}}{5x}+\frac{2x\sqrt{5+x^4}}{5\left(\sqrt{5}+x^2\right)} - \frac{2\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{5^{3/4}\sqrt{5+x^4}} + \frac{\left(2+3\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\,5^{3/4}\sqrt{5+x^4}}$$

output
```
-2/5*(x^4+5)^(1/2)/x+2/5*x*(x^4+5)^(1/2)/(x^2+5^(1/2))-2/5*5^(1/4)*(cos(2*
arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticE(sin
(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2)
)^2)^(1/2)/(x^4+5)^(1/2)+1/10*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2
*arctan(1/5*x*5^(3/4)))*EllipticF(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2)
)*(x^2+5^(1/2))*(2+3*5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)*5^(1/4)/(x^4
+5)^(1/2)
```

3.42.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.31

$$\int \frac{2+3x^2}{x^2\sqrt{5+x^4}}\,dx = -\frac{2\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-\frac{x^4}{5}\right)}{\sqrt{5}x} + \frac{3x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{x^4}{5}\right)}{\sqrt{5}}$$

input
```
Integrate[(2 + 3*x^2)/(x^2*Sqrt[5 + x^4]),x]
```

output
```
(-2*Hypergeometric2F1[-1/4, 1/2, 3/4, -1/5*x^4])/(Sqrt[5]*x) + (3*x*Hyperg
eometric2F1[1/4, 1/2, 5/4, -1/5*x^4])/Sqrt[5]
```

3.42.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1605, 25, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+2}{x^2\sqrt{x^4+5}}dx$$

$$\Big\downarrow \text{ 1605}$$

$$-\frac{1}{5}\int -\frac{2x^2+15}{\sqrt{x^4+5}}dx-\frac{2\sqrt{x^4+5}}{5x}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{5}\int \frac{2x^2+15}{\sqrt{x^4+5}}dx-\frac{2\sqrt{x^4+5}}{5x}$$

$$\Big\downarrow \text{ 1512}$$

$$\frac{1}{5}\left(\left(15+2\sqrt{5}\right)\int \frac{1}{\sqrt{x^4+5}}dx-2\sqrt{5}\int \frac{\sqrt{5}-x^2}{\sqrt{5}\sqrt{x^4+5}}dx\right)-\frac{2\sqrt{x^4+5}}{5x}$$

$$\Big\downarrow 27$$

$$\frac{1}{5}\left(\left(15+2\sqrt{5}\right)\int \frac{1}{\sqrt{x^4+5}}dx-2\int \frac{\sqrt{5}-x^2}{\sqrt{x^4+5}}dx\right)-\frac{2\sqrt{x^4+5}}{5x}$$

$$\Big\downarrow 761$$

$$\frac{1}{5}\left(\frac{\left(15+2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}-2\int \frac{\sqrt{5}-x^2}{\sqrt{x^4+5}}dx\right)-\frac{2\sqrt{x^4+5}}{5x}$$

$$\Big\downarrow 1510$$

$$\frac{1}{5}\left(\frac{\left(15+2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}-2\left(\frac{\sqrt[4]{5}(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\,\text{arcta}\right.}{\sqrt{x^4+5}}\right.\right.$$

$$\frac{2\sqrt{x^4+5}}{5x}$$

input
```
Int[(2 + 3*x^2)/(x^2*Sqrt[5 + x^4]),x]
```

output
```
(-2*Sqrt[5 + x^4])/(5*x) + (-2*(-((x*Sqrt[5 + x^4])/(Sqrt[5] + x^2)) + (5^
(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan
[x/5^(1/4)], 1/2])/Sqrt[5 + x^4]) + ((15 + 2*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt
[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(2*5^(1
/4)*Sqrt[5 + x^4]))/5
```

3.42.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

rule 1605
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + S
imp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*
(m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
 IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.42.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.32 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.22

method	result
meijerg	$-\frac{2\sqrt{5}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{x^4}{5}\right)}{5x}+\frac{3\sqrt{5}\,x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{x^4}{5}\right)}{5}$
default	$\frac{3\sqrt{5}\sqrt{25-5i\sqrt{5}\,x^2}\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)}{25\sqrt{i\sqrt{5}}\sqrt{x^4+5}}-\frac{2\sqrt{x^4+5}}{5x}+\frac{2i\sqrt{25-5i\sqrt{5}\,x^2}\sqrt{25+5i\sqrt{5}\,x^2}\left(F\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}\sqrt{\cdots}}{5}\cdots\right.\right.}{25\sqrt{i\sqrt{5}}\sqrt{x^4+5}}$
risch	$\frac{3\sqrt{5}\sqrt{25-5i\sqrt{5}\,x^2}\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)}{25\sqrt{i\sqrt{5}}\sqrt{x^4+5}}-\frac{2\sqrt{x^4+5}}{5x}+\frac{2i\sqrt{25-5i\sqrt{5}\,x^2}\sqrt{25+5i\sqrt{5}\,x^2}\left(F\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}\sqrt{\cdots}}{5}\cdots\right.\right.}{25\sqrt{i\sqrt{5}}\sqrt{x^4+5}}$
elliptic	$\frac{3\sqrt{5}\sqrt{25-5i\sqrt{5}\,x^2}\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)}{25\sqrt{i\sqrt{5}}\sqrt{x^4+5}}-\frac{2\sqrt{x^4+5}}{5x}+\frac{2i\sqrt{25-5i\sqrt{5}\,x^2}\sqrt{25+5i\sqrt{5}\,x^2}\left(F\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}\sqrt{\cdots}}{5}\cdots\right.\right.}{25\sqrt{i\sqrt{5}}\sqrt{x^4+5}}$

input
```
int((3*x^2+2)/x^2/(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*5^(1/2)/x*hypergeom([-1/4,1/2],[3/4],-1/5*x^4)+3/5*5^(1/2)*x*hypergeo
m([1/4,1/2],[5/4],-1/5*x^4)
```

3.42.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.42

$$\int \frac{2+3x^2}{x^2\sqrt{5+x^4}}\,dx = \frac{-2i\sqrt{5}x\sqrt{i\sqrt{5}}E(\arcsin\left(\frac{1}{5}\sqrt{5}x\sqrt{i\sqrt{5}}\right)|-1) - 13i\sqrt{5}x\sqrt{i\sqrt{5}}F(\arcsin\left(\frac{1}{5}\sqrt{5}x\sqrt{i\sqrt{5}}\right)|-1) - 10\sqrt{x^{4}}}{25\,x}$$

input
```
integrate((3*x^2+2)/x^2/(x^4+5)^(1/2),x, algorithm="fricas")
```

output
```
1/25*(-2*I*sqrt(5)*x*sqrt(I*sqrt(5))*elliptic_e(arcsin(1/5*sqrt(5)*x*sqrt(
I*sqrt(5))), -1) - 13*I*sqrt(5)*x*sqrt(I*sqrt(5))*elliptic_f(arcsin(1/5*sq
rt(5)*x*sqrt(I*sqrt(5))), -1) - 10*sqrt(x^4 + 5))/x
```

3.42.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.43

$$\int \frac{2+3x^2}{x^2\sqrt{5+x^4}}\,dx = \frac{3\sqrt{5}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\right|\frac{x^4e^{i\pi}}{5}\right)}{20\Gamma\left(\frac{5}{4}\right)} + \frac{\sqrt{5}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{4},\frac{1}{2}\\\frac{3}{4}\end{matrix}\right|\frac{x^4e^{i\pi}}{5}\right)}{10x\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((3*x**2+2)/x**2/(x**4+5)**(1/2),x)
```

output
```
3*sqrt(5)*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), x**4*exp_polar(I*pi)/5)/(
20*gamma(5/4)) + sqrt(5)*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), x**4*exp_p
olar(I*pi)/5)/(10*x*gamma(3/4))
```

3.42.7 Maxima [F]

$$\int \frac{2+3x^2}{x^2\sqrt{5+x^4}}\,dx = \int \frac{3\,x^2+2}{\sqrt{x^4+5}x^2}\,dx$$

input
```
integrate((3*x^2+2)/x^2/(x^4+5)^(1/2),x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 2)/(sqrt(x^4 + 5)*x^2), x)
```

3.42.8 Giac [F]

$$\int \frac{2+3x^2}{x^2\sqrt{5+x^4}}\,dx = \int \frac{3\,x^2+2}{\sqrt{x^4+5}x^2}\,dx$$

input
```
integrate((3*x^2+2)/x^2/(x^4+5)^(1/2),x, algorithm="giac")
```

output
```
integrate((3*x^2 + 2)/(sqrt(x^4 + 5)*x^2), x)
```

3.42.9 Mupad [B] (verification not implemented)

Time = 8.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.28

$$\int \frac{2+3x^2}{x^2\sqrt{5+x^4}}\,dx = \frac{3\sqrt{5}\,x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ -\frac{x^4}{5}\right)}{5} - \frac{2\sqrt{\frac{5}{x^4}+1}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\ \frac{7}{4};\ -\frac{5}{x^4}\right)}{3\,x\,\sqrt{x^4+5}}$$

input
```
int((3*x^2 + 2)/(x^2*(x^4 + 5)^(1/2)),x)
```

output
```
(3*5^(1/2)*x*hypergeom([1/4, 1/2], 5/4, -x^4/5))/5 - (2*(5/x^4 + 1)^(1/2)*
hypergeom([1/2, 3/4], 7/4, -5/x^4))/(3*x*(x^4 + 5)^(1/2))
```

3.43 $\int \frac{2+3x^2}{x^4\sqrt{5+x^4}}\,dx$

3.43.1 Optimal result

Integrand size = 20, antiderivative size = 189

$$\int \frac{2+3x^2}{x^4\sqrt{5+x^4}}\,dx = -\frac{2\sqrt{5+x^4}}{15x^3}-\frac{3\sqrt{5+x^4}}{5x}+\frac{3x\sqrt{5+x^4}}{5\left(\sqrt{5}+x^2\right)} - \frac{3\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{5^{3/4}\sqrt{5+x^4}} - \frac{\left(2-9\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{30\sqrt[4]{5}\sqrt{5+x^4}}$$

output
```
-2/15*(x^4+5)^(1/2)/x^3-3/5*(x^4+5)^(1/2)/x+3/5*x*(x^4+5)^(1/2)/(x^2+5^(1/
2))-3/5*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*
5^(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2)
)*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)-1/150*(cos(2*arctan(1/5*x*
5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticF(sin(2*arctan(1/5
*x*5^(3/4))),1/2*2^(1/2))*(2-9*5^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2
))^2)^(1/2)*5^(3/4)/(x^4+5)^(1/2)
```

3.43.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.29

$$\int \frac{2+3x^2}{x^4\sqrt{5+x^4}}\,dx = -\frac{2\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},-\frac{x^4}{5}\right)+9x^2\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-\frac{x^4}{5}\right)}{3\sqrt{5}x^3}$$

input
```
Integrate[(2 + 3*x^2)/(x^4*Sqrt[5 + x^4]),x]
```

output
```
-1/3*(2*Hypergeometric2F1[-3/4, 1/2, 1/4, -1/5*x^4] + 9*x^2*Hypergeometric
2F1[-1/4, 1/2, 3/4, -1/5*x^4])/(Sqrt[5]*x^3)
```

3.43.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1605, 25, 1605, 27, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+2}{x^4\sqrt{x^4+5}}\,dx$$

$$\Big\downarrow \text{1605}$$

$$-\frac{1}{15}\int -\frac{45-2x^2}{x^2\sqrt{x^4+5}}dx-\frac{2\sqrt{x^4+5}}{15x^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{15}\int \frac{45-2x^2}{x^2\sqrt{x^4+5}}dx-\frac{2\sqrt{x^4+5}}{15x^3}$$

$$\Big\downarrow \text{1605}$$

$$\frac{1}{15}\left(-\frac{1}{5}\int \frac{5(2-9x^2)}{\sqrt{x^4+5}}dx-\frac{9\sqrt{x^4+5}}{x}\right)-\frac{2\sqrt{x^4+5}}{15x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{15}\left(-\int \frac{2-9x^2}{\sqrt{x^4+5}}dx-\frac{9\sqrt{x^4+5}}{x}\right)-\frac{2\sqrt{x^4+5}}{15x^3}$$

$$\Big\downarrow \text{1512}$$

$$\frac{1}{15}\left(-\left(\left(2-9\sqrt{5}\right)\int \frac{1}{\sqrt{x^4+5}}dx\right)-9\sqrt{5}\int \frac{\sqrt{5}-x^2}{\sqrt{5}\sqrt{x^4+5}}dx-\frac{9\sqrt{x^4+5}}{x}\right)-\frac{2\sqrt{x^4+5}}{15x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{15}\left(-\left(\left(2-9\sqrt{5}\right)\int \frac{1}{\sqrt{x^4+5}}dx\right)-9\int \frac{\sqrt{5}-x^2}{\sqrt{x^4+5}}dx-\frac{9\sqrt{x^4+5}}{x}\right)-\frac{2\sqrt{x^4+5}}{15x^3}$$

$$\Big\downarrow \text{761}$$

$$\frac{1}{15}\left(-9\int \frac{\sqrt{5}-x^2}{\sqrt{x^4+5}}dx-\frac{\left(2-9\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}-\frac{9\sqrt{x^4+5}}{x}\right)-\frac{2\sqrt{x^4+5}}{15x^3}$$

$$\Big\downarrow \text{1510}$$

$$\frac{1}{15}\left(-\frac{\left(2-9\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}-9\left(\frac{\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\,\text{arc}\right.}{\sqrt{x^4+5}}\right.\right.$$

$$\frac{2\sqrt{x^4+5}}{15x^3}$$

input
```
Int[(2 + 3*x^2)/(x^4*Sqrt[5 + x^4]),x]
```

output
```
(-2*Sqrt[5 + x^4])/(15*x^3) + ((-9*Sqrt[5 + x^4])/x - 9*(-((x*Sqrt[5 + x^4
])/(Sqrt[5] + x^2)) + (5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x
^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4]) - ((2 - 9*Sqrt[
5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x
/5^(1/4)], 1/2])/(2*5^(1/4)*Sqrt[5 + x^4]))/15
```

3.43. $\int \frac{2+3x^2}{x^4\sqrt{5+x^4}}\,dx$

3.43.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

rule 1605
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + S
imp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*
(m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
 IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.43.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.99 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.21

method	result
meijerg	$-\frac{2\sqrt{5}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};-\frac{x^4}{5}\right)}{15x^3}-\frac{3\sqrt{5}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{x^4}{5}\right)}{5x}$
default	$-\frac{3\sqrt{x^4+5}}{5x}+\frac{3i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)\right)}{25\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}-\frac{2\sqrt{x^4+5}}{15x^3}-\frac{2\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5}}{375\sqrt{i\sqrt{5}}}$
elliptic	$-\frac{3\sqrt{x^4+5}}{5x}+\frac{3i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)\right)}{25\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}-\frac{2\sqrt{x^4+5}}{15x^3}-\frac{2\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5}}{375\sqrt{i\sqrt{5}}}$
risch	$-\frac{9x^6+2x^4+45x^2+10}{15x^3\sqrt{x^4+5}}-\frac{2\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{375\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{3i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right.\right.}{25\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$

input
```
int((3*x^2+2)/x^4/(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*5^(1/2)/x^3*hypergeom([-3/4,1/2],[1/4],-1/5*x^4)-3/5*5^(1/2)/x*hyper
geom([-1/4,1/2],[3/4],-1/5*x^4)
```

3.43.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.44

$$\int \frac{2+3x^2}{x^4\sqrt{5+x^4}}\,dx$$
$$=\frac{-9i\sqrt{5}x^3\sqrt{i\sqrt{5}}E(\arcsin\left(\frac{1}{5}\sqrt{5}x\sqrt{i\sqrt{5}}\right)\mid-1)+11i\sqrt{5}x^3\sqrt{i\sqrt{5}}F(\arcsin\left(\frac{1}{5}\sqrt{5}x\sqrt{i\sqrt{5}}\right)\mid-1)-5\sqrt{x}}{75\,x^3}$$

input
```
integrate((3*x^2+2)/x^4/(x^4+5)^(1/2),x, algorithm="fricas")
```

output
```
1/75*(-9*I*sqrt(5)*x^3*sqrt(I*sqrt(5))*elliptic_e(arcsin(1/5*sqrt(5)*x*sqr
t(I*sqrt(5))), -1) + 11*I*sqrt(5)*x^3*sqrt(I*sqrt(5))*elliptic_f(arcsin(1/
5*sqrt(5)*x*sqrt(I*sqrt(5))), -1) - 5*sqrt(x^4 + 5)*(9*x^2 + 2))/x^3
```

3.43.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.99 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.42

$$\int \frac{2+3x^2}{x^4\sqrt{5+x^4}}\,dx = \frac{3\sqrt{5}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{20x\Gamma\left(\frac{3}{4}\right)} + \frac{\sqrt{5}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{10x^3\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate((3*x**2+2)/x**4/(x**4+5)**(1/2),x)
```

output
```
3*sqrt(5)*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), x**4*exp_polar(I*pi)/5)/(
20*x*gamma(3/4)) + sqrt(5)*gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), x**4*exp
_polar(I*pi)/5)/(10*x**3*gamma(1/4))
```

3.43.7 Maxima [F]

$$\int \frac{2+3x^2}{x^4\sqrt{5+x^4}}\,dx = \int \frac{3\,x^2+2}{\sqrt{x^4+5}x^4}\,dx$$

input
```
integrate((3*x^2+2)/x^4/(x^4+5)^(1/2),x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 2)/(sqrt(x^4 + 5)*x^4), x)
```

3.43.8 Giac [F]

$$\int \frac{2+3x^2}{x^4\sqrt{5+x^4}}\,dx = \int \frac{3\,x^2+2}{\sqrt{x^4+5}x^4}\,dx$$

input
```
integrate((3*x^2+2)/x^4/(x^4+5)^(1/2),x, algorithm="giac")
```

output
```
integrate((3*x^2 + 2)/(sqrt(x^4 + 5)*x^4), x)
```

3.43.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x^2}{x^4\sqrt{5+x^4}}\,dx = \int \frac{3\,x^2+2}{x^4\,\sqrt{x^4+5}}\,dx$$

input
```
int((3*x^2 + 2)/(x^4*(x^4 + 5)^(1/2)),x)
```

output
```
int((3*x^2 + 2)/(x^4*(x^4 + 5)^(1/2)), x)
```

3.44 $\int \frac{x^7(2+3x^2)}{(5+x^4)^{3/2}}\,dx$

3.44.1 Optimal result

Integrand size = 20, antiderivative size = 58

$$\int \frac{x^7(2+3x^2)}{(5+x^4)^{3/2}}\,dx = -\frac{x^4(2+3x^2)}{2\sqrt{5+x^4}}+\frac{1}{4}(8+9x^2)\sqrt{5+x^4}-\frac{45}{4}\operatorname{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)$$

output
```
-45/4*arcsinh(1/5*x^2*5^(1/2))-1/2*x^4*(3*x^2+2)/(x^4+5)^(1/2)+1/4*(9*x^2+
8)*(x^4+5)^(1/2)
```

3.44.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88

$$\int \frac{x^7(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \frac{40+45x^2+4x^4+3x^6}{4\sqrt{5+x^4}}+\frac{45}{4}\log\left(-x^2+\sqrt{5+x^4}\right)$$

input
```
Integrate[(x^7*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

output
```
(40 + 45*x^2 + 4*x^4 + 3*x^6)/(4*Sqrt[5 + x^4]) + (45*Log[-x^2 + Sqrt[5 +
x^4]])/4
```

3.44.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1579, 530, 27, 2346, 455, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7(3x^2+2)}{(x^4+5)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1579}$$

$$\frac{1}{2}\int \frac{x^6(3x^2+2)}{(x^4+5)^{3/2}}dx^2$$

$$\Big\downarrow \text{ 530}$$

$$\frac{1}{2}\left(\frac{5(3x^2+2)}{\sqrt{x^4+5}}-\frac{1}{5}\int \frac{5(-3x^4-2x^2+15)}{\sqrt{x^4+5}}dx^2\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(\frac{5(3x^2+2)}{\sqrt{x^4+5}}-\int \frac{-3x^4-2x^2+15}{\sqrt{x^4+5}}dx^2\right)$$

$$\Big\downarrow \text{ 2346}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{45-4x^2}{\sqrt{x^4+5}}dx^2+\frac{3}{2}\sqrt{x^4+5}x^2+\frac{5(3x^2+2)}{\sqrt{x^4+5}}\right)$$

$$\Big\downarrow \text{ 455}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(4\sqrt{x^4+5}-45\int \frac{1}{\sqrt{x^4+5}}dx^2\right)+\frac{3}{2}\sqrt{x^4+5}x^2+\frac{5(3x^2+2)}{\sqrt{x^4+5}}\right)$$

$$\Big\downarrow \text{ 222}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(4\sqrt{x^4+5}-45\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)\right)+\frac{3}{2}\sqrt{x^4+5}x^2+\frac{5(3x^2+2)}{\sqrt{x^4+5}}\right)$$

input
```
Int[(x^7*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

output
```
((5*(2 + 3*x^2))/Sqrt[5 + x^4] + (3*x^2*Sqrt[5 + x^4])/2 + (4*Sqrt[5 + x^4
] - 45*ArcSinh[x^2/Sqrt[5]])/2)/2
```

3.44.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 530
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Co
eff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Po
lynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x
)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(a
 + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Qx + e*(2*p + 3), x], x], x]] /;
FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && EqQ[n,
1] && IntegerQ[2*p]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

3.44. $\int \frac{x^7(2+3x^2)}{(5+x^4)^{3/2}}\,dx$

3.44.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.67

method	result	size
risch	$\frac{3x^6+4x^4+45x^2+40}{4\sqrt{x^4+5}}-\frac{45\,\text{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{4}$	39
trager	$\frac{3x^6+4x^4+45x^2+40}{4\sqrt{x^4+5}}+\frac{45\ln\left(x^2-\sqrt{x^4+5}\right)}{4}$	44
pseudoelliptic	$\frac{3x^6+4x^4-45\,\text{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)\sqrt{x^4+5}+45x^2+40}{4\sqrt{x^4+5}}$	45
default	$\frac{3x^6}{4\sqrt{x^4+5}}+\frac{45x^2}{4\sqrt{x^4+5}}-\frac{45\,\text{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{4}+\frac{x^4+10}{\sqrt{x^4+5}}$	50
elliptic	$\frac{3x^6}{4\sqrt{x^4+5}}+\frac{45x^2}{4\sqrt{x^4+5}}-\frac{45\,\text{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{4}+\frac{x^4}{\sqrt{x^4+5}}+\frac{10}{\sqrt{x^4+5}}$	57
meijerg	$\frac{\frac{3\sqrt{\pi}\,x^2\sqrt{5}\,\left(x^4+15\right)}{20\sqrt{1+\frac{x^4}{5}}}-\frac{45\sqrt{\pi}\,\text{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{4}}{\sqrt{\pi}}+\frac{\sqrt{5}\,\left(-2\sqrt{\pi}+\frac{\sqrt{\pi}\,\left(8+\frac{4x^4}{5}\right)}{4\sqrt{1+\frac{x^4}{5}}}\right)}{\sqrt{\pi}}$	81

input
```
int(x^7*(3*x^2+2)/(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(3*x^6+4*x^4+45*x^2+40)/(x^4+5)^(1/2)-45/4*arcsinh(1/5*x^2*5^(1/2))
```

3.44.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.07

$$\int \frac{x^7(2+3x^2)}{(5+x^4)^{3/2}}\,dx=\frac{30\,x^4+45\,(x^4+5)\log\left(-x^2+\sqrt{x^4+5}\right)+(3\,x^6+4\,x^4+45\,x^2+40)\sqrt{x^4+5}+150}{4\,(x^4+5)}$$

input
```
integrate(x^7*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="fricas")
```

output
```
1/4*(30*x^4 + 45*(x^4 + 5)*log(-x^2 + sqrt(x^4 + 5)) + (3*x^6 + 4*x^4 + 45
*x^2 + 40)*sqrt(x^4 + 5) + 150)/(x^4 + 5)
```

3.44.6 Sympy [A] (verification not implemented)

Time = 6.47 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.14

$$\int \frac{x^7(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \frac{3x^6}{4\sqrt{x^4+5}} + \frac{x^4}{\sqrt{x^4+5}} + \frac{45x^2}{4\sqrt{x^4+5}} - \frac{45\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{4} + \frac{10}{\sqrt{x^4+5}}$$

input
```
integrate(x**7*(3*x**2+2)/(x**4+5)**(3/2),x)
```

output
```
3*x**6/(4*sqrt(x**4 + 5)) + x**4/sqrt(x**4 + 5) + 45*x**2/(4*sqrt(x**4 + 5
)) - 45*asinh(sqrt(5)*x**2/5)/4 + 10/sqrt(x**4 + 5)
```

3.44.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.53

$$\int \frac{x^7(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \sqrt{x^4+5} - \frac{15\left(\frac{3\,(x^4+5)}{x^4} - 2\right)}{4\left(\frac{\sqrt{x^4+5}}{x^2} - \frac{(x^4+5)^{\frac{3}{2}}}{x^6}\right)} + \frac{5}{\sqrt{x^4+5}} - \frac{45}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2} + 1\right) + \frac{45}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2} - 1\right)$$

input
```
integrate(x^7*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="maxima")
```

output
```
sqrt(x^4 + 5) - 15/4*(3*(x^4 + 5)/x^4 - 2)/(sqrt(x^4 + 5)/x^2 - (x^4 + 5)^
(3/2)/x^6) + 5/sqrt(x^4 + 5) - 45/8*log(sqrt(x^4 + 5)/x^2 + 1) + 45/8*log(
sqrt(x^4 + 5)/x^2 - 1)
```

3.44.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.78

$$\int \frac{x^7(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \frac{((3\,x^2+4)x^2+45)x^2+40}{4\sqrt{x^4+5}} + \frac{45}{4}\log\left(-x^2+\sqrt{x^4+5}\right)$$

input
```
integrate(x^7*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="giac")
```

output
```
1/4*(((3*x^2 + 4)*x^2 + 45)*x^2 + 40)/sqrt(x^4 + 5) + 45/4*log(-x^2 + sqrt
(x^4 + 5))
```

3.44.9 Mupad [B] (verification not implemented)

Time = 8.23 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.67

$$\int \frac{x^7(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \sqrt{x^4+5}\left(\frac{3\,x^2}{4}+1\right)-\frac{45\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{4}+\frac{\sqrt{5}\,\left(10+\sqrt{5}\,15\mathrm{i}\right)\,\sqrt{x^4+5}\,1\mathrm{i}}{20\,\left(-x^2+\sqrt{5}\,1\mathrm{i}\right)}-\frac{\sqrt{5}\,\left(-10+\sqrt{5}\,15\mathrm{i}\right)\,\sqrt{x^4+5}\,1\mathrm{i}}{20\,\left(x^2+\sqrt{5}\,1\mathrm{i}\right)}$$

input
```
int((x^7*(3*x^2 + 2))/(x^4 + 5)^(3/2),x)
```

output
```
(x^4 + 5)^(1/2)*((3*x^2)/4 + 1) - (45*asinh((5^(1/2)*x^2)/5))/4 + (5^(1/2)
*(5^(1/2)*15i + 10)*(x^4 + 5)^(1/2)*1i)/(20*(5^(1/2)*1i - x^2)) - (5^(1/2)
*(5^(1/2)*15i - 10)*(x^4 + 5)^(1/2)*1i)/(20*(5^(1/2)*1i + x^2))
```

3.45 $\int \frac{x^5(2+3x^2)}{(5+x^4)^{3/2}}\,dx$

3.45.1 Optimal result

Integrand size = 20, antiderivative size = 45

$$\int \frac{x^5(2+3x^2)}{(5+x^4)^{3/2}}\,dx = -\frac{x^2(2+3x^2)}{2\sqrt{5+x^4}}+3\sqrt{5+x^4}+\operatorname{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)$$

output
```
arcsinh(1/5*x^2*5^(1/2))-1/2*x^2*(3*x^2+2)/(x^4+5)^(1/2)+3*(x^4+5)^(1/2)
```

3.45.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{x^5(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \frac{30-2x^2+3x^4}{2\sqrt{5+x^4}}-\log\left(-x^2+\sqrt{5+x^4}\right)$$

input
```
Integrate[(x^5*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

output
```
(30 - 2*x^2 + 3*x^4)/(2*Sqrt[5 + x^4]) - Log[-x^2 + Sqrt[5 + x^4]]
```

3.45.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1579, 530, 27, 455, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(3x^2+2)}{(x^4+5)^{3/2}}dx$$

$$\Big\downarrow \text{ 1579}$$

$$\frac{1}{2}\int \frac{x^4(3x^2+2)}{(x^4+5)^{3/2}}dx^2$$

$$\Big\downarrow \text{ 530}$$

$$\frac{1}{2}\left(\frac{15-2x^2}{\sqrt{x^4+5}}-\frac{1}{5}\int -\frac{5(3x^2+2)}{\sqrt{x^4+5}}dx^2\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(\int \frac{3x^2+2}{\sqrt{x^4+5}}dx^2+\frac{15-2x^2}{\sqrt{x^4+5}}\right)$$

$$\Big\downarrow \text{ 455}$$

$$\frac{1}{2}\left(2\int \frac{1}{\sqrt{x^4+5}}dx^2+3\sqrt{x^4+5}+\frac{15-2x^2}{\sqrt{x^4+5}}\right)$$

$$\Big\downarrow \text{ 222}$$

$$\frac{1}{2}\left(2\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)+3\sqrt{x^4+5}+\frac{15-2x^2}{\sqrt{x^4+5}}\right)$$

input
```
Int[(x^5*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

output
```
((15 - 2*x^2)/Sqrt[5 + x^4] + 3*Sqrt[5 + x^4] + 2*ArcSinh[x^2/Sqrt[5]])/2
```

3.45.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 530
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Co
eff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Po
lynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x
)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(a
 + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Qx + e*(2*p + 3), x], x], x]] /;
FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && EqQ[n,
1] && IntegerQ[2*p]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

3.45.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.71

method	result	size
risch	$\frac{3x^4-2x^2+30}{2\sqrt{x^4+5}}+\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)$	32
trager	$\frac{3x^4-2x^2+30}{2\sqrt{x^4+5}}+\ln\left(x^2+\sqrt{x^4+5}\right)$	35
default	$\frac{\frac{3x^4}{2}+15}{\sqrt{x^4+5}}-\frac{x^2}{\sqrt{x^4+5}}+\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)$	37
pseudoelliptic	$\frac{3x^4+2\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)\sqrt{x^4+5}-2x^2+30}{2\sqrt{x^4+5}}$	40
elliptic	$\frac{3x^4}{2\sqrt{x^4+5}}+\frac{15}{\sqrt{x^4+5}}-\frac{x^2}{\sqrt{x^4+5}}+\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)$	44
meijerg	$\frac{3\sqrt{5}\left(-2\sqrt{\pi}+\frac{\sqrt{\pi}\left(8+\frac{4x^4}{5}\right)}{4\sqrt{1+\frac{x^4}{5}}}\right)}{2\sqrt{\pi}}+\frac{-\frac{\sqrt{\pi}\,x^2\sqrt{5}}{5\sqrt{1+\frac{x^4}{5}}}+\sqrt{\pi}\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{\sqrt{\pi}}$	75

input
```
int(x^5*(3*x^2+2)/(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(3*x^4-2*x^2+30)/(x^4+5)^(1/2)+arcsinh(1/5*x^2*5^(1/2))
```

3.45.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.29

$$\int \frac{x^5(2+3x^2)}{(5+x^4)^{3/2}}\,dx = -\frac{2\,x^4+2\,(x^4+5)\log\left(-x^2+\sqrt{x^4+5}\right)-(3\,x^4-2\,x^2+30)\sqrt{x^4+5}+10}{2\,(x^4+5)}$$

input
```
integrate(x^5*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(2*x^4 + 2*(x^4 + 5)*log(-x^2 + sqrt(x^4 + 5)) - (3*x^4 - 2*x^2 + 30)
*sqrt(x^4 + 5) + 10)/(x^4 + 5)
```

3.45.6 Sympy [A] (verification not implemented)

Time = 5.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.07

$$\int \frac{x^5(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \frac{3x^4}{2\sqrt{x^4+5}} - \frac{x^2}{\sqrt{x^4+5}} + \operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right) + \frac{15}{\sqrt{x^4+5}}$$

input
```
integrate(x**5*(3*x**2+2)/(x**4+5)**(3/2),x)
```

output
```
3*x**4/(2*sqrt(x**4 + 5)) - x**2/sqrt(x**4 + 5) + asinh(sqrt(5)*x**2/5) +
15/sqrt(x**4 + 5)
```

3.45.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.40

$$\int \frac{x^5(2+3x^2)}{(5+x^4)^{3/2}}\,dx = -\frac{x^2}{\sqrt{x^4+5}} + \frac{3}{2}\sqrt{x^4+5} + \frac{15}{2\sqrt{x^4+5}} + \frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right) - \frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

input
```
integrate(x^5*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="maxima")
```

output
```
-x^2/sqrt(x^4 + 5) + 3/2*sqrt(x^4 + 5) + 15/2/sqrt(x^4 + 5) + 1/2*log(sqrt
(x^4 + 5)/x^2 + 1) - 1/2*log(sqrt(x^4 + 5)/x^2 - 1)
```

3.45.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \frac{x^5(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \frac{(3\,x^2-2)x^2+30}{2\sqrt{x^4+5}} - \log\left(-x^2+\sqrt{x^4+5}\right)$$

input
```
integrate(x^5*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="giac")
```

output
```
1/2*((3*x^2 - 2)*x^2 + 30)/sqrt(x^4 + 5) - log(-x^2 + sqrt(x^4 + 5))
```

3.45.9 Mupad [B] (verification not implemented)

Time = 8.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.98

$$\int \frac{x^5(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \operatorname{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right) + \frac{3\sqrt{x^4+5}}{2} - \frac{\sqrt{5}\left(-15+\sqrt{5}\,2\mathrm{i}\right)\sqrt{x^4+5}\,1\mathrm{i}}{20\left(-x^2+\sqrt{5}\,1\mathrm{i}\right)} + \frac{\sqrt{5}\left(15+\sqrt{5}\,2\mathrm{i}\right)\sqrt{x^4+5}\,1\mathrm{i}}{20\left(x^2+\sqrt{5}\,1\mathrm{i}\right)}$$

input
```
int((x^5*(3*x^2 + 2))/(x^4 + 5)^(3/2),x)
```

output
```
asinh((5^(1/2)*x^2)/5) + (3*(x^4 + 5)^(1/2))/2 - (5^(1/2)*(5^(1/2)*2i - 15
)*(x^4 + 5)^(1/2)*1i)/(20*(5^(1/2)*1i - x^2)) + (5^(1/2)*(5^(1/2)*2i + 15)
*(x^4 + 5)^(1/2)*1i)/(20*(5^(1/2)*1i + x^2))
```

3.46 $\int \frac{x^3(2+3x^2)}{(5+x^4)^{3/2}}\,dx$

3.46.1 Optimal result

Integrand size = 20, antiderivative size = 35

$$\int \frac{x^3(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \frac{-2-3x^2}{2\sqrt{5+x^4}} + \frac{3}{2}\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)$$

output
```
3/2*arcsinh(1/5*x^2*5^(1/2))+1/2*(-3*x^2-2)/(x^4+5)^(1/2)
```

3.46.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

$$\int \frac{x^3(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \frac{-2-3x^2}{2\sqrt{5+x^4}} - \frac{3}{2}\log\left(-x^2+\sqrt{5+x^4}\right)$$

input
```
Integrate[(x^3*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

output
```
(-2 - 3*x^2)/(2*Sqrt[5 + x^4]) - (3*Log[-x^2 + Sqrt[5 + x^4]])/2
```

3.46.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1579, 530, 27, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(3x^2+2)}{(x^4+5)^{3/2}}\,dx$$
$$\Big\downarrow \; 1579$$
$$\frac{1}{2}\int \frac{x^2(3x^2+2)}{(x^4+5)^{3/2}}dx^2$$
$$\Big\downarrow \; 530$$
$$\frac{1}{2}\left(-\frac{1}{5}\int -\frac{15}{\sqrt{x^4+5}}dx^2-\frac{3x^2+2}{\sqrt{x^4+5}}\right)$$
$$\Big\downarrow \; 27$$
$$\frac{1}{2}\left(3\int \frac{1}{\sqrt{x^4+5}}dx^2-\frac{3x^2+2}{\sqrt{x^4+5}}\right)$$
$$\Big\downarrow \; 222$$
$$\frac{1}{2}\left(3\text{arcsinh}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{3x^2+2}{\sqrt{x^4+5}}\right)$$

input
```
Int[(x^3*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

output
```
(-((2 + 3*x^2)/Sqrt[5 + x^4]) + 3*ArcSinh[x^2/Sqrt[5]])/2
```

3.46.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 530
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Co
eff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Po
lynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x
)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(a
 + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Qx + e*(2*p + 3), x], x], x]] /;
FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && EqQ[n,
1] && IntegerQ[2*p]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

3.46.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

method	result	size
risch	$-\frac{3x^{2}+2}{2\sqrt{x^{4}+5}}+\frac{3 \operatorname{arcsinh}\left(\frac{x^{2}\sqrt{5}}{5}\right)}{2}$	29
trager	$-\frac{3x^{2}+2}{2\sqrt{x^{4}+5}}+\frac{3\ln\left(x^{2}+\sqrt{x^{4}+5}\right)}{2}$	32
pseudoelliptic	$-\frac{3\left(x^{2}-\operatorname{arcsinh}\left(\frac{x^{2}\sqrt{5}}{5}\right)\sqrt{x^{4}+5}+\frac{2}{3}\right)}{2\sqrt{x^{4}+5}}$	33
default	$-\frac{3x^{2}}{2\sqrt{x^{4}+5}}+\frac{3 \operatorname{arcsinh}\left(\frac{x^{2}\sqrt{5}}{5}\right)}{2}-\frac{1}{\sqrt{x^{4}+5}}$	34
elliptic	$-\frac{3x^{2}}{2\sqrt{x^{4}+5}}+\frac{3 \operatorname{arcsinh}\left(\frac{x^{2}\sqrt{5}}{5}\right)}{2}-\frac{1}{\sqrt{x^{4}+5}}$	34
meijerg	$\frac{-\frac{3\sqrt{\pi}\,x^{2}\sqrt{5}}{10\sqrt{1+\frac{x^{4}}{5}}}+\frac{3\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{x^{2}\sqrt{5}}{5}\right)}{2}}{\sqrt{\pi}}+\frac{\sqrt{5}\left(\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{1+\frac{x^{4}}{5}}}\right)}{5\sqrt{\pi}}$	67

input
```
int(x^3*(3*x^2+2)/(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(3*x^2+2)/(x^4+5)^(1/2)+3/2*arcsinh(1/5*x^2*5^(1/2))
```

3.46.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.49

$$\int \frac{x^3(2+3x^2)}{(5+x^4)^{3/2}}\,dx = -\frac{3\,x^4+3\,(x^4+5)\log\left(-x^2+\sqrt{x^4+5}\right)+\sqrt{x^4+5}(3\,x^2+2)+15}{2\,(x^4+5)}$$

input
```
integrate(x^3*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(3*x^4 + 3*(x^4 + 5)*log(-x^2 + sqrt(x^4 + 5)) + sqrt(x^4 + 5)*(3*x^2
 + 2) + 15)/(x^4 + 5)
```

3.46.6 Sympy [A] (verification not implemented)

Time = 4.34 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{x^3(2+3x^2)}{(5+x^4)^{3/2}}\,dx = -\frac{3x^2}{2\sqrt{x^4+5}}+\frac{3\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{2}-\frac{1}{\sqrt{x^4+5}}$$

input
```
integrate(x**3*(3*x**2+2)/(x**4+5)**(3/2),x)
```

output
```
-3*x**2/(2*sqrt(x**4 + 5)) + 3*asinh(sqrt(5)*x**2/5)/2 - 1/sqrt(x**4 + 5)
```

3.46.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.54

$$\int \frac{x^3(2+3x^2)}{(5+x^4)^{3/2}}\,dx = -\frac{3\,x^2}{2\,\sqrt{x^4+5}}-\frac{1}{\sqrt{x^4+5}}+\frac{3}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{3}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

input
```
integrate(x^3*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="maxima")
```

output
```
-3/2*x^2/sqrt(x^4 + 5) - 1/sqrt(x^4 + 5) + 3/4*log(sqrt(x^4 + 5)/x^2 + 1)
- 3/4*log(sqrt(x^4 + 5)/x^2 - 1)
```

3.46.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{x^3(2+3x^2)}{(5+x^4)^{3/2}}\,dx = -\frac{3\,x^2+2}{2\,\sqrt{x^4+5}} - \frac{3}{2}\,\log\left(-x^2+\sqrt{x^4+5}\right)$$

input
```
integrate(x^3*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="giac")
```

output
```
-1/2*(3*x^2 + 2)/sqrt(x^4 + 5) - 3/2*log(-x^2 + sqrt(x^4 + 5))
```

3.46.9 Mupad [B] (verification not implemented)

Time = 8.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.34

$$\int \frac{x^3(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \frac{3\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{2} - \frac{\sqrt{5}\,\left(2+\sqrt{5}\,3\mathrm{i}\right)\,\sqrt{x^4+5}\,1\mathrm{i}}{20\,\left(-x^2+\sqrt{5}\,1\mathrm{i}\right)} + \frac{\sqrt{5}\,\left(-2+\sqrt{5}\,3\mathrm{i}\right)\,\sqrt{x^4+5}\,1\mathrm{i}}{20\,\left(x^2+\sqrt{5}\,1\mathrm{i}\right)}$$

input
```
int((x^3*(3*x^2 + 2))/(x^4 + 5)^(3/2),x)
```

output
```
(3*asinh((5^(1/2)*x^2)/5))/2 - (5^(1/2)*(5^(1/2)*3i + 2)*(x^4 + 5)^(1/2)*1
i)/(20*(5^(1/2)*1i - x^2)) + (5^(1/2)*(5^(1/2)*3i - 2)*(x^4 + 5)^(1/2)*1i)
/(20*(5^(1/2)*1i + x^2))
```

3.47 $\int \frac{x(2+3x^2)}{(5+x^4)^{3/2}}\,dx$

3.47.1 Optimal result

Integrand size = 18, antiderivative size = 20

$$\int \frac{x(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \frac{-15+2x^2}{10\sqrt{5+x^4}}$$

output
```
1/10*(2*x^2-15)/(x^4+5)^(1/2)
```

3.47.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \frac{-15+2x^2}{10\sqrt{5+x^4}}$$

input
```
Integrate[(x*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

output
```
(-15 + 2*x^2)/(10*Sqrt[5 + x^4])
```

3.47.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1577, 453}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(3x^2+2)}{(x^4+5)^{3/2}}dx$$
$$\Big\downarrow \text{1577}$$
$$\frac{1}{2}\int \frac{3x^2+2}{(x^4+5)^{3/2}}dx^2$$
$$\Big\downarrow \text{453}$$
$$-\frac{15-2x^2}{10\sqrt{x^4+5}}$$

input
```
Int[(x*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

output
```
-1/10*(15 - 2*x^2)/Sqrt[5 + x^4]
```

3.47.3.1 Defintions of rubi rules used

rule 453
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-(a*
d - b*c*x)/(a*b*Sqrt[a + b*x^2]), x] /; FreeQ[{a, b, c, d}, x]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

3.47.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
gosper	$\frac{2x^2-15}{10\sqrt{x^4+5}}$	17
trager	$\frac{2x^2-15}{10\sqrt{x^4+5}}$	17
risch	$\frac{2x^2-15}{10\sqrt{x^4+5}}$	17
elliptic	$\frac{2x^2-15}{10\sqrt{x^4+5}}$	17
pseudoelliptic	$\frac{2x^2-15}{10\sqrt{x^4+5}}$	17
default	$\frac{x^2}{5\sqrt{x^4+5}}-\frac{3}{2\sqrt{x^4+5}}$	23
meijerg	$\frac{3\sqrt{5}\left(\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{1+\frac{x^4}{5}}}\right)}{10\sqrt{\pi}}+\frac{\sqrt{5}\,x^2}{25\sqrt{1+\frac{x^4}{5}}}$	45

input
```
int(x*(3*x^2+2)/(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/10*(2*x^2-15)/(x^4+5)^(1/2)
```

3.47.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \frac{x(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \frac{2\,x^4+\sqrt{x^4+5}(2\,x^2-15)+10}{10\,(x^4+5)}$$

input
```
integrate(x*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="fricas")
```

output
```
1/10*(2*x^4 + sqrt(x^4 + 5)*(2*x^2 - 15) + 10)/(x^4 + 5)
```

3.47.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(15) = 30.

Time = 3.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \frac{x(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \frac{\sqrt{5}x^2}{5\sqrt{5x^4+25}} - \frac{3}{2\sqrt{x^4+5}}$$

input
```
integrate(x*(3*x**2+2)/(x**4+5)**(3/2),x)
```

output
```
sqrt(5)*x**2/(5*sqrt(5*x**4 + 25)) - 3/(2*sqrt(x**4 + 5))
```

3.47.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \frac{x^2}{5\sqrt{x^4+5}} - \frac{3}{2\sqrt{x^4+5}}$$

input
```
integrate(x*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="maxima")
```

output
```
1/5*x^2/sqrt(x^4 + 5) - 3/2/sqrt(x^4 + 5)
```

3.47.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{x(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \frac{2\,x^2-15}{10\sqrt{x^4+5}}$$

input
```
integrate(x*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="giac")
```

output
```
1/10*(2*x^2 - 15)/sqrt(x^4 + 5)
```

3.47.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{x(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \frac{2\,x^2-15}{10\,\sqrt{x^4+5}}$$

input
```
int((x*(3*x^2 + 2))/(x^4 + 5)^(3/2),x)
```

output
```
(2*x^2 - 15)/(10*(x^4 + 5)^(1/2))
```

3.48 $\int \frac{2+3x^2}{x\left(5+x^4\right)^{3/2}}\,dx$

3.48.1 Optimal result

Integrand size = 20, antiderivative size = 46

$$\int \frac{2+3x^2}{x\left(5+x^4\right)^{3/2}}\,dx=\frac{2+3x^2}{10\sqrt{5+x^4}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{5\sqrt{5}}$$

output
```
-1/25*arctanh(1/5*(x^4+5)^(1/2)*5^(1/2))*5^(1/2)+1/10*(3*x^2+2)/(x^4+5)^(1
/2)
```

3.48.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int \frac{2+3x^2}{x\left(5+x^4\right)^{3/2}}\,dx=\frac{1}{50}\left(\frac{5(2+3x^2)}{\sqrt{5+x^4}}+4\sqrt{5}\operatorname{arctanh}\left(\frac{x^2-\sqrt{5+x^4}}{\sqrt{5}}\right)\right)$$

input
```
Integrate[(2 + 3*x^2)/(x*(5 + x^4)^(3/2)),x]
```

output
```
((5*(2 + 3*x^2))/Sqrt[5 + x^4] + 4*Sqrt[5]*ArcTanh[(x^2 - Sqrt[5 + x^4])/S
qrt[5]])/50
```

3.48.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1579, 532, 27, 243, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+2}{x\left(x^4+5\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{1579}$$

$$\frac{1}{2}\int \frac{3x^2+2}{x^2\left(x^4+5\right)^{3/2}}dx^2$$

$$\Big\downarrow \text{532}$$

$$\frac{1}{2}\left(\frac{3x^2+2}{5\sqrt{x^4+5}}-\frac{1}{5}\int -\frac{2}{x^2\sqrt{x^4+5}}dx^2\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{2}{5}\int \frac{1}{x^2\sqrt{x^4+5}}dx^2+\frac{3x^2+2}{5\sqrt{x^4+5}}\right)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\left(\frac{1}{5}\int \frac{1}{x^2\sqrt{x^4+5}}dx^4+\frac{3x^2+2}{5\sqrt{x^4+5}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{2}{5}\int \frac{1}{\sqrt{x^4+5}-5}d\sqrt{x^4+5}+\frac{3x^2+2}{5\sqrt{x^4+5}}\right)$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{2}\left(\frac{3x^2+2}{5\sqrt{x^4+5}}-\frac{2\text{arctanh}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{5\sqrt{5}}\right)$$

input
```
Int[(2 + 3*x^2)/(x*(5 + x^4)^(3/2)),x]
```

output
```
((2 + 3*x^2)/(5*Sqrt[5 + x^4]) - (2*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/(5*Sqr
t[5]))/2
```

3.48.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

3.48.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{3x^2+2}{10\sqrt{x^4+5}}-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{25}$	35
default	$\frac{3x^2}{10\sqrt{x^4+5}}+\frac{1}{5\sqrt{x^4+5}}-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{25}$	40
elliptic	$\frac{3x^2}{10\sqrt{x^4+5}}+\frac{1}{5\sqrt{x^4+5}}-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{25}$	40
pseudoelliptic	$\frac{-2\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)\sqrt{x^4+5}+15x^2+10}{50\sqrt{x^4+5}}$	41
trager	$\frac{3x^2+2}{10\sqrt{x^4+5}}+\frac{\operatorname{RootOf}\left(\_Z^2-5\right)\ln\left(\frac{\sqrt{x^4+5}-\operatorname{RootOf}\left(\_Z^2-5\right)}{x^2}\right)}{25}$	47
meijerg	$\frac{\sqrt{5}\left(-\sqrt{\pi}+\frac{\sqrt{\pi}}{\sqrt{1+\frac{x^4}{5}}}-\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{x^4}{5}}}{2}\right)+\frac{(2-2\ln(2)+4\ln(x)-\ln(5))\sqrt{\pi}}{2}\right)}{25\sqrt{\pi}}+\frac{3\sqrt{5}\,x^2}{50\sqrt{1+\frac{x^4}{5}}}$	84

input
```
int((3*x^2+2)/x/(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/10*(3*x^2+2)/(x^4+5)^(1/2)-1/25*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.33

$$\int \frac{2+3x^2}{x\,(5+x^4)^{3/2}}\,dx = \frac{15\,x^4+2\,\sqrt{5}(x^4+5)\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)+5\,\sqrt{x^4+5}(3\,x^2+2)+75}{50\,(x^4+5)}$$

input
```
integrate((3*x^2+2)/x/(x^4+5)^(3/2),x, algorithm="fricas")
```

output
```
1/50*(15*x^4 + 2*sqrt(5)*(x^4 + 5)*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) + 5
*sqrt(x^4 + 5)*(3*x^2 + 2) + 75)/(x^4 + 5)
```

3.48.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(41) = 82.

Time = 7.46 (sec) , antiderivative size = 212, normalized size of antiderivative = 4.61

$$\int \frac{2+3x^2}{x\left(5+x^4\right)^{3/2}}\,dx = \frac{2x^4\log\left(x^4\right)}{20\sqrt{5}x^4+100\sqrt{5}} - \frac{4x^4\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{20\sqrt{5}x^4+100\sqrt{5}} - \frac{2x^4\log\left(5\right)}{20\sqrt{5}x^4+100\sqrt{5}} + \frac{3x^2}{10\sqrt{x^4+5}} + \frac{4\sqrt{5}\sqrt{x^4+5}}{20\sqrt{5}x^4+100\sqrt{5}} + \frac{10\log\left(x^4\right)}{20\sqrt{5}x^4+100\sqrt{5}} - \frac{20\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{20\sqrt{5}x^4+100\sqrt{5}} - \frac{10\log\left(5\right)}{20\sqrt{5}x^4+100\sqrt{5}}$$

input
```
integrate((3*x**2+2)/x/(x**4+5)**(3/2),x)
```

output
```
2*x**4*log(x**4)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 4*x**4*log(sqrt(x**4/5
+ 1) + 1)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 2*x**4*log(5)/(20*sqrt(5)*x**4
 + 100*sqrt(5)) + 3*x**2/(10*sqrt(x**4 + 5)) + 4*sqrt(5)*sqrt(x**4 + 5)/(2
0*sqrt(5)*x**4 + 100*sqrt(5)) + 10*log(x**4)/(20*sqrt(5)*x**4 + 100*sqrt(5
)) - 20*log(sqrt(x**4/5 + 1) + 1)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 10*log
(5)/(20*sqrt(5)*x**4 + 100*sqrt(5))
```

3.48.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.22

$$\int \frac{2+3x^2}{x\left(5+x^4\right)^{3/2}}\,dx = \frac{3\,x^2}{10\,\sqrt{x^4+5}} + \frac{1}{50}\,\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right) + \frac{1}{5\,\sqrt{x^4+5}}$$

input
```
integrate((3*x^2+2)/x/(x^4+5)^(3/2),x, algorithm="maxima")
```

output
```
3/10*x^2/sqrt(x^4 + 5) + 1/50*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt
(5) + sqrt(x^4 + 5))) + 1/5/sqrt(x^4 + 5)
```

3.48.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.33

$$\int \frac{2+3x^2}{x\left(5+x^4\right)^{3/2}}\,dx = \frac{1}{25}\sqrt{5}\log\left(x^2+\sqrt{5}-\sqrt{x^4+5}\right) - \frac{1}{25}\sqrt{5}\log\left(-x^2+\sqrt{5}+\sqrt{x^4+5}\right) + \frac{3\,x^2+2}{10\,\sqrt{x^4+5}}$$

input
```
integrate((3*x^2+2)/x/(x^4+5)^(3/2),x, algorithm="giac")
```

output
```
1/25*sqrt(5)*log(x^2 + sqrt(5) - sqrt(x^4 + 5)) - 1/25*sqrt(5)*log(-x^2 +
sqrt(5) + sqrt(x^4 + 5)) + 1/10*(3*x^2 + 2)/sqrt(x^4 + 5)
```

3.48.9 Mupad [B] (verification not implemented)

Time = 7.97 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{2+3x^2}{x\left(5+x^4\right)^{3/2}}\,dx = \frac{1}{5\,\sqrt{x^4+5}} - \frac{\sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}}{5}\right)}{25} + \frac{3\,x^2}{10\,\sqrt{x^4+5}}$$

input
```
int((3*x^2 + 2)/(x*(x^4 + 5)^(3/2)),x)
```

output
```
1/(5*(x^4 + 5)^(1/2)) - (5^(1/2)*atanh((5^(1/2)*(x^4 + 5)^(1/2))/5))/25 +
(3*x^2)/(10*(x^4 + 5)^(1/2))
```

3.49 $\int \frac{2+3x^2}{x^3\left(5+x^4\right)^{3/2}}\,dx$

3.49.1 Optimal result

Integrand size = 20, antiderivative size = 65

$$\int \frac{2+3x^2}{x^3\left(5+x^4\right)^{3/2}}\,dx=\frac{2+3x^2}{10x^2\sqrt{5+x^4}}-\frac{2\sqrt{5+x^4}}{25x^2}-\frac{3\text{arctanh}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{10\sqrt{5}}$$

output
```
-3/50*arctanh(1/5*(x^4+5)^(1/2)*5^(1/2))*5^(1/2)+1/10*(3*x^2+2)/x^2/(x^4+5
)^(1/2)-2/25*(x^4+5)^(1/2)/x^2
```

3.49.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.91

$$\int \frac{2+3x^2}{x^3\left(5+x^4\right)^{3/2}}\,dx=\frac{1}{50}\left(\frac{-10+15x^2-4x^4}{x^2\sqrt{5+x^4}}+6\sqrt{5}\text{arctanh}\left(\frac{x^2-\sqrt{5+x^4}}{\sqrt{5}}\right)\right)$$

input
```
Integrate[(2 + 3*x^2)/(x^3*(5 + x^4)^(3/2)),x]
```

output
```
((-10 + 15*x^2 - 4*x^4)/(x^2*Sqrt[5 + x^4]) + 6*Sqrt[5]*ArcTanh[(x^2 - Sqr
t[5 + x^4])/Sqrt[5]])/50
```

3.49.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1579, 532, 25, 534, 243, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+2}{x^3 \left(x^4+5\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{1579}$$

$$\frac{1}{2} \int \frac{3x^2+2}{x^4 \left(x^4+5\right)^{3/2}}dx^2$$

$$\Big\downarrow \text{532}$$

$$\frac{1}{2} \left(\frac{15-2x^2}{25\sqrt{x^4+5}}-\frac{1}{5} \int -\frac{3x^2+2}{x^4\sqrt{x^4+5}}dx^2\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2} \left(\frac{1}{5} \int \frac{3x^2+2}{x^4\sqrt{x^4+5}}dx^2+\frac{15-2x^2}{25\sqrt{x^4+5}}\right)$$

$$\Big\downarrow \text{534}$$

$$\frac{1}{2} \left(\frac{1}{5} \left(3 \int \frac{1}{x^2\sqrt{x^4+5}}dx^2-\frac{2\sqrt{x^4+5}}{5x^2}\right)+\frac{15-2x^2}{25\sqrt{x^4+5}}\right)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2} \left(\frac{1}{5} \left(\frac{3}{2} \int \frac{1}{x^2\sqrt{x^4+5}}dx^4-\frac{2\sqrt{x^4+5}}{5x^2}\right)+\frac{15-2x^2}{25\sqrt{x^4+5}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2} \left(\frac{1}{5} \left(3 \int \frac{1}{\sqrt{x^4+5}-5}d\sqrt{x^4+5}-\frac{2\sqrt{x^4+5}}{5x^2}\right)+\frac{15-2x^2}{25\sqrt{x^4+5}}\right)$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{2} \left(\frac{1}{5} \left(-\frac{3\text{arctanh}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{\sqrt{5}}-\frac{2\sqrt{x^4+5}}{5x^2}\right)+\frac{15-2x^2}{25\sqrt{x^4+5}}\right)$$

input
```
Int[(2 + 3*x^2)/(x^3*(5 + x^4)^(3/2)),x]
```

output
```
((15 - 2*x^2)/(25*Sqrt[5 + x^4]) + ((-2*Sqrt[5 + x^4])/(5*x^2) - (3*ArcTan
h[Sqrt[5 + x^4]/Sqrt[5]])/Sqrt[5])/5)/2
```

3.49.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

3.49.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.66

method	result	size
risch	$-\frac{4x^4-15x^2+10}{50x^2\sqrt{x^4+5}}-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{50}$	43
default	$\frac{3}{10\sqrt{x^4+5}}-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{50}-\frac{2x^4+5}{25x^2\sqrt{x^4+5}}$	47
elliptic	$-\frac{1}{5x^2\sqrt{x^4+5}}-\frac{2x^2}{25\sqrt{x^4+5}}+\frac{3}{10\sqrt{x^4+5}}-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{50}$	52
pseudoelliptic	$\frac{-3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)x^2\sqrt{x^4+5}-4x^4+15x^2-10}{50x^2\sqrt{x^4+5}}$	52
trager	$-\frac{4x^4-15x^2+10}{50x^2\sqrt{x^4+5}}+\frac{3\,\operatorname{RootOf}\left(\_Z^2-5\right)\ln\left(\frac{\sqrt{x^4+5}-\operatorname{RootOf}\left(\_Z^2-5\right)}{x^2}\right)}{50}$	55
meijerg	$-\frac{\sqrt{5}\left(1+\frac{2x^4}{5}\right)}{25x^2\sqrt{1+\frac{x^4}{5}}}+\frac{3\sqrt{5}\left(-\sqrt{\pi}+\frac{\sqrt{\pi}}{\sqrt{1+\frac{x^4}{5}}}-\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{x^4}{5}}}{2}\right)+\frac{(2-2\ln(2)+4\ln(x)-\ln(5))\sqrt{\pi}}{2}\right)}{50\sqrt{\pi}}$	91

input
```
int((3*x^2+2)/x^3/(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/50*(4*x^4-15*x^2+10)/x^2/(x^4+5)^(1/2)-3/50*5^(1/2)*arctanh(5^(1/2)/(x^
4+5)^(1/2))
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.18

$$\int \frac{2+3x^2}{x^3 (5+x^4)^{3/2}}\,dx = -\frac{4\,x^6 - 3\,\sqrt{5}(x^6+5\,x^2)\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right) + 20\,x^2 + (4\,x^4 - 15\,x^2 + 10)\sqrt{x^4+5}}{50\,(x^6+5\,x^2)}$$

input
```
integrate((3*x^2+2)/x^3/(x^4+5)^(3/2),x, algorithm="fricas")
```

output
```
-1/50*(4*x^6 - 3*sqrt(5)*(x^6 + 5*x^2)*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2)
 + 20*x^2 + (4*x^4 - 15*x^2 + 10)*sqrt(x^4 + 5))/(x^6 + 5*x^2)
```

3.49.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(61) = 122.

Time = 4.62 (sec) , antiderivative size = 228, normalized size of antiderivative = 3.51

$$\begin{aligned}\int \frac{2+3x^2}{x^3 (5+x^4)^{3/2}}\,dx &= \frac{3x^4\log(x^4)}{20\sqrt{5}x^4+100\sqrt{5}} - \frac{6x^4\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{20\sqrt{5}x^4+100\sqrt{5}} \\ &\quad - \frac{3x^4\log(5)}{20\sqrt{5}x^4+100\sqrt{5}} + \frac{6\sqrt{5}\sqrt{x^4+5}}{20\sqrt{5}x^4+100\sqrt{5}} + \frac{15\log(x^4)}{20\sqrt{5}x^4+100\sqrt{5}} \\ &\quad - \frac{30\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{20\sqrt{5}x^4+100\sqrt{5}} - \frac{15\log(5)}{20\sqrt{5}x^4+100\sqrt{5}} - \frac{2}{25\sqrt{1+\frac{5}{x^4}}} - \frac{1}{5x^4\sqrt{1+\frac{5}{x^4}}}\end{aligned}$$

input
```
integrate((3*x**2+2)/x**3/(x**4+5)**(3/2),x)
```

output
```
3*x**4*log(x**4)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 6*x**4*log(sqrt(x**4/5
+ 1) + 1)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 3*x**4*log(5)/(20*sqrt(5)*x**4
 + 100*sqrt(5)) + 6*sqrt(5)*sqrt(x**4 + 5)/(20*sqrt(5)*x**4 + 100*sqrt(5))
 + 15*log(x**4)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 30*log(sqrt(x**4/5 + 1)
+ 1)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 15*log(5)/(20*sqrt(5)*x**4 + 100*sq
rt(5)) - 2/(25*sqrt(1 + 5/x**4)) - 1/(5*x**4*sqrt(1 + 5/x**4))
```

3.49.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05

$$\int \frac{2+3x^2}{x^3(5+x^4)^{3/2}}\,dx = -\frac{x^2}{25\sqrt{x^4+5}} + \frac{3}{100}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right) + \frac{3}{10\sqrt{x^4+5}} - \frac{\sqrt{x^4+5}}{25\,x^2}$$

input
```
integrate((3*x^2+2)/x^3/(x^4+5)^(3/2),x, algorithm="maxima")
```

output
```
-1/25*x^2/sqrt(x^4 + 5) + 3/100*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sq
rt(5) + sqrt(x^4 + 5))) + 3/10/sqrt(x^4 + 5) - 1/25*sqrt(x^4 + 5)/x^2
```

3.49.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.26

$$\int \frac{2+3x^2}{x^3(5+x^4)^{3/2}}\,dx = \frac{3}{50}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right) - \frac{2\,x^2-15}{50\sqrt{x^4+5}} + \frac{2}{5\left(\left(x^2-\sqrt{x^4+5}\right)^2-5\right)}$$

input
```
integrate((3*x^2+2)/x^3/(x^4+5)^(3/2),x, algorithm="giac")
```

output
```
3/50*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^
4 + 5))) - 1/50*(2*x^2 - 15)/sqrt(x^4 + 5) + 2/5/((x^2 - sqrt(x^4 + 5))^2
- 5)
```

3.49.9 Mupad [B] (verification not implemented)

Time = 7.98 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.72

$$\int \frac{2+3x^2}{x^3 (5+x^4)^{3/2}}\,dx = \frac{3}{10\sqrt{x^4+5}} - \frac{3\sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}}{5}\right)}{50} - \frac{2\,x^4+5}{25\,x^2\,\sqrt{x^4+5}}$$

input
```
int((3*x^2 + 2)/(x^3*(x^4 + 5)^(3/2)),x)
```

output
```
3/(10*(x^4 + 5)^(1/2)) - (3*5^(1/2)*atanh((5^(1/2)*(x^4 + 5)^(1/2))/5))/50
 - (2*x^4 + 5)/(25*x^2*(x^4 + 5)^(1/2))
```

3.50 $\int \frac{x^4(2+3x^2)}{(5+x^4)^{3/2}}\,dx$

3.50.1 Optimal result

Integrand size = 20, antiderivative size = 196

$$\int \frac{x^4(2+3x^2)}{(5+x^4)^{3/2}}\,dx = -\frac{x^3(15-2x^2)}{10\sqrt{5+x^4}}-\frac{1}{5}x\sqrt{5+x^4}+\frac{9x\sqrt{5+x^4}}{2\left(\sqrt{5}+x^2\right)} - \frac{9\sqrt[4]{5}(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{2\sqrt{5+x^4}} + \frac{\left(2+9\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{4\sqrt[4]{5}\sqrt{5+x^4}}$$

output
```
-1/10*x^3*(-2*x^2+15)/(x^4+5)^(1/2)-1/5*x*(x^4+5)^(1/2)+9/2*x*(x^4+5)^(1/2
)/(x^2+5^(1/2))-9/2*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*a
rctan(1/5*x*5^(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*
(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)+1/20*(cos(2*ar
ctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticF(sin(2
*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*(2+9*5^(1/2))*((x^4+5)/
(x^2+5^(1/2))^2)^(1/2)*5^(3/4)/(x^4+5)^(1/2)
```

3.50.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.36

$$\int \frac{x^4(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \frac{x(-1+3x^2)}{\sqrt{5+x^4}} + \frac{x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{x^4}{5}\right)}{\sqrt{5}} - \frac{3x^3\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\frac{x^4}{5}\right)}{\sqrt{5}}$$

input
```
Integrate[(x^4*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

output
```
(x*(-1 + 3*x^2))/Sqrt[5 + x^4] + (x*Hypergeometric2F1[1/4, 1/2, 5/4, -1/5*
x^4])/Sqrt[5] - (3*x^3*Hypergeometric2F1[3/4, 3/2, 7/4, -1/5*x^4])/Sqrt[5]
```

3.50.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1599, 27, 1603, 27, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(3x^2+2)}{(x^4+5)^{3/2}}\,dx$$

$$\Big\downarrow \ 1599$$

$$\frac{1}{10}\int \frac{3x^2(15-2x^2)}{\sqrt{x^4+5}}dx - \frac{x^3(15-2x^2)}{10\sqrt{x^4+5}}$$

$$\Big\downarrow \ 27$$

$$\frac{3}{10}\int \frac{x^2(15-2x^2)}{\sqrt{x^4+5}}dx - \frac{x^3(15-2x^2)}{10\sqrt{x^4+5}}$$

$$\Big\downarrow \ 1603$$

$$\frac{3}{10}\left(-\frac{1}{3}\int -\frac{5(9x^2+2)}{\sqrt{x^4+5}}dx - \frac{2}{3}\sqrt{x^4+5}x\right) - \frac{x^3(15-2x^2)}{10\sqrt{x^4+5}}$$

$$\Big\downarrow \ 27$$

$$\frac{3}{10}\left(\frac{5}{3}\int \frac{9x^2+2}{\sqrt{x^4+5}}dx-\frac{2}{3}x\sqrt{x^4+5}\right)-\frac{x^3(15-2x^2)}{10\sqrt{x^4+5}}$$

$$\Big\downarrow \ 1512$$

$$\frac{3}{10}\left(\frac{5}{3}\left(\left(2+9\sqrt{5}\right)\int \frac{1}{\sqrt{x^4+5}}dx-9\sqrt{5}\int \frac{\sqrt{5}-x^2}{\sqrt{5}\sqrt{x^4+5}}dx\right)-\frac{2}{3}x\sqrt{x^4+5}\right)-\frac{x^3(15-2x^2)}{10\sqrt{x^4+5}}$$

$$\Big\downarrow \ 27$$

$$\frac{3}{10}\left(\frac{5}{3}\left(\left(2+9\sqrt{5}\right)\int \frac{1}{\sqrt{x^4+5}}dx-9\int \frac{\sqrt{5}-x^2}{\sqrt{x^4+5}}dx\right)-\frac{2}{3}x\sqrt{x^4+5}\right)-\frac{x^3(15-2x^2)}{10\sqrt{x^4+5}}$$

$$\Big\downarrow \ 761$$

$$\frac{3}{10}\left(\frac{5}{3}\left(\frac{(2+9\sqrt{5})\,(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}-9\int \frac{\sqrt{5}-x^2}{\sqrt{x^4+5}}dx\right)-\frac{2}{3}x\sqrt{x^4+5}\right)-$$

$$\frac{x^3(15-2x^2)}{10\sqrt{x^4+5}}$$

$$\Big\downarrow \ 1510$$

$$\frac{3}{10}\left(\frac{5}{3}\left(\frac{(2+9\sqrt{5})\,(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}-9\left(\frac{\sqrt[4]{5}(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\,\mathrm{a}\right.}{\sqrt{x^4+5}}\right.\right.\right.$$

$$\frac{x^3(15-2x^2)}{10\sqrt{x^4+5}}$$

input
```
Int[(x^4*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

output
```
-1/10*(x^3*(15 - 2*x^2))/Sqrt[5 + x^4] + (3*((-2*x*Sqrt[5 + x^4])/3 + (5*(
-9*(-((x*Sqrt[5 + x^4])/(Sqrt[5] + x^2)) + (5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(
5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 +
x^4]) + ((2 + 9*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]
*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(2*5^(1/4)*Sqrt[5 + x^4])))/3))/10
```

3.50. $\int \frac{x^4(2+3x^2)}{(5+x^4)^{3/2}}\,dx$

3.50.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

rule 1599
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1)*((a*e - c*d*x^2)/(4*a*
c*(p + 1))), x] - Simp[f^2/(4*a*c*(p + 1))   Int[(f*x)^(m - 2)*(a + c*x^4)^
(p + 1)*(a*e*(m - 1) - c*d*(4*p + 4 + m + 1)*x^2), x], x] /; FreeQ[{a, c, d
, e, f}, x] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

rule 1603
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
 1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ
[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[
m])
```

3.50.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 3.57 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.20

method	result
meijerg	$\frac{3\sqrt{5}\,x^{7}{}_{2}F_{1}\left(\frac{3}{2},\frac{7}{4};\frac{11}{4};-\frac{x^{4}}{5}\right)}{175}+\frac{2\sqrt{5}\,x^{5}{}_{2}F_{1}\left(\frac{5}{4},\frac{3}{2};\frac{9}{4};-\frac{x^{4}}{5}\right)}{125}$
risch	$-\frac{x(3x^{2}+2)}{2\sqrt{x^{4}+5}}+\frac{\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+5i\sqrt{5}\,x^{2}}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{25\sqrt{i\sqrt{5}}\,\sqrt{x^{4}+5}}+\frac{9i\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+5i\sqrt{5}\,x^{2}}\,\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}}{}\right.\right.}{10\sqrt{i\sqrt{5}}\,\sqrt{x^{4}+5}}$
elliptic	$-\frac{2\left(\frac{3}{4}x^{3}+\frac{1}{2}x\right)}{\sqrt{x^{4}+5}}+\frac{\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+5i\sqrt{5}\,x^{2}}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{25\sqrt{i\sqrt{5}}\,\sqrt{x^{4}+5}}+\frac{9i\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+5i\sqrt{5}\,x^{2}}\,\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x}{}\right.\right.}{10\sqrt{i\sqrt{5}}\,\sqrt{x^{4}+5}}$
default	$-\frac{3x^{3}}{2\sqrt{x^{4}+5}}+\frac{9i\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+5i\sqrt{5}\,x^{2}}\,\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)\right)}{10\sqrt{i\sqrt{5}}\,\sqrt{x^{4}+5}}-\frac{x}{\sqrt{x^{4}+5}}+\frac{\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+5i}}{25\sqrt{i\sqrt{5}}\,\sqrt{}}$

input
```
int(x^4*(3*x^2+2)/(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
3/175*5^(1/2)*x^7*hypergeom([3/2,7/4],[11/4],-1/5*x^4)+2/125*5^(1/2)*x^5*h
ypergeom([5/4,3/2],[9/4],-1/5*x^4)
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.39

$$\int\frac{x^{4}(2+3x^{2})}{(5+x^{4})^{3/2}}\,dx=\frac{45\,(-5)^{\frac{3}{4}}\,(x^{5}+5\,x)E(\arcsin\left(\frac{(-5)^{\frac{1}{4}}}{x}\right)\mid-1)-43\,(-5)^{\frac{3}{4}}\,(x^{5}+5\,x)F(\arcsin\left(\frac{(-5)^{\frac{1}{4}}}{x}\right)\mid-}{10\,(x^{5}+5\,x)}$$

input
```
integrate(x^4*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="fricas")
```

output
```
1/10*(45*(-5)^(3/4)*(x^5 + 5*x)*elliptic_e(arcsin((-5)^(1/4)/x), -1) - 43*
(-5)^(3/4)*(x^5 + 5*x)*elliptic_f(arcsin((-5)^(1/4)/x), -1) + 5*(6*x^4 - 2
*x^2 + 45)*sqrt(x^4 + 5))/(x^5 + 5*x)
```

3.50.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.38

$$\int \frac{x^4(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \frac{3\sqrt{5}x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{100\Gamma\left(\frac{11}{4}\right)}+\frac{\sqrt{5}x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{50\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x**4*(3*x**2+2)/(x**4+5)**(3/2),x)
```

output
```
3*sqrt(5)*x**7*gamma(7/4)*hyper((3/2, 7/4), (11/4,), x**4*exp_polar(I*pi)/
5)/(100*gamma(11/4)) + sqrt(5)*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), x
**4*exp_polar(I*pi)/5)/(50*gamma(9/4))
```

3.50.7 Maxima [F]

$$\int \frac{x^4(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \int \frac{(3\,x^2+2)x^4}{(x^4+5)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 2)*x^4/(x^4 + 5)^(3/2), x)
```

3.50.8 Giac [F]

$$\int \frac{x^4(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \int \frac{(3\,x^2+2)x^4}{(x^4+5)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="giac")
```

output
```
integrate((3*x^2 + 2)*x^4/(x^4 + 5)^(3/2), x)
```

3.50.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \int \frac{x^4\,(3\,x^2+2)}{(x^4+5)^{3/2}}\,dx$$

input
```
int((x^4*(3*x^2 + 2))/(x^4 + 5)^(3/2),x)
```

output
```
int((x^4*(3*x^2 + 2))/(x^4 + 5)^(3/2), x)
```

3.51 $\int \frac{x^2(2+3x^2)}{(5+x^4)^{3/2}}\,dx$

3.51.1 Optimal result

Integrand size = 20, antiderivative size = 177

$$\int \frac{x^2(2+3x^2)}{(5+x^4)^{3/2}}\,dx = -\frac{x(15-2x^2)}{10\sqrt{5+x^4}} - \frac{x\sqrt{5+x^4}}{5(\sqrt{5}+x^2)} + \frac{(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{(\sqrt{5}+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{5^{3/4}\sqrt{5+x^4}} - \frac{(2-3\sqrt{5})(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{(\sqrt{5}+x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{4\ 5^{3/4}\sqrt{5+x^4}}$$

output
```
-1/10*x*(-2*x^2+15)/(x^4+5)^(1/2)-1/5*x*(x^4+5)^(1/2)/(x^2+5^(1/2))+1/5*5^
(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*
EllipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*((x^4+5)
/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)-1/20*(cos(2*arctan(1/5*x*5^(3/4)))^2
)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticF(sin(2*arctan(1/5*x*5^(3/4))
),1/2*2^(1/2))*(2-3*5^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)
*5^(1/4)/(x^4+5)^(1/2)
```

3.51.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.38

$$\int \frac{x^2(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \frac{1}{150}x\left(-\frac{225}{\sqrt{5+x^4}}+45\sqrt{5}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{x^4}{5}\right)\right.$$
$$\left.+4\sqrt{5}x^2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\frac{x^4}{5}\right)\right)$$

input
```
Integrate[(x^2*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

output
```
(x*(-225/Sqrt[5 + x^4] + 45*Sqrt[5]*Hypergeometric2F1[1/4, 1/2, 5/4, -1/5*
x^4] + 4*Sqrt[5]*x^2*Hypergeometric2F1[3/4, 3/2, 7/4, -1/5*x^4]))/150
```

3.51.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1599, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(3x^2+2)}{(x^4+5)^{3/2}}dx$$

$$\Big\downarrow \text{ 1599}$$

$$\frac{1}{10}\int \frac{15-2x^2}{\sqrt{x^4+5}}dx-\frac{x(15-2x^2)}{10\sqrt{x^4+5}}$$

$$\Big\downarrow \text{ 1512}$$

$$\frac{1}{10}\left(\left(15-2\sqrt{5}\right)\int \frac{1}{\sqrt{x^4+5}}dx+2\sqrt{5}\int \frac{\sqrt{5}-x^2}{\sqrt{5}\sqrt{x^4+5}}dx\right)-\frac{x(15-2x^2)}{10\sqrt{x^4+5}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{10}\left(\left(15-2\sqrt{5}\right)\int \frac{1}{\sqrt{x^4+5}}dx+2\int \frac{\sqrt{5}-x^2}{\sqrt{x^4+5}}dx\right)-\frac{x(15-2x^2)}{10\sqrt{x^4+5}}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{1}{10}\left(2\int \frac{\sqrt{5}-x^2}{\sqrt{x^4+5}}dx+\frac{\left(15-2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}\right)-\frac{x\left(15-2x^2\right)}{10\sqrt{x^4+5}}$$

$\big\downarrow$ 1510

$$\frac{1}{10}\left(\frac{\left(15-2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}+2\left(\frac{\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\arct\right.}{\sqrt{x^4+5}}\right.\right.$$

$$\frac{x\left(15-2x^2\right)}{10\sqrt{x^4+5}}$$

input
```
Int[(x^2*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

output
```
-1/10*(x*(15 - 2*x^2))/Sqrt[5 + x^4] + (2*(-((x*Sqrt[5 + x^4])/(Sqrt[5] +
x^2)) + (5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*Ellipti
cE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4]) + ((15 - 2*Sqrt[5])*(Sqrt[5]
+ x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/
2])/(2*5^(1/4)*Sqrt[5 + x^4]))/10
```

3.51.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

rule 1599
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1)*((a*e - c*d*x^2)/(4*a*
c*(p + 1))), x] - Simp[f^2/(4*a*c*(p + 1))   Int[(f*x)^(m - 2)*(a + c*x^4)^
(p + 1)*(a*e*(m - 1) - c*d*(4*p + 4 + m + 1)*x^2), x], x] /; FreeQ[{a, c, d
, e, f}, x] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

3.51.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 2.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.23

method	result
meijerg	$\frac{3\sqrt{5}\,x^{5}{}_{2}F_{1}\left(\frac{5}{4},\frac{3}{2};\frac{9}{4};-\frac{x^{4}}{5}\right)}{125}+\frac{2\sqrt{5}\,x^{3}{}_{2}F_{1}\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{x^{4}}{5}\right)}{75}$
risch	$\frac{x(2x^{2}-15)}{10\sqrt{x^{4}+5}}+\frac{3\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+5i\sqrt{5}\,x^{2}}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{50\sqrt{i\sqrt{5}}\,\sqrt{x^{4}+5}}-\frac{i\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+5i\sqrt{5}\,x^{2}}\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\ldots\right)\right)}{25\sqrt{i\sqrt{5}}\,\sqrt{x^{4}+5}}$ …
elliptic	$-\frac{2\left(-\frac{1}{10}x^{3}+\frac{3}{4}x\right)}{\sqrt{x^{4}+5}}+\frac{3\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+5i\sqrt{5}\,x^{2}}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{50\sqrt{i\sqrt{5}}\,\sqrt{x^{4}+5}}-\frac{i\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+5i\sqrt{5}\,x^{2}}\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\ldots\right)}{25\sqrt{i\sqrt{5}}\,\sqrt{x^{4}+5}}$ …
default	$-\frac{3x}{2\sqrt{x^{4}+5}}+\frac{3\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+5i\sqrt{5}\,x^{2}}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{50\sqrt{i\sqrt{5}}\,\sqrt{x^{4}+5}}+\frac{x^{3}}{5\sqrt{x^{4}+5}}-\frac{i\sqrt{25-5i\sqrt{5}\,x^{2}}\,\sqrt{25+5i\sqrt{5}\,x^{2}}\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},\ldots\right)\right)}{25\sqrt{i\sqrt{5}}\,\sqrt{x^{4}+5}}$ …

input
```
int(x^2*(3*x^2+2)/(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
3/125*5^(1/2)*x^5*hypergeom([5/4,3/2],[9/4],-1/5*x^4)+2/75*5^(1/2)*x^3*hyp
ergeom([3/4,3/2],[7/4],-1/5*x^4)
```

3.51.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.55

$$\int \frac{x^2(2+3x^2)}{(5+x^4)^{3/2}}\,dx =$$
$$-\frac{2\sqrt{5}(-i\,x^4-5i)\sqrt{i\sqrt{5}}E(\arcsin\left(\frac{1}{5}\sqrt{5}x\sqrt{i\sqrt{5}}\right)|-1)+17\sqrt{5}(i\,x^4+5i)\sqrt{i\sqrt{5}}F(\arcsin\left(\frac{1}{5}\sqrt{5}x\sqrt{i\sqrt{5}}\right)}{50\,(x^4+5)}$$

input
```
integrate(x^2*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="fricas")
```

output
```
-1/50*(2*sqrt(5)*(-I*x^4 - 5*I)*sqrt(I*sqrt(5))*elliptic_e(arcsin(1/5*sqrt
(5)*x*sqrt(I*sqrt(5))), -1) + 17*sqrt(5)*(I*x^4 + 5*I)*sqrt(I*sqrt(5))*ell
iptic_f(arcsin(1/5*sqrt(5)*x*sqrt(I*sqrt(5))), -1) - 5*sqrt(x^4 + 5)*(2*x^
3 - 15*x))/(x^4 + 5)
```

3.51.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.99 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.42

$$\int \frac{x^2(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \frac{3\sqrt{5}x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{100\Gamma\left(\frac{9}{4}\right)}+\frac{\sqrt{5}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{50\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x**2*(3*x**2+2)/(x**4+5)**(3/2),x)
```

output
```
3*sqrt(5)*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), x**4*exp_polar(I*pi)/5
)/(100*gamma(9/4)) + sqrt(5)*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), x**
4*exp_polar(I*pi)/5)/(50*gamma(7/4))
```

3.51.7 Maxima [F]

$$\int \frac{x^2(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \int \frac{(3\,x^2+2)x^2}{(x^4+5)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 2)*x^2/(x^4 + 5)^(3/2), x)
```

3.51.8 Giac [F]

$$\int \frac{x^2(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \int \frac{(3\,x^2+2)x^2}{(x^4+5)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="giac")
```

output
```
integrate((3*x^2 + 2)*x^2/(x^4 + 5)^(3/2), x)
```

3.51.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \int \frac{x^2\,(3\,x^2+2)}{(x^4+5)^{3/2}}\,dx$$

input
```
int((x^2*(3*x^2 + 2))/(x^4 + 5)^(3/2),x)
```

output
```
int((x^2*(3*x^2 + 2))/(x^4 + 5)^(3/2), x)
```

3.52 $\int \frac{2+3x^2}{(5+x^4)^{3/2}}\,dx$

3.52.1 Optimal result

Integrand size = 17, antiderivative size = 180

$$\int \frac{2+3x^2}{(5+x^4)^{3/2}}\,dx=\frac{x(2+3x^2)}{10\sqrt{5+x^4}}-\frac{3x\sqrt{5+x^4}}{10\left(\sqrt{5}+x^2\right)}+\frac{3(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{2\ 5^{3/4}\sqrt{5+x^4}}+\frac{\left(2-3\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{20\sqrt[4]{5}\sqrt{5+x^4}}$$

output
```
1/10*x*(3*x^2+2)/(x^4+5)^(1/2)-3/10*x*(x^4+5)^(1/2)/(x^2+5^(1/2))+3/10*5^(
1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*E
llipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*((x^4+5)/
(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)+1/100*(cos(2*arctan(1/5*x*5^(3/4)))^2
)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticF(sin(2*arctan(1/5*x*5^(3/4))
),1/2*2^(1/2))*(2-3*5^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)
*5^(3/4)/(x^4+5)^(1/2)
```

3.52.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.37

$$\int \frac{2+3x^2}{(5+x^4)^{3/2}}\,dx = \frac{1}{25}x\left(\frac{5}{\sqrt{5+x^4}}+\sqrt{5}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{x^4}{5}\right)\right.$$
$$\left.+\sqrt{5}x^2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\frac{x^4}{5}\right)\right)$$

input
```
Integrate[(2 + 3*x^2)/(5 + x^4)^(3/2),x]
```

output
```
(x*(5/Sqrt[5 + x^4] + Sqrt[5]*Hypergeometric2F1[1/4, 1/2, 5/4, -1/5*x^4] +
 Sqrt[5]*x^2*Hypergeometric2F1[3/4, 3/2, 7/4, -1/5*x^4]))/25
```

3.52.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {1493, 25, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+2}{(x^4+5)^{3/2}}dx$$

$$\Big\downarrow \text{1493}$$

$$\frac{x(3x^2+2)}{10\sqrt{x^4+5}}-\frac{1}{10}\int -\frac{2-3x^2}{\sqrt{x^4+5}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{10}\int \frac{2-3x^2}{\sqrt{x^4+5}}dx+\frac{x(3x^2+2)}{10\sqrt{x^4+5}}$$

$$\Big\downarrow \text{1512}$$

$$\frac{1}{10}\left(\left(2-3\sqrt{5}\right)\int \frac{1}{\sqrt{x^4+5}}dx+3\sqrt{5}\int \frac{\sqrt{5}-x^2}{\sqrt{5}\sqrt{x^4+5}}dx\right)+\frac{x(3x^2+2)}{10\sqrt{x^4+5}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{10}\left(\left(2-3\sqrt{5}\right)\int \frac{1}{\sqrt{x^4+5}}dx+3\int \frac{\sqrt{5}-x^2}{\sqrt{x^4+5}}dx\right)+\frac{x\left(3x^2+2\right)}{10\sqrt{x^4+5}}$$

$$\Big\downarrow \text{761}$$

$$\frac{1}{10}\left(3\int \frac{\sqrt{5}-x^2}{\sqrt{x^4+5}}dx+\frac{\left(2-3\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}\right)+$$
$$\frac{x\left(3x^2+2\right)}{10\sqrt{x^4+5}}$$

$$\Big\downarrow \text{1510}$$

$$\frac{1}{10}\left(\frac{\left(2-3\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}+3\left(\frac{\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\,\text{arcta}\right.}{\sqrt{x^4+5}}\right.\right.$$
$$\frac{x\left(3x^2+2\right)}{10\sqrt{x^4+5}}$$

input
```
Int[(2 + 3*x^2)/(5 + x^4)^(3/2),x]
```

output
```
(x*(2 + 3*x^2))/(10*Sqrt[5 + x^4]) + (3*(-((x*Sqrt[5 + x^4])/(Sqrt[5] + x^
2)) + (5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE
[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4]) + ((2 - 3*Sqrt[5])*(Sqrt[5] + x
^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])
/(2*5^(1/4)*Sqrt[5 + x^4]))/10
```

3.52.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1493
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Simp[1/(4*a*(p + 1)
)   Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && Integer
Q[2*p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

3.52.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.57 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.21

method	result
meijerg	$\frac{2\sqrt{5}\, x\, {}_2F_1\left(\frac{1}{4},\frac{3}{2};\frac{5}{4};-\frac{x^4}{5}\right)}{25}+\frac{\sqrt{5}\, x^3\, {}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{x^4}{5}\right)}{25}$
risch	$\frac{x(3x^2+2)}{10\sqrt{x^4+5}}+\frac{\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{125\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}-\frac{3i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}\,\sqrt{}}{5}\right.\right.}{50\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$
elliptic	$-\frac{2\left(-\frac{3}{20}x^3-\frac{1}{10}x\right)}{\sqrt{x^4+5}}+\frac{\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{125\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}-\frac{3i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\right.}{50\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$
default	$\frac{x}{5\sqrt{x^4+5}}+\frac{\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{125\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{3x^3}{10\sqrt{x^4+5}}-\frac{3i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right.\right.}{50\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$

input
```
int((3*x^2+2)/(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/25*5^(1/2)*x*hypergeom([1/4,3/2],[5/4],-1/5*x^4)+1/25*5^(1/2)*x^3*hyperg
eom([3/4,3/2],[7/4],-1/5*x^4)
```

3.52.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.54

$$\int \frac{2+3x^2}{(5+x^4)^{3/2}}\,dx = -\frac{3\sqrt{5}(-i\,x^4-5i)\sqrt{i\sqrt{5}}E\left(\arcsin\left(\frac{1}{5}\sqrt{5}x\sqrt{i\sqrt{5}}\right)|-1\right)+5\sqrt{5}(i\,x^4+5i)\sqrt{i\sqrt{5}}F\left(\arcsin\left(\frac{1}{5}\sqrt{5}x\sqrt{i\sqrt{5}}\right)\right.}{50\,(x^4+5)}$$

input
```
integrate((3*x^2+2)/(x^4+5)^(3/2),x, algorithm="fricas")
```

output
```
-1/50*(3*sqrt(5)*(-I*x^4 - 5*I)*sqrt(I*sqrt(5))*elliptic_e(arcsin(1/5*sqrt
(5)*x*sqrt(I*sqrt(5))), -1) + 5*sqrt(5)*(I*x^4 + 5*I)*sqrt(I*sqrt(5))*elli
ptic_f(arcsin(1/5*sqrt(5)*x*sqrt(I*sqrt(5))), -1) - 5*sqrt(x^4 + 5)*(3*x^3
 + 2*x))/(x^4 + 5)
```

3.52.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.97 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.41

$$\int \frac{2+3x^2}{(5+x^4)^{3/2}}\,dx = \frac{3\sqrt{5}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{100\Gamma\left(\frac{7}{4}\right)}+\frac{\sqrt{5}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{50\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((3*x**2+2)/(x**4+5)**(3/2),x)
```

output
```
3*sqrt(5)*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), x**4*exp_polar(I*pi)/5
)/(100*gamma(7/4)) + sqrt(5)*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), x**4*e
xp_polar(I*pi)/5)/(50*gamma(5/4))
```

3.52.7 Maxima [F]

$$\int \frac{2+3x^2}{(5+x^4)^{3/2}}\,dx = \int \frac{3\,x^2+2}{(x^4+5)^{\frac{3}{2}}}\,dx$$

input
```
integrate((3*x^2+2)/(x^4+5)^(3/2),x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 2)/(x^4 + 5)^(3/2), x)
```

3.52.8 Giac [F]

$$\int \frac{2+3x^2}{(5+x^4)^{3/2}}\,dx = \int \frac{3\,x^2+2}{(x^4+5)^{\frac{3}{2}}}\,dx$$

input
```
integrate((3*x^2+2)/(x^4+5)^(3/2),x, algorithm="giac")
```

output
```
integrate((3*x^2 + 2)/(x^4 + 5)^(3/2), x)
```

3.52.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x^2}{(5+x^4)^{3/2}}\,dx = \int \frac{3\,x^2+2}{(x^4+5)^{3/2}}\,dx$$

input
```
int((3*x^2 + 2)/(x^4 + 5)^(3/2),x)
```

output
```
int((3*x^2 + 2)/(x^4 + 5)^(3/2), x)
```

3.53 $\int \frac{2+3x^2}{x^2(5+x^4)^{3/2}}\,dx$

3.53.1 Optimal result

Integrand size = 20, antiderivative size = 196

$$\int \frac{2+3x^2}{x^2(5+x^4)^{3/2}}\,dx = \frac{2+3x^2}{10x\sqrt{5+x^4}} - \frac{3\sqrt{5+x^4}}{25x} + \frac{3x\sqrt{5+x^4}}{25\left(\sqrt{5}+x^2\right)} - \frac{3\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{5\ 5^{3/4}\sqrt{5+x^4}} + \frac{3\left(2+\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{20\ 5^{3/4}\sqrt{5+x^4}}$$

output
```
1/10*(3*x^2+2)/x/(x^4+5)^(1/2)-3/25*(x^4+5)^(1/2)/x+3/25*x*(x^4+5)^(1/2)/(
x^2+5^(1/2))-3/25*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arc
tan(1/5*x*5^(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x
^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)+3/100*5^(1/4)*(c
os(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*Elliptic
F(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(2+5^(1/2))*(x^2+5^(1/2))*((x^
4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)
```

3.53.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.36

$$\int \frac{2+3x^2}{x^2 (5+x^4)^{3/2}}\,dx = \frac{3x}{10\sqrt{5+x^4}} - \frac{2\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{3}{2},\frac{3}{4},-\frac{x^4}{5}\right)}{5\sqrt{5}x} + \frac{3x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{x^4}{5}\right)}{10\sqrt{5}}$$

input
```
Integrate[(2 + 3*x^2)/(x^2*(5 + x^4)^(3/2)),x]
```

output
```
(3*x)/(10*Sqrt[5 + x^4]) - (2*Hypergeometric2F1[-1/4, 3/2, 3/4, -1/5*x^4])
/(5*Sqrt[5]*x) + (3*x*Hypergeometric2F1[1/4, 1/2, 5/4, -1/5*x^4])/(10*Sqrt
[5])
```

3.53.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1601, 27, 1605, 25, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+2}{x^2 (x^4+5)^{3/2}}dx$$

$$\Big\downarrow \text{1601}$$

$$\frac{3x^2+2}{10x\sqrt{x^4+5}} - \frac{1}{10}\int -\frac{3(x^2+2)}{x^2\sqrt{x^4+5}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{10}\int \frac{x^2+2}{x^2\sqrt{x^4+5}}dx + \frac{3x^2+2}{10x\sqrt{x^4+5}}$$

$$\Big\downarrow \text{1605}$$

$$\frac{3}{10}\left(-\frac{1}{5}\int -\frac{2x^2+5}{\sqrt{x^4+5}}dx - \frac{2\sqrt{x^4+5}}{5x}\right) + \frac{3x^2+2}{10x\sqrt{x^4+5}}$$

$$\Big\downarrow \ 25$$

$$\frac{3}{10}\left(\frac{1}{5}\int \frac{2x^2+5}{\sqrt{x^4+5}}dx-\frac{2\sqrt{x^4+5}}{5x}\right)+\frac{3x^2+2}{10x\sqrt{x^4+5}}$$

$$\Big\downarrow \ 1512$$

$$\frac{3}{10}\left(\frac{1}{5}\left(\left(5+2\sqrt{5}\right)\int \frac{1}{\sqrt{x^4+5}}dx-2\sqrt{5}\int \frac{\sqrt{5}-x^2}{\sqrt{5}\sqrt{x^4+5}}dx\right)-\frac{2\sqrt{x^4+5}}{5x}\right)+\frac{3x^2+2}{10x\sqrt{x^4+5}}$$

$$\Big\downarrow \ 27$$

$$\frac{3}{10}\left(\frac{1}{5}\left(\left(5+2\sqrt{5}\right)\int \frac{1}{\sqrt{x^4+5}}dx-2\int \frac{\sqrt{5}-x^2}{\sqrt{x^4+5}}dx\right)-\frac{2\sqrt{x^4+5}}{5x}\right)+\frac{3x^2+2}{10x\sqrt{x^4+5}}$$

$$\Big\downarrow \ 761$$

$$\frac{3}{10}\left(\frac{1}{5}\left(\frac{\left(5+2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}-2\int \frac{\sqrt{5}-x^2}{\sqrt{x^4+5}}dx\right)-\frac{2\sqrt{x^4+5}}{5x}\right)+$$
$$\frac{3x^2+2}{10x\sqrt{x^4+5}}$$

$$\Big\downarrow \ 1510$$

$$\frac{3}{10}\left(\frac{1}{5}\left(\frac{\left(5+2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}-2\left(\frac{\sqrt[4]{5}(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\text{a}\right.}{\sqrt{x^4+5}}\right.\right.\right.$$
$$\frac{3x^2+2}{10x\sqrt{x^4+5}}$$

input
```
Int[(2 + 3*x^2)/(x^2*(5 + x^4)^(3/2)),x]
```

output
```
(2 + 3*x^2)/(10*x*Sqrt[5 + x^4]) + (3*((-2*Sqrt[5 + x^4])/(5*x) + (-2*(-((
x*Sqrt[5 + x^4])/(Sqrt[5] + x^2)) + (5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4
)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4]) +
 ((5 + 2*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*Ellipt
icF[2*ArcTan[x/5^(1/4)], 1/2])/(2*5^(1/4)*Sqrt[5 + x^4]))/5))/10
```

3.53. $\int \frac{2+3x^2}{x^2(5+x^4)^{3/2}}\,dx$

3.53.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

rule 1601
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(-(f*x)^(m + 1))*(a + c*x^4)^(p + 1)*((d + e*x^2)/(4*a*f*(p
 + 1))), x] + Simp[1/(4*a*(p + 1))   Int[(f*x)^m*(a + c*x^4)^(p + 1)*Simp[d
*(m + 4*(p + 1) + 1) + e*(m + 2*(2*p + 3) + 1)*x^2, x], x], x] /; FreeQ[{a,
 c, d, e, f, m}, x] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || Integ
erQ[m])
```

rule 1605
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + S
imp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*
(m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
 IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.53. $\int \frac{2+3x^2}{x^2(5+x^4)^{3/2}}\,dx$

3.53.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.94 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.19

method	result
meijerg	$-\frac{2\sqrt{5}\,{}_2F_1\left(-\frac{1}{4},\frac{3}{2};\frac{3}{4};-\frac{x^4}{5}\right)}{25x}+\frac{3\sqrt{5}\,x\,{}_2F_1\left(\frac{1}{4},\frac{3}{2};\frac{5}{4};-\frac{x^4}{5}\right)}{25}$
risch	$-\frac{6x^4-15x^2+20}{50x\sqrt{x^4+5}}+\frac{3\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{250\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{3i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\right.}{125\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$
elliptic	$-\frac{2\sqrt{x^4+5}}{25x}-\frac{2\left(\frac{1}{50}x^3-\frac{3}{20}x\right)}{\sqrt{x^4+5}}+\frac{3\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{250\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{3i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(F\left(\frac{x\sqrt{5}\right.\right.}{125\sqrt{i\sqrt{5}}\,\sqrt{x^4}}$
default	$\frac{3x}{10\sqrt{x^4+5}}+\frac{3\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{250\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}-\frac{2\sqrt{x^4+5}}{25x}-\frac{x^3}{25\sqrt{x^4+5}}+\frac{3i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(\right.}{125\sqrt{i}}$

input
```
int((3*x^2+2)/x^2/(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/25*5^(1/2)/x*hypergeom([-1/4,3/2],[3/4],-1/5*x^4)+3/25*5^(1/2)*x*hyperg
eom([1/4,3/2],[5/4],-1/5*x^4)
```

3.53.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.55

$$\int \frac{2+3x^2}{x^2\left(5+x^4\right)^{3/2}}\,dx =$$
$$-\frac{6\sqrt{5}(i\,x^5+5i\,x)\sqrt{i\,\sqrt{5}}E(\arcsin\left(\frac{1}{5}\,\sqrt{5}x\sqrt{i\,\sqrt{5}}\right)\,|-1)+9\sqrt{5}(i\,x^5+5i\,x)\sqrt{i\,\sqrt{5}}F(\arcsin\left(\frac{1}{5}\,\sqrt{5}x\sqrt{i\,\sqrt{5}}\right.}{250\left(x^5+5\,x\right)}$$

input
```
integrate((3*x^2+2)/x^2/(x^4+5)^(3/2),x, algorithm="fricas")
```

output
```
-1/250*(6*sqrt(5)*(I*x^5 + 5*I*x)*sqrt(I*sqrt(5))*elliptic_e(arcsin(1/5*sq
rt(5)*x*sqrt(I*sqrt(5))), -1) + 9*sqrt(5)*(I*x^5 + 5*I*x)*sqrt(I*sqrt(5))*
elliptic_f(arcsin(1/5*sqrt(5)*x*sqrt(I*sqrt(5))), -1) + 5*(6*x^4 - 15*x^2
+ 20)*sqrt(x^4 + 5))/(x^5 + 5*x)
```

3.53.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.69 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.38

$$\int \frac{2+3x^2}{x^2(5+x^4)^{3/2}}\,dx = \frac{3\sqrt{5}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{100\Gamma\left(\frac{5}{4}\right)} + \frac{\sqrt{5}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\\frac{3}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{50x\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((3*x**2+2)/x**2/(x**4+5)**(3/2),x)
```

output
```
3*sqrt(5)*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), x**4*exp_polar(I*pi)/5)/(
100*gamma(5/4)) + sqrt(5)*gamma(-1/4)*hyper((-1/4, 3/2), (3/4,), x**4*exp_
polar(I*pi)/5)/(50*x*gamma(3/4))
```

3.53.7 Maxima [F]

$$\int \frac{2+3x^2}{x^2(5+x^4)^{3/2}}\,dx = \int \frac{3\,x^2+2}{(x^4+5)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((3*x^2+2)/x^2/(x^4+5)^(3/2),x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 2)/((x^4 + 5)^(3/2)*x^2), x)
```

3.53.8 Giac [F]

$$\int \frac{2+3x^2}{x^2(5+x^4)^{3/2}}\,dx = \int \frac{3\,x^2+2}{(x^4+5)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((3*x^2+2)/x^2/(x^4+5)^(3/2),x, algorithm="giac")
```

output
```
integrate((3*x^2 + 2)/((x^4 + 5)^(3/2)*x^2), x)
```

3.53.9 Mupad [B] (verification not implemented)

Time = 7.95 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.24

$$\int \frac{2+3x^2}{x^2\left(5+x^4\right)^{3/2}}\,dx = \frac{3\sqrt{5}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{2};\ \frac{5}{4};\ -\frac{x^4}{5}\right)}{25} - \frac{2\left(\frac{5}{x^4}+1\right)^{3/2}{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{7}{4};\ \frac{11}{4};\ -\frac{5}{x^4}\right)}{7\,x\left(x^4+5\right)^{3/2}}$$

input
```
int((3*x^2 + 2)/(x^2*(x^4 + 5)^(3/2)),x)
```

output
```
(3*5^(1/2)*x*hypergeom([1/4, 3/2], 5/4, -x^4/5))/25 - (2*(5/x^4 + 1)^(3/2)
*hypergeom([3/2, 7/4], 11/4, -5/x^4))/(7*x*(x^4 + 5)^(3/2))
```

3.54 $\int \frac{2+3x^2}{x^4(5+x^4)^{3/2}}\,dx$

3.54.1 Optimal result

Integrand size = 20, antiderivative size = 214

$$\int \frac{2+3x^2}{x^4\left(5+x^4\right)^{3/2}}\,dx = \frac{2+3x^2}{10x^3\sqrt{5+x^4}}-\frac{\sqrt{5+x^4}}{15x^3}-\frac{9\sqrt{5+x^4}}{50x}+\frac{9x\sqrt{5+x^4}}{50\left(\sqrt{5}+x^2\right)}-\frac{9\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{10\ 5^{3/4}\sqrt{5+x^4}}+\frac{\left(27-2\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{60\ 5^{3/4}\sqrt{5+x^4}}$$

output
```
1/10*(3*x^2+2)/x^3/(x^4+5)^(1/2)-1/15*(x^4+5)^(1/2)/x^3-9/50*(x^4+5)^(1/2)
/x+9/50*x*(x^4+5)^(1/2)/(x^2+5^(1/2))-9/50*5^(1/4)*(cos(2*arctan(1/5*x*5^(
3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticE(sin(2*arctan(1/5*x*
5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+
5)^(1/2)+1/300*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5
^(3/4)))*EllipticF(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(27-2*5^(1/2)
)*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)*5^(1/4)/(x^4+5)^(1/2)
```

3.54.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.25

$$\int \frac{2+3x^2}{x^4(5+x^4)^{3/2}}\,dx = -\frac{2\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{3}{2},\frac{1}{4},-\frac{x^4}{5}\right)+9x^2\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{3}{2},\frac{3}{4},-\frac{x^4}{5}\right)}{15\sqrt{5}x^3}$$

input
```
Integrate[(2 + 3*x^2)/(x^4*(5 + x^4)^(3/2)),x]
```

output
```
-1/15*(2*Hypergeometric2F1[-3/4, 3/2, 1/4, -1/5*x^4] + 9*x^2*Hypergeometri
c2F1[-1/4, 3/2, 3/4, -1/5*x^4])/(Sqrt[5]*x^3)
```

3.54.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {1601, 25, 1605, 27, 1605, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+2}{x^4(x^4+5)^{3/2}}\,dx$$

$$\Big\downarrow \text{1601}$$

$$\frac{3x^2+2}{10x^3\sqrt{x^4+5}}-\frac{1}{10}\int -\frac{9x^2+10}{x^4\sqrt{x^4+5}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{10}\int \frac{9x^2+10}{x^4\sqrt{x^4+5}}dx+\frac{3x^2+2}{10x^3\sqrt{x^4+5}}$$

$$\Big\downarrow \text{1605}$$

$$\frac{1}{10}\left(-\frac{1}{15}\int -\frac{5(27-2x^2)}{x^2\sqrt{x^4+5}}dx-\frac{2\sqrt{x^4+5}}{3x^3}\right)+\frac{3x^2+2}{10x^3\sqrt{x^4+5}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{10}\left(\frac{1}{3}\int \frac{27-2x^2}{x^2\sqrt{x^4+5}}dx-\frac{2\sqrt{x^4+5}}{3x^3}\right)+\frac{3x^2+2}{10x^3\sqrt{x^4+5}}$$

$\big\downarrow$ 1605

$$\frac{1}{10}\left(\frac{1}{3}\left(-\frac{1}{5}\int \frac{10-27x^2}{\sqrt{x^4+5}}dx-\frac{27\sqrt{x^4+5}}{5x}\right)-\frac{2\sqrt{x^4+5}}{3x^3}\right)+\frac{3x^2+2}{10x^3\sqrt{x^4+5}}$$

$\big\downarrow$ 1512

$$\frac{1}{10}\left(\frac{1}{3}\left(\frac{1}{5}\left(-\left(\left(10-27\sqrt{5}\right)\int \frac{1}{\sqrt{x^4+5}}dx\right)-27\sqrt{5}\int \frac{\sqrt{5}-x^2}{\sqrt{5}\sqrt{x^4+5}}dx\right)-\frac{27\sqrt{x^4+5}}{5x}\right)-\frac{2\sqrt{x^4+5}}{3x^3}\right)+\frac{3x^2+2}{10x^3\sqrt{x^4+5}}$$

$\big\downarrow$ 27

$$\frac{1}{10}\left(\frac{1}{3}\left(\frac{1}{5}\left(-\left(\left(10-27\sqrt{5}\right)\int \frac{1}{\sqrt{x^4+5}}dx\right)-27\int \frac{\sqrt{5}-x^2}{\sqrt{x^4+5}}dx\right)-\frac{27\sqrt{x^4+5}}{5x}\right)-\frac{2\sqrt{x^4+5}}{3x^3}\right)+\frac{3x^2+2}{10x^3\sqrt{x^4+5}}$$

$\big\downarrow$ 761

$$\frac{1}{10}\left(\frac{1}{3}\left(\frac{1}{5}\left(-27\int \frac{\sqrt{5}-x^2}{\sqrt{x^4+5}}dx-\frac{\left(10-27\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}\right)-\frac{27\sqrt{x^4+5}}{5x}\right)-\ldots\right)+\frac{3x^2+2}{10x^3\sqrt{x^4+5}}$$

$\big\downarrow$ 1510

$$\frac{1}{10}\left(\frac{1}{3}\left(\frac{1}{5}\left(-\frac{\left(10-27\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}-27\left(\frac{\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\ldots}{\sqrt{x\ldots}}\ldots\right.\right.\right.\right.+\frac{3x^2+2}{10x^3\sqrt{x^4+5}}$$

input
```
Int[(2 + 3*x^2)/(x^4*(5 + x^4)^(3/2)),x]
```

3.54. $\int \frac{2+3x^2}{x^4(5+x^4)^{3/2}}\,dx$

output
```
(2 + 3*x^2)/(10*x^3*Sqrt[5 + x^4]) + ((-2*Sqrt[5 + x^4])/(3*x^3) + ((-27*S
qrt[5 + x^4])/(5*x) + (-27*(-((x*Sqrt[5 + x^4])/(Sqrt[5] + x^2)) + (5^(1/4
)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5
^(1/4)], 1/2])/Sqrt[5 + x^4]) - ((10 - 27*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5
 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(2*5^(1/4)
*Sqrt[5 + x^4]))/5)/3)/10
```

3.54.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

rule 1601
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(-(f*x)^(m + 1))*(a + c*x^4)^(p + 1)*((d + e*x^2)/(4*a*f*(p
 + 1))), x] + Simp[1/(4*a*(p + 1))   Int[(f*x)^m*(a + c*x^4)^(p + 1)*Simp[d
*(m + 4*(p + 1) + 1) + e*(m + 2*(2*p + 3) + 1)*x^2, x], x], x] /; FreeQ[{a,
 c, d, e, f, m}, x] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || Integ
erQ[m])
```

rule 1605
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + S
imp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*
(m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
 IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.54.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 2.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.19

method	result
meijerg	$-\frac{2\sqrt{5}\,{}_2F_1\left(-\frac{3}{4},\frac{3}{2};\frac{1}{4};-\frac{x^4}{5}\right)}{75x^3}-\frac{3\sqrt{5}\,{}_2F_1\left(-\frac{1}{4},\frac{3}{2};\frac{3}{4};-\frac{x^4}{5}\right)}{25x}$
risch	$-\frac{27x^6+10x^4+90x^2+20}{150x^3\sqrt{x^4+5}}-\frac{\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{375\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{9i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5}\right.\right.}{250\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$
elliptic	$-\frac{2\sqrt{x^4+5}}{75x^3}-\frac{3\sqrt{x^4+5}}{25x}-\frac{2\left(\frac{3}{100}x^3+\frac{1}{50}x\right)}{\sqrt{x^4+5}}-\frac{\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{375\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{9i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{}}}{2}$
default	$-\frac{3\sqrt{x^4+5}}{25x}-\frac{3x^3}{50\sqrt{x^4+5}}+\frac{9i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(F\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-E\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)\right)}{250\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}-\frac{x}{25\sqrt{x^4+5}}-\frac{2\sqrt{x^4+5}}{75x^3}-$

input
```
int((3*x^2+2)/x^4/(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/75*5^(1/2)/x^3*hypergeom([-3/4,3/2],[1/4],-1/5*x^4)-3/25*5^(1/2)/x*hype
rgeom([-1/4,3/2],[3/4],-1/5*x^4)
```

3.54.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.55

$$\int \frac{2+3x^2}{x^4\left(5+x^4\right)^{3/2}}\,dx =$$

$$-\frac{27\sqrt{5}(i\,x^7+5i\,x^3)\sqrt{i\sqrt{5}}E(\arcsin\left(\frac{1}{5}\sqrt{5}x\sqrt{i\sqrt{5}}\right)\mid -1)+37\sqrt{5}(-i\,x^7-5i\,x^3)\sqrt{i\sqrt{5}}F(\arcsin\left(\frac{1}{5}\sqrt{5}x\sqrt{\right.}\right.}{750\left(x^7+5\,x^3\right)}$$

input
```
integrate((3*x^2+2)/x^4/(x^4+5)^(3/2),x, algorithm="fricas")
```

output
```
-1/750*(27*sqrt(5)*(I*x^7 + 5*I*x^3)*sqrt(I*sqrt(5))*elliptic_e(arcsin(1/5
*sqrt(5)*x*sqrt(I*sqrt(5))), -1) + 37*sqrt(5)*(-I*x^7 - 5*I*x^3)*sqrt(I*sq
rt(5))*elliptic_f(arcsin(1/5*sqrt(5)*x*sqrt(I*sqrt(5))), -1) + 5*(27*x^6 +
 10*x^4 + 90*x^2 + 20)*sqrt(x^4 + 5))/(x^7 + 5*x^3)
```

3.54.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.00 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.37

$$\int \frac{2+3x^2}{x^4\left(5+x^4\right)^{3/2}}\,dx = \frac{3\sqrt{5}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{100x\Gamma\left(\frac{3}{4}\right)} + \frac{\sqrt{5}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{3}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{50x^3\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate((3*x**2+2)/x**4/(x**4+5)**(3/2),x)
```

output
```
3*sqrt(5)*gamma(-1/4)*hyper((-1/4, 3/2), (3/4,), x**4*exp_polar(I*pi)/5)/(
100*x*gamma(3/4)) + sqrt(5)*gamma(-3/4)*hyper((-3/4, 3/2), (1/4,), x**4*ex
p_polar(I*pi)/5)/(50*x**3*gamma(1/4))
```

3.54.7 Maxima [F]

$$\int \frac{2+3x^2}{x^4\left(5+x^4\right)^{3/2}}\,dx = \int \frac{3\,x^2+2}{\left(x^4+5\right)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate((3*x^2+2)/x^4/(x^4+5)^(3/2),x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 2)/((x^4 + 5)^(3/2)*x^4), x)
```

3.54.8 Giac [F]

$$\int \frac{2+3x^2}{x^4\left(5+x^4\right)^{3/2}}\,dx = \int \frac{3\,x^2+2}{\left(x^4+5\right)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate((3*x^2+2)/x^4/(x^4+5)^(3/2),x, algorithm="giac")
```

output
```
integrate((3*x^2 + 2)/((x^4 + 5)^(3/2)*x^4), x)
```

3.54.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x^2}{x^4\left(5+x^4\right)^{3/2}}\,dx = \int \frac{3\,x^2+2}{x^4\,\left(x^4+5\right)^{3/2}}\,dx$$

input
```
int((3*x^2 + 2)/(x^4*(x^4 + 5)^(3/2)),x)
```

output
```
int((3*x^2 + 2)/(x^4*(x^4 + 5)^(3/2)), x)
```

3.55 $\int (fx)^m (d+ex^2) (1+2x^2+x^4)^5 \, dx$

3.55.1 Optimal result

Integrand size = 25, antiderivative size = 269

$$\begin{aligned}\int (fx)^m (d+ex^2) (1+2x^2+x^4)^5 \, dx = {} & \frac{d(fx)^{1+m}}{f(1+m)}+\frac{(10d+e)(fx)^{3+m}}{f^3(3+m)} \\ & +\frac{5(9d+2e)(fx)^{5+m}}{f^5(5+m)}+\frac{15(8d+3e)(fx)^{7+m}}{f^7(7+m)} \\ & +\frac{30(7d+4e)(fx)^{9+m}}{f^9(9+m)}+\frac{42(6d+5e)(fx)^{11+m}}{f^{11}(11+m)} \\ & +\frac{42(5d+6e)(fx)^{13+m}}{f^{13}(13+m)}+\frac{30(4d+7e)(fx)^{15+m}}{f^{15}(15+m)} \\ & +\frac{15(3d+8e)(fx)^{17+m}}{f^{17}(17+m)}+\frac{5(2d+9e)(fx)^{19+m}}{f^{19}(19+m)} \\ & +\frac{(d+10e)(fx)^{21+m}}{f^{21}(21+m)}+\frac{e(fx)^{23+m}}{f^{23}(23+m)}\end{aligned}$$

output
```
d*(f*x)^(1+m)/f/(1+m)+(10*d+e)*(f*x)^(3+m)/f^3/(3+m)+5*(9*d+2*e)*(f*x)^(5+
m)/f^5/(5+m)+15*(8*d+3*e)*(f*x)^(7+m)/f^7/(7+m)+30*(7*d+4*e)*(f*x)^(9+m)/f
^9/(9+m)+42*(6*d+5*e)*(f*x)^(11+m)/f^11/(11+m)+42*(5*d+6*e)*(f*x)^(13+m)/f
^13/(13+m)+30*(4*d+7*e)*(f*x)^(15+m)/f^15/(15+m)+15*(3*d+8*e)*(f*x)^(17+m)
/f^17/(17+m)+5*(2*d+9*e)*(f*x)^(19+m)/f^19/(19+m)+(d+10*e)*(f*x)^(21+m)/f^
21/(21+m)+e*(f*x)^(23+m)/f^23/(23+m)
```

3.55.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.70

$$\int (fx)^m \left(d+ex^2\right) \left(1+2x^2+x^4\right)^5 \, dx = x(fx)^m \left(\frac{d}{1+m}+\frac{(10d+e)x^2}{3+m}+\frac{5(9d+2e)x^4}{5+m}+\frac{15(8d+3e)x^6}{7+m}+\frac{30(7d+4e)x^8}{9+m}+\frac{42(6d+5e)x^{10}}{11+m}+\frac{42(5d+6e)x^{12}}{13+m}+\frac{30(4d+7e)x^{14}}{15+m}+\frac{15(3d+8e)x^{16}}{17+m}+\frac{5(2d+9e)x^{18}}{19+m}+\frac{(d+10e)x^{20}}{21+m}+\frac{ex^{22}}{23+m}\right)$$

input
```
Integrate[(f*x)^m*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

output
```
x*(f*x)^m*(d/(1 + m) + ((10*d + e)*x^2)/(3 + m) + (5*(9*d + 2*e)*x^4)/(5 +
 m) + (15*(8*d + 3*e)*x^6)/(7 + m) + (30*(7*d + 4*e)*x^8)/(9 + m) + (42*(6
*d + 5*e)*x^10)/(11 + m) + (42*(5*d + 6*e)*x^12)/(13 + m) + (30*(4*d + 7*e
)*x^14)/(15 + m) + (15*(3*d + 8*e)*x^16)/(17 + m) + (5*(2*d + 9*e)*x^18)/(
19 + m) + ((d + 10*e)*x^20)/(21 + m) + (e*x^22)/(23 + m))
```

3.55.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1380, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^4+2x^2+1\right)^5 \left(d+ex^2\right) (fx)^m \, dx$$

$$\Big\downarrow \text{1380}$$

$$\int \left(x^2+1\right)^{10} \left(d+ex^2\right) (fx)^m dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(\frac{(d+10e)(fx)^{m+20}}{f^{20}} + \frac{5(2d+9e)(fx)^{m+18}}{f^{18}} + \frac{15(3d+8e)(fx)^{m+16}}{f^{16}} + \frac{30(4d+7e)(fx)^{m+14}}{f^{14}} + \frac{42(5d+6e)(f}{f^{12}}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{(d+10e)(fx)^{m+21}}{f^{21}(m+21)} + \frac{5(2d+9e)(fx)^{m+19}}{f^{19}(m+19)} + \frac{15(3d+8e)(fx)^{m+17}}{f^{17}(m+17)} + \frac{30(4d+7e)(fx)^{m+15}}{f^{15}(m+15)} + \frac{42(5d+6e)(fx)^{m+13}}{f^{13}(m+13)} + \frac{42(6d+5e)(fx)^{m+11}}{f^{11}(m+11)} + \frac{30(7d+4e)(fx)^{m+9}}{f^{9}(m+9)} + \frac{15(8d+3e)(fx)^{m+7}}{f^{7}(m+7)} + \frac{5(9d+2e)(fx)^{m+5}}{f^{5}(m+5)} + \frac{(10d+e)(fx)^{m+3}}{f^{3}(m+3)} + \frac{d(fx)^{m+1}}{f(m+1)} + \frac{e(fx)^{m+23}}{f^{23}(m+23)}$$

input
```
Int[(f*x)^m*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

output
```
(d*(f*x)^(1 + m))/(f*(1 + m)) + ((10*d + e)*(f*x)^(3 + m))/(f^3*(3 + m)) +
 (5*(9*d + 2*e)*(f*x)^(5 + m))/(f^5*(5 + m)) + (15*(8*d + 3*e)*(f*x)^(7 +
m))/(f^7*(7 + m)) + (30*(7*d + 4*e)*(f*x)^(9 + m))/(f^9*(9 + m)) + (42*(6*
d + 5*e)*(f*x)^(11 + m))/(f^11*(11 + m)) + (42*(5*d + 6*e)*(f*x)^(13 + m))
/(f^13*(13 + m)) + (30*(4*d + 7*e)*(f*x)^(15 + m))/(f^15*(15 + m)) + (15*(
3*d + 8*e)*(f*x)^(17 + m))/(f^17*(17 + m)) + (5*(2*d + 9*e)*(f*x)^(19 + m)
)/(f^19*(19 + m)) + ((d + 10*e)*(f*x)^(21 + m))/(f^21*(21 + m)) + (e*(f*x)
^(23 + m))/(f^23*(23 + m))
```

3.55.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.55.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2294 vs. 2(269) = 538.

Time = 0.75 (sec) , antiderivative size = 2295, normalized size of antiderivative = 8.53

method	result	size
gosper	Expression too large to display	2295
risch	Expression too large to display	2295
parallelrisch	Expression too large to display	3621

input
```
int((f*x)^m*(e*x^2+d)*(x^4+2*x^2+1)^5,x,method=_RETURNVERBOSE)
```

output
```
(f*x)^m*(e*m^11*x^22+121*e*m^10*x^22+d*m^11*x^20+10*e*m^11*x^20+6435*e*m^9
*x^22+123*d*m^10*x^20+1230*e*m^10*x^20+197835*e*m^8*x^22+10*d*m^11*x^18+66
35*d*m^9*x^20+45*e*m^11*x^18+66350*e*m^9*x^20+3889578*e*m^7*x^22+1250*d*m^
10*x^18+206505*d*m^8*x^20+5625*e*m^10*x^18+2065050*e*m^8*x^20+51069018*e*m
^6*x^22+45*d*m^11*x^16+68430*d*m^9*x^18+4103178*d*m^7*x^20+120*e*m^11*x^16
+307935*e*m^9*x^18+41031780*e*m^7*x^20+453714470*e*m^5*x^22+5715*d*m^10*x^
16+2158230*d*m^8*x^18+54362574*d*m^6*x^20+15240*e*m^10*x^16+9712035*e*m^8*
x^18+543625740*e*m^6*x^20+2702025590*e*m^4*x^22+120*d*m^11*x^14+317655*d*m
^9*x^16+43391460*d*m^7*x^18+486687830*d*m^5*x^20+210*e*m^11*x^14+847080*e*
m^9*x^16+195261570*e*m^7*x^18+4866878300*e*m^5*x^20+10431670821*e*m^3*x^22
+15480*d*m^10*x^14+10162665*d*m^8*x^16+580855380*d*m^6*x^18+2917013970*d*m
^4*x^20+27090*e*m^10*x^14+27100440*e*m^8*x^16+2613849210*e*m^6*x^18+291701
39700*e*m^4*x^20+24372200061*e*m^2*x^22+210*d*m^11*x^12+873960*d*m^9*x^14+
207024930*d*m^7*x^16+5246766620*d*m^5*x^18+11320966021*d*m^3*x^20+252*e*m^
11*x^12+1529430*e*m^9*x^14+552066480*e*m^7*x^16+23610449790*e*m^5*x^18+113
209660210*e*m^3*x^20+29985521895*e*m*x^22+27510*d*m^10*x^12+28391400*d*m^8
*x^14+2804395230*d*m^6*x^16+31686018220*d*m^4*x^18+26560342503*d*m^2*x^20+
33012*e*m^10*x^12+49684950*e*m^8*x^14+7478387280*e*m^6*x^16+142587081990*e
*m^4*x^18+265603425030*e*m^2*x^20+13749310575*e*x^22+252*d*m^11*x^10+15781
50*d*m^9*x^12+586902960*d*m^7*x^14+25598865870*d*m^5*x^16+123748247730*...
```

3.55.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1571 vs. 2(269) = 538.

Time = 0.26 (sec) , antiderivative size = 1571, normalized size of antiderivative = 5.84

$$\int (fx)^m (d+ex^2) \left(1+2x^2+x^4\right)^5 \, dx = \text{Too large to display}$$

input
```
integrate((f*x)^m*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

output
```
((e*m^11 + 121*e*m^10 + 6435*e*m^9 + 197835*e*m^8 + 3889578*e*m^7 + 510690
18*e*m^6 + 453714470*e*m^5 + 2702025590*e*m^4 + 10431670821*e*m^3 + 243722
00061*e*m^2 + 29985521895*e*m + 13749310575*e)*x^23 + ((d + 10*e)*m^11 + 1
23*(d + 10*e)*m^10 + 6635*(d + 10*e)*m^9 + 206505*(d + 10*e)*m^8 + 4103178
*(d + 10*e)*m^7 + 54362574*(d + 10*e)*m^6 + 486687830*(d + 10*e)*m^5 + 291
7013970*(d + 10*e)*m^4 + 11320966021*(d + 10*e)*m^3 + 26560342503*(d + 10*
e)*m^2 + 32778930735*(d + 10*e)*m + 15058768725*d + 150587687250*e)*x^21 +
 5*((2*d + 9*e)*m^11 + 125*(2*d + 9*e)*m^10 + 6843*(2*d + 9*e)*m^9 + 21582
3*(2*d + 9*e)*m^8 + 4339146*(2*d + 9*e)*m^7 + 58085538*(2*d + 9*e)*m^6 + 5
24676662*(2*d + 9*e)*m^5 + 3168601822*(2*d + 9*e)*m^4 + 12374824773*(2*d +
 9*e)*m^3 + 29178958257*(2*d + 9*e)*m^2 + 36145916415*(2*d + 9*e)*m + 3328
7804550*d + 149795120475*e)*x^19 + 15*((3*d + 8*e)*m^11 + 127*(3*d + 8*e)*
m^10 + 7059*(3*d + 8*e)*m^9 + 225837*(3*d + 8*e)*m^8 + 4600554*(3*d + 8*e)
*m^7 + 62319894*(3*d + 8*e)*m^6 + 568863686*(3*d + 8*e)*m^5 + 3466775738*(
3*d + 8*e)*m^4 + 13643071845*(3*d + 8*e)*m^3 + 32368407579*(3*d + 8*e)*m^2
 + 40283194455*(3*d + 8*e)*m + 55806025275*d + 148816067400*e)*x^17 + 30*(
(4*d + 7*e)*m^11 + 129*(4*d + 7*e)*m^10 + 7283*(4*d + 7*e)*m^9 + 236595*(4
*d + 7*e)*m^8 + 4890858*(4*d + 7*e)*m^7 + 67166442*(4*d + 7*e)*m^6 + 62080
5254*(4*d + 7*e)*m^5 + 3825379590*(4*d + 7*e)*m^4 + 15197565541*(4*d + 7*e
)*m^3 + 36337145829*(4*d + 7*e)*m^2 + 45488935863*(4*d + 7*e)*m + 84329...
```

3.55.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21612 vs. 2(228) = 456.

Time = 2.91 (sec) , antiderivative size = 21612, normalized size of antiderivative = 80.34

$$\int (fx)^m (d+ex^2) \left(1+2x^2+x^4\right)^5 \, dx = \text{Too large to display}$$

input
```
integrate((f*x)**m*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

output
```
Piecewise(((-d/(2*x**2) - 5*d/(2*x**4) - 15*d/(2*x**6) - 15*d/x**8 - 21*d/
x**10 - 21*d/x**12 - 15*d/x**14 - 15*d/(2*x**16) - 5*d/(2*x**18) - d/(2*x*
*20) - d/(22*x**22) + e*log(x) - 5*e/x**2 - 45*e/(4*x**4) - 20*e/x**6 - 10
5*e/(4*x**8) - 126*e/(5*x**10) - 35*e/(2*x**12) - 60*e/(7*x**14) - 45*e/(1
6*x**16) - 5*e/(9*x**18) - e/(20*x**20))/f**23, Eq(m, -23)), ((d*log(x) -
5*d/x**2 - 45*d/(4*x**4) - 20*d/x**6 - 105*d/(4*x**8) - 126*d/(5*x**10) -
35*d/(2*x**12) - 60*d/(7*x**14) - 45*d/(16*x**16) - 5*d/(9*x**18) - d/(20*
x**20) + e*x**2/2 + 10*e*log(x) - 45*e/(2*x**2) - 30*e/x**4 - 35*e/x**6 -
63*e/(2*x**8) - 21*e/x**10 - 10*e/x**12 - 45*e/(14*x**14) - 5*e/(8*x**16)
- e/(18*x**18))/f**21, Eq(m, -21)), ((d*x**2/2 + 10*d*log(x) - 45*d/(2*x**
2) - 30*d/x**4 - 35*d/x**6 - 63*d/(2*x**8) - 21*d/x**10 - 10*d/x**12 - 45*
d/(14*x**14) - 5*d/(8*x**16) - d/(18*x**18) + e*x**4/4 + 5*e*x**2 + 45*e*l
og(x) - 60*e/x**2 - 105*e/(2*x**4) - 42*e/x**6 - 105*e/(4*x**8) - 12*e/x**
10 - 15*e/(4*x**12) - 5*e/(7*x**14) - e/(16*x**16))/f**19, Eq(m, -19)), ((
d*x**4/4 + 5*d*x**2 + 45*d*log(x) - 60*d/x**2 - 105*d/(2*x**4) - 42*d/x**6
 - 105*d/(4*x**8) - 12*d/x**10 - 15*d/(4*x**12) - 5*d/(7*x**14) - d/(16*x*
*16) + e*x**6/6 + 5*e*x**4/2 + 45*e*x**2/2 + 120*e*log(x) - 105*e/x**2 - 6
3*e/x**4 - 35*e/x**6 - 15*e/x**8 - 9*e/(2*x**10) - 5*e/(6*x**12) - e/(14*x
**14))/f**17, Eq(m, -17)), ((d*x**6/6 + 5*d*x**4/2 + 45*d*x**2/2 + 120*d*l
og(x) - 105*d/x**2 - 63*d/x**4 - 35*d/x**6 - 15*d/x**8 - 9*d/(2*x**10) ...
```

3.55.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.38

$$\int (fx)^m (d+ex^2) (1+2x^2+x^4)^5 \, dx = \frac{ef^m x^{23} x^m}{m+23} + \frac{df^m x^{21} x^m}{m+21} + \frac{10\, ef^m x^{21} x^m}{m+21} + \frac{10\, df^m x^{19} x^m}{m+19} + \frac{45\, ef^m x^{19} x^m}{m+19} + \frac{45\, df^m x^{17} x^m}{m+17} + \frac{120\, ef^m x^{17} x^m}{m+17} + \frac{120\, df^m x^{15} x^m}{m+15} + \frac{210\, ef^m x^{15} x^m}{m+15} + \frac{210\, df^m x^{13} x^m}{m+13} + \frac{252\, ef^m x^{13} x^m}{m+13} + \frac{252\, df^m x^{11} x^m}{m+11} + \frac{210\, ef^m x^{11} x^m}{m+11} + \frac{210\, df^m x^9 x^m}{m+9} + \frac{120\, ef^m x^9 x^m}{m+9} + \frac{120\, df^m x^7 x^m}{m+7} + \frac{45\, ef^m x^7 x^m}{m+7} + \frac{45\, df^m x^5 x^m}{m+5} + \frac{10\, ef^m x^5 x^m}{m+5} + \frac{10\, df^m x^3 x^m}{m+3} + \frac{ef^m x^3 x^m}{m+3} + \frac{(fx)^{m+1} d}{f(m+1)}$$

input
```
integrate((f*x)^m*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

output
```
e*f^m*x^23*x^m/(m + 23) + d*f^m*x^21*x^m/(m + 21) + 10*e*f^m*x^21*x^m/(m +
 21) + 10*d*f^m*x^19*x^m/(m + 19) + 45*e*f^m*x^19*x^m/(m + 19) + 45*d*f^m*
x^17*x^m/(m + 17) + 120*e*f^m*x^17*x^m/(m + 17) + 120*d*f^m*x^15*x^m/(m +
15) + 210*e*f^m*x^15*x^m/(m + 15) + 210*d*f^m*x^13*x^m/(m + 13) + 252*e*f^
m*x^13*x^m/(m + 13) + 252*d*f^m*x^11*x^m/(m + 11) + 210*e*f^m*x^11*x^m/(m
+ 11) + 210*d*f^m*x^9*x^m/(m + 9) + 120*e*f^m*x^9*x^m/(m + 9) + 120*d*f^m*
x^7*x^m/(m + 7) + 45*e*f^m*x^7*x^m/(m + 7) + 45*d*f^m*x^5*x^m/(m + 5) + 10
*e*f^m*x^5*x^m/(m + 5) + 10*d*f^m*x^3*x^m/(m + 3) + e*f^m*x^3*x^m/(m + 3)
+ (f*x)^(m + 1)*d/(f*(m + 1))
```

3.55.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3620 vs. $2(269) = 538$.

Time $= 0.35$ (sec) , antiderivative size $= 3620$, normalized size of antiderivative $= 13.46$

$$\int (fx)^m \left(d+ex^2\right) \left(1+2x^2+x^4\right)^5 \, dx = \text{Too large to display}$$

input
```
integrate((f*x)^m*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

output
```
((f*x)^m*e*m^11*x^23 + 121*(f*x)^m*e*m^10*x^23 + (f*x)^m*d*m^11*x^21 + 10*
(f*x)^m*e*m^11*x^21 + 6435*(f*x)^m*e*m^9*x^23 + 123*(f*x)^m*d*m^10*x^21 +
1230*(f*x)^m*e*m^10*x^21 + 197835*(f*x)^m*e*m^8*x^23 + 10*(f*x)^m*d*m^11*x
^19 + 45*(f*x)^m*e*m^11*x^19 + 6635*(f*x)^m*d*m^9*x^21 + 66350*(f*x)^m*e*m
^9*x^21 + 3889578*(f*x)^m*e*m^7*x^23 + 1250*(f*x)^m*d*m^10*x^19 + 5625*(f*
x)^m*e*m^10*x^19 + 206505*(f*x)^m*d*m^8*x^21 + 2065050*(f*x)^m*e*m^8*x^21
+ 51069018*(f*x)^m*e*m^6*x^23 + 45*(f*x)^m*d*m^11*x^17 + 120*(f*x)^m*e*m^1
1*x^17 + 68430*(f*x)^m*d*m^9*x^19 + 307935*(f*x)^m*e*m^9*x^19 + 4103178*(f
*x)^m*d*m^7*x^21 + 41031780*(f*x)^m*e*m^7*x^21 + 453714470*(f*x)^m*e*m^5*x
^23 + 5715*(f*x)^m*d*m^10*x^17 + 15240*(f*x)^m*e*m^10*x^17 + 2158230*(f*x)
^m*d*m^8*x^19 + 9712035*(f*x)^m*e*m^8*x^19 + 54362574*(f*x)^m*d*m^6*x^21 +
 543625740*(f*x)^m*e*m^6*x^21 + 2702025590*(f*x)^m*e*m^4*x^23 + 120*(f*x)^
m*d*m^11*x^15 + 210*(f*x)^m*e*m^11*x^15 + 317655*(f*x)^m*d*m^9*x^17 + 8470
80*(f*x)^m*e*m^9*x^17 + 43391460*(f*x)^m*d*m^7*x^19 + 195261570*(f*x)^m*e*
m^7*x^19 + 486687830*(f*x)^m*d*m^5*x^21 + 4866878300*(f*x)^m*e*m^5*x^21 +
10431670821*(f*x)^m*e*m^3*x^23 + 15480*(f*x)^m*d*m^10*x^15 + 27090*(f*x)^m
*e*m^10*x^15 + 10162665*(f*x)^m*d*m^8*x^17 + 27100440*(f*x)^m*e*m^8*x^17 +
 580855380*(f*x)^m*d*m^6*x^19 + 2613849210*(f*x)^m*e*m^6*x^19 + 2917013970
*(f*x)^m*d*m^4*x^21 + 29170139700*(f*x)^m*e*m^4*x^21 + 24372200061*(f*x)^m
*e*m^2*x^23 + 210*(f*x)^m*d*m^11*x^13 + 252*(f*x)^m*e*m^11*x^13 + 87396...
```

3.55.9 Mupad [B] (verification not implemented)

Time = 8.62 (sec) , antiderivative size = 1539, normalized size of antiderivative = 5.72

$$\int (fx)^m \left(d+ex^2\right) \left(1+2x^2+x^4\right)^5 \, dx = \text{Too large to display}$$

input
```
int((f*x)^m*(d + e*x^2)*(2*x^2 + x^4 + 1)^5,x)
```

output
```
(d*x*(f*x)^m*(387182170935*m + 203363952363*m^2 + 60936676581*m^3 + 116415
82810*m^4 + 1495875590*m^5 + 132426294*m^6 + 8103018*m^7 + 336765*m^8 + 90
75*m^9 + 143*m^10 + m^11 + 316234143225))/(703416314160*m + 590546123298*m
^2 + 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 1628301884*m^6
 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m^12
+ 316234143225) + (e*x^23*(f*x)^m*(29985521895*m + 24372200061*m^2 + 10431
670821*m^3 + 2702025590*m^4 + 453714470*m^5 + 51069018*m^6 + 3889578*m^7 +
 197835*m^8 + 6435*m^9 + 121*m^10 + m^11 + 13749310575))/(703416314160*m +
 590546123298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 +
 1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 1
44*m^11 + m^12 + 316234143225) + (30*x^15*(f*x)^m*(4*d + 7*e)*(45488935863
*m + 36337145829*m^2 + 15197565541*m^3 + 3825379590*m^4 + 620805254*m^5 +
67166442*m^6 + 4890858*m^7 + 236595*m^8 + 7283*m^9 + 129*m^10 + m^11 + 210
82276215))/(703416314160*m + 590546123298*m^2 + 264300628944*m^3 + 7257825
9391*m^4 + 13137458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8
+ 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225) + (42*x^13*(f*x
)^m*(5*d + 6*e)*(52237739295*m + 41408337231*m^2 + 17145560901*m^3 + 42640
53730*m^4 + 682569590*m^5 + 72748638*m^6 + 5213898*m^7 + 248145*m^8 + 7515
*m^9 + 131*m^10 + m^11 + 24325703325))/(703416314160*m + 590546123298*m^2
+ 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 1628301884*m^6...
```

3.55. $\int (fx)^m \left(d+ex^2\right) \left(1+2x^2+x^4\right)^5 \, dx$

3.56 $\int x^5(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx$

3.56.1 Optimal result

Integrand size = 23, antiderivative size = 63

$$\int x^5(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx=\frac{1}{22}(d-e)\left(1+x^2\right)^{11}-\frac{1}{24}(2d-3e)\left(1+x^2\right)^{12}+\frac{1}{26}(d-3e)\left(1+x^2\right)^{13}+\frac{1}{28}e\left(1+x^2\right)^{14}$$

output
```
1/22*(d-e)*(x^2+1)^11-1/24*(2*d-3*e)*(x^2+1)^12+1/26*(d-3*e)*(x^2+1)^13+1/
28*e*(x^2+1)^14
```

3.56.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 153 vs. 2(63) = 126.

Time = 0.02 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.43

$$\int x^5(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx=\frac{dx^6}{6}+\frac{1}{8}(10d+e)x^8+\frac{1}{2}(9d+2e)x^{10}+\frac{5}{4}(8d+3e)x^{12}+\frac{15}{7}(7d+4e)x^{14}+\frac{21}{8}(6d+5e)x^{16}+\frac{7}{3}(5d+6e)x^{18}+\frac{3}{2}(4d+7e)x^{20}+\frac{15}{22}(3d+8e)x^{22}+\frac{5}{24}(2d+9e)x^{24}+\frac{1}{26}(d+10e)x^{26}+\frac{ex^{28}}{28}$$

input
```
Integrate[x^5*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

output
```
(d*x^6)/6 + ((10*d + e)*x^8)/8 + ((9*d + 2*e)*x^10)/2 + (5*(8*d + 3*e)*x^1
2)/4 + (15*(7*d + 4*e)*x^14)/7 + (21*(6*d + 5*e)*x^16)/8 + (7*(5*d + 6*e)*
x^18)/3 + (3*(4*d + 7*e)*x^20)/2 + (15*(3*d + 8*e)*x^22)/22 + (5*(2*d + 9*
e)*x^24)/24 + ((d + 10*e)*x^26)/26 + (e*x^28)/28
```

3.56.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1380, 354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(x^4+2x^2+1\right)^5 \left(d+ex^2\right) \, dx$$

$$\Big\downarrow \; 1380$$

$$\int x^5 \left(x^2+1\right)^{10} \left(d+ex^2\right) dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2}\int x^4 \left(x^2+1\right)^{10} \left(ex^2+d\right) dx^2$$

$$\Big\downarrow \; 85$$

$$\frac{1}{2}\int \left(e\left(x^2+1\right)^{13}+(d-3e)\left(x^2+1\right)^{12}+(3e-2d)\left(x^2+1\right)^{11}+(d-e)\left(x^2+1\right)^{10}\right) dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{1}{13}\left(x^2+1\right)^{13}(d-3e)-\frac{1}{12}\left(x^2+1\right)^{12}(2d-3e)+\frac{1}{11}\left(x^2+1\right)^{11}(d-e)+\frac{1}{14}e\left(x^2+1\right)^{14}\right)$$

input
```
Int[x^5*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

output
```
(((d - e)*(1 + x^2)^11)/11 - ((2*d - 3*e)*(1 + x^2)^12)/12 + ((d - 3*e)*(1
 + x^2)^13)/13 + (e*(1 + x^2)^14)/14)/2
```

3.56.3.1 Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.56.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(55) = 110$.

Time = 0.14 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.94

method	result
norman	$\left(\frac{5d}{12}+\frac{15e}{8}\right)x^{24}+\left(\frac{d}{26}+\frac{5e}{13}\right)x^{26}+\frac{e\,x^{28}}{28}+\left(\frac{35d}{3}+14e\right)x^{18}+\left(6d+\frac{21e}{2}\right)x^{20}+\left(\frac{45d}{22}+\frac{60e}{11}\right)x^{22}+$
default	$\frac{e\,x^{28}}{28}+\frac{(d+10e)x^{26}}{26}+\frac{(10d+45e)x^{24}}{24}+\frac{(45d+120e)x^{22}}{22}+\frac{(120d+210e)x^{20}}{20}+\frac{(210d+252e)x^{18}}{18}+\frac{(252d+210e)x^{16}}{16}$
risch	$\frac{1}{28}e\,x^{28}+\frac{1}{26}x^{26}d+\frac{5}{13}x^{26}e+\frac{5}{12}x^{24}d+\frac{15}{8}x^{24}e+\frac{45}{22}x^{22}d+\frac{60}{11}e\,x^{22}+6d\,x^{20}+\frac{21}{2}e\,x^{20}+\frac{35}{3}d\,x^{1}$
parallelrisch	$\frac{1}{28}e\,x^{28}+\frac{1}{26}x^{26}d+\frac{5}{13}x^{26}e+\frac{5}{12}x^{24}d+\frac{15}{8}x^{24}e+\frac{45}{22}x^{22}d+\frac{60}{11}e\,x^{22}+6d\,x^{20}+\frac{21}{2}e\,x^{20}+\frac{35}{3}d\,x^{1}$
gosper	$x^{6}\left(858e\,x^{22}+924d\,x^{20}+9240e\,x^{20}+10010d\,x^{18}+45045e\,x^{18}+49140d\,x^{16}+131040e\,x^{16}+144144d\,x^{14}+252252e\,x^{14}+280280d\,x\right.$

input
```
int(x^5*(e*x^2+d)*(x^4+2*x^2+1)^5,x,method=_RETURNVERBOSE)
```

output
```
(5/12*d+15/8*e)*x^24+(1/26*d+5/13*e)*x^26+1/28*e*x^28+(35/3*d+14*e)*x^18+(
6*d+21/2*e)*x^20+(45/22*d+60/11*e)*x^22+(63/4*d+105/8*e)*x^16+(9/2*d+e)*x^
10+(10*d+15/4*e)*x^12+(15*d+60/7*e)*x^14+1/6*d*x^6+(5/4*d+1/8*e)*x^8
```

3.56.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(55) = 110.

Time = 0.23 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.05

$$\int x^5(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{28}\,ex^{28} + \frac{1}{26}\,(d+10\,e)x^{26} + \frac{5}{24}\,(2\,d+9\,e)x^{24} + \frac{15}{22}\,(3\,d+8\,e)x^{22} + \frac{3}{2}\,(4\,d+7\,e)x^{20} + \frac{7}{3}\,(5\,d+6\,e)x^{18} + \frac{21}{8}\,(6\,d+5\,e)x^{16} + \frac{15}{7}\,(7\,d+4\,e)x^{14} + \frac{5}{4}\,(8\,d+3\,e)x^{12} + \frac{1}{2}\,(9\,d+2\,e)x^{10} + \frac{1}{8}\,(10\,d+e)x^8 + \frac{1}{6}\,dx^6$$

input
```
integrate(x^5*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

output
```
1/28*e*x^28 + 1/26*(d + 10*e)*x^26 + 5/24*(2*d + 9*e)*x^24 + 15/22*(3*d +
8*e)*x^22 + 3/2*(4*d + 7*e)*x^20 + 7/3*(5*d + 6*e)*x^18 + 21/8*(6*d + 5*e)
*x^16 + 15/7*(7*d + 4*e)*x^14 + 5/4*(8*d + 3*e)*x^12 + 1/2*(9*d + 2*e)*x^1
0 + 1/8*(10*d + e)*x^8 + 1/6*d*x^6
```

3.56.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(51) = 102.

Time = 0.03 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.13

$$\int x^5(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{dx^6}{6} + \frac{ex^{28}}{28} + x^{26}\left(\frac{d}{26}+\frac{5e}{13}\right) + x^{24}\cdot\left(\frac{5d}{12}+\frac{15e}{8}\right) + x^{22}\cdot\left(\frac{45d}{22}+\frac{60e}{11}\right) + x^{20}\cdot\left(6d+\frac{21e}{2}\right) + x^{18}\cdot\left(\frac{35d}{3}+14e\right) + x^{16}\cdot\left(\frac{63d}{4}+\frac{105e}{8}\right) + x^{14}\cdot\left(15d+\frac{60e}{7}\right) + x^{12}\cdot\left(10d+\frac{15e}{4}\right) + x^{10}\cdot\left(\frac{9d}{2}+e\right) + x^8\cdot\left(\frac{5d}{4}+\frac{e}{8}\right)$$

input
```
integrate(x**5*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

output
```
d*x**6/6 + e*x**28/28 + x**26*(d/26 + 5*e/13) + x**24*(5*d/12 + 15*e/8) +
x**22*(45*d/22 + 60*e/11) + x**20*(6*d + 21*e/2) + x**18*(35*d/3 + 14*e) +
 x**16*(63*d/4 + 105*e/8) + x**14*(15*d + 60*e/7) + x**12*(10*d + 15*e/4)
+ x**10*(9*d/2 + e) + x**8*(5*d/4 + e/8)
```

3.56.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(55) = 110$.

Time = 0.18 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.05

$$\int x^5(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{28}\,ex^{28} + \frac{1}{26}\,(d+10\,e)x^{26} + \frac{5}{24}\,(2\,d+9\,e)x^{24} + \frac{15}{22}\,(3\,d+8\,e)x^{22} + \frac{3}{2}\,(4\,d+7\,e)x^{20} + \frac{7}{3}\,(5\,d+6\,e)x^{18} + \frac{21}{8}\,(6\,d+5\,e)x^{16} + \frac{15}{7}\,(7\,d+4\,e)x^{14} + \frac{5}{4}\,(8\,d+3\,e)x^{12} + \frac{1}{2}\,(9\,d+2\,e)x^{10} + \frac{1}{8}\,(10\,d+e)x^8 + \frac{1}{6}\,dx^6$$

input
```
integrate(x^5*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

output
```
1/28*e*x^28 + 1/26*(d + 10*e)*x^26 + 5/24*(2*d + 9*e)*x^24 + 15/22*(3*d +
8*e)*x^22 + 3/2*(4*d + 7*e)*x^20 + 7/3*(5*d + 6*e)*x^18 + 21/8*(6*d + 5*e)
*x^16 + 15/7*(7*d + 4*e)*x^14 + 5/4*(8*d + 3*e)*x^12 + 1/2*(9*d + 2*e)*x^1
0 + 1/8*(10*d + e)*x^8 + 1/6*d*x^6
```

3.56.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(55) = 110$.

Time = 0.27 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.10

$$\int x^5(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{28}\,ex^{28} + \frac{1}{26}\,dx^{26} + \frac{5}{13}\,ex^{26} + \frac{5}{12}\,dx^{24} + \frac{15}{8}\,ex^{24} + \frac{45}{22}\,dx^{22} + \frac{60}{11}\,ex^{22} + 6\,dx^{20} + \frac{21}{2}\,ex^{20} + \frac{35}{3}\,dx^{18} + 14\,ex^{18} + \frac{63}{4}\,dx^{16} + \frac{105}{8}\,ex^{16} + 15\,dx^{14} + \frac{60}{7}\,ex^{14} + 10\,dx^{12} + \frac{15}{4}\,ex^{12} + \frac{9}{2}\,dx^{10} + ex^{10} + \frac{5}{4}\,dx^8 + \frac{1}{8}\,ex^8 + \frac{1}{6}\,dx^6$$

input
```
integrate(x^5*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

output
```
1/28*e*x^28 + 1/26*d*x^26 + 5/13*e*x^26 + 5/12*d*x^24 + 15/8*e*x^24 + 45/2
2*d*x^22 + 60/11*e*x^22 + 6*d*x^20 + 21/2*e*x^20 + 35/3*d*x^18 + 14*e*x^18
 + 63/4*d*x^16 + 105/8*e*x^16 + 15*d*x^14 + 60/7*e*x^14 + 10*d*x^12 + 15/4
*e*x^12 + 9/2*d*x^10 + e*x^10 + 5/4*d*x^8 + 1/8*e*x^8 + 1/6*d*x^6
```

3.56.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.92

$$\int x^5\left(d+ex^2\right)\left(1+2x^2+x^4\right)^5\,dx=\frac{e\,x^{28}}{28}+\left(\frac{d}{26}+\frac{5\,e}{13}\right)x^{26}+\left(\frac{5\,d}{12}+\frac{15\,e}{8}\right)x^{24}+\left(\frac{45\,d}{22}+\frac{60\,e}{11}\right)x^{22}+\left(6\,d+\frac{21\,e}{2}\right)x^{20}+\left(\frac{35\,d}{3}+14\,e\right)x^{18}+\left(\frac{63\,d}{4}+\frac{105\,e}{8}\right)x^{16}+\left(15\,d+\frac{60\,e}{7}\right)x^{14}+\left(10\,d+\frac{15\,e}{4}\right)x^{12}+\left(\frac{9\,d}{2}+e\right)x^{10}+\left(\frac{5\,d}{4}+\frac{e}{8}\right)x^{8}+\frac{d\,x^6}{6}$$

input
```
int(x^5*(d + e*x^2)*(2*x^2 + x^4 + 1)^5,x)
```

output
```
x^8*((5*d)/4 + e/8) + x^12*(10*d + (15*e)/4) + x^20*(6*d + (21*e)/2) + x^2
4*((5*d)/12 + (15*e)/8) + x^18*((35*d)/3 + 14*e) + x^26*(d/26 + (5*e)/13)
+ x^14*(15*d + (60*e)/7) + x^22*((45*d)/22 + (60*e)/11) + x^16*((63*d)/4 +
 (105*e)/8) + (d*x^6)/6 + (e*x^28)/28 + x^10*((9*d)/2 + e)
```

3.57 $\int x^4(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx$

3.57.1 Optimal result

Integrand size = 23, antiderivative size = 153

$$\int x^4(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx=\frac{dx^5}{5}+\frac{1}{7}(10d+e)x^7+\frac{5}{9}(9d+2e)x^9+\frac{15}{11}(8d+3e)x^{11}+\frac{30}{13}(7d+4e)x^{13}+\frac{14}{5}(6d+5e)x^{15}+\frac{42}{17}(5d+6e)x^{17}+\frac{30}{19}(4d+7e)x^{19}+\frac{5}{7}(3d+8e)x^{21}+\frac{5}{23}(2d+9e)x^{23}+\frac{1}{25}(d+10e)x^{25}+\frac{ex^{27}}{27}$$

output
```
1/5*d*x^5+1/7*(10*d+e)*x^7+5/9*(9*d+2*e)*x^9+15/11*(8*d+3*e)*x^11+30/13*(7
*d+4*e)*x^13+14/5*(6*d+5*e)*x^15+42/17*(5*d+6*e)*x^17+30/19*(4*d+7*e)*x^19
+5/7*(3*d+8*e)*x^21+5/23*(2*d+9*e)*x^23+1/25*(d+10*e)*x^25+1/27*e*x^27
```

3.57.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00

$$\int x^4(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx=\frac{dx^5}{5}+\frac{1}{7}(10d+e)x^7+\frac{5}{9}(9d+2e)x^9+\frac{15}{11}(8d+3e)x^{11}+\frac{30}{13}(7d+4e)x^{13}+\frac{14}{5}(6d+5e)x^{15}+\frac{42}{17}(5d+6e)x^{17}+\frac{30}{19}(4d+7e)x^{19}+\frac{5}{7}(3d+8e)x^{21}+\frac{5}{23}(2d+9e)x^{23}+\frac{1}{25}(d+10e)x^{25}+\frac{ex^{27}}{27}$$

input
```
Integrate[x^4*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

output
```
(d*x^5)/5 + ((10*d + e)*x^7)/7 + (5*(9*d + 2*e)*x^9)/9 + (15*(8*d + 3*e)*x
^11)/11 + (30*(7*d + 4*e)*x^13)/13 + (14*(6*d + 5*e)*x^15)/5 + (42*(5*d +
6*e)*x^17)/17 + (30*(4*d + 7*e)*x^19)/19 + (5*(3*d + 8*e)*x^21)/7 + (5*(2*
d + 9*e)*x^23)/23 + ((d + 10*e)*x^25)/25 + (e*x^27)/27
```

3.57.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1380, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(x^4+2 x^2+1\right)^5 \left(d+e x^2\right) \, dx$$

$$\Big\downarrow \ 1380$$

$$\int x^4 \left(x^2+1\right)^{10} \left(d+e x^2\right) dx$$

$$\Big\downarrow \ 355$$

$$\int \left(x^{24} (d+10 e)+5 x^{22} (2 d+9 e)+15 x^{20} (3 d+8 e)+30 x^{18} (4 d+7 e)+42 x^{16} (5 d+6 e)+42 x^{14} (6 d+5 e)+30 x^{12} (\right.$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{25} x^{25} (d+10 e)+\frac{5}{23} x^{23} (2 d+9 e)+\frac{5}{7} x^{21} (3 d+8 e)+\frac{30}{19} x^{19} (4 d+7 e)+\frac{42}{17} x^{17} (5 d+6 e)+\frac{14}{5} x^{15} (6 d+5 e)+\frac{30}{13} x^{13} (7 d+4 e)+\frac{15}{11} x^{11} (8 d+3 e)+\frac{5}{9} x^9 (9 d+2 e)+\frac{1}{7} x^7 (10 d+e)+\frac{d x^5}{5}+\frac{e x^{27}}{27}$$

input
```
Int[x^4*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

output
```
(d*x^5)/5 + ((10*d + e)*x^7)/7 + (5*(9*d + 2*e)*x^9)/9 + (15*(8*d + 3*e)*x
^11)/11 + (30*(7*d + 4*e)*x^13)/13 + (14*(6*d + 5*e)*x^15)/5 + (42*(5*d +
6*e)*x^17)/17 + (30*(4*d + 7*e)*x^19)/19 + (5*(3*d + 8*e)*x^21)/7 + (5*(2*
d + 9*e)*x^23)/23 + ((d + 10*e)*x^25)/25 + (e*x^27)/27
```

3.57.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.57.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.81

method	result
norman	$\left(\frac{10d}{23}+\frac{45e}{23}\right)x^{23}+\left(\frac{d}{25}+\frac{2e}{5}\right)x^{25}+\frac{e\,x^{27}}{27}+\left(\frac{15d}{7}+\frac{40e}{7}\right)x^{21}+\left(\frac{210d}{17}+\frac{252e}{17}\right)x^{17}+\left(\frac{120d}{19}+\frac{210e}{19}\right)$
default	$\frac{e\,x^{27}}{27}+\frac{(d+10e)x^{25}}{25}+\frac{(10d+45e)x^{23}}{23}+\frac{(45d+120e)x^{21}}{21}+\frac{(120d+210e)x^{19}}{19}+\frac{(210d+252e)x^{17}}{17}+\frac{(252d+210e)x^{15}}{15}$
risch	$\frac{1}{27}e\,x^{27}+\frac{1}{25}x^{25}d+\frac{2}{5}e\,x^{25}+\frac{10}{23}x^{23}d+\frac{45}{23}e\,x^{23}+\frac{15}{7}x^{21}d+\frac{40}{7}e\,x^{21}+\frac{120}{19}x^{19}d+\frac{210}{19}x^{19}e+\frac{210}{17}x$
parallelrisch	$\frac{1}{27}e\,x^{27}+\frac{1}{25}x^{25}d+\frac{2}{5}e\,x^{25}+\frac{10}{23}x^{23}d+\frac{45}{23}e\,x^{23}+\frac{15}{7}x^{21}d+\frac{40}{7}e\,x^{21}+\frac{120}{19}x^{19}d+\frac{210}{19}x^{19}e+\frac{210}{17}x$
gosper	$x^5(185910725e\,x^{22}+200783583d\,x^{20}+2007835830e\,x^{20}+2182430250d\,x^{18}+9820936125e\,x^{18}+10756263375d\,x^{16}+2868336900$

input
```
int(x^4*(e*x^2+d)*(x^4+2*x^2+1)^5,x,method=_RETURNVERBOSE)
```

output
```
(10/23*d+45/23*e)*x^23+(1/25*d+2/5*e)*x^25+1/27*e*x^27+(15/7*d+40/7*e)*x^2
1+(210/17*d+252/17*e)*x^17+(120/19*d+210/19*e)*x^19+(5*d+10/9*e)*x^9+(120/
11*d+45/11*e)*x^11+(210/13*d+120/13*e)*x^13+(84/5*d+14*e)*x^15+1/5*x^5*d+(
10/7*d+1/7*e)*x^7
```

3.57.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.84

$$\int x^4(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{27}\,ex^{27} + \frac{1}{25}\,(d+10\,e)x^{25} + \frac{5}{23}\,(2\,d+9\,e)x^{23} + \frac{5}{7}\,(3\,d+8\,e)x^{21} + \frac{30}{19}\,(4\,d+7\,e)x^{19} + \frac{42}{17}\,(5\,d+6\,e)x^{17} + \frac{14}{5}\,(6\,d+5\,e)x^{15} + \frac{30}{13}\,(7\,d+4\,e)x^{13} + \frac{15}{11}\,(8\,d+3\,e)x^{11} + \frac{5}{9}\,(9\,d+2\,e)x^9 + \frac{1}{7}\,(10\,d+e)x^7 + \frac{1}{5}\,dx^5$$

input
```
integrate(x^4*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

output
```
1/27*e*x^27 + 1/25*(d + 10*e)*x^25 + 5/23*(2*d + 9*e)*x^23 + 5/7*(3*d + 8*
e)*x^21 + 30/19*(4*d + 7*e)*x^19 + 42/17*(5*d + 6*e)*x^17 + 14/5*(6*d + 5*
e)*x^15 + 30/13*(7*d + 4*e)*x^13 + 15/11*(8*d + 3*e)*x^11 + 5/9*(9*d + 2*e
)*x^9 + 1/7*(10*d + e)*x^7 + 1/5*d*x^5
```

3.57.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.92

$$\int x^4(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{dx^5}{5} + \frac{ex^{27}}{27} + x^{25}\left(\frac{d}{25}+\frac{2e}{5}\right) + x^{23}\cdot\left(\frac{10d}{23}+\frac{45e}{23}\right) + x^{21}\cdot\left(\frac{15d}{7}+\frac{40e}{7}\right) + x^{19}\cdot\left(\frac{120d}{19}+\frac{210e}{19}\right) + x^{17}\cdot\left(\frac{210d}{17}+\frac{252e}{17}\right) + x^{15}\cdot\left(\frac{84d}{5}+14e\right) + x^{13}\cdot\left(\frac{210d}{13}+\frac{120e}{13}\right) + x^{11}\cdot\left(\frac{120d}{11}+\frac{45e}{11}\right) + x^9\cdot\left(5d+\frac{10e}{9}\right) + x^7\cdot\left(\frac{10d}{7}+\frac{e}{7}\right)$$

input
```
integrate(x**4*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

output
```
d*x**5/5 + e*x**27/27 + x**25*(d/25 + 2*e/5) + x**23*(10*d/23 + 45*e/23) +
 x**21*(15*d/7 + 40*e/7) + x**19*(120*d/19 + 210*e/19) + x**17*(210*d/17 +
 252*e/17) + x**15*(84*d/5 + 14*e) + x**13*(210*d/13 + 120*e/13) + x**11*(
120*d/11 + 45*e/11) + x**9*(5*d + 10*e/9) + x**7*(10*d/7 + e/7)
```

3.57.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.84

$$\int x^4(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{27}\,ex^{27}+\frac{1}{25}\,(d+10\,e)x^{25}+\frac{5}{23}\,(2\,d+9\,e)x^{23} + \frac{5}{7}\,(3\,d+8\,e)x^{21}+\frac{30}{19}\,(4\,d+7\,e)x^{19} + \frac{42}{17}\,(5\,d+6\,e)x^{17}+\frac{14}{5}\,(6\,d+5\,e)x^{15} + \frac{30}{13}\,(7\,d+4\,e)x^{13}+\frac{15}{11}\,(8\,d+3\,e)x^{11} + \frac{5}{9}\,(9\,d+2\,e)x^{9}+\frac{1}{7}\,(10\,d+e)x^{7}+\frac{1}{5}\,dx^{5}$$

input
```
integrate(x^4*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

output
```
1/27*e*x^27 + 1/25*(d + 10*e)*x^25 + 5/23*(2*d + 9*e)*x^23 + 5/7*(3*d + 8*
e)*x^21 + 30/19*(4*d + 7*e)*x^19 + 42/17*(5*d + 6*e)*x^17 + 14/5*(6*d + 5*
e)*x^15 + 30/13*(7*d + 4*e)*x^13 + 15/11*(8*d + 3*e)*x^11 + 5/9*(9*d + 2*e
)*x^9 + 1/7*(10*d + e)*x^7 + 1/5*d*x^5
```

3.57.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.87

$$\int x^4(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{27}\,ex^{27}+\frac{1}{25}\,dx^{25}+\frac{2}{5}\,ex^{25}+\frac{10}{23}\,dx^{23}+\frac{45}{23}\,ex^{23}+\frac{15}{7}\,dx^{21} + \frac{40}{7}\,ex^{21}+\frac{120}{19}\,dx^{19}+\frac{210}{19}\,ex^{19}+\frac{210}{17}\,dx^{17}+\frac{252}{17}\,ex^{17} + \frac{84}{5}\,dx^{15}+14\,ex^{15}+\frac{210}{13}\,dx^{13}+\frac{120}{13}\,ex^{13}+\frac{120}{11}\,dx^{11} + \frac{45}{11}\,ex^{11}+5\,dx^{9}+\frac{10}{9}\,ex^{9}+\frac{10}{7}\,dx^{7}+\frac{1}{7}\,ex^{7}+\frac{1}{5}\,dx^{5}$$

input
```
integrate(x^4*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

output
```
1/27*e*x^27 + 1/25*d*x^25 + 2/5*e*x^25 + 10/23*d*x^23 + 45/23*e*x^23 + 15/
7*d*x^21 + 40/7*e*x^21 + 120/19*d*x^19 + 210/19*e*x^19 + 210/17*d*x^17 + 2
52/17*e*x^17 + 84/5*d*x^15 + 14*e*x^15 + 210/13*d*x^13 + 120/13*e*x^13 + 1
20/11*d*x^11 + 45/11*e*x^11 + 5*d*x^9 + 10/9*e*x^9 + 10/7*d*x^7 + 1/7*e*x^
7 + 1/5*d*x^5
```

3.57.9 Mupad [B] (verification not implemented)

Time = 7.62 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.80

$$\int x^4(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{e\,x^{27}}{27}+\left(\frac{d}{25}+\frac{2\,e}{5}\right)x^{25}+\left(\frac{10\,d}{23}+\frac{45\,e}{23}\right)x^{23} + \left(\frac{15\,d}{7}+\frac{40\,e}{7}\right)x^{21}+\left(\frac{120\,d}{19}+\frac{210\,e}{19}\right)x^{19} + \left(\frac{210\,d}{17}+\frac{252\,e}{17}\right)x^{17}+\left(\frac{84\,d}{5}+14\,e\right)x^{15} + \left(\frac{210\,d}{13}+\frac{120\,e}{13}\right)x^{13}+\left(\frac{120\,d}{11}+\frac{45\,e}{11}\right)x^{11} + \left(5\,d+\frac{10\,e}{9}\right)x^9+\left(\frac{10\,d}{7}+\frac{e}{7}\right)x^7+\frac{d\,x^5}{5}$$

input
```
int(x^4*(d + e*x^2)*(2*x^2 + x^4 + 1)^5,x)
```

output
```
x^7*((10*d)/7 + e/7) + x^9*(5*d + (10*e)/9) + x^25*(d/25 + (2*e)/5) + x^21
*((15*d)/7 + (40*e)/7) + x^15*((84*d)/5 + 14*e) + x^23*((10*d)/23 + (45*e)
/23) + x^11*((120*d)/11 + (45*e)/11) + x^13*((210*d)/13 + (120*e)/13) + x^
19*((120*d)/19 + (210*e)/19) + x^17*((210*d)/17 + (252*e)/17) + (d*x^5)/5
+ (e*x^27)/27
```

3.58 $\int x^3(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx$

3.58.1 Optimal result

Integrand size = 23, antiderivative size = 45

$$\int x^3\left(d+ex^2\right)\left(1+2x^2+x^4\right)^5\,dx=-\frac{1}{22}(d-e)\left(1+x^2\right)^{11}+\frac{1}{24}(d-2e)\left(1+x^2\right)^{12}+\frac{1}{26}e\left(1+x^2\right)^{13}$$

output
```
-1/22*(d-e)*(x^2+1)^11+1/24*(d-2*e)*(x^2+1)^12+1/26*e*(x^2+1)^13
```

3.58.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 151 vs. 2(45) = 90.

Time = 0.02 (sec) , antiderivative size = 151, normalized size of antiderivative = 3.36

$$\int x^3\left(d+ex^2\right)\left(1+2x^2+x^4\right)^5\,dx=\frac{dx^4}{4}+\frac{1}{6}(10d+e)x^6+\frac{5}{8}(9d+2e)x^8+\frac{3}{2}(8d+3e)x^{10}+\frac{5}{2}(7d+4e)x^{12}+3(6d+5e)x^{14}+\frac{21}{8}(5d+6e)x^{16}+\frac{5}{3}(4d+7e)x^{18}+\frac{3}{4}(3d+8e)x^{20}+\frac{5}{22}(2d+9e)x^{22}+\frac{1}{24}(d+10e)x^{24}+\frac{ex^{26}}{26}$$

input
```
Integrate[x^3*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

output
```
(d*x^4)/4 + ((10*d + e)*x^6)/6 + (5*(9*d + 2*e)*x^8)/8 + (3*(8*d + 3*e)*x^
10)/2 + (5*(7*d + 4*e)*x^12)/2 + 3*(6*d + 5*e)*x^14 + (21*(5*d + 6*e)*x^16
)/8 + (5*(4*d + 7*e)*x^18)/3 + (3*(3*d + 8*e)*x^20)/4 + (5*(2*d + 9*e)*x^2
2)/22 + ((d + 10*e)*x^24)/24 + (e*x^26)/26
```

3.58.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1380, 354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(x^4+2 x^2+1\right)^5 \left(d+e x^2\right) \, dx$$

$$\Big\downarrow \text{1380}$$

$$\int x^3 \left(x^2+1\right)^{10} \left(d+e x^2\right)dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2} \int x^2 \left(x^2+1\right)^{10} \left(e x^2+d\right)dx^2$$

$$\Big\downarrow \text{85}$$

$$\frac{1}{2} \int \left(e \left(x^2+1\right)^{12}+(d-2 e) \left(x^2+1\right)^{11}+(e-d) \left(x^2+1\right)^{10}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(\frac{1}{12} \left(x^2+1\right)^{12} (d-2 e)-\frac{1}{11} \left(x^2+1\right)^{11} (d-e)+\frac{1}{13} e \left(x^2+1\right)^{13}\right)$$

input
```
Int[x^3*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

output
```
(-1/11*((d - e)*(1 + x^2)^11) + ((d - 2*e)*(1 + x^2)^12)/12 + (e*(1 + x^2)
^13)/13)/2
```

3.58.3.1 Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.58.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(39) = 78$.

Time = 0.14 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.76

method	result
norman	$\frac{x^{26}e}{26}+\left(\frac{5d}{11}+\frac{45e}{22}\right)x^{22}+\left(\frac{d}{24}+\frac{5e}{12}\right)x^{24}+\left(\frac{105d}{8}+\frac{63e}{4}\right)x^{16}+\left(\frac{20d}{3}+\frac{35e}{3}\right)x^{18}+\left(\frac{9d}{4}+6e\right)x^{20}+$
default	$\frac{x^{26}e}{26}+\frac{(d+10e)x^{24}}{24}+\frac{(10d+45e)x^{22}}{22}+\frac{(45d+120e)x^{20}}{20}+\frac{(120d+210e)x^{18}}{18}+\frac{(210d+252e)x^{16}}{16}+\frac{(252d+210e)x^{14}}{14}$
risch	$\frac{1}{26}x^{26}e+\frac{1}{24}x^{24}d+\frac{5}{12}x^{24}e+\frac{5}{11}x^{22}d+\frac{45}{22}e\,x^{22}+\frac{9}{4}d\,x^{20}+6e\,x^{20}+\frac{20}{3}d\,x^{18}+\frac{35}{3}e\,x^{18}+\frac{105}{8}d\,x$
parallelrisch	$\frac{1}{26}x^{26}e+\frac{1}{24}x^{24}d+\frac{5}{12}x^{24}e+\frac{5}{11}x^{22}d+\frac{45}{22}e\,x^{22}+\frac{9}{4}d\,x^{20}+6e\,x^{20}+\frac{20}{3}d\,x^{18}+\frac{35}{3}e\,x^{18}+\frac{105}{8}d\,x$
gosper	$x^4(132e\,x^{22}+143d\,x^{20}+1430e\,x^{20}+1560d\,x^{18}+7020e\,x^{18}+7722d\,x^{16}+20592e\,x^{16}+22880d\,x^{14}+40040e\,x^{14}+45045d\,x^{12}+540$

input
```
int(x^3*(e*x^2+d)*(x^4+2*x^2+1)^5,x,method=_RETURNVERBOSE)
```

output
```
1/26*x^26*e+(5/11*d+45/22*e)*x^22+(1/24*d+5/12*e)*x^24+(105/8*d+63/4*e)*x^
16+(20/3*d+35/3*e)*x^18+(9/4*d+6*e)*x^20+(35/2*d+10*e)*x^12+(18*d+15*e)*x^
14+(45/8*d+5/4*e)*x^8+(12*d+9/2*e)*x^10+1/4*d*x^4+(5/3*d+1/6*e)*x^6
```

3.58.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(39) = 78.

Time = 0.23 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.87

$$\int x^3(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{26}\,ex^{26} + \frac{1}{24}\,(d+10\,e)x^{24} + \frac{5}{22}\,(2\,d+9\,e)x^{22} + \frac{3}{4}\,(3\,d+8\,e)x^{20} + \frac{5}{3}\,(4\,d+7\,e)x^{18} + \frac{21}{8}\,(5\,d+6\,e)x^{16} + 3\,(6\,d+5\,e)x^{14} + \frac{5}{2}\,(7\,d+4\,e)x^{12} + \frac{3}{2}\,(8\,d+3\,e)x^{10} + \frac{5}{8}\,(9\,d+2\,e)x^{8} + \frac{1}{6}\,(10\,d+e)x^{6} + \frac{1}{4}\,dx^{4}$$

input
```
integrate(x^3*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

output
```
1/26*e*x^26 + 1/24*(d + 10*e)*x^24 + 5/22*(2*d + 9*e)*x^22 + 3/4*(3*d + 8*
e)*x^20 + 5/3*(4*d + 7*e)*x^18 + 21/8*(5*d + 6*e)*x^16 + 3*(6*d + 5*e)*x^1
4 + 5/2*(7*d + 4*e)*x^12 + 3/2*(8*d + 3*e)*x^10 + 5/8*(9*d + 2*e)*x^8 + 1/
6*(10*d + e)*x^6 + 1/4*d*x^4
```

3.58.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(36) = 72.

Time = 0.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 3.02

$$\int x^3(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{dx^4}{4} + \frac{ex^{26}}{26} + x^{24}\left(\frac{d}{24}+\frac{5e}{12}\right) + x^{22}\cdot\left(\frac{5d}{11}+\frac{45e}{22}\right) + x^{20}\cdot\left(\frac{9d}{4}+6e\right) + x^{18}\cdot\left(\frac{20d}{3}+\frac{35e}{3}\right) + x^{16}\cdot\left(\frac{105d}{8}+\frac{63e}{4}\right) + x^{14}\cdot(18d+15e) + x^{12}\cdot\left(\frac{35d}{2}+10e\right) + x^{10}\cdot\left(12d+\frac{9e}{2}\right) + x^{8}\cdot\left(\frac{45d}{8}+\frac{5e}{4}\right) + x^{6}\cdot\left(\frac{5d}{3}+\frac{e}{6}\right)$$

input
```
integrate(x**3*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

output
```
d*x**4/4 + e*x**26/26 + x**24*(d/24 + 5*e/12) + x**22*(5*d/11 + 45*e/22) +
 x**20*(9*d/4 + 6*e) + x**18*(20*d/3 + 35*e/3) + x**16*(105*d/8 + 63*e/4)
+ x**14*(18*d + 15*e) + x**12*(35*d/2 + 10*e) + x**10*(12*d + 9*e/2) + x**
8*(45*d/8 + 5*e/4) + x**6*(5*d/3 + e/6)
```

3.58.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(39) = 78$.

Time = 0.19 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.87

$$\int x^3(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{26}\,ex^{26} + \frac{1}{24}\,(d+10\,e)x^{24} + \frac{5}{22}\,(2\,d+9\,e)x^{22} + \frac{3}{4}\,(3\,d+8\,e)x^{20} + \frac{5}{3}\,(4\,d+7\,e)x^{18} + \frac{21}{8}\,(5\,d+6\,e)x^{16} + 3\,(6\,d+5\,e)x^{14} + \frac{5}{2}\,(7\,d+4\,e)x^{12} + \frac{3}{2}\,(8\,d+3\,e)x^{10} + \frac{5}{8}\,(9\,d+2\,e)x^{8} + \frac{1}{6}\,(10\,d+e)x^{6} + \frac{1}{4}\,dx^{4}$$

input
```
integrate(x^3*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

output
```
1/26*e*x^26 + 1/24*(d + 10*e)*x^24 + 5/22*(2*d + 9*e)*x^22 + 3/4*(3*d + 8*
e)*x^20 + 5/3*(4*d + 7*e)*x^18 + 21/8*(5*d + 6*e)*x^16 + 3*(6*d + 5*e)*x^1
4 + 5/2*(7*d + 4*e)*x^12 + 3/2*(8*d + 3*e)*x^10 + 5/8*(9*d + 2*e)*x^8 + 1/
6*(10*d + e)*x^6 + 1/4*d*x^4
```

3.58.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(39) = 78$.

Time = 0.26 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.96

$$\int x^3(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{26}\,ex^{26} + \frac{1}{24}\,dx^{24} + \frac{5}{12}\,ex^{24} + \frac{5}{11}\,dx^{22} + \frac{45}{22}\,ex^{22} + \frac{9}{4}\,dx^{20} + 6\,ex^{20} + \frac{20}{3}\,dx^{18} + \frac{35}{3}\,ex^{18} + \frac{105}{8}\,dx^{16} + \frac{63}{4}\,ex^{16} + 18\,dx^{14} + 15\,ex^{14} + \frac{35}{2}\,dx^{12} + 10\,ex^{12} + 12\,dx^{10} + \frac{9}{2}\,ex^{10} + \frac{45}{8}\,dx^{8} + \frac{5}{4}\,ex^{8} + \frac{5}{3}\,dx^{6} + \frac{1}{6}\,ex^{6} + \frac{1}{4}\,dx^{4}$$

input
```
integrate(x^3*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

output
```
1/26*e*x^26 + 1/24*d*x^24 + 5/12*e*x^24 + 5/11*d*x^22 + 45/22*e*x^22 + 9/4
*d*x^20 + 6*e*x^20 + 20/3*d*x^18 + 35/3*e*x^18 + 105/8*d*x^16 + 63/4*e*x^1
6 + 18*d*x^14 + 15*e*x^14 + 35/2*d*x^12 + 10*e*x^12 + 12*d*x^10 + 9/2*e*x^
10 + 45/8*d*x^8 + 5/4*e*x^8 + 5/3*d*x^6 + 1/6*e*x^6 + 1/4*d*x^4
```

3.58.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.73

$$\int x^3(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{e\,x^{26}}{26}+\left(\frac{d}{24}+\frac{5\,e}{12}\right)x^{24}+\left(\frac{5\,d}{11}+\frac{45\,e}{22}\right)x^{22} + \left(\frac{9\,d}{4}+6\,e\right)x^{20}+\left(\frac{20\,d}{3}+\frac{35\,e}{3}\right)x^{18} + \left(\frac{105\,d}{8}+\frac{63\,e}{4}\right)x^{16}+(18\,d+15\,e)\,x^{14} + \left(\frac{35\,d}{2}+10\,e\right)x^{12}+\left(12\,d+\frac{9\,e}{2}\right)x^{10} + \left(\frac{45\,d}{8}+\frac{5\,e}{4}\right)x^{8}+\left(\frac{5\,d}{3}+\frac{e}{6}\right)x^{6}+\frac{d\,x^4}{4}$$

input
```
int(x^3*(d + e*x^2)*(2*x^2 + x^4 + 1)^5,x)
```

output
```
x^6*((5*d)/3 + e/6) + x^10*(12*d + (9*e)/2) + x^20*((9*d)/4 + 6*e) + x^14*
(18*d + 15*e) + x^12*((35*d)/2 + 10*e) + x^24*(d/24 + (5*e)/12) + x^8*((45
*d)/8 + (5*e)/4) + x^18*((20*d)/3 + (35*e)/3) + x^22*((5*d)/11 + (45*e)/22
) + x^16*((105*d)/8 + (63*e)/4) + (d*x^4)/4 + (e*x^26)/26
```

3.59 $\int x^2(d+ex^2)(1+2x^2+x^4)^5\,dx$

3.59.1 Optimal result

Integrand size = 23, antiderivative size = 153

$$\begin{aligned}\int x^2(d+ex^2)(1+2x^2+x^4)^5\,dx = {}& \frac{dx^3}{3}+\frac{1}{5}(10d+e)x^5+\frac{5}{7}(9d+2e)x^7\\ &+\frac{5}{3}(8d+3e)x^9+\frac{30}{11}(7d+4e)x^{11}+\frac{42}{13}(6d+5e)x^{13}\\ &+\frac{14}{5}(5d+6e)x^{15}+\frac{30}{17}(4d+7e)x^{17}+\frac{15}{19}(3d+8e)x^{19}\\ &+\frac{5}{21}(2d+9e)x^{21}+\frac{1}{23}(d+10e)x^{23}+\frac{ex^{25}}{25}\end{aligned}$$

output
```
1/3*d*x^3+1/5*(10*d+e)*x^5+5/7*(9*d+2*e)*x^7+5/3*(8*d+3*e)*x^9+30/11*(7*d+
4*e)*x^11+42/13*(6*d+5*e)*x^13+14/5*(5*d+6*e)*x^15+30/17*(4*d+7*e)*x^17+15
/19*(3*d+8*e)*x^19+5/21*(2*d+9*e)*x^21+1/23*(d+10*e)*x^23+1/25*e*x^25
```

3.59.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00

$$\begin{aligned}\int x^2(d+ex^2)(1+2x^2+x^4)^5\,dx = {}& \frac{dx^3}{3}+\frac{1}{5}(10d+e)x^5+\frac{5}{7}(9d+2e)x^7\\ &+\frac{5}{3}(8d+3e)x^9+\frac{30}{11}(7d+4e)x^{11}+\frac{42}{13}(6d+5e)x^{13}\\ &+\frac{14}{5}(5d+6e)x^{15}+\frac{30}{17}(4d+7e)x^{17}+\frac{15}{19}(3d+8e)x^{19}\\ &+\frac{5}{21}(2d+9e)x^{21}+\frac{1}{23}(d+10e)x^{23}+\frac{ex^{25}}{25}\end{aligned}$$

input
```
Integrate[x^2*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

output
```
(d*x^3)/3 + ((10*d + e)*x^5)/5 + (5*(9*d + 2*e)*x^7)/7 + (5*(8*d + 3*e)*x^
9)/3 + (30*(7*d + 4*e)*x^11)/11 + (42*(6*d + 5*e)*x^13)/13 + (14*(5*d + 6*
e)*x^15)/5 + (30*(4*d + 7*e)*x^17)/17 + (15*(3*d + 8*e)*x^19)/19 + (5*(2*d
 + 9*e)*x^21)/21 + ((d + 10*e)*x^23)/23 + (e*x^25)/25
```

3.59.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1380, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(x^4+2x^2+1\right)^5\left(d+ex^2\right)dx$$

$$\Big\downarrow \text{ 1380}$$

$$\int x^2\left(x^2+1\right)^{10}\left(d+ex^2\right)dx$$

$$\Big\downarrow \text{ 355}$$

$$\int \left(x^{22}(d+10e)+5x^{20}(2d+9e)+15x^{18}(3d+8e)+30x^{16}(4d+7e)+42x^{14}(5d+6e)+42x^{12}(6d+5e)+30x^{10}(\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{23}x^{23}(d+10e)+\frac{5}{21}x^{21}(2d+9e)+\frac{15}{19}x^{19}(3d+8e)+\frac{30}{17}x^{17}(4d+7e)+\frac{14}{5}x^{15}(5d+6e)+\frac{42}{13}x^{13}(6d+5e)+\frac{30}{11}x^{11}(7d+4e)+\frac{5}{3}x^9(8d+3e)+\frac{5}{7}x^7(9d+2e)+\frac{1}{5}x^5(10d+e)+\frac{dx^3}{3}+\frac{ex^{25}}{25}$$

input
```
Int[x^2*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

output
```
(d*x^3)/3 + ((10*d + e)*x^5)/5 + (5*(9*d + 2*e)*x^7)/7 + (5*(8*d + 3*e)*x^
9)/3 + (30*(7*d + 4*e)*x^11)/11 + (42*(6*d + 5*e)*x^13)/13 + (14*(5*d + 6*
e)*x^15)/5 + (30*(4*d + 7*e)*x^17)/17 + (15*(3*d + 8*e)*x^19)/19 + (5*(2*d
 + 9*e)*x^21)/21 + ((d + 10*e)*x^23)/23 + (e*x^25)/25
```

3.59.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.59.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.81

method	result
norman	$\left(\frac{d}{23}+\frac{10e}{23}\right)x^{23}+\frac{e\,x^{25}}{25}+\left(\frac{10d}{21}+\frac{15e}{7}\right)x^{21}+\left(14d+\frac{84e}{5}\right)x^{15}+\left(\frac{120d}{17}+\frac{210e}{17}\right)x^{17}+\left(\frac{45d}{19}+\frac{120e}{19}\right)$
default	$\frac{e\,x^{25}}{25}+\frac{(d+10e)x^{23}}{23}+\frac{(10d+45e)x^{21}}{21}+\frac{(45d+120e)x^{19}}{19}+\frac{(120d+210e)x^{17}}{17}+\frac{(210d+252e)x^{15}}{15}+\frac{(252d+210e)x^{13}}{13}$
risch	$\frac{1}{25}e\,x^{25}+\frac{1}{23}x^{23}d+\frac{10}{23}e\,x^{23}+\frac{10}{21}x^{21}d+\frac{15}{7}e\,x^{21}+\frac{45}{19}x^{19}d+\frac{120}{19}x^{19}e+\frac{120}{17}x^{17}d+\frac{210}{17}x^{17}e+14$
parallelrisch	$\frac{1}{25}e\,x^{25}+\frac{1}{23}x^{23}d+\frac{10}{23}e\,x^{23}+\frac{10}{21}x^{21}d+\frac{15}{7}e\,x^{21}+\frac{45}{19}x^{19}d+\frac{120}{19}x^{19}e+\frac{120}{17}x^{17}d+\frac{210}{17}x^{17}e+14$
gosper	$x^{3}(22309287e\,x^{22}+24249225d\,x^{20}+242492250e\,x^{20}+265586750d\,x^{18}+1195140375e\,x^{18}+1320944625d\,x^{16}+3522519000e\,x^{16}$

input
```
int(x^2*(e*x^2+d)*(x^4+2*x^2+1)^5,x,method=_RETURNVERBOSE)
```

output
```
(1/23*d+10/23*e)*x^23+1/25*e*x^25+(10/21*d+15/7*e)*x^21+(14*d+84/5*e)*x^15
+(120/17*d+210/17*e)*x^17+(45/19*d+120/19*e)*x^19+(45/7*d+10/7*e)*x^7+(40/
3*d+5*e)*x^9+(210/11*d+120/11*e)*x^11+(252/13*d+210/13*e)*x^13+1/3*x^3*d+(
2*d+1/5*e)*x^5
```

3.59.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.84

$$\int x^2(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{25}\,ex^{25} + \frac{1}{23}\,(d+10\,e)x^{23} + \frac{5}{21}\,(2\,d+9\,e)x^{21} + \frac{15}{19}\,(3\,d+8\,e)x^{19} + \frac{30}{17}\,(4\,d+7\,e)x^{17} + \frac{14}{5}\,(5\,d+6\,e)x^{15} + \frac{42}{13}\,(6\,d+5\,e)x^{13} + \frac{30}{11}\,(7\,d+4\,e)x^{11} + \frac{5}{3}\,(8\,d+3\,e)x^9 + \frac{5}{7}\,(9\,d+2\,e)x^7 + \frac{1}{5}\,(10\,d+e)x^5 + \frac{1}{3}\,dx^3$$

input
```
integrate(x^2*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

output
```
1/25*e*x^25 + 1/23*(d + 10*e)*x^23 + 5/21*(2*d + 9*e)*x^21 + 15/19*(3*d +
8*e)*x^19 + 30/17*(4*d + 7*e)*x^17 + 14/5*(5*d + 6*e)*x^15 + 42/13*(6*d +
5*e)*x^13 + 30/11*(7*d + 4*e)*x^11 + 5/3*(8*d + 3*e)*x^9 + 5/7*(9*d + 2*e)
*x^7 + 1/5*(10*d + e)*x^5 + 1/3*d*x^3
```

3.59.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.91

$$\int x^2(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{dx^3}{3} + \frac{ex^{25}}{25} + x^{23}\left(\frac{d}{23}+\frac{10e}{23}\right) + x^{21}\cdot\left(\frac{10d}{21}+\frac{15e}{7}\right) + x^{19}\cdot\left(\frac{45d}{19}+\frac{120e}{19}\right) + x^{17}\cdot\left(\frac{120d}{17}+\frac{210e}{17}\right) + x^{15}\cdot\left(14d+\frac{84e}{5}\right) + x^{13}\cdot\left(\frac{252d}{13}+\frac{210e}{13}\right) + x^{11}\cdot\left(\frac{210d}{11}+\frac{120e}{11}\right) + x^9\cdot\left(\frac{40d}{3}+5e\right) + x^7\cdot\left(\frac{45d}{7}+\frac{10e}{7}\right) + x^5\cdot\left(2d+\frac{e}{5}\right)$$

input
```
integrate(x**2*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

output
```
d*x**3/3 + e*x**25/25 + x**23*(d/23 + 10*e/23) + x**21*(10*d/21 + 15*e/7)
+ x**19*(45*d/19 + 120*e/19) + x**17*(120*d/17 + 210*e/17) + x**15*(14*d +
 84*e/5) + x**13*(252*d/13 + 210*e/13) + x**11*(210*d/11 + 120*e/11) + x**
9*(40*d/3 + 5*e) + x**7*(45*d/7 + 10*e/7) + x**5*(2*d + e/5)
```

3.59.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.84

$$\int x^2(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{25}\,ex^{25}+\frac{1}{23}\,(d+10\,e)x^{23}+\frac{5}{21}\,(2\,d+9\,e)x^{21}+\frac{15}{19}\,(3\,d+8\,e)x^{19}+\frac{30}{17}\,(4\,d+7\,e)x^{17}+\frac{14}{5}\,(5\,d+6\,e)x^{15}+\frac{42}{13}\,(6\,d+5\,e)x^{13}+\frac{30}{11}\,(7\,d+4\,e)x^{11}+\frac{5}{3}\,(8\,d+3\,e)x^{9}+\frac{5}{7}\,(9\,d+2\,e)x^{7}+\frac{1}{5}\,(10\,d+e)x^{5}+\frac{1}{3}\,dx^{3}$$

input
```
integrate(x^2*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

output
```
1/25*e*x^25 + 1/23*(d + 10*e)*x^23 + 5/21*(2*d + 9*e)*x^21 + 15/19*(3*d +
8*e)*x^19 + 30/17*(4*d + 7*e)*x^17 + 14/5*(5*d + 6*e)*x^15 + 42/13*(6*d +
5*e)*x^13 + 30/11*(7*d + 4*e)*x^11 + 5/3*(8*d + 3*e)*x^9 + 5/7*(9*d + 2*e)
*x^7 + 1/5*(10*d + e)*x^5 + 1/3*d*x^3
```

3.59.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.87

$$\int x^2(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{25}\,ex^{25}+\frac{1}{23}\,dx^{23}+\frac{10}{23}\,ex^{23}+\frac{10}{21}\,dx^{21}+\frac{15}{7}\,ex^{21}+\frac{45}{19}\,dx^{19}+\frac{120}{19}\,ex^{19}+\frac{120}{17}\,dx^{17}+\frac{210}{17}\,ex^{17}+14\,dx^{15}+\frac{84}{5}\,ex^{15}+\frac{252}{13}\,dx^{13}+\frac{210}{13}\,ex^{13}+\frac{210}{11}\,dx^{11}+\frac{120}{11}\,ex^{11}+\frac{40}{3}\,dx^{9}+5\,ex^{9}+\frac{45}{7}\,dx^{7}+\frac{10}{7}\,ex^{7}+2\,dx^{5}+\frac{1}{5}\,ex^{5}+\frac{1}{3}\,dx^{3}$$

input
```
integrate(x^2*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

output
```
1/25*e*x^25 + 1/23*d*x^23 + 10/23*e*x^23 + 10/21*d*x^21 + 15/7*e*x^21 + 45
/19*d*x^19 + 120/19*e*x^19 + 120/17*d*x^17 + 210/17*e*x^17 + 14*d*x^15 + 8
4/5*e*x^15 + 252/13*d*x^13 + 210/13*e*x^13 + 210/11*d*x^11 + 120/11*e*x^11
 + 40/3*d*x^9 + 5*e*x^9 + 45/7*d*x^7 + 10/7*e*x^7 + 2*d*x^5 + 1/5*e*x^5 +
1/3*d*x^3
```

3.59.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.80

$$\int x^2(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{e\,x^{25}}{25}+\left(\frac{d}{23}+\frac{10\,e}{23}\right)x^{23}+\left(\frac{10\,d}{21}+\frac{15\,e}{7}\right)x^{21} + \left(\frac{45\,d}{19}+\frac{120\,e}{19}\right)x^{19}+\left(\frac{120\,d}{17}+\frac{210\,e}{17}\right)x^{17} + \left(14\,d+\frac{84\,e}{5}\right)x^{15}+\left(\frac{252\,d}{13}+\frac{210\,e}{13}\right)x^{13} + \left(\frac{210\,d}{11}+\frac{120\,e}{11}\right)x^{11}+\left(\frac{40\,d}{3}+5\,e\right)x^{9} + \left(\frac{45\,d}{7}+\frac{10\,e}{7}\right)x^{7}+\left(2\,d+\frac{e}{5}\right)x^{5}+\frac{d\,x^3}{3}$$

input
```
int(x^2*(d + e*x^2)*(2*x^2 + x^4 + 1)^5,x)
```

output
```
x^5*(2*d + e/5) + x^9*((40*d)/3 + 5*e) + x^21*((10*d)/21 + (15*e)/7) + x^7
*((45*d)/7 + (10*e)/7) + x^23*(d/23 + (10*e)/23) + x^15*(14*d + (84*e)/5)
+ x^19*((45*d)/19 + (120*e)/19) + x^11*((210*d)/11 + (120*e)/11) + x^17*((
120*d)/17 + (210*e)/17) + x^13*((252*d)/13 + (210*e)/13) + (d*x^3)/3 + (e*
x^25)/25
```

3.60 $\int x(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx$

3.60.1 Optimal result

Integrand size = 21, antiderivative size = 29

$$\int x(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx=\frac{1}{22}(d-e)\left(1+x^2\right)^{11}+\frac{1}{24}e\left(1+x^2\right)^{12}$$

output
```
1/22*(d-e)*(x^2+1)^11+1/24*e*(x^2+1)^12
```

3.60.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 149 vs. 2(29) = 58.

Time = 0.01 (sec) , antiderivative size = 149, normalized size of antiderivative = 5.14

$$\int x(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx=\frac{dx^2}{2}+\frac{1}{4}(10d+e)x^4+\frac{5}{6}(9d+2e)x^6+\frac{15}{8}(8d+3e)x^8+3(7d+4e)x^{10}+\frac{7}{2}(6d+5e)x^{12}+3(5d+6e)x^{14}+\frac{15}{8}(4d+7e)x^{16}+\frac{5}{6}(3d+8e)x^{18}+\frac{1}{4}(2d+9e)x^{20}+\frac{1}{22}(d+10e)x^{22}+\frac{ex^{24}}{24}$$

input
```
Integrate[x*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

output
```
(d*x^2)/2 + ((10*d + e)*x^4)/4 + (5*(9*d + 2*e)*x^6)/6 + (15*(8*d + 3*e)*x
^8)/8 + 3*(7*d + 4*e)*x^10 + (7*(6*d + 5*e)*x^12)/2 + 3*(5*d + 6*e)*x^14 +
 (15*(4*d + 7*e)*x^16)/8 + (5*(3*d + 8*e)*x^18)/6 + ((2*d + 9*e)*x^20)/4 +
 ((d + 10*e)*x^22)/22 + (e*x^24)/24
```

3.60.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1380, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(x^4+2x^2+1\right)^5\left(d+ex^2\right)\,dx$$

$$\Big\downarrow \text{1380}$$

$$\int x\left(x^2+1\right)^{10}\left(d+ex^2\right)dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \left(x^2+1\right)^{10}\left(ex^2+d\right)dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(e\left(x^2+1\right)^{11}+(d-e)\left(x^2+1\right)^{10}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{11}\left(x^2+1\right)^{11}(d-e)+\frac{1}{12}e\left(x^2+1\right)^{12}\right)$$

input
```
Int[x*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

output
```
(((d - e)*(1 + x^2)^11)/11 + (e*(1 + x^2)^12)/12)/2
```

3.60.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.60.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(25) = 50$.

Time = 0.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 4.28

method	result
norman	$\frac{x^{24}e}{24}+\left(\frac{d}{2}+\frac{9e}{4}\right)x^{20}+\left(\frac{d}{22}+\frac{5e}{11}\right)x^{22}+\left(\frac{5d}{2}+\frac{20e}{3}\right)x^{18}+(15d+18e)x^{14}+\left(\frac{15d}{2}+\frac{105e}{8}\right)x^{16}+$
default	$\frac{x^{24}e}{24}+\frac{(d+10e)x^{22}}{22}+\frac{(10d+45e)x^{20}}{20}+\frac{(45d+120e)x^{18}}{18}+\frac{(120d+210e)x^{16}}{16}+\frac{(210d+252e)x^{14}}{14}+\frac{(252d+210e)x^{12}}{12}$
risch	$\frac{1}{24}x^{24}e+\frac{1}{22}x^{22}d+\frac{5}{11}e\,x^{22}+\frac{1}{2}d\,x^{20}+\frac{9}{4}e\,x^{20}+\frac{5}{2}d\,x^{18}+\frac{20}{3}e\,x^{18}+\frac{15}{2}d\,x^{16}+\frac{105}{8}e\,x^{16}+15d\,x$
parallelrisch	$\frac{1}{24}x^{24}e+\frac{1}{22}x^{22}d+\frac{5}{11}e\,x^{22}+\frac{1}{2}d\,x^{20}+\frac{9}{4}e\,x^{20}+\frac{5}{2}d\,x^{18}+\frac{20}{3}e\,x^{18}+\frac{15}{2}d\,x^{16}+\frac{105}{8}e\,x^{16}+15d\,x$
gosper	$\frac{x^{2}\left(11e\,x^{22}+12d\,x^{20}+120e\,x^{20}+132d\,x^{18}+594e\,x^{18}+660d\,x^{16}+1760e\,x^{16}+1980d\,x^{14}+3465e\,x^{14}+3960d\,x^{12}+4752e\,x^{12}+5544\right.}{264}$

input
```
int(x*(e*x^2+d)*(x^4+2*x^2+1)^5,x,method=_RETURNVERBOSE)
```

output
```
1/24*x^24*e+(1/2*d+9/4*e)*x^20+(1/22*d+5/11*e)*x^22+(5/2*d+20/3*e)*x^18+(1
5*d+18*e)*x^14+(15/2*d+105/8*e)*x^16+(15/2*d+5/3*e)*x^6+(15*d+45/8*e)*x^8+
(21*d+12*e)*x^10+(21*d+35/2*e)*x^12+1/2*d*x^2+(5/2*d+1/4*e)*x^4
```

3.60.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(25) = 50$.

Time = 0.26 (sec) , antiderivative size = 129, normalized size of antiderivative = 4.45

$$\int x(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{24}\,ex^{24} + \frac{1}{22}\,(d+10\,e)x^{22} + \frac{1}{4}\,(2\,d+9\,e)x^{20} + \frac{5}{6}\,(3\,d+8\,e)x^{18} + \frac{15}{8}\,(4\,d+7\,e)x^{16} + 3\,(5\,d+6\,e)x^{14} + \frac{7}{2}\,(6\,d+5\,e)x^{12} + 3\,(7\,d+4\,e)x^{10} + \frac{15}{8}\,(8\,d+3\,e)x^{8} + \frac{5}{6}\,(9\,d+2\,e)x^{6} + \frac{1}{4}\,(10\,d+e)x^{4} + \frac{1}{2}\,dx^{2}$$

input
```
integrate(x*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

output
```
1/24*e*x^24 + 1/22*(d + 10*e)*x^22 + 1/4*(2*d + 9*e)*x^20 + 5/6*(3*d + 8*e
)*x^18 + 15/8*(4*d + 7*e)*x^16 + 3*(5*d + 6*e)*x^14 + 7/2*(6*d + 5*e)*x^12
 + 3*(7*d + 4*e)*x^10 + 15/8*(8*d + 3*e)*x^8 + 5/6*(9*d + 2*e)*x^6 + 1/4*(
10*d + e)*x^4 + 1/2*d*x^2
```

3.60.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(22) = 44$.

Time = 0.04 (sec) , antiderivative size = 133, normalized size of antiderivative = 4.59

$$\int x(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{dx^2}{2} + \frac{ex^{24}}{24} + x^{22}\left(\frac{d}{22}+\frac{5e}{11}\right) + x^{20}\left(\frac{d}{2}+\frac{9e}{4}\right) + x^{18}\cdot\left(\frac{5d}{2}+\frac{20e}{3}\right) + x^{16}\cdot\left(\frac{15d}{2}+\frac{105e}{8}\right) + x^{14}\cdot(15d+18e) + x^{12}\cdot\left(21d+\frac{35e}{2}\right) + x^{10}\cdot(21d+12e) + x^{8}\cdot\left(15d+\frac{45e}{8}\right) + x^{6}\cdot\left(\frac{15d}{2}+\frac{5e}{3}\right) + x^{4}\cdot\left(\frac{5d}{2}+\frac{e}{4}\right)$$

input
```
integrate(x*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

output
```
d*x**2/2 + e*x**24/24 + x**22*(d/22 + 5*e/11) + x**20*(d/2 + 9*e/4) + x**1
8*(5*d/2 + 20*e/3) + x**16*(15*d/2 + 105*e/8) + x**14*(15*d + 18*e) + x**1
2*(21*d + 35*e/2) + x**10*(21*d + 12*e) + x**8*(15*d + 45*e/8) + x**6*(15*
d/2 + 5*e/3) + x**4*(5*d/2 + e/4)
```

3.60.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(25) = 50$.

Time = 0.19 (sec) , antiderivative size = 129, normalized size of antiderivative = 4.45

$$\int x(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{24}\,ex^{24}+\frac{1}{22}\,(d+10\,e)x^{22}+\frac{1}{4}\,(2\,d+9\,e)x^{20}+\frac{5}{6}\,(3\,d+8\,e)x^{18}+\frac{15}{8}\,(4\,d+7\,e)x^{16}+3\,(5\,d+6\,e)x^{14}+\frac{7}{2}\,(6\,d+5\,e)x^{12}+3\,(7\,d+4\,e)x^{10}+\frac{15}{8}\,(8\,d+3\,e)x^{8}+\frac{5}{6}\,(9\,d+2\,e)x^{6}+\frac{1}{4}\,(10\,d+e)x^{4}+\frac{1}{2}\,dx^{2}$$

input
```
integrate(x*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

output
```
1/24*e*x^24 + 1/22*(d + 10*e)*x^22 + 1/4*(2*d + 9*e)*x^20 + 5/6*(3*d + 8*e
)*x^18 + 15/8*(4*d + 7*e)*x^16 + 3*(5*d + 6*e)*x^14 + 7/2*(6*d + 5*e)*x^12
 + 3*(7*d + 4*e)*x^10 + 15/8*(8*d + 3*e)*x^8 + 5/6*(9*d + 2*e)*x^6 + 1/4*(
10*d + e)*x^4 + 1/2*d*x^2
```

3.60.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(25) = 50$.

Time = 0.26 (sec) , antiderivative size = 133, normalized size of antiderivative = 4.59

$$\int x(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{24}\,ex^{24}+\frac{1}{22}\,dx^{22}+\frac{5}{11}\,ex^{22}+\frac{1}{2}\,dx^{20}+\frac{9}{4}\,ex^{20}+\frac{5}{2}\,dx^{18}+\frac{20}{3}\,ex^{18}+\frac{15}{2}\,dx^{16}+\frac{105}{8}\,ex^{16}+15\,dx^{14}+18\,ex^{14}+21\,dx^{12}+\frac{35}{2}\,ex^{12}+21\,dx^{10}+12\,ex^{10}+15\,dx^{8}+\frac{45}{8}\,ex^{8}+\frac{15}{2}\,dx^{6}+\frac{5}{3}\,ex^{6}+\frac{5}{2}\,dx^{4}+\frac{1}{4}\,ex^{4}+\frac{1}{2}\,dx^{2}$$

input
```
integrate(x*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

output
```
1/24*e*x^24 + 1/22*d*x^22 + 5/11*e*x^22 + 1/2*d*x^20 + 9/4*e*x^20 + 5/2*d*
x^18 + 20/3*e*x^18 + 15/2*d*x^16 + 105/8*e*x^16 + 15*d*x^14 + 18*e*x^14 +
21*d*x^12 + 35/2*e*x^12 + 21*d*x^10 + 12*e*x^10 + 15*d*x^8 + 45/8*e*x^8 +
15/2*d*x^6 + 5/3*e*x^6 + 5/2*d*x^4 + 1/4*e*x^4 + 1/2*d*x^2
```

3.60.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 123, normalized size of antiderivative = 4.24

$$\begin{aligned}\int x(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx &= \frac{e\,x^{24}}{24}+\left(\frac{d}{22}+\frac{5\,e}{11}\right)x^{22}+\left(\frac{d}{2}+\frac{9\,e}{4}\right)x^{20}\\&\quad+\left(\frac{5\,d}{2}+\frac{20\,e}{3}\right)x^{18}+\left(\frac{15\,d}{2}+\frac{105\,e}{8}\right)x^{16}\\&\quad+(15\,d+18\,e)\,x^{14}+\left(21\,d+\frac{35\,e}{2}\right)x^{12}\\&\quad+(21\,d+12\,e)\,x^{10}+\left(15\,d+\frac{45\,e}{8}\right)x^{8}\\&\quad+\left(\frac{15\,d}{2}+\frac{5\,e}{3}\right)x^{6}+\left(\frac{5\,d}{2}+\frac{e}{4}\right)x^{4}+\frac{d\,x^2}{2}\end{aligned}$$

input
```
int(x*(d + e*x^2)*(2*x^2 + x^4 + 1)^5,x)
```

output
```
x^4*((5*d)/2 + e/4) + x^6*((15*d)/2 + (5*e)/3) + x^20*(d/2 + (9*e)/4) + x^
10*(21*d + 12*e) + x^14*(15*d + 18*e) + x^18*((5*d)/2 + (20*e)/3) + x^22*(
d/22 + (5*e)/11) + x^12*(21*d + (35*e)/2) + x^8*(15*d + (45*e)/8) + x^16*(
(15*d)/2 + (105*e)/8) + (d*x^2)/2 + (e*x^24)/24
```

3.61 $\int (d+ex^2)(1+2x^2+x^4)^5\,dx$

3.61.1 Optimal result

Integrand size = 20, antiderivative size = 143

$$\int (d+ex^2)(1+2x^2+x^4)^5\,dx = dx+\frac{1}{3}(10d+e)x^3+(9d+2e)x^5+\frac{15}{7}(8d+3e)x^7+\frac{10}{3}(7d+4e)x^9+\frac{42}{11}(6d+5e)x^{11}+\frac{42}{13}(5d+6e)x^{13}+2(4d+7e)x^{15}+\frac{15}{17}(3d+8e)x^{17}+\frac{5}{19}(2d+9e)x^{19}+\frac{1}{21}(d+10e)x^{21}+\frac{ex^{23}}{23}$$

output
```
d*x+1/3*(10*d+e)*x^3+(9*d+2*e)*x^5+15/7*(8*d+3*e)*x^7+10/3*(7*d+4*e)*x^9+4
2/11*(6*d+5*e)*x^11+42/13*(5*d+6*e)*x^13+2*(4*d+7*e)*x^15+15/17*(3*d+8*e)*
x^17+5/19*(2*d+9*e)*x^19+1/21*(d+10*e)*x^21+1/23*e*x^23
```

3.61.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00

$$\int (d+ex^2)(1+2x^2+x^4)^5\,dx = dx+\frac{1}{3}(10d+e)x^3+(9d+2e)x^5+\frac{15}{7}(8d+3e)x^7+\frac{10}{3}(7d+4e)x^9+\frac{42}{11}(6d+5e)x^{11}+\frac{42}{13}(5d+6e)x^{13}+2(4d+7e)x^{15}+\frac{15}{17}(3d+8e)x^{17}+\frac{5}{19}(2d+9e)x^{19}+\frac{1}{21}(d+10e)x^{21}+\frac{ex^{23}}{23}$$

input
```
Integrate[(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

output
```
d*x + ((10*d + e)*x^3)/3 + (9*d + 2*e)*x^5 + (15*(8*d + 3*e)*x^7)/7 + (10*
(7*d + 4*e)*x^9)/3 + (42*(6*d + 5*e)*x^11)/11 + (42*(5*d + 6*e)*x^13)/13 +
 2*(4*d + 7*e)*x^15 + (15*(3*d + 8*e)*x^17)/17 + (5*(2*d + 9*e)*x^19)/19 +
 ((d + 10*e)*x^21)/21 + (e*x^23)/23
```

3.61.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1380, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^4+2x^2+1\right)^5 \left(d+ex^2\right) dx$$

$$\Big\downarrow \text{1380}$$

$$\int \left(x^2+1\right)^{10} \left(d+ex^2\right) dx$$

$$\Big\downarrow \text{290}$$

$$\int \left(x^{20}(d+10e)+5x^{18}(2d+9e)+15x^{16}(3d+8e)+30x^{14}(4d+7e)+42x^{12}(5d+6e)+42x^{10}(6d+5e)+30x^8(\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{21}x^{21}(d+10e)+\frac{5}{19}x^{19}(2d+9e)+\frac{15}{17}x^{17}(3d+8e)+2x^{15}(4d+7e)+\frac{42}{13}x^{13}(5d+6e)+\frac{42}{11}x^{11}(6d+5e)+\frac{10}{3}x^9(7d+4e)+\frac{15}{7}x^7(8d+3e)+x^5(9d+2e)+\frac{1}{3}x^3(10d+e)+dx+\frac{ex^{23}}{23}$$

input
```
Int[(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

output
```
d*x + ((10*d + e)*x^3)/3 + (9*d + 2*e)*x^5 + (15*(8*d + 3*e)*x^7)/7 + (10*
(7*d + 4*e)*x^9)/3 + (42*(6*d + 5*e)*x^11)/11 + (42*(5*d + 6*e)*x^13)/13 +
 2*(4*d + 7*e)*x^15 + (15*(3*d + 8*e)*x^17)/17 + (5*(2*d + 9*e)*x^19)/19 +
 ((d + 10*e)*x^21)/21 + (e*x^23)/23
```

3.61.3.1 Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.61.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.85

method	result
norman	$dx+\left(\frac{10d}{3}+\frac{e}{3}\right)x^{3}+\left(9d+2e\right)x^{5}+\left(\frac{120d}{7}+\frac{45e}{7}\right)x^{7}+\left(\frac{70d}{3}+\frac{40e}{3}\right)x^{9}+\left(\frac{252d}{11}+\frac{210e}{11}\right)x^{11}+\left(\right.$
default	$\frac{e\,x^{23}}{23}+\frac{(d+10e)x^{21}}{21}+\frac{(10d+45e)x^{19}}{19}+\frac{(45d+120e)x^{17}}{17}+\frac{(120d+210e)x^{15}}{15}+\frac{(210d+252e)x^{13}}{13}+\frac{(252d+210e)x^{11}}{11}$
risch	$\frac{1}{23}e\,x^{23}+\frac{1}{21}x^{21}d+\frac{10}{21}e\,x^{21}+\frac{10}{19}x^{19}d+\frac{45}{19}x^{19}e+\frac{45}{17}x^{17}d+\frac{120}{17}x^{17}e+8x^{15}d+14x^{15}e+\frac{210}{13}x^{13}$
parallelrisch	$\frac{1}{23}e\,x^{23}+\frac{1}{21}x^{21}d+\frac{10}{21}e\,x^{21}+\frac{10}{19}x^{19}d+\frac{45}{19}x^{19}e+\frac{45}{17}x^{17}d+\frac{120}{17}x^{17}e+8x^{15}d+14x^{15}e+\frac{210}{13}x^{13}$
gosper	$x\left(969969e\,x^{22}+1062347d\,x^{20}+10623470e\,x^{20}+11741730d\,x^{18}+52837785e\,x^{18}+59053995d\,x^{16}+157477320e\,x^{16}+178474296d\right.$

input
```
int((e*x^2+d)*(x^4+2*x^2+1)^5,x,method=_RETURNVERBOSE)
```

output
```
d*x+(10/3*d+1/3*e)*x^3+(9*d+2*e)*x^5+(120/7*d+45/7*e)*x^7+(70/3*d+40/3*e)*
x^9+(252/11*d+210/11*e)*x^11+(210/13*d+252/13*e)*x^13+(8*d+14*e)*x^15+(45/
17*d+120/17*e)*x^17+(10/19*d+45/19*e)*x^19+(1/21*d+10/21*e)*x^21+1/23*e*x^
23
```

3.61.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.87

$$\int (d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{23}\,ex^{23}+\frac{1}{21}\,(d+10\,e)x^{21}+\frac{5}{19}\,(2\,d+9\,e)x^{19}+\frac{15}{17}\,(3\,d+8\,e)x^{17}+2\,(4\,d+7\,e)x^{15}+\frac{42}{13}\,(5\,d+6\,e)x^{13}+\frac{42}{11}\,(6\,d+5\,e)x^{11}+\frac{10}{3}\,(7\,d+4\,e)x^{9}+\frac{15}{7}\,(8\,d+3\,e)x^{7}+(9\,d+2\,e)x^{5}+\frac{1}{3}\,(10\,d+e)x^{3}+dx$$

input
```
integrate((e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

output
```
1/23*e*x^23 + 1/21*(d + 10*e)*x^21 + 5/19*(2*d + 9*e)*x^19 + 15/17*(3*d +
8*e)*x^17 + 2*(4*d + 7*e)*x^15 + 42/13*(5*d + 6*e)*x^13 + 42/11*(6*d + 5*e
)*x^11 + 10/3*(7*d + 4*e)*x^9 + 15/7*(8*d + 3*e)*x^7 + (9*d + 2*e)*x^5 + 1
/3*(10*d + e)*x^3 + d*x
```

3.61.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.94

$$\int (d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = dx+\frac{ex^{23}}{23}+x^{21}\left(\frac{d}{21}+\frac{10e}{21}\right)+x^{19}\cdot\left(\frac{10d}{19}+\frac{45e}{19}\right)+x^{17}\cdot\left(\frac{45d}{17}+\frac{120e}{17}\right)+x^{15}\cdot(8d+14e)+x^{13}\cdot\left(\frac{210d}{13}+\frac{252e}{13}\right)+x^{11}\cdot\left(\frac{252d}{11}+\frac{210e}{11}\right)+x^{9}\cdot\left(\frac{70d}{3}+\frac{40e}{3}\right)+x^{7}\cdot\left(\frac{120d}{7}+\frac{45e}{7}\right)+x^{5}\cdot(9d+2e)+x^{3}\cdot\left(\frac{10d}{3}+\frac{e}{3}\right)$$

input
```
integrate((e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

output
```
d*x + e*x**23/23 + x**21*(d/21 + 10*e/21) + x**19*(10*d/19 + 45*e/19) + x*
*17*(45*d/17 + 120*e/17) + x**15*(8*d + 14*e) + x**13*(210*d/13 + 252*e/13
) + x**11*(252*d/11 + 210*e/11) + x**9*(70*d/3 + 40*e/3) + x**7*(120*d/7 +
 45*e/7) + x**5*(9*d + 2*e) + x**3*(10*d/3 + e/3)
```

3.61.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.87

$$\int (d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{23}\,ex^{23}+\frac{1}{21}\,(d+10\,e)x^{21}+\frac{5}{19}\,(2\,d+9\,e)x^{19} + \frac{15}{17}\,(3\,d+8\,e)x^{17}+2\,(4\,d+7\,e)x^{15}+\frac{42}{13}\,(5\,d+6\,e)x^{13} + \frac{42}{11}\,(6\,d+5\,e)x^{11}+\frac{10}{3}\,(7\,d+4\,e)x^{9} + \frac{15}{7}\,(8\,d+3\,e)x^{7}+(9\,d+2\,e)x^{5}+\frac{1}{3}\,(10\,d+e)x^{3}+dx$$

input
```
integrate((e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

output
```
1/23*e*x^23 + 1/21*(d + 10*e)*x^21 + 5/19*(2*d + 9*e)*x^19 + 15/17*(3*d +
8*e)*x^17 + 2*(4*d + 7*e)*x^15 + 42/13*(5*d + 6*e)*x^13 + 42/11*(6*d + 5*e
)*x^11 + 10/3*(7*d + 4*e)*x^9 + 15/7*(8*d + 3*e)*x^7 + (9*d + 2*e)*x^5 + 1
/3*(10*d + e)*x^3 + d*x
```

3.61.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.91

$$\int (d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{23}\,ex^{23}+\frac{1}{21}\,dx^{21}+\frac{10}{21}\,ex^{21}+\frac{10}{19}\,dx^{19}+\frac{45}{19}\,ex^{19} + \frac{45}{17}\,dx^{17}+\frac{120}{17}\,ex^{17}+8\,dx^{15}+14\,ex^{15}+\frac{210}{13}\,dx^{13} + \frac{252}{13}\,ex^{13}+\frac{252}{11}\,dx^{11}+\frac{210}{11}\,ex^{11}+\frac{70}{3}\,dx^{9}+\frac{40}{3}\,ex^{9} + \frac{120}{7}\,dx^{7}+\frac{45}{7}\,ex^{7}+9\,dx^{5}+2\,ex^{5}+\frac{10}{3}\,dx^{3}+\frac{1}{3}\,ex^{3}+dx$$

input
```
integrate((e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

output
```
1/23*e*x^23 + 1/21*d*x^21 + 10/21*e*x^21 + 10/19*d*x^19 + 45/19*e*x^19 + 4
5/17*d*x^17 + 120/17*e*x^17 + 8*d*x^15 + 14*e*x^15 + 210/13*d*x^13 + 252/1
3*e*x^13 + 252/11*d*x^11 + 210/11*e*x^11 + 70/3*d*x^9 + 40/3*e*x^9 + 120/7
*d*x^7 + 45/7*e*x^7 + 9*d*x^5 + 2*e*x^5 + 10/3*d*x^3 + 1/3*e*x^3 + d*x
```

3.61.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.84

$$\int (d+ex^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{e\,x^{23}}{23}+\left(\frac{d}{21}+\frac{10\,e}{21}\right)x^{21}+\left(\frac{10\,d}{19}+\frac{45\,e}{19}\right)x^{19}+\left(\frac{45\,d}{17}+\frac{120\,e}{17}\right)x^{17}+(8\,d+14\,e)\,x^{15}+\left(\frac{210\,d}{13}+\frac{252\,e}{13}\right)x^{13}+\left(\frac{252\,d}{11}+\frac{210\,e}{11}\right)x^{11}+\left(\frac{70\,d}{3}+\frac{40\,e}{3}\right)x^9+\left(\frac{120\,d}{7}+\frac{45\,e}{7}\right)x^7+(9\,d+2\,e)\,x^5+\left(\frac{10\,d}{3}+\frac{e}{3}\right)x^3+d\,x$$

input
```
int((d + e*x^2)*(2*x^2 + x^4 + 1)^5,x)
```

output
```
x^5*(9*d + 2*e) + x^3*((10*d)/3 + e/3) + x^15*(8*d + 14*e) + x^21*(d/21 +
(10*e)/21) + x^19*((10*d)/19 + (45*e)/19) + x^9*((70*d)/3 + (40*e)/3) + x^
7*((120*d)/7 + (45*e)/7) + x^17*((45*d)/17 + (120*e)/17) + x^11*((252*d)/1
1 + (210*e)/11) + x^13*((210*d)/13 + (252*e)/13) + d*x + (e*x^23)/23
```

3.62 $\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x} dx$

3.62.1 Optimal result

Integrand size = 23, antiderivative size = 93

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x} dx = 5dx^2 + \frac{45dx^4}{4} + 20dx^6 + \frac{105dx^8}{4} + \frac{126dx^{10}}{5} + \frac{35dx^{12}}{2} + \frac{60dx^{14}}{7} + \frac{45dx^{16}}{16} + \frac{5dx^{18}}{9} + \frac{dx^{20}}{20} + \frac{1}{22}e(1+x^2)^{11} + d\log(x)$$

output
```
5*d*x^2+45/4*d*x^4+20*d*x^6+105/4*d*x^8+126/5*d*x^10+35/2*d*x^12+60/7*d*x^
14+45/16*d*x^16+5/9*d*x^18+1/20*d*x^20+1/22*e*(x^2+1)^11+d*ln(x)
```

3.62.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.60

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x} dx = \frac{1}{2}(10d+e)x^2 + \frac{5}{4}(9d+2e)x^4 + \frac{5}{2}(8d+3e)x^6 + \frac{15}{4}(7d+4e)x^8 + \frac{21}{5}(6d+5e)x^{10} + \frac{7}{2}(5d+6e)x^{12} + \frac{15}{7}(4d+7e)x^{14} + \frac{15}{16}(3d+8e)x^{16} + \frac{5}{18}(2d+9e)x^{18} + \frac{1}{20}(d+10e)x^{20} + \frac{ex^{22}}{22} + d\log(x)$$

input
```
Integrate[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x,x]
```

output
```
((10*d + e)*x^2)/2 + (5*(9*d + 2*e)*x^4)/4 + (5*(8*d + 3*e)*x^6)/2 + (15*(
7*d + 4*e)*x^8)/4 + (21*(6*d + 5*e)*x^10)/5 + (7*(5*d + 6*e)*x^12)/2 + (15
*(4*d + 7*e)*x^14)/7 + (15*(3*d + 8*e)*x^16)/16 + (5*(2*d + 9*e)*x^18)/18
+ ((d + 10*e)*x^20)/20 + (e*x^22)/22 + d*Log[x]
```

3.62.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1380, 354, 90, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x^4+2x^2+1\right)^5\left(d+ex^2\right)}{x}dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{\left(x^2+1\right)^{10}\left(d+ex^2\right)}{x}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{\left(x^2+1\right)^{10}\left(ex^2+d\right)}{x^2}dx^2$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}\left(d\int \frac{\left(x^2+1\right)^{10}}{x^2}dx^2+\frac{1}{11}e\left(x^2+1\right)^{11}\right)$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\left(d\int \left(x^{18}+10x^{16}+45x^{14}+120x^{12}+210x^{10}+252x^8+210x^6+120x^4+45x^2+10+\frac{1}{x^2}\right)dx^2+\frac{1}{11}e\left(x^2+1\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(d\left(\frac{x^{20}}{10}+\frac{10x^{18}}{9}+\frac{45x^{16}}{8}+\frac{120x^{14}}{7}+35x^{12}+\frac{252x^{10}}{5}+\frac{105x^8}{2}+40x^6+\frac{45x^4}{2}+10x^2+\log\left(x^2\right)\right)+\frac{1}{11}e\left(x^2\right.\right.$$

input
```
Int[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x,x]
```

output
```
((e*(1 + x^2)^11)/11 + d*(10*x^2 + (45*x^4)/2 + 40*x^6 + (105*x^8)/2 + (25
2*x^10)/5 + 35*x^12 + (120*x^14)/7 + (45*x^16)/8 + (10*x^18)/9 + x^20/10 +
 Log[x^2]))/2
```

3.62.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.62.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.31

method	result
norman	$\left(5d+\frac{e}{2}\right)x^{2}+\left(20d+\frac{15e}{2}\right)x^{6}+\left(\frac{d}{20}+\frac{e}{2}\right)x^{20}+\left(\frac{5d}{9}+\frac{5e}{2}\right)x^{18}+\left(\frac{35d}{2}+21e\right)x^{12}+\left(\frac{45d}{4}+\frac{5e}{2}\right)$
default	$\frac{e\,x^{22}}{22}+\frac{d\,x^{20}}{20}+\frac{e\,x^{20}}{2}+\frac{5d\,x^{18}}{9}+\frac{5e\,x^{18}}{2}+\frac{45d\,x^{16}}{16}+\frac{15e\,x^{16}}{2}+\frac{60d\,x^{14}}{7}+15e\,x^{14}+\frac{35d\,x^{12}}{2}+21e\,x^{12}+$
risch	$\frac{e\,x^{22}}{22}+\frac{d\,x^{20}}{20}+\frac{e\,x^{20}}{2}+\frac{5d\,x^{18}}{9}+\frac{5e\,x^{18}}{2}+\frac{45d\,x^{16}}{16}+\frac{15e\,x^{16}}{2}+\frac{60d\,x^{14}}{7}+15e\,x^{14}+\frac{35d\,x^{12}}{2}+21e\,x^{12}+$
parallelrisch	$\frac{e\,x^{22}}{22}+\frac{d\,x^{20}}{20}+\frac{e\,x^{20}}{2}+\frac{5d\,x^{18}}{9}+\frac{5e\,x^{18}}{2}+\frac{45d\,x^{16}}{16}+\frac{15e\,x^{16}}{2}+\frac{60d\,x^{14}}{7}+15e\,x^{14}+\frac{35d\,x^{12}}{2}+21e\,x^{12}+$

input
```
int((e*x^2+d)*(x^4+2*x^2+1)^5/x,x,method=_RETURNVERBOSE)
```

output
```
(5*d+1/2*e)*x^2+(20*d+15/2*e)*x^6+(1/20*d+1/2*e)*x^20+(5/9*d+5/2*e)*x^18+(
35/2*d+21*e)*x^12+(45/4*d+5/2*e)*x^4+(45/16*d+15/2*e)*x^16+(60/7*d+15*e)*x
^14+(105/4*d+15*e)*x^8+(126/5*d+21*e)*x^10+1/22*e*x^22+d*ln(x)
```

3.62.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.37

$$\begin{aligned}\int \frac{(d+ex^2)\left(1+2x^2+x^4\right)^5}{x}\,dx &= \frac{1}{22}\,ex^{22}+\frac{1}{20}\,(d+10\,e)x^{20}+\frac{5}{18}\,(2\,d+9\,e)x^{18}\\&\quad+\frac{15}{16}\,(3\,d+8\,e)x^{16}+\frac{15}{7}\,(4\,d+7\,e)x^{14}+\frac{7}{2}\,(5\,d+6\,e)x^{12}\\&\quad+\frac{21}{5}\,(6\,d+5\,e)x^{10}+\frac{15}{4}\,(7\,d+4\,e)x^{8}+\frac{5}{2}\,(8\,d+3\,e)x^{6}\\&\quad+\frac{5}{4}\,(9\,d+2\,e)x^{4}+\frac{1}{2}\,(10\,d+e)x^{2}+d\log(x)\end{aligned}$$

input
```
integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x,x, algorithm="fricas")
```

output
```
1/22*e*x^22 + 1/20*(d + 10*e)*x^20 + 5/18*(2*d + 9*e)*x^18 + 15/16*(3*d +
8*e)*x^16 + 15/7*(4*d + 7*e)*x^14 + 7/2*(5*d + 6*e)*x^12 + 21/5*(6*d + 5*e
)*x^10 + 15/4*(7*d + 4*e)*x^8 + 5/2*(8*d + 3*e)*x^6 + 5/4*(9*d + 2*e)*x^4
+ 1/2*(10*d + e)*x^2 + d*log(x)
```

3.62.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.41

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x}\,dx = d\log(x) + \frac{ex^{22}}{22} + x^{20}\left(\frac{d}{20}+\frac{e}{2}\right) + x^{18}\cdot\left(\frac{5d}{9}+\frac{5e}{2}\right) + x^{16}\cdot\left(\frac{45d}{16}+\frac{15e}{2}\right) + x^{14}\cdot\left(\frac{60d}{7}+15e\right) + x^{12}\cdot\left(\frac{35d}{2}+21e\right) + x^{10}\cdot\left(\frac{126d}{5}+21e\right) + x^{8}\cdot\left(\frac{105d}{4}+15e\right) + x^{6}\cdot\left(20d+\frac{15e}{2}\right) + x^{4}\cdot\left(\frac{45d}{4}+\frac{5e}{2}\right) + x^{2}\cdot\left(5d+\frac{e}{2}\right)$$

input
```
integrate((e*x**2+d)*(x**4+2*x**2+1)**5/x,x)
```

output
```
d*log(x) + e*x**22/22 + x**20*(d/20 + e/2) + x**18*(5*d/9 + 5*e/2) + x**16
*(45*d/16 + 15*e/2) + x**14*(60*d/7 + 15*e) + x**12*(35*d/2 + 21*e) + x**1
0*(126*d/5 + 21*e) + x**8*(105*d/4 + 15*e) + x**6*(20*d + 15*e/2) + x**4*(
45*d/4 + 5*e/2) + x**2*(5*d + e/2)
```

3.62.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.40

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x}\,dx = \frac{1}{22}\,ex^{22} + \frac{1}{20}\,(d+10\,e)x^{20} + \frac{5}{18}\,(2\,d+9\,e)x^{18} + \frac{15}{16}\,(3\,d+8\,e)x^{16} + \frac{15}{7}\,(4\,d+7\,e)x^{14} + \frac{7}{2}\,(5\,d+6\,e)x^{12} + \frac{21}{5}\,(6\,d+5\,e)x^{10} + \frac{15}{4}\,(7\,d+4\,e)x^{8} + \frac{5}{2}\,(8\,d+3\,e)x^{6} + \frac{5}{4}\,(9\,d+2\,e)x^{4} + \frac{1}{2}\,(10\,d+e)x^{2} + \frac{1}{2}\,d\log\left(x^2\right)$$

input
```
integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x,x, algorithm="maxima")
```

output
```
1/22*e*x^22 + 1/20*(d + 10*e)*x^20 + 5/18*(2*d + 9*e)*x^18 + 15/16*(3*d +
8*e)*x^16 + 15/7*(4*d + 7*e)*x^14 + 7/2*(5*d + 6*e)*x^12 + 21/5*(6*d + 5*e
)*x^10 + 15/4*(7*d + 4*e)*x^8 + 5/2*(8*d + 3*e)*x^6 + 5/4*(9*d + 2*e)*x^4
+ 1/2*(10*d + e)*x^2 + 1/2*d*log(x^2)
```

3.62.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.44

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x}\,dx = \frac{1}{22}\,ex^{22} + \frac{1}{20}\,dx^{20} + \frac{1}{2}\,ex^{20} + \frac{5}{9}\,dx^{18} + \frac{5}{2}\,ex^{18} + \frac{45}{16}\,dx^{16} + \frac{15}{2}\,ex^{16} + \frac{60}{7}\,dx^{14} + 15\,ex^{14} + \frac{35}{2}\,dx^{12} + 21\,ex^{12} + \frac{126}{5}\,dx^{10} + 21\,ex^{10} + \frac{105}{4}\,dx^{8} + 15\,ex^{8} + 20\,dx^{6} + \frac{15}{2}\,ex^{6} + \frac{45}{4}\,dx^{4} + \frac{5}{2}\,ex^{4} + 5\,dx^{2} + \frac{1}{2}\,ex^{2} + \frac{1}{2}\,d\log\left(x^2\right)$$

input
```
integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x,x, algorithm="giac")
```

output
```
1/22*e*x^22 + 1/20*d*x^20 + 1/2*e*x^20 + 5/9*d*x^18 + 5/2*e*x^18 + 45/16*d
*x^16 + 15/2*e*x^16 + 60/7*d*x^14 + 15*e*x^14 + 35/2*d*x^12 + 21*e*x^12 +
126/5*d*x^10 + 21*e*x^10 + 105/4*d*x^8 + 15*e*x^8 + 20*d*x^6 + 15/2*e*x^6
+ 45/4*d*x^4 + 5/2*e*x^4 + 5*d*x^2 + 1/2*e*x^2 + 1/2*d*log(x^2)
```

3.62.9 Mupad [B] (verification not implemented)

Time = 7.72 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.30

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x}\,dx = x^2\left(5\,d+\frac{e}{2}\right) + x^{18}\left(\frac{5\,d}{9}+\frac{5\,e}{2}\right) + x^6\left(20\,d+\frac{15\,e}{2}\right) + x^{20}\left(\frac{d}{20}+\frac{e}{2}\right) + x^4\left(\frac{45\,d}{4}+\frac{5\,e}{2}\right) + x^{12}\left(\frac{35\,d}{2}+21\,e\right) + x^{16}\left(\frac{45\,d}{16}+\frac{15\,e}{2}\right) + x^{14}\left(\frac{60\,d}{7}+15\,e\right) + x^8\left(\frac{105\,d}{4}+15\,e\right) + x^{10}\left(\frac{126\,d}{5}+21\,e\right) + \frac{e\,x^{22}}{22} + d\,\ln\left(x\right)$$

input
```
int(((d + e*x^2)*(2*x^2 + x^4 + 1)^5)/x,x)
```

output
```
x^2*(5*d + e/2) + x^18*((5*d)/9 + (5*e)/2) + x^6*(20*d + (15*e)/2) + x^20*
(d/20 + e/2) + x^4*((45*d)/4 + (5*e)/2) + x^12*((35*d)/2 + 21*e) + x^16*((
45*d)/16 + (15*e)/2) + x^14*((60*d)/7 + 15*e) + x^8*((105*d)/4 + 15*e) + x
^10*((126*d)/5 + 21*e) + (e*x^22)/22 + d*log(x)
```

3.63 $\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^2} dx$

3.63.1 Optimal result

Integrand size = 23, antiderivative size = 141

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^2} dx = -\frac{d}{x} + (10d+e)x + \frac{5}{3}(9d+2e)x^3 + 3(8d+3e)x^5 + \frac{30}{7}(7d+4e)x^7 + \frac{14}{3}(6d+5e)x^9 + \frac{42}{11}(5d+6e)x^{11} + \frac{30}{13}(4d+7e)x^{13} + (3d+8e)x^{15} + \frac{5}{17}(2d+9e)x^{17} + \frac{1}{19}(d+10e)x^{19} + \frac{ex^{21}}{21}$$

output
```
-d/x+(10*d+e)*x+5/3*(9*d+2*e)*x^3+3*(8*d+3*e)*x^5+30/7*(7*d+4*e)*x^7+14/3*
(6*d+5*e)*x^9+42/11*(5*d+6*e)*x^11+30/13*(4*d+7*e)*x^13+(3*d+8*e)*x^15+5/1
7*(2*d+9*e)*x^17+1/19*(d+10*e)*x^19+1/21*e*x^21
```

3.63.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^2} dx = -\frac{d}{x} + (10d+e)x + \frac{5}{3}(9d+2e)x^3 + 3(8d+3e)x^5 + \frac{30}{7}(7d+4e)x^7 + \frac{14}{3}(6d+5e)x^9 + \frac{42}{11}(5d+6e)x^{11} + \frac{30}{13}(4d+7e)x^{13} + (3d+8e)x^{15} + \frac{5}{17}(2d+9e)x^{17} + \frac{1}{19}(d+10e)x^{19} + \frac{ex^{21}}{21}$$

input
```
Integrate[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x^2,x]
```

output
```
-(d/x) + (10*d + e)*x + (5*(9*d + 2*e)*x^3)/3 + 3*(8*d + 3*e)*x^5 + (30*(7
*d + 4*e)*x^7)/7 + (14*(6*d + 5*e)*x^9)/3 + (42*(5*d + 6*e)*x^11)/11 + (30
*(4*d + 7*e)*x^13)/13 + (3*d + 8*e)*x^15 + (5*(2*d + 9*e)*x^17)/17 + ((d +
 10*e)*x^19)/19 + (e*x^21)/21
```

3.63.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1380, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^4+2x^2+1)^5 (d+ex^2)}{x^2} dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{(x^2+1)^{10} (d+ex^2)}{x^2} dx$$

$$\Big\downarrow \text{355}$$

$$\int \Big(x^{18}(d+10e)+5x^{16}(2d+9e)+15x^{14}(3d+8e)+30x^{12}(4d+7e)+42x^{10}(5d+6e)+42x^8(6d+5e)+30x^6(7$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{19}x^{19}(d+10e)+\frac{5}{17}x^{17}(2d+9e)+x^{15}(3d+8e)+\frac{30}{13}x^{13}(4d+7e)+\frac{42}{11}x^{11}(5d+6e)+\frac{14}{3}x^9(6d+5e)+\frac{30}{7}x^7(7d+4e)+3x^5(8d+3e)+\frac{5}{3}x^3(9d+2e)+x(10d+e)-\frac{d}{x}+\frac{ex^{21}}{21}$$

input
```
Int[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x^2,x]
```

output
```
-(d/x) + (10*d + e)*x + (5*(9*d + 2*e)*x^3)/3 + 3*(8*d + 3*e)*x^5 + (30*(7
*d + 4*e)*x^7)/7 + (14*(6*d + 5*e)*x^9)/3 + (42*(5*d + 6*e)*x^11)/11 + (30
*(4*d + 7*e)*x^13)/13 + (3*d + 8*e)*x^15 + (5*(2*d + 9*e)*x^17)/17 + ((d +
 10*e)*x^19)/19 + (e*x^21)/21
```

3.63.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.63.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.87

method	result
norman	$\frac{-d+(10d+e)x^2+\left(15d+\frac{10e}{3}\right)x^4+(24d+9e)x^6+\left(30d+\frac{120e}{7}\right)x^8+\left(28d+\frac{70e}{3}\right)x^{10}+\left(\frac{210d}{11}+\frac{252e}{11}\right)x^{12}+\left(\frac{120d}{13}+\frac{210e}{13}\right)x^{14}+(3d+8e}{x}$
default	$\frac{e\,x^{21}}{21}+\frac{x^{19}d}{19}+\frac{10x^{19}e}{19}+\frac{10x^{17}d}{17}+\frac{45x^{17}e}{17}+3x^{15}d+8x^{15}e+\frac{120x^{13}d}{13}+\frac{210x^{13}e}{13}+\frac{210x^{11}d}{11}+\frac{252e\,x^{11}}{11}$
risch	$\frac{e\,x^{21}}{21}+\frac{x^{19}d}{19}+\frac{10x^{19}e}{19}+\frac{10x^{17}d}{17}+\frac{45x^{17}e}{17}+3x^{15}d+8x^{15}e+\frac{120x^{13}d}{13}+\frac{210x^{13}e}{13}+\frac{210x^{11}d}{11}+\frac{252e\,x^{11}}{11}$
gosper	$\underline{46189e\,x^{22}+51051d\,x^{20}+510510e\,x^{20}+570570d\,x^{18}+2567565e\,x^{18}+2909907d\,x^{16}+7759752e\,x^{16}+8953560d\,x^{14}+15668730e\,x}$
parallelrisch	$\underline{46189e\,x^{22}+51051d\,x^{20}+510510e\,x^{20}+570570d\,x^{18}+2567565e\,x^{18}+2909907d\,x^{16}+7759752e\,x^{16}+8953560d\,x^{14}+15668730e\,x}$

input
```
int((e*x^2+d)*(x^4+2*x^2+1)^5/x^2,x,method=_RETURNVERBOSE)
```

output
```
(-d+(10*d+e)*x^2+(15*d+10/3*e)*x^4+(24*d+9*e)*x^6+(30*d+120/7*e)*x^8+(28*d
+70/3*e)*x^10+(210/11*d+252/11*e)*x^12+(120/13*d+210/13*e)*x^14+(3*d+8*e)*
x^16+(10/17*d+45/17*e)*x^18+(1/19*d+10/19*e)*x^20+1/21*e*x^22)/x
```

3.63.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^2}\,dx$$
$$= \frac{46189\,ex^{22} + 51051\,(d+10\,e)x^{20} + 285285\,(2\,d+9\,e)x^{18} + 969969\,(3\,d+8\,e)x^{16} + 2238390\,(4\,d+7\,e)x^{14}}{}$$

input
```
integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x^2,x, algorithm="fricas")
```

output
```
1/969969*(46189*e*x^22 + 51051*(d + 10*e)*x^20 + 285285*(2*d + 9*e)*x^18 +
 969969*(3*d + 8*e)*x^16 + 2238390*(4*d + 7*e)*x^14 + 3703518*(5*d + 6*e)*
x^12 + 4526522*(6*d + 5*e)*x^10 + 4157010*(7*d + 4*e)*x^8 + 2909907*(8*d +
 3*e)*x^6 + 1616615*(9*d + 2*e)*x^4 + 969969*(10*d + e)*x^2 - 969969*d)/x
```

3.63.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^2}\,dx = -\frac{d}{x} + \frac{ex^{21}}{21} + x^{19}\left(\frac{d}{19} + \frac{10e}{19}\right) + x^{17}\cdot\left(\frac{10d}{17} + \frac{45e}{17}\right) + x^{15}\cdot(3d+8e) + x^{13}\cdot\left(\frac{120d}{13} + \frac{210e}{13}\right) + x^{11} \cdot\left(\frac{210d}{11} + \frac{252e}{11}\right) + x^9\cdot\left(28d + \frac{70e}{3}\right) + x^7\cdot\left(30d + \frac{120e}{7}\right) + x^5\cdot(24d+9e) + x^3\cdot\left(15d + \frac{10e}{3}\right) + x(10d+e)$$

input
```
integrate((e*x**2+d)*(x**4+2*x**2+1)**5/x**2,x)
```

output
```
-d/x + e*x**21/21 + x**19*(d/19 + 10*e/19) + x**17*(10*d/17 + 45*e/17) + x
**15*(3*d + 8*e) + x**13*(120*d/13 + 210*e/13) + x**11*(210*d/11 + 252*e/1
1) + x**9*(28*d + 70*e/3) + x**7*(30*d + 120*e/7) + x**5*(24*d + 9*e) + x*
*3*(15*d + 10*e/3) + x*(10*d + e)
```

3.63.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.89

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^2}\,dx = \frac{1}{21}ex^{21} + \frac{1}{19}(d+10e)x^{19} + \frac{5}{17}(2d+9e)x^{17} + (3d+8e)x^{15} + \frac{30}{13}(4d+7e)x^{13} + \frac{42}{11}(5d+6e)x^{11} + \frac{14}{3}(6d+5e)x^9 + \frac{30}{7}(7d+4e)x^7 + 3(8d+3e)x^5 + \frac{5}{3}(9d+2e)x^3 + (10d+e)x - \frac{d}{x}$$

input
```
integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x^2,x, algorithm="maxima")
```

output
```
1/21*e*x^21 + 1/19*(d + 10*e)*x^19 + 5/17*(2*d + 9*e)*x^17 + (3*d + 8*e)*x
^15 + 30/13*(4*d + 7*e)*x^13 + 42/11*(5*d + 6*e)*x^11 + 14/3*(6*d + 5*e)*x
^9 + 30/7*(7*d + 4*e)*x^7 + 3*(8*d + 3*e)*x^5 + 5/3*(9*d + 2*e)*x^3 + (10*
d + e)*x - d/x
```

3.63.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^2}\,dx = \frac{1}{21}ex^{21} + \frac{1}{19}dx^{19} + \frac{10}{19}ex^{19} + \frac{10}{17}dx^{17} + \frac{45}{17}ex^{17} + 3dx^{15} + 8ex^{15} + \frac{120}{13}dx^{13} + \frac{210}{13}ex^{13} + \frac{210}{11}dx^{11} + \frac{252}{11}ex^{11} + 28dx^9 + \frac{70}{3}ex^9 + 30dx^7 + \frac{120}{7}ex^7 + 24dx^5 + 9ex^5 + 15dx^3 + \frac{10}{3}ex^3 + 10dx + ex - \frac{d}{x}$$

input
```
integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x^2,x, algorithm="giac")
```

output
```
1/21*e*x^21 + 1/19*d*x^19 + 10/19*e*x^19 + 10/17*d*x^17 + 45/17*e*x^17 + 3
*d*x^15 + 8*e*x^15 + 120/13*d*x^13 + 210/13*e*x^13 + 210/11*d*x^11 + 252/1
1*e*x^11 + 28*d*x^9 + 70/3*e*x^9 + 30*d*x^7 + 120/7*e*x^7 + 24*d*x^5 + 9*e
*x^5 + 15*d*x^3 + 10/3*e*x^3 + 10*d*x + e*x - d/x
```

3.63.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^2}\,dx = x^{15}\,(3\,d+8\,e)+x^3\,\left(15\,d+\frac{10\,e}{3}\right)+x^5\,(24\,d+9\,e)+x^{19}\,\left(\frac{d}{19}+\frac{10\,e}{19}\right)+x^{17}\,\left(\frac{10\,d}{17}+\frac{45\,e}{17}\right)+x^9\,\left(28\,d+\frac{70\,e}{3}\right)+x^7\,\left(30\,d+\frac{120\,e}{7}\right)+x^{13}\,\left(\frac{120\,d}{13}+\frac{210\,e}{13}\right)+x^{11}\,\left(\frac{210\,d}{11}+\frac{252\,e}{11}\right)+x\,(10\,d+e)-\frac{d}{x}+\frac{e\,x^{21}}{21}$$

input
```
int(((d + e*x^2)*(2*x^2 + x^4 + 1)^5)/x^2,x)
```

output
```
x^15*(3*d + 8*e) + x^3*(15*d + (10*e)/3) + x^5*(24*d + 9*e) + x^19*(d/19 +
 (10*e)/19) + x^17*((10*d)/17 + (45*e)/17) + x^9*(28*d + (70*e)/3) + x^7*(
30*d + (120*e)/7) + x^13*((120*d)/13 + (210*e)/13) + x^11*((210*d)/11 + (2
52*e)/11) + x*(10*d + e) - d/x + (e*x^21)/21
```

3.64 $\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^3} dx$

3.64.1 Optimal result

Integrand size = 23, antiderivative size = 147

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^3} dx = -\frac{d}{2x^2} + \frac{5}{2}(9d+2e)x^2 + \frac{15}{4}(8d+3e)x^4 + 5(7d+4e)x^6 + \frac{21}{4}(6d+5e)x^8 + \frac{21}{5}(5d+6e)x^{10} + \frac{5}{2}(4d+7e)x^{12} + \frac{15}{14}(3d+8e)x^{14} + \frac{5}{16}(2d+9e)x^{16} + \frac{1}{18}(d+10e)x^{18} + \frac{ex^{20}}{20} + (10d+e)\log(x)$$

output
```
-1/2*d/x^2+5/2*(9*d+2*e)*x^2+15/4*(8*d+3*e)*x^4+5*(7*d+4*e)*x^6+21/4*(6*d+
5*e)*x^8+21/5*(5*d+6*e)*x^10+5/2*(4*d+7*e)*x^12+15/14*(3*d+8*e)*x^14+5/16*
(2*d+9*e)*x^16+1/18*(d+10*e)*x^18+1/20*e*x^20+(10*d+e)*ln(x)
```

3.64.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^3} dx = -\frac{d}{2x^2} + \frac{5}{2}(9d+2e)x^2 + \frac{15}{4}(8d+3e)x^4 + 5(7d+4e)x^6 + \frac{21}{4}(6d+5e)x^8 + \frac{21}{5}(5d+6e)x^{10} + \frac{5}{2}(4d+7e)x^{12} + \frac{15}{14}(3d+8e)x^{14} + \frac{5}{16}(2d+9e)x^{16} + \frac{1}{18}(d+10e)x^{18} + \frac{ex^{20}}{20} + (10d+e)\log(x)$$

input
```
Integrate[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x^3,x]
```

output
```
-1/2*d/x^2 + (5*(9*d + 2*e)*x^2)/2 + (15*(8*d + 3*e)*x^4)/4 + 5*(7*d + 4*e
)*x^6 + (21*(6*d + 5*e)*x^8)/4 + (21*(5*d + 6*e)*x^10)/5 + (5*(4*d + 7*e)*
x^12)/2 + (15*(3*d + 8*e)*x^14)/14 + (5*(2*d + 9*e)*x^16)/16 + ((d + 10*e)
*x^18)/18 + (e*x^20)/20 + (10*d + e)*Log[x]
```

3.64.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1380, 354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x^4+2x^2+1\right)^5\left(d+ex^2\right)}{x^3}dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{\left(x^2+1\right)^{10}\left(d+ex^2\right)}{x^3}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{\left(x^2+1\right)^{10}\left(ex^2+d\right)}{x^4}dx^2$$

$$\Big\downarrow \text{85}$$

$$\frac{1}{2}\int \Big(ex^{18}+(d+10e)x^{16}+5(2d+9e)x^{14}+15(3d+8e)x^{12}+30(4d+7e)x^{10}+42(5d+6e)x^8+42(6d+5e)x^6$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\Big(\frac{1}{9}x^{18}(d+10e)+\frac{5}{8}x^{16}(2d+9e)+\frac{15}{7}x^{14}(3d+8e)+5x^{12}(4d+7e)+\frac{42}{5}x^{10}(5d+6e)+\frac{21}{2}x^8(6d+5e)+10x^6$$

input
```
Int[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x^3,x]
```

output
```
(-(d/x^2) + 5*(9*d + 2*e)*x^2 + (15*(8*d + 3*e)*x^4)/2 + 10*(7*d + 4*e)*x^
6 + (21*(6*d + 5*e)*x^8)/2 + (42*(5*d + 6*e)*x^10)/5 + 5*(4*d + 7*e)*x^12
+ (15*(3*d + 8*e)*x^14)/7 + (5*(2*d + 9*e)*x^16)/8 + ((d + 10*e)*x^18)/9 +
 (e*x^20)/10 + (10*d + e)*Log[x^2])/2
```

3.64.3.1 Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.64.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.84

method	result
norman	$\frac{\left(10d+\frac{35e}{2}\right)x^{14}+\left(21d+\frac{126e}{5}\right)x^{12}+\left(30d+\frac{45e}{4}\right)x^{6}+\left(35d+20e\right)x^{8}+\left(\frac{d}{18}+\frac{5e}{9}\right)x^{20}+\left(\frac{5d}{8}+\frac{45e}{16}\right)x^{18}+\left(\frac{45d}{2}+5e\right)x^{4}+\left(\frac{45d}{14}+\frac{60e}{7}\right)x^{1}}{x^{2}}$
default	$\frac{e\,x^{20}}{20}+\frac{d\,x^{18}}{18}+\frac{5e\,x^{18}}{9}+\frac{5d\,x^{16}}{8}+\frac{45e\,x^{16}}{16}+\frac{45d\,x^{14}}{14}+\frac{60e\,x^{14}}{7}+10d\,x^{12}+\frac{35e\,x^{12}}{2}+21d\,x^{10}+\frac{126e\,x^{10}}{5}$
risch	$\frac{e\,x^{20}}{20}+\frac{d\,x^{18}}{18}+\frac{5e\,x^{18}}{9}+\frac{5d\,x^{16}}{8}+\frac{45e\,x^{16}}{16}+\frac{45d\,x^{14}}{14}+\frac{60e\,x^{14}}{7}+10d\,x^{12}+\frac{35e\,x^{12}}{2}+21d\,x^{10}+\frac{126e\,x^{10}}{5}$
parallelrisch	$252e\,x^{22}+280d\,x^{20}+2800e\,x^{20}+3150d\,x^{18}+14175e\,x^{18}+16200d\,x^{16}+43200e\,x^{16}+50400d\,x^{14}+88200e\,x^{14}+105840d\,x^{12}+127$

input
```
int((e*x^2+d)*(x^4+2*x^2+1)^5/x^3,x,method=_RETURNVERBOSE)
```

output
```
((10*d+35/2*e)*x^14+(21*d+126/5*e)*x^12+(30*d+45/4*e)*x^6+(35*d+20*e)*x^8+
(1/18*d+5/9*e)*x^20+(5/8*d+45/16*e)*x^18+(45/2*d+5*e)*x^4+(45/14*d+60/7*e)
*x^16+(63/2*d+105/4*e)*x^10-1/2*d+1/20*e*x^22)/x^2+(10*d+e)*ln(x)
```

3.64.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^3}\,dx = \frac{252\,ex^{22} + 280\,(d+10\,e)x^{20} + 1575\,(2\,d+9\,e)x^{18} + 5400\,(3\,d+8\,e)x^{16} + 12600\,(4\,d+7\,e)x^{14} + 21168\,(5$$

input
```
integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x^3,x, algorithm="fricas")
```

output
```
1/5040*(252*e*x^22 + 280*(d + 10*e)*x^20 + 1575*(2*d + 9*e)*x^18 + 5400*(3
*d + 8*e)*x^16 + 12600*(4*d + 7*e)*x^14 + 21168*(5*d + 6*e)*x^12 + 26460*(
6*d + 5*e)*x^10 + 25200*(7*d + 4*e)*x^8 + 18900*(8*d + 3*e)*x^6 + 12600*(9
*d + 2*e)*x^4 + 5040*(10*d + e)*x^2*log(x) - 2520*d)/x^2
```

3.64.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.89

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^3}\,dx = -\frac{d}{2x^2} + \frac{ex^{20}}{20} + x^{18}\left(\frac{d}{18}+\frac{5e}{9}\right) + x^{16}\cdot\left(\frac{5d}{8}+\frac{45e}{16}\right) + x^{14}\cdot\left(\frac{45d}{14}+\frac{60e}{7}\right) + x^{12}\cdot\left(10d+\frac{35e}{2}\right) + x^{10}\cdot\left(21d+\frac{126e}{5}\right) + x^8\cdot\left(\frac{63d}{2}+\frac{105e}{4}\right) + x^6\cdot(35d+20e) + x^4\cdot\left(30d+\frac{45e}{4}\right) + x^2\cdot\left(\frac{45d}{2}+5e\right) + (10d+e)\log(x)$$

input
```
integrate((e*x**2+d)*(x**4+2*x**2+1)**5/x**3,x)
```

output
```
-d/(2*x**2) + e*x**20/20 + x**18*(d/18 + 5*e/9) + x**16*(5*d/8 + 45*e/16)
+ x**14*(45*d/14 + 60*e/7) + x**12*(10*d + 35*e/2) + x**10*(21*d + 126*e/5
) + x**8*(63*d/2 + 105*e/4) + x**6*(35*d + 20*e) + x**4*(30*d + 45*e/4) +
x**2*(45*d/2 + 5*e) + (10*d + e)*log(x)
```

3.64.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^3}\,dx = \frac{1}{20}\,ex^{20} + \frac{1}{18}\,(d+10\,e)x^{18} + \frac{5}{16}\,(2\,d+9\,e)x^{16} + \frac{15}{14}\,(3\,d+8\,e)x^{14} + \frac{5}{2}\,(4\,d+7\,e)x^{12} + \frac{21}{5}\,(5\,d+6\,e)x^{10} + \frac{21}{4}\,(6\,d+5\,e)x^{8} + 5\,(7\,d+4\,e)x^{6} + \frac{15}{4}\,(8\,d+3\,e)x^{4} + \frac{5}{2}\,(9\,d+2\,e)x^{2} + \frac{1}{2}\,(10\,d+e)\log\left(x^2\right) - \frac{d}{2\,x^2}$$

input
```
integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x^3,x, algorithm="maxima")
```

output
```
1/20*e*x^20 + 1/18*(d + 10*e)*x^18 + 5/16*(2*d + 9*e)*x^16 + 15/14*(3*d +
8*e)*x^14 + 5/2*(4*d + 7*e)*x^12 + 21/5*(5*d + 6*e)*x^10 + 21/4*(6*d + 5*e
)*x^8 + 5*(7*d + 4*e)*x^6 + 15/4*(8*d + 3*e)*x^4 + 5/2*(9*d + 2*e)*x^2 + 1
/2*(10*d + e)*log(x^2) - 1/2*d/x^2
```

3.64.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^3}\,dx = \frac{1}{20}\,ex^{20} + \frac{1}{18}\,dx^{18} + \frac{5}{9}\,ex^{18} + \frac{5}{8}\,dx^{16} + \frac{45}{16}\,ex^{16} + \frac{45}{14}\,dx^{14} + \frac{60}{7}\,ex^{14} + 10\,dx^{12} + \frac{35}{2}\,ex^{12} + 21\,dx^{10} + \frac{126}{5}\,ex^{10} + \frac{63}{2}\,dx^{8} + \frac{105}{4}\,ex^{8} + 35\,dx^{6} + 20\,ex^{6} + 30\,dx^{4} + \frac{45}{4}\,ex^{4} + \frac{45}{2}\,dx^{2} + 5\,ex^{2} + \frac{1}{2}\,(10\,d+e)\log\left(x^2\right) - \frac{10\,dx^2+ex^2+d}{2\,x^2}$$

input
```
integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x^3,x, algorithm="giac")
```

output
```
1/20*e*x^20 + 1/18*d*x^18 + 5/9*e*x^18 + 5/8*d*x^16 + 45/16*e*x^16 + 45/14
*d*x^14 + 60/7*e*x^14 + 10*d*x^12 + 35/2*e*x^12 + 21*d*x^10 + 126/5*e*x^10
 + 63/2*d*x^8 + 105/4*e*x^8 + 35*d*x^6 + 20*e*x^6 + 30*d*x^4 + 45/4*e*x^4
+ 45/2*d*x^2 + 5*e*x^2 + 1/2*(10*d + e)*log(x^2) - 1/2*(10*d*x^2 + e*x^2 +
 d)/x^2
```

3.64.9 Mupad [B] (verification not implemented)

Time = 7.72 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.82

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^3}\,dx = x^{18}\left(\frac{d}{18}+\frac{5\,e}{9}\right)+x^2\left(\frac{45\,d}{2}+5\,e\right)+x^{12}\left(10\,d+\frac{35\,e}{2}\right) + x^6\,(35\,d+20\,e)+x^4\left(30\,d+\frac{45\,e}{4}\right)+x^{16}\left(\frac{5\,d}{8}+\frac{45\,e}{16}\right) + x^{14}\left(\frac{45\,d}{14}+\frac{60\,e}{7}\right)+x^{10}\left(21\,d+\frac{126\,e}{5}\right) + x^8\left(\frac{63\,d}{2}+\frac{105\,e}{4}\right)-\frac{d}{2\,x^2}+\frac{e\,x^{20}}{20}+\ln(x)\,(10\,d+e)$$

input
```
int(((d + e*x^2)*(2*x^2 + x^4 + 1)^5)/x^3,x)
```

output
```
x^18*(d/18 + (5*e)/9) + x^2*((45*d)/2 + 5*e) + x^12*(10*d + (35*e)/2) + x^
6*(35*d + 20*e) + x^4*(30*d + (45*e)/4) + x^16*((5*d)/8 + (45*e)/16) + x^1
4*((45*d)/14 + (60*e)/7) + x^10*(21*d + (126*e)/5) + x^8*((63*d)/2 + (105*
e)/4) - d/(2*x^2) + (e*x^20)/20 + log(x)*(10*d + e)
```

3.65 $\int (fx)^m (1+x^2) (1+2x^2+x^4)^5 \, dx$

3.65.1 Optimal result

Integrand size = 23, antiderivative size = 203

$$\int (fx)^m \left(1+x^2\right) \left(1+2x^2+x^4\right)^5 \, dx = \frac{(fx)^{1+m}}{f(1+m)} + \frac{11(fx)^{3+m}}{f^3(3+m)} + \frac{55(fx)^{5+m}}{f^5(5+m)} + \frac{165(fx)^{7+m}}{f^7(7+m)} + \frac{330(fx)^{9+m}}{f^9(9+m)} + \frac{462(fx)^{11+m}}{f^{11}(11+m)} + \frac{462(fx)^{13+m}}{f^{13}(13+m)} + \frac{330(fx)^{15+m}}{f^{15}(15+m)} + \frac{165(fx)^{17+m}}{f^{17}(17+m)} + \frac{55(fx)^{19+m}}{f^{19}(19+m)} + \frac{11(fx)^{21+m}}{f^{21}(21+m)} + \frac{(fx)^{23+m}}{f^{23}(23+m)}$$

output
```
(f*x)^(1+m)/f/(1+m)+11*(f*x)^(3+m)/f^3/(3+m)+55*(f*x)^(5+m)/f^5/(5+m)+165*
(f*x)^(7+m)/f^7/(7+m)+330*(f*x)^(9+m)/f^9/(9+m)+462*(f*x)^(11+m)/f^11/(11+
m)+462*(f*x)^(13+m)/f^13/(13+m)+330*(f*x)^(15+m)/f^15/(15+m)+165*(f*x)^(17
+m)/f^17/(17+m)+55*(f*x)^(19+m)/f^19/(19+m)+11*(f*x)^(21+m)/f^21/(21+m)+(f
*x)^(23+m)/f^23/(23+m)
```

3.65.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.60

$$\int (fx)^m (1+x^2) (1+2x^2+x^4)^5 \, dx = x(fx)^m \left(\frac{1}{1+m}+\frac{11x^2}{3+m}+\frac{55x^4}{5+m}+\frac{165x^6}{7+m}+\frac{330x^8}{9+m} +\frac{462x^{10}}{11+m}+\frac{462x^{12}}{13+m}+\frac{330x^{14}}{15+m}+\frac{165x^{16}}{17+m} +\frac{55x^{18}}{19+m}+\frac{11x^{20}}{21+m}+\frac{x^{22}}{23+m}\right)$$

input
```
Integrate[(f*x)^m*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

output
```
x*(f*x)^m*((1 + m)^(-1) + (11*x^2)/(3 + m) + (55*x^4)/(5 + m) + (165*x^6)/
(7 + m) + (330*x^8)/(9 + m) + (462*x^10)/(11 + m) + (462*x^12)/(13 + m) +
(330*x^14)/(15 + m) + (165*x^16)/(17 + m) + (55*x^18)/(19 + m) + (11*x^20)
/(21 + m) + x^22/(23 + m))
```

3.65.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1380, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x^2+1) (x^4+2x^2+1)^5 (fx)^m \, dx$$

$$\Big\downarrow \text{ 1380}$$

$$\int (x^2+1)^{11} (fx)^m dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(\frac{(fx)^{m+22}}{f^{22}}+\frac{11(fx)^{m+20}}{f^{20}}+\frac{55(fx)^{m+18}}{f^{18}}+\frac{165(fx)^{m+16}}{f^{16}}+\frac{330(fx)^{m+14}}{f^{14}}+\frac{462(fx)^{m+12}}{f^{12}}+\frac{462(fx)^{m+10}}{f^{10}}+\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(fx)^{m+23}}{f^{23}(m+23)}+\frac{11(fx)^{m+21}}{f^{21}(m+21)}+\frac{55(fx)^{m+19}}{f^{19}(m+19)}+\frac{165(fx)^{m+17}}{f^{17}(m+17)}+\frac{330(fx)^{m+15}}{f^{15}(m+15)}+\frac{462(fx)^{m+13}}{f^{13}(m+13)}+\frac{462(fx)^{m+11}}{f^{11}(m+11)}+\frac{330(fx)^{m+9}}{f^{9}(m+9)}+\frac{165(fx)^{m+7}}{f^{7}(m+7)}+\frac{55(fx)^{m+5}}{f^{5}(m+5)}+\frac{11(fx)^{m+3}}{f^{3}(m+3)}+\frac{(fx)^{m+1}}{f(m+1)}$$

input
```
Int[(f*x)^m*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

output
```
(f*x)^(1 + m)/(f*(1 + m)) + (11*(f*x)^(3 + m))/(f^3*(3 + m)) + (55*(f*x)^(
5 + m))/(f^5*(5 + m)) + (165*(f*x)^(7 + m))/(f^7*(7 + m)) + (330*(f*x)^(9
+ m))/(f^9*(9 + m)) + (462*(f*x)^(11 + m))/(f^11*(11 + m)) + (462*(f*x)^(1
3 + m))/(f^13*(13 + m)) + (330*(f*x)^(15 + m))/(f^15*(15 + m)) + (165*(f*x
)^(17 + m))/(f^17*(17 + m)) + (55*(f*x)^(19 + m))/(f^19*(19 + m)) + (11*(f
*x)^(21 + m))/(f^21*(21 + m)) + (f*x)^(23 + m)/(f^23*(23 + m))
```

3.65.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.65.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1120 vs. 2(203) = 406.

Time = 0.65 (sec) , antiderivative size = 1121, normalized size of antiderivative = 5.52

method	result	size
gosper	Expression too large to display	1121
risch	Expression too large to display	1121
parallelrisch	Expression too large to display	1849

input
```
int((f*x)^m*(x^2+1)*(x^4+2*x^2+1)^5,x,method=_RETURNVERBOSE)
```

output
```
(f*x)^m*(m^11*x^22+121*m^10*x^22+11*m^11*x^20+6435*m^9*x^22+1353*m^10*x^20
+197835*m^8*x^22+55*m^11*x^18+72985*m^9*x^20+3889578*m^7*x^22+6875*m^10*x^
18+2271555*m^8*x^20+51069018*m^6*x^22+165*m^11*x^16+376365*m^9*x^18+451349
58*m^7*x^20+453714470*m^5*x^22+20955*m^10*x^16+11870265*m^8*x^18+597988314
*m^6*x^20+2702025590*m^4*x^22+330*m^11*x^14+1164735*m^9*x^16+238653030*m^7
*x^18+5353566130*m^5*x^20+10431670821*m^3*x^22+42570*m^10*x^14+37263105*m^
8*x^16+3194704590*m^6*x^18+32087153670*m^4*x^20+24372200061*m^2*x^22+462*m
^11*x^12+2403390*m^9*x^14+759091410*m^7*x^16+28857216410*m^5*x^18+12453062
6231*m^3*x^20+29985521895*m*x^22+60522*m^10*x^12+78076350*m^8*x^14+1028278
2510*m^6*x^16+174273100210*m^4*x^18+292163767533*m^2*x^20+13749310575*x^22
+462*m^11*x^10+3471930*m^9*x^12+1613983140*m^7*x^14+93862508190*m^5*x^16+6
80615362515*m^3*x^18+360568238085*m*x^20+61446*m^10*x^10+114642990*m^8*x^1
2+22164925860*m^6*x^14+572017996770*m^4*x^16+1604842704135*m^2*x^18+165646
455975*x^20+330*m^11*x^8+3582810*m^9*x^10+2408820876*m^7*x^12+204865733820
*m^5*x^14+2251106854425*m^3*x^16+1988025402825*m*x^18+44550*m^10*x^8+12036
7170*m^8*x^10+33609870756*m^6*x^12+1262375264700*m^4*x^14+5340787250535*m^
2*x^16+915414625125*x^18+165*m^11*x^6+2640990*m^9*x^8+2575140876*m^7*x^10+
315347150580*m^5*x^12+5015196628530*m^3*x^14+6646727085075*m*x^16+22605*m^
10*x^6+90358290*m^8*x^8+36597992508*m^6*x^10+1969992823260*m^4*x^12+119912
58123570*m^2*x^14+3069331390125*x^16+55*m^11*x^4+1362735*m^9*x^6+197190...
```

3.65.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 759 vs. $2(203) = 406$.

Time = 0.26 (sec) , antiderivative size = 759, normalized size of antiderivative = 3.74

$$\int (fx)^m \left(1+x^2\right) \left(1+2x^2+x^4\right)^5 \, dx = \text{Too large to display}$$

input
```
integrate((f*x)^m*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

output
```
((m^11 + 121*m^10 + 6435*m^9 + 197835*m^8 + 3889578*m^7 + 51069018*m^6 + 4
53714470*m^5 + 2702025590*m^4 + 10431670821*m^3 + 24372200061*m^2 + 299855
21895*m + 13749310575)*x^23 + 11*(m^11 + 123*m^10 + 6635*m^9 + 206505*m^8
+ 4103178*m^7 + 54362574*m^6 + 486687830*m^5 + 2917013970*m^4 + 1132096602
1*m^3 + 26560342503*m^2 + 32778930735*m + 15058768725)*x^21 + 55*(m^11 + 1
25*m^10 + 6843*m^9 + 215823*m^8 + 4339146*m^7 + 58085538*m^6 + 524676662*m
^5 + 3168601822*m^4 + 12374824773*m^3 + 29178958257*m^2 + 36145916415*m +
16643902275)*x^19 + 165*(m^11 + 127*m^10 + 7059*m^9 + 225837*m^8 + 4600554
*m^7 + 62319894*m^6 + 568863686*m^5 + 3466775738*m^4 + 13643071845*m^3 + 3
2368407579*m^2 + 40283194455*m + 18602008425)*x^17 + 330*(m^11 + 129*m^10
+ 7283*m^9 + 236595*m^8 + 4890858*m^7 + 67166442*m^6 + 620805254*m^5 + 382
5379590*m^4 + 15197565541*m^3 + 36337145829*m^2 + 45488935863*m + 21082276
215)*x^15 + 462*(m^11 + 131*m^10 + 7515*m^9 + 248145*m^8 + 5213898*m^7 + 7
2748638*m^6 + 682569590*m^5 + 4264053730*m^4 + 17145560901*m^3 + 414083372
31*m^2 + 52237739295*m + 24325703325)*x^13 + 462*(m^11 + 133*m^10 + 7755*m
^9 + 260535*m^8 + 5573898*m^7 + 79216434*m^6 + 756921110*m^5 + 4811326190*
m^4 + 19653671301*m^3 + 48110244633*m^2 + 61333432335*m + 28748558475)*x^1
1 + 330*(m^11 + 135*m^10 + 8003*m^9 + 273813*m^8 + 5975466*m^7 + 86750118*
m^6 + 847550822*m^5 + 5509501002*m^4 + 22992750373*m^3 + 57365875587*m^2 +
 74253243015*m + 35137127025)*x^9 + 165*(m^11 + 137*m^10 + 8259*m^9 + 2...
```

3.65.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11387 vs. 2(177) = 354.

Time = 2.32 (sec) , antiderivative size = 11387, normalized size of antiderivative = 56.09

$$\int (fx)^m \left(1+x^2\right) \left(1+2x^2+x^4\right)^5 \, dx = \text{Too large to display}$$

input
```
integrate((f*x)**m*(x**2+1)*(x**4+2*x**2+1)**5,x)
```

output
```
Piecewise(((log(x) - 11/(2*x**2) - 55/(4*x**4) - 55/(2*x**6) - 165/(4*x**8
) - 231/(5*x**10) - 77/(2*x**12) - 165/(7*x**14) - 165/(16*x**16) - 55/(18
*x**18) - 11/(20*x**20) - 1/(22*x**22))/f**23, Eq(m, -23)), ((x**2/2 + 11*
log(x) - 55/(2*x**2) - 165/(4*x**4) - 55/x**6 - 231/(4*x**8) - 231/(5*x**1
0) - 55/(2*x**12) - 165/(14*x**14) - 55/(16*x**16) - 11/(18*x**18) - 1/(20
*x**20))/f**21, Eq(m, -21)), ((x**4/4 + 11*x**2/2 + 55*log(x) - 165/(2*x**
2) - 165/(2*x**4) - 77/x**6 - 231/(4*x**8) - 33/x**10 - 55/(4*x**12) - 55/
(14*x**14) - 11/(16*x**16) - 1/(18*x**18))/f**19, Eq(m, -19)), ((x**6/6 +
11*x**4/4 + 55*x**2/2 + 165*log(x) - 165/x**2 - 231/(2*x**4) - 77/x**6 - 1
65/(4*x**8) - 33/(2*x**10) - 55/(12*x**12) - 11/(14*x**14) - 1/(16*x**16))
/f**17, Eq(m, -17)), ((x**8/8 + 11*x**6/6 + 55*x**4/4 + 165*x**2/2 + 330*l
og(x) - 231/x**2 - 231/(2*x**4) - 55/x**6 - 165/(8*x**8) - 11/(2*x**10) -
11/(12*x**12) - 1/(14*x**14))/f**15, Eq(m, -15)), ((x**10/10 + 11*x**8/8 +
 55*x**6/6 + 165*x**4/4 + 165*x**2 + 462*log(x) - 231/x**2 - 165/(2*x**4)
- 55/(2*x**6) - 55/(8*x**8) - 11/(10*x**10) - 1/(12*x**12))/f**13, Eq(m, -
13)), ((x**12/12 + 11*x**10/10 + 55*x**8/8 + 55*x**6/2 + 165*x**4/2 + 231*
x**2 + 462*log(x) - 165/x**2 - 165/(4*x**4) - 55/(6*x**6) - 11/(8*x**8) -
1/(10*x**10))/f**11, Eq(m, -11)), ((x**14/14 + 11*x**12/12 + 11*x**10/2 +
165*x**8/8 + 55*x**6 + 231*x**4/2 + 231*x**2 + 330*log(x) - 165/(2*x**2) -
 55/(4*x**4) - 11/(6*x**6) - 1/(8*x**8))/f**9, Eq(m, -9)), ((x**16/16 +...
```

3.65.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.95

$$\int (fx)^m (1+x^2) (1+2x^2+x^4)^5 \, dx = \frac{f^m x^{23} x^m}{m+23} + \frac{11 f^m x^{21} x^m}{m+21} + \frac{55 f^m x^{19} x^m}{m+19} + \frac{165 f^m x^{17} x^m}{m+17} + \frac{330 f^m x^{15} x^m}{m+15} + \frac{462 f^m x^{13} x^m}{m+13} + \frac{462 f^m x^{11} x^m}{m+11} + \frac{330 f^m x^9 x^m}{m+9} + \frac{165 f^m x^7 x^m}{m+7} + \frac{55 f^m x^5 x^m}{m+5} + \frac{11 f^m x^3 x^m}{m+3} + \frac{(fx)^{m+1}}{f(m+1)}$$

input
```
integrate((f*x)^m*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

output
```
f^m*x^23*x^m/(m + 23) + 11*f^m*x^21*x^m/(m + 21) + 55*f^m*x^19*x^m/(m + 19
) + 165*f^m*x^17*x^m/(m + 17) + 330*f^m*x^15*x^m/(m + 15) + 462*f^m*x^13*x
^m/(m + 13) + 462*f^m*x^11*x^m/(m + 11) + 330*f^m*x^9*x^m/(m + 9) + 165*f^
m*x^7*x^m/(m + 7) + 55*f^m*x^5*x^m/(m + 5) + 11*f^m*x^3*x^m/(m + 3) + (f*x
)^(m + 1)/(f*(m + 1))
```

3.65.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1848 vs. $2(203) = 406$.

Time = 0.30 (sec) , antiderivative size = 1848, normalized size of antiderivative = 9.10

$$\int (fx)^m (1+x^2) (1+2x^2+x^4)^5 \, dx = \text{Too large to display}$$

input
```
integrate((f*x)^m*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

output
```
((f*x)^m*m^11*x^23 + 121*(f*x)^m*m^10*x^23 + 11*(f*x)^m*m^11*x^21 + 6435*(
f*x)^m*m^9*x^23 + 1353*(f*x)^m*m^10*x^21 + 197835*(f*x)^m*m^8*x^23 + 55*(f
*x)^m*m^11*x^19 + 72985*(f*x)^m*m^9*x^21 + 3889578*(f*x)^m*m^7*x^23 + 6875
*(f*x)^m*m^10*x^19 + 2271555*(f*x)^m*m^8*x^21 + 51069018*(f*x)^m*m^6*x^23
+ 165*(f*x)^m*m^11*x^17 + 376365*(f*x)^m*m^9*x^19 + 45134958*(f*x)^m*m^7*x
^21 + 453714470*(f*x)^m*m^5*x^23 + 20955*(f*x)^m*m^10*x^17 + 11870265*(f*x
)^m*m^8*x^19 + 597988314*(f*x)^m*m^6*x^21 + 2702025590*(f*x)^m*m^4*x^23 +
330*(f*x)^m*m^11*x^15 + 1164735*(f*x)^m*m^9*x^17 + 238653030*(f*x)^m*m^7*x
^19 + 5353566130*(f*x)^m*m^5*x^21 + 10431670821*(f*x)^m*m^3*x^23 + 42570*(
f*x)^m*m^10*x^15 + 37263105*(f*x)^m*m^8*x^17 + 3194704590*(f*x)^m*m^6*x^19
 + 32087153670*(f*x)^m*m^4*x^21 + 24372200061*(f*x)^m*m^2*x^23 + 462*(f*x)
^m*m^11*x^13 + 2403390*(f*x)^m*m^9*x^15 + 759091410*(f*x)^m*m^7*x^17 + 288
57216410*(f*x)^m*m^5*x^19 + 124530626231*(f*x)^m*m^3*x^21 + 29985521895*(f
*x)^m*m*x^23 + 60522*(f*x)^m*m^10*x^13 + 78076350*(f*x)^m*m^8*x^15 + 10282
782510*(f*x)^m*m^6*x^17 + 174273100210*(f*x)^m*m^4*x^19 + 292163767533*(f*
x)^m*m^2*x^21 + 13749310575*(f*x)^m*x^23 + 462*(f*x)^m*m^11*x^11 + 3471930
*(f*x)^m*m^9*x^13 + 1613983140*(f*x)^m*m^7*x^15 + 93862508190*(f*x)^m*m^5*
x^17 + 680615362515*(f*x)^m*m^3*x^19 + 360568238085*(f*x)^m*m*x^21 + 61446
*(f*x)^m*m^10*x^11 + 114642990*(f*x)^m*m^8*x^13 + 22164925860*(f*x)^m*m^6*
x^15 + 572017996770*(f*x)^m*m^4*x^17 + 1604842704135*(f*x)^m*m^2*x^19 +...
```

3.65.9 Mupad [B] (verification not implemented)

Time = 8.34 (sec) , antiderivative size = 1483, normalized size of antiderivative = 7.31

$$\int (fx)^m \left(1+x^2\right) \left(1+2x^2+x^4\right)^5 \, dx = \text{Too large to display}$$

input
```
int((x^2 + 1)*(f*x)^m*(2*x^2 + x^4 + 1)^5,x)
```

output
```
(x^3*(f*x)^m*(2192684754645*m + 1434440867211*m^2 + 490955350391*m^3 + 102
468500970*m^4 + 14014513810*m^5 + 1298935638*m^6 + 82295598*m^7 + 3514005*
m^8 + 96745*m^9 + 1551*m^10 + 11*m^11 + 1159525191825))/(703416314160*m +
590546123298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 +
1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 14
4*m^11 + m^12 + 316234143225) + (x^19*(f*x)^m*(1988025402825*m + 160484270
4135*m^2 + 680615362515*m^3 + 174273100210*m^4 + 28857216410*m^5 + 3194704
590*m^6 + 238653030*m^7 + 11870265*m^8 + 376365*m^9 + 6875*m^10 + 55*m^11
+ 915414625125))/(703416314160*m + 590546123298*m^2 + 264300628944*m^3 + 7
2578259391*m^4 + 13137458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 843978
3*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225) + (x^11*(
f*x)^m*(28336045738770*m + 22226933020446*m^2 + 9079996141062*m^3 + 222283
2699780*m^4 + 349697552820*m^5 + 36597992508*m^6 + 2575140876*m^7 + 120367
170*m^8 + 3582810*m^9 + 61446*m^10 + 462*m^11 + 13281834015450))/(70341631
4160*m + 590546123298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 131374584
00*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*
m^10 + 144*m^11 + m^12 + 316234143225) + (x^21*(f*x)^m*(360568238085*m + 2
92163767533*m^2 + 124530626231*m^3 + 32087153670*m^4 + 5353566130*m^5 + 59
7988314*m^6 + 45134958*m^7 + 2271555*m^8 + 72985*m^9 + 1353*m^10 + 11*m^11
 + 165646455975))/(703416314160*m + 590546123298*m^2 + 264300628944*m^3...
```

3.66 $\int x^5(1+x^2)\left(1+2x^2+x^4\right)^5\,dx$

3.66.1 Optimal result

Integrand size = 21, antiderivative size = 34

$$\int x^5(1+x^2)\left(1+2x^2+x^4\right)^5\,dx=\frac{1}{24}\left(1+x^2\right)^{12}-\frac{1}{13}\left(1+x^2\right)^{13}+\frac{1}{28}\left(1+x^2\right)^{14}$$

output
```
1/24*(x^2+1)^12-1/13*(x^2+1)^13+1/28*(x^2+1)^14
```

3.66.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 85 vs. 2(34) = 68.

Time = 0.00 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.50

$$\int x^5(1+x^2)\left(1+2x^2+x^4\right)^5\,dx=\frac{x^6}{6}+\frac{11x^8}{8}+\frac{11x^{10}}{2}+\frac{55x^{12}}{4}+\frac{165x^{14}}{7}+\frac{231x^{16}}{8}+\frac{77x^{18}}{3}+\frac{33x^{20}}{2}+\frac{15x^{22}}{2}+\frac{55x^{24}}{24}+\frac{11x^{26}}{26}+\frac{x^{28}}{28}$$

input
```
Integrate[x^5*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

output
```
x^6/6 + (11*x^8)/8 + (11*x^10)/2 + (55*x^12)/4 + (165*x^14)/7 + (231*x^16)
/8 + (77*x^18)/3 + (33*x^20)/2 + (15*x^22)/2 + (55*x^24)/24 + (11*x^26)/26
 + x^28/28
```

3.66.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1380, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 (x^2+1) (x^4+2x^2+1)^5 \, dx$$

$$\Big\downarrow \text{1380}$$

$$\int x^5 (x^2+1)^{11} \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int x^4 (x^2+1)^{11} \, dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left((x^2+1)^{13} - 2(x^2+1)^{12} + (x^2+1)^{11}\right) dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{14}(x^2+1)^{14} - \frac{2}{13}(x^2+1)^{13} + \frac{1}{12}(x^2+1)^{12}\right)$$

input
```
Int[x^5*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

output
```
((1 + x^2)^12/12 - (2*(1 + x^2)^13)/13 + (1 + x^2)^14/14)/2
```

3.66.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.66.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

method	result
default	$\frac{(x^2+1)^{12}}{24} - \frac{(x^2+1)^{13}}{13} + \frac{(x^2+1)^{14}}{28}$
norman	$\frac{15}{2}x^{22} + \frac{55}{24}x^{24} + \frac{11}{26}x^{26} + \frac{1}{28}x^{28} + \frac{1}{6}x^{6} + \frac{11}{8}x^{8} + \frac{11}{2}x^{10} + \frac{55}{4}x^{12} + \frac{165}{7}x^{14} + \frac{231}{8}x^{16} + \frac{77}{3}x^{18} + \frac{33}{2}$
parallelrisch	$\frac{15}{2}x^{22} + \frac{55}{24}x^{24} + \frac{11}{26}x^{26} + \frac{1}{28}x^{28} + \frac{1}{6}x^{6} + \frac{11}{8}x^{8} + \frac{11}{2}x^{10} + \frac{55}{4}x^{12} + \frac{165}{7}x^{14} + \frac{231}{8}x^{16} + \frac{77}{3}x^{18} + \frac{33}{2}$
gosper	$\frac{x^6(78x^{22}+924x^{20}+5005x^{18}+16380x^{16}+36036x^{14}+56056x^{12}+63063x^{10}+51480x^8+30030x^6+12012x^4+3003x^2+364)}{2184}$
risch	$\frac{11}{2}x^{10} + \frac{55}{4}x^{12} + \frac{165}{7}x^{14} + \frac{11}{8}x^{8} + \frac{1}{6}x^{6} + \frac{1}{2184} + \frac{55}{24}x^{24} + \frac{11}{26}x^{26} + \frac{1}{28}x^{28} + \frac{33}{2}x^{20} + \frac{77}{3}x^{18} + \frac{231}{8}$

input
```
int(x^5*(x^2+1)*(x^4+2*x^2+1)^5,x,method=_RETURNVERBOSE)
```

output
```
1/24*(x^2+1)^12-1/13*(x^2+1)^13+1/28*(x^2+1)^14
```

3.66.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(28) = 56.

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.79

$$\int x^5(1+x^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{28}x^{28} + \frac{11}{26}x^{26} + \frac{55}{24}x^{24} + \frac{15}{2}x^{22} + \frac{33}{2}x^{20} + \frac{77}{3}x^{18} + \frac{231}{8}x^{16} + \frac{165}{7}x^{14} + \frac{55}{4}x^{12} + \frac{11}{2}x^{10} + \frac{11}{8}x^{8} + \frac{1}{6}x^{6}$$

input
```
integrate(x^5*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

output
```
1/28*x^28 + 11/26*x^26 + 55/24*x^24 + 15/2*x^22 + 33/2*x^20 + 77/3*x^18 +
231/8*x^16 + 165/7*x^14 + 55/4*x^12 + 11/2*x^10 + 11/8*x^8 + 1/6*x^6
```

3.66.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(24) = 48.

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.24

$$\int x^5(1+x^2)\left(1+2x^2+x^4\right)^5\,dx=\frac{x^{28}}{28}+\frac{11x^{26}}{26}+\frac{55x^{24}}{24}+\frac{15x^{22}}{2}+\frac{33x^{20}}{2}+\frac{77x^{18}}{3}+\frac{231x^{16}}{8}+\frac{165x^{14}}{7}+\frac{55x^{12}}{4}+\frac{11x^{10}}{2}+\frac{11x^{8}}{8}+\frac{x^{6}}{6}$$

input
```
integrate(x**5*(x**2+1)*(x**4+2*x**2+1)**5,x)
```

output
```
x**28/28 + 11*x**26/26 + 55*x**24/24 + 15*x**22/2 + 33*x**20/2 + 77*x**18/
3 + 231*x**16/8 + 165*x**14/7 + 55*x**12/4 + 11*x**10/2 + 11*x**8/8 + x**6
/6
```

3.66.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(28) = 56.

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.79

$$\int x^5(1+x^2)\left(1+2x^2+x^4\right)^5\,dx=\frac{1}{28}x^{28}+\frac{11}{26}x^{26}+\frac{55}{24}x^{24}+\frac{15}{2}x^{22}+\frac{33}{2}x^{20}+\frac{77}{3}x^{18}+\frac{231}{8}x^{16}+\frac{165}{7}x^{14}+\frac{55}{4}x^{12}+\frac{11}{2}x^{10}+\frac{11}{8}x^{8}+\frac{1}{6}x^{6}$$

input
```
integrate(x^5*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

output
```
1/28*x^28 + 11/26*x^26 + 55/24*x^24 + 15/2*x^22 + 33/2*x^20 + 77/3*x^18 +
231/8*x^16 + 165/7*x^14 + 55/4*x^12 + 11/2*x^10 + 11/8*x^8 + 1/6*x^6
```

3.66.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(28) = 56$.

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.79

$$\int x^5(1+x^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{28}x^{28}+\frac{11}{26}x^{26}+\frac{55}{24}x^{24}+\frac{15}{2}x^{22}+\frac{33}{2}x^{20}+\frac{77}{3}x^{18}+\frac{231}{8}x^{16}+\frac{165}{7}x^{14}+\frac{55}{4}x^{12}+\frac{11}{2}x^{10}+\frac{11}{8}x^8+\frac{1}{6}x^6$$

input
```
integrate(x^5*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

output
```
1/28*x^28 + 11/26*x^26 + 55/24*x^24 + 15/2*x^22 + 33/2*x^20 + 77/3*x^18 +
231/8*x^16 + 165/7*x^14 + 55/4*x^12 + 11/2*x^10 + 11/8*x^8 + 1/6*x^6
```

3.66.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.79

$$\int x^5(1+x^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{x^{28}}{28}+\frac{11\,x^{26}}{26}+\frac{55\,x^{24}}{24}+\frac{15\,x^{22}}{2}+\frac{33\,x^{20}}{2}+\frac{77\,x^{18}}{3}+\frac{231\,x^{16}}{8}+\frac{165\,x^{14}}{7}+\frac{55\,x^{12}}{4}+\frac{11\,x^{10}}{2}+\frac{11\,x^8}{8}+\frac{x^6}{6}$$

input
```
int(x^5*(x^2 + 1)*(2*x^2 + x^4 + 1)^5,x)
```

output
```
x^6/6 + (11*x^8)/8 + (11*x^10)/2 + (55*x^12)/4 + (165*x^14)/7 + (231*x^16)
/8 + (77*x^18)/3 + (33*x^20)/2 + (15*x^22)/2 + (55*x^24)/24 + (11*x^26)/26
 + x^28/28
```

3.67 $\int x^4(1+x^2)\left(1+2x^2+x^4\right)^5\,dx$

3.67.1 Optimal result

Integrand size = 21, antiderivative size = 83

$$\int x^4(1+x^2)\left(1+2x^2+x^4\right)^5\,dx=\frac{x^5}{5}+\frac{11x^7}{7}+\frac{55x^9}{9}+15x^{11}+\frac{330x^{13}}{13}+\frac{154x^{15}}{5}+\frac{462x^{17}}{17}+\frac{330x^{19}}{19}+\frac{55x^{21}}{7}+\frac{55x^{23}}{23}+\frac{11x^{25}}{25}+\frac{x^{27}}{27}$$

output
```
1/5*x^5+11/7*x^7+55/9*x^9+15*x^11+330/13*x^13+154/5*x^15+462/17*x^17+330/1
9*x^19+55/7*x^21+55/23*x^23+11/25*x^25+1/27*x^27
```

3.67.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int x^4(1+x^2)\left(1+2x^2+x^4\right)^5\,dx=\frac{x^5}{5}+\frac{11x^7}{7}+\frac{55x^9}{9}+15x^{11}+\frac{330x^{13}}{13}+\frac{154x^{15}}{5}+\frac{462x^{17}}{17}+\frac{330x^{19}}{19}+\frac{55x^{21}}{7}+\frac{55x^{23}}{23}+\frac{11x^{25}}{25}+\frac{x^{27}}{27}$$

input
```
Integrate[x^4*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

output
```
x^5/5 + (11*x^7)/7 + (55*x^9)/9 + 15*x^11 + (330*x^13)/13 + (154*x^15)/5 +
 (462*x^17)/17 + (330*x^19)/19 + (55*x^21)/7 + (55*x^23)/23 + (11*x^25)/25
 + x^27/27
```

3.67.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1380, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 (x^2+1) \left(x^4+2x^2+1\right)^5 dx$$

$$\Big\downarrow \text{ 1380}$$

$$\int x^4 \left(x^2+1\right)^{11} dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(x^{26}+11x^{24}+55x^{22}+165x^{20}+330x^{18}+462x^{16}+462x^{14}+330x^{12}+165x^{10}+55x^8+11x^6+x^4\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^{27}}{27}+\frac{11x^{25}}{25}+\frac{55x^{23}}{23}+\frac{55x^{21}}{7}+\frac{330x^{19}}{19}+\frac{462x^{17}}{17}+\frac{154x^{15}}{5}+\frac{330x^{13}}{13}+15x^{11}+\frac{55x^9}{9}+\frac{11x^7}{7}+\frac{x^5}{5}$$

input
```
Int[x^4*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

output
```
x^5/5 + (11*x^7)/7 + (55*x^9)/9 + 15*x^11 + (330*x^13)/13 + (154*x^15)/5 +
 (462*x^17)/17 + (330*x^19)/19 + (55*x^21)/7 + (55*x^23)/23 + (11*x^25)/25
 + x^27/27
```

3.67.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.67.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.75

method	result
default	$\frac{1}{5}x^5+\frac{11}{7}x^7+\frac{55}{9}x^9+15x^{11}+\frac{330}{13}x^{13}+\frac{154}{5}x^{15}+\frac{462}{17}x^{17}+\frac{330}{19}x^{19}+\frac{55}{7}x^{21}+\frac{55}{23}x^{23}+\frac{11}{25}x^{25}+$
norman	$\frac{1}{5}x^5+\frac{11}{7}x^7+\frac{55}{9}x^9+15x^{11}+\frac{330}{13}x^{13}+\frac{154}{5}x^{15}+\frac{462}{17}x^{17}+\frac{330}{19}x^{19}+\frac{55}{7}x^{21}+\frac{55}{23}x^{23}+\frac{11}{25}x^{25}+$
risch	$\frac{1}{5}x^5+\frac{11}{7}x^7+\frac{55}{9}x^9+15x^{11}+\frac{330}{13}x^{13}+\frac{154}{5}x^{15}+\frac{462}{17}x^{17}+\frac{330}{19}x^{19}+\frac{55}{7}x^{21}+\frac{55}{23}x^{23}+\frac{11}{25}x^{25}+$
parallelrisch	$\frac{1}{5}x^5+\frac{11}{7}x^7+\frac{55}{9}x^9+15x^{11}+\frac{330}{13}x^{13}+\frac{154}{5}x^{15}+\frac{462}{17}x^{17}+\frac{330}{19}x^{19}+\frac{55}{7}x^{21}+\frac{55}{23}x^{23}+\frac{11}{25}x^{25}+$
gosper	$\frac{x^5(16900975x^{22}+200783583x^{20}+1091215125x^{18}+3585421125x^{16}+7925667750x^{14}+12401338950x^{12}+14054850810x^{10}+115}{456326325}$

input
```
int(x^4*(x^2+1)*(x^4+2*x^2+1)^5,x,method=_RETURNVERBOSE)
```

output
```
1/5*x^5+11/7*x^7+55/9*x^9+15*x^11+330/13*x^13+154/5*x^15+462/17*x^17+330/1
9*x^19+55/7*x^21+55/23*x^23+11/25*x^25+1/27*x^27
```

3.67.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int x^4(1+x^2)\left(1+2x^2+x^4\right)^5\,dx=\frac{1}{27}\,x^{27}+\frac{11}{25}\,x^{25}+\frac{55}{23}\,x^{23}+\frac{55}{7}\,x^{21}+\frac{330}{19}\,x^{19}+\frac{462}{17}\,x^{17}+\frac{154}{5}\,x^{15}+\frac{330}{13}\,x^{13}+15\,x^{11}+\frac{55}{9}\,x^{9}+\frac{11}{7}\,x^{7}+\frac{1}{5}\,x^{5}$$

input
```
integrate(x^4*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

output
```
1/27*x^27 + 11/25*x^25 + 55/23*x^23 + 55/7*x^21 + 330/19*x^19 + 462/17*x^1
7 + 154/5*x^15 + 330/13*x^13 + 15*x^11 + 55/9*x^9 + 11/7*x^7 + 1/5*x^5
```

3.67.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int x^4(1+x^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{x^{27}}{27}+\frac{11x^{25}}{25}+\frac{55x^{23}}{23}+\frac{55x^{21}}{7}+\frac{330x^{19}}{19}+\frac{462x^{17}}{17}+\frac{154x^{15}}{5}+\frac{330x^{13}}{13}+15x^{11}+\frac{55x^9}{9}+\frac{11x^7}{7}+\frac{x^5}{5}$$

input
```
integrate(x**4*(x**2+1)*(x**4+2*x**2+1)**5,x)
```

output
```
x**27/27 + 11*x**25/25 + 55*x**23/23 + 55*x**21/7 + 330*x**19/19 + 462*x**
17/17 + 154*x**15/5 + 330*x**13/13 + 15*x**11 + 55*x**9/9 + 11*x**7/7 + x*
*5/5
```

3.67.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int x^4(1+x^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{27}x^{27}+\frac{11}{25}x^{25}+\frac{55}{23}x^{23}+\frac{55}{7}x^{21}+\frac{330}{19}x^{19}+\frac{462}{17}x^{17}+\frac{154}{5}x^{15}+\frac{330}{13}x^{13}+15x^{11}+\frac{55}{9}x^9+\frac{11}{7}x^7+\frac{1}{5}x^5$$

input
```
integrate(x^4*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

output
```
1/27*x^27 + 11/25*x^25 + 55/23*x^23 + 55/7*x^21 + 330/19*x^19 + 462/17*x^1
7 + 154/5*x^15 + 330/13*x^13 + 15*x^11 + 55/9*x^9 + 11/7*x^7 + 1/5*x^5
```

3.67.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int x^4(1+x^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{27}x^{27}+\frac{11}{25}x^{25}+\frac{55}{23}x^{23}+\frac{55}{7}x^{21}+\frac{330}{19}x^{19}+\frac{462}{17}x^{17}+\frac{154}{5}x^{15}+\frac{330}{13}x^{13}+15x^{11}+\frac{55}{9}x^9+\frac{11}{7}x^7+\frac{1}{5}x^5$$

input
```
integrate(x^4*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

output
```
1/27*x^27 + 11/25*x^25 + 55/23*x^23 + 55/7*x^21 + 330/19*x^19 + 462/17*x^1
7 + 154/5*x^15 + 330/13*x^13 + 15*x^11 + 55/9*x^9 + 11/7*x^7 + 1/5*x^5
```

3.67.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int x^4(1+x^2)\left(1+2x^2+x^4\right)^5\,dx=\frac{x^{27}}{27}+\frac{11\,x^{25}}{25}+\frac{55\,x^{23}}{23}+\frac{55\,x^{21}}{7}+\frac{330\,x^{19}}{19}+\frac{462\,x^{17}}{17}+\frac{154\,x^{15}}{5}+\frac{330\,x^{13}}{13}+15\,x^{11}+\frac{55\,x^9}{9}+\frac{11\,x^7}{7}+\frac{x^5}{5}$$

input
```
int(x^4*(x^2 + 1)*(2*x^2 + x^4 + 1)^5,x)
```

output
```
x^5/5 + (11*x^7)/7 + (55*x^9)/9 + 15*x^11 + (330*x^13)/13 + (154*x^15)/5 +
 (462*x^17)/17 + (330*x^19)/19 + (55*x^21)/7 + (55*x^23)/23 + (11*x^25)/25
 + x^27/27
```

3.68 $\int x^3(1+x^2)\left(1+2x^2+x^4\right)^5\,dx$

3.68.1 Optimal result

Integrand size = 21, antiderivative size = 23

$$\int x^3(1+x^2)\left(1+2x^2+x^4\right)^5\,dx=-\frac{1}{24}\left(1+x^2\right)^{12}+\frac{1}{26}\left(1+x^2\right)^{13}$$

output
```
-1/24*(x^2+1)^12+1/26*(x^2+1)^13
```

3.68.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 83 vs. 2(23) = 46.

Time = 0.00 (sec) , antiderivative size = 83, normalized size of antiderivative = 3.61

$$\int x^3(1+x^2)\left(1+2x^2+x^4\right)^5\,dx=\frac{x^4}{4}+\frac{11x^6}{6}+\frac{55x^8}{8}+\frac{33x^{10}}{2}+\frac{55x^{12}}{2}+33x^{14}+\frac{231x^{16}}{8}+\frac{55x^{18}}{3}+\frac{33x^{20}}{4}+\frac{5x^{22}}{2}+\frac{11x^{24}}{24}+\frac{x^{26}}{26}$$

input
```
Integrate[x^3*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

output
```
x^4/4 + (11*x^6)/6 + (55*x^8)/8 + (33*x^10)/2 + (55*x^12)/2 + 33*x^14 + (2
31*x^16)/8 + (55*x^18)/3 + (33*x^20)/4 + (5*x^22)/2 + (11*x^24)/24 + x^26/
26
```

3.68.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1380, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (x^2+1) (x^4+2x^2+1)^5 \, dx$$

$$\Big\downarrow \text{ 1380}$$

$$\int x^3 (x^2+1)^{11} \, dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2} \int x^2 (x^2+1)^{11} \, dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2} \int \left((x^2+1)^{12} - (x^2+1)^{11}\right) dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2} \left(\frac{1}{13} (x^2+1)^{13} - \frac{1}{12} (x^2+1)^{12}\right)$$

input
```
Int[x^3*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

output
```
(-1/12*(1 + x^2)^12 + (1 + x^2)^13/13)/2
```

3.68.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.68.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result
default	$-\frac{(x^2+1)^{12}}{24}+\frac{(x^2+1)^{13}}{26}$
norman	$\frac{5}{2}x^{22}+\frac{11}{24}x^{24}+\frac{1}{26}x^{26}+\frac{1}{4}x^4+\frac{11}{6}x^6+\frac{55}{8}x^8+\frac{33}{2}x^{10}+\frac{55}{2}x^{12}+33x^{14}+\frac{231}{8}x^{16}+\frac{55}{3}x^{18}+\frac{33}{4}x$
parallelrisch	$\frac{5}{2}x^{22}+\frac{11}{24}x^{24}+\frac{1}{26}x^{26}+\frac{1}{4}x^4+\frac{11}{6}x^6+\frac{55}{8}x^8+\frac{33}{2}x^{10}+\frac{55}{2}x^{12}+33x^{14}+\frac{231}{8}x^{16}+\frac{55}{3}x^{18}+\frac{33}{4}x$
gosper	$\frac{x^4(12x^{22}+143x^{20}+780x^{18}+2574x^{16}+5720x^{14}+9009x^{12}+10296x^{10}+8580x^8+5148x^6+2145x^4+572x^2+78)}{312}$
risch	$\frac{33}{2}x^{10}+\frac{55}{2}x^{12}+\frac{1}{4}x^4+33x^{14}+\frac{55}{8}x^8+\frac{11}{6}x^6+\frac{11}{24}x^{24}+\frac{1}{26}x^{26}+\frac{33}{4}x^{20}+\frac{55}{3}x^{18}+\frac{231}{8}x^{16}+\frac{5}{2}x$

input
```
int(x^3*(x^2+1)*(x^4+2*x^2+1)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/24*(x^2+1)^12+1/26*(x^2+1)^13
```

3.68.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(19) = 38.

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.65

$$\int x^3(1+x^2)\left(1+2x^2+x^4\right)^5\,dx=\frac{1}{26}\,x^{26}+\frac{11}{24}\,x^{24}+\frac{5}{2}\,x^{22}+\frac{33}{4}\,x^{20}+\frac{55}{3}\,x^{18}+\frac{231}{8}\,x^{16}+33\,x^{14}+\frac{55}{2}\,x^{12}+\frac{33}{2}\,x^{10}+\frac{55}{8}\,x^8+\frac{11}{6}\,x^6+\frac{1}{4}\,x^4$$

input
```
integrate(x^3*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

output
```
1/26*x^26 + 11/24*x^24 + 5/2*x^22 + 33/4*x^20 + 55/3*x^18 + 231/8*x^16 + 3
3*x^14 + 55/2*x^12 + 33/2*x^10 + 55/8*x^8 + 11/6*x^6 + 1/4*x^4
```

3.68.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(15) = 30$.

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 3.26

$$\int x^3(1+x^2)\left(1+2x^2+x^4\right)^5\,dx=\frac{x^{26}}{26}+\frac{11x^{24}}{24}+\frac{5x^{22}}{2}+\frac{33x^{20}}{4}+\frac{55x^{18}}{3}+\frac{231x^{16}}{8}+33x^{14}+\frac{55x^{12}}{2}+\frac{33x^{10}}{2}+\frac{55x^8}{8}+\frac{11x^6}{6}+\frac{x^4}{4}$$

input
```
integrate(x**3*(x**2+1)*(x**4+2*x**2+1)**5,x)
```

output
```
x**26/26 + 11*x**24/24 + 5*x**22/2 + 33*x**20/4 + 55*x**18/3 + 231*x**16/8
 + 33*x**14 + 55*x**12/2 + 33*x**10/2 + 55*x**8/8 + 11*x**6/6 + x**4/4
```

3.68.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(19) = 38$.

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.65

$$\int x^3(1+x^2)\left(1+2x^2+x^4\right)^5\,dx=\frac{1}{26}x^{26}+\frac{11}{24}x^{24}+\frac{5}{2}x^{22}+\frac{33}{4}x^{20}+\frac{55}{3}x^{18}+\frac{231}{8}x^{16}+33\,x^{14}+\frac{55}{2}x^{12}+\frac{33}{2}x^{10}+\frac{55}{8}x^8+\frac{11}{6}x^6+\frac{1}{4}x^4$$

input
```
integrate(x^3*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

output
```
1/26*x^26 + 11/24*x^24 + 5/2*x^22 + 33/4*x^20 + 55/3*x^18 + 231/8*x^16 + 3
3*x^14 + 55/2*x^12 + 33/2*x^10 + 55/8*x^8 + 11/6*x^6 + 1/4*x^4
```

3.68.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(19) = 38.

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.65

$$\int x^3(1+x^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{26}x^{26}+\frac{11}{24}x^{24}+\frac{5}{2}x^{22}+\frac{33}{4}x^{20}+\frac{55}{3}x^{18}+\frac{231}{8}x^{16}+33\,x^{14}+\frac{55}{2}x^{12}+\frac{33}{2}x^{10}+\frac{55}{8}x^{8}+\frac{11}{6}x^{6}+\frac{1}{4}x^{4}$$

input
```
integrate(x^3*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

output
```
1/26*x^26 + 11/24*x^24 + 5/2*x^22 + 33/4*x^20 + 55/3*x^18 + 231/8*x^16 + 3
3*x^14 + 55/2*x^12 + 33/2*x^10 + 55/8*x^8 + 11/6*x^6 + 1/4*x^4
```

3.68.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.65

$$\int x^3(1+x^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{x^{26}}{26}+\frac{11\,x^{24}}{24}+\frac{5\,x^{22}}{2}+\frac{33\,x^{20}}{4}+\frac{55\,x^{18}}{3}+\frac{231\,x^{16}}{8}+33\,x^{14}+\frac{55\,x^{12}}{2}+\frac{33\,x^{10}}{2}+\frac{55\,x^{8}}{8}+\frac{11\,x^{6}}{6}+\frac{x^4}{4}$$

input
```
int(x^3*(x^2 + 1)*(2*x^2 + x^4 + 1)^5,x)
```

output
```
x^4/4 + (11*x^6)/6 + (55*x^8)/8 + (33*x^10)/2 + (55*x^12)/2 + 33*x^14 + (2
31*x^16)/8 + (55*x^18)/3 + (33*x^20)/4 + (5*x^22)/2 + (11*x^24)/24 + x^26/
26
```

3.69 $\int x^2(1+x^2)(1+2x^2+x^4)^5\,dx$

3.69.1 Optimal result

Integrand size = 21, antiderivative size = 83

$$\int x^2(1+x^2)(1+2x^2+x^4)^5\,dx=\frac{x^3}{3}+\frac{11x^5}{5}+\frac{55x^7}{7}+\frac{55x^9}{3}+30x^{11}+\frac{462x^{13}}{13}+\frac{154x^{15}}{5}+\frac{330x^{17}}{17}+\frac{165x^{19}}{19}+\frac{55x^{21}}{21}+\frac{11x^{23}}{23}+\frac{x^{25}}{25}$$

output
```
1/3*x^3+11/5*x^5+55/7*x^7+55/3*x^9+30*x^11+462/13*x^13+154/5*x^15+330/17*x
^17+165/19*x^19+55/21*x^21+11/23*x^23+1/25*x^25
```

3.69.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int x^2(1+x^2)(1+2x^2+x^4)^5\,dx=\frac{x^3}{3}+\frac{11x^5}{5}+\frac{55x^7}{7}+\frac{55x^9}{3}+30x^{11}+\frac{462x^{13}}{13}+\frac{154x^{15}}{5}+\frac{330x^{17}}{17}+\frac{165x^{19}}{19}+\frac{55x^{21}}{21}+\frac{11x^{23}}{23}+\frac{x^{25}}{25}$$

input
```
Integrate[x^2*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

output
```
x^3/3 + (11*x^5)/5 + (55*x^7)/7 + (55*x^9)/3 + 30*x^11 + (462*x^13)/13 + (
154*x^15)/5 + (330*x^17)/17 + (165*x^19)/19 + (55*x^21)/21 + (11*x^23)/23
+ x^25/25
```

3.69.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1380, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(x^2+1)\left(x^4+2x^2+1\right)^5 dx$$

$$\Big\downarrow \text{1380}$$

$$\int x^2(x^2+1)^{11} dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(x^{24}+11x^{22}+55x^{20}+165x^{18}+330x^{16}+462x^{14}+462x^{12}+330x^{10}+165x^8+55x^6+11x^4+x^2\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^{25}}{25}+\frac{11x^{23}}{23}+\frac{55x^{21}}{21}+\frac{165x^{19}}{19}+\frac{330x^{17}}{17}+\frac{154x^{15}}{5}+\frac{462x^{13}}{13}+30x^{11}+\frac{55x^9}{3}+\frac{55x^7}{7}+\frac{11x^5}{5}+\frac{x^3}{3}$$

input
```
Int[x^2*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

output
```
x^3/3 + (11*x^5)/5 + (55*x^7)/7 + (55*x^9)/3 + 30*x^11 + (462*x^13)/13 + (
154*x^15)/5 + (330*x^17)/17 + (165*x^19)/19 + (55*x^21)/21 + (11*x^23)/23
+ x^25/25
```

3.69.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.69.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.75

method	result
default	$\frac{1}{3}x^3+\frac{11}{5}x^5+\frac{55}{7}x^7+\frac{55}{3}x^9+30x^{11}+\frac{462}{13}x^{13}+\frac{154}{5}x^{15}+\frac{330}{17}x^{17}+\frac{165}{19}x^{19}+\frac{55}{21}x^{21}+\frac{11}{23}x^{23}+$
norman	$\frac{1}{3}x^3+\frac{11}{5}x^5+\frac{55}{7}x^7+\frac{55}{3}x^9+30x^{11}+\frac{462}{13}x^{13}+\frac{154}{5}x^{15}+\frac{330}{17}x^{17}+\frac{165}{19}x^{19}+\frac{55}{21}x^{21}+\frac{11}{23}x^{23}+$
risch	$\frac{1}{3}x^3+\frac{11}{5}x^5+\frac{55}{7}x^7+\frac{55}{3}x^9+30x^{11}+\frac{462}{13}x^{13}+\frac{154}{5}x^{15}+\frac{330}{17}x^{17}+\frac{165}{19}x^{19}+\frac{55}{21}x^{21}+\frac{11}{23}x^{23}+$
parallelrisch	$\frac{1}{3}x^3+\frac{11}{5}x^5+\frac{55}{7}x^7+\frac{55}{3}x^9+30x^{11}+\frac{462}{13}x^{13}+\frac{154}{5}x^{15}+\frac{330}{17}x^{17}+\frac{165}{19}x^{19}+\frac{55}{21}x^{21}+\frac{11}{23}x^{23}+$
gosper	$\frac{x^3(2028117x^{22}+24249225x^{20}+132793375x^{18}+440314875x^{16}+984233250x^{14}+1561650090x^{12}+1801903950x^{10}+1521087750}{50702925}$

input
```
int(x^2*(x^2+1)*(x^4+2*x^2+1)^5,x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3+11/5*x^5+55/7*x^7+55/3*x^9+30*x^11+462/13*x^13+154/5*x^15+330/17*x
^17+165/19*x^19+55/21*x^21+11/23*x^23+1/25*x^25
```

3.69.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int x^2(1+x^2)\left(1+2x^2+x^4\right)^5\,dx=\frac{1}{25}\,x^{25}+\frac{11}{23}\,x^{23}+\frac{55}{21}\,x^{21}+\frac{165}{19}\,x^{19}+\frac{330}{17}\,x^{17}+\frac{154}{5}\,x^{15}+\frac{462}{13}\,x^{13}+30\,x^{11}+\frac{55}{3}\,x^9+\frac{55}{7}\,x^7+\frac{11}{5}\,x^5+\frac{1}{3}\,x^3$$

input
```
integrate(x^2*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

output
```
1/25*x^25 + 11/23*x^23 + 55/21*x^21 + 165/19*x^19 + 330/17*x^17 + 154/5*x^
15 + 462/13*x^13 + 30*x^11 + 55/3*x^9 + 55/7*x^7 + 11/5*x^5 + 1/3*x^3
```

3.69.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int x^2(1+x^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{x^{25}}{25}+\frac{11x^{23}}{23}+\frac{55x^{21}}{21}+\frac{165x^{19}}{19}+\frac{330x^{17}}{17}+\frac{154x^{15}}{5}+\frac{462x^{13}}{13}+30x^{11}+\frac{55x^9}{3}+\frac{55x^7}{7}+\frac{11x^5}{5}+\frac{x^3}{3}$$

input
```
integrate(x**2*(x**2+1)*(x**4+2*x**2+1)**5,x)
```

output
```
x**25/25 + 11*x**23/23 + 55*x**21/21 + 165*x**19/19 + 330*x**17/17 + 154*x
**15/5 + 462*x**13/13 + 30*x**11 + 55*x**9/3 + 55*x**7/7 + 11*x**5/5 + x**
3/3
```

3.69.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int x^2(1+x^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{25}\,x^{25}+\frac{11}{23}\,x^{23}+\frac{55}{21}\,x^{21}+\frac{165}{19}\,x^{19}+\frac{330}{17}\,x^{17}+\frac{154}{5}\,x^{15}+\frac{462}{13}\,x^{13}+30\,x^{11}+\frac{55}{3}\,x^9+\frac{55}{7}\,x^7+\frac{11}{5}\,x^5+\frac{1}{3}\,x^3$$

input
```
integrate(x^2*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

output
```
1/25*x^25 + 11/23*x^23 + 55/21*x^21 + 165/19*x^19 + 330/17*x^17 + 154/5*x^
15 + 462/13*x^13 + 30*x^11 + 55/3*x^9 + 55/7*x^7 + 11/5*x^5 + 1/3*x^3
```

3.69.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int x^2(1+x^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{25}\,x^{25}+\frac{11}{23}\,x^{23}+\frac{55}{21}\,x^{21}+\frac{165}{19}\,x^{19}+\frac{330}{17}\,x^{17}+\frac{154}{5}\,x^{15}+\frac{462}{13}\,x^{13}+30\,x^{11}+\frac{55}{3}\,x^9+\frac{55}{7}\,x^7+\frac{11}{5}\,x^5+\frac{1}{3}\,x^3$$

input
```
integrate(x^2*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

output
```
1/25*x^25 + 11/23*x^23 + 55/21*x^21 + 165/19*x^19 + 330/17*x^17 + 154/5*x^
15 + 462/13*x^13 + 30*x^11 + 55/3*x^9 + 55/7*x^7 + 11/5*x^5 + 1/3*x^3
```

3.69.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int x^2(1+x^2)\left(1+2x^2+x^4\right)^5\,dx=\frac{x^{25}}{25}+\frac{11\,x^{23}}{23}+\frac{55\,x^{21}}{21}+\frac{165\,x^{19}}{19}+\frac{330\,x^{17}}{17}+\frac{154\,x^{15}}{5}+\frac{462\,x^{13}}{13}+30\,x^{11}+\frac{55\,x^9}{3}+\frac{55\,x^7}{7}+\frac{11\,x^5}{5}+\frac{x^3}{3}$$

input
```
int(x^2*(x^2 + 1)*(2*x^2 + x^4 + 1)^5,x)
```

output
```
x^3/3 + (11*x^5)/5 + (55*x^7)/7 + (55*x^9)/3 + 30*x^11 + (462*x^13)/13 + (
154*x^15)/5 + (330*x^17)/17 + (165*x^19)/19 + (55*x^21)/21 + (11*x^23)/23
+ x^25/25
```

3.70 $\int x(1+x^2)\left(1+2x^2+x^4\right)^5\,dx$

3.70.1 Optimal result

Integrand size = 19, antiderivative size = 11

$$\int x(1+x^2)\left(1+2x^2+x^4\right)^5\,dx=\frac{1}{24}\left(1+x^2\right)^{12}$$

output
```
1/24*(x^2+1)^12
```

3.70.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int x(1+x^2)\left(1+2x^2+x^4\right)^5\,dx=\frac{1}{24}\left(1+x^2\right)^{12}$$

input
```
Integrate[x*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

output
```
(1 + x^2)^12/24
```

3.70.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1380, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(x^2+1)\left(x^4+2x^2+1\right)^5\,dx$$

$$\Big\downarrow \text{1380}$$

$$\int x\left(x^2+1\right)^{11}dx$$

$$\Big\downarrow \text{241}$$

$$\frac{1}{24}\left(x^2+1\right)^{12}$$

input
```
Int[x*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

output
```
(1 + x^2)^12/24
```

3.70.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

3.70.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result
default	$\frac{(x^2+1)^{12}}{24}$
gosper	$\frac{x^2(x^{22}+12x^{20}+66x^{18}+220x^{16}+495x^{14}+792x^{12}+924x^{10}+792x^8+495x^6+220x^4+66x^2+12)}{24}$
norman	$\frac{1}{24}x^{24}+\frac{1}{2}x^{22}+\frac{11}{4}x^{20}+\frac{55}{6}x^{18}+33x^{14}+\frac{165}{8}x^{16}+\frac{165}{8}x^8+33x^{10}+\frac{77}{2}x^{12}+\frac{1}{2}x^2+\frac{11}{4}x^4+\frac{55}{6}$
parallelrisch	$\frac{1}{24}x^{24}+\frac{1}{2}x^{22}+\frac{11}{4}x^{20}+\frac{55}{6}x^{18}+33x^{14}+\frac{165}{8}x^{16}+\frac{165}{8}x^8+33x^{10}+\frac{77}{2}x^{12}+\frac{1}{2}x^2+\frac{11}{4}x^4+\frac{55}{6}$
risch	$\frac{1}{24}x^{24}+\frac{1}{2}x^{22}+\frac{11}{4}x^{20}+\frac{55}{6}x^{18}+\frac{165}{8}x^{16}+33x^{14}+\frac{77}{2}x^{12}+33x^{10}+\frac{165}{8}x^8+\frac{55}{6}x^6+\frac{11}{4}x^4+\frac{1}{2}$

input
```
int(x*(x^2+1)*(x^4+2*x^2+1)^5,x,method=_RETURNVERBOSE)
```

output
```
1/24*(x^2+1)^12
```

3.70.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(9) = 18.

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 5.55

$$\int x(1+x^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{24}x^{24}+\frac{1}{2}x^{22}+\frac{11}{4}x^{20}+\frac{55}{6}x^{18}+\frac{165}{8}x^{16}+33\,x^{14}+\frac{77}{2}x^{12}+33\,x^{10}+\frac{165}{8}x^8+\frac{55}{6}x^6+\frac{11}{4}x^4+\frac{1}{2}x^2$$

input
```
integrate(x*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

output
```
1/24*x^24 + 1/2*x^22 + 11/4*x^20 + 55/6*x^18 + 165/8*x^16 + 33*x^14 + 77/2
*x^12 + 33*x^10 + 165/8*x^8 + 55/6*x^6 + 11/4*x^4 + 1/2*x^2
```

3.70.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(7) = 14.

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 6.45

$$\int x(1+x^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{x^{24}}{24}+\frac{x^{22}}{2}+\frac{11x^{20}}{4}+\frac{55x^{18}}{6}+\frac{165x^{16}}{8}+33x^{14}+\frac{77x^{12}}{2}+33x^{10}+\frac{165x^8}{8}+\frac{55x^6}{6}+\frac{11x^4}{4}+\frac{x^2}{2}$$

input
```
integrate(x*(x**2+1)*(x**4+2*x**2+1)**5,x)
```

output
```
x**24/24 + x**22/2 + 11*x**20/4 + 55*x**18/6 + 165*x**16/8 + 33*x**14 + 77
*x**12/2 + 33*x**10 + 165*x**8/8 + 55*x**6/6 + 11*x**4/4 + x**2/2
```

3.70.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(9) = 18.

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 5.55

$$\int x(1+x^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{24}\,x^{24}+\frac{1}{2}\,x^{22}+\frac{11}{4}\,x^{20}+\frac{55}{6}\,x^{18}+\frac{165}{8}\,x^{16}+33\,x^{14}+\frac{77}{2}\,x^{12}+33\,x^{10}+\frac{165}{8}\,x^8+\frac{55}{6}\,x^6+\frac{11}{4}\,x^4+\frac{1}{2}\,x^2$$

input
```
integrate(x*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

output
```
1/24*x^24 + 1/2*x^22 + 11/4*x^20 + 55/6*x^18 + 165/8*x^16 + 33*x^14 + 77/2
*x^12 + 33*x^10 + 165/8*x^8 + 55/6*x^6 + 11/4*x^4 + 1/2*x^2
```

3.70.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(9) = 18.

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 6.91

$$\int x(1+x^2)\left(1+2x^2+x^4\right)^5 dx = \frac{1}{24}\left(x^4+2x^2\right)^6+\frac{1}{4}\left(x^4+2x^2\right)^5+\frac{5}{8}\left(x^4+2x^2\right)^4+\frac{1}{4}x^4+\frac{5}{6}\left(x^4+2x^2\right)^3+\frac{5}{8}\left(x^4+2x^2\right)^2+\frac{1}{2}x^2$$

input
```
integrate(x*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

output
```
1/24*(x^4 + 2*x^2)^6 + 1/4*(x^4 + 2*x^2)^5 + 5/8*(x^4 + 2*x^2)^4 + 1/4*x^4
 + 5/6*(x^4 + 2*x^2)^3 + 5/8*(x^4 + 2*x^2)^2 + 1/2*x^2
```

3.70.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 5.55

$$\int x(1+x^2)\left(1+2x^2+x^4\right)^5 dx = \frac{x^{24}}{24}+\frac{x^{22}}{2}+\frac{11\,x^{20}}{4}+\frac{55\,x^{18}}{6}+\frac{165\,x^{16}}{8}+33\,x^{14}+\frac{77\,x^{12}}{2}+33\,x^{10}+\frac{165\,x^8}{8}+\frac{55\,x^6}{6}+\frac{11\,x^4}{4}+\frac{x^2}{2}$$

input
```
int(x*(x^2 + 1)*(2*x^2 + x^4 + 1)^5,x)
```

output
```
x^2/2 + (11*x^4)/4 + (55*x^6)/6 + (165*x^8)/8 + 33*x^10 + (77*x^12)/2 + 33
*x^14 + (165*x^16)/8 + (55*x^18)/6 + (11*x^20)/4 + x^22/2 + x^24/24
```

3.71 $\int (1+x^2) (1+2x^2+x^4)^5 \, dx$

3.71.1 Optimal result

Integrand size = 18, antiderivative size = 73

$$\int (1+x^2) (1+2x^2+x^4)^5 \, dx = x+\frac{11x^3}{3}+11x^5+\frac{165x^7}{7}+\frac{110x^9}{3}+42x^{11}+\frac{462x^{13}}{13}+22x^{15}+\frac{165x^{17}}{17}+\frac{55x^{19}}{19}+\frac{11x^{21}}{21}+\frac{x^{23}}{23}$$

output
```
x+11/3*x^3+11*x^5+165/7*x^7+110/3*x^9+42*x^11+462/13*x^13+22*x^15+165/17*x
^17+55/19*x^19+11/21*x^21+1/23*x^23
```

3.71.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int (1+x^2) (1+2x^2+x^4)^5 \, dx = x+\frac{11x^3}{3}+11x^5+\frac{165x^7}{7}+\frac{110x^9}{3}+42x^{11}+\frac{462x^{13}}{13}+22x^{15}+\frac{165x^{17}}{17}+\frac{55x^{19}}{19}+\frac{11x^{21}}{21}+\frac{x^{23}}{23}$$

input
```
Integrate[(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

output
```
x + (11*x^3)/3 + 11*x^5 + (165*x^7)/7 + (110*x^9)/3 + 42*x^11 + (462*x^13)
/13 + 22*x^15 + (165*x^17)/17 + (55*x^19)/19 + (11*x^21)/21 + x^23/23
```

3.71.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1380, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x^2+1)(x^4+2x^2+1)^5 dx$$

$$\Big\downarrow \text{1380}$$

$$\int (x^2+1)^{11} dx$$

$$\Big\downarrow \text{210}$$

$$\int (x^{22}+11x^{20}+55x^{18}+165x^{16}+330x^{14}+462x^{12}+462x^{10}+330x^8+165x^6+55x^4+11x^2+1) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^{23}}{23}+\frac{11x^{21}}{21}+\frac{55x^{19}}{19}+\frac{165x^{17}}{17}+22x^{15}+\frac{462x^{13}}{13}+42x^{11}+\frac{110x^9}{3}+\frac{165x^7}{7}+11x^5+\frac{11x^3}{3}+x$$

input
```
Int[(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

output
```
x + (11*x^3)/3 + 11*x^5 + (165*x^7)/7 + (110*x^9)/3 + 42*x^11 + (462*x^13)
/13 + 22*x^15 + (165*x^17)/17 + (55*x^19)/19 + (11*x^21)/21 + x^23/23
```

3.71.3.1 Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.71.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.79

method	result
default	$x+\frac{11}{3}x^{3}+11x^{5}+\frac{165}{7}x^{7}+\frac{110}{3}x^{9}+42x^{11}+\frac{462}{13}x^{13}+22x^{15}+\frac{165}{17}x^{17}+\frac{55}{19}x^{19}+\frac{11}{21}x^{21}+\frac{1}{23}x^{2}$
norman	$x+\frac{11}{3}x^{3}+11x^{5}+\frac{165}{7}x^{7}+\frac{110}{3}x^{9}+42x^{11}+\frac{462}{13}x^{13}+22x^{15}+\frac{165}{17}x^{17}+\frac{55}{19}x^{19}+\frac{11}{21}x^{21}+\frac{1}{23}x^{2}$
risch	$x+\frac{11}{3}x^{3}+11x^{5}+\frac{165}{7}x^{7}+\frac{110}{3}x^{9}+42x^{11}+\frac{462}{13}x^{13}+22x^{15}+\frac{165}{17}x^{17}+\frac{55}{19}x^{19}+\frac{11}{21}x^{21}+\frac{1}{23}x^{2}$
parallelrisch	$x+\frac{11}{3}x^{3}+11x^{5}+\frac{165}{7}x^{7}+\frac{110}{3}x^{9}+42x^{11}+\frac{462}{13}x^{13}+22x^{15}+\frac{165}{17}x^{17}+\frac{55}{19}x^{19}+\frac{11}{21}x^{21}+\frac{1}{23}x^{2}$
gosper	$\frac{x(88179x^{22}+1062347x^{20}+5870865x^{18}+19684665x^{16}+44618574x^{14}+72076158x^{12}+85180914x^{10}+74364290x^{8}+47805615x^{6}}{2028117}$

input
```
int((x^2+1)*(x^4+2*x^2+1)^5,x,method=_RETURNVERBOSE)
```

output
```
x+11/3*x^3+11*x^5+165/7*x^7+110/3*x^9+42*x^11+462/13*x^13+22*x^15+165/17*x
^17+55/19*x^19+11/21*x^21+1/23*x^23
```

3.71.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int (1+x^2)\left(1+2x^2+x^4\right)^5\,dx=\frac{1}{23}\,x^{23}+\frac{11}{21}\,x^{21}+\frac{55}{19}\,x^{19}+\frac{165}{17}\,x^{17}+22\,x^{15}+\frac{462}{13}\,x^{13}+42\,x^{11}+\frac{110}{3}\,x^{9}+\frac{165}{7}\,x^{7}+11\,x^{5}+\frac{11}{3}\,x^{3}+x$$

input
```
integrate((x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

output
```
1/23*x^23 + 11/21*x^21 + 55/19*x^19 + 165/17*x^17 + 22*x^15 + 462/13*x^13
+ 42*x^11 + 110/3*x^9 + 165/7*x^7 + 11*x^5 + 11/3*x^3 + x
```

3.71.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.93

$$\int (1+x^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{x^{23}}{23}+\frac{11x^{21}}{21}+\frac{55x^{19}}{19}+\frac{165x^{17}}{17}+22x^{15}+\frac{462x^{13}}{13}+42x^{11}+\frac{110x^9}{3}+\frac{165x^7}{7}+11x^5+\frac{11x^3}{3}+x$$

input
```
integrate((x**2+1)*(x**4+2*x**2+1)**5,x)
```

output
```
x**23/23 + 11*x**21/21 + 55*x**19/19 + 165*x**17/17 + 22*x**15 + 462*x**13
/13 + 42*x**11 + 110*x**9/3 + 165*x**7/7 + 11*x**5 + 11*x**3/3 + x
```

3.71.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int (1+x^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{23}\,x^{23}+\frac{11}{21}\,x^{21}+\frac{55}{19}\,x^{19}+\frac{165}{17}\,x^{17}+22\,x^{15}+\frac{462}{13}\,x^{13}+42\,x^{11}+\frac{110}{3}\,x^9+\frac{165}{7}\,x^7+11\,x^5+\frac{11}{3}\,x^3+x$$

input
```
integrate((x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

output
```
1/23*x^23 + 11/21*x^21 + 55/19*x^19 + 165/17*x^17 + 22*x^15 + 462/13*x^13
+ 42*x^11 + 110/3*x^9 + 165/7*x^7 + 11*x^5 + 11/3*x^3 + x
```

3.71.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int (1+x^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{1}{23}\,x^{23}+\frac{11}{21}\,x^{21}+\frac{55}{19}\,x^{19}+\frac{165}{17}\,x^{17}+22\,x^{15}+\frac{462}{13}\,x^{13}+42\,x^{11}+\frac{110}{3}\,x^9+\frac{165}{7}\,x^7+11\,x^5+\frac{11}{3}\,x^3+x$$

input
```
integrate((x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

output
```
1/23*x^23 + 11/21*x^21 + 55/19*x^19 + 165/17*x^17 + 22*x^15 + 462/13*x^13
+ 42*x^11 + 110/3*x^9 + 165/7*x^7 + 11*x^5 + 11/3*x^3 + x
```

3.71.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int (1+x^2)\left(1+2x^2+x^4\right)^5\,dx = \frac{x^{23}}{23}+\frac{11\,x^{21}}{21}+\frac{55\,x^{19}}{19}+\frac{165\,x^{17}}{17}+22\,x^{15}+\frac{462\,x^{13}}{13}+42\,x^{11}+\frac{110\,x^9}{3}+\frac{165\,x^7}{7}+11\,x^5+\frac{11\,x^3}{3}+x$$

input
```
int((x^2 + 1)*(2*x^2 + x^4 + 1)^5,x)
```

output
```
x + (11*x^3)/3 + 11*x^5 + (165*x^7)/7 + (110*x^9)/3 + 42*x^11 + (462*x^13)
/13 + 22*x^15 + (165*x^17)/17 + (55*x^19)/19 + (11*x^21)/21 + x^23/23
```

3.72 $\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x} \, dx$

3.72.1 Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x} \, dx = \frac{11x^2}{2}+\frac{55x^4}{4}+\frac{55x^6}{2}+\frac{165x^8}{4}+\frac{231x^{10}}{5}+\frac{77x^{12}}{2}+\frac{165x^{14}}{7}+\frac{165x^{16}}{16}+\frac{55x^{18}}{18}+\frac{11x^{20}}{20}+\frac{x^{22}}{22}+\log(x)$$

output
```
11/2*x^2+55/4*x^4+55/2*x^6+165/4*x^8+231/5*x^10+77/2*x^12+165/7*x^14+165/1
6*x^16+55/18*x^18+11/20*x^20+1/22*x^22+ln(x)
```

3.72.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x} \, dx = \frac{11x^2}{2}+\frac{55x^4}{4}+\frac{55x^6}{2}+\frac{165x^8}{4}+\frac{231x^{10}}{5}+\frac{77x^{12}}{2}+\frac{165x^{14}}{7}+\frac{165x^{16}}{16}+\frac{55x^{18}}{18}+\frac{11x^{20}}{20}+\frac{x^{22}}{22}+\log(x)$$

input
```
Integrate[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x,x]
```

output
```
(11*x^2)/2 + (55*x^4)/4 + (55*x^6)/2 + (165*x^8)/4 + (231*x^10)/5 + (77*x^
12)/2 + (165*x^14)/7 + (165*x^16)/16 + (55*x^18)/18 + (11*x^20)/20 + x^22/
22 + Log[x]
```

3.72.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1380, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+1)(x^4+2x^2+1)^5}{x}dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{(x^2+1)^{11}}{x}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(x^2+1)^{11}}{x^2}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(x^{20}+11x^{18}+55x^{16}+165x^{14}+330x^{12}+462x^{10}+462x^8+330x^6+165x^4+55x^2+11+\frac{1}{x^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{x^{22}}{11}+\frac{11x^{20}}{10}+\frac{55x^{18}}{9}+\frac{165x^{16}}{8}+\frac{330x^{14}}{7}+77x^{12}+\frac{462x^{10}}{5}+\frac{165x^8}{2}+55x^6+\frac{55x^4}{2}+11x^2+\log(x^2)\right)$$

input
```
Int[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x,x]
```

output
```
(11*x^2 + (55*x^4)/2 + 55*x^6 + (165*x^8)/2 + (462*x^10)/5 + 77*x^12 + (33
0*x^14)/7 + (165*x^16)/8 + (55*x^18)/9 + (11*x^20)/10 + x^22/11 + Log[x^2]
)/2
```

3.72.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.72.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.74

method	result	s
default	$\frac{11x^{2}}{2}+\frac{55x^{4}}{4}+\frac{55x^{6}}{2}+\frac{165x^{8}}{4}+\frac{231x^{10}}{5}+\frac{77x^{12}}{2}+\frac{165x^{14}}{7}+\frac{165x^{16}}{16}+\frac{55x^{18}}{18}+\frac{11x^{20}}{20}+\frac{x^{22}}{22}+\ln\left(x\right)$	5
norman	$\frac{11x^{2}}{2}+\frac{55x^{4}}{4}+\frac{55x^{6}}{2}+\frac{165x^{8}}{4}+\frac{231x^{10}}{5}+\frac{77x^{12}}{2}+\frac{165x^{14}}{7}+\frac{165x^{16}}{16}+\frac{55x^{18}}{18}+\frac{11x^{20}}{20}+\frac{x^{22}}{22}+\ln\left(x\right)$	5
risch	$\frac{11x^{2}}{2}+\frac{55x^{4}}{4}+\frac{55x^{6}}{2}+\frac{165x^{8}}{4}+\frac{231x^{10}}{5}+\frac{77x^{12}}{2}+\frac{165x^{14}}{7}+\frac{165x^{16}}{16}+\frac{55x^{18}}{18}+\frac{11x^{20}}{20}+\frac{x^{22}}{22}+\ln\left(x\right)$	5
parallelrisch	$\frac{11x^{2}}{2}+\frac{55x^{4}}{4}+\frac{55x^{6}}{2}+\frac{165x^{8}}{4}+\frac{231x^{10}}{5}+\frac{77x^{12}}{2}+\frac{165x^{14}}{7}+\frac{165x^{16}}{16}+\frac{55x^{18}}{18}+\frac{11x^{20}}{20}+\frac{x^{22}}{22}+\ln\left(x\right)$	5

input
```
int((x^2+1)*(x^4+2*x^2+1)^5/x,x,method=_RETURNVERBOSE)
```

output
```
11/2*x^2+55/4*x^4+55/2*x^6+165/4*x^8+231/5*x^10+77/2*x^12+165/7*x^14+165/1
6*x^16+55/18*x^18+11/20*x^20+1/22*x^22+ln(x)
```

3.72.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.72

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x}\,dx = \frac{1}{22}x^{22} + \frac{11}{20}x^{20} + \frac{55}{18}x^{18} + \frac{165}{16}x^{16} + \frac{165}{7}x^{14} + \frac{77}{2}x^{12} + \frac{231}{5}x^{10} + \frac{165}{4}x^{8} + \frac{55}{2}x^{6} + \frac{55}{4}x^{4} + \frac{11}{2}x^{2} + \log(x)$$

input
```
integrate((x^2+1)*(x^4+2*x^2+1)^5/x,x, algorithm="fricas")
```

output
```
1/22*x^22 + 11/20*x^20 + 55/18*x^18 + 165/16*x^16 + 165/7*x^14 + 77/2*x^12
 + 231/5*x^10 + 165/4*x^8 + 55/2*x^6 + 55/4*x^4 + 11/2*x^2 + log(x)
```

3.72.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x}\,dx = \frac{x^{22}}{22} + \frac{11x^{20}}{20} + \frac{55x^{18}}{18} + \frac{165x^{16}}{16} + \frac{165x^{14}}{7} + \frac{77x^{12}}{2} + \frac{231x^{10}}{5} + \frac{165x^{8}}{4} + \frac{55x^{6}}{2} + \frac{55x^{4}}{4} + \frac{11x^{2}}{2} + \log(x)$$

input
```
integrate((x**2+1)*(x**4+2*x**2+1)**5/x,x)
```

output
```
x**22/22 + 11*x**20/20 + 55*x**18/18 + 165*x**16/16 + 165*x**14/7 + 77*x**
12/2 + 231*x**10/5 + 165*x**8/4 + 55*x**6/2 + 55*x**4/4 + 11*x**2/2 + log(
x)
```

3.72.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.78

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x}\,dx = \frac{1}{22}x^{22} + \frac{11}{20}x^{20} + \frac{55}{18}x^{18} + \frac{165}{16}x^{16} + \frac{165}{7}x^{14} + \frac{77}{2}x^{12} + \frac{231}{5}x^{10} + \frac{165}{4}x^{8} + \frac{55}{2}x^{6} + \frac{55}{4}x^{4} + \frac{11}{2}x^{2} + \frac{1}{2}\log(x^2)$$

input
```
integrate((x^2+1)*(x^4+2*x^2+1)^5/x,x, algorithm="maxima")
```

output
```
1/22*x^22 + 11/20*x^20 + 55/18*x^18 + 165/16*x^16 + 165/7*x^14 + 77/2*x^12
 + 231/5*x^10 + 165/4*x^8 + 55/2*x^6 + 55/4*x^4 + 11/2*x^2 + 1/2*log(x^2)
```

3.72.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.78

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x}\,dx = \frac{1}{22}x^{22} + \frac{11}{20}x^{20} + \frac{55}{18}x^{18} + \frac{165}{16}x^{16} + \frac{165}{7}x^{14} + \frac{77}{2}x^{12} + \frac{231}{5}x^{10} + \frac{165}{4}x^8 + \frac{55}{2}x^6 + \frac{55}{4}x^4 + \frac{11}{2}x^2 + \frac{1}{2}\log(x^2)$$

input
```
integrate((x^2+1)*(x^4+2*x^2+1)^5/x,x, algorithm="giac")
```

output
```
1/22*x^22 + 11/20*x^20 + 55/18*x^18 + 165/16*x^16 + 165/7*x^14 + 77/2*x^12
 + 231/5*x^10 + 165/4*x^8 + 55/2*x^6 + 55/4*x^4 + 11/2*x^2 + 1/2*log(x^2)
```

3.72.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.72

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x}\,dx = \ln(x) + \frac{11\,x^2}{2} + \frac{55\,x^4}{4} + \frac{55\,x^6}{2} + \frac{165\,x^8}{4} + \frac{231\,x^{10}}{5} + \frac{77\,x^{12}}{2} + \frac{165\,x^{14}}{7} + \frac{165\,x^{16}}{16} + \frac{55\,x^{18}}{18} + \frac{11\,x^{20}}{20} + \frac{x^{22}}{22}$$

input
```
int(((x^2 + 1)*(2*x^2 + x^4 + 1)^5)/x,x)
```

output
```
log(x) + (11*x^2)/2 + (55*x^4)/4 + (55*x^6)/2 + (165*x^8)/4 + (231*x^10)/5
 + (77*x^12)/2 + (165*x^14)/7 + (165*x^16)/16 + (55*x^18)/18 + (11*x^20)/2
0 + x^22/22
```

3.73 $\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^2} dx$

3.73.1 Optimal result

Integrand size = 21, antiderivative size = 73

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^2} dx = -\frac{1}{x}+11x+\frac{55x^3}{3}+33x^5+\frac{330x^7}{7}+\frac{154x^9}{3}+42x^{11}+\frac{330x^{13}}{13}+11x^{15}+\frac{55x^{17}}{17}+\frac{11x^{19}}{19}+\frac{x^{21}}{21}$$

output
```
-1/x+11*x+55/3*x^3+33*x^5+330/7*x^7+154/3*x^9+42*x^11+330/13*x^13+11*x^15+
55/17*x^17+11/19*x^19+1/21*x^21
```

3.73.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^2} dx = -\frac{1}{x}+11x+\frac{55x^3}{3}+33x^5+\frac{330x^7}{7}+\frac{154x^9}{3}+42x^{11}+\frac{330x^{13}}{13}+11x^{15}+\frac{55x^{17}}{17}+\frac{11x^{19}}{19}+\frac{x^{21}}{21}$$

input
```
Integrate[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x^2,x]
```

output
```
-x^(-1) + 11*x + (55*x^3)/3 + 33*x^5 + (330*x^7)/7 + (154*x^9)/3 + 42*x^11
 + (330*x^13)/13 + 11*x^15 + (55*x^17)/17 + (11*x^19)/19 + x^21/21
```

3.73.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1380, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+1)(x^4+2x^2+1)^5}{x^2}dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{(x^2+1)^{11}}{x^2}dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(x^{20}+11x^{18}+55x^{16}+165x^{14}+330x^{12}+462x^{10}+462x^8+330x^6+165x^4+55x^2+\frac{1}{x^2}+11\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^{21}}{21}+\frac{11x^{19}}{19}+\frac{55x^{17}}{17}+11x^{15}+\frac{330x^{13}}{13}+42x^{11}+\frac{154x^9}{3}+\frac{330x^7}{7}+33x^5+\frac{55x^3}{3}+11x-\frac{1}{x}$$

input
```
Int[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x^2,x]
```

output
```
-x^(-1) + 11*x + (55*x^3)/3 + 33*x^5 + (330*x^7)/7 + (154*x^9)/3 + 42*x^11
 + (330*x^13)/13 + 11*x^15 + (55*x^17)/17 + (11*x^19)/19 + x^21/21
```

3.73.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.73.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

method	result
default	$-\frac{1}{x}+11x+\frac{55x^3}{3}+33x^5+\frac{330x^7}{7}+\frac{154x^9}{3}+42x^{11}+\frac{330x^{13}}{13}+11x^{15}+\frac{55x^{17}}{17}+\frac{11x^{19}}{19}+\frac{x^{21}}{21}$
risch	$-\frac{1}{x}+11x+\frac{55x^3}{3}+33x^5+\frac{330x^7}{7}+\frac{154x^9}{3}+42x^{11}+\frac{330x^{13}}{13}+11x^{15}+\frac{55x^{17}}{17}+\frac{11x^{19}}{19}+\frac{x^{21}}{21}$
norman	$\frac{-1+11x^2+\frac{55}{3}x^4+33x^6+\frac{330}{7}x^8+\frac{154}{3}x^{10}+42x^{12}+\frac{330}{13}x^{14}+11x^{16}+\frac{55}{17}x^{18}+\frac{11}{19}x^{20}+\frac{1}{21}x^{22}}{x}$
gosper	$\frac{4199x^{22}+51051x^{20}+285285x^{18}+969969x^{16}+2238390x^{14}+3703518x^{12}+4526522x^{10}+4157010x^8+2909907x^6+1616615x^4+9}{88179x}$
parallelrisch	$\frac{4199x^{22}+51051x^{20}+285285x^{18}+969969x^{16}+2238390x^{14}+3703518x^{12}+4526522x^{10}+4157010x^8+2909907x^6+1616615x^4+9}{88179x}$

input
```
int((x^2+1)*(x^4+2*x^2+1)^5/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/x+11*x+55/3*x^3+33*x^5+330/7*x^7+154/3*x^9+42*x^11+330/13*x^13+11*x^15+
55/17*x^17+11/19*x^19+1/21*x^21
```

3.73.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.85

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^2}\,dx = \frac{4199\,x^{22}+51051\,x^{20}+285285\,x^{18}+969969\,x^{16}+2238390\,x^{14}+3703518\,x^{12}+4526522\,x^{10}+4157010\,x^{8}}{88179\,x}$$

input
```
integrate((x^2+1)*(x^4+2*x^2+1)^5/x^2,x, algorithm="fricas")
```

output
```
1/88179*(4199*x^22 + 51051*x^20 + 285285*x^18 + 969969*x^16 + 2238390*x^14
 + 3703518*x^12 + 4526522*x^10 + 4157010*x^8 + 2909907*x^6 + 1616615*x^4 +
 969969*x^2 - 88179)/x
```

3.73.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^2}\,dx = \frac{x^{21}}{21}+\frac{11x^{19}}{19}+\frac{55x^{17}}{17}+11x^{15}+\frac{330x^{13}}{13}+42x^{11}+\frac{154x^9}{3}+\frac{330x^7}{7}+33x^5+\frac{55x^3}{3}+11x-\frac{1}{x}$$

input
```
integrate((x**2+1)*(x**4+2*x**2+1)**5/x**2,x)
```

output
```
x**21/21 + 11*x**19/19 + 55*x**17/17 + 11*x**15 + 330*x**13/13 + 42*x**11
+ 154*x**9/3 + 330*x**7/7 + 33*x**5 + 55*x**3/3 + 11*x - 1/x
```

3.73.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.81

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^2}\,dx = \frac{1}{21}\,x^{21}+\frac{11}{19}\,x^{19}+\frac{55}{17}\,x^{17}+11\,x^{15}+\frac{330}{13}\,x^{13}+42\,x^{11}+\frac{154}{3}\,x^9+\frac{330}{7}\,x^7+33\,x^5+\frac{55}{3}\,x^3+11\,x-\frac{1}{x}$$

input
```
integrate((x^2+1)*(x^4+2*x^2+1)^5/x^2,x, algorithm="maxima")
```

output
```
1/21*x^21 + 11/19*x^19 + 55/17*x^17 + 11*x^15 + 330/13*x^13 + 42*x^11 + 15
4/3*x^9 + 330/7*x^7 + 33*x^5 + 55/3*x^3 + 11*x - 1/x
```

3.73.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.81

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^2}\,dx = \frac{1}{21}\,x^{21}+\frac{11}{19}\,x^{19}+\frac{55}{17}\,x^{17}+11\,x^{15}+\frac{330}{13}\,x^{13}+42\,x^{11}+\frac{154}{3}\,x^9+\frac{330}{7}\,x^7+33\,x^5+\frac{55}{3}\,x^3+11\,x-\frac{1}{x}$$

input
```
integrate((x^2+1)*(x^4+2*x^2+1)^5/x^2,x, algorithm="giac")
```

output
```
1/21*x^21 + 11/19*x^19 + 55/17*x^17 + 11*x^15 + 330/13*x^13 + 42*x^11 + 15
4/3*x^9 + 330/7*x^7 + 33*x^5 + 55/3*x^3 + 11*x - 1/x
```

3.73.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.81

$$\int \frac{(1+x^2)\,(1+2x^2+x^4)^5}{x^2}\,dx = 11\,x-\frac{1}{x}+\frac{55\,x^3}{3}+33\,x^5+\frac{330\,x^7}{7}+\frac{154\,x^9}{3} +42\,x^{11}+\frac{330\,x^{13}}{13}+11\,x^{15}+\frac{55\,x^{17}}{17}+\frac{11\,x^{19}}{19}+\frac{x^{21}}{21}$$

input
```
int(((x^2 + 1)*(2*x^2 + x^4 + 1)^5)/x^2,x)
```

output
```
11*x - 1/x + (55*x^3)/3 + 33*x^5 + (330*x^7)/7 + (154*x^9)/3 + 42*x^11 + (
330*x^13)/13 + 11*x^15 + (55*x^17)/17 + (11*x^19)/19 + x^21/21
```

3.74 $\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^3} dx$

3.74.1 Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^3} dx = -\frac{1}{2x^2} + \frac{55x^2}{2} + \frac{165x^4}{4} + 55x^6 + \frac{231x^8}{4} + \frac{231x^{10}}{5} + \frac{55x^{12}}{2} + \frac{165x^{14}}{14} + \frac{55x^{16}}{16} + \frac{11x^{18}}{18} + \frac{x^{20}}{20} + 11\log(x)$$

output
```
-1/2/x^2+55/2*x^2+165/4*x^4+55*x^6+231/4*x^8+231/5*x^10+55/2*x^12+165/14*x
^14+55/16*x^16+11/18*x^18+1/20*x^20+11*ln(x)
```

3.74.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^3} dx = -\frac{1}{2x^2} + \frac{55x^2}{2} + \frac{165x^4}{4} + 55x^6 + \frac{231x^8}{4} + \frac{231x^{10}}{5} + \frac{55x^{12}}{2} + \frac{165x^{14}}{14} + \frac{55x^{16}}{16} + \frac{11x^{18}}{18} + \frac{x^{20}}{20} + 11\log(x)$$

input
```
Integrate[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x^3,x]
```

output
```
-1/2*1/x^2 + (55*x^2)/2 + (165*x^4)/4 + 55*x^6 + (231*x^8)/4 + (231*x^10)/
5 + (55*x^12)/2 + (165*x^14)/14 + (55*x^16)/16 + (11*x^18)/18 + x^20/20 +
11*Log[x]
```

3.74.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1380, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+1)(x^4+2x^2+1)^5}{x^3}dx$$

$$\Big\downarrow \ 1380$$

$$\int \frac{(x^2+1)^{11}}{x^3}dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2}\int \frac{(x^2+1)^{11}}{x^4}dx^2$$

$$\Big\downarrow \ 49$$

$$\frac{1}{2}\int \left(x^{18}+11x^{16}+55x^{14}+165x^{12}+330x^{10}+462x^8+462x^6+330x^4+165x^2+55+\frac{11}{x^2}+\frac{1}{x^4}\right)dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{x^{20}}{10}+\frac{11x^{18}}{9}+\frac{55x^{16}}{8}+\frac{165x^{14}}{7}+55x^{12}+\frac{462x^{10}}{5}+\frac{231x^8}{2}+110x^6+\frac{165x^4}{2}+55x^2-\frac{1}{x^2}+11\log(x^2)\right)$$

input
```
Int[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x^3,x]
```

output
```
(-x^(-2) + 55*x^2 + (165*x^4)/2 + 110*x^6 + (231*x^8)/2 + (462*x^10)/5 + 5
5*x^12 + (165*x^14)/7 + (55*x^16)/8 + (11*x^18)/9 + x^20/10 + 11*Log[x^2])
/2
```

3.74.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.74.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.76

method	result
default	$-\frac{1}{2x^2}+\frac{55x^2}{2}+\frac{165x^4}{4}+55x^6+\frac{231x^8}{4}+\frac{231x^{10}}{5}+\frac{55x^{12}}{2}+\frac{165x^{14}}{14}+\frac{55x^{16}}{16}+\frac{11x^{18}}{18}+\frac{x^{20}}{20}+11\ln(x$
risch	$-\frac{1}{2x^2}+\frac{55x^2}{2}+\frac{165x^4}{4}+55x^6+\frac{231x^8}{4}+\frac{231x^{10}}{5}+\frac{55x^{12}}{2}+\frac{165x^{14}}{14}+\frac{55x^{16}}{16}+\frac{11x^{18}}{18}+\frac{x^{20}}{20}+11\ln(x$
norman	$\frac{-\frac{1}{2}+\frac{55}{2}x^4+\frac{165}{4}x^6+55x^8+\frac{231}{4}x^{10}+\frac{231}{5}x^{12}+\frac{55}{2}x^{14}+\frac{165}{14}x^{16}+\frac{55}{16}x^{18}+\frac{11}{18}x^{20}+\frac{1}{20}x^{22}}{x^2}+11\ln(x)$
parallelrisch	$\frac{252x^{22}+3080x^{20}+17325x^{18}+59400x^{16}+138600x^{14}+232848x^{12}+291060x^{10}+277200x^8+207900x^6+138600x^4+55440\ln(x)x}{5040x^2}$

input
```
int((x^2+1)*(x^4+2*x^2+1)^5/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2+55/2*x^2+165/4*x^4+55*x^6+231/4*x^8+231/5*x^10+55/2*x^12+165/14*x
^14+55/16*x^16+11/18*x^18+1/20*x^20+11*ln(x)
```

3.74.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^3}\,dx = \frac{252\,x^{22}+3080\,x^{20}+17325\,x^{18}+59400\,x^{16}+138600\,x^{14}+232848\,x^{12}+291060\,x^{10}+277200\,x^8+20790}{5040\,x^2}$$

input
```
integrate((x^2+1)*(x^4+2*x^2+1)^5/x^3,x, algorithm="fricas")
```

output
```
1/5040*(252*x^22 + 3080*x^20 + 17325*x^18 + 59400*x^16 + 138600*x^14 + 232
848*x^12 + 291060*x^10 + 277200*x^8 + 207900*x^6 + 138600*x^4 + 55440*x^2*
log(x) - 2520)/x^2
```

3.74.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^3}\,dx = \frac{x^{20}}{20}+\frac{11x^{18}}{18}+\frac{55x^{16}}{16}+\frac{165x^{14}}{14}+\frac{55x^{12}}{2}+\frac{231x^{10}}{5} + \frac{231x^8}{4}+55x^6+\frac{165x^4}{4}+\frac{55x^2}{2}+11\log(x)-\frac{1}{2x^2}$$

input
```
integrate((x**2+1)*(x**4+2*x**2+1)**5/x**3,x)
```

output
```
x**20/20 + 11*x**18/18 + 55*x**16/16 + 165*x**14/14 + 55*x**12/2 + 231*x**
10/5 + 231*x**8/4 + 55*x**6 + 165*x**4/4 + 55*x**2/2 + 11*log(x) - 1/(2*x*
*2)
```

3.74.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.78

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^3}\,dx = \frac{1}{20}\,x^{20}+\frac{11}{18}\,x^{18}+\frac{55}{16}\,x^{16}+\frac{165}{14}\,x^{14}+\frac{55}{2}\,x^{12}+\frac{231}{5}\,x^{10} + \frac{231}{4}\,x^8+55\,x^6+\frac{165}{4}\,x^4+\frac{55}{2}\,x^2-\frac{1}{2\,x^2}+\frac{11}{2}\,\log\left(x^2\right)$$

input
```
integrate((x^2+1)*(x^4+2*x^2+1)^5/x^3,x, algorithm="maxima")
```

output
```
1/20*x^20 + 11/18*x^18 + 55/16*x^16 + 165/14*x^14 + 55/2*x^12 + 231/5*x^10
 + 231/4*x^8 + 55*x^6 + 165/4*x^4 + 55/2*x^2 - 1/2/x^2 + 11/2*log(x^2)
```

3.74.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.86

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^3}\,dx = \frac{1}{20}\,x^{20} + \frac{11}{18}\,x^{18} + \frac{55}{16}\,x^{16} + \frac{165}{14}\,x^{14} + \frac{55}{2}\,x^{12} + \frac{231}{5}\,x^{10} + \frac{231}{4}\,x^8 + 55\,x^6 + \frac{165}{4}\,x^4 + \frac{55}{2}\,x^2 - \frac{11\,x^2+1}{2\,x^2} + \frac{11}{2}\,\log\left(x^2\right)$$

input
```
integrate((x^2+1)*(x^4+2*x^2+1)^5/x^3,x, algorithm="giac")
```

output
```
1/20*x^20 + 11/18*x^18 + 55/16*x^16 + 165/14*x^14 + 55/2*x^12 + 231/5*x^10
 + 231/4*x^8 + 55*x^6 + 165/4*x^4 + 55/2*x^2 - 1/2*(11*x^2 + 1)/x^2 + 11/2
*log(x^2)
```

3.74.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.75

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^3}\,dx = 11\,\ln(x) - \frac{1}{2\,x^2} + \frac{55\,x^2}{2} + \frac{165\,x^4}{4} + 55\,x^6 + \frac{231\,x^8}{4} + \frac{231\,x^{10}}{5} + \frac{55\,x^{12}}{2} + \frac{165\,x^{14}}{14} + \frac{55\,x^{16}}{16} + \frac{11\,x^{18}}{18} + \frac{x^{20}}{20}$$

input
```
int(((x^2 + 1)*(2*x^2 + x^4 + 1)^5)/x^3,x)
```

output
```
11*log(x) - 1/(2*x^2) + (55*x^2)/2 + (165*x^4)/4 + 55*x^6 + (231*x^8)/4 +
(231*x^10)/5 + (55*x^12)/2 + (165*x^14)/14 + (55*x^16)/16 + (11*x^18)/18 +
 x^20/20
```

3.75 $\int \frac{x^2(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

3.75.1 Optimal result

Integrand size = 33, antiderivative size = 145

$$\int \frac{x^2(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{(bd-ae)x(a+bx^2)}{b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{ex^3(a+bx^2)}{3b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{a}(bd-ae)\left(a+bx^2\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
(-a*e+b*d)*x*(b*x^2+a)/b^2/((b*x^2+a)^2)^(1/2)+1/3*e*x^3*(b*x^2+a)/b/((b*x
^2+a)^2)^(1/2)-(-a*e+b*d)*(b*x^2+a)*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(5
/2)/((b*x^2+a)^2)^(1/2)
```

3.75.2 Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.55

$$\int \frac{x^2(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{(a+bx^2)\left(\sqrt{b}x(3bd-3ae+bex^2)+3\sqrt{a}(-bd+ae)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\right)}{3b^{5/2}\sqrt{(a+bx^2)^2}}$$

input
```
Integrate[(x^2*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
((a + b*x^2)*(Sqrt[b]*x*(3*b*d - 3*a*e + b*e*x^2) + 3*Sqrt[a]*(-(b*d) + a*
e)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]))/(3*b^(5/2)*Sqrt[(a + b*x^2)^2])
```

3.75.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.59, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1384, 27, 363, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b(a+bx^2)\int \frac{x^2(ex^2+d)}{b(bx^2+a)}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^2)\int \frac{x^2(ex^2+d)}{bx^2+a}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{363}$$

$$\frac{(a+bx^2)\left(\frac{(bd-ae)\int \frac{x^2}{bx^2+a}dx}{b}+\frac{ex^3}{3b}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{262}$$

$$\frac{(a+bx^2)\left(\frac{(bd-ae)\left(\frac{x}{b}-\frac{a\int \frac{1}{bx^2+a}dx}{b}\right)}{b}+\frac{ex^3}{3b}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{218}$$

$$\frac{(a+bx^2)\left(\frac{\left(\frac{x}{b}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}\right)(bd-ae)}{b}+\frac{ex^3}{3b}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[(x^2*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
((a + b*x^2)*((e*x^3)/(3*b) + ((b*d - a*e)*(x/b - (Sqrt[a]*ArcTan[(Sqrt[b]
*x)/Sqrt[a]])/b^(3/2)))/b))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
```

3.75.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.75.4 Maple [A] (verified)

Time = 2.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.63

method	result
default	$-\frac{(b\,x^2+a)\left(-\sqrt{ab}\,be\,x^3+3\sqrt{ab}\,aex-3\sqrt{ab}\,bdx-3\arctan\left(\frac{bx}{\sqrt{ab}}\right)a^2e+3\arctan\left(\frac{bx}{\sqrt{ab}}\right)abd\right)}{3\sqrt{(b\,x^2+a)^2}\,b^2\sqrt{ab}}$
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,\left(\frac{1}{3}e\,x^3b-aex+bdx\right)}{(b\,x^2+a)b^2}+\frac{\sqrt{(b\,x^2+a)^2}\,\sqrt{-ab}\,(ae-bd)\ln\left(-\sqrt{-ab}\,x+a\right)}{2(b\,x^2+a)b^3}-\frac{\sqrt{(b\,x^2+a)^2}\,\sqrt{-ab}\,(ae-bd)\ln\left(\sqrt{-ab}\,x+a\right)}{2(b\,x^2+a)b^3}$

input
```
int(x^2*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(b*x^2+a)*(-(a*b)^(1/2)*b*e*x^3+3*(a*b)^(1/2)*a*e*x-3*(a*b)^(1/2)*b*d
*x-3*arctan(b*x/(a*b)^(1/2))*a^2*e+3*arctan(b*x/(a*b)^(1/2))*a*b*d)/((b*x^
2+a)^2)^(1/2)/b^2/(a*b)^(1/2)
```

3.75.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.89

$$\int \frac{x^2(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$$
$$=\left[\frac{2\,bex^3-3\,(bd-ae)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+6\,(bd-ae)x}{6\,b^2},\frac{bex^3-3\,(bd-ae)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+\right.$$

input
```
integrate(x^2*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(2*b*e*x^3 - 3*(b*d - a*e)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) -
 a)/(b*x^2 + a)) + 6*(b*d - a*e)*x)/b^2, 1/3*(b*e*x^3 - 3*(b*d - a*e)*sqrt
(a/b)*arctan(b*x*sqrt(a/b)/a) + 3*(b*d - a*e)*x)/b^2]
```

3.75.6 Sympy [F]

$$\int \frac{x^2(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{x^2(d+ex^2)}{\sqrt{(a+bx^2)^2}}\,dx$$

input
```
integrate(x**2*(e*x**2+d)/((b*x**2+a)**2)**(1/2),x)
```

output
```
Integral(x**2*(d + e*x**2)/sqrt((a + b*x**2)**2), x)
```

3.75.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.37

$$\int \frac{x^2(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = -\frac{(abd-a^2e)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{bex^3+3\,(bd-ae)x}{3\,b^2}$$

input
```
integrate(x^2*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
-(a*b*d - a^2*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(b*e*x^3 + 3*
(b*d - a*e)*x)/b^2
```

3.75.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.68

$$\int \frac{x^2(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = -\frac{(abd\text{sgn}(bx^2+a)-a^2e\text{sgn}(bx^2+a))\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{b^2ex^3\text{sgn}(bx^2+a)+3\,b^2dx\text{sgn}(bx^2+a)-3\,abex\text{sgn}(bx^2+a)}{3\,b^3}$$

input
```
integrate(x^2*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

output
```
-(a*b*d*sgn(b*x^2 + a) - a^2*e*sgn(b*x^2 + a))*arctan(b*x/sqrt(a*b))/(sqrt
(a*b)*b^2) + 1/3*(b^2*e*x^3*sgn(b*x^2 + a) + 3*b^2*d*x*sgn(b*x^2 + a) - 3*
a*b*e*x*sgn(b*x^2 + a))/b^3
```

3.75.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{x^2\,(e\,x^2+d)}{\sqrt{(b\,x^2+a)^2}}\,dx$$

input
```
int((x^2*(d + e*x^2))/((a + b*x^2)^2)^(1/2),x)
```

output
```
int((x^2*(d + e*x^2))/((a + b*x^2)^2)^(1/2), x)
```

3.76 $\int \frac{x(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}} \, dx$

3.76.1 Optimal result

Integrand size = 31, antiderivative size = 83

$$\int \frac{x(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}} \, dx = \frac{e\sqrt{a^2+2abx^2+b^2x^4}}{2b^2} + \frac{(bd-ae)\,(a+bx^2)\log\,(a+bx^2)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
1/2*(-a*e+b*d)*(b*x^2+a)*ln(b*x^2+a)/b^2/((b*x^2+a)^2)^(1/2)+1/2*e*((b*x^2
+a)^2)^(1/2)/b^2
```

3.76.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 276 vs. 2(83) = 166.

Time = 0.55 (sec) , antiderivative size = 276, normalized size of antiderivative = 3.33

$$\int \frac{x(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}} \, dx =$$
$$-\frac{(2a+bx^2)\left(bex^2\left(\sqrt{a^2}bx^2+a\left(\sqrt{a^2}-\sqrt{(a+bx^2)^2}\right)\right)-(-bd+ae)\left(-a^2-abx^2+\sqrt{a^2}\sqrt{(a+bx^2)^2}\right.\right.}{2b^2\left(\sqrt{a^2}-\sqrt{(a+b\right.}}$$

input
```
Integrate[(x*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
-1/2*((2*a + b*x^2)*(b*e*x^2*(Sqrt[a^2]*b*x^2 + a*(Sqrt[a^2] - Sqrt[(a + b
*x^2)^2])) - (-(b*d) + a*e)*(-a^2 - a*b*x^2 + Sqrt[a^2]*Sqrt[(a + b*x^2)^2
])*Log[Sqrt[a^2] - b*x^2 - Sqrt[(a + b*x^2)^2]] + (-(b*d) + a*e)*(-a^2 - a
*b*x^2 + Sqrt[a^2]*Sqrt[(a + b*x^2)^2])*Log[b^2*(Sqrt[a^2] + b*x^2 - Sqrt[
(a + b*x^2)^2])]))/(b^2*(Sqrt[a^2] - Sqrt[(a + b*x^2)^2])*(Sqrt[a^2]*b*x^2
 + a*(Sqrt[a^2] - Sqrt[(a + b*x^2)^2])))
```

3.76.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.75, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1384, 27, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b(a+bx^2)\int \frac{x(ex^2+d)}{b(bx^2+a)}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^2)\int \frac{x(ex^2+d)}{bx^2+a}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{353}$$

$$\frac{(a+bx^2)\int \frac{ex^2+d}{bx^2+a}dx^2}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{49}$$

$$\frac{(a+bx^2)\int \left(\frac{e}{b}+\frac{bd-ae}{b(bx^2+a)}\right)dx^2}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx^2)\left(\frac{(bd-ae)\log(a+bx^2)}{b^2}+\frac{ex^2}{b}\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[(x*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
((a + b*x^2)*((e*x^2)/b + ((b*d - a*e)*Log[a + b*x^2])/b^2))/(2*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])
```

3.76.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.76.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.31 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.54

method	result	size
pseudoelliptic	$-\frac{(-e\,x^{2} b+\ln(b\,x^{2}+a) a e-\ln(b\,x^{2}+a) b d)\,\mathrm{csgn}(b\,x^{2}+a)}{2 b^{2}}$	45
default	$-\frac{(b\,x^{2}+a)(-e\,x^{2} b+\ln(b\,x^{2}+a) a e-\ln(b\,x^{2}+a) b d)}{2\sqrt{(b\,x^{2}+a)^{2}}\, b^{2}}$	55
risch	$\frac{\sqrt{(b\,x^{2}+a)^{2}}\, e\,x^{2}}{2(b\,x^{2}+a) b}-\frac{\sqrt{(b\,x^{2}+a)^{2}}\,(a e-b d) \ln(b\,x^{2}+a)}{2(b\,x^{2}+a) b^{2}}$	72

input
```
int(x*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-e*x^2*b+ln(b*x^2+a)*a*e-ln(b*x^2+a)*b*d)*csgn(b*x^2+a)/b^2
```

3.76.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.35

$$\int \frac{x(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx=\frac{bex^2+(bd-ae)\log(bx^2+a)}{2\,b^2}$$

input
```
integrate(x*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(b*e*x^2 + (b*d - a*e)*log(b*x^2 + a))/b^2
```

3.76.6 Sympy [A] (verification not implemented)

Time = 3.91 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.81

$$\int \frac{x(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx=\frac{\begin{cases}\frac{\left(\frac{a}{b}+x^2\right)\left(-\frac{ae}{b}+d\right)\log\left(\frac{a}{b}+x^2\right)}{\sqrt{b^2\left(\frac{a}{b}+x^2\right)^2}}+\frac{e\sqrt{a^2+2abx^2+b^2x^4}}{b^2} & \text{for } b^2\neq 0\\ \frac{2d\sqrt{a^2+2abx^2}+\frac{e\left(-a^2\sqrt{a^2+2abx^2}+\frac{\left(a^2+2abx^2\right)^{\frac{3}{2}}}{3}\right)}{ab}}{2ab} & \text{for } ab\neq 0\\ \frac{dx^2+\frac{ex^4}{2}}{\sqrt{a^2}} & \text{otherwise}\end{cases}}{2}$$

input
```
integrate(x*(e*x**2+d)/((b*x**2+a)**2)**(1/2),x)
```

output
```
Piecewise(((a/b + x**2)*(-a*e/b + d)*log(a/b + x**2)/sqrt(b**2*(a/b + x**2
)**2) + e*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/b**2, Ne(b**2, 0)), ((2*d*sq
rt(a**2 + 2*a*b*x**2) + e*(-a**2*sqrt(a**2 + 2*a*b*x**2) + (a**2 + 2*a*b*x
**2)**(3/2)/3)/(a*b))/(2*a*b), Ne(a*b, 0)), ((d*x**2 + e*x**4/2)/sqrt(a**2
), True))/2
```

3.76.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.37

$$\int \frac{x(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{ex^2}{2\,b}+\frac{(bd-ae)\log\left(bx^2+a\right)}{2\,b^2}$$

input
```
integrate(x*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*e*x^2/b + 1/2*(b*d - a*e)*log(b*x^2 + a)/b^2
```

3.76.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.48

$$\int \frac{x(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{1}{2}\left(\frac{ex^2}{b}+\frac{(bd-ae)\log\left(|bx^2+a|\right)}{b^2}\right)\mathrm{sgn}\left(bx^2+a\right)$$

input
```
integrate(x*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*(e*x^2/b + (b*d - a*e)*log(abs(b*x^2 + a))/b^2)*sgn(b*x^2 + a)
```

3.76.9 Mupad [B] (verification not implemented)

Time = 8.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.24

$$\int \frac{x(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{e\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2\,b^2} - \frac{a\,b\,e\,\ln\left(a\,b+\sqrt{{\left(b\,x^2+a\right)}^2}\,\sqrt{b^2}+b^2\,x^2\right)}{2\,{\left(b^2\right)}^{3/2}} + \frac{b^2\,d\,\ln\left(b^2\,x^2+a\,b\right)\,\mathrm{sign}\left(2\,b^2\,x^2+2\,a\,b\right)}{2\,{\left(b^2\right)}^{3/2}}$$

input
```
int((x*(d + e*x^2))/((a + b*x^2)^2)^(1/2),x)
```

output
```
(e*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(2*b^2) - (a*b*e*log(a*b + ((a + b*x
^2)^2)^(1/2)*(b^2)^(1/2) + b^2*x^2))/(2*(b^2)^(3/2)) + (b^2*d*log(a*b + b^
2*x^2)*sign(2*a*b + 2*b^2*x^2))/(2*(b^2)^(3/2))
```

3.76. $\int \frac{x(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

3.77 $\int \frac{d+ex^2}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

3.77.1 Optimal result

Integrand size = 30, antiderivative size = 97

$$\int \frac{d+ex^2}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx=\frac{ex(a+bx^2)}{b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(bd-ae)\,(a+bx^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
e*x*(b*x^2+a)/b/((b*x^2+a)^2)^(1/2)+(-a*e+b*d)*(b*x^2+a)*arctan(x*b^(1/2)/
a^(1/2))/b^(3/2)/a^(1/2)/((b*x^2+a)^2)^(1/2)
```

3.77.2 Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.71

$$\int \frac{d+ex^2}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx=-\frac{(a+bx^2)\left(-\sqrt{a}\sqrt{b}ex+(-bd+ae)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\right)}{\sqrt{a}b^{3/2}\sqrt{(a+bx^2)^2}}$$

input
```
Integrate[(d + e*x^2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
-(((a + b*x^2)*(-(Sqrt[a]*Sqrt[b]*e*x) + (-(b*d) + a*e)*ArcTan[(Sqrt[b]*x)
/Sqrt[a]]))/(Sqrt[a]*b^(3/2)*Sqrt[(a + b*x^2)^2]))
```

3.77.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.71, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1384, 27, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{\sqrt{a^2+2abx^2+b^2x^4}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b(a+bx^2)\int \frac{ex^2+d}{b(bx^2+a)}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^2)\int \frac{ex^2+d}{bx^2+a}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{299}$$

$$\frac{(a+bx^2)\left(\frac{(bd-ae)\int \frac{1}{bx^2+a}dx}{b}+\frac{ex}{b}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{218}$$

$$\frac{(a+bx^2)\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bd-ae)}{\sqrt{a}b^{3/2}}+\frac{ex}{b}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[(d + e*x^2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
((a + b*x^2)*((e*x)/b + ((b*d - a*e)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]
*b^(3/2))))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
```

3.77.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.77.4 Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.64

method	result	size
default	$\frac{\left(b\,x^{2}+a\right)\left(e x \sqrt{a b}-\arctan\left(\frac{b x}{\sqrt{a b}}\right) a e+\arctan\left(\frac{b x}{\sqrt{a b}}\right) b d\right)}{\sqrt{\left(b\,x^{2}+a\right)^{2}}\, b \sqrt{a b}}$	62
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\, e x}{\left(b\,x^{2}+a\right) b}-\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(a e-b d\right) \ln\left(b x-\sqrt{-a b}\right)}{2\left(b\,x^{2}+a\right) b \sqrt{-a b}}+\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(a e-b d\right) \ln\left(-b x-\sqrt{-a b}\right)}{2\left(b\,x^{2}+a\right) b \sqrt{-a b}}$	133

input
```
int((e*x^2+d)/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(b*x^2+a)*(e*x*(a*b)^(1/2)-arctan(b*x/(a*b)^(1/2))*a*e+arctan(b*x/(a*b)^(1
/2))*b*d)/((b*x^2+a)^2)^(1/2)/b/(a*b)^(1/2)
```

3.77.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.01

$$\int \frac{d+ex^2}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \left[\frac{2\,abex+\sqrt{-ab}(bd-ae)\log\left(\frac{bx^2+2\,\sqrt{-ab}x-a}{bx^2+a}\right)}{2\,ab^2}, \frac{abex+\sqrt{ab}(bd-ae)\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{ab^2}\right]$$

input
```
integrate((e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(2*a*b*e*x + sqrt(-a*b)*(b*d - a*e)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/
(b*x^2 + a)))/(a*b^2), (a*b*e*x + sqrt(a*b)*(b*d - a*e)*arctan(sqrt(a*b)*x
/a))/(a*b^2)]
```

3.77.6 Sympy [F]

$$\int \frac{d+ex^2}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{d+ex^2}{\sqrt{(a+bx^2)^2}}\,dx$$

input
```
integrate((e*x**2+d)/((b*x**2+a)**2)**(1/2),x)
```

output
```
Integral((d + e*x**2)/sqrt((a + b*x**2)**2), x)
```

3.77.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.34

$$\int \frac{d+ex^2}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{ex}{b} + \frac{(bd-ae)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b}$$

input
```
integrate((e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
e*x/b + (b*d - a*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

3.77.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.59

$$\int \frac{d+ex^2}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{ex\text{sgn}(bx^2+a)}{b} + \frac{(bd\text{sgn}(bx^2+a) - ae\text{sgn}(bx^2+a))\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b}$$

input
```
integrate((e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

output
```
e*x*sgn(b*x^2 + a)/b + (b*d*sgn(b*x^2 + a) - a*e*sgn(b*x^2 + a))*arctan(b*
x/sqrt(a*b))/(sqrt(a*b)*b)
```

3.77.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{e\,x^2+d}{\sqrt{(b\,x^2+a)^2}}\,dx$$

input
```
int((d + e*x^2)/((a + b*x^2)^2)^(1/2),x)
```

output
```
int((d + e*x^2)/((a + b*x^2)^2)^(1/2), x)
```

3.78 $\int \frac{d+ex^2}{x\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

3.78.1 Optimal result

Integrand size = 33, antiderivative size = 92

$$\int \frac{d+ex^2}{x\sqrt{a^2+2abx^2+b^2x^4}}\,dx=\frac{d(a+bx^2)\log(x)}{a\sqrt{a^2+2abx^2+b^2x^4}}-\frac{(bd-ae)\,(a+bx^2)\log\left(a+bx^2\right)}{2ab\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
d*(b*x^2+a)*ln(x)/a/((b*x^2+a)^2)^(1/2)-1/2*(-a*e+b*d)*(b*x^2+a)*ln(b*x^2+
a)/a/b/((b*x^2+a)^2)^(1/2)
```

3.78.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.36

$$\int \frac{d+ex^2}{x\sqrt{a^2+2abx^2+b^2x^4}}\,dx=\frac{1}{4}\left(\left(\frac{2d}{a}-\frac{4e}{b}\right)\operatorname{arctanh}\left(\frac{\sqrt{a^2}-\sqrt{(a+bx^2)^2}}{bx^2}\right)+\frac{d\left(-2\log\left(x^2\right)+\log\left(\sqrt{a^2}-bx^2-\sqrt{(a+bx^2)^2}\right)+\log\left(\sqrt{a^2}+bx^2-\sqrt{(a+bx^2)^2}\right)\right)}{\sqrt{a^2}}\right)$$

input
```
Integrate[(d + e*x^2)/(x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

output
```
(((2*d)/a - (4*e)/b)*ArcTanh[(Sqrt[a^2] - Sqrt[(a + b*x^2)^2])/(b*x^2)] +
(d*(-2*Log[x^2] + Log[Sqrt[a^2] - b*x^2 - Sqrt[(a + b*x^2)^2]] + Log[Sqrt[
a^2] + b*x^2 - Sqrt[(a + b*x^2)^2]]))/Sqrt[a^2])/4
```

3.78.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.73, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1384, 27, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{x\sqrt{a^2+2abx^2+b^2x^4}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b(a+bx^2)\int \frac{ex^2+d}{bx(bx^2+a)}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^2)\int \frac{ex^2+d}{x(bx^2+a)}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{354}$$

$$\frac{(a+bx^2)\int \frac{ex^2+d}{x^2(bx^2+a)}dx^2}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{86}$$

$$\frac{(a+bx^2)\int \left(\frac{d}{ax^2}+\frac{ae-bd}{a(bx^2+a)}\right)dx^2}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx^2)\left(\frac{d\log(x^2)}{a}-\frac{(bd-ae)\log(a+bx^2)}{ab}\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[(d + e*x^2)/(x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

output
```
((a + b*x^2)*((d*Log[x^2])/a - ((b*d - a*e)*Log[a + b*x^2])/(a*b)))/(2*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])
```

3.78.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.78.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.34 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.52

method	result	size
pseudoelliptic	$\frac{\left(d\ln\left(x^{2}\right)b+\ln\left(b\,x^{2}+a\right)ae-\ln\left(b\,x^{2}+a\right)bd\right)\operatorname{csgn}\left(b\,x^{2}+a\right)}{2ab}$	48
default	$\frac{\left(b\,x^{2}+a\right)\left(2d\ln\left(x\right)b+\ln\left(b\,x^{2}+a\right)ae-\ln\left(b\,x^{2}+a\right)bd\right)}{2\sqrt{\left(b\,x^{2}+a\right)^{2}}\,ab}$	57
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,d\ln\left(x\right)}{\left(b\,x^{2}+a\right)a}+\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(ae-bd\right)\ln\left(-b\,x^{2}-a\right)}{2\left(b\,x^{2}+a\right)ab}$	76

input
```
int((e*x^2+d)/x/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(d*ln(x^2)*b+ln(b*x^2+a)*a*e-ln(b*x^2+a)*b*d)*csgn(b*x^2+a)/a/b
```

3.78.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.36

$$\int \frac{d+ex^2}{x\sqrt{a^2+2abx^2+b^2x^4}}\,dx=\frac{2\,bd\log\left(x\right)-\left(bd-ae\right)\log\left(bx^2+a\right)}{2\,ab}$$

input
```
integrate((e*x^2+d)/x/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(2*b*d*log(x) - (b*d - a*e)*log(b*x^2 + a))/(a*b)
```

3.78.6 Sympy [F]

$$\int \frac{d+ex^2}{x\sqrt{a^2+2abx^2+b^2x^4}}\,dx=\int \frac{d+ex^2}{x\sqrt{\left(a+bx^2\right)^2}}\,dx$$

input
```
integrate((e*x**2+d)/x/((b*x**2+a)**2)**(1/2),x)
```

output
```
Integral((d + e*x**2)/(x*sqrt((a + b*x**2)**2)), x)
```

3.78.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.38

$$\int \frac{d+ex^2}{x\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{d\log(x^2)}{2a} - \frac{(bd-ae)\log(bx^2+a)}{2ab}$$

input
```
integrate((e*x^2+d)/x/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*d*log(x^2)/a - 1/2*(b*d - a*e)*log(b*x^2 + a)/(a*b)
```

3.78.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.65

$$\int \frac{d+ex^2}{x\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{d\log(x^2)\,\mathrm{sgn}(bx^2+a)}{2a} - \frac{(bd\mathrm{sgn}(bx^2+a) - ae\mathrm{sgn}(bx^2+a))\log(|bx^2+a|)}{2ab}$$

input
```
integrate((e*x^2+d)/x/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*d*log(x^2)*sgn(b*x^2 + a)/a - 1/2*(b*d*sgn(b*x^2 + a) - a*e*sgn(b*x^2
+ a))*log(abs(b*x^2 + a))/(a*b)
```

3.78.9 Mupad [B] (verification not implemented)

Time = 8.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.90

$$\int \frac{d+ex^2}{x\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{e\,\ln(b^2\,x^2+a\,b)\,\mathrm{sign}(2\,b^2\,x^2+2\,a\,b)}{2\,\sqrt{b^2}} - \frac{d\,\ln\left(\frac{1}{x^2}\right)}{2\,\sqrt{a^2}} - \frac{d\,\ln\left(\sqrt{(b\,x^2+a)^2}\,\sqrt{a^2}+a^2+a\,b\,x^2\right)}{2\,\sqrt{a^2}}$$

input
```
int((d + e*x^2)/(x*((a + b*x^2)^2)^(1/2)),x)
```

output
```
(e*log(a*b + b^2*x^2)*sign(2*a*b + 2*b^2*x^2))/(2*(b^2)^(1/2)) - (d*log(1/
x^2))/(2*(a^2)^(1/2)) - (d*log(((a + b*x^2)^2)^(1/2)*(a^2)^(1/2) + a^2 + a
*b*x^2))/(2*(a^2)^(1/2))
```

3.78. $\int \frac{d+ex^2}{x\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

3.79 $\int \frac{d+ex^2}{x^2\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

3.79.1 Optimal result

Integrand size = 33, antiderivative size = 101

$$\int \frac{d+ex^2}{x^2\sqrt{a^2+2abx^2+b^2x^4}}\,dx=-\frac{d(a+bx^2)}{ax\sqrt{a^2+2abx^2+b^2x^4}}-\frac{(bd-ae)\,(a+bx^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
-d*(b*x^2+a)/a/x/((b*x^2+a)^2)^(1/2)-(-a*e+b*d)*(b*x^2+a)*arctan(x*b^(1/2)
/a^(1/2))/a^(3/2)/b^(1/2)/((b*x^2+a)^2)^(1/2)
```

3.79.2 Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.71

$$\int \frac{d+ex^2}{x^2\sqrt{a^2+2abx^2+b^2x^4}}\,dx=\frac{(a+bx^2)\left(-\sqrt{a}\sqrt{b}d+(-bdx+aex)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\right)}{a^{3/2}\sqrt{b}x\sqrt{(a+bx^2)^2}}$$

input
```
Integrate[(d + e*x^2)/(x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

output
```
((a + b*x^2)*(-(Sqrt[a]*Sqrt[b]*d) + (-(b*d*x) + a*e*x)*ArcTan[(Sqrt[b]*x)
/Sqrt[a]]))/(a^(3/2)*Sqrt[b]*x*Sqrt[(a + b*x^2)^2])
```

3.79.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.72, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1384, 27, 359, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{x^2\sqrt{a^2+2abx^2+b^2x^4}}dx$$

$$\Big\downarrow \; 1384$$

$$\frac{b\left(a+bx^2\right)\int \frac{ex^2+d}{bx^2(bx^2+a)}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(a+bx^2\right)\int \frac{ex^2+d}{x^2(bx^2+a)}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 359$$

$$\frac{\left(a+bx^2\right)\left(-\frac{(bd-ae)\int \frac{1}{bx^2+a}dx}{a}-\frac{d}{ax}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 218$$

$$\frac{\left(a+bx^2\right)\left(-\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bd-ae)}{a^{3/2}\sqrt{b}}-\frac{d}{ax}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[(d + e*x^2)/(x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

output
```
((a + b*x^2)*(-(d/(a*x)) - ((b*d - a*e)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3
/2)*Sqrt[b])))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
```

3.79.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.79.4 Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.66

method	result	size
default	$-\frac{\left(b\,x^{2}+a\right)\left(-\arctan\left(\frac{b x}{\sqrt{a b}}\right) a e x+\arctan\left(\frac{b x}{\sqrt{a b}}\right) b d x+d\sqrt{a b}\right)}{\sqrt{\left(b\,x^{2}+a\right)^{2}}\, a\sqrt{a b}\, x}$	67
risch	$-\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\, d}{\left(b\,x^{2}+a\right) a x}-\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(a e-b d\right)\ln\left(-\sqrt{-a b}\, x+a\right)}{2\left(b\,x^{2}+a\right)\sqrt{-a b}\, a}+\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(a e-b d\right)\ln\left(-\sqrt{-a b}\, x-a\right)}{2\left(b\,x^{2}+a\right)\sqrt{-a b}\, a}$	135

input
```
int((e*x^2+d)/x^2/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(b*x^2+a)*(-arctan(b*x/(a*b)^(1/2))*a*e*x+arctan(b*x/(a*b)^(1/2))*b*d*x+d
*(a*b)^(1/2))/((b*x^2+a)^2)^(1/2)/a/(a*b)^(1/2)/x
```

3.79.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.04

$$\int \frac{d+ex^2}{x^2\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \left[\frac{\sqrt{-ab}(bd-ae)x\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)-2\,abd}{2\,a^2bx},\ -\frac{\sqrt{ab}(bd-ae)x\arctan\left(\frac{\sqrt{ab}x}{a}\right)+abd}{a^2bx}\right]$$

input
```
integrate((e*x^2+d)/x^2/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(-a*b)*(b*d - a*e)*x*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a
)) - 2*a*b*d)/(a^2*b*x), -(sqrt(a*b)*(b*d - a*e)*x*arctan(sqrt(a*b)*x/a) +
 a*b*d)/(a^2*b*x)]
```

3.79.6 Sympy [F]

$$\int \frac{d+ex^2}{x^2\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{d+ex^2}{x^2\sqrt{(a+bx^2)^2}}\,dx$$

input
```
integrate((e*x**2+d)/x**2/((b*x**2+a)**2)**(1/2),x)
```

output
```
Integral((d + e*x**2)/(x**2*sqrt((a + b*x**2)**2)), x)
```

3.79.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.37

$$\int \frac{d+ex^2}{x^2\sqrt{a^2+2abx^2+b^2x^4}}\,dx = -\frac{(bd-ae)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a}-\frac{d}{ax}$$

input
```
integrate((e*x^2+d)/x^2/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
-(b*d - a*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - d/(a*x)
```

3.79.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.60

$$\int \frac{d+ex^2}{x^2\sqrt{a^2+2abx^2+b^2x^4}}\,dx = -\frac{(bd\text{sgn}(bx^2+a)-ae\text{sgn}(bx^2+a))\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a} - \frac{d\text{sgn}(bx^2+a)}{ax}$$

input
```
integrate((e*x^2+d)/x^2/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

output
```
-(b*d*sgn(b*x^2 + a) - a*e*sgn(b*x^2 + a))*arctan(b*x/sqrt(a*b))/(sqrt(a*b
)*a) - d*sgn(b*x^2 + a)/(a*x)
```

3.79.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{x^2\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{e\,x^2+d}{x^2\,\sqrt{{\left(b\,x^2+a\right)}^2}}\,dx$$

input
```
int((d + e*x^2)/(x^2*((a + b*x^2)^2)^(1/2)),x)
```

output
```
int((d + e*x^2)/(x^2*((a + b*x^2)^2)^(1/2)), x)
```

3.80 $\int \frac{d+ex^2}{x^3\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

3.80.1 Optimal result

Integrand size = 33, antiderivative size = 137

$$\int \frac{d+ex^2}{x^3\sqrt{a^2+2abx^2+b^2x^4}}\,dx=-\frac{d(a+bx^2)}{2ax^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{(bd-ae)\,(a+bx^2)\log(x)}{a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(bd-ae)\,(a+bx^2)\log\left(a+bx^2\right)}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
-1/2*d*(b*x^2+a)/a/x^2/((b*x^2+a)^2)^(1/2)-(-a*e+b*d)*(b*x^2+a)*ln(x)/a^2/
((b*x^2+a)^2)^(1/2)+1/2*(-a*e+b*d)*(b*x^2+a)*ln(b*x^2+a)/a^2/((b*x^2+a)^2)
^(1/2)
```

3.80.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.20

$$\int \frac{d+ex^2}{x^3\sqrt{a^2+2abx^2+b^2x^4}}\,dx$$
$$=\frac{a^2d-\sqrt{a^2}d\sqrt{(a+bx^2)^2}+2a(bd-ae)x^2\log\left(x^2\right)-\left(-a+\sqrt{a^2}\right)(-bd+ae)x^2\log\left(\sqrt{a^2}-bx^2-\sqrt{(a+\cdots}\right.}{4\left(a^2\right)^{3/2}x^2}$$

input
```
Integrate[(d + e*x^2)/(x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

output
```
(a^2*d - Sqrt[a^2]*d*Sqrt[(a + b*x^2)^2] + 2*a*(b*d - a*e)*x^2*Log[x^2] -
(-a + Sqrt[a^2])*(-(b*d) + a*e)*x^2*Log[Sqrt[a^2] - b*x^2 - Sqrt[(a + b*x^
2)^2]] + (a + Sqrt[a^2])*(-(b*d) + a*e)*x^2*Log[Sqrt[a^2] + b*x^2 - Sqrt[(
a + b*x^2)^2]])/(4*(a^2)^(3/2)*x^2)
```

3.80.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.58, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1384, 27, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{x^3\sqrt{a^2+2abx^2+b^2x^4}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b(a+bx^2)\int \frac{ex^2+d}{bx^3(bx^2+a)}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^2)\int \frac{ex^2+d}{x^3(bx^2+a)}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{354}$$

$$\frac{(a+bx^2)\int \frac{ex^2+d}{x^4(bx^2+a)}dx^2}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{86}$$

$$\frac{(a+bx^2)\int \left(\frac{d}{ax^4}-\frac{b(ae-bd)}{a^2(bx^2+a)}+\frac{ae-bd}{a^2x^2}\right)dx^2}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx^2)\left(-\frac{\log(x^2)(bd-ae)}{a^2}+\frac{(bd-ae)\log(a+bx^2)}{a^2}-\frac{d}{ax^2}\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[(d + e*x^2)/(x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

output
```
((a + b*x^2)*(-(d/(a*x^2)) - ((b*d - a*e)*Log[x^2])/a^2 + ((b*d - a*e)*Log
[a + b*x^2])/a^2))/(2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

3.80.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.80.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.34 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.42

method	result	size
pseudoelliptic	$-\frac{\left(x^{2}\left(ae-bd\right)\ln\left(b\,x^{2}+a\right)-x^{2}\left(ae-bd\right)\ln\left(x^{2}\right)+da\right)\operatorname{csgn}\left(b\,x^{2}+a\right)}{2a^{2}x^{2}}$	58
default	$\frac{\left(b\,x^{2}+a\right)\left(2\ln\left(x\right)ae\,x^{2}-2\ln\left(x\right)bd\,x^{2}-\ln\left(b\,x^{2}+a\right)ae\,x^{2}+\ln\left(b\,x^{2}+a\right)bd\,x^{2}-da\right)}{2\sqrt{\left(b\,x^{2}+a\right)^{2}}\,a^{2}x^{2}}$	79
risch	$-\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,d}{2\left(b\,x^{2}+a\right)a\,x^{2}}+\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(ae-bd\right)\ln\left(x\right)}{\left(b\,x^{2}+a\right)a^{2}}-\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(ae-bd\right)\ln\left(b\,x^{2}+a\right)}{2\left(b\,x^{2}+a\right)a^{2}}$	106

input
```
int((e*x^2+d)/x^3/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(x^2*(a*e-b*d)*ln(b*x^2+a)-x^2*(a*e-b*d)*ln(x^2)+d*a)*csgn(b*x^2+a)/a
^2/x^2
```

3.80.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.35

$$\int \frac{d+ex^2}{x^3\sqrt{a^2+2abx^2+b^2x^4}}\,dx=\frac{(bd-ae)x^2\log\left(bx^2+a\right)-2\,(bd-ae)x^2\log\left(x\right)-ad}{2\,a^2x^2}$$

input
```
integrate((e*x^2+d)/x^3/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*((b*d - a*e)*x^2*log(b*x^2 + a) - 2*(b*d - a*e)*x^2*log(x) - a*d)/(a^2
*x^2)
```

3.80.6 Sympy [F]

$$\int \frac{d+ex^2}{x^3\sqrt{a^2+2abx^2+b^2x^4}}\,dx=\int \frac{d+ex^2}{x^3\sqrt{\left(a+bx^2\right)^2}}\,dx$$

input
```
integrate((e*x**2+d)/x**3/((b*x**2+a)**2)**(1/2),x)
```

output
```
Integral((d + e*x**2)/(x**3*sqrt((a + b*x**2)**2)), x)
```

3.80.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.35

$$\int \frac{d+ex^2}{x^3\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{(bd-ae)\log(bx^2+a)}{2\,a^2} - \frac{(bd-ae)\log(x^2)}{2\,a^2} - \frac{d}{2\,ax^2}$$

input
```
integrate((e*x^2+d)/x^3/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(b*d - a*e)*log(b*x^2 + a)/a^2 - 1/2*(b*d - a*e)*log(x^2)/a^2 - 1/2*d/
(a*x^2)
```

3.80.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.93

$$\int \frac{d+ex^2}{x^3\sqrt{a^2+2abx^2+b^2x^4}}\,dx = -\frac{(bd\text{sgn}(bx^2+a)-ae\text{sgn}(bx^2+a))\log(x^2)}{2\,a^2} + \frac{(b^2d\text{sgn}(bx^2+a)-abe\text{sgn}(bx^2+a))\log(|bx^2+a|)}{2\,a^2b} + \frac{bdx^2\text{sgn}(bx^2+a)-aex^2\text{sgn}(bx^2+a)-ad\text{sgn}(bx^2+a)}{2\,a^2x^2}$$

input
```
integrate((e*x^2+d)/x^3/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(b*d*sgn(b*x^2 + a) - a*e*sgn(b*x^2 + a))*log(x^2)/a^2 + 1/2*(b^2*d*s
gn(b*x^2 + a) - a*b*e*sgn(b*x^2 + a))*log(abs(b*x^2 + a))/(a^2*b) + 1/2*(b
*d*x^2*sgn(b*x^2 + a) - a*e*x^2*sgn(b*x^2 + a) - a*d*sgn(b*x^2 + a))/(a^2*
x^2)
```

3.80.9 Mupad [B] (verification not implemented)

Time = 8.32 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.91

$$\int \frac{d+ex^2}{x^3\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{a\,b\,d\,\mathrm{atanh}\left(\frac{a^2+b\,a\,x^2}{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}\right)}{2\,{\left(a^2\right)}^{3/2}} - \frac{e\,\ln\left(\frac{1}{x^2}\right)}{2\,\sqrt{a^2}} - \frac{d\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2\,a^2\,x^2} - \frac{e\,\ln\left(\sqrt{{\left(b\,x^2+a\right)}^2}\,\sqrt{a^2}+a^2+a\,b\,x^2\right)}{2\,\sqrt{a^2}}$$

input
```
int((d + e*x^2)/(x^3*((a + b*x^2)^2)^(1/2)),x)
```

output
```
(a*b*d*atanh((a^2 + a*b*x^2)/((a^2)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2
))))/(2*(a^2)^(3/2)) - (e*log(1/x^2))/(2*(a^2)^(1/2)) - (d*(a^2 + b^2*x^4
+ 2*a*b*x^2)^(1/2))/(2*a^2*x^2) - (e*log(((a + b*x^2)^2)^(1/2)*(a^2)^(1/2)
 + a^2 + a*b*x^2))/(2*(a^2)^(1/2))
```

3.81 $\int \frac{x^2(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

3.81.1 Optimal result

Integrand size = 33, antiderivative size = 153

$$\int \frac{x^2(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{(bd-5ae)x}{8ab^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(bd-ae)x}{4b^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(bd+3ae)(a+bx^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
1/8*(-5*a*e+b*d)*x/a/b^2/((b*x^2+a)^2)^(1/2)-1/4*(-a*e+b*d)*x/b^2/(b*x^2+a
)/((b*x^2+a)^2)^(1/2)+1/8*(3*a*e+b*d)*(b*x^2+a)*arctan(x*b^(1/2)/a^(1/2))/
a^(3/2)/b^(5/2)/((b*x^2+a)^2)^(1/2)
```

3.81.2 Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.71

$$\int \frac{x^2(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{-\sqrt{a}\sqrt{b}x(3a^2e-b^2dx^2+ab(d+5ex^2))+(bd+3ae)(a+bx^2)^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right.}{8a^{3/2}b^{5/2}(a+bx^2)\sqrt{(a+bx^2)^2}}$$

input
```
Integrate[(x^2*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
(-(Sqrt[a]*Sqrt[b]*x*(3*a^2*e - b^2*d*x^2 + a*b*(d + 5*e*x^2))) + (b*d + 3
*a*e)*(a + b*x^2)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(5/2)*(a + b
*x^2)*Sqrt[(a + b*x^2)^2])
```

3.81.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.81, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1384, 27, 360, 25, 298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^3(a+bx^2)\int \frac{x^2(ex^2+d)}{b^3(bx^2+a)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^2)\int \frac{x^2(ex^2+d)}{(bx^2+a)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{360}$$

$$\frac{(a+bx^2)\left(-\frac{\int -\frac{4bex^2+bd-ae}{(bx^2+a)^2}dx}{4b^2}-\frac{x(bd-ae)}{4b^2(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{25}$$

$$\frac{(a+bx^2)\left(\frac{\int \frac{4bex^2+bd-ae}{(bx^2+a)^2}dx}{4b^2}-\frac{x(bd-ae)}{4b^2(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{298}$$

$$\frac{(a+bx^2)\left(\frac{\frac{(3ae+bd)\int \frac{1}{bx^2+a}dx}{2a}+\frac{x(bd-5ae)}{2a(a+bx^2)}}{4b^2}-\frac{x(bd-ae)}{4b^2(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{218}$$

$$\frac{(a+bx^2)\left(\frac{\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(3ae+bd)}{2a^{3/2}\sqrt{b}}+\frac{x(bd-5ae)}{2a\left(a+bx^2\right)}}{4b^2}-\frac{x(bd-ae)}{4b^2(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[(x^2*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
((a + b*x^2)*(-1/4*((b*d - a*e)*x)/(b^2*(a + b*x^2)^2) + (((b*d - 5*a*e)*x
)/(2*a*(a + b*x^2)) + ((b*d + 3*a*e)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/
2)*Sqrt[b]))/(4*b^2)))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
```

3.81.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.81.4 Maple [A] (verified)

Time = 2.09 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.08

method	result
risch	$\frac{\sqrt{(b\,x^2+a)^2}\left(-\frac{(5ae-bd)x^3}{8ab}-\frac{(3ae+bd)x}{8b^2}\right)}{(b\,x^2+a)^3}-\frac{\sqrt{(b\,x^2+a)^2}\,(3ae+bd)\ln\left(bx+\sqrt{-ab}\right)}{16(b\,x^2+a)\sqrt{-ab}\,b^2a}+\frac{\sqrt{(b\,x^2+a)^2}\,(3ae+bd)\ln\left(-bx+\sqrt{-ab}\right)}{16(b\,x^2+a)\sqrt{-ab}\,b^2a}$
default	$-\frac{\left(-3\arctan\left(\frac{bx}{\sqrt{ab}}\right)a\,b^2e\,x^4-\arctan\left(\frac{bx}{\sqrt{ab}}\right)b^3d\,x^4+5\sqrt{ab}\,abe\,x^3-\sqrt{ab}\,b^2d\,x^3-6\arctan\left(\frac{bx}{\sqrt{ab}}\right)a^2be\,x^2-2\arctan\left(\frac{bx}{\sqrt{ab}}\right)a\,b^2d\,x^2+3\sqrt{\right.}{8\sqrt{ab}\,a\,b^2\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}$

input
```
int(x^2*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)^2)^(1/2)/(b*x^2+a)^3*(-1/8*(5*a*e-b*d)/a/b*x^3-1/8*(3*a*e+b*d)/
b^2*x)-1/16*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/(-a*b)^(1/2)*(3*a*e+b*d)/b^2/a*l
n(b*x+(-a*b)^(1/2))+1/16*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/(-a*b)^(1/2)*(3*a*e
+b*d)/b^2/a*ln(-b*x+(-a*b)^(1/2))
```

3.81.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.96

$$\int \frac{x^2(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \left[\frac{2\,(ab^3d-5\,a^2b^2e)x^3-((b^3d+3\,ab^2e)x^4+a^2bd+3\,a^3e+2\,(ab^2d+3\,a^2be)}{16\,(a^2b^5x^4+2\,a^3b^4x^2+a^4}\right.$$

input
```
integrate(x^2*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas
")
```

output
```
[1/16*(2*(a*b^3*d - 5*a^2*b^2*e)*x^3 - ((b^3*d + 3*a*b^2*e)*x^4 + a^2*b*d
+ 3*a^3*e + 2*(a*b^2*d + 3*a^2*b*e)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a
*b)*x - a)/(b*x^2 + a)) - 2*(a^2*b^2*d + 3*a^3*b*e)*x)/(a^2*b^5*x^4 + 2*a^
3*b^4*x^2 + a^4*b^3), 1/8*((a*b^3*d - 5*a^2*b^2*e)*x^3 + ((b^3*d + 3*a*b^2
*e)*x^4 + a^2*b*d + 3*a^3*e + 2*(a*b^2*d + 3*a^2*b*e)*x^2)*sqrt(a*b)*arcta
n(sqrt(a*b)*x/a) - (a^2*b^2*d + 3*a^3*b*e)*x)/(a^2*b^5*x^4 + 2*a^3*b^4*x^2
 + a^4*b^3)]
```

3.81.6 Sympy [F]

$$\int \frac{x^2(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{x^2(d+ex^2)}{\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*(e*x**2+d)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Integral(x**2*(d + e*x**2)/((a + b*x**2)**2)**(3/2), x)
```

3.81.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.82

$$\int \frac{x^2(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = -\frac{1}{8}\,e\left(\frac{5\,bx^3+3\,ax}{b^4x^4+2\,ab^3x^2+a^2b^2}-\frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2}\right) + \frac{1}{8}\,d\left(\frac{bx^3-ax}{ab^3x^4+2\,a^2b^2x^2+a^3b}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}ab}\right)$$

input
```
integrate(x^2*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima
")
```

output
```
-1/8*e*((5*b*x^3 + 3*a*x)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2) - 3*arctan(b*x
/sqrt(a*b))/(sqrt(a*b)*b^2)) + 1/8*d*((b*x^3 - a*x)/(a*b^3*x^4 + 2*a^2*b^2
*x^2 + a^3*b) + arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b))
```

3.81.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.64

$$\int \frac{x^2(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{(bd+3\,ae)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}ab^2\mathrm{sgn}\left(bx^2+a\right)} + \frac{b^2dx^3-5\,abex^3-abdx-3\,a^2ex}{8\,(bx^2+a)^2ab^2\mathrm{sgn}\left(bx^2+a\right)}$$

input
```
integrate(x^2*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/8*(b*d + 3*a*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2*sgn(b*x^2 + a)) +
 1/8*(b^2*d*x^3 - 5*a*b*e*x^3 - a*b*d*x - 3*a^2*e*x)/((b*x^2 + a)^2*a*b^2*
sgn(b*x^2 + a))
```

3.81.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{x^2\,(e\,x^2+d)}{(a^2+2\,a\,b\,x^2+b^2\,x^4)^{3/2}}\,dx$$

input
```
int((x^2*(d + e*x^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

output
```
int((x^2*(d + e*x^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

3.82 $\int \frac{x(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

3.82.1 Optimal result

Integrand size = 31, antiderivative size = 77

$$\int \frac{x(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = -\frac{e}{2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{bd-ae}{4b^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
-1/2*e/b^2/((b*x^2+a)^2)^(1/2)+1/4*(a*e-b*d)/b^2/(b*x^2+a)/((b*x^2+a)^2)^(
1/2)
```

3.82.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 181 vs. 2(77) = 154.

Time = 0.65 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.35

$$\int \frac{x(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx =$$

$$-\frac{x^2\left(a^3bdx^2+a^2b^2ex^6+a^4(2d+ex^2)+a\left(-b^3dx^6+\sqrt{a^2}bx^2\sqrt{(a+bx^2)^2}(d+ex^2)\right)-\sqrt{a^2}\sqrt{(a+bx^2)^2}(\ldots}{4a^4(a+bx^2)\left(\sqrt{a^2}bx^2+a\left(\sqrt{a^2}-\sqrt{(a+bx^2)^2}\right)\right)}$$

input
```
Integrate[(x*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
-1/4*(x^2*(a^3*b*d*x^2 + a^2*b^2*e*x^6 + a^4*(2*d + e*x^2) + a*(-(b^3*d*x^
6) + Sqrt[a^2]*b*x^2*Sqrt[(a + b*x^2)^2]*(d + e*x^2)) - Sqrt[a^2]*Sqrt[(a
+ b*x^2)^2]*(b^2*d*x^4 + a^2*(2*d + e*x^2))))/(a^4*(a + b*x^2)*(Sqrt[a^2]*
b*x^2 + a*(Sqrt[a^2] - Sqrt[(a + b*x^2)^2])))
```

3.82.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.70, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1384, 27, 353, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{b^3(a+bx^2)\int \frac{x(ex^2+d)}{b^3(bx^2+a)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx^2)\int \frac{x(ex^2+d)}{(bx^2+a)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 353}$$

$$\frac{(a+bx^2)\int \frac{ex^2+d}{(bx^2+a)^3}dx^2}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 48}$$

$$-\frac{(d+ex^2)^2}{4(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}(bd-ae)}$$

input
```
Int[(x*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
-1/4*(d + e*x^2)^2/((b*d - a*e)*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])
```

3.82.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.82.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.48

method	result	size
pseudoelliptic	$-\frac{\left(\left(2 e \,x^{2}+d \right) b +a e \right) \operatorname{csgn}\left(b \,x^{2}+a \right)}{4 \left(b \,x^{2}+a \right)^{2} b^{2}}$	37
gosper	$-\frac{\left(b \,x^{2}+a \right) \left(2 e \,x^{2} b +a e +b d \right)}{4 b^{2} \left(\left(b \,x^{2}+a \right)^{2}\right)^{\frac{3}{2}}}$	38
default	$-\frac{\left(b \,x^{2}+a \right) \left(2 e \,x^{2} b +a e +b d \right)}{4 b^{2} \left(\left(b \,x^{2}+a \right)^{2}\right)^{\frac{3}{2}}}$	38
risch	$\frac{\sqrt{\left(b \,x^{2}+a \right)^{2}}\, \left(-\frac{e \,x^{2}}{2 b}-\frac{a e +b d}{4 b^{2}}\right)}{\left(b \,x^{2}+a \right)^{3}}$	44

input
```
int(x*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*((2*e*x^2+d)*b+a*e)*csgn(b*x^2+a)/(b*x^2+a)^2/b^2
```

3.82.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.55

$$\int \frac{x(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = -\frac{2\,bex^2+bd+ae}{4\,(b^4x^4+2\,ab^3x^2+a^2b^2)}$$

input
```
integrate(x*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/4*(2*b*e*x^2 + b*d + a*e)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

3.82.6 Sympy [F]

$$\int \frac{x(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{x(d+ex^2)}{\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(e*x**2+d)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Integral(x*(d + e*x**2)/((a + b*x**2)**2)**(3/2), x)
```

3.82.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.84

$$\int \frac{x(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = -\frac{(2\,bx^2+a)e}{4\,(b^4x^4+2\,ab^3x^2+a^2b^2)} - \frac{d}{4\,(b^3x^4+2\,ab^2x^2+a^2b)}$$

input
```
integrate(x*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(2*b*x^2 + a)*e/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2) - 1/4*d/(b^3*x^4 +
2*a*b^2*x^2 + a^2*b)
```

3.82.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.49

$$\int \frac{x(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = -\frac{2\,bex^2+bd+ae}{4\,(bx^2+a)^2b^2\mathrm{sgn}\,(bx^2+a)}$$

input
```
integrate(x*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
-1/4*(2*b*e*x^2 + b*d + a*e)/((b*x^2 + a)^2*b^2*sgn(b*x^2 + a))
```

3.82.9 Mupad [B] (verification not implemented)

Time = 7.73 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.62

$$\int \frac{x(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = -\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\,(2\,b\,e\,x^2+a\,e+b\,d)}{4\,b^2\,(b\,x^2+a)^3}$$

input
```
int((x*(d + e*x^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

output
```
-((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*(a*e + b*d + 2*b*e*x^2))/(4*b^2*(a + b
*x^2)^3)
```

3.83 $\int \frac{d+ex^2}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

3.83.1 Optimal result

Integrand size = 30, antiderivative size = 156

$$\int \frac{d+ex^2}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{(3bd+ae)x}{8a^2b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(bd-ae)x}{4ab\,(a+bx^2)\,\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(3bd+ae)\,(a+bx^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
1/8*(a*e+3*b*d)*x/a^2/b/((b*x^2+a)^2)^(1/2)+1/4*(-a*e+b*d)*x/a/b/(b*x^2+a)
/((b*x^2+a)^2)^(1/2)+1/8*(a*e+3*b*d)*(b*x^2+a)*arctan(x*b^(1/2)/a^(1/2))/a
^(5/2)/b^(3/2)/((b*x^2+a)^2)^(1/2)
```

3.83.2 Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.69

$$\int \frac{d+ex^2}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{\sqrt{a}\sqrt{b}x(-a^2e+3b^2dx^2+ab(5d+ex^2))+(3bd+ae)\,(a+bx^2)^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}\,(a+bx^2)\,\sqrt{(a+bx^2)^2}}$$

input
```
Integrate[(d + e*x^2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
(Sqrt[a]*Sqrt[b]*x*(-(a^2*e) + 3*b^2*d*x^2 + a*b*(5*d + e*x^2)) + (3*b*d +
 a*e)*(a + b*x^2)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(3/2)*(a + b
*x^2)*Sqrt[(a + b*x^2)^2])
```

3.83.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.78, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1384, 27, 298, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^3(a+bx^2)\int \frac{ex^2+d}{b^3(bx^2+a)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^2)\int \frac{ex^2+d}{(bx^2+a)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{298}$$

$$\frac{(a+bx^2)\left(\frac{(ae+3bd)\int \frac{1}{(bx^2+a)^2}dx}{4ab}+\frac{x(bd-ae)}{4ab(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{215}$$

$$\frac{(a+bx^2)\left(\frac{(ae+3bd)\left(\frac{\int \frac{1}{bx^2+a}dx}{2a}+\frac{x}{2a(a+bx^2)}\right)}{4ab}+\frac{x(bd-ae)}{4ab(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{218}$$

$$\frac{(a+bx^2)\left(\frac{\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}\right)(ae+3bd)}{4ab}+\frac{x(bd-ae)}{4ab(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[(d + e*x^2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
((a + b*x^2)*(((b*d - a*e)*x)/(4*a*b*(a + b*x^2)^2) + ((3*b*d + a*e)*(x/(2
*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])))/(4*a*b
)))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
```

3.83.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.83.4 Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.05

method	result
risch	$\frac{\sqrt{(b\,x^2+a)^2}\left(\frac{(ae+3bd)x^3}{8a^2}-\frac{(ae-5bd)x}{8ab}\right)}{(b\,x^2+a)^3}-\frac{\sqrt{(b\,x^2+a)^2}\,(ae+3bd)\ln\left(bx+\sqrt{-ab}\right)}{16(b\,x^2+a)\sqrt{-ab}\,b\,a^2}+\frac{\sqrt{(b\,x^2+a)^2}\,(ae+3bd)\ln\left(-bx+\sqrt{-ab}\right)}{16(b\,x^2+a)\sqrt{-ab}\,b\,a^2}$
default	$-\frac{\left(-\arctan\left(\frac{bx}{\sqrt{ab}}\right)a\,b^2e\,x^4-3\arctan\left(\frac{bx}{\sqrt{ab}}\right)b^3d\,x^4-\sqrt{ab}\,abe\,x^3-3\sqrt{ab}\,b^2d\,x^3-2\arctan\left(\frac{bx}{\sqrt{ab}}\right)a^2be\,x^2-6\arctan\left(\frac{bx}{\sqrt{ab}}\right)a\,b^2d\,x^2+\sqrt{a}\right.}{8\sqrt{ab}\,b\,a^2\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}$

input
```
int((e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)^2)^(1/2)/(b*x^2+a)^3*(1/8*(a*e+3*b*d)/a^2*x^3-1/8*(a*e-5*b*d)/a
/b*x)-1/16*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/(-a*b)^(1/2)*(a*e+3*b*d)/b/a^2*ln
(b*x+(-a*b)^(1/2))+1/16*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/(-a*b)^(1/2)*(a*e+3*
b*d)/b/a^2*ln(-b*x+(-a*b)^(1/2))
```

3.83.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.93

$$\int \frac{d+ex^2}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \left[\frac{2\left(3\,ab^3d+a^2b^2e\right)x^3-\left(\left(3\,b^3d+ab^2e\right)x^4+3\,a^2bd+a^3e+2\left(3\,ab^2d+a^2be\right)\right.}{16\left(a^3b^4x^4+2\,a^4b^3x^2+a^5\right.}\right.$$

input
```
integrate((e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(2*(3*a*b^3*d + a^2*b^2*e)*x^3 - ((3*b^3*d + a*b^2*e)*x^4 + 3*a^2*b*
d + a^3*e + 2*(3*a*b^2*d + a^2*b*e)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a
*b)*x - a)/(b*x^2 + a)) + 2*(5*a^2*b^2*d - a^3*b*e)*x)/(a^3*b^4*x^4 + 2*a^
4*b^3*x^2 + a^5*b^2), 1/8*((3*a*b^3*d + a^2*b^2*e)*x^3 + ((3*b^3*d + a*b^2
*e)*x^4 + 3*a^2*b*d + a^3*e + 2*(3*a*b^2*d + a^2*b*e)*x^2)*sqrt(a*b)*arcta
n(sqrt(a*b)*x/a) + (5*a^2*b^2*d - a^3*b*e)*x)/(a^3*b^4*x^4 + 2*a^4*b^3*x^2
 + a^5*b^2)]
```

3.83.6 Sympy [F]

$$\int \frac{d+ex^2}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{d+ex^2}{((a+bx^2)^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x**2+d)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Integral((d + e*x**2)/((a + b*x**2)**2)**(3/2), x)
```

3.83.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.79

$$\int \frac{d+ex^2}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{1}{8}\,d\left(\frac{3\,bx^3+5\,ax}{a^2b^2x^4+2\,a^3bx^2+a^4}+\frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2}\right) + \frac{1}{8}\,e\left(\frac{bx^3-ax}{ab^3x^4+2\,a^2b^2x^2+a^3b}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}ab}\right)$$

input
```
integrate((e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/8*d*((3*b*x^3 + 5*a*x)/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) + 3*arctan(b*x/
sqrt(a*b))/(sqrt(a*b)*a^2)) + 1/8*e*((b*x^3 - a*x)/(a*b^3*x^4 + 2*a^2*b^2*
x^2 + a^3*b) + arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b))
```

3.83.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.63

$$\int \frac{d+ex^2}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{(3\,bd+ae)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b\mathrm{sgn}\left(bx^2+a\right)}+\frac{3\,b^2dx^3+abex^3+5\,abdx-a^2ex}{8\,(bx^2+a)^2a^2b\mathrm{sgn}\left(bx^2+a\right)}$$

input
```
integrate((e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/8*(3*b*d + a*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b*sgn(b*x^2 + a)) +
 1/8*(3*b^2*d*x^3 + a*b*e*x^3 + 5*a*b*d*x - a^2*e*x)/((b*x^2 + a)^2*a^2*b*
sgn(b*x^2 + a))
```

3.83.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{e\,x^2+d}{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}}\,dx$$

input
```
int((d + e*x^2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

output
```
int((d + e*x^2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

3.84 $\int \frac{d+ex^2}{x(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

3.84.1 Optimal result

Integrand size = 33, antiderivative size = 161

$$\int \frac{d+ex^2}{x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx=\frac{d}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{bd-ae}{4ab\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{d(a+bx^2)\log(x)}{a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(a+bx^2)\log\left(a+bx^2\right)}{2a^3\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
1/2*d/a^2/((b*x^2+a)^2)^(1/2)+1/4*(-a*e+b*d)/a/b/(b*x^2+a)/((b*x^2+a)^2)^(
1/2)+d*(b*x^2+a)*ln(x)/a^3/((b*x^2+a)^2)^(1/2)-1/2*d*(b*x^2+a)*ln(b*x^2+a)
/a^3/((b*x^2+a)^2)^(1/2)
```

3.84.2 Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.57

$$\int \frac{d+ex^2}{x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx=\frac{a(3abd-a^2e+2b^2dx^2)+4bd(a+bx^2)^2\log(x)-2bd(a+bx^2)^2\log\left(a+ba\right.}{4a^3b\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

input
```
Integrate[(d + e*x^2)/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

output
```
(a*(3*a*b*d - a^2*e + 2*b^2*d*x^2) + 4*b*d*(a + b*x^2)^2*Log[x] - 2*b*d*(a
 + b*x^2)^2*Log[a + b*x^2])/(4*a^3*b*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

3.84.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.61, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1384, 27, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{x \left(a^2+2abx^2+b^2x^4\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^3 \left(a+bx^2\right) \int \frac{ex^2+d}{b^3 x \left(bx^2+a\right)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(a+bx^2\right) \int \frac{ex^2+d}{x \left(bx^2+a\right)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{354}$$

$$\frac{\left(a+bx^2\right) \int \frac{ex^2+d}{x^2 \left(bx^2+a\right)^3}dx^2}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{86}$$

$$\frac{\left(a+bx^2\right) \int \left(-\frac{bd}{a^3 \left(bx^2+a\right)}+\frac{d}{a^3 x^2}-\frac{bd}{a^2 \left(bx^2+a\right)^2}+\frac{ae-bd}{a \left(bx^2+a\right)^3}\right)dx^2}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(a+bx^2\right) \left(-\frac{d \log \left(a+bx^2\right)}{a^3}+\frac{d \log \left(x^2\right)}{a^3}+\frac{d}{a^2 \left(a+bx^2\right)}+\frac{bd-ae}{2ab \left(a+bx^2\right)^2}\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[(d + e*x^2)/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

output
```
((a + b*x^2)*((b*d - a*e)/(2*a*b*(a + b*x^2)^2) + d/(a^2*(a + b*x^2)) + (d
*Log[x^2])/a^3 - (d*Log[a + b*x^2])/a^3))/(2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4])
```

3.84.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.84.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.54

method	result
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{2}+a\right)\left(2db\left(b\,x^{2}+a\right)^{2}\ln\left(b\,x^{2}+a\right)-2db\left(b\,x^{2}+a\right)^{2}\ln\left(x^{2}\right)+a\left(-2b^{2}d\,x^{2}+e\,a^{2}-3dab\right)\right)}{4a^{3}b\left(b\,x^{2}+a\right)^{2}}$
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\left(\frac{bd\,x^{2}}{2a^{2}}-\frac{ae-3bd}{4ab}\right)}{\left(b\,x^{2}+a\right)^{3}}+\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,d\ln\left(x\right)}{\left(b\,x^{2}+a\right)a^{3}}-\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,d\ln\left(b\,x^{2}+a\right)}{2\left(b\,x^{2}+a\right)a^{3}}$
default	$\frac{(4\ln(x)b^{3}d\,x^{4}-2\ln\left(b\,x^{2}+a\right)b^{3}d\,x^{4}+8\ln(x)a\,b^{2}d\,x^{2}-4\ln\left(b\,x^{2}+a\right)a\,b^{2}d\,x^{2}+2b^{2}d\,x^{2}a+4\ln(x)a^{2}bd-2\ln\left(b\,x^{2}+a\right)a^{2}bd-a^{3}e+}{4b\,a^{3}\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}$

input
```
int((e*x^2+d)/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*csgn(b*x^2+a)*(2*d*b*(b*x^2+a)^2*ln(b*x^2+a)-2*d*b*(b*x^2+a)^2*ln(x^2
)+a*(-2*b^2*d*x^2+a^2*e-3*a*b*d))/a^3/b/(b*x^2+a)^2
```

3.84.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.74

$$\int \frac{d+ex^2}{x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{2\,ab^2dx^2+3\,a^2bd-a^3e-2\left(b^3dx^4+2\,ab^2dx^2+a^2bd\right)\log\left(bx^2+a\right)+4\left(b^3\right.}{4\left(a^3b^3x^4+2\,a^4b^2x^2+a^5b\right)}$$

input
```
integrate((e*x^2+d)/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/4*(2*a*b^2*d*x^2 + 3*a^2*b*d - a^3*e - 2*(b^3*d*x^4 + 2*a*b^2*d*x^2 + a^
2*b*d)*log(b*x^2 + a) + 4*(b^3*d*x^4 + 2*a*b^2*d*x^2 + a^2*b*d)*log(x))/(a
^3*b^3*x^4 + 2*a^4*b^2*x^2 + a^5*b)
```

3.84.6 Sympy [F]

$$\int \frac{d+ex^2}{x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \int \frac{d+ex^2}{x\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x**2+d)/x/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Integral((d + e*x**2)/(x*((a + b*x**2)**2)**(3/2)), x)
```

3.84.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.55

$$\int \frac{d+ex^2}{x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{1}{4}\,d\left(\frac{2\,bx^2+3\,a}{a^2b^2x^4+2\,a^3bx^2+a^4}-\frac{2\,\log\left(bx^2+a\right)}{a^3}+\frac{4\,\log\left(x\right)}{a^3}\right) - \frac{e}{4\,\left(b^3x^4+2\,ab^2x^2+a^2b\right)}$$

input
```
integrate((e*x^2+d)/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/4*d*((2*b*x^2 + 3*a)/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) - 2*log(b*x^2 + a
)/a^3 + 4*log(x)/a^3) - 1/4*e/(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)
```

3.84.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.66

$$\int \frac{d+ex^2}{x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{d\log\left(x^2\right)}{2\,a^3\mathrm{sgn}\left(bx^2+a\right)} - \frac{d\log\left(\left|bx^2+a\right|\right)}{2\,a^3\mathrm{sgn}\left(bx^2+a\right)} + \frac{3\,b^3dx^4+8\,ab^2dx^2+6\,a^2bd-a^3e}{4\,\left(bx^2+a\right)^2a^3b\mathrm{sgn}\left(bx^2+a\right)}$$

input
```
integrate((e*x^2+d)/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/2*d*log(x^2)/(a^3*sgn(b*x^2 + a)) - 1/2*d*log(abs(b*x^2 + a))/(a^3*sgn(b
*x^2 + a)) + 1/4*(3*b^3*d*x^4 + 8*a*b^2*d*x^2 + 6*a^2*b*d - a^3*e)/((b*x^2
 + a)^2*a^3*b*sgn(b*x^2 + a))
```

3.84.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \int \frac{e\,x^2+d}{x\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}\,dx$$

input
```
int((d + e*x^2)/(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
```

output
```
int((d + e*x^2)/(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

3.85 $\int \frac{d+ex^2}{x^2(a^2+2abx^2+b^2x^4)^{3/2}} dx$

3.85.1 Optimal result

Integrand size = 33, antiderivative size = 190

$$\int \frac{d+ex^2}{x^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx=-\frac{(7bd-3ae)x}{8a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{(bd-ae)x}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(a+bx^2)}{a^3x\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3(5bd-ae)\left(a+bx^2\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
-1/8*(-3*a*e+7*b*d)*x/a^3/((b*x^2+a)^2)^(1/2)-1/4*(-a*e+b*d)*x/a^2/(b*x^2+
a)/((b*x^2+a)^2)^(1/2)-d*(b*x^2+a)/a^3/x/((b*x^2+a)^2)^(1/2)-3/8*(-a*e+5*b
*d)*(b*x^2+a)*arctan(x*b^(1/2)/a^(1/2))/a^(7/2)/b^(1/2)/((b*x^2+a)^2)^(1/2
)
```

3.85.2 Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.65

$$\int \frac{d+ex^2}{x^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx=\frac{\sqrt{a}\sqrt{b}(-15b^2dx^4+a^2(-8d+5ex^2)+ab(-25dx^2+3ex^4))+3(-5bd+a}{8a^{7/2}\sqrt{b}x\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

input
```
Integrate[(d + e*x^2)/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

output
```
(Sqrt[a]*Sqrt[b]*(-15*b^2*d*x^4 + a^2*(-8*d + 5*e*x^2) + a*b*(-25*d*x^2 +
3*e*x^4)) + 3*(-5*b*d + a*e)*x*(a + b*x^2)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(8*a^(7/2)*Sqrt[b]*x*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

3.85.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.73, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1384, 27, 361, 25, 27, 361, 25, 359, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{x^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^3\left(a+bx^2\right)\int \frac{ex^2+d}{b^3x^2\left(bx^2+a\right)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(a+bx^2\right)\int \frac{ex^2+d}{x^2\left(bx^2+a\right)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{361}$$

$$\frac{\left(a+bx^2\right)\left(-\frac{1}{4}\int -\frac{4ad-3(bd-ae)x^2}{a^2x^2\left(bx^2+a\right)^2}dx-\frac{x(bd-ae)}{4a^2\left(a+bx^2\right)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\left(a+bx^2\right)\left(\frac{1}{4}\int \frac{4ad-3(bd-ae)x^2}{a^2x^2\left(bx^2+a\right)^2}dx-\frac{x(bd-ae)}{4a^2\left(a+bx^2\right)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(a+bx^2\right)\left(\frac{\int \frac{4ad-3(bd-ae)x^2}{x^2\left(bx^2+a\right)^2}dx}{4a^2}-\frac{x(bd-ae)}{4a^2\left(a+bx^2\right)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 361$$

$$\frac{(a+bx^2)\left(\frac{-\frac{1}{2}\int -\frac{8d-\left(\frac{7bd}{a}-3e\right)x^2}{x^2\left(bx^2+a\right)}dx-\frac{x(7bd-3ae)}{2a\left(a+bx^2\right)}}{4a^2}-\frac{x(bd-ae)}{4a^2(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 25$$

$$\frac{(a+bx^2)\left(\frac{\frac{1}{2}\int \frac{8d-\left(\frac{7bd}{a}-3e\right)x^2}{x^2\left(bx^2+a\right)}dx-\frac{x(7bd-3ae)}{2a\left(a+bx^2\right)}}{4a^2}-\frac{x(bd-ae)}{4a^2(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 359$$

$$\frac{(a+bx^2)\left(\frac{\frac{1}{2}\left(-\frac{3(5bd-ae)\int \frac{1}{bx^2+a}dx}{a}-\frac{8d}{ax}\right)-\frac{x(7bd-3ae)}{2a\left(a+bx^2\right)}}{4a^2}-\frac{x(bd-ae)}{4a^2(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 218$$

$$\frac{(a+bx^2)\left(\frac{\frac{1}{2}\left(-\frac{3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(5bd-ae)}{a^{3/2}\sqrt{b}}-\frac{8d}{ax}\right)-\frac{x(7bd-3ae)}{2a\left(a+bx^2\right)}}{4a^2}-\frac{x(bd-ae)}{4a^2(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[(d + e*x^2)/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

output
```
((a + b*x^2)*(-1/4*((b*d - a*e)*x)/(a^2*(a + b*x^2)^2) + (-1/2*((7*b*d - 3
*a*e)*x)/(a*(a + b*x^2)) + ((-8*d)/(a*x) - (3*(5*b*d - a*e)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b]))/2)/(4*a^2)))/Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4]
```

3.85. $\int \frac{d+ex^2}{x^2(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

3.85.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.85.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.22 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.93

method	result
risch	$\frac{\sqrt{(b\,x^2+a)^2}\left(\frac{3b(ae-5bd)x^4}{8a^3}+\frac{5(ae-5bd)x^2}{8a^2}-\frac{d}{a}\right)}{(b\,x^2+a)^3x}+\frac{3\sqrt{(b\,x^2+a)^2}\left(\sum_{\_R=\mathrm{RootOf}\left(a^7\_Z^2b+e^2a^2-10abde+25b^2d^2\right)}\_R\ln\left(\left(3\_R^2a^7b\right.\right.\right.}{16(b\,x^2+a)}$
default	$-\frac{\left(-3\arctan\left(\frac{bx}{\sqrt{ab}}\right)a\,b^2e\,x^5+15\arctan\left(\frac{bx}{\sqrt{ab}}\right)b^3d\,x^5-3\sqrt{ab}\,abe\,x^4+15\sqrt{ab}\,b^2d\,x^4-6\arctan\left(\frac{bx}{\sqrt{ab}}\right)a^2be\,x^3+30\arctan\left(\frac{bx}{\sqrt{ab}}\right)a\,b^2d\,x\right.}{8\sqrt{ab}\,x\,a^3\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}$

input
```
int((e*x^2+d)/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)^2)^(1/2)/(b*x^2+a)^3*(3/8*b*(a*e-5*b*d)/a^3*x^4+5/8/a^2*(a*e-5*
b*d)*x^2-d/a)/x+3/16*((b*x^2+a)^2)^(1/2)/(b*x^2+a)*sum(_R*ln((3*_R^2*a^7*b
+2*a^2*e^2-20*a*b*d*e+50*b^2*d^2)*x+(-a^5*e+5*a^4*b*d)*_R),_R=RootOf(_Z^2*
a^7*b+a^2*e^2-10*a*b*d*e+25*b^2*d^2))
```

3.85.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.76

$$\int \frac{d+ex^2}{x^2\,(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \left[-\frac{16\,a^3bd+6\,(5\,ab^3d-a^2b^2e)x^4+10\,(5\,a^2b^2d-a^3be)x^2-3\,((5\,b^3d-ab\right.}{16\,(a^4b^3x^5+}$$

$$-\frac{8\,a^3bd+3\,(5\,ab^3d-a^2b^2e)x^4+5\,(5\,a^2b^2d-a^3be)x^2+3\,((5\,b^3d-ab^2e)x^5+2\,(5\,ab^2d-a^2be)x^3+(5\,a^2bd}{8\,(a^4b^3x^5+2\,a^5b^2x^3+a^6bx)}$$

input
```
integrate((e*x^2+d)/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas
")
```

output
```
[-1/16*(16*a^3*b*d + 6*(5*a*b^3*d - a^2*b^2*e)*x^4 + 10*(5*a^2*b^2*d - a^3
*b*e)*x^2 - 3*((5*b^3*d - a*b^2*e)*x^5 + 2*(5*a*b^2*d - a^2*b*e)*x^3 + (5*
a^2*b*d - a^3*e)*x)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a
)))/(a^4*b^3*x^5 + 2*a^5*b^2*x^3 + a^6*b*x), -1/8*(8*a^3*b*d + 3*(5*a*b^3*
d - a^2*b^2*e)*x^4 + 5*(5*a^2*b^2*d - a^3*b*e)*x^2 + 3*((5*b^3*d - a*b^2*e
)*x^5 + 2*(5*a*b^2*d - a^2*b*e)*x^3 + (5*a^2*b*d - a^3*e)*x)*sqrt(a*b)*arc
tan(sqrt(a*b)*x/a))/(a^4*b^3*x^5 + 2*a^5*b^2*x^3 + a^6*b*x)]
```

3.85.6 Sympy [F]

$$\int \frac{d+ex^2}{x^2 \left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \int \frac{d+ex^2}{x^2 \left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x**2+d)/x**2/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Integral((d + e*x**2)/(x**2*((a + b*x**2)**2)**(3/2)), x)
```

3.85.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.71

$$\int \frac{d+ex^2}{x^2 \left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = -\frac{1}{8}\,d\left(\frac{15\,b^2x^4+25\,abx^2+8\,a^2}{a^3b^2x^5+2\,a^4bx^3+a^5x}+\frac{15\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^3}\right) + \frac{1}{8}\,e\left(\frac{3\,bx^3+5\,ax}{a^2b^2x^4+2\,a^3bx^2+a^4}+\frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2}\right)$$

input
```
integrate((e*x^2+d)/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima
")
```

output
```
-1/8*d*((15*b^2*x^4 + 25*a*b*x^2 + 8*a^2)/(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5
*x) + 15*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)) + 1/8*e*((3*b*x^3 + 5*a*
x)/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) + 3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*
a^2))
```

3.85.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.59

$$\int \frac{d+ex^2}{x^2 (a^2+2abx^2+b^2x^4)^{3/2}}\,dx = -\frac{3\,(5\,bd-ae)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^3\mathrm{sgn}\,(bx^2+a)} - \frac{d}{a^3x\mathrm{sgn}\,(bx^2+a)} - \frac{7\,b^2dx^3-3\,abex^3+9\,abdx-5\,a^2ex}{8\,(bx^2+a)^2a^3\mathrm{sgn}\,(bx^2+a)}$$

input
```
integrate((e*x^2+d)/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
-3/8*(5*b*d - a*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3*sgn(b*x^2 + a)) -
d/(a^3*x*sgn(b*x^2 + a)) - 1/8*(7*b^2*d*x^3 - 3*a*b*e*x^3 + 9*a*b*d*x - 5*
a^2*e*x)/((b*x^2 + a)^2*a^3*sgn(b*x^2 + a))
```

3.85.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{x^2 (a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{e\,x^2+d}{x^2\,(a^2+2\,a\,b\,x^2+b^2\,x^4)^{3/2}}\,dx$$

input
```
int((d + e*x^2)/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
```

output
```
int((d + e*x^2)/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

3.86 $\int \frac{d+ex^2}{x^3\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

3.86.1 Optimal result

Integrand size = 33, antiderivative size = 223

$$\int \frac{d+ex^2}{x^3\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = -\frac{2bd-ae}{2a^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{bd-ae}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(a+bx^2)}{2a^3x^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(3bd-ae)\left(a+bx^2\right)\log(x)}{a^4\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(3bd-ae)\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a^4\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
1/2*(a*e-2*b*d)/a^3/((b*x^2+a)^2)^(1/2)+1/4*(a*e-b*d)/a^2/(b*x^2+a)/((b*x^
2+a)^2)^(1/2)-1/2*d*(b*x^2+a)/a^3/x^2/((b*x^2+a)^2)^(1/2)-(-a*e+3*b*d)*(b*
x^2+a)*ln(x)/a^4/((b*x^2+a)^2)^(1/2)+1/2*(-a*e+3*b*d)*(b*x^2+a)*ln(b*x^2+a
)/a^4/((b*x^2+a)^2)^(1/2)
```

3.86.2 Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.58

$$\int \frac{d+ex^2}{x^3\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{a(-6b^2dx^4+a^2(-2d+3ex^2)+ab(-9dx^2+2ex^4))+4(-3bd+ae)x^2(a}{4a^4x^2\left(a+bx^2\right)\sqrt{(a+b}}$$

input
```
Integrate[(d + e*x^2)/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

output
```
(a*(-6*b^2*d*x^4 + a^2*(-2*d + 3*e*x^2) + a*b*(-9*d*x^2 + 2*e*x^4)) + 4*(-
3*b*d + a*e)*x^2*(a + b*x^2)^2*Log[x] + 2*(3*b*d - a*e)*x^2*(a + b*x^2)^2*
Log[a + b*x^2])/(4*a^4*x^2*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

3.86.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.58, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1384, 27, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{x^3 (a^2+2abx^2+b^2x^4)^{3/2}} \, dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^3 (a+bx^2) \int \frac{ex^2+d}{b^3x^3(bx^2+a)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^2) \int \frac{ex^2+d}{x^3(bx^2+a)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{354}$$

$$\frac{(a+bx^2) \int \frac{ex^2+d}{x^4(bx^2+a)^3}dx^2}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{86}$$

$$\frac{(a+bx^2) \int \left(\frac{d}{a^3x^4}-\frac{b(ae-3bd)}{a^4(bx^2+a)}+\frac{ae-3bd}{a^4x^2}-\frac{b(ae-2bd)}{a^3(bx^2+a)^2}-\frac{b(ae-bd)}{a^2(bx^2+a)^3}\right)dx^2}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx^2) \left(-\frac{\log(x^2)(3bd-ae)}{a^4}+\frac{(3bd-ae)\log(a+bx^2)}{a^4}-\frac{2bd-ae}{a^3(a+bx^2)}-\frac{d}{a^3x^2}-\frac{bd-ae}{2a^2(a+bx^2)^2}\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[(d + e*x^2)/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

output
```
((a + b*x^2)*(-(d/(a^3*x^2)) - (b*d - a*e)/(2*a^2*(a + b*x^2)^2) - (2*b*d
- a*e)/(a^3*(a + b*x^2)) - ((3*b*d - a*e)*Log[x^2])/a^4 + ((3*b*d - a*e)*L
og[a + b*x^2])/a^4))/(2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

3.86.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.86.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.54

method	result
pseudoelliptic	$-\frac{\operatorname{csgn}(b\,x^2+a)\left(x^2\left(b\,x^2+a\right)^2(ae-3bd)\ln(b\,x^2+a)-x^2\left(b\,x^2+a\right)^2(ae-3bd)\ln(x^2)+a\left(\left(-abe+3b^2d\right)x^4-\frac{3a(ae-3bd)x^2}{2}+d\,a\right.\right.}{2\left(b\,x^2+a\right)^2x^2a^4}$
risch	$\frac{\sqrt{\left(b\,x^2+a\right)^2}\left(\frac{b(ae-3bd)x^4}{2a^3}+\frac{3(ae-3bd)x^2}{4a^2}-\frac{d}{2a}\right)}{\left(b\,x^2+a\right)^3x^2}+\frac{\sqrt{\left(b\,x^2+a\right)^2}\,(ae-3bd)\ln(x)}{\left(b\,x^2+a\right)a^4}-\frac{\sqrt{\left(b\,x^2+a\right)^2}\,(ae-3bd)\ln\left(b\,x^2+a\right)}{2\left(b\,x^2+a\right)a^4}$
default	$\frac{(4\ln(x)a\,b^2e\,x^6-12\ln(x)b^3d\,x^6-2\ln(b\,x^2+a)a\,b^2e\,x^6+6\ln(b\,x^2+a)b^3d\,x^6+8\ln(x)a^2be\,x^4-24\ln(x)a\,b^2d\,x^4-4\ln(b\,x^2+a)a}{}$

input
```
int((e*x^2+d)/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*csgn(b*x^2+a)*(x^2*(b*x^2+a)^2*(a*e-3*b*d)*ln(b*x^2+a)-x^2*(b*x^2+a)^
2*(a*e-3*b*d)*ln(x^2)+a*((-a*b*e+3*b^2*d)*x^4-3/2*a*(a*e-3*b*d)*x^2+d*a^2)
)/(b*x^2+a)^2/x^2/a^4
```

3.86.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.92

$$\int \frac{d+ex^2}{x^3\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx =$$
$$-\frac{2\,(3\,ab^2d-a^2be)x^4+2\,a^3d+3\,(3\,a^2bd-a^3e)x^2-2\,((3\,b^3d-ab^2e)x^6+2\,(3\,ab^2d-a^2be)x^4+(3\,a^2bd-a}{4\,(a^4b^2x^6+2\,a^5bx^4+}$$

input
```
integrate((e*x^2+d)/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas
")
```

output
```
-1/4*(2*(3*a*b^2*d - a^2*b*e)*x^4 + 2*a^3*d + 3*(3*a^2*b*d - a^3*e)*x^2 -
2*((3*b^3*d - a*b^2*e)*x^6 + 2*(3*a*b^2*d - a^2*b*e)*x^4 + (3*a^2*b*d - a^
3*e)*x^2)*log(b*x^2 + a) + 4*((3*b^3*d - a*b^2*e)*x^6 + 2*(3*a*b^2*d - a^2
*b*e)*x^4 + (3*a^2*b*d - a^3*e)*x^2)*log(x))/(a^4*b^2*x^6 + 2*a^5*b*x^4 +
a^6*x^2)
```

3.86.6 Sympy [F]

$$\int \frac{d+ex^2}{x^3(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{d+ex^2}{x^3\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x**2+d)/x**3/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Integral((d + e*x**2)/(x**3*((a + b*x**2)**2)**(3/2)), x)
```

3.86.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.62

$$\int \frac{d+ex^2}{x^3(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = -\frac{1}{4}\,d\left(\frac{6\,b^2x^4+9\,abx^2+2\,a^2}{a^3b^2x^6+2\,a^4bx^4+a^5x^2}-\frac{6\,b\log\left(bx^2+a\right)}{a^4}+\frac{12\,b\log\left(x\right)}{a^4}\right)+\frac{1}{4}\,e\left(\frac{2\,bx^2+3\,a}{a^2b^2x^4+2\,a^3bx^2+a^4}-\frac{2\,\log\left(bx^2+a\right)}{a^3}+\frac{4\,\log\left(x\right)}{a^3}\right)$$

input
```
integrate((e*x^2+d)/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima
")
```

output
```
-1/4*d*((6*b^2*x^4 + 9*a*b*x^2 + 2*a^2)/(a^3*b^2*x^6 + 2*a^4*b*x^4 + a^5*x
^2) - 6*b*log(b*x^2 + a)/a^4 + 12*b*log(x)/a^4) + 1/4*e*((2*b*x^2 + 3*a)/(
a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) - 2*log(b*x^2 + a)/a^3 + 4*log(x)/a^3)
```

3.86.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.82

$$\int \frac{d+ex^2}{x^3(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = -\frac{(3\,bd-ae)\log\left(x^2\right)}{2\,a^4\mathrm{sgn}\left(bx^2+a\right)}+\frac{(3\,b^2d-abe)\log\left(|bx^2+a|\right)}{2\,a^4b\mathrm{sgn}\left(bx^2+a\right)}-\frac{9\,b^3dx^4-3\,ab^2ex^4+22\,ab^2dx^2-8\,a^2bex^2+14\,a^2bd-6\,a^3e}{4\,(bx^2+a)^2a^4\mathrm{sgn}\left(bx^2+a\right)}+\frac{3\,bdx^2-aex^2-ad}{2\,a^4x^2\mathrm{sgn}\left(bx^2+a\right)}$$

input
```
integrate((e*x^2+d)/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
-1/2*(3*b*d - a*e)*log(x^2)/(a^4*sgn(b*x^2 + a)) + 1/2*(3*b^2*d - a*b*e)*l
og(abs(b*x^2 + a))/(a^4*b*sgn(b*x^2 + a)) - 1/4*(9*b^3*d*x^4 - 3*a*b^2*e*x
^4 + 22*a*b^2*d*x^2 - 8*a^2*b*e*x^2 + 14*a^2*b*d - 6*a^3*e)/((b*x^2 + a)^2
*a^4*sgn(b*x^2 + a)) + 1/2*(3*b*d*x^2 - a*e*x^2 - a*d)/(a^4*x^2*sgn(b*x^2
+ a))
```

3.86.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{x^3 (a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{e\,x^2+d}{x^3\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}\,dx$$

input
```
int((d + e*x^2)/(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
```

output
```
int((d + e*x^2)/(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

3.87 $\int (fx)^m (d+ex^2) \left(a^2+2abx^2+b^2x^4\right)^{5/2} \, dx$

3.87.1 Optimal result

Integrand size = 35, antiderivative size = 400

$$\begin{aligned}
\int (fx)^m (d+ex^2) \left(a^2+2abx^2+b^2x^4\right)^{5/2} \, dx &= \frac{a^5 d (fx)^{1+m} \sqrt{a^2+2abx^2+b^2x^4}}{f(1+m)(a+bx^2)} \\
&+ \frac{a^4(5bd+ae)(fx)^{3+m} \sqrt{a^2+2abx^2+b^2x^4}}{f^3(3+m)(a+bx^2)} \\
&+ \frac{5a^3b(2bd+ae)(fx)^{5+m} \sqrt{a^2+2abx^2+b^2x^4}}{f^5(5+m)(a+bx^2)} \\
&+ \frac{10a^2b^2(bd+ae)(fx)^{7+m} \sqrt{a^2+2abx^2+b^2x^4}}{f^7(7+m)(a+bx^2)} \\
&+ \frac{5ab^3(bd+2ae)(fx)^{9+m} \sqrt{a^2+2abx^2+b^2x^4}}{f^9(9+m)(a+bx^2)} \\
&+ \frac{b^4(bd+5ae)(fx)^{11+m} \sqrt{a^2+2abx^2+b^2x^4}}{f^{11}(11+m)(a+bx^2)} + \frac{b^5 e (fx)^{13+m} \sqrt{a^2+2abx^2+b^2x^4}}{f^{13}(13+m)(a+bx^2)}
\end{aligned}$$

output
```
a^5*d*(f*x)^(1+m)*((b*x^2+a)^2)^(1/2)/f/(1+m)/(b*x^2+a)+a^4*(a*e+5*b*d)*(f
*x)^(3+m)*((b*x^2+a)^2)^(1/2)/f^3/(3+m)/(b*x^2+a)+5*a^3*b*(a*e+2*b*d)*(f*x
)^(5+m)*((b*x^2+a)^2)^(1/2)/f^5/(5+m)/(b*x^2+a)+10*a^2*b^2*(a*e+b*d)*(f*x)
^(7+m)*((b*x^2+a)^2)^(1/2)/f^7/(7+m)/(b*x^2+a)+5*a*b^3*(2*a*e+b*d)*(f*x)^(
9+m)*((b*x^2+a)^2)^(1/2)/f^9/(9+m)/(b*x^2+a)+b^4*(5*a*e+b*d)*(f*x)^(11+m)*
((b*x^2+a)^2)^(1/2)/f^11/(11+m)/(b*x^2+a)+b^5*e*(f*x)^(13+m)*((b*x^2+a)^2)
^(1/2)/f^13/(13+m)/(b*x^2+a)
```

3.87.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.40

$$\int (fx)^m (d+ex^2) (a^2+2abx^2+b^2x^4)^{5/2} dx = \frac{x(fx)^m\sqrt{(a+bx^2)^2}\left(\frac{a^5d}{1+m}+\frac{a^4(5bd+ae)x^2}{3+m}+\frac{5a^3b(2bd+ae)x^4}{5+m}+\frac{10a^2b^2(bd+ae)x^6}{7+m}+\frac{5ab^3(bd+2ae)x^8}{9+m}+\frac{b^4(\ldots}{}\right.}{a+bx^2}$$

input
```
Integrate[(f*x)^m*(d + e*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(x*(f*x)^m*Sqrt[(a + b*x^2)^2]*((a^5*d)/(1 + m) + (a^4*(5*b*d + a*e)*x^2)/
(3 + m) + (5*a^3*b*(2*b*d + a*e)*x^4)/(5 + m) + (10*a^2*b^2*(b*d + a*e)*x^
6)/(7 + m) + (5*a*b^3*(b*d + 2*a*e)*x^8)/(9 + m) + (b^4*(b*d + 5*a*e)*x^10
)/(11 + m) + (b^5*e*x^12)/(13 + m)))/(a + b*x^2)
```

3.87.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.54, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1384, 27, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^2+2abx^2+b^2x^4)^{5/2} (d+ex^2) (fx)^m dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int b^5(fx)^m (bx^2+a)^5 (ex^2+d) dx}{b^5 (a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int (fx)^m (bx^2+a)^5 (ex^2+d) dx}{a+bx^2}$$

$$\Big\downarrow \text{355}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(a^5d(fx)^m+\frac{a^4(5bd+ae)(fx)^{m+2}}{f^2}+\frac{5a^3b(2bd+ae)(fx)^{m+4}}{f^4}+\frac{10a^2b^2(bd+ae)(fx)^{m+6}}{f^6}+\frac{5ab^3(bd+2ae)(fx\ldots}{f^8}\right.}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{a^5d(fx)^{m+1}}{f(m+1)}+\frac{a^4(fx)^{m+3}(ae+5bd)}{f^3(m+3)}+\frac{5a^3b(fx)^{m+5}(ae+2bd)}{f^5(m+5)}+\frac{10a^2b^2(fx)^{m+7}(ae+bd)}{f^7(m+7)}+\frac{b^4(fx)^{m+11}(5ae+b\cdots}{f^{11}(m+11)}\right.}{a+bx^2}$$

input
```
Int[(f*x)^m*(d + e*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((a^5*d*(f*x)^(1 + m))/(f*(1 + m)) + (a^4
*(5*b*d + a*e)*(f*x)^(3 + m))/(f^3*(3 + m)) + (5*a^3*b*(2*b*d + a*e)*(f*x)
^(5 + m))/(f^5*(5 + m)) + (10*a^2*b^2*(b*d + a*e)*(f*x)^(7 + m))/(f^7*(7 +
 m)) + (5*a*b^3*(b*d + 2*a*e)*(f*x)^(9 + m))/(f^9*(9 + m)) + (b^4*(b*d + 5
*a*e)*(f*x)^(11 + m))/(f^11*(11 + m)) + (b^5*e*(f*x)^(13 + m))/(f^13*(13 +
 m))))/(a + b*x^2)
```

3.87.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.87.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1098 vs. 2(323) = 646.

Time = 0.12 (sec) , antiderivative size = 1099, normalized size of antiderivative = 2.75

method	result	size
gosper	Expression too large to display	1099
risch	Expression too large to display	1099

input
```
int((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOS
E)
```

output
```
x*(b^5*e*m^6*x^12+36*b^5*e*m^5*x^12+5*a*b^4*e*m^6*x^10+b^5*d*m^6*x^10+505*
b^5*e*m^4*x^12+190*a*b^4*e*m^5*x^10+38*b^5*d*m^5*x^10+3480*b^5*e*m^3*x^12+
10*a^2*b^3*e*m^6*x^8+5*a*b^4*d*m^6*x^8+2775*a*b^4*e*m^4*x^10+555*b^5*d*m^4
*x^10+12139*b^5*e*m^2*x^12+400*a^2*b^3*e*m^5*x^8+200*a*b^4*d*m^5*x^8+19700
*a*b^4*e*m^3*x^10+3940*b^5*d*m^3*x^10+19524*b^5*e*m*x^12+10*a^3*b^2*e*m^6*
x^6+10*a^2*b^3*d*m^6*x^6+6130*a^2*b^3*e*m^4*x^8+3065*a*b^4*d*m^4*x^8+70195
*a*b^4*e*m^2*x^10+14039*b^5*d*m^2*x^10+10395*b^5*e*x^12+420*a^3*b^2*e*m^5*
x^6+420*a^2*b^3*d*m^5*x^6+45280*a^2*b^3*e*m^3*x^8+22640*a*b^4*d*m^3*x^8+11
4510*a*b^4*e*m*x^10+22902*b^5*d*m*x^10+5*a^4*b*e*m^6*x^4+10*a^3*b^2*d*m^6*
x^4+6790*a^3*b^2*e*m^4*x^6+6790*a^2*b^3*d*m^4*x^6+166270*a^2*b^3*e*m^2*x^8
+83135*a*b^4*d*m^2*x^8+61425*a*b^4*e*x^10+12285*b^5*d*x^10+220*a^4*b*e*m^5
*x^4+440*a^3*b^2*d*m^5*x^4+52920*a^3*b^2*e*m^3*x^6+52920*a^2*b^3*d*m^3*x^6
+276880*a^2*b^3*e*m*x^8+138440*a*b^4*d*m*x^8+a^5*e*m^6*x^2+5*a^4*b*d*m^6*x
^2+3765*a^4*b*e*m^4*x^4+7530*a^3*b^2*d*m^4*x^4+203350*a^3*b^2*e*m^2*x^6+20
3350*a^2*b^3*d*m^2*x^6+150150*a^2*b^3*e*x^8+75075*a*b^4*d*x^8+46*a^5*e*m^5
*x^2+230*a^4*b*d*m^5*x^2+31400*a^4*b*e*m^3*x^4+62800*a^3*b^2*d*m^3*x^4+349
860*a^3*b^2*e*m*x^6+349860*a^2*b^3*d*m*x^6+a^5*d*m^6+835*a^5*e*m^4*x^2+417
5*a^4*b*d*m^4*x^2+129895*a^4*b*e*m^2*x^4+259790*a^3*b^2*d*m^2*x^4+193050*a
^3*b^2*e*x^6+193050*a^2*b^3*d*x^6+48*a^5*d*m^5+7540*a^5*e*m^3*x^2+37700*a^
4*b*d*m^3*x^2+237180*a^4*b*e*m*x^4+474360*a^3*b^2*d*m*x^4+925*a^5*d*m^4...
```

3.87.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 853 vs. $2(323) = 646$.

Time = 0.28 (sec) , antiderivative size = 853, normalized size of antiderivative = 2.13

$$\int (fx)^m (d+ex^2) (a^2+2abx^2+b^2x^4)^{5/2} dx = \frac{((b^5em^6+36b^5em^5+505b^5em^4+3480b^5em^3+12139b^5em^2+19524b^5em+10395b^5e)x^{1\cdots}}{}$$

input
```
integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fr
icas")
```

output
```
((b^5*e*m^6 + 36*b^5*e*m^5 + 505*b^5*e*m^4 + 3480*b^5*e*m^3 + 12139*b^5*e*
m^2 + 19524*b^5*e*m + 10395*b^5*e)*x^13 + ((b^5*d + 5*a*b^4*e)*m^6 + 12285
*b^5*d + 61425*a*b^4*e + 38*(b^5*d + 5*a*b^4*e)*m^5 + 555*(b^5*d + 5*a*b^4
*e)*m^4 + 3940*(b^5*d + 5*a*b^4*e)*m^3 + 14039*(b^5*d + 5*a*b^4*e)*m^2 + 2
2902*(b^5*d + 5*a*b^4*e)*m)*x^11 + 5*((a*b^4*d + 2*a^2*b^3*e)*m^6 + 15015*
a*b^4*d + 30030*a^2*b^3*e + 40*(a*b^4*d + 2*a^2*b^3*e)*m^5 + 613*(a*b^4*d
+ 2*a^2*b^3*e)*m^4 + 4528*(a*b^4*d + 2*a^2*b^3*e)*m^3 + 16627*(a*b^4*d + 2
*a^2*b^3*e)*m^2 + 27688*(a*b^4*d + 2*a^2*b^3*e)*m)*x^9 + 10*((a^2*b^3*d +
a^3*b^2*e)*m^6 + 19305*a^2*b^3*d + 19305*a^3*b^2*e + 42*(a^2*b^3*d + a^3*b
^2*e)*m^5 + 679*(a^2*b^3*d + a^3*b^2*e)*m^4 + 5292*(a^2*b^3*d + a^3*b^2*e)
*m^3 + 20335*(a^2*b^3*d + a^3*b^2*e)*m^2 + 34986*(a^2*b^3*d + a^3*b^2*e)*m
)*x^7 + 5*((2*a^3*b^2*d + a^4*b*e)*m^6 + 54054*a^3*b^2*d + 27027*a^4*b*e +
 44*(2*a^3*b^2*d + a^4*b*e)*m^5 + 753*(2*a^3*b^2*d + a^4*b*e)*m^4 + 6280*(
2*a^3*b^2*d + a^4*b*e)*m^3 + 25979*(2*a^3*b^2*d + a^4*b*e)*m^2 + 47436*(2*
a^3*b^2*d + a^4*b*e)*m)*x^5 + ((5*a^4*b*d + a^5*e)*m^6 + 225225*a^4*b*d +
45045*a^5*e + 46*(5*a^4*b*d + a^5*e)*m^5 + 835*(5*a^4*b*d + a^5*e)*m^4 + 7
540*(5*a^4*b*d + a^5*e)*m^3 + 34759*(5*a^4*b*d + a^5*e)*m^2 + 73054*(5*a^4
*b*d + a^5*e)*m)*x^3 + (a^5*d*m^6 + 48*a^5*d*m^5 + 925*a^5*d*m^4 + 9120*a^
5*d*m^3 + 48259*a^5*d*m^2 + 129072*a^5*d*m + 135135*a^5*d)*x)*(f*x)^m/(m^7
 + 49*m^6 + 973*m^5 + 10045*m^4 + 57379*m^3 + 177331*m^2 + 264207*m + 1...
```

3.87.6 Sympy [F]

$$\int (fx)^m (d+ex^2) (a^2+2abx^2+b^2x^4)^{5/2} \, dx = \int (fx)^m (d+ex^2) \left((a+bx^2)^2\right)^{\frac{5}{2}} \, dx$$

input
```
integrate((f*x)**m*(e*x**2+d)*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral((f*x)**m*(d + e*x**2)*((a + b*x**2)**2)**(5/2), x)
```

3.87.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.23

$$\int (fx)^m (d+ex^2) (a^2+2abx^2 +b^2x^4)^{5/2} \, dx = \frac{((m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945)b^5f^mx^{11}+5\,(m^5+27\,m^4+262\,m^3+112}{} + \frac{((m^5+35\,m^4+470\,m^3+3010\,m^2+9129\,m+10395)b^5f^mx^{13}+5\,(m^5+37\,m^4+518\,m^3+3422\,m^2+10}{}$$

input
```
integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="ma
xima")
```

output
```
((m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)*b^5*f^m*x^11 + 5*(m^5 +
 27*m^4 + 262*m^3 + 1122*m^2 + 2041*m + 1155)*a*b^4*f^m*x^9 + 10*(m^5 + 29
*m^4 + 302*m^3 + 1366*m^2 + 2577*m + 1485)*a^2*b^3*f^m*x^7 + 10*(m^5 + 31*
m^4 + 350*m^3 + 1730*m^2 + 3489*m + 2079)*a^3*b^2*f^m*x^5 + 5*(m^5 + 33*m^
4 + 406*m^3 + 2262*m^2 + 5353*m + 3465)*a^4*b*f^m*x^3 + (m^5 + 35*m^4 + 47
0*m^3 + 3010*m^2 + 9129*m + 10395)*a^5*f^m*x)*d*x^m/(m^6 + 36*m^5 + 505*m^
4 + 3480*m^3 + 12139*m^2 + 19524*m + 10395) + ((m^5 + 35*m^4 + 470*m^3 + 3
010*m^2 + 9129*m + 10395)*b^5*f^m*x^13 + 5*(m^5 + 37*m^4 + 518*m^3 + 3422*
m^2 + 10617*m + 12285)*a*b^4*f^m*x^11 + 10*(m^5 + 39*m^4 + 574*m^3 + 3954*
m^2 + 12673*m + 15015)*a^2*b^3*f^m*x^9 + 10*(m^5 + 41*m^4 + 638*m^3 + 4654
*m^2 + 15681*m + 19305)*a^3*b^2*f^m*x^7 + 5*(m^5 + 43*m^4 + 710*m^3 + 5570
*m^2 + 20409*m + 27027)*a^4*b*f^m*x^5 + (m^5 + 45*m^4 + 790*m^3 + 6750*m^2
 + 28009*m + 45045)*a^5*f^m*x^3)*e*x^m/(m^6 + 48*m^5 + 925*m^4 + 9120*m^3
+ 48259*m^2 + 129072*m + 135135)
```

3.87.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2171 vs. $2(323) = 646$.

Time = 0.36 (sec) , antiderivative size = 2171, normalized size of antiderivative = 5.43

$$\int (fx)^m (d + ex^2) (a^2 + 2abx^2 + b^2x^4)^{5/2} dx = \text{Too large to display}$$

input
```
integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="gi
ac")
```

output
```
((f*x)^m*b^5*e*m^6*x^13*sgn(b*x^2 + a) + 36*(f*x)^m*b^5*e*m^5*x^13*sgn(b*x
^2 + a) + (f*x)^m*b^5*d*m^6*x^11*sgn(b*x^2 + a) + 5*(f*x)^m*a*b^4*e*m^6*x^
11*sgn(b*x^2 + a) + 505*(f*x)^m*b^5*e*m^4*x^13*sgn(b*x^2 + a) + 38*(f*x)^m
*b^5*d*m^5*x^11*sgn(b*x^2 + a) + 190*(f*x)^m*a*b^4*e*m^5*x^11*sgn(b*x^2 +
a) + 3480*(f*x)^m*b^5*e*m^3*x^13*sgn(b*x^2 + a) + 5*(f*x)^m*a*b^4*d*m^6*x^
9*sgn(b*x^2 + a) + 10*(f*x)^m*a^2*b^3*e*m^6*x^9*sgn(b*x^2 + a) + 555*(f*x)
^m*b^5*d*m^4*x^11*sgn(b*x^2 + a) + 2775*(f*x)^m*a*b^4*e*m^4*x^11*sgn(b*x^2
 + a) + 12139*(f*x)^m*b^5*e*m^2*x^13*sgn(b*x^2 + a) + 200*(f*x)^m*a*b^4*d*
m^5*x^9*sgn(b*x^2 + a) + 400*(f*x)^m*a^2*b^3*e*m^5*x^9*sgn(b*x^2 + a) + 39
40*(f*x)^m*b^5*d*m^3*x^11*sgn(b*x^2 + a) + 19700*(f*x)^m*a*b^4*e*m^3*x^11*
sgn(b*x^2 + a) + 19524*(f*x)^m*b^5*e*m*x^13*sgn(b*x^2 + a) + 10*(f*x)^m*a^
2*b^3*d*m^6*x^7*sgn(b*x^2 + a) + 10*(f*x)^m*a^3*b^2*e*m^6*x^7*sgn(b*x^2 +
a) + 3065*(f*x)^m*a*b^4*d*m^4*x^9*sgn(b*x^2 + a) + 6130*(f*x)^m*a^2*b^3*e*
m^4*x^9*sgn(b*x^2 + a) + 14039*(f*x)^m*b^5*d*m^2*x^11*sgn(b*x^2 + a) + 701
95*(f*x)^m*a*b^4*e*m^2*x^11*sgn(b*x^2 + a) + 10395*(f*x)^m*b^5*e*x^13*sgn(
b*x^2 + a) + 420*(f*x)^m*a^2*b^3*d*m^5*x^7*sgn(b*x^2 + a) + 420*(f*x)^m*a^
3*b^2*e*m^5*x^7*sgn(b*x^2 + a) + 22640*(f*x)^m*a*b^4*d*m^3*x^9*sgn(b*x^2 +
 a) + 45280*(f*x)^m*a^2*b^3*e*m^3*x^9*sgn(b*x^2 + a) + 22902*(f*x)^m*b^5*d
*m*x^11*sgn(b*x^2 + a) + 114510*(f*x)^m*a*b^4*e*m*x^11*sgn(b*x^2 + a) + 10
*(f*x)^m*a^3*b^2*d*m^6*x^5*sgn(b*x^2 + a) + 5*(f*x)^m*a^4*b*e*m^6*x^5*s...
```

3.87.9 Mupad [F(-1)]

Timed out.

$$\int (fx)^m (d+ex^2) (a^2+2abx^2+b^2x^4)^{5/2} dx = \int (f\,x)^m\,(e\,x^2+d)\,(a^2+2\,a\,b\,x^2+b^2\,x^4)^{5/2}\,dx$$

input
```
int((f*x)^m*(d + e*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

output
```
int((f*x)^m*(d + e*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

3.87. $\int (fx)^m \left(d+ex^2\right) \left(a^2+2abx^2+b^2x^4\right)^{5/2} \, dx$

3.88 $\int (fx)^m (d+ex^2) \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx$

3.88.1 Optimal result

Integrand size = 35, antiderivative size = 276

$$\int (fx)^m (d+ex^2) \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx = \frac{a^3d(fx)^{1+m}\sqrt{a^2+2abx^2+b^2x^4}}{f(1+m)(a+bx^2)} + \frac{a^2(3bd+ae)(fx)^{3+m}\sqrt{a^2+2abx^2+b^2x^4}}{f^3(3+m)(a+bx^2)} + \frac{3ab(bd+ae)(fx)^{5+m}\sqrt{a^2+2abx^2+b^2x^4}}{f^5(5+m)(a+bx^2)} + \frac{b^2(bd+3ae)(fx)^{7+m}\sqrt{a^2+2abx^2+b^2x^4}}{f^7(7+m)(a+bx^2)} + \frac{b^3e(fx)^{9+m}\sqrt{a^2+2abx^2+b^2x^4}}{f^9(9+m)(a+bx^2)}$$

output
```
a^3*d*(f*x)^(1+m)*((b*x^2+a)^2)^(1/2)/f/(1+m)/(b*x^2+a)+a^2*(a*e+3*b*d)*(f
*x)^(3+m)*((b*x^2+a)^2)^(1/2)/f^3/(3+m)/(b*x^2+a)+3*a*b*(a*e+b*d)*(f*x)^(5
+m)*((b*x^2+a)^2)^(1/2)/f^5/(5+m)/(b*x^2+a)+b^2*(3*a*e+b*d)*(f*x)^(7+m)*((
b*x^2+a)^2)^(1/2)/f^7/(7+m)/(b*x^2+a)+b^3*e*(f*x)^(9+m)*((b*x^2+a)^2)^(1/2
)/f^9/(9+m)/(b*x^2+a)
```

3.88.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.41

$$\int (fx)^m \left(d+ex^2\right) \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx = \frac{x(fx)^m \left(\left(a+bx^2\right)^2\right)^{3/2} \left(\frac{a^3d}{1+m}+\frac{a^2(3bd+ae)x^2}{3+m}+\frac{3ab(bd+ae)x^4}{5+m}+\frac{b^2(bd+3ae)x^6}{7+m}+\frac{b^3ex^8}{9+m}\right)}{\left(a+bx^2\right)^3}$$

input
```
Integrate[(f*x)^m*(d + e*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
(x*(f*x)^m*((a + b*x^2)^2)^(3/2)*((a^3*d)/(1 + m) + (a^2*(3*b*d + a*e)*x^2
)/(3 + m) + (3*a*b*(b*d + a*e)*x^4)/(5 + m) + (b^2*(b*d + 3*a*e)*x^6)/(7 +
 m) + (b^3*e*x^8)/(9 + m)))/(a + b*x^2)^3
```

3.88.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.55, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1384, 27, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx^2+b^2x^4\right)^{3/2} \left(d+ex^2\right) (fx)^m \, dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4} \int b^3(fx)^m \left(bx^2+a\right)^3 \left(ex^2+d\right)dx}{b^3 \left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4} \int (fx)^m \left(bx^2+a\right)^3 \left(ex^2+d\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{355}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4} \int \left(a^3d(fx)^m+\frac{a^2(3bd+ae)(fx)^{m+2}}{f^2}+\frac{3ab(bd+ae)(fx)^{m+4}}{f^4}+\frac{b^2(bd+3ae)(fx)^{m+6}}{f^6}+\frac{b^3e(fx)^{m+8}}{f^8}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{a^3d(fx)^{m+1}}{f(m+1)}+\frac{a^2(fx)^{m+3}(ae+3bd)}{f^3(m+3)}+\frac{b^2(fx)^{m+7}(3ae+bd)}{f^7(m+7)}+\frac{3ab(fx)^{m+5}(ae+bd)}{f^5(m+5)}+\frac{b^3e(fx)^{m+9}}{f^9(m+9)}\right)}{a+bx^2}$$

input
```
Int[(f*x)^m*(d + e*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((a^3*d*(f*x)^(1 + m))/(f*(1 + m)) + (a^2
*(3*b*d + a*e)*(f*x)^(3 + m))/(f^3*(3 + m)) + (3*a*b*(b*d + a*e)*(f*x)^(5
+ m))/(f^5*(5 + m)) + (b^2*(b*d + 3*a*e)*(f*x)^(7 + m))/(f^7*(7 + m)) + (b
^3*e*(f*x)^(9 + m))/(f^9*(9 + m))))/(a + b*x^2)
```

3.88.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.88.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 494 vs. $2(221) = 442$.

Time = 0.03 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.79

method	result
gosper	$\frac{x\left(b^{3}e\,m^{4}x^{8}+16b^{3}e\,m^{3}x^{8}+3a\,b^{2}e\,m^{4}x^{6}+b^{3}d\,m^{4}x^{6}+86b^{3}e\,m^{2}x^{8}+54a\,b^{2}e\,m^{3}x^{6}+18b^{3}d\,m^{3}x^{6}+176m\,x^{8}b^{3}e+3a^{2}be\,m^{4}x^{4}+3a\,b^{2}d\,m^{4}\right.}{}$
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(b^{3}e\,m^{4}x^{8}+16b^{3}e\,m^{3}x^{8}+3a\,b^{2}e\,m^{4}x^{6}+b^{3}d\,m^{4}x^{6}+86b^{3}e\,m^{2}x^{8}+54a\,b^{2}e\,m^{3}x^{6}+18b^{3}d\,m^{3}x^{6}+176m\,x^{8}b^{3}e+3a^{2}be\,m^{4}x^{4}\right.}{}$

input
```
int((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
x*(b^3*e*m^4*x^8+16*b^3*e*m^3*x^8+3*a*b^2*e*m^4*x^6+b^3*d*m^4*x^6+86*b^3*e
*m^2*x^8+54*a*b^2*e*m^3*x^6+18*b^3*d*m^3*x^6+176*b^3*e*m*x^8+3*a^2*b*e*m^4
*x^4+3*a*b^2*d*m^4*x^4+312*a*b^2*e*m^2*x^6+104*b^3*d*m^2*x^6+105*b^3*e*x^8
+60*a^2*b*e*m^3*x^4+60*a*b^2*d*m^3*x^4+666*a*b^2*e*m*x^6+222*b^3*d*m*x^6+a
^3*e*m^4*x^2+3*a^2*b*d*m^4*x^2+390*a^2*b*e*m^2*x^4+390*a*b^2*d*m^2*x^4+405
*a*b^2*e*x^6+135*b^3*d*x^6+22*a^3*e*m^3*x^2+66*a^2*b*d*m^3*x^2+900*a^2*b*e
*m*x^4+900*a*b^2*d*m*x^4+a^3*d*m^4+164*a^3*e*m^2*x^2+492*a^2*b*d*m^2*x^2+5
67*a^2*b*e*x^4+567*a*b^2*d*x^4+24*a^3*d*m^3+458*a^3*e*m*x^2+1374*a^2*b*d*m
*x^2+206*a^3*d*m^2+315*a^3*e*x^2+945*a^2*b*d*x^2+744*a^3*d*m+945*a^3*d)*(f
*x)^m*((b*x^2+a)^2)^(3/2)/(9+m)/(7+m)/(5+m)/(3+m)/(1+m)/(b*x^2+a)^3
```

3.88.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.38

$$\int (fx)^m \left(d+ex^2\right) \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{((b^3em^4+16\,b^3em^3+86\,b^3em^2+176\,b^3em+105\,b^3e)x^9+((b^3d+3\,ab^2e)m^4+135\,b^3d+4}{}$$

input
```
integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fr
icas")
```

output
```
((b^3*e*m^4 + 16*b^3*e*m^3 + 86*b^3*e*m^2 + 176*b^3*e*m + 105*b^3*e)*x^9 +
 ((b^3*d + 3*a*b^2*e)*m^4 + 135*b^3*d + 405*a*b^2*e + 18*(b^3*d + 3*a*b^2*
e)*m^3 + 104*(b^3*d + 3*a*b^2*e)*m^2 + 222*(b^3*d + 3*a*b^2*e)*m)*x^7 + 3*
((a*b^2*d + a^2*b*e)*m^4 + 189*a*b^2*d + 189*a^2*b*e + 20*(a*b^2*d + a^2*b
*e)*m^3 + 130*(a*b^2*d + a^2*b*e)*m^2 + 300*(a*b^2*d + a^2*b*e)*m)*x^5 + (
(3*a^2*b*d + a^3*e)*m^4 + 945*a^2*b*d + 315*a^3*e + 22*(3*a^2*b*d + a^3*e)
*m^3 + 164*(3*a^2*b*d + a^3*e)*m^2 + 458*(3*a^2*b*d + a^3*e)*m)*x^3 + (a^3
*d*m^4 + 24*a^3*d*m^3 + 206*a^3*d*m^2 + 744*a^3*d*m + 945*a^3*d)*x)*(f*x)^
m/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

3.88.6 Sympy [F]

$$\int (fx)^m\left(d+ex^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx=\int (fx)^m\left(d+ex^2\right)\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((f*x)**m*(e*x**2+d)*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Integral((f*x)**m*(d + e*x**2)*((a + b*x**2)**2)**(3/2), x)
```

3.88.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.88

$$\int (fx)^m\left(d+ex^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx=\frac{\left((m^3+9\,m^2+23\,m+15)b^3f^mx^7+3\left(m^3+11\,m^2+31\,m+21\right)ab^2f^mx^5+3\left(m^3+13\,m^2+\right.}{m^4+16\,m^3+86\,m^2+176\,m+}+\frac{\left((m^3+15\,m^2+71\,m+105)b^3f^mx^9+3\left(m^3+17\,m^2+87\,m+135\right)ab^2f^mx^7+3\left(m^3+19\,m^2+111\,m+\right.}{m^4+24\,m^3+206\,m^2+744\,m+945}$$

input
```
integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="ma
xima")
```

output
```
((m^3 + 9*m^2 + 23*m + 15)*b^3*f^m*x^7 + 3*(m^3 + 11*m^2 + 31*m + 21)*a*b^
2*f^m*x^5 + 3*(m^3 + 13*m^2 + 47*m + 35)*a^2*b*f^m*x^3 + (m^3 + 15*m^2 + 7
1*m + 105)*a^3*f^m*x)*d*x^m/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105) + ((m^3
+ 15*m^2 + 71*m + 105)*b^3*f^m*x^9 + 3*(m^3 + 17*m^2 + 87*m + 135)*a*b^2*f
^m*x^7 + 3*(m^3 + 19*m^2 + 111*m + 189)*a^2*b*f^m*x^5 + (m^3 + 21*m^2 + 14
3*m + 315)*a^3*f^m*x^3)*e*x^m/(m^4 + 24*m^3 + 206*m^2 + 744*m + 945)
```

3.88.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 993 vs. $2(221) = 442$.

Time = 0.30 (sec) , antiderivative size = 993, normalized size of antiderivative = 3.60

$$\int (fx)^m \left(d+ex^2\right) \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="gi
ac")
```

output
```
((f*x)^m*b^3*e*m^4*x^9*sgn(b*x^2 + a) + 16*(f*x)^m*b^3*e*m^3*x^9*sgn(b*x^2
 + a) + (f*x)^m*b^3*d*m^4*x^7*sgn(b*x^2 + a) + 3*(f*x)^m*a*b^2*e*m^4*x^7*s
gn(b*x^2 + a) + 86*(f*x)^m*b^3*e*m^2*x^9*sgn(b*x^2 + a) + 18*(f*x)^m*b^3*d
*m^3*x^7*sgn(b*x^2 + a) + 54*(f*x)^m*a*b^2*e*m^3*x^7*sgn(b*x^2 + a) + 176*
(f*x)^m*b^3*e*m*x^9*sgn(b*x^2 + a) + 3*(f*x)^m*a*b^2*d*m^4*x^5*sgn(b*x^2 +
 a) + 3*(f*x)^m*a^2*b*e*m^4*x^5*sgn(b*x^2 + a) + 104*(f*x)^m*b^3*d*m^2*x^7
*sgn(b*x^2 + a) + 312*(f*x)^m*a*b^2*e*m^2*x^7*sgn(b*x^2 + a) + 105*(f*x)^m
*b^3*e*x^9*sgn(b*x^2 + a) + 60*(f*x)^m*a*b^2*d*m^3*x^5*sgn(b*x^2 + a) + 60
*(f*x)^m*a^2*b*e*m^3*x^5*sgn(b*x^2 + a) + 222*(f*x)^m*b^3*d*m*x^7*sgn(b*x^
2 + a) + 666*(f*x)^m*a*b^2*e*m*x^7*sgn(b*x^2 + a) + 3*(f*x)^m*a^2*b*d*m^4*
x^3*sgn(b*x^2 + a) + (f*x)^m*a^3*e*m^4*x^3*sgn(b*x^2 + a) + 390*(f*x)^m*a*
b^2*d*m^2*x^5*sgn(b*x^2 + a) + 390*(f*x)^m*a^2*b*e*m^2*x^5*sgn(b*x^2 + a)
+ 135*(f*x)^m*b^3*d*x^7*sgn(b*x^2 + a) + 405*(f*x)^m*a*b^2*e*x^7*sgn(b*x^2
 + a) + 66*(f*x)^m*a^2*b*d*m^3*x^3*sgn(b*x^2 + a) + 22*(f*x)^m*a^3*e*m^3*x
^3*sgn(b*x^2 + a) + 900*(f*x)^m*a*b^2*d*m*x^5*sgn(b*x^2 + a) + 900*(f*x)^m
*a^2*b*e*m*x^5*sgn(b*x^2 + a) + (f*x)^m*a^3*d*m^4*x*sgn(b*x^2 + a) + 492*(
f*x)^m*a^2*b*d*m^2*x^3*sgn(b*x^2 + a) + 164*(f*x)^m*a^3*e*m^2*x^3*sgn(b*x^
2 + a) + 567*(f*x)^m*a*b^2*d*x^5*sgn(b*x^2 + a) + 567*(f*x)^m*a^2*b*e*x^5*
sgn(b*x^2 + a) + 24*(f*x)^m*a^3*d*m^3*x*sgn(b*x^2 + a) + 1374*(f*x)^m*a^2*
b*d*m*x^3*sgn(b*x^2 + a) + 458*(f*x)^m*a^3*e*m*x^3*sgn(b*x^2 + a) + 206...
```

3.88.9 Mupad [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right) \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx = \int (f\,x)^m \left(e\,x^2+d\right) \left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2} \, dx$$

input
```
int((f*x)^m*(d + e*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

output
```
int((f*x)^m*(d + e*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

3.89 $\int (fx)^m (d+ex^2) \sqrt{a^2+2abx^2+b^2x^4}\,dx$

3.89.1 Optimal result

Integrand size = 35, antiderivative size = 153

$$\int (fx)^m (d+ex^2) \sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{ad(fx)^{1+m}\sqrt{a^2+2abx^2+b^2x^4}}{f(1+m)(a+bx^2)} + \frac{(bd+ae)(fx)^{3+m}\sqrt{a^2+2abx^2+b^2x^4}}{f^3(3+m)(a+bx^2)} + \frac{be(fx)^{5+m}\sqrt{a^2+2abx^2+b^2x^4}}{f^5(5+m)(a+bx^2)}$$

output
```
a*d*(f*x)^(1+m)*((b*x^2+a)^2)^(1/2)/f/(1+m)/(b*x^2+a)+(a*e+b*d)*(f*x)^(3+m
)*((b*x^2+a)^2)^(1/2)/f^3/(3+m)/(b*x^2+a)+b*e*(f*x)^(5+m)*((b*x^2+a)^2)^(1
/2)/f^5/(5+m)/(b*x^2+a)
```

3.89.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.56

$$\int (fx)^m (d+ex^2) \sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{x(fx)^m\sqrt{(a+bx^2)^2}(a(5+m)(d(3+m)+e(1+m)x^2)+b(1+m)x^2(d(5+m)+e(3+m)x^2))}{(1+m)(3+m)(5+m)(a+bx^2)}$$

input
```
Integrate[(f*x)^m*(d + e*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
(x*(f*x)^m*Sqrt[(a + b*x^2)^2]*(a*(5 + m)*(d*(3 + m) + e*(1 + m)*x^2) + b*
(1 + m)*x^2*(d*(5 + m) + e*(3 + m)*x^2)))/((1 + m)*(3 + m)*(5 + m)*(a + b*
x^2))
```

3.89.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.60, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1384, 27, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a^2+2abx^2+b^2x^4}\left(d+ex^2\right)(fx)^m\,dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int b(fx)^m\left(bx^2+a\right)\left(ex^2+d\right)dx}{b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int (fx)^m\left(bx^2+a\right)\left(ex^2+d\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{ 355}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(ad(fx)^m+\frac{(bd+ae)(fx)^{m+2}}{f^2}+\frac{be(fx)^{m+4}}{f^4}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{(fx)^{m+3}(ae+bd)}{f^3(m+3)}+\frac{ad(fx)^{m+1}}{f(m+1)}+\frac{be(fx)^{m+5}}{f^5(m+5)}\right)}{a+bx^2}$$

input
```
Int[(f*x)^m*(d + e*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((a*d*(f*x)^(1 + m))/(f*(1 + m)) + ((b*d
+ a*e)*(f*x)^(3 + m))/(f^3*(3 + m)) + (b*e*(f*x)^(5 + m))/(f^5*(5 + m))))/
(a + b*x^2)
```

3.89.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.89.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.86

method	result
gosper	$\frac{x\left(b e\,m^{2} x^{4}+4 b e m\,x^{4}+a e\,m^{2} x^{2}+b d\,m^{2} x^{2}+3 b e\,x^{4}+6 a e m\,x^{2}+6 b d m\,x^{2}+a d\,m^{2}+5 a e\,x^{2}+5 b d\,x^{2}+8 a d m+15 d a\right)\left(f x\right)^{m} \sqrt{\left(b\,x^{2}+a\right)^{2}}}{\left(5+m\right)\left(3+m\right)\left(1+m\right)\left(b\,x^{2}+a\right)}$
risch	$\frac{x\left(b e\,m^{2} x^{4}+4 b e m\,x^{4}+a e\,m^{2} x^{2}+b d\,m^{2} x^{2}+3 b e\,x^{4}+6 a e m\,x^{2}+6 b d m\,x^{2}+a d\,m^{2}+5 a e\,x^{2}+5 b d\,x^{2}+8 a d m+15 d a\right)\left(f x\right)^{m} \sqrt{\left(b\,x^{2}+a\right)^{2}}}{\left(5+m\right)\left(3+m\right)\left(1+m\right)\left(b\,x^{2}+a\right)}$

input
```
int((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
x*(b*e*m^2*x^4+4*b*e*m*x^4+a*e*m^2*x^2+b*d*m^2*x^2+3*b*e*x^4+6*a*e*m*x^2+6
*b*d*m*x^2+a*d*m^2+5*a*e*x^2+5*b*d*x^2+8*a*d*m+15*a*d)*(f*x)^m*((b*x^2+a)^
2)^(1/2)/(5+m)/(3+m)/(1+m)/(b*x^2+a)
```

3.89.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.61

$$\int (fx)^m (d+ex^2) \sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{((bem^2+4\,bem+3\,be)x^5+((bd+ae)m^2+5\,bd+5\,ae+6\,(bd+ae)m)x^3+(adm^2+8\,adm+15\,ad)x)(}{m^3+9\,m^2+23\,m+15}$$

input
```
integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="fr
icas")
```

output
```
((b*e*m^2 + 4*b*e*m + 3*b*e)*x^5 + ((b*d + a*e)*m^2 + 5*b*d + 5*a*e + 6*(b
*d + a*e)*m)*x^3 + (a*d*m^2 + 8*a*d*m + 15*a*d)*x)*(f*x)^m/(m^3 + 9*m^2 +
23*m + 15)
```

3.89.6 Sympy [F]

$$\int (fx)^m (d+ex^2) \sqrt{a^2+2abx^2+b^2x^4}\,dx = \int (fx)^m (d+ex^2) \sqrt{(a+bx^2)^2}\,dx$$

input
```
integrate((f*x)**m*(e*x**2+d)*(b**2*x**4+2*a*b*x**2+a**2)**(1/2),x)
```

output
```
Integral((f*x)**m*(d + e*x**2)*sqrt((a + b*x**2)**2), x)
```

3.89.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.49

$$\int (fx)^m (d+ex^2) \sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{(bf^m(m+1)x^3+af^m(m+3)x)dx^m}{m^2+4\,m+3} + \frac{(bf^m(m+3)x^5+af^m(m+5)x^3)ex^m}{m^2+8\,m+15}$$

input
```
integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="ma
xima")
```

output
```
(b*f^m*(m + 1)*x^3 + a*f^m*(m + 3)*x)*d*x^m/(m^2 + 4*m + 3) + (b*f^m*(m +
3)*x^5 + a*f^m*(m + 5)*x^3)*e*x^m/(m^2 + 8*m + 15)
```

3.89.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(120) = 240$.

Time = 0.27 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.72

$$\int (fx)^m (d+ex^2) \sqrt{a^2+2abx^2+b^2x^4}\, dx$$
$$= \frac{(fx)^m bem^2x^5\text{sgn}(bx^2+a) + 4(fx)^m bemx^5\text{sgn}(bx^2+a) + (fx)^m bdm^2x^3\text{sgn}(bx^2+a) + (fx)^m aem^2x^3\text{s}}{}$$

input
```
integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="gi
ac")
```

output
```
((f*x)^m*b*e*m^2*x^5*sgn(b*x^2 + a) + 4*(f*x)^m*b*e*m*x^5*sgn(b*x^2 + a) +
 (f*x)^m*b*d*m^2*x^3*sgn(b*x^2 + a) + (f*x)^m*a*e*m^2*x^3*sgn(b*x^2 + a) +
 3*(f*x)^m*b*e*x^5*sgn(b*x^2 + a) + 6*(f*x)^m*b*d*m*x^3*sgn(b*x^2 + a) + 6
*(f*x)^m*a*e*m*x^3*sgn(b*x^2 + a) + (f*x)^m*a*d*m^2*x*sgn(b*x^2 + a) + 5*(
f*x)^m*b*d*x^3*sgn(b*x^2 + a) + 5*(f*x)^m*a*e*x^3*sgn(b*x^2 + a) + 8*(f*x)
^m*a*d*m*x*sgn(b*x^2 + a) + 15*(f*x)^m*a*d*x*sgn(b*x^2 + a))/(m^3 + 9*m^2
+ 23*m + 15)
```

3.89.9 Mupad [F(-1)]

Timed out.

$$\int (fx)^m (d+ex^2) \sqrt{a^2+2abx^2+b^2x^4}\, dx = \int (f\,x)^m\,\left(e\,x^2+d\right)\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\, dx$$

input
```
int((f*x)^m*(d + e*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2),x)
```

output
```
int((f*x)^m*(d + e*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2), x)
```

3.90 $\int \frac{(fx)^m(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

3.90.1 Optimal result

Integrand size = 35, antiderivative size = 134

$$\int \frac{(fx)^m(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{e(fx)^{1+m}(a+bx^2)}{bf(1+m)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(bd-ae)(fx)^{1+m}(a+bx^2)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{abf(1+m)\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
e*(f*x)^(1+m)*(b*x^2+a)/b/f/(1+m)/((b*x^2+a)^2)^(1/2)+(-a*e+b*d)*(f*x)^(1+
m)*(b*x^2+a)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a/b/f/(1+m)/((
b*x^2+a)^2)^(1/2)
```

3.90.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.58

$$\int \frac{(fx)^m(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = -\frac{x(fx)^m(a+bx^2)\left(-ae+(-bd+ae)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)\right)}{ab(1+m)\sqrt{(a+bx^2)^2}}$$

input
```
Integrate[((f*x)^m*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
-((x*(f*x)^m*(a + b*x^2)*(-(a*e) + (-(b*d) + a*e)*Hypergeometric2F1[1, (1
+ m)/2, (3 + m)/2, -((b*x^2)/a)]))/(a*b*(1 + m)*Sqrt[(a + b*x^2)^2]))
```

3.90.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.79, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1384, 27, 363, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(fx)^m}{\sqrt{a^2+2abx^2+b^2x^4}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{b(a+bx^2)\int \frac{(fx)^m(ex^2+d)}{b(bx^2+a)}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx^2)\int \frac{(fx)^m(ex^2+d)}{bx^2+a}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 363}$$

$$\frac{(a+bx^2)\left(\frac{(bd-ae)\int \frac{(fx)^m}{bx^2+a}dx}{b}+\frac{e(fx)^{m+1}}{bf(m+1)}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 278}$$

$$\frac{(a+bx^2)\left(\frac{(fx)^{m+1}(bd-ae)\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{abf(m+1)}+\frac{e(fx)^{m+1}}{bf(m+1)}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[((f*x)^m*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
((a + b*x^2)*((e*(f*x)^(1 + m))/(b*f*(1 + m)) + ((b*d - a*e)*(f*x)^(1 + m)
*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*b*f*(1 + m))
))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
```

3.90.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.90.4 Maple [F]

$$\int \frac{(fx)^m\,(e\,x^2+d)}{\sqrt{b^2x^4+2ab\,x^2+a^2}}dx$$

input
```
int((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x)
```

output
```
int((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x)
```

3.90.5 Fricas [F]

$$\int \frac{(fx)^m (d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{(ex^2+d)(fx)^m}{\sqrt{b^2x^4+2\,abx^2+a^2}}\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="fr
icas")
```

output
```
integral((e*x^2 + d)*(f*x)^m/sqrt(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

3.90.6 Sympy [F]

$$\int \frac{(fx)^m (d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{(fx)^m (d+ex^2)}{\sqrt{(a+bx^2)^2}}\,dx$$

input
```
integrate((f*x)**m*(e*x**2+d)/(b**2*x**4+2*a*b*x**2+a**2)**(1/2),x)
```

output
```
Integral((f*x)**m*(d + e*x**2)/sqrt((a + b*x**2)**2), x)
```

3.90.7 Maxima [F]

$$\int \frac{(fx)^m (d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{(ex^2+d)(fx)^m}{\sqrt{b^2x^4+2\,abx^2+a^2}}\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((e*x^2 + d)*(f*x)^m/sqrt(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

3.90.8 Giac [F]

$$\int \frac{(fx)^m (d + ex^2)}{\sqrt{a^2 + 2abx^2 + b^2x^4}} \, dx = \int \frac{(ex^2 + d)(fx)^m}{\sqrt{b^2x^4 + 2\,abx^2 + a^2}} \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((e*x^2 + d)*(f*x)^m/sqrt(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

3.90.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^m (d + ex^2)}{\sqrt{a^2 + 2abx^2 + b^2x^4}} \, dx = \int \frac{(f\,x)^m\,(e\,x^2 + d)}{\sqrt{a^2 + 2\,a\,b\,x^2 + b^2\,x^4}} \, dx$$

input
```
int(((f*x)^m*(d + e*x^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2),x)
```

output
```
int(((f*x)^m*(d + e*x^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2), x)
```

3.91 $\int \frac{(fx)^m(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

3.91.1 Optimal result

Integrand size = 35, antiderivative size = 154

$$\int \frac{(fx)^m(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{(bd-ae)(fx)^{1+m}}{4abf(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(bd(3-m)+ae(1+m))(fx)^{1+m}(a+bx^2)\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{4a^3bf(1+m)\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
1/4*(-a*e+b*d)*(f*x)^(1+m)/a/b/f/(b*x^2+a)/((b*x^2+a)^2)^(1/2)+1/4*(b*d*(3
-m)+a*e*(1+m))*(f*x)^(1+m)*(b*x^2+a)*hypergeom([2, 1/2+1/2*m],[3/2+1/2*m],
-b*x^2/a)/a^3/b/f/(1+m)/((b*x^2+a)^2)^(1/2)
```

3.91.2 Mathematica [A] (verified)

Time = 0.11 (sec), antiderivative size = 101, normalized size of antiderivative = 0.66

$$\int \frac{(fx)^m(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{x(fx)^m(a+bx^2)\left(ae\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)+(bd-ae)\right.}{a^3b(1+m)\sqrt{(a+bx^2)^2}}$$

input
```
Integrate[((f*x)^m*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
(x*(f*x)^m*(a + b*x^2)*(a*e*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((
b*x^2)/a)] + (b*d - a*e)*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, -((b*x
^2)/a)]))/(a^3*b*(1 + m)*Sqrt[(a + b*x^2)^2])
```

3.91.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1384, 27, 362, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(fx)^m}{(a^2+2abx^2+b^2x^4)^{3/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^3(a+bx^2)\int \frac{(fx)^m(ex^2+d)}{b^3(bx^2+a)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^2)\int \frac{(fx)^m(ex^2+d)}{(bx^2+a)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{362}$$

$$\frac{(a+bx^2)\left(\frac{(ae(m+1)+bd(3-m))\int \frac{(fx)^m}{(bx^2+a)^2}dx}{4ab}+\frac{(fx)^{m+1}(bd-ae)}{4abf(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{278}$$

$$\frac{(a+bx^2)\left(\frac{(fx)^{m+1}(ae(m+1)+bd(3-m))\,\text{Hypergeometric2F1}\left(2,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{4a^3bf(m+1)}+\frac{(fx)^{m+1}(bd-ae)}{4abf(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[((f*x)^m*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

3.91. $\int \frac{(fx)^m(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}dx$

output
```
((a + b*x^2)*(((b*d - a*e)*(f*x)^(1 + m))/(4*a*b*f*(a + b*x^2)^2) + ((b*d*
(3 - m) + a*e*(1 + m))*(f*x)^(1 + m)*Hypergeometric2F1[2, (1 + m)/2, (3 +
m)/2, -((b*x^2)/a)])/(4*a^3*b*f*(1 + m))))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
```

3.91.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.91.4 Maple [F]

$$\int \frac{(fx)^m (e\,x^2+d)}{(b^2x^4+2ab\,x^2+a^2)^{\frac{3}{2}}}dx$$

input
```
int((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

output
```
int((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

3.91.5 Fricas [F]

$$\int \frac{(fx)^m\,(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{(ex^2+d)(fx)^m}{(b^2x^4+2\,abx^2+a^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b^2*x^4 + 2*a*b*x^2 + a^2)*(e*x^2 + d)*(f*x)^m/(b^4*x^8 + 4*
a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4), x)
```

3.91.6 Sympy [F]

$$\int \frac{(fx)^m\,(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{(fx)^m\,(d+ex^2)}{\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x)**m*(e*x**2+d)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Integral((f*x)**m*(d + e*x**2)/((a + b*x**2)**2)**(3/2), x)
```

3.91.7 Maxima [F]

$$\int \frac{(fx)^m\,(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{(ex^2+d)(fx)^m}{(b^2x^4+2\,abx^2+a^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate((e*x^2 + d)*(f*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2), x)
```

3.91.8 Giac [F]

$$\int \frac{(fx)^m (d + ex^2)}{(a^2 + 2abx^2 + b^2x^4)^{3/2}} \, dx = \int \frac{(ex^2 + d)(fx)^m}{(b^2x^4 + 2\,abx^2 + a^2)^{\frac{3}{2}}} \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate((e*x^2 + d)*(f*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2), x)
```

3.91.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^m (d + ex^2)}{(a^2 + 2abx^2 + b^2x^4)^{3/2}} \, dx = \int \frac{(f\,x)^m\,(e\,x^2 + d)}{(a^2 + 2\,a\,b\,x^2 + b^2\,x^4)^{3/2}} \, dx$$

input
```
int(((f*x)^m*(d + e*x^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

output
```
int(((f*x)^m*(d + e*x^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

3.92 $\int x(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p\,dx$

3.92.1 Optimal result

Integrand size = 29, antiderivative size = 34

$$\int x\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p\,dx=\frac{\left(a^2+2abx^2+b^2x^4\right)^{1+p}}{4b(1+p)}$$

output
```
1/4*(b^2*x^4+2*a*b*x^2+a^2)^(p+1)/b/(p+1)
```

3.92.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int x\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p\,dx=\frac{\left(\left(a+bx^2\right)^2\right)^{1+p}}{4b(1+p)}$$

input
```
Integrate[x*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

output
```
((a + b*x^2)^2)^(1 + p)/(4*b*(1 + p))
```

3.92.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.44, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1383, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p\,dx$$

$$\Big\downarrow \; 1383$$

$$(a+bx^2)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\int x\left(bx^2+a\right)^{2p+1}dx$$

$$\Big\downarrow \; 241$$

$$\frac{(a+bx^2)^{2(p+1)-2p}\left(a^2+2abx^2+b^2x^4\right)^p}{4b(p+1)}$$

input
```
Int[x*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

output
```
((a + b*x^2)^(-2*p + 2*(1 + p))*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b*(1 + p
))
```

3.92.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 1383
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e_.)*(
x_)^(n_))^(q_.), x_Symbol] :> Simp[(a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^(2
*p)   Int[u*(d + e*x^n)^(q + 2*p), x], x] /; FreeQ[{a, b, c, d, e, n, p, q}
, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &&  !Int
egerQ[p]
```

3.92.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.18

method	result	size
gosper	$\frac{(b\,x^2+a)^2(b^2x^4+2ab\,x^2+a^2)^p}{4b(1+p)}$	40
risch	$\frac{(b^2x^4+2ab\,x^2+a^2)\left((b\,x^2+a)^2\right)^p}{4b(1+p)}$	40
parallelrisch	$\frac{x^4(b^2x^4+2ab\,x^2+a^2)^p a\,b^2+2x^2(b^2x^4+2ab\,x^2+a^2)^p a^2b+a^3(b^2x^4+2ab\,x^2+a^2)^p}{4ab(1+p)}$	96
norman	$\frac{a\,x^2\mathrm{e}^{p\ln\left(b^2x^4+2ab\,x^2+a^2\right)}}{2p+2}+\frac{a^2\mathrm{e}^{p\ln\left(b^2x^4+2ab\,x^2+a^2\right)}}{4b(1+p)}+\frac{b\,x^4\mathrm{e}^{p\ln\left(b^2x^4+2ab\,x^2+a^2\right)}}{4+4p}$	103

input
```
int(x*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x,method=_RETURNVERBOSE)
```

output
```
1/4*(b*x^2+a)^2/b/(1+p)*(b^2*x^4+2*a*b*x^2+a^2)^p
```

3.92.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.38

$$\int x(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p\,dx=\frac{(b^2x^4+2\,abx^2+a^2)(b^2x^4+2\,abx^2+a^2)^p}{4\,(bp+b)}$$

input
```
integrate(x*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

output
```
1/4*(b^2*x^4 + 2*a*b*x^2 + a^2)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(b*p + b)
```

3.92.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. 2(27) = 54.

Time = 4.39 (sec) , antiderivative size = 155, normalized size of antiderivative = 4.56

$$\int x(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p\,dx = \begin{cases} \frac{x^2}{2a} & \text{for } b=0\wedge p=-1 \\ \frac{ax^2(a^2)^p}{2} & \text{for } b=0 \\ \frac{\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2b}+\frac{\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2b} & \text{for } p=-1 \\ \frac{a^2(a^2+2abx^2+b^2x^4)^p}{4bp+4b}+\frac{2abx^2(a^2+2abx^2+b^2x^4)^p}{4bp+4b}+\frac{b^2x^4(a^2+2abx^2+b^2x^4)^p}{4bp+4b} & \text{otherwise} \end{cases}$$

input
```
integrate(x*(b*x**2+a)*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

output
```
Piecewise((x**2/(2*a), Eq(b, 0) & Eq(p, -1)), (a*x**2*(a**2)**p/2, Eq(b, 0
)), (log(x - sqrt(-a/b))/(2*b) + log(x + sqrt(-a/b))/(2*b), Eq(p, -1)), (a
**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(4*b*p + 4*b) + 2*a*b*x**2*(a**2 +
2*a*b*x**2 + b**2*x**4)**p/(4*b*p + 4*b) + b**2*x**4*(a**2 + 2*a*b*x**2 +
b**2*x**4)**p/(4*b*p + 4*b), True))
```

3.92.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(32) = 64.

Time = 0.21 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.53

$$\int x(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p\,dx = \frac{(bx^2+a)(bx^2+a)^{2p}a}{2b(2p+1)} + \frac{(b^2(2p+1)x^4+2abpx^2-a^2)(bx^2+a)^{2p}}{4(2p^2+3p+1)b}$$

input
```
integrate(x*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

output
```
1/2*(b*x^2 + a)*(b*x^2 + a)^(2*p)*a/(b*(2*p + 1)) + 1/4*(b^2*(2*p + 1)*x^4
 + 2*a*b*p*x^2 - a^2)*(b*x^2 + a)^(2*p)/((2*p^2 + 3*p + 1)*b)
```

3.92.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int x(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p\,dx=\frac{(b^2x^4+2\,abx^2+a^2)^{p+1}}{4\,b(p+1)}$$

input
```
integrate(x*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

output
```
1/4*(b^2*x^4 + 2*a*b*x^2 + a^2)^(p + 1)/(b*(p + 1))
```

3.92.9 Mupad [B] (verification not implemented)

Time = 7.81 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.74

$$\int x(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p\,dx=\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^p\left(\frac{a^2}{4\,b\,(p+1)}+\frac{a\,x^2}{2\,(p+1)}+\frac{b\,x^4}{4\,(p+1)}\right)$$

input
```
int(x*(a + b*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

output
```
(a^2 + b^2*x^4 + 2*a*b*x^2)^p*(a^2/(4*b*(p + 1)) + (a*x^2)/(2*(p + 1)) + (
b*x^4)/(4*(p + 1)))
```

3.93 $\int x^3(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p\,dx$

3.93.1 Optimal result

Integrand size = 31, antiderivative size = 86

$$\int x^3\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p\,dx=-\frac{a(a+bx^2)^2\left(a^2+2abx^2+b^2x^4\right)^p}{4b^2(1+p)}+\frac{\left(a+bx^2\right)^3\left(a^2+2abx^2+b^2x^4\right)^p}{2b^2(3+2p)}$$

output
```
-1/4*a*(b*x^2+a)^2*(b^2*x^4+2*a*b*x^2+a^2)^p/b^2/(p+1)+1/2*(b*x^2+a)^3*(b^
2*x^4+2*a*b*x^2+a^2)^p/b^2/(3+2*p)
```

3.93.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.52

$$\int x^3\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p\,dx=\frac{\left(\left(a+bx^2\right)^2\right)^{1+p}\left(-a+2b(1+p)x^2\right)}{4b^2(1+p)(3+2p)}$$

input
```
Integrate[x^3*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

output
```
(((a + b*x^2)^2)^(1 + p)*(-a + 2*b*(1 + p)*x^2))/(4*b^2*(1 + p)*(3 + 2*p))
```

3.93.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1383, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (a+bx^2) \left(a^2+2abx^2+b^2x^4\right)^p \, dx$$

$$\Big\downarrow \; 1383$$

$$(a+bx^2)^{-2p} \left(a^2+2abx^2+b^2x^4\right)^p \int x^3 \left(bx^2+a\right)^{2p+1} dx$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2}(a+bx^2)^{-2p} \left(a^2+2abx^2+b^2x^4\right)^p \int x^2 \left(bx^2+a\right)^{2p+1} dx^2$$

$$\Big\downarrow \; 53$$

$$\frac{1}{2}(a+bx^2)^{-2p} \left(a^2+2abx^2+b^2x^4\right)^p \int \left(\frac{\left(bx^2+a\right)^{2p+2}}{b} - \frac{a\left(bx^2+a\right)^{2p+1}}{b}\right) dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}(a+bx^2)^{-2p} \left(a^2+2abx^2+b^2x^4\right)^p \left(\frac{\left(a+bx^2\right)^{2p+3}}{b^2(2p+3)} - \frac{a\left(a+bx^2\right)^{2(p+1)}}{2b^2(p+1)}\right)$$

input
```
Int[x^3*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

output
```
((a^2 + 2*a*b*x^2 + b^2*x^4)^p*(-1/2*(a*(a + b*x^2)^(2*(1 + p)))/(b^2*(1 +
 p)) + (a + b*x^2)^(3 + 2*p)/(b^2*(3 + 2*p))))/(2*(a + b*x^2)^(2*p))
```

3.93.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1383
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e_.)*(
x_)^(n_))^(q_.), x_Symbol] :> Simp[(a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^(2
*p)   Int[u*(d + e*x^n)^(q + 2*p), x], x] /; FreeQ[{a, b, c, d, e, n, p, q}
, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &&  !Int
egerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.93.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.72

method	result
gosper	$-\frac{\left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{p} \left(-2 x^{2} p b -2 b \,x^{2}+a \right) \left(b \,x^{2}+a \right)^{2}}{4 b^{2} \left(2 p^{2}+5 p +3\right)}$
risch	$-\frac{\left(-2 b^{3} p \,x^{6}-2 b^{3} x^{6}-4 a \,b^{2} p \,x^{4}-3 b^{2} x^{4} a -2 a^{2} b p \,x^{2}+a^{3}\right) \left(\left(b \,x^{2}+a \right)^{2}\right)^{p}}{4 \left(1+p \right) \left(3+2 p \right) b^{2}}$
norman	$-\frac{a^{3} {\mathrm e}^{p \ln \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)}}{4 b^{2} \left(2 p^{2}+5 p +3\right)}+\frac{b \,x^{6} {\mathrm e}^{p \ln \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)}}{6+4 p}+\frac{a \left(4 p +3\right) x^{4} {\mathrm e}^{p \ln \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)}}{8 p^{2}+20 p +12}+\frac{p \,a^{2} x^{2} {\mathrm e}^{p \ln \left(b^{2} x^{4}+2 a b \,x^{2}+\right.}}{2 b \left(2 p^{2}+5 p +3\right)}$
parallelrisch	$\frac{2 x^{6} \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{p} b^{3} p +2 x^{6} \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{p} b^{3}+4 x^{4} \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{p} a \,b^{2} p +3 x^{4} \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{p} a \,b^{2}+2 x^{2} \left(b^{2} x\right.}{4 b^{2} \left(2 p^{2}+5 p +3\right)}$

input
```
int(x^3*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(b^2*x^4+2*a*b*x^2+a^2)^p*(-2*b*p*x^2-2*b*x^2+a)*(b*x^2+a)^2/b^2/(2*p
^2+5*p+3)
```

3.93.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.07

$$\int x^3(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p\,dx = \frac{(2\,(b^3p+b^3)x^6+2\,a^2bpx^2+(4\,ab^2p+3\,ab^2)x^4-a^3)(b^2x^4+2\,abx^2+a^2)^p}{4\,(2\,b^2p^2+5\,b^2p+3\,b^2)}$$

input
```
integrate(x^3*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

output
```
1/4*(2*(b^3*p + b^3)*x^6 + 2*a^2*b*p*x^2 + (4*a*b^2*p + 3*a*b^2)*x^4 - a^3
)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(2*b^2*p^2 + 5*b^2*p + 3*b^2)
```

3.93.6 Sympy [F]

$$\int x^3(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p\,dx = \begin{cases} \frac{ax^4(a^2)^p}{4} \\ \int \frac{x^3(a+bx^2)}{\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx \\ -\frac{a\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2b^2}-\frac{a\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2b^2}+\frac{x^2}{2b} \\ -\frac{a^3(a^2+2abx^2+b^2x^4)^p}{8b^2p^2+20b^2p+12b^2}+\frac{2a^2bpx^2(a^2+2abx^2+b^2x^4)^p}{8b^2p^2+20b^2p+12b^2}+\frac{4ab^2px^4(a^2+2abx^2+b^2x^4)^p}{8b^2p^2+20b^2p+12b^2}+\frac{3ab^2x^4(a^2+2abx^2+b^2x^4)^p}{8b^2p^2+20b^2p+12b^2}+\frac{2b^3px^6(a^2+2a}{8b^2p^2+20b} \end{cases}$$

input
```
integrate(x**3*(b*x**2+a)*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

output
```
Piecewise((a*x**4*(a**2)**p/4, Eq(b, 0)), (Integral(x**3*(a + b*x**2)/((a
+ b*x**2)**2)**(3/2), x), Eq(p, -3/2)), (-a*log(x - sqrt(-a/b))/(2*b**2) -
 a*log(x + sqrt(-a/b))/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**3*(a**2 + 2
*a*b*x**2 + b**2*x**4)**p/(8*b**2*p**2 + 20*b**2*p + 12*b**2) + 2*a**2*b*p
*x**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**2*p**2 + 20*b**2*p + 12*b**
2) + 4*a*b**2*p*x**4*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**2*p**2 + 20*
b**2*p + 12*b**2) + 3*a*b**2*x**4*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b*
*2*p**2 + 20*b**2*p + 12*b**2) + 2*b**3*p*x**6*(a**2 + 2*a*b*x**2 + b**2*x
**4)**p/(8*b**2*p**2 + 20*b**2*p + 12*b**2) + 2*b**3*x**6*(a**2 + 2*a*b*x*
*2 + b**2*x**4)**p/(8*b**2*p**2 + 20*b**2*p + 12*b**2), True))
```

3.93.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.57

$$\int x^3(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p\,dx$$
$$=\frac{(b^2(2\,p+1)x^4+2\,abpx^2-a^2)(bx^2+a)^{2\,p}a}{4\,(2\,p^2+3\,p+1)b^2}$$
$$+\frac{((2\,p^2+3\,p+1)b^3x^6+(2\,p^2+p)ab^2x^4-2\,a^2bpx^2+a^3)(bx^2+a)^{2\,p}}{2\,(4\,p^3+12\,p^2+11\,p+3)b^2}$$

input
```
integrate(x^3*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

output
```
1/4*(b^2*(2*p + 1)*x^4 + 2*a*b*p*x^2 - a^2)*(b*x^2 + a)^(2*p)*a/((2*p^2 +
3*p + 1)*b^2) + 1/2*((2*p^2 + 3*p + 1)*b^3*x^6 + (2*p^2 + p)*a*b^2*x^4 - 2
*a^2*b*p*x^2 + a^3)*(b*x^2 + a)^(2*p)/((4*p^3 + 12*p^2 + 11*p + 3)*b^2)
```

3.93.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(82) = 164.

Time = 0.30 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.28

$$\int x^3(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p\,dx$$
$$=\frac{2\,(b^2x^4+2\,abx^2+a^2)^pb^3px^6+2\,(b^2x^4+2\,abx^2+a^2)^pb^3x^6+4\,(b^2x^4+2\,abx^2+a^2)^pab^2px^4+3\,(b^2x^4+2\,a}{4\,(2\,b^2p^2+5\,b^2p+3\,b^2)}$$

input
```
integrate(x^3*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

output
```
1/4*(2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*b^3*p*x^6 + 2*(b^2*x^4 + 2*a*b*x^2 +
a^2)^p*b^3*x^6 + 4*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*b^2*p*x^4 + 3*(b^2*x^4
+ 2*a*b*x^2 + a^2)^p*a*b^2*x^4 + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^2*b*p*x
^2 - (b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^3)/(2*b^2*p^2 + 5*b^2*p + 3*b^2)
```

3.93.9 Mupad [B] (verification not implemented)

Time = 7.93 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.26

$$\int x^3(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p\,dx = \left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^p\left(\frac{b\,x^6\,(p+1)}{2\,\left(2\,p^2+5\,p+3\right)} - \frac{a^3}{4\,b^2\,\left(2\,p^2+5\,p+3\right)} + \frac{a\,x^4\,(4\,p+3)}{4\,\left(2\,p^2+5\,p+3\right)} + \frac{a^2\,p\,x^2}{2\,b\,\left(2\,p^2+5\,p+3\right)}\right)$$

input
```
int(x^3*(a + b*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

output
```
(a^2 + b^2*x^4 + 2*a*b*x^2)^p*((b*x^6*(p + 1))/(2*(5*p + 2*p^2 + 3)) - a^3
/(4*b^2*(5*p + 2*p^2 + 3)) + (a*x^4*(4*p + 3))/(4*(5*p + 2*p^2 + 3)) + (a^
2*p*x^2)/(2*b*(5*p + 2*p^2 + 3)))
```

3.94 $\int x^5(a+bx^2)(a^2+2abx^2+b^2x^4)^p\,dx$

3.94.1 Optimal result

Integrand size = 31, antiderivative size = 128

$$\int x^5(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p\,dx=\frac{a^2(a+bx^2)^2\left(a^2+2abx^2+b^2x^4\right)^p}{4b^3(1+p)}-\frac{a(a+bx^2)^3\left(a^2+2abx^2+b^2x^4\right)^p}{b^3(3+2p)}+\frac{(a+bx^2)^4\left(a^2+2abx^2+b^2x^4\right)^p}{4b^3(2+p)}$$

output
```
1/4*a^2*(b*x^2+a)^2*(b^2*x^4+2*a*b*x^2+a^2)^p/b^3/(p+1)-a*(b*x^2+a)^3*(b^2
*x^4+2*a*b*x^2+a^2)^p/b^3/(3+2*p)+1/4*(b*x^2+a)^4*(b^2*x^4+2*a*b*x^2+a^2)^
p/b^3/(2+p)
```

3.94.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.53

$$\int x^5(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p\,dx=\frac{\left((a+bx^2)^2\right)^{1+p}\left(a^2-2ab(1+p)x^2+b^2(3+5p+2p^2)\,x^4\right)}{4b^3(1+p)(2+p)(3+2p)}$$

input
```
Integrate[x^5*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

output
```
(((a + b*x^2)^2)^(1 + p)*(a^2 - 2*a*b*(1 + p)*x^2 + b^2*(3 + 5*p + 2*p^2)*
x^4))/(4*b^3*(1 + p)*(2 + p)*(3 + 2*p))
```

3.94.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1383, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p\,dx$$

$$\Big\downarrow \text{1383}$$

$$\left(a+bx^2\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\int x^5\left(bx^2+a\right)^{2p+1}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\left(a+bx^2\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\int x^4\left(bx^2+a\right)^{2p+1}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\left(a+bx^2\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\int\left(\frac{a^2\left(bx^2+a\right)^{2p+1}}{b^2}-\frac{2a\left(bx^2+a\right)^{2p+2}}{b^2}+\frac{\left(bx^2+a\right)^{2p+3}}{b^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(a+bx^2\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\left(\frac{a^2\left(a+bx^2\right)^{2(p+1)}}{2b^3(p+1)}+\frac{\left(a+bx^2\right)^{2(p+2)}}{2b^3(p+2)}-\frac{2a\left(a+bx^2\right)^{2p+3}}{b^3(2p+3)}\right)$$

input
```
Int[x^5*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

output
```
((a^2 + 2*a*b*x^2 + b^2*x^4)^p*((a^2*(a + b*x^2)^(2*(1 + p)))/(2*b^3*(1 +
p)) + (a + b*x^2)^(2*(2 + p))/(2*b^3*(2 + p)) - (2*a*(a + b*x^2)^(3 + 2*p)
)/(b^3*(3 + 2*p))))/(2*(a + b*x^2)^(2*p))
```

3.94.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1383
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e_.)*(
x_)^(n_))^(q_.), x_Symbol] :> Simp[(a + b*x^n + c*x^(2*n))^p/(d + e*x^n)^(2
*p)   Int[u*(d + e*x^n)^(q + 2*p), x], x] /; FreeQ[{a, b, c, d, e, n, p, q}
, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &&  !Int
egerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.94.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.77

method	result
gosper	$\frac{\left(b\,x^{2}+a\right)^{2}\left(2b^{2}p^{2}x^{4}+5b^{2}p\,x^{4}+3b^{2}x^{4}-2abp\,x^{2}-2ab\,x^{2}+a^{2}\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{p}}{4b^{3}\left(2p^{3}+9p^{2}+13p+6\right)}$
risch	$\frac{\left(2b^{4}p^{2}x^{8}+5b^{4}p\,x^{8}+4a\,b^{3}p^{2}x^{6}+3b^{4}x^{8}+8a\,b^{3}p\,x^{6}+2a^{2}b^{2}p^{2}x^{4}+4a\,b^{3}x^{6}+a^{2}b^{2}p\,x^{4}-2a^{3}p\,x^{2}b+a^{4}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{p}}{4(3+2p)(2+p)(1+p)b^{3}}$
norman	$\frac{a(1+p)x^{6}\mathrm{e}^{p\ln\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)}}{2p^{2}+7p+6}+\frac{a^{4}\mathrm{e}^{p\ln\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)}}{4b^{3}\left(2p^{3}+9p^{2}+13p+6\right)}+\frac{b\,x^{8}\mathrm{e}^{p\ln\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)}}{8+4p}-\frac{p\,a^{3}x^{2}\mathrm{e}^{p\ln\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)}}{2b^{2}\left(2p^{3}+9p^{2}+13p+6\right)}$
parallelrisch	$2x^{8}\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{p}a\,b^{4}p^{2}+5x^{8}\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{p}a\,b^{4}p+3x^{8}\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{p}a\,b^{4}+4x^{6}\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{p}a^{2}b^{3}p^{2}+$

input
```
int(x^5*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x,method=_RETURNVERBOSE)
```

output
```
1/4*(b*x^2+a)^2*(2*b^2*p^2*x^4+5*b^2*p*x^4+3*b^2*x^4-2*a*b*p*x^2-2*a*b*x^2
+a^2)*(b^2*x^4+2*a*b*x^2+a^2)^p/b^3/(2*p^3+9*p^2+13*p+6)
```

3.94.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.09

$$\int x^5(a+bx^2)(a^2+2abx^2+b^2x^4)^p\,dx = \frac{((2b^4p^2+5b^4p+3b^4)x^8-2a^3bpx^2+4(ab^3p^2+2ab^3p+ab^3)x^6+(2a^2b^2p^2+a^2b^2p)x^4+a^4)(b^2x^4+2a}{4(2b^3p^3+9b^3p^2+13b^3p+6b^3)}$$

input
```
integrate(x^5*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

output
```
1/4*((2*b^4*p^2 + 5*b^4*p + 3*b^4)*x^8 - 2*a^3*b*p*x^2 + 4*(a*b^3*p^2 + 2*
a*b^3*p + a*b^3)*x^6 + (2*a^2*b^2*p^2 + a^2*b^2*p)*x^4 + a^4)*(b^2*x^4 + 2
*a*b*x^2 + a^2)^p/(2*b^3*p^3 + 9*b^3*p^2 + 13*b^3*p + 6*b^3)
```

3.94.6 Sympy [F]

$$\int x^5(a+bx^2)(a^2+2abx^2+b^2x^4)^p\,dx = \begin{cases} \frac{ax^6(a^2)^p}{6} \\ \frac{2a^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4}+\frac{2a^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4}+\frac{3a^2}{4a^2b^3+8ab^4x^2+4b^5x^4}+\frac{4abx^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4}+\frac{4abx^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4}+\frac{}{4a^2b^3+} \\ \int\frac{x^5(a+bx^2)}{\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx \\ \frac{a^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2b^3}+\frac{a^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2b^3}-\frac{ax^2}{2b^2}+\frac{x^4}{4b} \\ \frac{a^4(a^2+2abx^2+b^2x^4)^p}{8b^3p^3+36b^3p^2+52b^3p+24b^3}-\frac{2a^3bpx^2(a^2+2abx^2+b^2x^4)^p}{8b^3p^3+36b^3p^2+52b^3p+24b^3}+\frac{2a^2b^2p^2x^4(a^2+2abx^2+b^2x^4)^p}{8b^3p^3+36b^3p^2+52b^3p+24b^3}+\frac{a^2b^2px^4(a^2+2abx^2+b^2x^4)^p}{8b^3p^3+36b^3p^2+52b^3p+24b^3}+\frac{4ab^3p^2x}{8b^3p^3+} \end{cases}$$

input
```
integrate(x**5*(b*x**2+a)*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

output
```
Piecewise((a*x**6*(a**2)**p/6, Eq(b, 0)), (2*a**2*log(x - sqrt(-a/b))/(4*a
**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(x + sqrt(-a/b))/(4*a*
*2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 3*a**2/(4*a**2*b**3 + 8*a*b**4*x*
*2 + 4*b**5*x**4) + 4*a*b*x**2*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4
*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b
**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5
*x**4) + 2*b**2*x**4*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*
b**5*x**4) + 2*b**2*x**4*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2
+ 4*b**5*x**4), Eq(p, -2)), (Integral(x**5*(a + b*x**2)/((a + b*x**2)**2)*
*(3/2), x), Eq(p, -3/2)), (a**2*log(x - sqrt(-a/b))/(2*b**3) + a**2*log(x
+ sqrt(-a/b))/(2*b**3) - a*x**2/(2*b**2) + x**4/(4*b), Eq(p, -1)), (a**4*(
a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 36*b**3*p**2 + 52*b**3*p
+ 24*b**3) - 2*a**3*b*p*x**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p*
*3 + 36*b**3*p**2 + 52*b**3*p + 24*b**3) + 2*a**2*b**2*p**2*x**4*(a**2 + 2
*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 36*b**3*p**2 + 52*b**3*p + 24*b**
3) + a**2*b**2*p*x**4*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 36
*b**3*p**2 + 52*b**3*p + 24*b**3) + 4*a*b**3*p**2*x**6*(a**2 + 2*a*b*x**2
+ b**2*x**4)**p/(8*b**3*p**3 + 36*b**3*p**2 + 52*b**3*p + 24*b**3) + 8*a*b
**3*p*x**6*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 36*b**3*p**2
+ 52*b**3*p + 24*b**3) + 4*a*b**3*x**6*(a**2 + 2*a*b*x**2 + b**2*x**4)*...
```

3.94.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.53

$$\int x^5(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p\,dx$$
$$=\frac{((2\,p^2+3\,p+1)b^3x^6+(2\,p^2+p)ab^2x^4-2\,a^2bpx^2+a^3)(bx^2+a)^{2\,p}a}{2\,(4\,p^3+12\,p^2+11\,p+3)b^3}$$
$$+\frac{((4\,p^3+12\,p^2+11\,p+3)b^4x^8+2\,(2\,p^3+3\,p^2+p)ab^3x^6-3\,(2\,p^2+p)a^2b^2x^4+6\,a^3bpx^2-3\,a^4)(bx^2+a}{4\,(4\,p^4+20\,p^3+35\,p^2+25\,p+6)b^3}$$

input
```
integrate(x^5*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

output
```
1/2*((2*p^2 + 3*p + 1)*b^3*x^6 + (2*p^2 + p)*a*b^2*x^4 - 2*a^2*b*p*x^2 + a
^3)*(b*x^2 + a)^(2*p)*a/((4*p^3 + 12*p^2 + 11*p + 3)*b^3) + 1/4*((4*p^3 +
12*p^2 + 11*p + 3)*b^4*x^8 + 2*(2*p^3 + 3*p^2 + p)*a*b^3*x^6 - 3*(2*p^2 +
p)*a^2*b^2*x^4 + 6*a^3*b*p*x^2 - 3*a^4)*(b*x^2 + a)^(2*p)/((4*p^4 + 20*p^3
 + 35*p^2 + 25*p + 6)*b^3)
```

3.94.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 331 vs. $2(124) = 248$.

Time = 0.28 (sec) , antiderivative size = 331, normalized size of antiderivative = 2.59

$$\int x^5 (a+bx^2) \left(a^2+2abx^2+b^2x^4\right)^p \, dx$$
$$= \frac{2 \left(b^2x^4+2\,abx^2+a^2\right)^p b^4p^2x^8+5 \left(b^2x^4+2\,abx^2+a^2\right)^p b^4px^8+4 \left(b^2x^4+2\,abx^2+a^2\right)^p ab^3p^2x^6+3 \left(b^2x^4+\right.}{}$$

input
```
integrate(x^5*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

output
```
1/4*(2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*b^4*p^2*x^8 + 5*(b^2*x^4 + 2*a*b*x^2
+ a^2)^p*b^4*p*x^8 + 4*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*b^3*p^2*x^6 + 3*(b^
2*x^4 + 2*a*b*x^2 + a^2)^p*b^4*x^8 + 8*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*b^3
*p*x^6 + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^2*b^2*p^2*x^4 + 4*(b^2*x^4 + 2*
a*b*x^2 + a^2)^p*a*b^3*x^6 + (b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^2*b^2*p*x^4 -
 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^3*b*p*x^2 + (b^2*x^4 + 2*a*b*x^2 + a^2)
^p*a^4)/(2*b^3*p^3 + 9*b^3*p^2 + 13*b^3*p + 6*b^3)
```

3.94.9 Mupad [B] (verification not implemented)

Time = 7.97 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.32

$$\int x^5 (a+bx^2) \left(a^2+2abx^2+b^2x^4\right)^p \, dx$$
$$= \left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^p \left(\frac{a^4}{4\,b^3\,\left(2\,p^3+9\,p^2+13\,p+6\right)} + \frac{a\,x^6\,{\left(p+1\right)}^2}{2\,p^3+9\,p^2+13\,p+6} + \frac{b\,x^8\,\left(2\,p^2+5\,p+3\right)}{4\,\left(2\,p^3+9\,p^2+13\,p+6\right)} - \frac{a^3\,p\,x^2}{2\,b^2\,\left(2\,p^3+9\,p^2+13\,p+6\right)} + \frac{a^2\,p\,x^4\,\left(2\,p+1\right)}{4\,b\,\left(2\,p^3+9\,p^2+13\,p+6\right)}\right)$$

input
```
int(x^5*(a + b*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

output
```
(a^2 + b^2*x^4 + 2*a*b*x^2)^p*(a^4/(4*b^3*(13*p + 9*p^2 + 2*p^3 + 6)) + (a
*x^6*(p + 1)^2)/(13*p + 9*p^2 + 2*p^3 + 6) + (b*x^8*(5*p + 2*p^2 + 3))/(4*
(13*p + 9*p^2 + 2*p^3 + 6)) - (a^3*p*x^2)/(2*b^2*(13*p + 9*p^2 + 2*p^3 + 6
)) + (a^2*p*x^4*(2*p + 1))/(4*b*(13*p + 9*p^2 + 2*p^3 + 6)))
```

3.94. $\int x^5(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p\,dx$

3.95 $\int x^3(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx$

3.95.1 Optimal result

Integrand size = 25, antiderivative size = 166

$$\int x^3(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx = \frac{1}{4}a^3Ax^4+\frac{1}{6}a^2(3Ab+aB)x^6+\frac{3}{8}a\left(abB+A\left(b^2+ac\right)\right)x^8 + \frac{1}{10}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^{10} + \frac{1}{12}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^{12} + \frac{3}{14}c\left(b^2B+Abc+aBc\right)x^{14} + \frac{1}{16}c^2(3bB+Ac)x^{16}+\frac{1}{18}Bc^3x^{18}$$

output
```
1/4*a^3*A*x^4+1/6*a^2*(3*A*b+B*a)*x^6+3/8*a*(a*b*B+A*(a*c+b^2))*x^8+1/10*(
3*a*B*(a*c+b^2)+A*(6*a*b*c+b^3))*x^10+1/12*(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+
B*b^3)*x^12+3/14*c*(A*b*c+B*a*c+B*b^2)*x^14+1/16*c^2*(A*c+3*B*b)*x^16+1/18
*B*c^3*x^18
```

3.95.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00

$$\int x^3(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx=\frac{1}{4}a^3Ax^4+\frac{1}{6}a^2(3Ab+aB)x^6+\frac{3}{8}a\left(abB+A\left(b^2+ac\right)\right)x^8+\frac{1}{10}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^{10}+\frac{1}{12}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^{12}+\frac{3}{14}c\left(b^2B+Abc+aBc\right)x^{14}+\frac{1}{16}c^2(3bB+Ac)x^{16}+\frac{1}{18}Bc^3x^{18}$$

input
```
Integrate[x^3*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

output
```
(a^3*A*x^4)/4 + (a^2*(3*A*b + a*B)*x^6)/6 + (3*a*(a*b*B + A*(b^2 + a*c))*x
^8)/8 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^10)/10 + ((b^3*B + 3*A*
b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x^12)/12 + (3*c*(b^2*B + A*b*c + a*B*c)*x^1
4)/14 + (c^2*(3*b*B + A*c)*x^16)/16 + (B*c^3*x^18)/18
```

3.95.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx$$

$$\Big\downarrow \ 1578$$

$$\frac{1}{2}\int x^2(Bx^2+A)\left(cx^4+bx^2+a\right)^3dx^2$$

$$\Big\downarrow \ 1195$$

$$\frac{1}{2}\int \left(Bc^3x^{16}+c^2(3bB+Ac)x^{14}+3c\left(Bb^2+Acb+aBc\right)x^{12}+\left(Bb^3+3Acb^2+6aBcb+3aAc^2\right)x^{10}+\left(3aB\left(b^2\right.\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{2}a^3Ax^4+\frac{1}{3}a^2x^6(aB+3Ab)+\frac{3}{7}cx^{14}(aBc+Abc+b^2B)+\frac{3}{4}ax^8\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{6}x^{12}\left(3aAc^2+6abBc\right.\right.$$

input
```
Int[x^3*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

output
```
((a^3*A*x^4)/2 + (a^2*(3*A*b + a*B)*x^6)/3 + (3*a*(a*b*B + A*(b^2 + a*c))*
x^8)/4 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^10)/5 + ((b^3*B + 3*A*
b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x^12)/6 + (3*c*(b^2*B + A*b*c + a*B*c)*x^14
)/7 + (c^2*(3*b*B + A*c)*x^16)/8 + (B*c^3*x^18)/9)/2
```

3.95.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.95.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.02

method	result
norman	$\frac{a^3 A x^4}{4}+\left(\frac{1}{2} A a^2 b+\frac{1}{6} B a^3\right) x^6+\left(\frac{3}{8} A c a^2+\frac{3}{8} A a b^2+\frac{3}{8} B a^2 b\right) x^8+\left(\frac{3}{5} A a b c+\frac{1}{10} A b^3+\frac{3}{10} a^2 B\right.$
gosper	$\frac{1}{4} a^3 A x^4+\frac{1}{2} x^6 A a^2 b+\frac{1}{6} x^6 B a^3+\frac{3}{8} x^8 A c a^2+\frac{3}{8} x^8 A a b^2+\frac{3}{8} x^8 B a^2 b+\frac{3}{5} x^{10} A a b c+\frac{1}{10} x^{10} A b^3$
risch	$\frac{1}{4} a^3 A x^4+\frac{1}{2} x^6 A a^2 b+\frac{1}{6} x^6 B a^3+\frac{3}{8} x^8 A c a^2+\frac{3}{8} x^8 A a b^2+\frac{3}{8} x^8 B a^2 b+\frac{3}{5} x^{10} A a b c+\frac{1}{10} x^{10} A b^3$
parallelrisch	$\frac{1}{4} a^3 A x^4+\frac{1}{2} x^6 A a^2 b+\frac{1}{6} x^6 B a^3+\frac{3}{8} x^8 A c a^2+\frac{3}{8} x^8 A a b^2+\frac{3}{8} x^8 B a^2 b+\frac{3}{5} x^{10} A a b c+\frac{1}{10} x^{10} A b^3$
default	$\frac{B c^3 x^{18}}{18}+\frac{\left(A c^3+3 B b c^2\right) x^{16}}{16}+\frac{\left(3 A b c^2+B\left(a c^2+2 b^2 c+c\left(2 a c+b^2\right)\right)\right) x^{14}}{14}+\frac{\left(A\left(a c^2+2 b^2 c+c\left(2 a c+b^2\right)\right)+B\left(4 a b c+b\left(2 a c\right.\right.\right.}{12}$

input
```
int(x^3*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^3*A*x^4+(1/2*A*a^2*b+1/6*B*a^3)*x^6+(3/8*A*c*a^2+3/8*A*a*b^2+3/8*B*a
^2*b)*x^8+(3/5*A*a*b*c+1/10*A*b^3+3/10*a^2*B*c+3/10*B*a*b^2)*x^10+(1/4*A*a
*c^2+1/4*A*b^2*c+1/2*B*a*b*c+1/12*B*b^3)*x^12+(3/14*A*b*c^2+3/14*B*a*c^2+3
/14*B*b^2*c)*x^14+(1/16*A*c^3+3/16*B*b*c^2)*x^16+1/18*B*c^3*x^18
```

3.95.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00

$$\begin{aligned}\int x^3\left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3\,dx &= \frac{1}{18}\,Bc^3x^{18}+\frac{1}{16}\,\left(3\,Bbc^2+Ac^3\right)x^{16}\\&\quad+\frac{3}{14}\,\left(Bb^2c+(Ba+Ab)c^2\right)x^{14}\\&\quad+\frac{1}{12}\,\left(Bb^3+3\,Aac^2+3\,\left(2\,Bab+Ab^2\right)c\right)x^{12}\\&\quad+\frac{1}{10}\,\left(3\,Bab^2+Ab^3+3\,\left(Ba^2+2\,Aab\right)c\right)x^{10}\\&\quad+\frac{3}{8}\,\left(Ba^2b+Aab^2+Aa^2c\right)x^8\\&\quad+\frac{1}{4}\,Aa^3x^4+\frac{1}{6}\,\left(Ba^3+3\,Aa^2b\right)x^6\end{aligned}$$

input
```
integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/18*B*c^3*x^18 + 1/16*(3*B*b*c^2 + A*c^3)*x^16 + 3/14*(B*b^2*c + (B*a + A
*b)*c^2)*x^14 + 1/12*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^12 + 1/
10*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^10 + 3/8*(B*a^2*b + A*a*b
^2 + A*a^2*c)*x^8 + 1/4*A*a^3*x^4 + 1/6*(B*a^3 + 3*A*a^2*b)*x^6
```

3.95.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.22

$$\int x^3(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx = \frac{Aa^3x^4}{4}+\frac{Bc^3x^{18}}{18}+x^{16}\left(\frac{Ac^3}{16}+\frac{3Bbc^2}{16}\right)+x^{14}\cdot\left(\frac{3Abc^2}{14}+\frac{3Bac^2}{14}+\frac{3Bb^2c}{14}\right)+x^{12}\left(\frac{Aac^2}{4}+\frac{Ab^2c}{4}+\frac{Babc}{2}+\frac{Bb^3}{12}\right)+x^{10}\cdot\left(\frac{3Aabc}{5}+\frac{Ab^3}{10}+\frac{3Ba^2c}{10}+\frac{3Bab^2}{10}\right)+x^8\cdot\left(\frac{3Aa^2c}{8}+\frac{3Aab^2}{8}+\frac{3Ba^2b}{8}\right)+x^6\left(\frac{Aa^2b}{2}+\frac{Ba^3}{6}\right)$$

input
```
integrate(x**3*(B*x**2+A)*(c*x**4+b*x**2+a)**3,x)
```

output
```
A*a**3*x**4/4 + B*c**3*x**18/18 + x**16*(A*c**3/16 + 3*B*b*c**2/16) + x**1
4*(3*A*b*c**2/14 + 3*B*a*c**2/14 + 3*B*b**2*c/14) + x**12*(A*a*c**2/4 + A*
b**2*c/4 + B*a*b*c/2 + B*b**3/12) + x**10*(3*A*a*b*c/5 + A*b**3/10 + 3*B*a
**2*c/10 + 3*B*a*b**2/10) + x**8*(3*A*a**2*c/8 + 3*A*a*b**2/8 + 3*B*a**2*b
/8) + x**6*(A*a**2*b/2 + B*a**3/6)
```

3.95.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00

$$\int x^3(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx = \frac{1}{18}Bc^3x^{18}+\frac{1}{16}\left(3Bbc^2+Ac^3\right)x^{16}+\frac{3}{14}\left(Bb^2c+(Ba+Ab)c^2\right)x^{14}+\frac{1}{12}\left(Bb^3+3Aac^2+3\left(2Bab+Ab^2\right)c\right)x^{12}+\frac{1}{10}\left(3Bab^2+Ab^3+3\left(Ba^2+2Aab\right)c\right)x^{10}+\frac{3}{8}\left(Ba^2b+Aab^2+Aa^2c\right)x^8+\frac{1}{4}Aa^3x^4+\frac{1}{6}\left(Ba^3+3Aa^2b\right)x^6$$

input
```
integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/18*B*c^3*x^18 + 1/16*(3*B*b*c^2 + A*c^3)*x^16 + 3/14*(B*b^2*c + (B*a + A
*b)*c^2)*x^14 + 1/12*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^12 + 1/
10*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^10 + 3/8*(B*a^2*b + A*a*b
^2 + A*a^2*c)*x^8 + 1/4*A*a^3*x^4 + 1/6*(B*a^3 + 3*A*a^2*b)*x^6
```

3.95.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.16

$$\int x^3(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx = \frac{1}{18}Bc^3x^{18}+\frac{3}{16}Bbc^2x^{16}+\frac{1}{16}Ac^3x^{16}+\frac{3}{14}Bb^2cx^{14}+\frac{3}{14}Bac^2x^{14}+\frac{3}{14}Abc^2x^{14}+\frac{1}{12}Bb^3x^{12}+\frac{1}{2}Babcx^{12}+\frac{1}{4}Ab^2cx^{12}+\frac{1}{4}Aac^2x^{12}+\frac{3}{10}Bab^2x^{10}+\frac{1}{10}Ab^3x^{10}+\frac{3}{10}Ba^2cx^{10}+\frac{3}{5}Aabcx^{10}+\frac{3}{8}Ba^2bx^8+\frac{3}{8}Aab^2x^8+\frac{3}{8}Aa^2cx^8+\frac{1}{6}Ba^3x^6+\frac{1}{2}Aa^2bx^6+\frac{1}{4}Aa^3x^4$$

input
```
integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/18*B*c^3*x^18 + 3/16*B*b*c^2*x^16 + 1/16*A*c^3*x^16 + 3/14*B*b^2*c*x^14
+ 3/14*B*a*c^2*x^14 + 3/14*A*b*c^2*x^14 + 1/12*B*b^3*x^12 + 1/2*B*a*b*c*x^
12 + 1/4*A*b^2*c*x^12 + 1/4*A*a*c^2*x^12 + 3/10*B*a*b^2*x^10 + 1/10*A*b^3*
x^10 + 3/10*B*a^2*c*x^10 + 3/5*A*a*b*c*x^10 + 3/8*B*a^2*b*x^8 + 3/8*A*a*b^
2*x^8 + 3/8*A*a^2*c*x^8 + 1/6*B*a^3*x^6 + 1/2*A*a^2*b*x^6 + 1/4*A*a^3*x^4
```

3.95.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.02

$$\int x^3(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx = x^{10}\left(\frac{3\,B\,c\,a^2}{10}+\frac{3\,B\,a\,b^2}{10}+\frac{3\,A\,c\,a\,b}{5}+\frac{A\,b^3}{10}\right) + x^{12}\left(\frac{B\,b^3}{12}+\frac{A\,b^2\,c}{4}+\frac{B\,a\,b\,c}{2}+\frac{A\,a\,c^2}{4}\right) + x^6\left(\frac{B\,a^3}{6}+\frac{A\,b\,a^2}{2}\right) + x^{16}\left(\frac{A\,c^3}{16}+\frac{3\,B\,b\,c^2}{16}\right) + x^8\left(\frac{3\,B\,a^2\,b}{8}+\frac{3\,A\,c\,a^2}{8}+\frac{3\,A\,a\,b^2}{8}\right) + x^{14}\left(\frac{3\,B\,b^2\,c}{14}+\frac{3\,A\,b\,c^2}{14}+\frac{3\,B\,a\,c^2}{14}\right) + \frac{A\,a^3\,x^4}{4} + \frac{B\,c^3\,x^{18}}{18}$$

input
```
int(x^3*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x)
```

output
```
x^10*((A*b^3)/10 + (3*B*a*b^2)/10 + (3*B*a^2*c)/10 + (3*A*a*b*c)/5) + x^12
*((B*b^3)/12 + (A*a*c^2)/4 + (A*b^2*c)/4 + (B*a*b*c)/2) + x^6*((B*a^3)/6 +
 (A*a^2*b)/2) + x^16*((A*c^3)/16 + (3*B*b*c^2)/16) + x^8*((3*A*a*b^2)/8 +
(3*A*a^2*c)/8 + (3*B*a^2*b)/8) + x^14*((3*A*b*c^2)/14 + (3*B*a*c^2)/14 + (
3*B*b^2*c)/14) + (A*a^3*x^4)/4 + (B*c^3*x^18)/18
```

3.96 $\int x^2(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx$

3.96.1 Optimal result

Integrand size = 25, antiderivative size = 166

$$\begin{aligned}\int x^2(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx =& \frac{1}{3}a^3Ax^3+\frac{1}{5}a^2(3Ab+aB)x^5+\frac{3}{7}a\left(abB+A\left(b^2+ac\right)\right)x^7\\&+\frac{1}{9}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^9\\&+\frac{1}{11}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^{11}\\&+\frac{3}{13}c\left(b^2B+Abc+aBc\right)x^{13}\\&+\frac{1}{15}c^2(3bB+Ac)x^{15}+\frac{1}{17}Bc^3x^{17}\end{aligned}$$

output
```
1/3*a^3*A*x^3+1/5*a^2*(3*A*b+B*a)*x^5+3/7*a*(a*b*B+A*(a*c+b^2))*x^7+1/9*(3
*a*B*(a*c+b^2)+A*(6*a*b*c+b^3))*x^9+1/11*(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*
b^3)*x^11+3/13*c*(A*b*c+B*a*c+B*b^2)*x^13+1/15*c^2*(A*c+3*B*b)*x^15+1/17*B
*c^3*x^17
```

3.96.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00

$$\int x^2(A+Bx^2)(a+bx^2+cx^4)^3\,dx = \frac{1}{3}a^3Ax^3+\frac{1}{5}a^2(3Ab+aB)x^5+\frac{3}{7}a(abB+A(b^2+ac))x^7 + \frac{1}{9}(3aB(b^2+ac)+A(b^3+6abc))x^9 + \frac{1}{11}(b^3B+3Ab^2c+6abBc+3aAc^2)x^{11} + \frac{3}{13}c(b^2B+Abc+aBc)x^{13} + \frac{1}{15}c^2(3bB+Ac)x^{15}+\frac{1}{17}Bc^3x^{17}$$

input
```
Integrate[x^2*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

output
```
(a^3*A*x^3)/3 + (a^2*(3*A*b + a*B)*x^5)/5 + (3*a*(a*b*B + A*(b^2 + a*c))*x
^7)/7 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^9)/9 + ((b^3*B + 3*A*b^
2*c + 6*a*b*B*c + 3*a*A*c^2)*x^11)/11 + (3*c*(b^2*B + A*b*c + a*B*c)*x^13)
/13 + (c^2*(3*b*B + A*c)*x^15)/15 + (B*c^3*x^17)/17
```

3.96.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(A+Bx^2)(a+bx^2+cx^4)^3\,dx$$

$$\Big\downarrow \text{1584}$$

$$\int (a^3Ax^2+a^2x^4(aB+3Ab)+3cx^{12}(aBc+Abc+b^2B)+3ax^6(A(ac+b^2)+abB)+x^{10}(3aAc^2+6abBc+3A$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}a^3Ax^3+\frac{1}{5}a^2x^5(aB+3Ab)+\frac{3}{13}cx^{13}(aBc+Abc+b^2B)+\frac{3}{7}ax^7\left(A(ac+b^2)+abB\right)+\frac{1}{11}x^{11}(3aAc^2+6abBc+3Ab^2c+b^3B)+\frac{1}{9}x^9\left(A(6abc+b^3)+3aB(ac+b^2)\right)+\frac{1}{15}c^2x^{15}(Ac+3bB)+\frac{1}{17}Bc^3x^{17}$$

input
```
Int[x^2*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

output
```
(a^3*A*x^3)/3 + (a^2*(3*A*b + a*B)*x^5)/5 + (3*a*(a*b*B + A*(b^2 + a*c))*x
^7)/7 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^9)/9 + ((b^3*B + 3*A*b^
2*c + 6*a*b*B*c + 3*a*A*c^2)*x^11)/11 + (3*c*(b^2*B + A*b*c + a*B*c)*x^13)
/13 + (c^2*(3*b*B + A*c)*x^15)/15 + (B*c^3*x^17)/17
```

3.96.3.1 Defintions of rubi rules used

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.96.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.02

method	result
norman	$\frac{a^3 A x^3}{3}+\left(\frac{3}{5}A\,a^2b+\frac{1}{5}B\,a^3\right)x^5+\left(\frac{3}{7}Ac\,a^2+\frac{3}{7}Aa\,b^2+\frac{3}{7}B\,a^2b\right)x^7+\left(\frac{2}{3}Aabc+\frac{1}{9}A\,b^3+\frac{1}{3}a^2Bc\right.$
gosper	$\frac{1}{3}a^3A\,x^3+\frac{3}{5}x^5A\,a^2b+\frac{1}{5}x^5B\,a^3+\frac{3}{7}x^7Ac\,a^2+\frac{3}{7}x^7Aa\,b^2+\frac{3}{7}x^7B\,a^2b+\frac{2}{3}x^9Aabc+\frac{1}{9}x^9A\,b^3+$
risch	$\frac{1}{3}a^3A\,x^3+\frac{3}{5}x^5A\,a^2b+\frac{1}{5}x^5B\,a^3+\frac{3}{7}x^7Ac\,a^2+\frac{3}{7}x^7Aa\,b^2+\frac{3}{7}x^7B\,a^2b+\frac{2}{3}x^9Aabc+\frac{1}{9}x^9A\,b^3+$
parallelrisch	$\frac{1}{3}a^3A\,x^3+\frac{3}{5}x^5A\,a^2b+\frac{1}{5}x^5B\,a^3+\frac{3}{7}x^7Ac\,a^2+\frac{3}{7}x^7Aa\,b^2+\frac{3}{7}x^7B\,a^2b+\frac{2}{3}x^9Aabc+\frac{1}{9}x^9A\,b^3+$
default	$\frac{B\,c^3x^{17}}{17}+\frac{(A\,c^3+3Bb\,c^2)x^{15}}{15}+\frac{(3Ab\,c^2+B(a\,c^2+2b^2c+c(2ac+b^2)))x^{13}}{13}+\frac{(A(a\,c^2+2b^2c+c(2ac+b^2))+B(4abc+b(2ac\ldots}{11}$

input
```
int(x^2*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^3*A*x^3+(3/5*A*a^2*b+1/5*B*a^3)*x^5+(3/7*A*c*a^2+3/7*A*a*b^2+3/7*B*a
^2*b)*x^7+(2/3*A*a*b*c+1/9*A*b^3+1/3*a^2*B*c+1/3*B*a*b^2)*x^9+(3/11*A*a*c^
2+3/11*A*b^2*c+6/11*B*a*b*c+1/11*B*b^3)*x^11+(3/13*A*b*c^2+3/13*B*a*c^2+3/
13*B*b^2*c)*x^13+(1/15*A*c^3+1/5*B*b*c^2)*x^15+1/17*B*c^3*x^17
```

3.96.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00

$$\begin{aligned}\int x^2(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx &= \frac{1}{17}\,Bc^3x^{17}+\frac{1}{15}\left(3\,Bbc^2+Ac^3\right)x^{15}\\ &+\frac{3}{13}\left(Bb^2c+(Ba+Ab)c^2\right)x^{13}\\ &+\frac{1}{11}\left(Bb^3+3\,Aac^2+3\left(2\,Bab+Ab^2\right)c\right)x^{11}\\ &+\frac{1}{9}\left(3\,Bab^2+Ab^3+3\left(Ba^2+2\,Aab\right)c\right)x^9\\ &+\frac{3}{7}\left(Ba^2b+Aab^2+Aa^2c\right)x^7\\ &+\frac{1}{3}\,Aa^3x^3+\frac{1}{5}\left(Ba^3+3\,Aa^2b\right)x^5\end{aligned}$$

input
```
integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/17*B*c^3*x^17 + 1/15*(3*B*b*c^2 + A*c^3)*x^15 + 3/13*(B*b^2*c + (B*a + A
*b)*c^2)*x^13 + 1/11*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^11 + 1/
9*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^9 + 3/7*(B*a^2*b + A*a*b^2
 + A*a^2*c)*x^7 + 1/3*A*a^3*x^3 + 1/5*(B*a^3 + 3*A*a^2*b)*x^5
```

3.96.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.23

$$\int x^2(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx = \frac{Aa^3x^3}{3}+\frac{Bc^3x^{17}}{17}+x^{15}\left(\frac{Ac^3}{15}+\frac{Bbc^2}{5}\right) + x^{13}\cdot\left(\frac{3Abc^2}{13}+\frac{3Bac^2}{13}+\frac{3Bb^2c}{13}\right)+x^{11} \cdot\left(\frac{3Aac^2}{11}+\frac{3Ab^2c}{11}+\frac{6Babc}{11}+\frac{Bb^3}{11}\right)+x^9 \cdot\left(\frac{2Aabc}{3}+\frac{Ab^3}{9}+\frac{Ba^2c}{3}+\frac{Bab^2}{3}\right)+x^7 \cdot\left(\frac{3Aa^2c}{7}+\frac{3Aab^2}{7}+\frac{3Ba^2b}{7}\right)+x^5\cdot\left(\frac{3Aa^2b}{5}+\frac{Ba^3}{5}\right)$$

input
```
integrate(x**2*(B*x**2+A)*(c*x**4+b*x**2+a)**3,x)
```

output
```
A*a**3*x**3/3 + B*c**3*x**17/17 + x**15*(A*c**3/15 + B*b*c**2/5) + x**13*(
3*A*b*c**2/13 + 3*B*a*c**2/13 + 3*B*b**2*c/13) + x**11*(3*A*a*c**2/11 + 3*
A*b**2*c/11 + 6*B*a*b*c/11 + B*b**3/11) + x**9*(2*A*a*b*c/3 + A*b**3/9 + B
*a**2*c/3 + B*a*b**2/3) + x**7*(3*A*a**2*c/7 + 3*A*a*b**2/7 + 3*B*a**2*b/7
) + x**5*(3*A*a**2*b/5 + B*a**3/5)
```

3.96.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00

$$\int x^2(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx = \frac{1}{17}\,Bc^3x^{17}+\frac{1}{15}\,\left(3\,Bbc^2+Ac^3\right)x^{15} + \frac{3}{13}\,\left(Bb^2c+(Ba+Ab)c^2\right)x^{13} + \frac{1}{11}\,\left(Bb^3+3\,Aac^2+3\,\left(2\,Bab+Ab^2\right)c\right)x^{11} + \frac{1}{9}\,\left(3\,Bab^2+Ab^3+3\,\left(Ba^2+2\,Aab\right)c\right)x^9 + \frac{3}{7}\,\left(Ba^2b+Aab^2+Aa^2c\right)x^7 + \frac{1}{3}\,Aa^3x^3+\frac{1}{5}\,\left(Ba^3+3\,Aa^2b\right)x^5$$

input
```
integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/17*B*c^3*x^17 + 1/15*(3*B*b*c^2 + A*c^3)*x^15 + 3/13*(B*b^2*c + (B*a + A
*b)*c^2)*x^13 + 1/11*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^11 + 1/
9*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^9 + 3/7*(B*a^2*b + A*a*b^2
 + A*a^2*c)*x^7 + 1/3*A*a^3*x^3 + 1/5*(B*a^3 + 3*A*a^2*b)*x^5
```

3.96.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.16

$$\int x^2(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx = \frac{1}{17}Bc^3x^{17}+\frac{1}{5}Bbc^2x^{15}+\frac{1}{15}Ac^3x^{15}+\frac{3}{13}Bb^2cx^{13}+\frac{3}{13}Bac^2x^{13}+\frac{3}{13}Abc^2x^{13}+\frac{1}{11}Bb^3x^{11}+\frac{6}{11}Babcx^{11}+\frac{3}{11}Ab^2cx^{11}+\frac{3}{11}Aac^2x^{11}+\frac{1}{3}Bab^2x^9+\frac{1}{9}Ab^3x^9+\frac{1}{3}Ba^2cx^9+\frac{2}{3}Aabcx^9+\frac{3}{7}Ba^2bx^7+\frac{3}{7}Aab^2x^7+\frac{3}{7}Aa^2cx^7+\frac{1}{5}Ba^3x^5+\frac{3}{5}Aa^2bx^5+\frac{1}{3}Aa^3x^3$$

input
```
integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/17*B*c^3*x^17 + 1/5*B*b*c^2*x^15 + 1/15*A*c^3*x^15 + 3/13*B*b^2*c*x^13 +
 3/13*B*a*c^2*x^13 + 3/13*A*b*c^2*x^13 + 1/11*B*b^3*x^11 + 6/11*B*a*b*c*x^
11 + 3/11*A*b^2*c*x^11 + 3/11*A*a*c^2*x^11 + 1/3*B*a*b^2*x^9 + 1/9*A*b^3*x
^9 + 1/3*B*a^2*c*x^9 + 2/3*A*a*b*c*x^9 + 3/7*B*a^2*b*x^7 + 3/7*A*a*b^2*x^7
 + 3/7*A*a^2*c*x^7 + 1/5*B*a^3*x^5 + 3/5*A*a^2*b*x^5 + 1/3*A*a^3*x^3
```

3.96.9 Mupad [B] (verification not implemented)

Time = 7.90 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.02

$$\int x^2(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx = x^9\left(\frac{B\,c\,a^2}{3}+\frac{B\,a\,b^2}{3}+\frac{2\,A\,c\,a\,b}{3}+\frac{A\,b^3}{9}\right) + x^{11}\left(\frac{B\,b^3}{11}+\frac{3\,A\,b^2\,c}{11}+\frac{6\,B\,a\,b\,c}{11}+\frac{3\,A\,a\,c^2}{11}\right) + x^5\left(\frac{B\,a^3}{5}+\frac{3\,A\,b\,a^2}{5}\right)+x^{15}\left(\frac{A\,c^3}{15}+\frac{B\,b\,c^2}{5}\right) + x^7\left(\frac{3\,B\,a^2\,b}{7}+\frac{3\,A\,c\,a^2}{7}+\frac{3\,A\,a\,b^2}{7}\right) + x^{13}\left(\frac{3\,B\,b^2\,c}{13}+\frac{3\,A\,b\,c^2}{13}+\frac{3\,B\,a\,c^2}{13}\right) + \frac{A\,a^3\,x^3}{3}+\frac{B\,c^3\,x^{17}}{17}$$

input
```
int(x^2*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x)
```

output
```
x^9*((A*b^3)/9 + (B*a*b^2)/3 + (B*a^2*c)/3 + (2*A*a*b*c)/3) + x^11*((B*b^3
)/11 + (3*A*a*c^2)/11 + (3*A*b^2*c)/11 + (6*B*a*b*c)/11) + x^5*((B*a^3)/5
+ (3*A*a^2*b)/5) + x^15*((A*c^3)/15 + (B*b*c^2)/5) + x^7*((3*A*a*b^2)/7 +
(3*A*a^2*c)/7 + (3*B*a^2*b)/7) + x^13*((3*A*b*c^2)/13 + (3*B*a*c^2)/13 + (
3*B*b^2*c)/13) + (A*a^3*x^3)/3 + (B*c^3*x^17)/17
```

3.97 $\int x(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx$

3.97.1 Optimal result

Integrand size = 23, antiderivative size = 166

$$\begin{aligned}\int x(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx &= \frac{1}{2}a^3Ax^2+\frac{1}{4}a^2(3Ab+aB)x^4+\frac{1}{2}a\left(abB+A\left(b^2+ac\right)\right)x^6\\&\quad+\frac{1}{8}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^8\\&\quad+\frac{1}{10}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^{10}\\&\quad+\frac{1}{4}c\left(b^2B+Abc+aBc\right)x^{12}\\&\quad+\frac{1}{14}c^2(3bB+Ac)x^{14}+\frac{1}{16}Bc^3x^{16}\end{aligned}$$

output
```
1/2*a^3*A*x^2+1/4*a^2*(3*A*b+B*a)*x^4+1/2*a*(a*b*B+A*(a*c+b^2))*x^6+1/8*(3
*a*B*(a*c+b^2)+A*(6*a*b*c+b^3))*x^8+1/10*(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*
b^3)*x^10+1/4*c*(A*b*c+B*a*c+B*b^2)*x^12+1/14*c^2*(A*c+3*B*b)*x^14+1/16*B*
c^3*x^16
```

3.97.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.93

$$\int x(A+Bx^2)(a+bx^2+cx^4)^3\,dx = \frac{1}{560}x^2\big(280a^3A+140a^2(3Ab+aB)x^2 + 280a\big(abB+A(b^2+ac)\big)x^4 + 70\big(3aB(b^2+ac)+A(b^3+6abc)\big)x^6 + 56\big(b^3B+3Ab^2c+6abBc+3aAc^2\big)x^8 + 140c\big(b^2B+Abc+aBc\big)x^{10}+40c^2(3bB+Ac)x^{12} + 35Bc^3x^{14}\big)$$

input
```
Integrate[x*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

output
```
(x^2*(280*a^3*A + 140*a^2*(3*A*b + a*B)*x^2 + 280*a*(a*b*B + A*(b^2 + a*c)
)*x^4 + 70*(3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^6 + 56*(b^3*B + 3*A*b
^2*c + 6*a*b*B*c + 3*a*A*c^2)*x^8 + 140*c*(b^2*B + A*b*c + a*B*c)*x^10 + 4
0*c^2*(3*b*B + A*c)*x^12 + 35*B*c^3*x^14))/560
```

3.97.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1576, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(A+Bx^2)(a+bx^2+cx^4)^3\,dx$$

$$\Big\downarrow \text{1576}$$

$$\frac{1}{2}\int (Bx^2+A)(cx^4+bx^2+a)^3\,dx^2$$

$$\Big\downarrow \text{1140}$$

$$\frac{1}{2}\int \big(Bc^3x^{14}+c^2(3bB+Ac)x^{12}+3c\big(Bb^2+Acb+aBc\big)x^{10}+\big(Bb^3+3Acb^2+6aBcb+3aAc^2\big)x^8+\big(3aB(b^2+$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(a^3Ax^2+\frac{1}{2}a^2x^4(aB+3Ab)+\frac{1}{2}cx^{12}(aBc+Abc+b^2B)+ax^6\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{5}x^{10}\left(3aAc^2+6abBc+\right.\right.$$

input
```
Int[x*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

output
```
(a^3*A*x^2 + (a^2*(3*A*b + a*B)*x^4)/2 + a*(a*b*B + A*(b^2 + a*c))*x^6 + (
(3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^8)/4 + ((b^3*B + 3*A*b^2*c + 6*a
*b*B*c + 3*a*A*c^2)*x^10)/5 + (c*(b^2*B + A*b*c + a*B*c)*x^12)/2 + (c^2*(3
*b*B + A*c)*x^14)/7 + (B*c^3*x^16)/8)/2
```

3.97.3.1 Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.97.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.02

method	result
norman	$\frac{a^3Ax^2}{2}+\left(\frac{3}{4}Aa^2b+\frac{1}{4}Ba^3\right)x^4+\left(\frac{1}{2}Aca^2+\frac{1}{2}Aab^2+\frac{1}{2}Ba^2b\right)x^6+\left(\frac{3}{4}Aabc+\frac{1}{8}Ab^3+\frac{3}{8}a^2Bc\right.$
gosper	$\frac{1}{2}a^3Ax^2+\frac{3}{4}x^4Aa^2b+\frac{1}{4}x^4Ba^3+\frac{1}{2}x^6Aca^2+\frac{1}{2}x^6Aab^2+\frac{1}{2}x^6Ba^2b+\frac{3}{4}x^8Aabc+\frac{1}{8}x^8Ab^3+$
risch	$\frac{1}{2}a^3Ax^2+\frac{3}{4}x^4Aa^2b+\frac{1}{4}x^4Ba^3+\frac{1}{2}x^6Aca^2+\frac{1}{2}x^6Aab^2+\frac{1}{2}x^6Ba^2b+\frac{3}{4}x^8Aabc+\frac{1}{8}x^8Ab^3+$
parallelrisch	$\frac{1}{2}a^3Ax^2+\frac{3}{4}x^4Aa^2b+\frac{1}{4}x^4Ba^3+\frac{1}{2}x^6Aca^2+\frac{1}{2}x^6Aab^2+\frac{1}{2}x^6Ba^2b+\frac{3}{4}x^8Aabc+\frac{1}{8}x^8Ab^3+$
default	$\frac{Bc^3x^{16}}{16}+\frac{(Ac^3+3Bbc^2)x^{14}}{14}+\frac{(3Abc^2+B(ac^2+2b^2c+c(2ac+b^2)))x^{12}}{12}+\frac{(A(ac^2+2b^2c+c(2ac+b^2))+B(4abc+b(2ac\right.}{10}$

input
```
int(x*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^3*A*x^2+(3/4*A*a^2*b+1/4*B*a^3)*x^4+(1/2*A*c*a^2+1/2*A*a*b^2+1/2*B*a
^2*b)*x^6+(3/4*A*a*b*c+1/8*A*b^3+3/8*a^2*B*c+3/8*B*a*b^2)*x^8+(3/10*A*a*c^
2+3/10*A*b^2*c+3/5*B*a*b*c+1/10*B*b^3)*x^10+(1/4*A*b*c^2+1/4*B*a*c^2+1/4*B
*b^2*c)*x^12+(1/14*A*c^3+3/14*B*b*c^2)*x^14+1/16*B*c^3*x^16
```

3.97.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00

$$
\begin{aligned}
\int x(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx =& \frac{1}{16}Bc^3x^{16}+\frac{1}{14}\left(3Bbc^2+Ac^3\right)x^{14} \\
&+\frac{1}{4}\left(Bb^2c+(Ba+Ab)c^2\right)x^{12} \\
&+\frac{1}{10}\left(Bb^3+3Aac^2+3\left(2Bab+Ab^2\right)c\right)x^{10} \\
&+\frac{1}{8}\left(3Bab^2+Ab^3+3\left(Ba^2+2Aab\right)c\right)x^8 \\
&+\frac{1}{2}\left(Ba^2b+Aab^2+Aa^2c\right)x^6 \\
&+\frac{1}{2}Aa^3x^2+\frac{1}{4}\left(Ba^3+3Aa^2b\right)x^4
\end{aligned}
$$

input
```
integrate(x*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/16*B*c^3*x^16 + 1/14*(3*B*b*c^2 + A*c^3)*x^14 + 1/4*(B*b^2*c + (B*a + A*
b)*c^2)*x^12 + 1/10*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^10 + 1/8
*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^8 + 1/2*(B*a^2*b + A*a*b^2
+ A*a^2*c)*x^6 + 1/2*A*a^3*x^2 + 1/4*(B*a^3 + 3*A*a^2*b)*x^4
```

3.97.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.20

$$\int x(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx = \frac{Aa^3x^2}{2}+\frac{Bc^3x^{16}}{16}+x^{14}\left(\frac{Ac^3}{14}+\frac{3Bbc^2}{14}\right)+x^{12}\left(\frac{Abc^2}{4}+\frac{Bac^2}{4}+\frac{Bb^2c}{4}\right)+x^{10}\cdot\left(\frac{3Aac^2}{10}+\frac{3Ab^2c}{10}+\frac{3Babc}{5}+\frac{Bb^3}{10}\right)+x^8\cdot\left(\frac{3Aabc}{4}+\frac{Ab^3}{8}+\frac{3Ba^2c}{8}+\frac{3Bab^2}{8}\right)+x^6\left(\frac{Aa^2c}{2}+\frac{Aab^2}{2}+\frac{Ba^2b}{2}\right)+x^4\cdot\left(\frac{3Aa^2b}{4}+\frac{Ba^3}{4}\right)$$

input
```
integrate(x*(B*x**2+A)*(c*x**4+b*x**2+a)**3,x)
```

output
```
A*a**3*x**2/2 + B*c**3*x**16/16 + x**14*(A*c**3/14 + 3*B*b*c**2/14) + x**1
2*(A*b*c**2/4 + B*a*c**2/4 + B*b**2*c/4) + x**10*(3*A*a*c**2/10 + 3*A*b**2
*c/10 + 3*B*a*b*c/5 + B*b**3/10) + x**8*(3*A*a*b*c/4 + A*b**3/8 + 3*B*a**2
*c/8 + 3*B*a*b**2/8) + x**6*(A*a**2*c/2 + A*a*b**2/2 + B*a**2*b/2) + x**4*
(3*A*a**2*b/4 + B*a**3/4)
```

3.97.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00

$$\int x(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx = \frac{1}{16}\,Bc^3x^{16}+\frac{1}{14}\,(3\,Bbc^2+Ac^3)x^{14}+\frac{1}{4}\,(Bb^2c+(Ba+Ab)c^2)x^{12}+\frac{1}{10}\,(Bb^3+3\,Aac^2+3\,(2\,Bab+Ab^2)c)x^{10}+\frac{1}{8}\,(3\,Bab^2+Ab^3+3\,(Ba^2+2\,Aab)c)x^8+\frac{1}{2}\,(Ba^2b+Aab^2+Aa^2c)x^6+\frac{1}{2}\,Aa^3x^2+\frac{1}{4}\,(Ba^3+3\,Aa^2b)x^4$$

input
```
integrate(x*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/16*B*c^3*x^16 + 1/14*(3*B*b*c^2 + A*c^3)*x^14 + 1/4*(B*b^2*c + (B*a + A*
b)*c^2)*x^12 + 1/10*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^10 + 1/8
*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^8 + 1/2*(B*a^2*b + A*a*b^2
+ A*a^2*c)*x^6 + 1/2*A*a^3*x^2 + 1/4*(B*a^3 + 3*A*a^2*b)*x^4
```

3.97.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.16

$$\int x(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx = \frac{1}{16}Bc^3x^{16}+\frac{3}{14}Bbc^2x^{14}+\frac{1}{14}Ac^3x^{14}+\frac{1}{4}Bb^2cx^{12}+\frac{1}{4}Bac^2x^{12}+\frac{1}{4}Abc^2x^{12}+\frac{1}{10}Bb^3x^{10}+\frac{3}{5}Babcx^{10}+\frac{3}{10}Ab^2cx^{10}+\frac{3}{10}Aac^2x^{10}+\frac{3}{8}Bab^2x^8+\frac{1}{8}Ab^3x^8+\frac{3}{8}Ba^2cx^8+\frac{3}{4}Aabcx^8+\frac{1}{2}Ba^2bx^6+\frac{1}{2}Aab^2x^6+\frac{1}{2}Aa^2cx^6+\frac{1}{4}Ba^3x^4+\frac{3}{4}Aa^2bx^4+\frac{1}{2}Aa^3x^2$$

input
```
integrate(x*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/16*B*c^3*x^16 + 3/14*B*b*c^2*x^14 + 1/14*A*c^3*x^14 + 1/4*B*b^2*c*x^12 +
 1/4*B*a*c^2*x^12 + 1/4*A*b*c^2*x^12 + 1/10*B*b^3*x^10 + 3/5*B*a*b*c*x^10
+ 3/10*A*b^2*c*x^10 + 3/10*A*a*c^2*x^10 + 3/8*B*a*b^2*x^8 + 1/8*A*b^3*x^8
+ 3/8*B*a^2*c*x^8 + 3/4*A*a*b*c*x^8 + 1/2*B*a^2*b*x^6 + 1/2*A*a*b^2*x^6 +
1/2*A*a^2*c*x^6 + 1/4*B*a^3*x^4 + 3/4*A*a^2*b*x^4 + 1/2*A*a^3*x^2
```

3.97.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.02

$$\int x(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx = x^8\left(\frac{3\,B\,c\,a^2}{8}+\frac{3\,B\,a\,b^2}{8}+\frac{3\,A\,c\,a\,b}{4}+\frac{A\,b^3}{8}\right) + x^{10}\left(\frac{B\,b^3}{10}+\frac{3\,A\,b^2\,c}{10}+\frac{3\,B\,a\,b\,c}{5}+\frac{3\,A\,a\,c^2}{10}\right) + x^4\left(\frac{B\,a^3}{4}+\frac{3\,A\,b\,a^2}{4}\right)+x^{14}\left(\frac{A\,c^3}{14}+\frac{3\,B\,b\,c^2}{14}\right) + x^6\left(\frac{B\,a^2\,b}{2}+\frac{A\,c\,a^2}{2}+\frac{A\,a\,b^2}{2}\right) + x^{12}\left(\frac{B\,b^2\,c}{4}+\frac{A\,b\,c^2}{4}+\frac{B\,a\,c^2}{4}\right)+\frac{A\,a^3\,x^2}{2}+\frac{B\,c^3\,x^{16}}{16}$$

input
```
int(x*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x)
```

output
```
x^8*((A*b^3)/8 + (3*B*a*b^2)/8 + (3*B*a^2*c)/8 + (3*A*a*b*c)/4) + x^10*((B
*b^3)/10 + (3*A*a*c^2)/10 + (3*A*b^2*c)/10 + (3*B*a*b*c)/5) + x^4*((B*a^3)
/4 + (3*A*a^2*b)/4) + x^14*((A*c^3)/14 + (3*B*b*c^2)/14) + x^6*((A*a*b^2)/
2 + (A*a^2*c)/2 + (B*a^2*b)/2) + x^12*((A*b*c^2)/4 + (B*a*c^2)/4 + (B*b^2*
c)/4) + (A*a^3*x^2)/2 + (B*c^3*x^16)/16
```

3.98 $\int (A+Bx^2)(a+bx^2+cx^4)^3\,dx$

3.98.1 Optimal result

Integrand size = 22, antiderivative size = 161

$$\int (A+Bx^2)(a+bx^2+cx^4)^3\,dx = a^3Ax+\frac{1}{3}a^2(3Ab+aB)x^3+\frac{3}{5}a\left(abB+A(b^2+ac)\right)x^5 +\frac{1}{7}\left(3aB(b^2+ac)+A(b^3+6abc)\right)x^7 +\frac{1}{9}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^9 +\frac{3}{11}c\left(b^2B+Abc+aBc\right)x^{11} +\frac{1}{13}c^2(3bB+Ac)x^{13}+\frac{1}{15}Bc^3x^{15}$$

output
```
a^3*A*x+1/3*a^2*(3*A*b+B*a)*x^3+3/5*a*(a*b*B+A*(a*c+b^2))*x^5+1/7*(3*a*B*(
a*c+b^2)+A*(6*a*b*c+b^3))*x^7+1/9*(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*b^3)*x^
9+3/11*c*(A*b*c+B*a*c+B*b^2)*x^11+1/13*c^2*(A*c+3*B*b)*x^13+1/15*B*c^3*x^1
5
```

3.98.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00

$$\int \left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3\,dx = a^3Ax+\frac{1}{3}a^2(3Ab+aB)x^3+\frac{3}{5}a\left(abB+A\left(b^2+ac\right)\right)x^5 +\frac{1}{7}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^7 +\frac{1}{9}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^9 +\frac{3}{11}c\left(b^2B+Abc+aBc\right)x^{11} +\frac{1}{13}c^2(3bB+Ac)x^{13}+\frac{1}{15}Bc^3x^{15}$$

input
```
Integrate[(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

output
```
a^3*A*x + (a^2*(3*A*b + a*B)*x^3)/3 + (3*a*(a*b*B + A*(b^2 + a*c))*x^5)/5
+ ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^7)/7 + ((b^3*B + 3*A*b^2*c +
6*a*b*B*c + 3*a*A*c^2)*x^9)/9 + (3*c*(b^2*B + A*b*c + a*B*c)*x^11)/11 + (c
^2*(3*b*B + A*c)*x^13)/13 + (B*c^3*x^15)/15
```

3.98.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3\,dx$$

$$\Big\downarrow \text{1467}$$

$$\int \left(a^3A+a^2x^2(aB+3Ab)+3cx^{10}\left(aBc+Abc+b^2B\right)+3ax^4\left(A\left(ac+b^2\right)+abB\right)+x^8\left(3aAc^2+6abBc+3Ab^2c\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$a^3Ax+\frac{1}{3}a^2x^3(aB+3Ab)+\frac{3}{11}cx^{11}(aBc+Abc+b^2B)+\frac{3}{5}ax^5(A(ac+b^2)+abB)+\frac{1}{9}x^9(3aAc^2+6abBc+3Ab^2c+b^3B)+\frac{1}{7}x^7(A(6abc+b^3)+3aB(ac+b^2))+\frac{1}{13}c^2x^{13}(Ac+3bB)+\frac{1}{15}Bc^3x^{15}$$

input
```
Int[(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

output
```
a^3*A*x + (a^2*(3*A*b + a*B)*x^3)/3 + (3*a*(a*b*B + A*(b^2 + a*c))*x^5)/5
+ ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^7)/7 + ((b^3*B + 3*A*b^2*c +
6*a*b*B*c + 3*a*A*c^2)*x^9)/9 + (3*c*(b^2*B + A*b*c + a*B*c)*x^11)/11 + (c
^2*(3*b*B + A*c)*x^13)/13 + (B*c^3*x^15)/15
```

3.98.3.1 Defintions of rubi rules used

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.98.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.03

method	result
norman	$a^3Ax+\left(A\,a^2b+\frac{1}{3}B\,a^3\right)x^3+\left(\frac{3}{5}Ac\,a^2+\frac{3}{5}Aa\,b^2+\frac{3}{5}B\,a^2b\right)x^5+\left(\frac{6}{7}Aabc+\frac{1}{7}A\,b^3+\frac{3}{7}a^2Bc+\right.$
gosper	$a^3Ax+x^3A\,a^2b+\frac{1}{3}x^3B\,a^3+\frac{3}{5}x^5Ac\,a^2+\frac{3}{5}x^5Aa\,b^2+\frac{3}{5}x^5B\,a^2b+\frac{6}{7}x^7Aabc+\frac{1}{7}x^7A\,b^3+\frac{3}{7}x^7$
risch	$a^3Ax+x^3A\,a^2b+\frac{1}{3}x^3B\,a^3+\frac{3}{5}x^5Ac\,a^2+\frac{3}{5}x^5Aa\,b^2+\frac{3}{5}x^5B\,a^2b+\frac{6}{7}x^7Aabc+\frac{1}{7}x^7A\,b^3+\frac{3}{7}x^7$
parallelrisch	$a^3Ax+x^3A\,a^2b+\frac{1}{3}x^3B\,a^3+\frac{3}{5}x^5Ac\,a^2+\frac{3}{5}x^5Aa\,b^2+\frac{3}{5}x^5B\,a^2b+\frac{6}{7}x^7Aabc+\frac{1}{7}x^7A\,b^3+\frac{3}{7}x^7$
default	$\frac{B\,c^3x^{15}}{15}+\frac{(A\,c^3+3Bb\,c^2)x^{13}}{13}+\frac{(3Ab\,c^2+B(a\,c^2+2b^2c+c(2ac+b^2)))x^{11}}{11}+\frac{(A(a\,c^2+2b^2c+c(2ac+b^2))+B(4abc+b(2ac}{9}$

input
```
int((B*x^2+A)*(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*A*x+(A*a^2*b+1/3*B*a^3)*x^3+(3/5*A*c*a^2+3/5*A*a*b^2+3/5*B*a^2*b)*x^5+
(6/7*A*a*b*c+1/7*A*b^3+3/7*a^2*B*c+3/7*B*a*b^2)*x^7+(1/3*A*a*c^2+1/3*A*b^2
*c+2/3*B*a*b*c+1/9*B*b^3)*x^9+(3/11*A*b*c^2+3/11*B*a*c^2+3/11*B*b^2*c)*x^1
1+(1/13*A*c^3+3/13*B*b*c^2)*x^13+1/15*B*c^3*x^15
```

3.98.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.01

$$\int (A+Bx^2)(a+bx^2+cx^4)^3\,dx = \frac{1}{15}Bc^3x^{15}+\frac{1}{13}(3Bbc^2+Ac^3)x^{13} + \frac{3}{11}(Bb^2c+(Ba+Ab)c^2)x^{11} + \frac{1}{9}(Bb^3+3Aac^2+3(2Bab+Ab^2)c)x^9 + \frac{1}{7}(3Bab^2+Ab^3+3(Ba^2+2Aab)c)x^7 + \frac{3}{5}(Ba^2b+Aab^2+Aa^2c)x^5 + Aa^3x+\frac{1}{3}(Ba^3+3Aa^2b)x^3$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/15*B*c^3*x^15 + 1/13*(3*B*b*c^2 + A*c^3)*x^13 + 3/11*(B*b^2*c + (B*a + A
*b)*c^2)*x^11 + 1/9*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^9 + 1/7*
(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^7 + 3/5*(B*a^2*b + A*a*b^2 +
 A*a^2*c)*x^5 + A*a^3*x + 1/3*(B*a^3 + 3*A*a^2*b)*x^3
```

3.98.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.24

$$\int (A+Bx^2)(a+bx^2+cx^4)^3\,dx = Aa^3x + \frac{Bc^3x^{15}}{15} + x^{13}\left(\frac{Ac^3}{13} + \frac{3Bbc^2}{13}\right) + x^{11}\cdot\left(\frac{3Abc^2}{11} + \frac{3Bac^2}{11} + \frac{3Bb^2c}{11}\right) + x^9\left(\frac{Aac^2}{3} + \frac{Ab^2c}{3} + \frac{2Babc}{3} + \frac{Bb^3}{9}\right) + x^7\cdot\left(\frac{6Aabc}{7} + \frac{Ab^3}{7} + \frac{3Ba^2c}{7} + \frac{3Bab^2}{7}\right) + x^5\cdot\left(\frac{3Aa^2c}{5} + \frac{3Aab^2}{5} + \frac{3Ba^2b}{5}\right) + x^3\left(Aa^2b + \frac{Ba^3}{3}\right)$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2+a)**3,x)
```

output
```
A*a**3*x + B*c**3*x**15/15 + x**13*(A*c**3/13 + 3*B*b*c**2/13) + x**11*(3*
A*b*c**2/11 + 3*B*a*c**2/11 + 3*B*b**2*c/11) + x**9*(A*a*c**2/3 + A*b**2*c
/3 + 2*B*a*b*c/3 + B*b**3/9) + x**7*(6*A*a*b*c/7 + A*b**3/7 + 3*B*a**2*c/7
 + 3*B*a*b**2/7) + x**5*(3*A*a**2*c/5 + 3*A*a*b**2/5 + 3*B*a**2*b/5) + x**
3*(A*a**2*b + B*a**3/3)
```

3.98.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.01

$$\int (A+Bx^2)(a+bx^2+cx^4)^3\,dx = \frac{1}{15}\,Bc^3x^{15} + \frac{1}{13}\,(3\,Bbc^2 + Ac^3)x^{13} + \frac{3}{11}\,(Bb^2c + (Ba+Ab)c^2)x^{11} + \frac{1}{9}\,(Bb^3 + 3\,Aac^2 + 3\,(2\,Bab + Ab^2)c)x^9 + \frac{1}{7}\,(3\,Bab^2 + Ab^3 + 3\,(Ba^2 + 2\,Aab)c)x^7 + \frac{3}{5}\,(Ba^2b + Aab^2 + Aa^2c)x^5 + Aa^3x + \frac{1}{3}\,(Ba^3 + 3\,Aa^2b)x^3$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/15*B*c^3*x^15 + 1/13*(3*B*b*c^2 + A*c^3)*x^13 + 3/11*(B*b^2*c + (B*a + A
*b)*c^2)*x^11 + 1/9*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^9 + 1/7*
(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^7 + 3/5*(B*a^2*b + A*a*b^2 +
 A*a^2*c)*x^5 + A*a^3*x + 1/3*(B*a^3 + 3*A*a^2*b)*x^3
```

3.98.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.17

$$\int (A+Bx^2)(a+bx^2+cx^4)^3\,dx = \frac{1}{15}Bc^3x^{15}+\frac{3}{13}Bbc^2x^{13}+\frac{1}{13}Ac^3x^{13}+\frac{3}{11}Bb^2cx^{11}+\frac{3}{11}Bac^2x^{11}+\frac{3}{11}Abc^2x^{11}+\frac{1}{9}Bb^3x^9+\frac{2}{3}Babcx^9+\frac{1}{3}Ab^2cx^9+\frac{1}{3}Aac^2x^9+\frac{3}{7}Bab^2x^7+\frac{1}{7}Ab^3x^7+\frac{3}{7}Ba^2cx^7+\frac{6}{7}Aabcx^7+\frac{3}{5}Ba^2bx^5+\frac{3}{5}Aab^2x^5+\frac{3}{5}Aa^2cx^5+\frac{1}{3}Ba^3x^3+Aa^2bx^3+Aa^3x$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/15*B*c^3*x^15 + 3/13*B*b*c^2*x^13 + 1/13*A*c^3*x^13 + 3/11*B*b^2*c*x^11
+ 3/11*B*a*c^2*x^11 + 3/11*A*b*c^2*x^11 + 1/9*B*b^3*x^9 + 2/3*B*a*b*c*x^9
+ 1/3*A*b^2*c*x^9 + 1/3*A*a*c^2*x^9 + 3/7*B*a*b^2*x^7 + 1/7*A*b^3*x^7 + 3/
7*B*a^2*c*x^7 + 6/7*A*a*b*c*x^7 + 3/5*B*a^2*b*x^5 + 3/5*A*a*b^2*x^5 + 3/5*
A*a^2*c*x^5 + 1/3*B*a^3*x^3 + A*a^2*b*x^3 + A*a^3*x
```

3.98.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.02

$$\int (A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx = x^7\left(\frac{3\,B\,c\,a^2}{7}+\frac{3\,B\,a\,b^2}{7}+\frac{6\,A\,c\,a\,b}{7}+\frac{A\,b^3}{7}\right) + x^9\left(\frac{B\,b^3}{9}+\frac{A\,b^2\,c}{3}+\frac{2\,B\,a\,b\,c}{3}+\frac{A\,a\,c^2}{3}\right) + x^3\left(\frac{B\,a^3}{3}+A\,b\,a^2\right)+x^{13}\left(\frac{A\,c^3}{13}+\frac{3\,B\,b\,c^2}{13}\right) + x^5\left(\frac{3\,B\,a^2\,b}{5}+\frac{3\,A\,c\,a^2}{5}+\frac{3\,A\,a\,b^2}{5}\right) + x^{11}\left(\frac{3\,B\,b^2\,c}{11}+\frac{3\,A\,b\,c^2}{11}+\frac{3\,B\,a\,c^2}{11}\right) + \frac{B\,c^3\,x^{15}}{15}+A\,a^3\,x$$

input
```
int((A + B*x^2)*(a + b*x^2 + c*x^4)^3,x)
```

output
```
x^7*((A*b^3)/7 + (3*B*a*b^2)/7 + (3*B*a^2*c)/7 + (6*A*a*b*c)/7) + x^9*((B*
b^3)/9 + (A*a*c^2)/3 + (A*b^2*c)/3 + (2*B*a*b*c)/3) + x^3*((B*a^3)/3 + A*a
^2*b) + x^13*((A*c^3)/13 + (3*B*b*c^2)/13) + x^5*((3*A*a*b^2)/5 + (3*A*a^2
*c)/5 + (3*B*a^2*b)/5) + x^11*((3*A*b*c^2)/11 + (3*B*a*c^2)/11 + (3*B*b^2*
c)/11) + (B*c^3*x^15)/15 + A*a^3*x
```

3.99 $\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x}\,dx$

3.99.1 Optimal result

Integrand size = 25, antiderivative size = 162

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x}\,dx = \frac{1}{2}a^2(3Ab+aB)x^2+\frac{3}{4}a\left(abB+A\left(b^2+ac\right)\right)x^4 + \frac{1}{6}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^6 + \frac{1}{8}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^8 + \frac{3}{10}c\left(b^2B+Abc+aBc\right)x^{10} + \frac{1}{12}c^2(3bB+Ac)x^{12}+\frac{1}{14}Bc^3x^{14}+a^3A\log(x)$$

output
```
1/2*a^2*(3*A*b+B*a)*x^2+3/4*a*(a*b*B+A*(a*c+b^2))*x^4+1/6*(3*a*B*(a*c+b^2)
+A*(6*a*b*c+b^3))*x^6+1/8*(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*b^3)*x^8+3/10*c
*(A*b*c+B*a*c+B*b^2)*x^10+1/12*c^2*(A*c+3*B*b)*x^12+1/14*B*c^3*x^14+a^3*A*
ln(x)
```

3.99.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x}\,dx = \frac{1}{2}a^2(3Ab+aB)x^2+\frac{3}{4}a\left(abB+A\left(b^2+ac\right)\right)x^4+\frac{1}{6}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^6+\frac{1}{8}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^8+\frac{3}{10}c\left(b^2B+Abc+aBc\right)x^{10}+\frac{1}{12}c^2(3bB+Ac)x^{12}+\frac{1}{14}Bc^3x^{14}+a^3A\log(x)$$

input
```
Integrate[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x,x]
```

output
```
(a^2*(3*A*b + a*B)*x^2)/2 + (3*a*(a*b*B + A*(b^2 + a*c))*x^4)/4 + ((3*a*B*
(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^6)/6 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c
+ 3*a*A*c^2)*x^8)/8 + (3*c*(b^2*B + A*b*c + a*B*c)*x^10)/10 + (c^2*(3*b*B
+ A*c)*x^12)/12 + (B*c^3*x^14)/14 + a^3*A*Log[x]
```

3.99.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x}\,dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{(Bx^2+A)(cx^4+bx^2+a)^3}{x^2}dx^2$$

$$\Big\downarrow \text{1195}$$

$$\frac{1}{2}\int \Big(Bc^3x^{12}+c^2(3bB+Ac)x^{10}+3c\left(Bb^2+Acb+aBc\right)x^8+\left(Bb^3+3Acb^2+6aBcb+3aAc^2\right)x^6+\left(3aB\left(b^2+\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(a^3A\log\left(x^2\right)+a^2x^2(aB+3Ab)+\frac{3}{5}cx^{10}\left(aBc+Abc+b^2B\right)+\frac{3}{2}ax^4\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{4}x^8\left(3aAc^2+6abB\right.\right.$$

input
```
Int[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x,x]
```

output
```
(a^2*(3*A*b + a*B)*x^2 + (3*a*(a*b*B + A*(b^2 + a*c))*x^4)/2 + ((3*a*B*(b^
2 + a*c) + A*(b^3 + 6*a*b*c))*x^6)/3 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3
*a*A*c^2)*x^8)/4 + (3*c*(b^2*B + A*b*c + a*B*c)*x^10)/5 + (c^2*(3*b*B + A*
c)*x^12)/6 + (B*c^3*x^14)/7 + a^3*A*Log[x^2])/2
```

3.99.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.99.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.03

method	result
norman	$\left(\frac{1}{12}A\,c^{3}+\frac{1}{4}Bb\,c^{2}\right)x^{12}+\left(\frac{3}{2}A\,a^{2}b+\frac{1}{2}B\,a^{3}\right)x^{2}+\left(\frac{3}{10}Ab\,c^{2}+\frac{3}{10}Ba\,c^{2}+\frac{3}{10}B\,b^{2}c\right)x^{10}+\left(\frac{3}{4}Ac\,a\right.$
default	$\frac{B\,c^{3}x^{14}}{14}+\frac{A\,c^{3}x^{12}}{12}+\frac{Bb\,c^{2}x^{12}}{4}+\frac{3Ab\,c^{2}x^{10}}{10}+\frac{3Ba\,c^{2}x^{10}}{10}+\frac{3B\,b^{2}c\,x^{10}}{10}+\frac{3Aa\,c^{2}x^{8}}{8}+\frac{3A\,b^{2}c\,x^{8}}{8}+\frac{3Babc\,x^{8}}{4}+$
risch	$\frac{B\,c^{3}x^{14}}{14}+\frac{A\,c^{3}x^{12}}{12}+\frac{Bb\,c^{2}x^{12}}{4}+\frac{3Ab\,c^{2}x^{10}}{10}+\frac{3Ba\,c^{2}x^{10}}{10}+\frac{3B\,b^{2}c\,x^{10}}{10}+\frac{3Aa\,c^{2}x^{8}}{8}+\frac{3A\,b^{2}c\,x^{8}}{8}+\frac{3Babc\,x^{8}}{4}+$
parallelrisch	$\frac{B\,c^{3}x^{14}}{14}+\frac{A\,c^{3}x^{12}}{12}+\frac{Bb\,c^{2}x^{12}}{4}+\frac{3Ab\,c^{2}x^{10}}{10}+\frac{3Ba\,c^{2}x^{10}}{10}+\frac{3B\,b^{2}c\,x^{10}}{10}+\frac{3Aa\,c^{2}x^{8}}{8}+\frac{3A\,b^{2}c\,x^{8}}{8}+\frac{3Babc\,x^{8}}{4}+$

input
```
int((B*x^2+A)*(c*x^4+b*x^2+a)^3/x,x,method=_RETURNVERBOSE)
```

output
```
(1/12*A*c^3+1/4*B*b*c^2)*x^12+(3/2*A*a^2*b+1/2*B*a^3)*x^2+(3/10*A*b*c^2+3/
10*B*a*c^2+3/10*B*b^2*c)*x^10+(3/4*A*c*a^2+3/4*A*a*b^2+3/4*B*a^2*b)*x^4+(3
/8*A*a*c^2+3/8*A*b^2*c+3/4*B*a*b*c+1/8*B*b^3)*x^8+(A*a*b*c+1/6*A*b^3+1/2*a
^2*B*c+1/2*B*a*b^2)*x^6+1/14*B*c^3*x^14+a^3*A*ln(x)
```

3.99.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.01

$$\begin{aligned}\int \frac{(A+Bx^2)\,(a+bx^2+cx^4)^3}{x}\,dx &= \frac{1}{14}\,Bc^3x^{14}+\frac{1}{12}\,(3\,Bbc^2+Ac^3)x^{12}\\ &+\frac{3}{10}\,(Bb^2c+(Ba+Ab)c^2)x^{10}\\ &+\frac{1}{8}\,(Bb^3+3\,Aac^2+3\,(2\,Bab+Ab^2)c)x^8\\ &+\frac{1}{6}\,(3\,Bab^2+Ab^3+3\,(Ba^2+2\,Aab)c)x^6\\ &+\frac{3}{4}\,(Ba^2b+Aab^2+Aa^2c)x^4\\ &+Aa^3\log{(x)}+\frac{1}{2}\,(Ba^3+3\,Aa^2b)x^2\end{aligned}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x,x, algorithm="fricas")
```

output
```
1/14*B*c^3*x^14 + 1/12*(3*B*b*c^2 + A*c^3)*x^12 + 3/10*(B*b^2*c + (B*a + A
*b)*c^2)*x^10 + 1/8*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^8 + 1/6*
(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^6 + 3/4*(B*a^2*b + A*a*b^2 +
 A*a^2*c)*x^4 + A*a^3*log(x) + 1/2*(B*a^3 + 3*A*a^2*b)*x^2
```

3.99.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.23

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x}\,dx = Aa^3\log(x) + \frac{Bc^3x^{14}}{14} + x^{12}\left(\frac{Ac^3}{12}+\frac{Bbc^2}{4}\right) + x^{10}\cdot\left(\frac{3Abc^2}{10}+\frac{3Bac^2}{10}+\frac{3Bb^2c}{10}\right) + x^8\cdot\left(\frac{3Aac^2}{8}+\frac{3Ab^2c}{8}+\frac{3Babc}{4}+\frac{Bb^3}{8}\right) + x^6\left(Aabc+\frac{Ab^3}{6}+\frac{Ba^2c}{2}+\frac{Bab^2}{2}\right) + x^4\cdot\left(\frac{3Aa^2c}{4}+\frac{3Aab^2}{4}+\frac{3Ba^2b}{4}\right) + x^2\cdot\left(\frac{3Aa^2b}{2}+\frac{Ba^3}{2}\right)$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2+a)**3/x,x)
```

output
```
A*a**3*log(x) + B*c**3*x**14/14 + x**12*(A*c**3/12 + B*b*c**2/4) + x**10*(
3*A*b*c**2/10 + 3*B*a*c**2/10 + 3*B*b**2*c/10) + x**8*(3*A*a*c**2/8 + 3*A*
b**2*c/8 + 3*B*a*b*c/4 + B*b**3/8) + x**6*(A*a*b*c + A*b**3/6 + B*a**2*c/2
 + B*a*b**2/2) + x**4*(3*A*a**2*c/4 + 3*A*a*b**2/4 + 3*B*a**2*b/4) + x**2*
(3*A*a**2*b/2 + B*a**3/2)
```

3.99.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x}\,dx = \frac{1}{14}Bc^3x^{14} + \frac{1}{12}(3Bbc^2+Ac^3)x^{12} + \frac{3}{10}(Bb^2c+(Ba+Ab)c^2)x^{10} + \frac{1}{8}\left(Bb^3+3Aac^2+3\left(2Bab+Ab^2\right)c\right)x^8 + \frac{1}{6}\left(3Bab^2+Ab^3+3\left(Ba^2+2Aab\right)c\right)x^6 + \frac{3}{4}(Ba^2b+Aab^2+Aa^2c)x^4 + \frac{1}{2}Aa^3\log\left(x^2\right) + \frac{1}{2}(Ba^3+3Aa^2b)x^2$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x,x, algorithm="maxima")
```

output
```
1/14*B*c^3*x^14 + 1/12*(3*B*b*c^2 + A*c^3)*x^12 + 3/10*(B*b^2*c + (B*a + A
*b)*c^2)*x^10 + 1/8*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^8 + 1/6*
(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^6 + 3/4*(B*a^2*b + A*a*b^2 +
 A*a^2*c)*x^4 + 1/2*A*a^3*log(x^2) + 1/2*(B*a^3 + 3*A*a^2*b)*x^2
```

3.99.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.19

$$\begin{aligned}\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x}\,dx &= \frac{1}{14}Bc^3x^{14}+\frac{1}{4}Bbc^2x^{12}+\frac{1}{12}Ac^3x^{12}+\frac{3}{10}Bb^2cx^{10}\\ &\quad+\frac{3}{10}Bac^2x^{10}+\frac{3}{10}Abc^2x^{10}+\frac{1}{8}Bb^3x^8+\frac{3}{4}Babcx^8\\ &\quad+\frac{3}{8}Ab^2cx^8+\frac{3}{8}Aac^2x^8+\frac{1}{2}Bab^2x^6+\frac{1}{6}Ab^3x^6\\ &\quad+\frac{1}{2}Ba^2cx^6+Aabcx^6+\frac{3}{4}Ba^2bx^4+\frac{3}{4}Aab^2x^4\\ &\quad+\frac{3}{4}Aa^2cx^4+\frac{1}{2}Ba^3x^2+\frac{3}{2}Aa^2bx^2+\frac{1}{2}Aa^3\log\left(x^2\right)\end{aligned}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x,x, algorithm="giac")
```

output
```
1/14*B*c^3*x^14 + 1/4*B*b*c^2*x^12 + 1/12*A*c^3*x^12 + 3/10*B*b^2*c*x^10 +
 3/10*B*a*c^2*x^10 + 3/10*A*b*c^2*x^10 + 1/8*B*b^3*x^8 + 3/4*B*a*b*c*x^8 +
 3/8*A*b^2*c*x^8 + 3/8*A*a*c^2*x^8 + 1/2*B*a*b^2*x^6 + 1/6*A*b^3*x^6 + 1/2
*B*a^2*c*x^6 + A*a*b*c*x^6 + 3/4*B*a^2*b*x^4 + 3/4*A*a*b^2*x^4 + 3/4*A*a^2
*c*x^4 + 1/2*B*a^3*x^2 + 3/2*A*a^2*b*x^2 + 1/2*A*a^3*log(x^2)
```

3.99.9 Mupad [B] (verification not implemented)

Time = 7.77 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.02

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x}\,dx = x^6\left(\frac{B\,c\,a^2}{2}+\frac{B\,a\,b^2}{2}+A\,c\,a\,b+\frac{A\,b^3}{6}\right) + x^8\left(\frac{B\,b^3}{8}+\frac{3\,A\,b^2\,c}{8}+\frac{3\,B\,a\,b\,c}{4}+\frac{3\,A\,a\,c^2}{8}\right) + x^2\left(\frac{B\,a^3}{2}+\frac{3\,A\,b\,a^2}{2}\right) + x^{12}\left(\frac{A\,c^3}{12}+\frac{B\,b\,c^2}{4}\right) + x^4\left(\frac{3\,B\,a^2\,b}{4}+\frac{3\,A\,c\,a^2}{4}+\frac{3\,A\,a\,b^2}{4}\right) + x^{10}\left(\frac{3\,B\,b^2\,c}{10}+\frac{3\,A\,b\,c^2}{10}+\frac{3\,B\,a\,c^2}{10}\right) + \frac{B\,c^3\,x^{14}}{14} + A\,a^3\,\ln(x)$$

input
```
int(((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x,x)
```

output
```
x^6*((A*b^3)/6 + (B*a*b^2)/2 + (B*a^2*c)/2 + A*a*b*c) + x^8*((B*b^3)/8 + (
3*A*a*c^2)/8 + (3*A*b^2*c)/8 + (3*B*a*b*c)/4) + x^2*((B*a^3)/2 + (3*A*a^2*
b)/2) + x^12*((A*c^3)/12 + (B*b*c^2)/4) + x^4*((3*A*a*b^2)/4 + (3*A*a^2*c)
/4 + (3*B*a^2*b)/4) + x^10*((3*A*b*c^2)/10 + (3*B*a*c^2)/10 + (3*B*b^2*c)/
10) + (B*c^3*x^14)/14 + A*a^3*log(x)
```

3.100 $\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^2}\,dx$

3.100.1 Optimal result

Integrand size = 25, antiderivative size = 156

$$\begin{aligned}\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^2}\,dx = &-\frac{a^3A}{x}+a^2(3Ab+aB)x+a\big(abB+A\big(b^2+ac\big)\big)x^3\\&+\frac{1}{5}\big(3aB\big(b^2+ac\big)+A\big(b^3+6abc\big)\big)x^5\\&+\frac{1}{7}\big(b^3B+3Ab^2c+6abBc+3aAc^2\big)x^7\\&+\frac{1}{3}c\big(b^2B+Abc+aBc\big)x^9\\&+\frac{1}{11}c^2(3bB+Ac)x^{11}+\frac{1}{13}Bc^3x^{13}\end{aligned}$$

output
```
-a^3*A/x+a^2*(3*A*b+B*a)*x+a*(a*b*B+A*(a*c+b^2))*x^3+1/5*(3*a*B*(a*c+b^2)+
A*(6*a*b*c+b^3))*x^5+1/7*(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*b^3)*x^7+1/3*c*(
A*b*c+B*a*c+B*b^2)*x^9+1/11*c^2*(A*c+3*B*b)*x^11+1/13*B*c^3*x^13
```

3.100.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^2}\,dx = -\frac{a^3A}{x} + a^2(3Ab+aB)x + a\left(abB + A\left(b^2+ac\right)\right)x^3 + \frac{1}{5}\left(3aB\left(b^2+ac\right) + A\left(b^3+6abc\right)\right)x^5 + \frac{1}{7}\left(b^3B + 3Ab^2c + 6abBc + 3aAc^2\right)x^7 + \frac{1}{3}c\left(b^2B + Abc + aBc\right)x^9 + \frac{1}{11}c^2(3bB+Ac)x^{11} + \frac{1}{13}Bc^3x^{13}$$

input
```
Integrate[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^2,x]
```

output
```
-((a^3*A)/x) + a^2*(3*A*b + a*B)*x + a*(a*b*B + A*(b^2 + a*c))*x^3 + ((3*a
*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^5)/5 + ((b^3*B + 3*A*b^2*c + 6*a*b*B
*c + 3*a*A*c^2)*x^7)/7 + (c*(b^2*B + A*b*c + a*B*c)*x^9)/3 + (c^2*(3*b*B +
 A*c)*x^11)/11 + (B*c^3*x^13)/13
```

3.100.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^2}\,dx$$

$$\Big\downarrow \text{1584}$$

$$\int \left(\frac{a^3A}{x^2} + a^2(aB+3Ab) + 3cx^8\left(aBc+Abc+b^2B\right) + 3ax^2\left(A\left(ac+b^2\right)+abB\right) + x^6\left(3aAc^2+6abBc+3Ab^2c+\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3A}{x}+a^2x(aB+3Ab)+\frac{1}{3}cx^9\left(aBc+Abc+b^2B\right)+ax^3\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{7}x^7\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{5}x^5\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{1}{11}c^2x^{11}(Ac+3bB)+\frac{1}{13}Bc^3x^{13}$$

input
```
Int[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^2,x]
```

output
```
-((a^3*A)/x) + a^2*(3*A*b + a*B)*x + a*(a*b*B + A*(b^2 + a*c))*x^3 + ((3*a
*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^5)/5 + ((b^3*B + 3*A*b^2*c + 6*a*b*B
*c + 3*a*A*c^2)*x^7)/7 + (c*(b^2*B + A*b*c + a*B*c)*x^9)/3 + (c^2*(3*b*B +
 A*c)*x^11)/11 + (B*c^3*x^13)/13
```

3.100.3.1 Defintions of rubi rules used

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.100.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.07

method	result
norman	$\frac{-A a^3+(3A a^2b+B a^3)x^2+(Ac a^2+Aa b^2+B a^2b)x^4+(\frac{6}{5}Aabc+\frac{1}{5}A b^3+\frac{3}{5}a^2Bc+\frac{3}{5}Ba b^2)x^6+(\frac{3}{7}Aa c^2+\frac{3}{7}A b^2c+\frac{6}{7}Babc+\frac{1}{7}B}{x}$
default	$\frac{B c^3x^{13}}{13}+\frac{A c^3x^{11}}{11}+\frac{3Bb c^2x^{11}}{11}+\frac{Ab c^2x^9}{3}+\frac{Ba c^2x^9}{3}+\frac{B b^2c x^9}{3}+\frac{3Aa c^2x^7}{7}+\frac{3A b^2c x^7}{7}+\frac{6Babc x^7}{7}+\frac{B b^3x}{7}$
risch	$\frac{B c^3x^{13}}{13}+\frac{A c^3x^{11}}{11}+\frac{3Bb c^2x^{11}}{11}+\frac{Ab c^2x^9}{3}+\frac{Ba c^2x^9}{3}+\frac{B b^2c x^9}{3}+\frac{3Aa c^2x^7}{7}+\frac{3A b^2c x^7}{7}+\frac{6Babc x^7}{7}+\frac{B b^3x}{7}$
gosper	$-\frac{-1155B c^3x^{14}-1365A c^3x^{12}-4095Bb c^2x^{12}-5005Ab c^2x^{10}-5005Ba c^2x^{10}-5005B b^2c x^{10}-6435Aa c^2x^8-6435A b^2c x^8-}{}$
parallelrisch	$\frac{1155B c^3x^{14}+1365A c^3x^{12}+4095Bb c^2x^{12}+5005Ab c^2x^{10}+5005Ba c^2x^{10}+5005B b^2c x^{10}+6435Aa c^2x^8+6435A b^2c x^8+1287}{}$

input
```
int((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/x*(-A*a^3+(3*A*a^2*b+B*a^3)*x^2+(A*a^2*c+A*a*b^2+B*a^2*b)*x^4+(6/5*A*a*b
*c+1/5*A*b^3+3/5*a^2*B*c+3/5*B*a*b^2)*x^6+(3/7*A*a*c^2+3/7*A*b^2*c+6/7*B*a
*b*c+1/7*B*b^3)*x^8+(1/3*A*b*c^2+1/3*B*a*c^2+1/3*B*b^2*c)*x^10+(1/11*A*c^3
+3/11*B*b*c^2)*x^12+1/13*B*c^3*x^14)
```

3.100.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^2}\,dx = \frac{1155\,Bc^3x^{14}+1365\,(3\,Bbc^2+Ac^3)x^{12}+5005\,(Bb^2c+(Ba+Ab)c^2)x^{10}+2145\,(Bb^3+3\,Aac^2+3\,(2\,Bab}{}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^2,x, algorithm="fricas")
```

output
```
1/15015*(1155*B*c^3*x^14 + 1365*(3*B*b*c^2 + A*c^3)*x^12 + 5005*(B*b^2*c +
 (B*a + A*b)*c^2)*x^10 + 2145*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*
x^8 + 3003*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^6 + 15015*(B*a^2*
b + A*a*b^2 + A*a^2*c)*x^4 - 15015*A*a^3 + 15015*(B*a^3 + 3*A*a^2*b)*x^2)/
x
```

3.100.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.19

$$\begin{aligned}\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^2}\,dx &= -\frac{Aa^3}{x}+\frac{Bc^3x^{13}}{13}+x^{11}\left(\frac{Ac^3}{11}+\frac{3Bbc^2}{11}\right)\\ &\quad+x^9\left(\frac{Abc^2}{3}+\frac{Bac^2}{3}+\frac{Bb^2c}{3}\right)+x^7\\ &\quad\cdot\left(\frac{3Aac^2}{7}+\frac{3Ab^2c}{7}+\frac{6Babc}{7}+\frac{Bb^3}{7}\right)+x^5\\ &\quad\cdot\left(\frac{6Aabc}{5}+\frac{Ab^3}{5}+\frac{3Ba^2c}{5}+\frac{3Bab^2}{5}\right)\\ &\quad+x^3\left(Aa^2c+Aab^2+Ba^2b\right)+x\left(3Aa^2b+Ba^3\right)\end{aligned}$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2+a)**3/x**2,x)
```

output
```
-A*a**3/x + B*c**3*x**13/13 + x**11*(A*c**3/11 + 3*B*b*c**2/11) + x**9*(A*
b*c**2/3 + B*a*c**2/3 + B*b**2*c/3) + x**7*(3*A*a*c**2/7 + 3*A*b**2*c/7 +
6*B*a*b*c/7 + B*b**3/7) + x**5*(6*A*a*b*c/5 + A*b**3/5 + 3*B*a**2*c/5 + 3*
B*a*b**2/5) + x**3*(A*a**2*c + A*a*b**2 + B*a**2*b) + x*(3*A*a**2*b + B*a*
*3)
```

3.100.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^2}\,dx = \frac{1}{13}Bc^3x^{13} + \frac{1}{11}(3Bbc^2+Ac^3)x^{11} + \frac{1}{3}(Bb^2c+(Ba+Ab)c^2)x^9 + \frac{1}{7}(Bb^3+3Aac^2+3(2Bab+Ab^2)c)x^7 + \frac{1}{5}(3Bab^2+Ab^3+3(Ba^2+2Aab)c)x^5 + (Ba^2b+Aab^2+Aa^2c)x^3 - \frac{Aa^3}{x} + (Ba^3+3Aa^2b)x$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^2,x, algorithm="maxima")
```

output
```
1/13*B*c^3*x^13 + 1/11*(3*B*b*c^2 + A*c^3)*x^11 + 1/3*(B*b^2*c + (B*a + A*
b)*c^2)*x^9 + 1/7*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^7 + 1/5*(3
*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^5 + (B*a^2*b + A*a*b^2 + A*a^2
*c)*x^3 - A*a^3/x + (B*a^3 + 3*A*a^2*b)*x
```

3.100.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.19

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^2}\,dx = \frac{1}{13}Bc^3x^{13} + \frac{3}{11}Bbc^2x^{11} + \frac{1}{11}Ac^3x^{11} + \frac{1}{3}Bb^2cx^9 + \frac{1}{3}Bac^2x^9 + \frac{1}{3}Abc^2x^9 + \frac{1}{7}Bb^3x^7 + \frac{6}{7}Babcx^7 + \frac{3}{7}Ab^2cx^7 + \frac{3}{7}Aac^2x^7 + \frac{3}{5}Bab^2x^5 + \frac{1}{5}Ab^3x^5 + \frac{3}{5}Ba^2cx^5 + \frac{6}{5}Aabcx^5 + Ba^2bx^3 + Aab^2x^3 + Aa^2cx^3 + Ba^3x + 3\,Aa^2bx - \frac{Aa^3}{x}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^2,x, algorithm="giac")
```

output
```
1/13*B*c^3*x^13 + 3/11*B*b*c^2*x^11 + 1/11*A*c^3*x^11 + 1/3*B*b^2*c*x^9 +
1/3*B*a*c^2*x^9 + 1/3*A*b*c^2*x^9 + 1/7*B*b^3*x^7 + 6/7*B*a*b*c*x^7 + 3/7*
A*b^2*c*x^7 + 3/7*A*a*c^2*x^7 + 3/5*B*a*b^2*x^5 + 1/5*A*b^3*x^5 + 3/5*B*a^
2*c*x^5 + 6/5*A*a*b*c*x^5 + B*a^2*b*x^3 + A*a*b^2*x^3 + A*a^2*c*x^3 + B*a^
3*x + 3*A*a^2*b*x - A*a^3/x
```

3.100.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^2}\,dx = x^5\left(\frac{3\,B\,c\,a^2}{5} + \frac{3\,B\,a\,b^2}{5} + \frac{6\,A\,c\,a\,b}{5} + \frac{A\,b^3}{5}\right) + x^7\left(\frac{B\,b^3}{7} + \frac{3\,A\,b^2\,c}{7} + \frac{6\,B\,a\,b\,c}{7} + \frac{3\,A\,a\,c^2}{7}\right) + x\,\left(B\,a^3 + 3\,A\,b\,a^2\right) + x^{11}\left(\frac{A\,c^3}{11} + \frac{3\,B\,b\,c^2}{11}\right) + x^3\,\left(B\,a^2\,b + A\,c\,a^2 + A\,a\,b^2\right) + x^9\left(\frac{B\,b^2\,c}{3} + \frac{A\,b\,c^2}{3} + \frac{B\,a\,c^2}{3}\right) - \frac{A\,a^3}{x} + \frac{B\,c^3\,x^{13}}{13}$$

input
```
int(((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^2,x)
```

output
```
x^5*((A*b^3)/5 + (3*B*a*b^2)/5 + (3*B*a^2*c)/5 + (6*A*a*b*c)/5) + x^7*((B*
b^3)/7 + (3*A*a*c^2)/7 + (3*A*b^2*c)/7 + (6*B*a*b*c)/7) + x*(B*a^3 + 3*A*a
^2*b) + x^11*((A*c^3)/11 + (3*B*b*c^2)/11) + x^3*(A*a*b^2 + A*a^2*c + B*a^
2*b) + x^9*((A*b*c^2)/3 + (B*a*c^2)/3 + (B*b^2*c)/3) - (A*a^3)/x + (B*c^3*
x^13)/13
```

3.100. $\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^2}\,dx$

3.101 $\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^3}\,dx$

3.101.1 Optimal result

Integrand size = 25, antiderivative size = 162

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^3}\,dx = -\frac{a^3A}{2x^2}+\frac{3}{2}a(abB+A(b^2+ac))x^2 + \frac{1}{4}(3aB(b^2+ac)+A(b^3+6abc))x^4 + \frac{1}{6}(b^3B+3Ab^2c+6abBc+3aAc^2)x^6 + \frac{3}{8}c(b^2B+Abc+aBc)x^8+\frac{1}{10}c^2(3bB+Ac)x^{10} + \frac{1}{12}Bc^3x^{12}+a^2(3Ab+aB)\log(x)$$

output
```
-1/2*a^3*A/x^2+3/2*a*(a*b*B+A*(a*c+b^2))*x^2+1/4*(3*a*B*(a*c+b^2)+A*(6*a*b
*c+b^3))*x^4+1/6*(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*b^3)*x^6+3/8*c*(A*b*c+B*
a*c+B*b^2)*x^8+1/10*c^2*(A*c+3*B*b)*x^10+1/12*B*c^3*x^12+a^2*(3*A*b+B*a)*l
n(x)
```

3.101.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^3}\,dx = -\frac{a^3A}{2x^2}+\frac{3}{2}a\left(abB+A\left(b^2+ac\right)\right)x^2 +\frac{1}{4}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^4 +\frac{1}{6}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^6 +\frac{3}{8}c\left(b^2B+Abc+aBc\right)x^8+\frac{1}{10}c^2(3bB+Ac)x^{10} +\frac{1}{12}Bc^3x^{12}+a^2(3Ab+aB)\log(x)$$

input

```
Integrate[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^3,x]
```

output

```
-1/2*(a^3*A)/x^2 + (3*a*(a*b*B + A*(b^2 + a*c))*x^2)/2 + ((3*a*B*(b^2 + a*
c) + A*(b^3 + 6*a*b*c))*x^4)/4 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c
^2)*x^6)/6 + (3*c*(b^2*B + A*b*c + a*B*c)*x^8)/8 + (c^2*(3*b*B + A*c)*x^10
)/10 + (B*c^3*x^12)/12 + a^2*(3*A*b + a*B)*Log[x]
```

3.101.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^3}\,dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{(Bx^2+A)(cx^4+bx^2+a)^3}{x^4}dx^2$$

$$\Big\downarrow \text{1195}$$

$$\frac{1}{2}\int \Big(Bc^3x^{10}+c^2(3bB+Ac)x^8+3c\left(Bb^2+Acb+aBc\right)x^6+\left(Bb^3+3Acb^2+6aBcb+3aAc^2\right)x^4+\left(3aB\left(b^2+\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^3A}{x^2}+a^2\log\left(x^2\right)(aB+3Ab)+\frac{3}{4}cx^8\left(aBc+Abc+b^2B\right)+3ax^2\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{3}x^6\left(3aAc^2+6abB\right.\right.$$

input
```
Int[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^3,x]
```

output
```
(-((a^3*A)/x^2) + 3*a*(a*b*B + A*(b^2 + a*c))*x^2 + ((3*a*B*(b^2 + a*c) +
A*(b^3 + 6*a*b*c))*x^4)/2 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x
^6)/3 + (3*c*(b^2*B + A*b*c + a*B*c)*x^8)/4 + (c^2*(3*b*B + A*c)*x^10)/5 +
 (B*c^3*x^12)/6 + a^2*(3*A*b + a*B)*Log[x^2])/2
```

3.101.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.101.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.04

method	result
norman	$\frac{\left(\frac{1}{10} A\,c^{3}+\frac{3}{10} B b\,c^{2}\right) x^{12}+\left(\frac{3}{8} A b\,c^{2}+\frac{3}{8} B a\,c^{2}+\frac{3}{8} B\,b^{2} c\right) x^{10}+\left(\frac{3}{2} A c\,a^{2}+\frac{3}{2} A a\,b^{2}+\frac{3}{2} B\,a^{2} b\right) x^{4}+\left(\frac{1}{2} A a\,c^{2}+\frac{1}{2} A\,b^{2} c+B a b c+\frac{1}{6} B\,b^{3}\right) x^{8}}{x^{2}}$
default	$\frac{B\,c^{3} x^{12}}{12}+\frac{A\,c^{3} x^{10}}{10}+\frac{3 B b\,c^{2} x^{10}}{10}+\frac{3 A b\,c^{2} x^{8}}{8}+\frac{3 B a\,c^{2} x^{8}}{8}+\frac{3 B\,b^{2} c\,x^{8}}{8}+\frac{A a\,c^{2} x^{6}}{2}+\frac{A\,b^{2} c\,x^{6}}{2}+B a b c\,x^{6}+$
risch	$\frac{B\,c^{3} x^{12}}{12}+\frac{A\,c^{3} x^{10}}{10}+\frac{3 B b\,c^{2} x^{10}}{10}+\frac{3 A b\,c^{2} x^{8}}{8}+\frac{3 B a\,c^{2} x^{8}}{8}+\frac{3 B\,b^{2} c\,x^{8}}{8}+\frac{A a\,c^{2} x^{6}}{2}+\frac{A\,b^{2} c\,x^{6}}{2}+B a b c\,x^{6}+$
parallelrisch	$10 B\,c^{3} x^{14}+12 A\,c^{3} x^{12}+36 B b\,c^{2} x^{12}+45 A b\,c^{2} x^{10}+45 B a\,c^{2} x^{10}+45 B\,b^{2} c\,x^{10}+60 A a\,c^{2} x^{8}+60 A\,b^{2} c\,x^{8}+120 B a b c\,x^{8}+20 B\,b^{3} x^{8}+$

input
```
int((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
((1/10*A*c^3+3/10*B*b*c^2)*x^12+(3/8*A*b*c^2+3/8*B*a*c^2+3/8*B*b^2*c)*x^10
+(3/2*A*c*a^2+3/2*A*a*b^2+3/2*B*a^2*b)*x^4+(1/2*A*a*c^2+1/2*A*b^2*c+B*a*b*
c+1/6*B*b^3)*x^8+(3/2*A*a*b*c+1/4*A*b^3+3/4*a^2*B*c+3/4*B*a*b^2)*x^6-1/2*A
*a^3+1/12*B*c^3*x^14)/x^2+(3*A*a^2*b+B*a^3)*ln(x)
```

3.101.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.05

$$\int \frac{(A+Bx^2)\,(a+bx^2+cx^4)^3}{x^3}\,dx$$
$$= \frac{10\,Bc^3x^{14}+12\,(3\,Bbc^2+Ac^3)x^{12}+45\,(Bb^2c+(Ba+Ab)c^2)x^{10}+20\,(Bb^3+3\,Aac^2+3\,(2\,Bab+Ab^2)c)}{}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^3,x, algorithm="fricas")
```

output
```
1/120*(10*B*c^3*x^14 + 12*(3*B*b*c^2 + A*c^3)*x^12 + 45*(B*b^2*c + (B*a +
A*b)*c^2)*x^10 + 20*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^8 + 30*(
3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^6 + 180*(B*a^2*b + A*a*b^2 +
A*a^2*c)*x^4 - 60*A*a^3 + 120*(B*a^3 + 3*A*a^2*b)*x^2*log(x))/x^2
```

3.101.6 Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.22

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^3}\,dx = -\frac{Aa^3}{2x^2}+\frac{Bc^3x^{12}}{12}+a^2\cdot(3Ab+Ba)\log(x) +x^{10}\left(\frac{Ac^3}{10}+\frac{3Bbc^2}{10}\right)+x^8\cdot\left(\frac{3Abc^2}{8}+\frac{3Bac^2}{8}+\frac{3Bb^2c}{8}\right) +x^6\left(\frac{Aac^2}{2}+\frac{Ab^2c}{2}+Babc+\frac{Bb^3}{6}\right)+x^4 \cdot\left(\frac{3Aabc}{2}+\frac{Ab^3}{4}+\frac{3Ba^2c}{4}+\frac{3Bab^2}{4}\right) +x^2\cdot\left(\frac{3Aa^2c}{2}+\frac{3Aab^2}{2}+\frac{3Ba^2b}{2}\right)$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2+a)**3/x**3,x)
```

output
```
-A*a**3/(2*x**2) + B*c**3*x**12/12 + a**2*(3*A*b + B*a)*log(x) + x**10*(A*
c**3/10 + 3*B*b*c**2/10) + x**8*(3*A*b*c**2/8 + 3*B*a*c**2/8 + 3*B*b**2*c/
8) + x**6*(A*a*c**2/2 + A*b**2*c/2 + B*a*b*c + B*b**3/6) + x**4*(3*A*a*b*c
/2 + A*b**3/4 + 3*B*a**2*c/4 + 3*B*a*b**2/4) + x**2*(3*A*a**2*c/2 + 3*A*a*
b**2/2 + 3*B*a**2*b/2)
```

3.101.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^3}\,dx = \frac{1}{12}Bc^3x^{12}+\frac{1}{10}\left(3\,Bbc^2+Ac^3\right)x^{10} +\frac{3}{8}\left(Bb^2c+(Ba+Ab)c^2\right)x^8 +\frac{1}{6}\left(Bb^3+3\,Aac^2+3\left(2\,Bab+Ab^2\right)c\right)x^6 +\frac{1}{4}\left(3\,Bab^2+Ab^3+3\left(Ba^2+2\,Aab\right)c\right)x^4 +\frac{3}{2}\left(Ba^2b+Aab^2+Aa^2c\right)x^2 -\frac{Aa^3}{2\,x^2}+\frac{1}{2}\left(Ba^3+3\,Aa^2b\right)\log\left(x^2\right)$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^3,x, algorithm="maxima")
```

output
```
1/12*B*c^3*x^12 + 1/10*(3*B*b*c^2 + A*c^3)*x^10 + 3/8*(B*b^2*c + (B*a + A*
b)*c^2)*x^8 + 1/6*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^6 + 1/4*(3
*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^4 + 3/2*(B*a^2*b + A*a*b^2 + A
*a^2*c)*x^2 - 1/2*A*a^3/x^2 + 1/2*(B*a^3 + 3*A*a^2*b)*log(x^2)
```

3.101.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.31

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^3}\,dx = \frac{1}{12}Bc^3x^{12} + \frac{3}{10}Bbc^2x^{10} + \frac{1}{10}Ac^3x^{10} + \frac{3}{8}Bb^2cx^8 + \frac{3}{8}Bac^2x^8 + \frac{3}{8}Abc^2x^8 + \frac{1}{6}Bb^3x^6 + Babcx^6 + \frac{1}{2}Ab^2cx^6 + \frac{1}{2}Aac^2x^6 + \frac{3}{4}Bab^2x^4 + \frac{1}{4}Ab^3x^4 + \frac{3}{4}Ba^2cx^4 + \frac{3}{2}Aabcx^4 + \frac{3}{2}Ba^2bx^2 + \frac{3}{2}Aab^2x^2 + \frac{3}{2}Aa^2cx^2 + \frac{1}{2}\left(Ba^3 + 3\,Aa^2b\right)\log\left(x^2\right) - \frac{Ba^3x^2 + 3\,Aa^2bx^2 + Aa^3}{2\,x^2}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^3,x, algorithm="giac")
```

output
```
1/12*B*c^3*x^12 + 3/10*B*b*c^2*x^10 + 1/10*A*c^3*x^10 + 3/8*B*b^2*c*x^8 +
3/8*B*a*c^2*x^8 + 3/8*A*b*c^2*x^8 + 1/6*B*b^3*x^6 + B*a*b*c*x^6 + 1/2*A*b^
2*c*x^6 + 1/2*A*a*c^2*x^6 + 3/4*B*a*b^2*x^4 + 1/4*A*b^3*x^4 + 3/4*B*a^2*c*
x^4 + 3/2*A*a*b*c*x^4 + 3/2*B*a^2*b*x^2 + 3/2*A*a*b^2*x^2 + 3/2*A*a^2*c*x^
2 + 1/2*(B*a^3 + 3*A*a^2*b)*log(x^2) - 1/2*(B*a^3*x^2 + 3*A*a^2*b*x^2 + A*
a^3)/x^2
```

3.101.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.02

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^3}\,dx = x^4\left(\frac{3\,B\,c\,a^2}{4}+\frac{3\,B\,a\,b^2}{4}+\frac{3\,A\,c\,a\,b}{2}+\frac{A\,b^3}{4}\right) + x^6\left(\frac{B\,b^3}{6}+\frac{A\,b^2\,c}{2}+B\,a\,b\,c+\frac{A\,a\,c^2}{2}\right) + x^{10}\left(\frac{A\,c^3}{10}+\frac{3\,B\,b\,c^2}{10}\right)+\ln(x)\,\left(B\,a^3+3\,A\,b\,a^2\right) + x^2\left(\frac{3\,B\,a^2\,b}{2}+\frac{3\,A\,c\,a^2}{2}+\frac{3\,A\,a\,b^2}{2}\right) + x^8\left(\frac{3\,B\,b^2\,c}{8}+\frac{3\,A\,b\,c^2}{8}+\frac{3\,B\,a\,c^2}{8}\right)-\frac{A\,a^3}{2\,x^2}+\frac{B\,c^3\,x^{12}}{12}$$

input
```
int(((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^3,x)
```

output
```
x^4*((A*b^3)/4 + (3*B*a*b^2)/4 + (3*B*a^2*c)/4 + (3*A*a*b*c)/2) + x^6*((B*
b^3)/6 + (A*a*c^2)/2 + (A*b^2*c)/2 + B*a*b*c) + x^10*((A*c^3)/10 + (3*B*b*
c^2)/10) + log(x)*(B*a^3 + 3*A*a^2*b) + x^2*((3*A*a*b^2)/2 + (3*A*a^2*c)/2
 + (3*B*a^2*b)/2) + x^8*((3*A*b*c^2)/8 + (3*B*a*c^2)/8 + (3*B*b^2*c)/8) -
(A*a^3)/(2*x^2) + (B*c^3*x^12)/12
```

3.102 $\int \frac{x^5(A+Bx^2)}{a+bx^2+cx^4}\,dx$

3.102.1 Optimal result

Integrand size = 25, antiderivative size = 133

$$\int \frac{x^5(A+Bx^2)}{a+bx^2+cx^4}\,dx = -\frac{(bB-Ac)x^2}{2c^2}+\frac{Bx^4}{4c}+\frac{(b^3B-Ab^2c-3abBc+2aAc^2)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}+\frac{(b^2B-Abc-aBc)\log(a+bx^2+cx^4)}{4c^3}$$

output
```
-1/2*(-A*c+B*b)*x^2/c^2+1/4*B*x^4/c+1/4*(-A*b*c-B*a*c+B*b^2)*ln(c*x^4+b*x^
2+a)/c^3+1/2*(2*A*a*c^2-A*b^2*c-3*B*a*b*c+B*b^3)*arctanh((2*c*x^2+b)/(-4*a
*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(1/2)
```

3.102.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.95

$$\int \frac{x^5(A+Bx^2)}{a+bx^2+cx^4}\,dx$$
$$=\frac{2c(-bB+Ac)x^2+Bc^2x^4+\frac{2(-b^3B+Ab^2c+3abBc-2aAc^2)\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+(b^2B-Abc-aBc)\log(a+bx^2+}{4c^3}$$

input
```
Integrate[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

output
```
(2*c*(-(b*B) + A*c)*x^2 + B*c^2*x^4 + (2*(-(b^3*B) + A*b^2*c + 3*a*b*B*c -
 2*a*A*c^2)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] +
 (b^2*B - A*b*c - a*B*c)*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

3.102.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1578, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(A+Bx^2)}{a+bx^2+cx^4}dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{x^4(Bx^2+A)}{cx^4+bx^2+a}dx^2$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{2}\int \left(\frac{Bx^2}{c}-\frac{bB-Ac}{c^2}+\frac{(Bb^2-Acb-aBc)\,x^2+a(bB-Ac)}{c^2\,(cx^4+bx^2+a)}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{(2aAc^2-3abBc-Ab^2c+b^3B)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}+\frac{(-aBc-Abc+b^2B)\log\left(a+bx^2+cx^4\right)}{2c^3}-\frac{x^2(bB-A}{c^2}\right.$$

input
```
Int[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

output
```
(-(((b*B - A*c)*x^2)/c^2) + (B*x^4)/(2*c) + ((b^3*B - A*b^2*c - 3*a*b*B*c
+ 2*a*A*c^2)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a
*c]) + ((b^2*B - A*b*c - a*B*c)*Log[a + b*x^2 + c*x^4])/(2*c^3))/2
```

3.102.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.102.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{\frac{1}{2}B\,x^4c+Ac\,x^2-Bb\,x^2}{2c^2}+\frac{\frac{\left(-Abc-Bac+B\,b^2\right)\ln\left(c\,x^4+b\,x^2+a\right)}{2c}+\frac{2\left(-Aac+abB-\frac{\left(-Abc-Bac+B\,b^2\right)b}{2c}\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2c^2}$	136
risch	Expression too large to display	2131

input
```
int(x^5*(B*x^2+A)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/c^2*(1/2*B*x^4*c+A*c*x^2-B*b*x^2)+1/2/c^2*(1/2*(-A*b*c-B*a*c+B*b^2)/c*
ln(c*x^4+b*x^2+a)+2*(-A*a*c+a*b*B-1/2*(-A*b*c-B*a*c+B*b^2)*b/c)/(4*a*c-b^2
)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

3.102.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 421, normalized size of antiderivative = 3.17

$$\int \frac{x^5(A+Bx^2)}{a+bx^2+cx^4}\,dx$$
$$= \left[\frac{(Bb^2c^2-4\,Bac^3)x^4-2\,(Bb^3c+4\,Aac^3-(4\,Bab+Ab^2)c^2)x^2+(Bb^3+2\,Aac^2-(3\,Bab+Ab^2)c)\sqrt{b^2-\cdots}}{4\,(b\cdots}\right.$$

input
```
integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
[1/4*((B*b^2*c^2 - 4*B*a*c^3)*x^4 - 2*(B*b^3*c + 4*A*a*c^3 - (4*B*a*b + A*
b^2)*c^2)*x^2 + (B*b^3 + 2*A*a*c^2 - (3*B*a*b + A*b^2)*c)*sqrt(b^2 - 4*a*c
)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*
c))/(c*x^4 + b*x^2 + a)) + (B*b^4 + 4*(B*a^2 + A*a*b)*c^2 - (5*B*a*b^2 + A
*b^3)*c)*log(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4), 1/4*((B*b^2*c^2 - 4*
B*a*c^3)*x^4 - 2*(B*b^3*c + 4*A*a*c^3 - (4*B*a*b + A*b^2)*c^2)*x^2 + 2*(B*
b^3 + 2*A*a*c^2 - (3*B*a*b + A*b^2)*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2
 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (B*b^4 + 4*(B*a^2 + A*a*b)*c^2 -
 (5*B*a*b^2 + A*b^3)*c)*log(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4)]
```

3.102.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(A+Bx^2)}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.102.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(A+Bx^2)}{a+bx^2+cx^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.102.8 Giac [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.95

$$\int \frac{x^5(A+Bx^2)}{a+bx^2+cx^4}\,dx = \frac{Bcx^4-2\,Bbx^2+2\,Acx^2}{4\,c^2}+\frac{(Bb^2-Bac-Abc)\log\left(cx^4+bx^2+a\right)}{4\,c^3} - \frac{(Bb^3-3\,Babc-Ab^2c+2\,Aac^2)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}c^3}$$

input
```
integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/4*(B*c*x^4 - 2*B*b*x^2 + 2*A*c*x^2)/c^2 + 1/4*(B*b^2 - B*a*c - A*b*c)*lo
g(c*x^4 + b*x^2 + a)/c^3 - 1/2*(B*b^3 - 3*B*a*b*c - A*b^2*c + 2*A*a*c^2)*a
rctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)
```

3.102.9 Mupad [B] (verification not implemented)

Time = 7.88 (sec) , antiderivative size = 1343, normalized size of antiderivative = 10.10

$$\int \frac{x^5(A+Bx^2)}{a+bx^2+cx^4}\,dx = x^2\left(\frac{A}{2\,c}-\frac{B\,b}{2\,c^2}\right)+\frac{B\,x^4}{4\,c}$$
$$-\frac{\ln\left(c\,x^4+b\,x^2+a\right)\,\left(8\,B\,a^2\,c^2-10\,B\,a\,b^2\,c+8\,A\,a\,b\,c^2+2\,B\,b^4-2\,A\,b^3\,c\right)}{2\,\left(16\,a\,c^4-4\,b^2\,c^3\right)}$$
$$+\frac{\mathrm{atan}\left(\frac{2\,c^4\,\left(4\,a\,c-b^2\right)\left(x^2\left(\frac{\frac{\left(\frac{-6\,B\,b^3\,c^3+6\,A\,b^2\,c^4+10\,B\,a\,b\,c^4-4\,A\,a\,c^5}{c^4}-\frac{4\,b\,c^2\,\left(8\,B\,a^2\,c^2-10\,B\,a\,b^2\,c+8\,A\,a\,b\,c^2+2\,B\,b^4-2\,A\,b^3\,c\right)}{16\,a\,c^4-4\,b^2\,c^3}\right)\left(B\,b^3-A\,b^2\,c-\cdots\right.}{8\,c^3\,\sqrt{4\,a\,c-b^2}}}{a}\cdots\right.\right.}{\cdots}\right)}{\cdots}$$

input

```
int((x^5*(A + B*x^2))/(a + b*x^2 + c*x^4),x)
```

output
```
x^2*(A/(2*c) - (B*b)/(2*c^2)) + (B*x^4)/(4*c) - (log(a + b*x^2 + c*x^4)*(2
*B*b^4 + 8*B*a^2*c^2 - 2*A*b^3*c + 8*A*a*b*c^2 - 10*B*a*b^2*c))/(2*(16*a*c
^4 - 4*b^2*c^3)) + (atan((2*c^4*(4*a*c - b^2)*(x^2*(((((6*A*b^2*c^4 - 6*B*
b^3*c^3 - 4*A*a*c^5 + 10*B*a*b*c^4)/c^4 - (4*b*c^2*(2*B*b^4 + 8*B*a^2*c^2
- 2*A*b^3*c + 8*A*a*b*c^2 - 10*B*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3))*(B*b^3
+ 2*A*a*c^2 - A*b^2*c - 3*B*a*b*c))/(8*c^3*(4*a*c - b^2)^(1/2)) - (b*(B*b^
3 + 2*A*a*c^2 - A*b^2*c - 3*B*a*b*c)*(2*B*b^4 + 8*B*a^2*c^2 - 2*A*b^3*c +
8*A*a*b*c^2 - 10*B*a*b^2*c))/(2*c*(4*a*c - b^2)^(1/2)*(16*a*c^4 - 4*b^2*c^
3)))/a - (b*((((6*A*b^2*c^4 - 6*B*b^3*c^3 - 4*A*a*c^5 + 10*B*a*b*c^4)/c^4
- (4*b*c^2*(2*B*b^4 + 8*B*a^2*c^2 - 2*A*b^3*c + 8*A*a*b*c^2 - 10*B*a*b^2*c
))/(16*a*c^4 - 4*b^2*c^3))*(2*B*b^4 + 8*B*a^2*c^2 - 2*A*b^3*c + 8*A*a*b*c^
2 - 10*B*a*b^2*c))/(2*(16*a*c^4 - 4*b^2*c^3)) - (B^2*b^5 + A^2*b^3*c^2 - 2
*A*B*b^4*c - A*B*a^2*c^3 - A^2*a*b*c^3 - 3*B^2*a*b^3*c + 2*B^2*a^2*b*c^2 +
 4*A*B*a*b^2*c^2)/c^4 + (b*(B*b^3 + 2*A*a*c^2 - A*b^2*c - 3*B*a*b*c)^2)/(2
*c^4*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))) + ((((8*B*a^2*c^4 + 8*A*a
*b*c^4 - 8*B*a*b^2*c^3)/c^4 - (8*a*c^2*(2*B*b^4 + 8*B*a^2*c^2 - 2*A*b^3*c
+ 8*A*a*b*c^2 - 10*B*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3))*(B*b^3 + 2*A*a*c^2
- A*b^2*c - 3*B*a*b*c))/(8*c^3*(4*a*c - b^2)^(1/2)) - (a*(B*b^3 + 2*A*a*c^
2 - A*b^2*c - 3*B*a*b*c)*(2*B*b^4 + 8*B*a^2*c^2 - 2*A*b^3*c + 8*A*a*b*c^2
- 10*B*a*b^2*c))/(c*(4*a*c - b^2)^(1/2)*(16*a*c^4 - 4*b^2*c^3)))/a - (b...
```

3.102. $\int \frac{x^5(A+Bx^2)}{a+bx^2+cx^4}\,dx$

3.103 $\int \frac{x^3(A+Bx^2)}{a+bx^2+cx^4}\,dx$

3.103.1 Optimal result

Integrand size = 25, antiderivative size = 97

$$\int \frac{x^3(A+Bx^2)}{a+bx^2+cx^4}\,dx=\frac{Bx^2}{2c}-\frac{(b^2B-Abc-2aBc)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{(bB-Ac)\log(a+bx^2+cx^4)}{4c^2}$$

output
```
1/2*B*x^2/c-1/4*(-A*c+B*b)*ln(c*x^4+b*x^2+a)/c^2-1/2*(-A*b*c-2*B*a*c+B*b^2
)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

3.103.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.96

$$\int \frac{x^3(A+Bx^2)}{a+bx^2+cx^4}\,dx=\frac{2Bcx^2+\frac{2(b^2B-Abc-2aBc)\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+(-bB+Ac)\log(a+bx^2+cx^4)}{4c^2}$$

input
```
Integrate[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

output
```
(2*B*c*x^2 + (2*(b^2*B - A*b*c - 2*a*B*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 +
 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (-(b*B) + A*c)*Log[a + b*x^2 + c*x^4])/(4*c
^2)
```

3.103.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1578, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (A+Bx^2)}{a+bx^2+cx^4} \, dx$$

$$\Big\downarrow \; 1578$$

$$\frac{1}{2} \int \frac{x^2 (Bx^2+A)}{cx^4+bx^2+a} dx^2$$

$$\Big\downarrow \; 1200$$

$$\frac{1}{2} \int \left(\frac{B}{c} - \frac{(bB-Ac)x^2+aB}{c\,(cx^4+bx^2+a)}\right) dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(-\frac{(-2aBc-Abc+b^2B)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}} - \frac{(bB-Ac)\log\left(a+bx^2+cx^4\right)}{2c^2} + \frac{Bx^2}{c}\right)$$

input
```
Int[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

output
```
((B*x^2)/c - ((b^2*B - A*b*c - 2*a*B*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4
*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]) - ((b*B - A*c)*Log[a + b*x^2 + c*x^4])/(2*
c^2))/2
```

3.103.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.103.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.01

method	result	size
default	$\frac{B x^2}{2c}+\frac{\frac{(Ac-Bb)\ln\left(c x^4+b x^2+a\right)}{2c}+\frac{2\left(-Ba-\frac{(Ac-Bb)b}{2c}\right)\arctan\left(\frac{2c x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2c}$	98
risch	Expression too large to display	1398

input
```
int(x^3*(B*x^2+A)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*B*x^2/c+1/2/c*(1/2*(A*c-B*b)/c*ln(c*x^4+b*x^2+a)+2*(-B*a-1/2*(A*c-B*b)
*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

3.103.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 312, normalized size of antiderivative = 3.22

$$\int \frac{x^3(A+Bx^2)}{a+bx^2+cx^4}\,dx = \left[\frac{2\,(Bb^2c-4\,Bac^2)x^2-(Bb^2-(2\,Ba+Ab)c)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-(Bb^3 \cdots}{4\,(b^2c^2-4\,ac^3)}\right.$$

input
```
integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
[1/4*(2*(B*b^2*c - 4*B*a*c^2)*x^2 - (B*b^2 - (2*B*a + A*b)*c)*sqrt(b^2 - 4
*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 -
4*a*c))/(c*x^4 + b*x^2 + a)) - (B*b^3 + 4*A*a*c^2 - (4*B*a*b + A*b^2)*c)*l
og(c*x^4 + b*x^2 + a))/(b^2*c^2 - 4*a*c^3), 1/4*(2*(B*b^2*c - 4*B*a*c^2)*x
^2 - 2*(B*b^2 - (2*B*a + A*b)*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*
sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (B*b^3 + 4*A*a*c^2 - (4*B*a*b + A*b^2)
*c)*log(c*x^4 + b*x^2 + a))/(b^2*c^2 - 4*a*c^3)]
```

3.103.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(90) = 180$.

Time = 72.99 (sec) , antiderivative size = 434, normalized size of antiderivative = 4.47

$$\int \frac{x^3(A+Bx^2)}{a+bx^2+cx^4}\,dx = \frac{Bx^2}{2c} + \left(-\frac{-Ac+Bb}{4c^2} - \frac{\sqrt{-4ac+b^2}\,(Abc+2Bac-Bb^2)}{4c^2\cdot(4ac-b^2)}\right)\log\left(x^2 + \frac{2Aac-Bab-8ac^2\left(-\frac{-Ac+Bb}{4c^2}-\frac{\sqrt{-4ac+b^2}(Abc+2Bac-Bb^2)}{4c^2\cdot(4ac-b^2)}\right.}{Abc+2Bac-}\right. + \left(-\frac{-Ac+Bb}{4c^2} + \frac{\sqrt{-4ac+b^2}\,(Abc+2Bac-Bb^2)}{4c^2\cdot(4ac-b^2)}\right)\log\left(x^2 + \frac{2Aac-Bab-8ac^2\left(-\frac{-Ac+Bb}{4c^2}+\frac{\sqrt{-4ac+b^2}(Abc+2Bac-Bb^2)}{4c^2\cdot(4ac-b^2)}\right.}{Abc+2Bac-}\right.$$

input
```
integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2+a),x)
```

output
```
B*x**2/(2*c) + (-(-A*c + B*b)/(4*c**2) - sqrt(-4*a*c + b**2)*(A*b*c + 2*B*
a*c - B*b**2)/(4*c**2*(4*a*c - b**2)))*log(x**2 + (2*A*a*c - B*a*b - 8*a*c
**2*(-(-A*c + B*b)/(4*c**2) - sqrt(-4*a*c + b**2)*(A*b*c + 2*B*a*c - B*b**
2)/(4*c**2*(4*a*c - b**2))) + 2*b**2*c*(-(-A*c + B*b)/(4*c**2) - sqrt(-4*a
*c + b**2)*(A*b*c + 2*B*a*c - B*b**2)/(4*c**2*(4*a*c - b**2))))/(A*b*c + 2
*B*a*c - B*b**2)) + (-(-A*c + B*b)/(4*c**2) + sqrt(-4*a*c + b**2)*(A*b*c +
 2*B*a*c - B*b**2)/(4*c**2*(4*a*c - b**2)))*log(x**2 + (2*A*a*c - B*a*b -
8*a*c**2*(-(-A*c + B*b)/(4*c**2) + sqrt(-4*a*c + b**2)*(A*b*c + 2*B*a*c -
B*b**2)/(4*c**2*(4*a*c - b**2))) + 2*b**2*c*(-(-A*c + B*b)/(4*c**2) + sqrt
(-4*a*c + b**2)*(A*b*c + 2*B*a*c - B*b**2)/(4*c**2*(4*a*c - b**2))))/(A*b*
c + 2*B*a*c - B*b**2))
```

3.103.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(A+Bx^2)}{a+bx^2+cx^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.103.8 Giac [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.94

$$\int \frac{x^3(A+Bx^2)}{a+bx^2+cx^4}\,dx = \frac{Bx^2}{2\,c} - \frac{(Bb-Ac)\log\left(cx^4+bx^2+a\right)}{4\,c^2} + \frac{(Bb^2-2\,Bac-Abc)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}c^2}$$

input
```
integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/2*B*x^2/c - 1/4*(B*b - A*c)*log(c*x^4 + b*x^2 + a)/c^2 + 1/2*(B*b^2 - 2*
B*a*c - A*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c
)*c^2)
```

3.103.9 Mupad [B] (verification not implemented)

Time = 8.01 (sec) , antiderivative size = 979, normalized size of antiderivative = 10.09

$$\int \frac{x^3(A+Bx^2)}{a+bx^2+cx^4}\,dx = \frac{B\,x^2}{2\,c} + \frac{\ln\left(c\,x^4+b\,x^2+a\right)\,\left(2\,B\,b^3-2\,A\,b^2\,c-8\,B\,a\,b\,c+8\,A\,a\,c^2\right)}{2\,\left(16\,a\,c^3-4\,b^2\,c^2\right)}$$

$$-\frac{\mathrm{atan}\left(\frac{2\,c^2\,\left(4\,a\,c-b^2\right)\left(\frac{\frac{\left(\frac{8\,A\,a\,c^3-8\,B\,a\,b\,c^2}{c^2}-\frac{8\,a\,c^2\,\left(2\,B\,b^3-2\,A\,b^2\,c-8\,B\,a\,b\,c+8\,A\,a\,c^2\right)}{16\,a\,c^3-4\,b^2\,c^2}\right)\left(-B\,b^2+A\,c\,b+2\,B\,a\,c\right)}{8\,c^2\,\sqrt{4\,a\,c-b^2}}-\frac{a\,\left(-B\,b^2+A\,c\,b+2\,B\,a\,c\right)\,\left(2\,B\,b^3\cdots\right.}{\sqrt{4\,a\,c-b^2}\,\left(16\,a\cdots\right.}}{a}\cdots\right.}{\cdots}\right)}{\cdots}$$

input
```
int((x^3*(A + B*x^2))/(a + b*x^2 + c*x^4),x)
```

output
```
(B*x^2)/(2*c) + (log(a + b*x^2 + c*x^4)*(2*B*b^3 + 8*A*a*c^2 - 2*A*b^2*c -
 8*B*a*b*c))/(2*(16*a*c^3 - 4*b^2*c^2)) - (atan((2*c^2*(4*a*c - b^2)*(((((
8*A*a*c^3 - 8*B*a*b*c^2)/c^2 - (8*a*c^2*(2*B*b^3 + 8*A*a*c^2 - 2*A*b^2*c -
 8*B*a*b*c))/(16*a*c^3 - 4*b^2*c^2))*(A*b*c - B*b^2 + 2*B*a*c))/(8*c^2*(4*
a*c - b^2)^(1/2)) - (a*(A*b*c - B*b^2 + 2*B*a*c)*(2*B*b^3 + 8*A*a*c^2 - 2*
A*b^2*c - 8*B*a*b*c))/((4*a*c - b^2)^(1/2)*(16*a*c^3 - 4*b^2*c^2)))/a + x^
2*(((((6*A*b*c^3 - 6*B*b^2*c^2 + 4*B*a*c^3)/c^2 - (4*b*c^2*(2*B*b^3 + 8*A*
a*c^2 - 2*A*b^2*c - 8*B*a*b*c))/(16*a*c^3 - 4*b^2*c^2))*(A*b*c - B*b^2 + 2
*B*a*c))/(8*c^2*(4*a*c - b^2)^(1/2)) - (b*(A*b*c - B*b^2 + 2*B*a*c)*(2*B*b
^3 + 8*A*a*c^2 - 2*A*b^2*c - 8*B*a*b*c))/(2*(4*a*c - b^2)^(1/2)*(16*a*c^3
 - 4*b^2*c^2)))/a + (b*((((6*A*b*c^3 - 6*B*b^2*c^2 + 4*B*a*c^3)/c^2 - (4*b*
c^2*(2*B*b^3 + 8*A*a*c^2 - 2*A*b^2*c - 8*B*a*b*c))/(16*a*c^3 - 4*b^2*c^2))
*(2*B*b^3 + 8*A*a*c^2 - 2*A*b^2*c - 8*B*a*b*c))/(2*(16*a*c^3 - 4*b^2*c^2))
 - (B^2*b^3 + A^2*b*c^2 + A*B*a*c^2 - 2*A*B*b^2*c - B^2*a*b*c)/c^2 + (b*(A
*b*c - B*b^2 + 2*B*a*c)^2)/(2*c^2*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2
))) + (b*((((8*A*a*c^3 - 8*B*a*b*c^2)/c^2 - (8*a*c^2*(2*B*b^3 + 8*A*a*c^2
 - 2*A*b^2*c - 8*B*a*b*c))/(16*a*c^3 - 4*b^2*c^2))*(2*B*b^3 + 8*A*a*c^2 - 2
*A*b^2*c - 8*B*a*b*c))/(2*(16*a*c^3 - 4*b^2*c^2)) - (A^2*a*c^2 + B^2*a*b^2
 - 2*A*B*a*b*c)/c^2 + (a*(A*b*c - B*b^2 + 2*B*a*c)^2)/(c^2*(4*a*c - b^2)))
)/(2*a*(4*a*c - b^2)^(1/2))))/(B^2*b^4 + A^2*b^2*c^2 + 4*B^2*a^2*c^2 - ...
```

3.103. $\int \frac{x^3(A+Bx^2)}{a+bx^2+cx^4}\,dx$

3.104 $\int \frac{x(A+Bx^2)}{a+bx^2+cx^4}\,dx$

3.104.1 Optimal result

Integrand size = 23, antiderivative size = 71

$$\int \frac{x(A+Bx^2)}{a+bx^2+cx^4}\,dx=\frac{(bB-2Ac)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{B\log\left(a+bx^2+cx^4\right)}{4c}$$

output
```
1/4*B*ln(c*x^4+b*x^2+a)/c+1/2*(-2*A*c+B*b)*arctanh((2*c*x^2+b)/(-4*a*c+b^2
)^(1/2))/c/(-4*a*c+b^2)^(1/2)
```

3.104.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int \frac{x(A+Bx^2)}{a+bx^2+cx^4}\,dx=\frac{-\frac{2(bB-2Ac)\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+B\log\left(a+bx^2+cx^4\right)}{4c}$$

input
```
Integrate[(x*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

output
```
((-2*(b*B - 2*A*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4
*a*c] + B*Log[a + b*x^2 + c*x^4])/(4*c)
```

3.104.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1576, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx^2)}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{ 1576}$$

$$\frac{1}{2}\int \frac{Bx^2+A}{cx^4+bx^2+a}dx^2$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{1}{2}\left(\frac{B\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{2c}-\frac{(bB-2Ac)\int \frac{1}{cx^4+bx^2+a}dx^2}{2c}\right)$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{1}{2}\left(\frac{(bB-2Ac)\int \frac{1}{-x^4+b^2-4ac}d(2cx^2+b)}{c}+\frac{B\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{2c}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}\left(\frac{B\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{2c}+\frac{(bB-2Ac)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}\right)$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{1}{2}\left(\frac{(bB-2Ac)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{B\log(a+bx^2+cx^4)}{2c}\right)$$

input
```
Int[(x*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

output
```
(((b*B - 2*A*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*
a*c]) + (B*Log[a + b*x^2 + c*x^4])/(2*c))/2
```

3.104.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

3.104.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

method	result
default	$\frac{B\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{4c}+\frac{\left(A-\frac{Bb}{2c}\right)\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}$
risch	$\frac{\ln\left(\left(-8Aa\,c^{2}+2A\,b^{2}c+4Babc-B\,b^{3}-\sqrt{-(4ac-b^{2})(2Ac-Bb)^{2}}\,b\right)x^{2}-2\sqrt{-(4ac-b^{2})(2Ac-Bb)^{2}}\,a\right)Ba}{4ac-b^{2}}-\frac{\ln\left(\left(-8Aa\,c^{2}+2A\,b^{2}c-\right.\right.}{}$

input
```
int(x*(B*x^2+A)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*B*ln(c*x^4+b*x^2+a)/c+(A-1/2*B*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+
b)/(4*a*c-b^2)^(1/2))
```

3.104.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 219, normalized size of antiderivative = 3.08

$$\int \frac{x(A+Bx^2)}{a+bx^2+cx^4}\,dx = \left[-\frac{(Bb-2\,Ac)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-(Bb^2-4\,Bac)\log\left(cx^4+bx^2+a\right)}{4\,(b^2c-4\,ac^2)},\right.$$

input
```
integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
[-1/4*((B*b - 2*A*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 -
2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (B*b^2 - 4
*B*a*c)*log(c*x^4 + b*x^2 + a))/(b^2*c - 4*a*c^2), 1/4*(2*(B*b - 2*A*c)*sq
rt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) +
 (B*b^2 - 4*B*a*c)*log(c*x^4 + b*x^2 + a))/(b^2*c - 4*a*c^2)]
```

3.104.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(61) = 122$.

Time = 5.22 (sec) , antiderivative size = 287, normalized size of antiderivative = 4.04

$$\int \frac{x(A+Bx^2)}{a+bx^2+cx^4}\,dx = \left(\frac{B}{4c} - \frac{(-2Ac+Bb)\sqrt{-4ac+b^2}}{4c\,(4ac-b^2)}\right)\log\left(x^2+\frac{-Ab+2Ba-8ac\left(\frac{B}{4c}-\frac{(-2Ac+Bb)\sqrt{-4ac+b^2}}{4c(4ac-b^2)}\right)+2b^2\left(\frac{B}{4c}-\frac{(-2Ac+}{4c(}\right.}{-2Ac+Bb}\right.$$
$$+\left(\frac{B}{4c} + \frac{(-2Ac+Bb)\sqrt{-4ac+b^2}}{4c\,(4ac-b^2)}\right)\log\left(x^2+\frac{-Ab+2Ba-8ac\left(\frac{B}{4c}+\frac{(-2Ac+Bb)\sqrt{-4ac+b^2}}{4c(4ac-b^2)}\right)+2b^2\left(\frac{B}{4c}+\frac{(-2Ac+}{4c(}\right.}{-2Ac+Bb}\right.$$

input
```
integrate(x*(B*x**2+A)/(c*x**4+b*x**2+a),x)
```

output
```
(B/(4*c) - (-2*A*c + B*b)*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)))*log(x*
*2 + (-A*b + 2*B*a - 8*a*c*(B/(4*c) - (-2*A*c + B*b)*sqrt(-4*a*c + b**2)/(
4*c*(4*a*c - b**2))) + 2*b**2*(B/(4*c) - (-2*A*c + B*b)*sqrt(-4*a*c + b**2
)/(4*c*(4*a*c - b**2))))/(-2*A*c + B*b)) + (B/(4*c) + (-2*A*c + B*b)*sqrt(
-4*a*c + b**2)/(4*c*(4*a*c - b**2)))*log(x**2 + (-A*b + 2*B*a - 8*a*c*(B/(
4*c) + (-2*A*c + B*b)*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2))) + 2*b**2*(
B/(4*c) + (-2*A*c + B*b)*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2))))/(-2*A*
c + B*b))
```

3.104.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(A+Bx^2)}{a+bx^2+cx^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.104.8 Giac [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.94

$$\int \frac{x(A+Bx^2)}{a+bx^2+cx^4}\,dx = \frac{B\log\left(cx^4+bx^2+a\right)}{4\,c} - \frac{(Bb-2\,Ac)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}c}$$

input
```
integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/4*B*log(c*x^4 + b*x^2 + a)/c - 1/2*(B*b - 2*A*c)*arctan((2*c*x^2 + b)/sq
rt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c)
```

3.104.9 Mupad [B] (verification not implemented)

Time = 7.71 (sec) , antiderivative size = 606, normalized size of antiderivative = 8.54

$$\int \frac{x(A+Bx^2)}{a+bx^2+cx^4}\,dx = -\frac{\ln\left(c\,x^4+b\,x^2+a\right)\,\left(2\,B\,b^2-8\,B\,a\,c\right)}{2\,\left(16\,a\,c^2-4\,b^2\,c\right)}$$

$$-\frac{\operatorname{atan}\left(\frac{2\,\left(4\,a\,c-b^2\right)\,\left(x^2\,\left(\frac{\frac{\left(2\,A\,c-B\,b\right)\,\left(6\,B\,b\,c-4\,A\,c^2+\frac{4\,b\,c^2\,\left(2\,B\,b^2-8\,B\,a\,c\right)}{16\,a\,c^2-4\,b^2\,c}\right)}{8\,c\,\sqrt{4\,a\,c-b^2}}+\frac{b\,c\,\left(2\,B\,b^2-8\,B\,a\,c\right)\,\left(2\,A\,c-B\,b\right)}{2\,\left(16\,a\,c^2-4\,b^2\,c\right)\,\sqrt{4\,a\,c-b^2}}}{a}+\frac{b\,\left(B^2\,b-A\,B\,c-\frac{b\,\left(2\,A\,c-B\,b\right)^2}{2\,\left(4\,a\,c-b^2\right)}+\cdots\right.}{\cdots}\right.\right.}{\cdots}\right)}{\cdots}$$

input
```
int((x*(A + B*x^2))/(a + b*x^2 + c*x^4),x)
```

output
```
- (log(a + b*x^2 + c*x^4)*(2*B*b^2 - 8*B*a*c))/(2*(16*a*c^2 - 4*b^2*c)) -
(atan((2*(4*a*c - b^2)*(x^2*((((2*A*c - B*b)*(6*B*b*c - 4*A*c^2 + (4*b*c^2
*(2*B*b^2 - 8*B*a*c))/(16*a*c^2 - 4*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) + (
b*c*(2*B*b^2 - 8*B*a*c)*(2*A*c - B*b))/(2*(16*a*c^2 - 4*b^2*c)*(4*a*c - b^
2)^(1/2)))/a + (b*(B^2*b - A*B*c - (b*(2*A*c - B*b)^2)/(2*(4*a*c - b^2)) +
 ((2*B*b^2 - 8*B*a*c)*(6*B*b*c - 4*A*c^2 + (4*b*c^2*(2*B*b^2 - 8*B*a*c))/(
16*a*c^2 - 4*b^2*c)))/(2*(16*a*c^2 - 4*b^2*c))))/(2*a*(4*a*c - b^2)^(1/2))
) + (((8*B*a*c + (8*a*c^2*(2*B*b^2 - 8*B*a*c))/(16*a*c^2 - 4*b^2*c))*(2*A*
c - B*b))/(8*c*(4*a*c - b^2)^(1/2)) + (a*c*(2*B*b^2 - 8*B*a*c)*(2*A*c - B*
b))/((16*a*c^2 - 4*b^2*c)*(4*a*c - b^2)^(1/2)))/a + (b*(B^2*a + ((2*B*b^2
- 8*B*a*c)*(8*B*a*c + (8*a*c^2*(2*B*b^2 - 8*B*a*c))/(16*a*c^2 - 4*b^2*c)))
/(2*(16*a*c^2 - 4*b^2*c)) - (a*(2*A*c - B*b)^2)/(4*a*c - b^2)))/(2*a*(4*a*
c - b^2)^(1/2))))/(4*A^2*c^2 + B^2*b^2 - 4*A*B*b*c))*(2*A*c - B*b))/(2*c*(
4*a*c - b^2)^(1/2))
```

3.105 $\int \frac{A+Bx^2}{x(a+bx^2+cx^4)}\,dx$

3.105.1 Optimal result

Integrand size = 25, antiderivative size = 78

$$\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)}\,dx=\frac{(Ab-2aB)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}+\frac{A\log(x)}{a}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a}$$

output
```
A*ln(x)/a-1/4*A*ln(c*x^4+b*x^2+a)/a+1/2*(A*b-2*B*a)*arctanh((2*c*x^2+b)/(-
4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(1/2)
```

3.105.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.64

$$\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)}\,dx$$
$$=\frac{4A\sqrt{b^2-4ac}\log(x)-\left(-2aB+A\left(b+\sqrt{b^2-4ac}\right)\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right)+\left(-2aB+A\left(b-\sqrt{b^2-\right.\right.}{4a\sqrt{b^2-4ac}}$$

input
```
Integrate[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)),x]
```

output
```
(4*A*Sqrt[b^2 - 4*a*c]*Log[x] - (-2*a*B + A*(b + Sqrt[b^2 - 4*a*c]))*Log[b
 - Sqrt[b^2 - 4*a*c] + 2*c*x^2] + (-2*a*B + A*(b - Sqrt[b^2 - 4*a*c]))*Log
[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(4*a*Sqrt[b^2 - 4*a*c])
```

3.105.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1578, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)}dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{Bx^2+A}{x^2\left(cx^4+bx^2+a\right)}dx^2$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{2}\int \left(\frac{A}{ax^2}+\frac{-Acx^2-Ab+aB}{a\left(cx^4+bx^2+a\right)}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{(Ab-2aB)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{A\log\left(a+bx^2+cx^4\right)}{2a}+\frac{A\log\left(x^2\right)}{a}\right)$$

input
```
Int[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)),x]
```

output
```
(((A*b - 2*a*B)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*
a*c]) + (A*Log[x^2])/a - (A*Log[a + b*x^2 + c*x^4])/(2*a))/2
```

3.105.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.105.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.97

method	result
default	$\frac{A\ln(x)}{a}-\frac{\frac{A\ln\left(c\,x^4+b\,x^2+a\right)}{2}+\frac{2\left(\frac{Ab}{2}-Ba\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2a}$
risch	$\frac{A\ln(x)}{a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4c\,a^2-b^2a\right)\_Z^2+\left(4Aac-A\,b^2\right)\_Z+A^2c-bBA+B^2a\right)}\_R\ln\left(\left(\left(10ac-3b^2\right)\_R^2+(5Ac-Bb)\_R+2B^2\right)x\right.\right.}{2}$

input
```
int((B*x^2+A)/x/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
A*ln(x)/a-1/2/a*(1/2*A*ln(c*x^4+b*x^2+a)+2*(1/2*A*b-B*a)/(4*a*c-b^2)^(1/2)
*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

3.105.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 249, normalized size of antiderivative = 3.19

$$\int\frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)}\,dx$$
$$=\left[-\frac{(2\,Ba-Ab)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)+\left(Ab^2-4\,Aac\right)\log\left(cx^4+bx^2+a\right)-}{4\left(ab^2-4\,a^2c\right)}\right.$$
$$-\frac{2\,(2\,Ba-Ab)\sqrt{-b^2+4\,ac}\arctan\left(-\frac{\left(2\,cx^2+b\right)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)+\left(Ab^2-4\,Aac\right)\log\left(cx^4+bx^2+a\right)-4\left(Ab^2-\right.}{4\left(ab^2-4\,a^2c\right)}$$

input
```
integrate((B*x^2+A)/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
[-1/4*((2*B*a - A*b)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 -
2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (A*b^2 - 4
*A*a*c)*log(c*x^4 + b*x^2 + a) - 4*(A*b^2 - 4*A*a*c)*log(x))/(a*b^2 - 4*a^
2*c), -1/4*(2*(2*B*a - A*b)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(
-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (A*b^2 - 4*A*a*c)*log(c*x^4 + b*x^2 + a) -
4*(A*b^2 - 4*A*a*c)*log(x))/(a*b^2 - 4*a^2*c)]
```

3.105.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)}\,dx=\text{Timed out}$$

input
```
integrate((B*x**2+A)/x/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.105.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((B*x^2+A)/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.105.8 Giac [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)}\,dx = -\frac{A\log\left(cx^4+bx^2+a\right)}{4\,a} + \frac{A\log\left(x^2\right)}{2\,a} + \frac{(2\,Ba-Ab)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}a}$$

input
```
integrate((B*x^2+A)/x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
-1/4*A*log(c*x^4 + b*x^2 + a)/a + 1/2*A*log(x^2)/a + 1/2*(2*B*a - A*b)*arc
tan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a)
```

3.105.9 Mupad [B] (verification not implemented)

Time = 10.10 (sec) , antiderivative size = 2424, normalized size of antiderivative = 31.08

$$\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^2)/(x*(a + b*x^2 + c*x^4)),x)
```

output
```
(A*log(x))/a - (log((A*B^2*c^2 + ((A + a*(-(A*b - 2*B*a)^2/(a^2*(4*a*c - b
^2)))^(1/2))*(B^2*a*c^2 + ((A + a*(-(A*b - 2*B*a)^2/(a^2*(4*a*c - b^2)))^(
1/2))*(4*A*b^2*c^2 + 2*c^2*x^2*(5*A*b*c - 4*B*b^2 + 10*B*a*c) - 4*B*a*b*c^
2 + (b*c^2*(A + a*(-(A*b - 2*B*a)^2/(a^2*(4*a*c - b^2)))^(1/2))*(a*b + 3*b
^2*x^2 - 10*a*c*x^2))/a))/(4*a) - 4*A*B*b*c^2 - B*c^2*x^2*(5*A*c + B*b)))/
(4*a) + B^3*c^2*x^2)*(A*B^2*c^2 + ((A - a*(-(A*b - 2*B*a)^2/(a^2*(4*a*c -
b^2)))^(1/2))*(B^2*a*c^2 + ((A - a*(-(A*b - 2*B*a)^2/(a^2*(4*a*c - b^2)))^
(1/2))*(4*A*b^2*c^2 + 2*c^2*x^2*(5*A*b*c - 4*B*b^2 + 10*B*a*c) - 4*B*a*b*c
^2 + (b*c^2*(A - a*(-(A*b - 2*B*a)^2/(a^2*(4*a*c - b^2)))^(1/2))*(a*b + 3*
b^2*x^2 - 10*a*c*x^2))/a))/(4*a) - 4*A*B*b*c^2 - B*c^2*x^2*(5*A*c + B*b)))
/(4*a) + B^3*c^2*x^2))*(2*A*b^2 - 8*A*a*c))/(2*(4*a*b^2 - 16*a^2*c)) - (at
an((2*(4*a*c - b^2)^(3/2)*(3*A*b^3 - B*a*b^2 + B*a^2*c - 8*A*a*b*c)*(A*B^2
*c^2 + ((2*A*b^2 - 8*A*a*c)*(((2*A*b^2 - 8*A*a*c)*(4*A*b^2*c^2 - 4*B*a*b*c
^2 + (2*a*b^2*c^2*(2*A*b^2 - 8*A*a*c))/(4*a*b^2 - 16*a^2*c)))/(2*(4*a*b^2
- 16*a^2*c)) + B^2*a*c^2 - 4*A*B*b*c^2))/(2*(4*a*b^2 - 16*a^2*c)) - ((A*b
- 2*B*a)*(((A*b - 2*B*a)*(4*A*b^2*c^2 - 4*B*a*b*c^2 + (2*a*b^2*c^2*(2*A*b^
2 - 8*A*a*c))/(4*a*b^2 - 16*a^2*c)))/(4*a*(4*a*c - b^2)^(1/2)) + (b^2*c^2*
(2*A*b^2 - 8*A*a*c)*(A*b - 2*B*a))/(2*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^2)^(
1/2))))/(4*a*(4*a*c - b^2)^(1/2)) - (b^2*c^2*(2*A*b^2 - 8*A*a*c)*(A*b - 2*
B*a)^2)/(8*a*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^2))))/(c^2*(A^2*b^2*c^2 + ...
```

3.105. $\int \frac{A+Bx^2}{x(a+bx^2+cx^4)}\,dx$

3.106 $\int \frac{A+Bx^2}{x^3(a+bx^2+cx^4)}\,dx$

3.106.1 Optimal result

Integrand size = 25, antiderivative size = 112

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx=-\frac{A}{2ax^2}-\frac{\left(Ab^2-abB-2aAc\right)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}-\frac{(Ab-aB)\log(x)}{a^2}+\frac{(Ab-aB)\log\left(a+bx^2+cx^4\right)}{4a^2}$$

output
```
-1/2*A/a/x^2-(A*b-B*a)*ln(x)/a^2+1/4*(A*b-B*a)*ln(c*x^4+b*x^2+a)/a^2-1/2*(
-2*A*a*c+A*b^2-B*a*b)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+
b^2)^(1/2)
```

3.106.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.66

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx$$
$$=\frac{-\frac{2aA}{x^2}+4(-Ab+aB)\log(x)+\frac{\left(-aB\left(b+\sqrt{b^2-4ac}\right)+A\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right)}{\sqrt{b^2-4ac}}+\frac{\left(aB\left(b-\sqrt{b^2-4ac}\right)+A\cdots\right.}{\cdots}}{4a^2}$$

input
```
Integrate[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)),x]
```

output
```
((-2*a*A)/x^2 + 4*(-(A*b) + a*B)*Log[x] + ((-(a*B*(b + Sqrt[b^2 - 4*a*c]))
 + A*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c]))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*
x^2])/Sqrt[b^2 - 4*a*c] + ((a*B*(b - Sqrt[b^2 - 4*a*c]) + A*(-b^2 + 2*a*c
+ b*Sqrt[b^2 - 4*a*c]))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4
*a*c])/(4*a^2)
```

3.106.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1578, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^3(a+bx^2+cx^4)}dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{Bx^2+A}{x^4(cx^4+bx^2+a)}dx^2$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{2}\int \left(\frac{A}{ax^4}+\frac{(Ab-aB)cx^2-abB+A(b^2-ac)}{a^2(cx^4+bx^2+a)}+\frac{aB-Ab}{a^2x^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{(-2aAc-abB+Ab^2)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}+\frac{(Ab-aB)\log(a+bx^2+cx^4)}{2a^2}-\frac{\log(x^2)(Ab-aB)}{a^2}-\frac{A}{ax^2}\right)$$

input
```
Int[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)),x]
```

output
```
(-(A/(a*x^2)) - ((A*b^2 - a*b*B - 2*a*A*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2
- 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]) - ((A*b - a*B)*Log[x^2])/a^2 + ((A*b -
a*B)*Log[a + b*x^2 + c*x^4])/(2*a^2))/2
```

3.106.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.106.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.12

method	result
default	$-\frac{A}{2a\,x^2}+\frac{(-Ab+Ba)\ln(x)}{a^2}-\frac{\frac{(-Abc+Bac)\ln\left(c\,x^4+b\,x^2+a\right)}{2c}+\frac{2\left(Aac-A\,b^2+abB-\frac{(-Abc+Bac)b}{2c}\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2a^2}$
risch	$-\frac{A}{2a\,x^2}-\frac{\ln(x)Ab}{a^2}+\frac{\ln(x)B}{a}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(4a^3c-a^2b^2\right)\_Z^2+\left(-4Aabc+A\,b^3+4a^2Bc-Ba\,b^2\right)\_Z+A^2c^2-bBAc+B^2ac\right)}\_R\ln\left(\right.\right.}{}$

input
```
int((B*x^2+A)/x^3/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*A/a/x^2+1/a^2*(-A*b+B*a)*ln(x)-1/2/a^2*(1/2*(-A*b*c+B*a*c)/c*ln(c*x^4
+b*x^2+a)+2*(A*a*c-A*b^2+a*b*B-1/2*(-A*b*c+B*a*c)*b/c)/(4*a*c-b^2)^(1/2)*a
rctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

3.106.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 385, normalized size of antiderivative = 3.44

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx$$
$$= \left[\frac{(Bab-Ab^2+2\,Aac)\sqrt{b^2-4\,ac}x^2\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-2\,Aab^2+8\,Aa^2c-(Bab^2}{4\,(a^2b^2-4\,a^3c}\right.$$

input
```
integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
[1/4*((B*a*b - A*b^2 + 2*A*a*c)*sqrt(b^2 - 4*a*c)*x^2*log((2*c^2*x^4 + 2*b
*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a
)) - 2*A*a*b^2 + 8*A*a^2*c - (B*a*b^2 - A*b^3 - 4*(B*a^2 - A*a*b)*c)*x^2*l
og(c*x^4 + b*x^2 + a) + 4*(B*a*b^2 - A*b^3 - 4*(B*a^2 - A*a*b)*c)*x^2*log(
x))/((a^2*b^2 - 4*a^3*c)*x^2), 1/4*(2*(B*a*b - A*b^2 + 2*A*a*c)*sqrt(-b^2
+ 4*a*c)*x^2*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 2*A
*a*b^2 + 8*A*a^2*c - (B*a*b^2 - A*b^3 - 4*(B*a^2 - A*a*b)*c)*x^2*log(c*x^4
 + b*x^2 + a) + 4*(B*a*b^2 - A*b^3 - 4*(B*a^2 - A*a*b)*c)*x^2*log(x))/((a^
2*b^2 - 4*a^3*c)*x^2)]
```

3.106.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx = \text{Timed out}$$

input
```
integrate((B*x**2+A)/x**3/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.106.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx^2}{x^3(a+bx^2+cx^4)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.106.8 Giac [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx^2}{x^3(a+bx^2+cx^4)}\,dx = -\frac{(Ba-Ab)\log(cx^4+bx^2+a)}{4\,a^2} + \frac{(Ba-Ab)\log(x^2)}{2\,a^2} - \frac{(Bab-Ab^2+2\,Aac)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}a^2} - \frac{Bax^2-Abx^2+Aa}{2\,a^2x^2}$$

input
```
integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
-1/4*(B*a - A*b)*log(c*x^4 + b*x^2 + a)/a^2 + 1/2*(B*a - A*b)*log(x^2)/a^2
 - 1/2*(B*a*b - A*b^2 + 2*A*a*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/
(sqrt(-b^2 + 4*a*c)*a^2) - 1/2*(B*a*x^2 - A*b*x^2 + A*a)/(a^2*x^2)
```

3.106.9 Mupad [B] (verification not implemented)

Time = 10.27 (sec) , antiderivative size = 3729, normalized size of antiderivative = 33.29

$$\int \frac{A+Bx^2}{x^3(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)),x)
```

output
```
- A/(2*a*x^2) - (log(x)*(A*b - B*a))/a^2 - (log(((A^3*c^5*x^2)/a^3 - (((((
4*b*c^2*(A*a*c - A*b^2 + B*a*b))/a - (2*c^3*x^2*(A*b^2 + 10*A*a*c - 5*B*a*
b))/a + (b*c^2*(a*b + 3*b^2*x^2 - 10*a*c*x^2)*(B*a - A*b + a^2*(-(2*A*a*c
- A*b^2 + B*a*b)^2/(a^4*(4*a*c - b^2)))^(1/2)))/a^2)*(B*a - A*b + a^2*(-(2
*A*a*c - A*b^2 + B*a*b)^2/(a^4*(4*a*c - b^2)))^(1/2)))/(4*a^2) + (A*c^3*(A
*a*c - 4*A*b^2 + 4*B*a*b))/a^2 - (A*c^4*x^2*(6*A*b - 5*B*a))/a^2)*(B*a - A
*b + a^2*(-(2*A*a*c - A*b^2 + B*a*b)^2/(a^4*(4*a*c - b^2)))^(1/2)))/(4*a^2
) + (A^2*c^4*(A*b - B*a))/a^3)*((((((2*c^3*x^2*(A*b^2 + 10*A*a*c - 5*B*a*b
))/a - (4*b*c^2*(A*a*c - A*b^2 + B*a*b))/a + (b*c^2*(a*b + 3*b^2*x^2 - 10*
a*c*x^2)*(A*b - B*a + a^2*(-(2*A*a*c - A*b^2 + B*a*b)^2/(a^4*(4*a*c - b^2)
))^(1/2)))/a^2)*(A*b - B*a + a^2*(-(2*A*a*c - A*b^2 + B*a*b)^2/(a^4*(4*a*c
 - b^2)))^(1/2)))/(4*a^2) + (A*c^3*(A*a*c - 4*A*b^2 + 4*B*a*b))/a^2 - (A*c
^4*x^2*(6*A*b - 5*B*a))/a^2)*(A*b - B*a + a^2*(-(2*A*a*c - A*b^2 + B*a*b)^
2/(a^4*(4*a*c - b^2)))^(1/2)))/(4*a^2) + (A^3*c^5*x^2)/a^3 + (A^2*c^4*(A*b
 - B*a))/a^3))*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*(16*a^3*c
 - 4*a^2*b^2)) - (atan((16*a^6*x^2*((((((5*A*B*a^2*c^4 - 6*A^2*a*b*c^4)/a^
3 - (((20*A*a^3*c^4 - 10*B*a^3*b*c^3 + 2*A*a^2*b^2*c^3)/a^3 + ((40*a^4*b*c
^3 - 12*a^3*b^3*c^2)*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*a^3
*(16*a^3*c - 4*a^2*b^2)))*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(
2*(16*a^3*c - 4*a^2*b^2)))*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c...
```

3.106. $\int \frac{A+Bx^2}{x^3(a+bx^2+cx^4)}\,dx$

3.107 $\int \frac{x^4(A+Bx^2)}{a+bx^2+cx^4} dx$

3.107.1 Optimal result

Integrand size = 25, antiderivative size = 261

$$\int \frac{x^4(A+Bx^2)}{a+bx^2+cx^4} dx = -\frac{(bB-Ac)x}{c^2}+\frac{Bx^3}{3c} + \frac{\left(b^2B-Abc-aBc-\frac{b^3B-Ab^2c-3abBc+2aAc^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(b^2B-Abc-aBc+\frac{b^3B-Ab^2c-3abBc+2aAc^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
-(-A*c+B*b)*x/c^2+1/3*B*x^3/c+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)
^(1/2))^(1/2))*(B*b^2-A*b*c-B*a*c+(-2*A*a*c^2+A*b^2*c+3*B*a*b*c-B*b^3)/(-4
*a*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan(x
*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(B*b^2-A*b*c-B*a*c+(2*A*a*c
^2-A*b^2*c-3*B*a*b*c+B*b^3)/(-4*a*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(b+(-4*a*c
+b^2)^(1/2))^(1/2)
```

3.107.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.25

$$\int \frac{x^4(A+Bx^2)}{a+bx^2+cx^4}\,dx = \frac{(-bB+Ac)x}{c^2}+\frac{Bx^3}{3c} + \frac{\left(-b^3B+Ab^2c+3abBc-2aAc^2+b^2B\sqrt{b^2-4ac}-Abc\sqrt{b^2-4ac}-aBc\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(b^3B-Ab^2c-3abBc+2aAc^2+b^2B\sqrt{b^2-4ac}-Abc\sqrt{b^2-4ac}-aBc\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

input
```
Integrate[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

output
```
((-(b*B) + A*c)*x)/c^2 + (B*x^3)/(3*c) + ((-(b^3*B) + A*b^2*c + 3*a*b*B*c
- 2*a*A*c^2 + b^2*B*Sqrt[b^2 - 4*a*c] - A*b*c*Sqrt[b^2 - 4*a*c] - a*B*c*Sq
rt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^3*B
- A*b^2*c - 3*a*b*B*c + 2*a*A*c^2 + b^2*B*Sqrt[b^2 - 4*a*c] - A*b*c*Sqrt[b
^2 - 4*a*c] - a*B*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
 Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2
 - 4*a*c]])
```

3.107.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1602, 27, 1602, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\left(A+Bx^2\right)}{a+bx^2+cx^4}dx$$

$$\Big\downarrow \text{1602}$$

$$\frac{Bx^3}{3c}-\frac{\int \frac{3x^2\left((bB-Ac)x^2+aB\right)}{cx^4+bx^2+a}dx}{3c}$$

$$\Big\downarrow \text{27}$$

$$\frac{Bx^3}{3c}-\frac{\int \frac{x^2\left((bB-Ac)x^2+aB\right)}{cx^4+bx^2+a}dx}{c}$$

$\Big\downarrow$ 1602

$$\frac{Bx^3}{3c}-\frac{\frac{x(bB-Ac)}{c}-\frac{\int \frac{\left(Bb^2-Acb-aBc\right)x^2+a(bB-Ac)}{cx^4+bx^2+a}dx}{c}}{c}$$

$\Big\downarrow$ 1480

$$\frac{Bx^3}{3c}-\frac{\frac{x(bB-Ac)}{c}-\frac{\frac{1}{2}\left(-\frac{2aAc^2-3abBc-Ab^2c+b^3B}{\sqrt{b^2-4ac}}-aBc-Abc+b^2B\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(\frac{2aAc^2-3abBc-Ab^2c+b^3B}{\sqrt{b^2-4ac}}-aBc-Abc+b^2B\right)\int \frac{}{cx^2+}}{c}}{c}$$

$\Big\downarrow$ 218

$$\frac{Bx^3}{3c}-\frac{\frac{x(bB-Ac)}{c}-\frac{\frac{\left(-\frac{2aAc^2-3abBc-Ab^2c+b^3B}{\sqrt{b^2-4ac}}-aBc-Abc+b^2B\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{2aAc^2-3abBc-Ab^2c+b^3B}{\sqrt{b^2-4ac}}-aBc-Abc+b^2B\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{\sqrt{b^2-4a}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{c}}{c}$$

input
```
Int[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

output
```
(B*x^3)/(3*c) - (((b*B - A*c)*x)/c - (((b^2*B - A*b*c - a*B*c - (b^3*B - A
*b^2*c - 3*a*b*B*c + 2*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]
*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a
*c]]) + ((b^2*B - A*b*c - a*B*c + (b^3*B - A*b^2*c - 3*a*b*B*c + 2*a*A*c^2
)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]
]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/c)/c
```

3.107.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

3.107.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.36

method	result
risch	$\frac{B\,x^{3}}{3c}+\frac{Ax}{c}-\frac{Bbx}{c^{2}}+\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{4}+\_Z^{2}b+a\right)}\frac{\left(\left(-Abc-Bac+B\,b^{2}\right)\_R^{2}-Aac+abB\right)\ln\left(x-\_R\right)}{2c\_R^{3}+\_Rb}}{2c^{2}}$
default	$\frac{\frac{1}{3}Bc\,x^{3}+Acx-Bbx}{c^{2}}+\frac{\frac{\left(-Abc\sqrt{-4ac+b^{2}}+2Aa\,c^{2}-A\,b^{2}c-Bac\sqrt{-4ac+b^{2}}+B\,b^{2}\sqrt{-4ac+b^{2}}-3Babc+B\,b^{3}\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{2c\sqrt{-4ac+b^{2}}\,\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}-}{}$

input
```
int(x^4*(B*x^2+A)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*B*x^3/c+1/c*A*x-1/c^2*B*b*x+1/2/c^2*sum(((-A*b*c-B*a*c+B*b^2)*_R^2-A*a
*c+a*b*B)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

3.107.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5140 vs. 2(225) = 450.

Time = 2.57 (sec) , antiderivative size = 5140, normalized size of antiderivative = 19.69

$$\int \frac{x^4(A+Bx^2)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.107.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^2)}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.107.7 Maxima [F]

$$\int \frac{x^4(A+Bx^2)}{a+bx^2+cx^4}\,dx = \int \frac{(Bx^2+A)x^4}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
1/3*(B*c*x^3 - 3*(B*b - A*c)*x)/c^2 - integrate(-(B*a*b - A*a*c + (B*b^2 -
 (B*a + A*b)*c)*x^2)/(c*x^4 + b*x^2 + a), x)/c^2
```

3.107.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4391 vs. $2(225) = 450$.

Time = 1.07 (sec) , antiderivative size = 4391, normalized size of antiderivative = 16.82

$$\int \frac{x^4(A + Bx^2)}{a + bx^2 + cx^4}\, dx = \text{Too large to display}$$

input
```
integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/8*((2*b^5*c^3 - 16*a*b^3*c^4 + 32*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 + 8*(b^2 - 4*a*c)*a*b*c^4)*A*
c^2 - (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^
2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*B*c^2 - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2...
```

3.107.9 Mupad [B] (verification not implemented)

Time = 8.40 (sec) , antiderivative size = 10177, normalized size of antiderivative = 38.99

$$\int \frac{x^4(A + Bx^2)}{a + bx^2 + cx^4}\, dx = \text{Too large to display}$$

input
```
int((x^4*(A + B*x^2))/(a + b*x^2 + c*x^4),x)
```

output
```
x*(A/c - (B*b)/c^2) - atan(((((16*A*a^2*c^5 - 4*A*a*b^2*c^4 + 4*B*a*b^3*c^
3 - 16*B*a^2*b*c^4)/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(B^2*b^7 + A^2*b
^5*c^2 + B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c
^2 + A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*c^2*(-(4*a*c - b^2)^3)
^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c
^4 - A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^
2*c^3 - 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 - 2*A*B*
b^3*c*(-(4*a*c - b^2)^3)^(1/2) + 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(
8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(B^2*b^7 + A^2*b^5*
c^2 + B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2
+ A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1
/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4
- A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c
^3 - 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 - 2*A*B*b^3
*c*(-(4*a*c - b^2)^3)^(1/2) + 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(
16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*x*(B^2*b^6 + 2*A^2*a^2*c^
4 + A^2*b^4*c^2 - 2*B^2*a^3*c^3 - 2*A*B*b^5*c + 9*B^2*a^2*b^2*c^2 - 6*B^2*
a*b^4*c - 4*A^2*a*b^2*c^3 + 10*A*B*a*b^3*c^2 - 10*A*B*a^2*b*c^3))/c^3)*(-(
B^2*b^7 + A^2*b^5*c^2 + B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 2
5*B^2*a^2*b^3*c^2 + A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*c^2*...
```

3.107. $\int \frac{x^4(A+Bx^2)}{a+bx^2+cx^4}\,dx$

3.108 $\int \frac{x^2(A+Bx^2)}{a+bx^2+cx^4}\,dx$

3.108.1 Optimal result

Integrand size = 25, antiderivative size = 208

$$\int \frac{x^2(A+Bx^2)}{a+bx^2+cx^4}\,dx = \frac{Bx}{c}-\frac{\left(bB-Ac-\frac{b^2B-Abc-2aBc}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(bB-Ac+\frac{b^2B-Abc-2aBc}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
B*x/c-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(B*b-A*c+
(A*b*c+2*B*a*c-B*b^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^
(1/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(B
*b-A*c+(-A*b*c-2*B*a*c+B*b^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b+(-4*a
*c+b^2)^(1/2))^(1/2)
```

3.108.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.21

$$\int \frac{x^2(A+Bx^2)}{a+bx^2+cx^4}\,dx = \frac{Bx}{c}-\frac{\left(-b^2B+Abc+2aBc+bB\sqrt{b^2-4ac}-Ac\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(b^2B-Abc-2aBc+bB\sqrt{b^2-4ac}-Ac\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

input
```
Integrate[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

output
```
(B*x)/c - ((-(b^2*B) + A*b*c + 2*a*B*c + b*B*Sqrt[b^2 - 4*a*c] - A*c*Sqrt[
b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sq
rt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((b^2*B - A
*b*c - 2*a*B*c + b*B*Sqrt[b^2 - 4*a*c] - A*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 -
 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

3.108.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1602, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (A+Bx^2)}{a+bx^2+cx^4} \, dx$$

$$\Big\downarrow \text{1602}$$

$$\frac{Bx}{c}-\frac{\int \frac{(bB-Ac)x^2+aB}{cx^4+bx^2+a}dx}{c}$$

$$\Big\downarrow \text{1480}$$

$$\frac{Bx}{c}-\frac{\frac{1}{2}\left(-\frac{-2aBc-Abc+b^2B}{\sqrt{b^2-4ac}}-Ac+bB\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(\frac{-2aBc-Abc+b^2B}{\sqrt{b^2-4ac}}-Ac+bB\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{c}$$

$$\Big\downarrow \text{218}$$

$$\frac{Bx}{c}-\frac{\frac{\left(-\frac{-2aBc-Abc+b^2B}{\sqrt{b^2-4ac}}-Ac+bB\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{-2aBc-Abc+b^2B}{\sqrt{b^2-4ac}}-Ac+bB\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{c}$$

input
```
Int[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

output
```
(B*x)/c - (((b*B - A*c - (b^2*B - A*b*c - 2*a*B*c)/Sqrt[b^2 - 4*a*c])*ArcT
an[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt
[b - Sqrt[b^2 - 4*a*c]]) + ((b*B - A*c + (b^2*B - A*b*c - 2*a*B*c)/Sqrt[b^
2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt
[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/c
```

3.108.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

3.108.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.31

method	result
risch	$\frac{Bx}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(\_R^2(Ac-Bb)-Ba\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}}{2c}$
default	$\frac{Bx}{c}+\frac{\left(Ac\sqrt{-4ac+b^2}+Abc-Bb\sqrt{-4ac+b^2}+2Bac-B\,b^2\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2c\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(Ac\sqrt{-4ac+b^2}-Abc-Bb\sqrt{-4ac+\right.}{2c\sqrt{-4a}}$

input
```
int(x^2*(B*x^2+A)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
B*x/c+1/2/c*sum((_R^2*(A*c-B*b)-B*a)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z
^4*c+_Z^2*b+a))
```

3.108.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2632 vs. 2(172) = 344.

Time = 0.62 (sec) , antiderivative size = 2632, normalized size of antiderivative = 12.65

$$\int \frac{x^2(A+Bx^2)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
1/2*(sqrt(1/2)*c*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b + 2*A
*B*b^2)*c + (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2
*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2
 + 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(2*(B^4*a
*b^2 - A*B^3*b^3 - 3*A^3*B*b*c^2 + A^4*c^3 - (B^4*a^2 + A*B^3*a*b - 3*A^2*
B^2*b^2)*c)*x + sqrt(1/2)*(B^3*b^4 - 4*A^2*B*a*c^3 + (4*B^3*a^2 + 8*A*B^2*
a*b + A^2*B*b^2)*c^2 - (5*B^3*a*b^2 + 2*A*B^2*b^3)*c - (B*b^3*c^3 + 8*A*a*
c^5 - 2*(2*B*a*b + A*b^2)*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*
A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2
+ 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)
*c^2 - (3*B^2*a*b + 2*A*B*b^2)*c + (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4
*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*
b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 -
 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*
a*b + 2*A*B*b^2)*c + (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*
B^2*a + 2*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(
B^4*a*b^2 + 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log
(2*(B^4*a*b^2 - A*B^3*b^3 - 3*A^3*B*b*c^2 + A^4*c^3 - (B^4*a^2 + A*B^3*a*b
 - 3*A^2*B^2*b^2)*c)*x - sqrt(1/2)*(B^3*b^4 - 4*A^2*B*a*c^3 + (4*B^3*a^2 +
 8*A*B^2*a*b + A^2*B*b^2)*c^2 - (5*B^3*a*b^2 + 2*A*B^2*b^3)*c - (B*b^3*...
```

3.108.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(A + Bx^2)}{a + bx^2 + cx^4} \, dx = \text{Timed out}$$

input
```
integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.108.7 Maxima [F]

$$\int \frac{x^2(A+Bx^2)}{a+bx^2+cx^4}\,dx = \int \frac{(Bx^2+A)x^2}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
B*x/c + integrate(-((B*b - A*c)*x^2 + B*a)/(c*x^4 + b*x^2 + a), x)/c
```

3.108.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3179 vs. $2(172) = 344$.

Time = 0.96 (sec) , antiderivative size = 3179, normalized size of antiderivative = 15.28

$$\int \frac{x^2(A+Bx^2)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
B*x/c + 1/8*((2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*A*c^
2 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*B*c^2 - 2
*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b^3*c^3 - 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^3*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*...
```

3.108.9 Mupad [B] (verification not implemented)

Time = 8.23 (sec) , antiderivative size = 6366, normalized size of antiderivative = 30.61

$$\int \frac{x^2(A+Bx^2)}{a+bx^2+cx^4} \, dx = \text{Too large to display}$$

input
```
int((x^2*(A + B*x^2))/(a + b*x^2 + c*x^4),x)
```

output
```
(B*x)/c - atan(((((16*B*a^2*c^3 - 4*B*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*
a*b*c^4)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2
*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c
^3 - 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2*b*c^2 +
 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*c^5 + b
^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*c^2*(-(4*
a*c - b^2)^3)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*
A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^2)^3)^(
1/2) + 12*B^2*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^
2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(B^2*b^4 - 2
*A^2*a*c^3 + A^2*b^2*c^2 + 2*B^2*a^2*c^2 - 2*A*B*b^3*c - 4*B^2*a*b^2*c + 6
*A*B*a*b*c^2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*c^2*(-(4*a*c - b^2)^3)^(1
/2) - B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*
A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^
2*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^
2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*B*a^2*c^3 - 4*B*a*b^2*c^
2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*c^2*(-
(4*a*c - b^2)^3)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c -
16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^2)^3
)^(1/2) + 12*B^2*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*...
```

3.109 $\int \frac{A+Bx^2}{a+bx^2+cx^4}\,dx$

3.109.1 Optimal result

Integrand size = 22, antiderivative size = 172

$$\int \frac{A+Bx^2}{a+bx^2+cx^4}\,dx=\frac{\left(B-\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(B+\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(B+(2*A*c-B*b)/
(-4*a*c+b^2)^(1/2))*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arcta
n(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(B+(-2*A*c+B*b)/(-4*a*c+
b^2)^(1/2))*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.109.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx^2}{a+bx^2+cx^4}\,dx=\frac{\frac{\left(-bB+2Ac+B\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(bB-2Ac+B\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}}}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

input
```
Integrate[(A + B*x^2)/(a + b*x^2 + c*x^4),x]
```

output
```
(((-(b*B) + 2*A*c + B*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
 - Sqrt[b^2 - 4*a*c]]])/Sqrt[b - Sqrt[b^2 - 4*a*c]] + ((b*B - 2*A*c + B*Sq
rt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/
Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

3.109.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{a+bx^2+cx^4}dx$$

$$\Big\downarrow \text{1480}$$

$$\frac{1}{2}\left(B-\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(\frac{bB-2Ac}{\sqrt{b^2-4ac}}+B\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx$$

$$\Big\downarrow \text{218}$$

$$\frac{\left(B-\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{bB-2Ac}{\sqrt{b^2-4ac}}+B\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

input
```
Int[(A + B*x^2)/(a + b*x^2 + c*x^4),x]
```

output
```
((B - (b*B - 2*A*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
 Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((B
+ (b*B - 2*A*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqr
t[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

3.109.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

3.109.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.26

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(c\_Z^4+\_Z^2 b+a\right)} \frac{\left(B\_R^2+A\right)\ln\left(x-\_R\right)}{2c\_R^3+\_R b}\right)}{2}$
default	$4c\left(\frac{\left(-2Ac+B\sqrt{-4ac+b^2}+Bb\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,c\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(2Ac+B\sqrt{-4ac+b^2}-Bb\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right.}{8\sqrt{-4ac+b^2}\,c\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right.$

input
```
int((B*x^2+A)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum((B*_R^2+A)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

3.109.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1569 vs. $2(138) = 276$.

Time = 0.44 (sec) , antiderivative size = 1569, normalized size of antiderivative = 9.12

$$\int \frac{A+Bx^2}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate((B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
1/2*sqrt(1/2)*sqrt(-(B^2*a*b - (4*A*B*a - A^2*b)*c + (a*b^2*c - 4*a^2*c^2)
*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b
^2*c - 4*a^2*c^2))*log(-2*(B^4*a^2 - A*B^3*a*b + A^3*B*b*c - A^4*c^2)*x +
sqrt(1/2)*(A*B^2*a*b^2 + 4*A^3*a*c^2 - (4*A*B^2*a^2 + A^3*b^2)*c + (4*(2*B
*a^3 - A*a^2*b)*c^2 - (2*B*a^2*b^2 - A*a*b^3)*c)*sqrt((B^4*a^2 - 2*A^2*B^2
*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(B^2*a*b - (4*A*B*a - A^
2*b)*c + (a*b^2*c - 4*a^2*c^2)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a
^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))) - 1/2*sqrt(1/2)*sqrt(-(B
^2*a*b - (4*A*B*a - A^2*b)*c + (a*b^2*c - 4*a^2*c^2)*sqrt((B^4*a^2 - 2*A^2
*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log
(-2*(B^4*a^2 - A*B^3*a*b + A^3*B*b*c - A^4*c^2)*x - sqrt(1/2)*(A*B^2*a*b^2
 + 4*A^3*a*c^2 - (4*A*B^2*a^2 + A^3*b^2)*c + (4*(2*B*a^3 - A*a^2*b)*c^2 -
(2*B*a^2*b^2 - A*a*b^3)*c)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b
^2*c^2 - 4*a^3*c^3)))*sqrt(-(B^2*a*b - (4*A*B*a - A^2*b)*c + (a*b^2*c - 4*
a^2*c^2)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3
)))/(a*b^2*c - 4*a^2*c^2))) + 1/2*sqrt(1/2)*sqrt(-(B^2*a*b - (4*A*B*a - A^
2*b)*c - (a*b^2*c - 4*a^2*c^2)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a
^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log(-2*(B^4*a^2 - A*B^3*a
*b + A^3*B*b*c - A^4*c^2)*x + sqrt(1/2)*(A*B^2*a*b^2 + 4*A^3*a*c^2 - (4*A*
B^2*a^2 + A^3*b^2)*c - (4*(2*B*a^3 - A*a^2*b)*c^2 - (2*B*a^2*b^2 - A*a*...
```

3.109.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate((B*x**2+A)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.109.7 Maxima [F]

$$\int \frac{A+Bx^2}{a+bx^2+cx^4}\,dx=\int \frac{Bx^2+A}{cx^4+bx^2+a}\,dx$$

input
```
integrate((B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/(c*x^4 + b*x^2 + a), x)
```

3.109.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1402 vs. 2(138) = 276.

Time = 0.78 (sec) , antiderivative size = 1402, normalized size of antiderivative = 8.15

$$\int \frac{A+Bx^2}{a+bx^2+cx^4}\,dx=\text{Too large to display}$$

input
```
integrate((B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/4*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^
3*c - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^
2 - 4*a*c)*b*c^2)*A - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*B)*arctan(2*sqrt(1/2)*x/sqrt
((b + sqrt(b^2 - 4*a*c))/c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^
2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*abs(c)) + 1/4*((sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^2*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c + 2*b^4*c + 16*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt...
```

3.109.9 Mupad [B] (verification not implemented)

Time = 8.07 (sec) , antiderivative size = 4109, normalized size of antiderivative = 23.89

$$\int \frac{A+Bx^2}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^2)/(a + b*x^2 + c*x^4),x)
```

output
```
- atan((((-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c
*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c
 - 4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*(x*(8*
b^3*c^2 - 32*a*b*c^3)*(-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*
b^3*c - A^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 -
4*B^2*a^2*b*c - 4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))
^(1/2) - 4*A*b^2*c^2 + 16*A*a*c^3) + x*(4*A^2*c^3 - 4*B^2*a*c^2 + 2*B^2*b^
2*c - 4*A*B*b*c^2))*(-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^
3*c - A^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*
B^2*a^2*b*c - 4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(
1/2)*1i + ((-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2
*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b
*c - 4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*(4*A
*b^2*c^2 + x*(8*b^3*c^2 - 32*a*b*c^3)*(-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)
^3)^(1/2) + A^2*b^3*c - A^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 -
4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*
c^2 + a*b^4*c)))^(1/2) - 16*A*a*c^3) + x*(4*A^2*c^3 - 4*B^2*a*c^2 + 2*B^2*
b^2*c - 4*A*B*b*c^2))*(-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*
b^3*c - A^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 -
4*B^2*a^2*b*c - 4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c...
```

3.110 $\int \frac{A+Bx^2}{x^2(a+bx^2+cx^4)}\,dx$

3.110.1 Optimal result

Integrand size = 25, antiderivative size = 189

$$\int \frac{A+Bx^2}{x^2(a+bx^2+cx^4)}\,dx = -\frac{A}{ax}-\frac{\sqrt{c}\left(A+\frac{Ab-2aB}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(A-\frac{Ab-2aB}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
-A/a/x-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*
(A+(A*b-2*B*a)/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-
1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(A+(-A*
b+2*B*a)/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.110.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx^2}{x^2(a+bx^2+cx^4)}\,dx = -\frac{\frac{2A}{x}+\frac{\sqrt{2}\sqrt{c}\left(-2aB+A\left(b+\sqrt{b^2-4ac}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(2aB+A\left(-b+\sqrt{b^2-4ac}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{2a}$$

input
```
Integrate[(A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)),x]
```

output
```
-1/2*((2*A)/x + (Sqrt[2]*Sqrt[c]*(-2*a*B + A*(b + Sqrt[b^2 - 4*a*c]))*ArcT
an[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sq
rt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(2*a*B + A*(-b + Sqrt[b^2 -
4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^
2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/a
```

3.110.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1604, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^2 (a+bx^2+cx^4)} \, dx$$

$$\Big\downarrow \text{1604}$$

$$-\frac{\int \frac{Acx^2+Ab-aB}{cx^4+bx^2+a}dx}{a}-\frac{A}{ax}$$

$$\Big\downarrow \text{1480}$$

$$-\frac{\frac{1}{2}c\left(\frac{Ab-2aB}{\sqrt{b^2-4ac}}+A\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}c\left(A-\frac{Ab-2aB}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{a}-\frac{A}{ax}$$

$$\Big\downarrow \text{218}$$

$$-\frac{\frac{\sqrt{c}\left(\frac{Ab-2aB}{\sqrt{b^2-4ac}}+A\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(A-\frac{Ab-2aB}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{a}-\frac{A}{ax}$$

input
```
Int[(A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)),x]
```

output
```
-(A/(a*x)) - ((Sqrt[c]*(A + (A*b - 2*a*B)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4
*a*c]]) + (Sqrt[c]*(A - (A*b - 2*a*B)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c
]]))/a
```

3.110.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.110.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.92

method	result
default	$\frac{4c\left(\frac{\left(-A\sqrt{-4ac+b^{2}}+Ab-2Ba\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{8\sqrt{-4ac+b^{2}}\,\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}-\frac{\left(-A\sqrt{-4ac+b^{2}}-Ab+2Ba\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{8\sqrt{-4ac+b^{2}}\,\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{a}-\frac{A}{ax}$
risch	$-\frac{A}{ax}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(16a^{5}c^{2}-8a^{4}b^{2}c+b^{4}a^{3}\right)\_Z^{4}+\left(12A^{2}a^{2}b\,c^{2}-7A^{2}a\,b^{3}c+A^{2}b^{5}-16AB\,a^{3}c^{2}+12AB\,a^{2}b^{2}c-2ABa\,b^{4}-4B^{2}a^{3}bc+B^{2}a^{2}b^{3}\right.\right.}\right.}{}$

input
```
int((B*x^2+A)/x^2/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
4/a*c*(1/8*(-A*(-4*a*c+b^2)^(1/2)+A*b-2*B*a)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2))-1/8*(-A*(-4*a*c+b^2)^(1/2)-A*b+2*B*a)/(-4*a*c+b^2)^(1/2)*2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)))-A/a/x
```

3.110.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2914 vs. $2(155) = 310$.

Time = 0.79 (sec) , antiderivative size = 2914, normalized size of antiderivative = 15.42

$$\int \frac{A+Bx^2}{x^2(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
1/2*(sqrt(1/2)*a*x*sqrt(-(B^2*a^2*b - 2*A*B*a*b^2 + A^2*b^3 + (4*A*B*a^2 -
 3*A^2*a*b)*c + (a^3*b^2 - 4*a^4*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*
B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A
^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(
2*(A^4*a*c^3 + (A^3*B*a*b - A^4*b^2)*c^2 - (B^4*a^3 - 3*A*B^3*a^2*b + 3*A^
2*B^2*a*b^2 - A^3*B*b^3)*c)*x + sqrt(1/2)*(B^3*a^3*b^2 - 3*A*B^2*a^2*b^3 +
 3*A^2*B*a*b^4 - A^3*b^5 + 4*(A^2*B*a^3 - A^3*a^2*b)*c^2 - (4*B^3*a^4 - 12
*A*B^2*a^3*b + 13*A^2*B*a^2*b^2 - 5*A^3*a*b^3)*c - (B*a^4*b^3 - A*a^3*b^4
- 8*A*a^5*c^2 - 2*(2*B*a^5*b - 3*A*a^4*b^2)*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3
*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^
2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(B^2*a^2
*b - 2*A*B*a*b^2 + A^2*b^3 + (4*A*B*a^2 - 3*A^2*a*b)*c + (a^3*b^2 - 4*a^4*
c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4
*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b
^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x*sqrt(-(B^2*a^2*b - 2
*A*B*a*b^2 + A^2*b^3 + (4*A*B*a^2 - 3*A^2*a*b)*c + (a^3*b^2 - 4*a^4*c)*sqr
t((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 +
 A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4
*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(2*(A^4*a*c^3 + (A^3*B*a*b - A^4*b^2)*c^
2 - (B^4*a^3 - 3*A*B^3*a^2*b + 3*A^2*B^2*a*b^2 - A^3*B*b^3)*c)*x - sqrt...
```

3.110.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx=\text{Timed out}$$

input
```
integrate((B*x**2+A)/x**2/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.110.7 Maxima [F]

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx=\int \frac{Bx^2+A}{(cx^4+bx^2+a)x^2}\,dx$$

input
```
integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
-integrate((A*c*x^2 - B*a + A*b)/(c*x^4 + b*x^2 + a), x)/a - A/(a*x)
```

3.110.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2805 vs. $2(155) = 310$.

Time = 1.05 (sec) , antiderivative size = 2805, normalized size of antiderivative = 14.84

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx=\text{Too large to display}$$

input
```
integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
-A/(a*x) + 1/8*((2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*A*a^2 -
 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a*b^4*c - 2*a*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2
 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 16*a^2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 32*a^3*b*c^3 + 2*(b^2 - 4*a*c)*a*b^3*c -
 8*(b^2 - 4*a*c)*a^2*b*c^2)*A*abs(a) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^2*b^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c - 2*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*a^2*b^4*c + 16*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^3*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c
^2 + 16*a^3*b^2*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3...
```

3.110.9 Mupad [B] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 6335, normalized size of antiderivative = 33.52

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)),x)
```

output
```
- atan(((x*(4*A^2*a^4*c^4 - 4*B^2*a^5*c^3 - 2*A^2*a^3*b^2*c^3 + 4*A*B*a^4*
b*c^3) + (-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + B^2
*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3
*c - A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 -
 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a
^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(A^2*b^5
 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*(-(4*a*c - b^2
)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c - A^2*a*c*(-(4*a
*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 - 2*A*B*a*b*(-(4*a*c
 - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*
c)))^(1/2) - 16*B*a^6*c^3 + 16*A*a^5*b*c^3 - 4*A*a^4*b^3*c^2 + 4*B*a^5*b^2
*c^2))*(-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a
^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c
 - A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 - 2
*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5
*c^2 - 8*a^4*b^2*c)))^(1/2)*1i + (x*(4*A^2*a^4*c^4 - 4*B^2*a^5*c^3 - 2*A^2
*a^3*b^2*c^3 + 4*A*B*a^4*b*c^3) + (-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*
a*c - b^2)^3)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*
A*B*a^3*c^2 - 7*A^2*a*b^3*c - A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3
*b*c + 12*A^2*a^2*b*c^2 - 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a...
```

3.110. $\int \frac{A+Bx^2}{x^2(a+bx^2+cx^4)}\,dx$

3.111 $\int \frac{A+Bx^2}{x^4(a+bx^2+cx^4)}\,dx$

3.111.1 Optimal result

Integrand size = 25, antiderivative size = 271

$$\begin{aligned}\int \frac{A+Bx^2}{x^4\left(a+bx^2+cx^4\right)}\,dx &= -\frac{A}{3ax^3}+\frac{Ab-aB}{a^2x} \\ &\quad -\frac{\sqrt{c}\left(aB\left(b+\sqrt{b^2-4ac}\right)-A\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} \\ &\quad +\frac{\sqrt{c}\left(aB\left(b-\sqrt{b^2-4ac}\right)-A\left(b^2-2ac-b\sqrt{b^2-4ac}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

output
```
-1/3*A/a/x^3+(A*b-B*a)/a^2/x-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^
(1/2))^(1/2))*c^(1/2)*(a*B*(b+(-4*a*c+b^2)^(1/2))-A*(b^2-2*a*c+b*(-4*a*c+b
^2)^(1/2)))/a^2*2^(1/2)/(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/
2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(a*B*(b-(
-4*a*c+b^2)^(1/2))-A*(b^2-2*a*c-b*(-4*a*c+b^2)^(1/2)))/a^2*2^(1/2)/(-4*a*c
+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.111.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx^2}{x^4(a+bx^2+cx^4)}\,dx$$
$$= \frac{-\frac{2aA}{x^3}+\frac{6Ab-6aB}{x}-\frac{3\sqrt{2}\sqrt{c}\left(aB\left(b+\sqrt{b^2-4ac}\right)-A\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(aB\left(b-\sqrt{b^2-4ac}\right)+A\left(-b\right.\right.}{\sqrt{b^2-4\right.}}}{6a^2}$$

input
```
Integrate[(A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)),x]
```

output
```
((-2*a*A)/x^3 + (6*A*b - 6*a*B)/x - (3*Sqrt[2]*Sqrt[c]*(a*B*(b + Sqrt[b^2
- 4*a*c]) - A*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]
*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4
*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(a*B*(b - Sqrt[b^2 - 4*a*c]) + A*(-b^2 + 2*a*
c + b*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4
*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(6*a^2)
```

3.111.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^4(a+bx^2+cx^4)}\,dx$$

$$\Big\downarrow \; 1604$$

$$-\frac{\int \frac{3(Acx^2+Ab-aB)}{x^2(cx^4+bx^2+a)}dx}{3a}-\frac{A}{3ax^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{Acx^2+Ab-aB}{x^2(cx^4+bx^2+a)}dx}{a}-\frac{A}{3ax^3}$$

$$\Big\downarrow \; 1604$$

$$-\frac{-\frac{\int -\frac{-((Ab-aB)cx^2)+abB-A(b^2-ac)}{cx^4+bx^2+a}dx}{a}-\frac{Ab-aB}{ax}}{a}-\frac{A}{3ax^3}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\int -\frac{Ab^2-aBb+(Ab-aB)cx^2-aAc}{cx^4+bx^2+a}dx}{a}-\frac{Ab-aB}{ax}}{a}-\frac{A}{3ax^3}$$

$$\Big\downarrow \; 25$$

$$-\frac{-\frac{\int -\frac{-\left((Ab-aB)cx^2\right)+abB-A\left(b^2-ac\right)}{cx^4+bx^2+a}dx}{a}-\frac{Ab-aB}{ax}}{a}-\frac{A}{3ax^3}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\int -\frac{Ab^2-aBb+(Ab-aB)cx^2-aAc}{cx^4+bx^2+a}dx}{a}-\frac{Ab-aB}{ax}}{a}-\frac{A}{3ax^3}$$

$$\Big\downarrow \; 25$$

$$-\frac{-\frac{\int -\frac{-\left((Ab-aB)cx^2\right)+abB-A\left(b^2-ac\right)}{cx^4+bx^2+a}dx}{a}-\frac{Ab-aB}{ax}}{a}-\frac{A}{3ax^3}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\int -\frac{Ab^2-aBb+(Ab-aB)cx^2-aAc}{cx^4+bx^2+a}dx}{a}-\frac{Ab-aB}{ax}}{a}-\frac{A}{3ax^3}$$

$$\Big\downarrow \; 25$$

$$-\frac{-\frac{\int -\frac{-\left((Ab-aB)cx^2\right)+abB-A\left(b^2-ac\right)}{cx^4+bx^2+a}dx}{a}-\frac{Ab-aB}{ax}}{a}-\frac{A}{3ax^3}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\int -\frac{Ab^2-aBb+(Ab-aB)cx^2-aAc}{cx^4+bx^2+a}dx}{a}-\frac{Ab-aB}{ax}}{a}-\frac{A}{3ax^3}$$

$$\Big\downarrow \; 25$$

$$-\frac{-\frac{\int -\frac{-\left((Ab-aB)cx^2\right)+abB-A\left(b^2-ac\right)}{cx^4+bx^2+a}dx}{a}-\frac{Ab-aB}{ax}}{a}-\frac{A}{3ax^3}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\int -\frac{Ab^2-aBb+(Ab-aB)cx^2-aAc}{cx^4+bx^2+a}dx}{a}-\frac{Ab-aB}{ax}}{a}-\frac{A}{3ax^3}$$

$$\Big\downarrow \; 25$$

3.111. $\int \frac{A+Bx^2}{x^4(a+bx^2+cx^4)}\,dx$

$$-\frac{-\frac{\int -\frac{-\left((Ab-aB)cx^2\right)+abB-A\left(b^2-ac\right)}{cx^4+bx^2+a}dx}{a}-\frac{Ab-aB}{ax}}{a}-\frac{A}{3ax^3}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\int -\frac{Ab^2-aBb+(Ab-aB)cx^2-aAc}{cx^4+bx^2+a}dx}{a}-\frac{Ab-aB}{ax}}{a}-\frac{A}{3ax^3}$$

$$\Big\downarrow \; 25$$

$$-\frac{-\frac{\int -\frac{-\left((Ab-aB)cx^2\right)+abB-A\left(b^2-ac\right)}{cx^4+bx^2+a}dx}{a}-\frac{Ab-aB}{ax}}{a}-\frac{A}{3ax^3}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\int -\frac{Ab^2-aBb+(Ab-aB)cx^2-aAc}{cx^4+bx^2+a}dx}{a}-\frac{Ab-aB}{ax}}{a}-\frac{A}{3ax^3}$$

$$\Big\downarrow \; 25$$

$$-\frac{-\frac{\int -\frac{-\left((Ab-aB)cx^2\right)+abB-A\left(b^2-ac\right)}{cx^4+bx^2+a}dx}{a}-\frac{Ab-aB}{ax}}{a}-\frac{A}{3ax^3}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\int -\frac{Ab^2-aBb+(Ab-aB)cx^2-aAc}{cx^4+bx^2+a}dx}{a}-\frac{Ab-aB}{ax}}{a}-\frac{A}{3ax^3}$$

$$\Big\downarrow \; 25$$

$$-\frac{-\frac{\int -\frac{-\left((Ab-aB)cx^2\right)+abB-A\left(b^2-ac\right)}{cx^4+bx^2+a}dx}{a}-\frac{Ab-aB}{ax}}{a}-\frac{A}{3ax^3}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\int -\frac{Ab^2-aBb+(Ab-aB)cx^2-aAc}{cx^4+bx^2+a}dx}{a}-\frac{Ab-aB}{ax}}{a}-\frac{A}{3ax^3}$$

$$\Big\downarrow \; 25$$

$$-\frac{-\frac{\int -\frac{-\left((Ab-aB)cx^2\right)+abB-A\left(b^2-ac\right)}{cx^4+bx^2+a}dx}{a}-\frac{Ab-aB}{ax}}{a}-\frac{A}{3ax^3}$$

$$\Big\downarrow \; 25$$

3.111. $\int \frac{A+Bx^2}{x^4\left(a+bx^2+cx^4\right)}\,dx$

$$-\frac{\frac{\int -\frac{Ab^2-aBb+(Ab-aB)cx^2-aAc}{cx^4+bx^2+a}dx}{a}-\frac{Ab-aB}{ax}}{a}-\frac{A}{3ax^3}$$

$$\Big\downarrow 25$$

$$-\frac{-\frac{\int -\frac{-\left((Ab-aB)cx^2\right)+abB-A\left(b^2-ac\right)}{cx^4+bx^2+a}dx}{a}-\frac{Ab-aB}{ax}}{a}-\frac{A}{3ax^3}$$

$$\Big\downarrow 25$$

$$-\frac{\frac{\int -\frac{Ab^2-aBb+(Ab-aB)cx^2-aAc}{cx^4+bx^2+a}dx}{a}-\frac{Ab-aB}{ax}}{a}-\frac{A}{3ax^3}$$

$$\Big\downarrow 25$$

$$-\frac{-\frac{\int -\frac{-\left((Ab-aB)cx^2\right)+abB-A\left(b^2-ac\right)}{cx^4+bx^2+a}dx}{a}-\frac{Ab-aB}{ax}}{a}-\frac{A}{3ax^3}$$

$$\Big\downarrow 25$$

$$-\frac{\frac{\int -\frac{Ab^2-aBb+(Ab-aB)cx^2-aAc}{cx^4+bx^2+a}dx}{a}-\frac{Ab-aB}{ax}}{a}-\frac{A}{3ax^3}$$

$$\Big\downarrow 25$$

$$-\frac{-\frac{\int -\frac{-\left((Ab-aB)cx^2\right)+abB-A\left(b^2-ac\right)}{cx^4+bx^2+a}dx}{a}-\frac{Ab-aB}{ax}}{a}-\frac{A}{3ax^3}$$

$$\Big\downarrow 25$$

$$-\frac{\frac{\int -\frac{Ab^2-aBb+(Ab-aB)cx^2-aAc}{cx^4+bx^2+a}dx}{a}-\frac{Ab-aB}{ax}}{a}-\frac{A}{3ax^3}$$

$$\Big\downarrow 25$$

$$-\frac{-\frac{\int -\frac{-\left((Ab-aB)cx^2\right)+abB-A\left(b^2-ac\right)}{cx^4+bx^2+a}dx}{a}-\frac{Ab-aB}{ax}}{a}-\frac{A}{3ax^3}$$

$$\Big\downarrow 25$$

$$-\frac{\frac{\int -\frac{Ab^2-aBb+(Ab-aB)cx^2-aAc}{cx^4+bx^2+a}dx}{a}-\frac{Ab-aB}{ax}}{a}-\frac{A}{3ax^3}$$

input
```
Int[(A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)),x]
```

3.111. $\int \frac{A+Bx^2}{x^4(a+bx^2+cx^4)}\,dx$

output
```
$Aborted
```

3.111.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.111.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.86

method	result
default	$-\frac{A}{3a\,x^{3}}-\frac{-Ab+Ba}{x\,a^{2}}+\frac{4c\left(\frac{\left(Ab\sqrt{-4ac+b^{2}}+2Aac-A\,b^{2}-Ba\sqrt{-4ac+b^{2}}+abB\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{8\sqrt{-4ac+b^{2}}\,\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}-\frac{\left(Ab\sqrt{-4ac+b^{2}}-2Aac+\right.}{}\right.}{a^{2}}$
risch	$\frac{\frac{(Ab-Ba)x^{2}}{a^{2}}-\frac{A}{3a}}{x^{3}}+\frac{\left(\_R=\operatorname{RootOf}\left(\left(16a^{7}c^{2}-8b^{2}c\,a^{6}+b^{4}a^{5}\right)\_Z^{4}+\left(-20A^{2}a^{3}b\,c^{3}+25A^{2}a^{2}b^{3}c^{2}-9A^{2}a\,b^{5}c+A^{2}b^{7}+16AB\,a^{4}c^{3}-36AB\,a^{3}b^{2}c^{2}-\right.\right.\right.}{}$

input
```
int((B*x^2+A)/x^4/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*A/a/x^3-(-A*b+B*a)/x/a^2+4/a^2*c*(1/8*(A*b*(-4*a*c+b^2)^(1/2)+2*A*a*c
-A*b^2-B*a*(-4*a*c+b^2)^(1/2)+a*b*B)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)-1/8*(A*b*(-4*a*c+b^2)^(1/2)-2*A*a*c+A*b^2-B*a*(-4*a*c+b^2)^(1/2)-a*b*B)/
(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

3.111.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5442 vs. $2(227) = 454$.

Time = 2.89 (sec) , antiderivative size = 5442, normalized size of antiderivative = 20.08

$$\int \frac{A+Bx^2}{x^4(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.111.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^4(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate((B*x**2+A)/x**4/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.111.7 Maxima [F]

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2+cx^4\right)}\,dx=\int \frac{Bx^2+A}{(cx^4+bx^2+a)x^4}\,dx$$

input
```
integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(-((B*a - A*b)*c*x^2 + B*a*b - A*b^2 + A*a*c)/(c*x^4 + b*x^2 + a)
, x)/a^2 - 1/3*(3*(B*a - A*b)*x^2 + A*a)/(a^2*x^3)
```

3.111.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2870 vs. $2(227) = 454$.

Time = 0.91 (sec) , antiderivative size = 2870, normalized size of antiderivative = 10.59

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2+cx^4\right)}\,dx=\text{Too large to display}$$

input
```
integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/4*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 9*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^
5*c - 2*b^6*c + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 1
0*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^4*c^2 + 18*a*b^4*c^2 + 2*b^5*c^2 - 16*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*b*c^3 - 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 48*a
^2*b^2*c^3 - 14*a*b^3*c^3 + 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*
c^4 + 32*a^3*c^4 + 24*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^5 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*b^4*c - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^2*b*c^2 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c
^2 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 +
 2*(b^2 - 4*a*c)*b^4*c - 10*(b^2 - 4*a*c)*a*b^2*c^2 - 2*(b^2 - 4*a*c)*b^3*
c^2 + 8*(b^2 - 4*a*c)*a^2*c^3 + 6*(b^2 - 4*a*c)*a*b*c^3)*A - (sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*a*b^
5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2)*...
```

3.111.9 Mupad [B] (verification not implemented)

Time = 8.69 (sec) , antiderivative size = 10101, normalized size of antiderivative = 37.27

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)),x)
```

output
```
- (A/(3*a) - (x^2*(A*b - B*a))/a^2)/x^3 - atan((((-(A^2*b^7 + B^2*a^2*b^5
+ A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 + A^
2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12
*B^2*a^4*b*c^2 - B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 -
 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(
1/2) + 16*A*B*a^2*b^4*c + 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*
b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*(16*A*a^10*c^4 + x*(32*a^11*b*c^3
- 8*a^10*b^3*c^2)*(-(A^2*b^7 + B^2*a^2*b^5 + A^2*b^4*(-(4*a*c - b^2)^3)^(1
/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 + A^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1
/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*
c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 - B^2*a^3*c*(-(4
*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 - 3*A^2*a*b^2*c*(-(4*a*c - b^2)^
3)^(1/2) - 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c + 4*A*B
*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)
))^(1/2) + 16*B*a^10*b*c^3 + 4*A*a^8*b^4*c^2 - 20*A*a^9*b^2*c^3 - 4*B*a^9*
b^3*c^2) - x*(4*A^2*a^8*c^5 - 4*B^2*a^9*c^4 + 2*A^2*a^6*b^4*c^3 - 8*A^2*a^
7*b^2*c^4 + 2*B^2*a^8*b^2*c^3 - 4*A*B*a^7*b^3*c^3 + 12*A*B*a^8*b*c^4))*(-(
A^2*b^7 + B^2*a^2*b^5 + A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 2
5*A^2*a^2*b^3*c^2 + A^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*b^2*...
```

3.111. $\int \frac{A+Bx^2}{x^4(a+bx^2+cx^4)}\,dx$

3.112 $\int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$

3.112.1 Optimal result

Integrand size = 25, antiderivative size = 212

$$\begin{aligned}\int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx &= \frac{(2b^2B-Abc-6aBc)\,x^2}{2c^2\,(b^2-4ac)}-\frac{x^4(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)}\\&\quad-\frac{(2b^4B-Ab^3c-12ab^2Bc+6aAbc^2+12a^2Bc^2)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\,(b^2-4ac)^{3/2}}\\&\quad-\frac{(2bB-Ac)\log\left(a+bx^2+cx^4\right)}{4c^3}\end{aligned}$$

output
```
1/2*(-A*b*c-6*B*a*c+2*B*b^2)*x^2/c^2/(-4*a*c+b^2)-1/2*x^4*(a*(-2*A*c+B*b)+
(-A*b*c-2*B*a*c+B*b^2)*x^2)/c/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*(6*A*a*b*c^
2-A*b^3*c+12*B*a^2*c^2-12*B*a*b^2*c+2*B*b^4)*arctanh((2*c*x^2+b)/(-4*a*c+b
^2)^(1/2))/c^3/(-4*a*c+b^2)^(3/2)-1/4*(-A*c+2*B*b)*ln(c*x^4+b*x^2+a)/c^3
```

3.112.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.98

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$$
$$= \frac{2Bcx^2 - \frac{2(b^3(bB-Ac)x^2+a^2c(-3bB+2c(A+Bx^2))+ab(b^2B+3Ac^2x^2-bc(A+4Bx^2)))}{(b^2-4ac)(a+bx^2+cx^4)} - \frac{2(2b^4B-Ab^3c-12ab^2Bc+6aAbc^2+12a^2Bc^2)\,a}{(-b^2+4ac)^{3/2}}}{4c^3}$$

input
```
Integrate[(x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(2*B*c*x^2 - (2*(b^3*(b*B - A*c)*x^2 + a^2*c*(-3*b*B + 2*c*(A + B*x^2)) +
a*b*(b^2*B + 3*A*c^2*x^2 - b*c*(A + 4*B*x^2))))/((b^2 - 4*a*c)*(a + b*x^2
+ c*x^4)) - (2*(2*b^4*B - A*b^3*c - 12*a*b^2*B*c + 6*a*A*b*c^2 + 12*a^2*B*
c^2)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + (-2*
b*B + A*c)*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

3.112.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1578, 1233, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{x^6\left(Bx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx^2$$

$$\Big\downarrow \text{1233}$$

$$\frac{1}{2}\left(\frac{\int \frac{x^2\left(\left(2Bb^2-Acb-6aBc\right)x^2+2a(bB-2Ac)\right)}{cx^4+bx^2+a}dx^2}{c\left(b^2-4ac\right)}-\frac{x^4\left(x^2\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{2}\left(\frac{\int\left(\frac{2Bb^2}{c}-Ab-6aB-\frac{(b^2-4ac)(2bB-Ac)x^2+a(2Bb^2-Acb-6aBc)}{c(cx^4+bx^2+a)}\right)dx^2}{c(b^2-4ac)}-\frac{x^4(x^2(-2aBc-Abc+b^2B)+a(bB-2\right.$$
$$c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)$$

$\downarrow$ 2009

$$\frac{1}{2}\left(\frac{-\frac{(12a^2Bc^2+6aAbc^2-12ab^2Bc-Ab^3c+2b^4B)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{(b^2-4ac)(2bB-Ac)\log(a+bx^2+cx^4)}{2c^2}-\left(x^2\left(6aB+Ab-\frac{2b}{}\right.\right.}{c(b^2-4ac)}\right.$$

input
```
Int[(x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(-((x^4*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2))/(c*(b^2 - 4*a*c
)*(a + b*x^2 + c*x^4))) + (-((A*b + 6*a*B - (2*b^2*B)/c)*x^2) - ((2*b^4*B
- A*b^3*c - 12*a*b^2*B*c + 6*a*A*b*c^2 + 12*a^2*B*c^2)*ArcTanh[(b + 2*c*x^
2)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]) - ((b^2 - 4*a*c)*(2*b*B - A
*c)*Log[a + b*x^2 + c*x^4])/(2*c^2))/(c*(b^2 - 4*a*c)))/2
```

3.112.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.112.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.33

method	result
default	$\frac{B\,x^2}{2c^2}+\frac{\frac{\left(3Aab\,c^2-A\,b^3c+2B\,a^2c^2-4Ba\,b^2c+B\,b^4\right)x^2}{c\left(4ac-b^2\right)}+\frac{a\left(2Aa\,c^2-A\,b^2c-3Babc+B\,b^3\right)}{c\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a}+\frac{\frac{\left(4Aa\,c^2-A\,b^2c-8Babc+2B\,b^3\right)\ln\left(c\,x^4+b\,x^2+a\right)}{2c}+}{2c^2}$
risch	Expression too large to display

input
```
int(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*B*x^2/c^2+1/2/c^2*(((3*A*a*b*c^2-A*b^3*c+2*B*a^2*c^2-4*B*a*b^2*c+B*b^4
)/c/(4*a*c-b^2)*x^2+a*(2*A*a*c^2-A*b^2*c-3*B*a*b*c+B*b^3)/c/(4*a*c-b^2))/(
c*x^4+b*x^2+a)+1/(4*a*c-b^2)*(1/2*(4*A*a*c^2-A*b^2*c-8*B*a*b*c+2*B*b^3)/c*
ln(c*x^4+b*x^2+a)+2*(-A*a*b*c-6*a^2*B*c+2*B*a*b^2-1/2*(4*A*a*c^2-A*b^2*c-8
*B*a*b*c+2*B*b^3)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1
/2))))
```

3.112.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 650 vs. $2(200) = 400$.

Time = 0.35 (sec) , antiderivative size = 1323, normalized size of antiderivative = 6.24

$$\int \frac{x^7(A+Bx^2)}{\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*B*a*b^5 - 16*A*a^3*c^3 - 2*(B*b^4*c^2 - 8*B*a*b^2*c^3 + 16*B*a^2*
c^4)*x^6 - 2*(B*b^5*c - 8*B*a*b^3*c^2 + 16*B*a^2*b*c^3)*x^4 + 12*(2*B*a^3*
b + A*a^2*b^2)*c^2 + 2*(B*b^6 - 12*(2*B*a^3 + A*a^2*b)*c^3 + (26*B*a^2*b^2
 + 7*A*a*b^3)*c^2 - (9*B*a*b^4 + A*b^5)*c)*x^2 + (2*B*a*b^4 + (2*B*b^4*c +
 6*(2*B*a^2 + A*a*b)*c^3 - (12*B*a*b^2 + A*b^3)*c^2)*x^4 + 6*(2*B*a^3 + A*
a^2*b)*c^2 + (2*B*b^5 + 6*(2*B*a^2*b + A*a*b^2)*c^2 - (12*B*a*b^3 + A*b^4)
*c)*x^2 - (12*B*a^2*b^2 + A*a*b^3)*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2
*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 +
 a)) - 2*(7*B*a^2*b^3 + A*a*b^4)*c + (2*B*a*b^5 - 16*A*a^3*c^3 + (2*B*b^5*
c - 16*A*a^2*c^4 + 8*(4*B*a^2*b + A*a*b^2)*c^3 - (16*B*a*b^3 + A*b^4)*c^2)
*x^4 + 8*(4*B*a^3*b + A*a^2*b^2)*c^2 + (2*B*b^6 - 16*A*a^2*b*c^3 + 8*(4*B*
a^2*b^2 + A*a*b^3)*c^2 - (16*B*a*b^4 + A*b^5)*c)*x^2 - (16*B*a^2*b^3 + A*a
*b^4)*c)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 +
 (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^4 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^
2*b*c^5)*x^2), -1/4*(2*B*a*b^5 - 16*A*a^3*c^3 - 2*(B*b^4*c^2 - 8*B*a*b^2*c
^3 + 16*B*a^2*c^4)*x^6 - 2*(B*b^5*c - 8*B*a*b^3*c^2 + 16*B*a^2*b*c^3)*x^4
+ 12*(2*B*a^3*b + A*a^2*b^2)*c^2 + 2*(B*b^6 - 12*(2*B*a^3 + A*a^2*b)*c^3 +
 (26*B*a^2*b^2 + 7*A*a*b^3)*c^2 - (9*B*a*b^4 + A*b^5)*c)*x^2 + 2*(2*B*a*b^
4 + (2*B*b^4*c + 6*(2*B*a^2 + A*a*b)*c^3 - (12*B*a*b^2 + A*b^3)*c^2)*x^4 +
 6*(2*B*a^3 + A*a^2*b)*c^2 + (2*B*b^5 + 6*(2*B*a^2*b + A*a*b^2)*c^2 - (...
```

3.112.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^2} dx = \text{Timed out}$$

input
```
integrate(x**7*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.112.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.112.8 Giac [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.13

$$\begin{aligned}\int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx &= \frac{Bx^2}{2\,c^2} + \frac{(2\,Bb^4 - 12\,Bab^2c - Ab^3c + 12\,Ba^2c^2 + 6\,Aabc^2)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(b^2c^3 - 4\,ac^4)\sqrt{-b^2+4\,ac}} \\ &\quad + \frac{2\,Bb^3x^4 - 8\,Babcx^4 - Ab^2cx^4 + 4\,Aac^2x^4 + Ab^3x^2 - 4\,Ba^2cx^2 - 2\,Aabcx^2 - 2\,Ba^2b + Aab^2}{4\,(cx^4+bx^2+a)(b^2c^2 - 4\,ac^3)} \\ &\quad - \frac{(2\,Bb - Ac)\log(cx^4+bx^2+a)}{4\,c^3}\end{aligned}$$

input
```
integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*B*x^2/c^2 + 1/2*(2*B*b^4 - 12*B*a*b^2*c - A*b^3*c + 12*B*a^2*c^2 + 6*A
*a*b*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^3 - 4*a*c^4)*sq
rt(-b^2 + 4*a*c)) + 1/4*(2*B*b^3*x^4 - 8*B*a*b*c*x^4 - A*b^2*c*x^4 + 4*A*a
*c^2*x^4 + A*b^3*x^2 - 4*B*a^2*c*x^2 - 2*A*a*b*c*x^2 - 2*B*a^2*b + A*a*b^2
)/((c*x^4 + b*x^2 + a)*(b^2*c^2 - 4*a*c^3)) - 1/4*(2*B*b - A*c)*log(c*x^4
+ b*x^2 + a)/c^3
```

3.112.9 Mupad [B] (verification not implemented)

Time = 8.06 (sec) , antiderivative size = 2282, normalized size of antiderivative = 10.76

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

output
```
((a*(B*b^3 + 2*A*a*c^2 - A*b^2*c - 3*B*a*b*c))/(2*c*(4*a*c - b^2)) + (x^2*
(B*b^4 + 2*B*a^2*c^2 - A*b^3*c + 3*A*a*b*c^2 - 4*B*a*b^2*c))/(2*c*(4*a*c -
 b^2)))/(a*c^2 + c^3*x^4 + b*c^2*x^2) + (B*x^2)/(2*c^2) + (log(a + b*x^2 +
 c*x^4)*(4*B*b^7 + 128*A*a^3*c^4 - 2*A*b^6*c - 48*B*a*b^5*c + 24*A*a*b^4*c
^2 - 256*B*a^3*b*c^3 - 96*A*a^2*b^2*c^3 + 192*B*a^2*b^3*c^2))/(2*(256*a^3*
c^6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5)) - (atan(((8*a*c^5*(4*a*
c - b^2)^3 - 2*b^2*c^4*(4*a*c - b^2)^3)*(x^2*(((((24*B*a^2*c^5 - 6*A*b^3*c
^4 + 12*B*b^4*c^3 + 28*A*a*b*c^5 - 56*B*a*b^2*c^4)/(4*a*c^5 - b^2*c^4) + (
(8*b^3*c^6 - 32*a*b*c^7)*(4*B*b^7 + 128*A*a^3*c^4 - 2*A*b^6*c - 48*B*a*b^5
*c + 24*A*a*b^4*c^2 - 256*B*a^3*b*c^3 - 96*A*a^2*b^2*c^3 + 192*B*a^2*b^3*c
^2))/(2*(4*a*c^5 - b^2*c^4)*(256*a^3*c^6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*
a^2*b^2*c^5)))*(2*B*b^4 + 12*B*a^2*c^2 - A*b^3*c + 6*A*a*b*c^2 - 12*B*a*b^
2*c))/(8*c^3*(4*a*c - b^2)^(3/2)) + ((8*b^3*c^6 - 32*a*b*c^7)*(2*B*b^4 + 1
2*B*a^2*c^2 - A*b^3*c + 6*A*a*b*c^2 - 12*B*a*b^2*c)*(4*B*b^7 + 128*A*a^3*c
^4 - 2*A*b^6*c - 48*B*a*b^5*c + 24*A*a*b^4*c^2 - 256*B*a^3*b*c^3 - 96*A*a^
2*b^2*c^3 + 192*B*a^2*b^3*c^2))/(16*c^3*(4*a*c - b^2)^(3/2)*(4*a*c^5 - b^2
*c^4)*(256*a^3*c^6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5)))/(a*(4*a
*c - b^2)) + (b*((4*B^2*b^5 + A^2*b^3*c^2 - 4*A*B*b^4*c - 6*A*B*a^2*c^3 -
5*A^2*a*b*c^3 - 20*B^2*a*b^3*c + 12*B^2*a^2*b*c^2 + 20*A*B*a*b^2*c^2)/(4*a
*c^5 - b^2*c^4) + (((24*B*a^2*c^5 - 6*A*b^3*c^4 + 12*B*b^4*c^3 + 28*A*a...
```

3.113 $\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$

3.113.1 Optimal result

Integrand size = 25, antiderivative size = 147

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = -\frac{x^2(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{(b^3B-6abBc+4aAc^2)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\,(b^2-4ac)^{3/2}} + \frac{B\log\left(a+bx^2+cx^4\right)}{4c^2}$$

output
```
-1/2*x^2*(a*(-2*A*c+B*b)+(-A*b*c-2*B*a*c+B*b^2)*x^2)/c/(-4*a*c+b^2)/(c*x^4
+b*x^2+a)+1/2*(4*A*a*c^2-6*B*a*b*c+B*b^3)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)
^(1/2))/c^2/(-4*a*c+b^2)^(3/2)+1/4*B*ln(c*x^4+b*x^2+a)/c^2
```

3.113.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.09

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \frac{-\frac{2(2a^2Bc+b^2(-bB+Ac)x^2+a(-b^2B-2Ac^2x^2+bc(A+3Bx^2)))}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{2(b^3B-6abBc+4aAc^2)\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}} + B\log\left(a+bx^2+c\right.}{4c^2}$$

input
```
Integrate[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((-2*(2*a^2*B*c + b^2*(-(b*B) + A*c)*x^2 + a*(-(b^2*B) - 2*A*c^2*x^2 + b*c
*(A + 3*B*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*(b^3*B - 6*a*b*
B*c + 4*a*A*c^2)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^
(3/2) + B*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

3.113.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1578, 1233, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 (A + Bx^2)}{(a + bx^2 + cx^4)^2} \, dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2} \int \frac{x^4 (Bx^2 + A)}{(cx^4 + bx^2 + a)^2} dx^2$$

$$\Big\downarrow \text{1233}$$

$$\frac{1}{2} \left(\frac{\int \frac{B(b^2-4ac)x^2+a(bB-2Ac)}{cx^4+bx^2+a} dx^2}{c(b^2 - 4ac)} - \frac{x^2 (x^2 (-2aBc - Abc + b^2 B) + a(bB - 2Ac))}{c(b^2 - 4ac)(a + bx^2 + cx^4)} \right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2} \left(\frac{\frac{B(b^2-4ac)\int \frac{2cx^2+b}{cx^4+bx^2+a} dx^2}{2c} - \frac{(4aAc^2-6abBc+b^3B)\int \frac{1}{cx^4+bx^2+a} dx^2}{2c}}{c(b^2 - 4ac)} - \frac{x^2 (x^2 (-2aBc - Abc + b^2 B) + a(bB - 2Ac))}{c(b^2 - 4ac)(a + bx^2 + cx^4)} \right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2} \left(\frac{\frac{(4aAc^2-6abBc+b^3B)\int \frac{1}{-x^4+b^2-4ac} d(2cx^2+b)}{c} + \frac{B(b^2-4ac)\int \frac{2cx^2+b}{cx^4+bx^2+a} dx^2}{2c}}{c(b^2 - 4ac)} - \frac{x^2 (x^2 (-2aBc - Abc + b^2 B) + a(bB - 2A}{c(b^2 - 4ac)(a + bx^2 + cx^4)}\right.$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{\frac{B(b^2-4ac)\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{2c}+\frac{(4aAc^2-6abBc+b^3B)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}}{c\left(b^2-4ac\right)}-\frac{x^2\left(x^2\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right.$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{\frac{(4aAc^2-6abBc+b^3B)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{B(b^2-4ac)\log(a+bx^2+cx^4)}{2c}}{c\left(b^2-4ac\right)}-\frac{x^2\left(x^2\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right.$$

input
```
Int[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(-((x^2*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2))/(c*(b^2 - 4*a*c
)*(a + b*x^2 + c*x^4))) + (((b^3*B - 6*a*b*B*c + 4*a*A*c^2)*ArcTanh[(b + 2
*c*x^2)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]) + (B*(b^2 - 4*a*c)*Log[a
 + b*x^2 + c*x^4])/(2*c))/(c*(b^2 - 4*a*c)))/2
```

3.113.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.113.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.44

method	result
default	$\frac{-\frac{\left(2Aa\,c^{2}-A\,b^{2}c-3Babc+B\,b^{3}\right)x^{2}}{c^{2}\left(4ac-b^{2}\right)}+\frac{a\left(Abc+2Bac-B\,b^{2}\right)}{c^{2}\left(4ac-b^{2}\right)}}{2c\,x^{4}+2b\,x^{2}+2a}+\frac{\frac{\left(4Bac-B\,b^{2}\right)\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{2c}+\frac{2\left(2Aac-abB-\frac{\left(4Bac-B\,b^{2}\right)b}{2c}\right)\arctan\left(\cdots\right)}{\sqrt{4ac-b^{2}}}}{2(4ac-b^{2})c}$
risch	Expression too large to display

input
```
int(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-1/c^2*(2*A*a*c^2-A*b^2*c-3*B*a*b*c+B*b^3)/(4*a*c-b^2)*x^2+a*(A*b*c+2
*B*a*c-B*b^2)/c^2/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/2/(4*a*c-b^2)/c*(1/2*(4*B
*a*c-B*b^2)/c*ln(c*x^4+b*x^2+a)+2*(2*A*a*c-a*b*B-1/2*(4*B*a*c-B*b^2)*b/c)/
(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

3.113.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(137) = 274$.

Time = 0.30 (sec) , antiderivative size = 849, normalized size of antiderivative = 5.78

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$$
$$= \left[\frac{2\,Bab^4+8\,(2\,Ba^3+Aa^2b)c^2+2\,(Bb^5-8\,Aa^2c^3+6\,(2\,Ba^2b+Aab^2)c^2-(7\,Bab^3+Ab^4)c)x^2-(Bab^3\right.$$

input
```
integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*B*a*b^4 + 8*(2*B*a^3 + A*a^2*b)*c^2 + 2*(B*b^5 - 8*A*a^2*c^3 + 6*(
2*B*a^2*b + A*a*b^2)*c^2 - (7*B*a*b^3 + A*b^4)*c)*x^2 - (B*a*b^3 - 6*B*a^2
*b*c + 4*A*a^2*c^2 + (B*b^3*c - 6*B*a*b*c^2 + 4*A*a*c^3)*x^4 + (B*b^4 - 6*
B*a*b^2*c + 4*A*a*b*c^2)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2
 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 2
*(6*B*a^2*b^2 + A*a*b^3)*c + (B*a*b^4 - 8*B*a^2*b^2*c + 16*B*a^3*c^2 + (B*
b^4*c - 8*B*a*b^2*c^2 + 16*B*a^2*c^3)*x^4 + (B*b^5 - 8*B*a*b^3*c + 16*B*a^
2*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*
c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 +
16*a^2*b*c^4)*x^2), 1/4*(2*B*a*b^4 + 8*(2*B*a^3 + A*a^2*b)*c^2 + 2*(B*b^5
- 8*A*a^2*c^3 + 6*(2*B*a^2*b + A*a*b^2)*c^2 - (7*B*a*b^3 + A*b^4)*c)*x^2 +
 2*(B*a*b^3 - 6*B*a^2*b*c + 4*A*a^2*c^2 + (B*b^3*c - 6*B*a*b*c^2 + 4*A*a*c
^3)*x^4 + (B*b^4 - 6*B*a*b^2*c + 4*A*a*b*c^2)*x^2)*sqrt(-b^2 + 4*a*c)*arct
an(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 2*(6*B*a^2*b^2 + A*a
*b^3)*c + (B*a*b^4 - 8*B*a^2*b^2*c + 16*B*a^3*c^2 + (B*b^4*c - 8*B*a*b^2*c
^2 + 16*B*a^2*c^3)*x^4 + (B*b^5 - 8*B*a*b^3*c + 16*B*a^2*b*c^2)*x^2)*log(c
*x^4 + b*x^2 + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*
a*b^2*c^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^2)]
```

3.113.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.113.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.113.8 Giac [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.32

$$\begin{aligned}\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx &= -\frac{(Bb^3-6\,Babc+4\,Aac^2)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(b^2c^2-4\,ac^3)\sqrt{-b^2+4\,ac}}+\frac{B\log\left(cx^4+bx^2+a\right)}{4\,c^2}\\&\quad-\frac{Bb^2cx^4-4\,Bac^2x^4-Bb^3x^2+2\,Babcx^2+2\,Ab^2cx^2-4\,Aac^2x^2-Bab^2+2\,Aabc}{4\,(cx^4+bx^2+a)(b^2c^2-4\,ac^3)}\end{aligned}$$

input
```
integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(B*b^3 - 6*B*a*b*c + 4*A*a*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*
c))/((b^2*c^2 - 4*a*c^3)*sqrt(-b^2 + 4*a*c)) + 1/4*B*log(c*x^4 + b*x^2 + a
)/c^2 - 1/4*(B*b^2*c*x^4 - 4*B*a*c^2*x^4 - B*b^3*x^2 + 2*B*a*b*c*x^2 + 2*A
*b^2*c*x^2 - 4*A*a*c^2*x^2 - B*a*b^2 + 2*A*a*b*c)/((c*x^4 + b*x^2 + a)*(b^
2*c^2 - 4*a*c^3))
```

3.113.9 Mupad [B] (verification not implemented)

Time = 8.16 (sec) , antiderivative size = 1527, normalized size of antiderivative = 10.39

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

output
```
- ((x^2*(B*b^3 + 2*A*a*c^2 - A*b^2*c - 3*B*a*b*c))/(2*c^2*(4*a*c - b^2)) -
 (a*(A*b*c - B*b^2 + 2*B*a*c))/(2*c^2*(4*a*c - b^2)))/(a + b*x^2 + c*x^4)
- (log(a + b*x^2 + c*x^4)*(2*B*b^6 - 128*B*a^3*c^3 - 24*B*a*b^4*c + 96*B*a
^2*b^2*c^2))/(2*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)
) - (atan(((8*a*c^3*(4*a*c - b^2)^3 - 2*b^2*c^2*(4*a*c - b^2)^3)*((((8*B*a
 + (8*a*c^2*(2*B*b^6 - 128*B*a^3*c^3 - 24*B*a*b^4*c + 96*B*a^2*b^2*c^2))/(
256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4))*(B*b^3 + 4*A*a*
c^2 - 6*B*a*b*c))/(8*c^2*(4*a*c - b^2)^(3/2)) + (a*(B*b^3 + 4*A*a*c^2 - 6*
B*a*b*c)*(2*B*b^6 - 128*B*a^3*c^3 - 24*B*a*b^4*c + 96*B*a^2*b^2*c^2))/((4*
a*c - b^2)^(3/2)*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4
)))/(a*(4*a*c - b^2)) - x^2*(((((6*B*b^3*c^2 + 8*A*a*c^4 - 28*B*a*b*c^3)/(
4*a*c^3 - b^2*c^2) + ((8*b^3*c^4 - 32*a*b*c^5)*(2*B*b^6 - 128*B*a^3*c^3 -
24*B*a*b^4*c + 96*B*a^2*b^2*c^2))/(2*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*
b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)))*(B*b^3 + 4*A*a*c^2 - 6*B*a*b*c
))/(8*c^2*(4*a*c - b^2)^(3/2)) + ((8*b^3*c^4 - 32*a*b*c^5)*(B*b^3 + 4*A*a*
c^2 - 6*B*a*b*c)*(2*B*b^6 - 128*B*a^3*c^3 - 24*B*a*b^4*c + 96*B*a^2*b^2*c^
2))/(16*c^2*(4*a*c - b^2)^(3/2)*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^6*c
^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)))/(a*(4*a*c - b^2)) + (b*((B^2*b^3 +
2*A*B*a*c^2 - 5*B^2*a*b*c)/(4*a*c^3 - b^2*c^2) + (((6*B*b^3*c^2 + 8*A*a*c^
4 - 28*B*a*b*c^3)/(4*a*c^3 - b^2*c^2) + ((8*b^3*c^4 - 32*a*b*c^5)*(2*B*...
```

3.114 $\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$

3.114.1 Optimal result

Integrand size = 25, antiderivative size = 107

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = -\frac{a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{(Ab-2aB)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

output
```
1/2*(-a*(-2*A*c+B*b)-(-A*b*c-2*B*a*c+B*b^2)*x^2)/c/(-4*a*c+b^2)/(c*x^4+b*x
^2+a)-(A*b-2*B*a)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/
2)
```

3.114.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.04

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \frac{abB+b(bB-Ac)x^2-2ac(A+Bx^2)}{2c\,(-b^2+4ac)\,(a+bx^2+cx^4)} - \frac{(Ab-2aB)\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}$$

input
```
Integrate[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(a*b*B + b*(b*B - A*c)*x^2 - 2*a*c*(A + B*x^2))/(2*c*(-b^2 + 4*a*c)*(a + b
*x^2 + c*x^4)) - ((A*b - 2*a*B)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/
(-b^2 + 4*a*c)^(3/2)
```

3.114.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1578, 1224, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (A+Bx^2)}{(a+bx^2+cx^4)^2} \, dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{x^2 (Bx^2+A)}{(cx^4+bx^2+a)^2}dx^2$$

$$\Big\downarrow \text{1224}$$

$$\frac{1}{2}\left(\frac{(Ab-2aB)\int \frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}-\frac{x^2 (-2aBc-Abc+b^2B)+a(bB-2Ac)}{c(b^2-4ac)(a+bx^2+cx^4)}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(-\frac{2(Ab-2aB)\int \frac{1}{-x^4+b^2-4ac}d(2cx^2+b)}{b^2-4ac}-\frac{x^2 (-2aBc-Abc+b^2B)+a(bB-2Ac)}{c(b^2-4ac)(a+bx^2+cx^4)}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(-\frac{2(Ab-2aB)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{x^2 (-2aBc-Abc+b^2B)+a(bB-2Ac)}{c(b^2-4ac)(a+bx^2+cx^4)}\right)$$

input
```
Int[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(-((a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2)/(c*(b^2 - 4*a*c)*(a +
 b*x^2 + c*x^4))) - (2*(A*b - 2*a*B)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*
c]])/(b^2 - 4*a*c)^(3/2))/2
```

3.114.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1224
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x))*((a + b*x + c*x^2)^(p
+ 1)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c
*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c))   Int[(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -
1] &&  !(IntegerQ[p] && NeQ[a, 0] && NiceSqrtQ[b^2 - 4*a*c])
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.114.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.18

method	result
default	$\frac{-\frac{\left(Abc+2Bac-B\,b^{2}\right)x^{2}}{c\left(4ac-b^{2}\right)}-\frac{a(2Ac-Bb)}{\left(4ac-b^{2}\right)c}}{2c\,x^{4}+2b\,x^{2}+2a}-\frac{(Ab-2Ba)\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{(4ac-b^{2})^{\frac{3}{2}}}$
risch	$\frac{-\frac{\left(Abc+2Bac-B\,b^{2}\right)x^{2}}{2c\left(4ac-b^{2}\right)}-\frac{a(2Ac-Bb)}{2\left(4ac-b^{2}\right)c}}{c\,x^{4}+b\,x^{2}+a}+\frac{\ln\left(\left(\left(-(-4ac+b^{2})^{\frac{3}{2}}+4abc-b^{3}\right)x^{2}+8c\,a^{2}-2b^{2}a\right)Ab}{2(-4ac+b^{2})^{\frac{3}{2}}}-\frac{\ln\left(\left(\left(-(-4ac+b^{2})^{\frac{3}{2}}+4abc-b^{3}\right)x\right.\right.}{(-4ac+b^{2})^{\frac{3}{2}}}$

input
```
int(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-(A*b*c+2*B*a*c-B*b^2)/c/(4*a*c-b^2)*x^2-a*(2*A*c-B*b)/(4*a*c-b^2)/c)
/(c*x^4+b*x^2+a)-(A*b-2*B*a)/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b
^2)^(1/2))
```

3.114.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(101) = 202$.

Time = 0.26 (sec) , antiderivative size = 538, normalized size of antiderivative = 5.03

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$$
$$= \left[-\frac{Bab^3 + 8\,Aa^2c^2 + (Bb^4 + 4\,(2\,Ba^2 + Aab)c^2 - (6\,Bab^2 + Ab^3)c)x^2 - ((2\,Ba - Ab)c^2x^4 + (2\,Bab - Ab}{2\,(ab^4c - 8\,a^2b^2c^2 + 16\,a^3c^3 + (b^4c^2 - 8\,ab^2c^3 +}\right.$$
$$-\frac{Bab^3 + 8\,Aa^2c^2 + (Bb^4 + 4\,(2\,Ba^2 + Aab)c^2 - (6\,Bab^2 + Ab^3)c)x^2 - 2\,((2\,Ba - Ab)c^2x^4 + (2\,Bab - A}{2\,(ab^4c - 8\,a^2b^2c^2 + 16\,a^3c^3 + (b^4c^2 - 8\,ab^2c^3 + 16\,a}$$

input
```
integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/2*(B*a*b^3 + 8*A*a^2*c^2 + (B*b^4 + 4*(2*B*a^2 + A*a*b)*c^2 - (6*B*a*b
^2 + A*b^3)*c)*x^2 - ((2*B*a - A*b)*c^2*x^4 + (2*B*a*b - A*b^2)*c*x^2 + (2
*B*a^2 - A*a*b)*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*
a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 2*(2*B*a^2*b
 + A*a*b^2)*c)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*
c^3 + 16*a^2*c^4)*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2), -1/2*(B
*a*b^3 + 8*A*a^2*c^2 + (B*b^4 + 4*(2*B*a^2 + A*a*b)*c^2 - (6*B*a*b^2 + A*b
^3)*c)*x^2 - 2*((2*B*a - A*b)*c^2*x^4 + (2*B*a*b - A*b^2)*c*x^2 + (2*B*a^2
 - A*a*b)*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(
b^2 - 4*a*c)) - 2*(2*B*a^2*b + A*a*b^2)*c)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a
^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + (b^5*c - 8*a*b^3*c^2 +
 16*a^2*b*c^3)*x^2)]
```

3.114.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(99) = 198$.

Time = 4.23 (sec) , antiderivative size = 394, normalized size of antiderivative = 3.68

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx =$$

$$-\frac{\sqrt{-\frac{1}{(4ac-b^2)^3}}(-Ab+2Ba)\log\left(x^2+\frac{-Ab^2+2Bab-16a^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}(-Ab+2Ba)+8ab^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}(-Ab+2Ba)-b^4\sqrt{\cdots}}{-2Abc+4Bac}\right.}{2}$$

$$+\frac{\sqrt{-\frac{1}{(4ac-b^2)^3}}(-Ab+2Ba)\log\left(x^2+\frac{-Ab^2+2Bab+16a^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}(-Ab+2Ba)-8ab^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}(-Ab+2Ba)+b^4\sqrt{\cdots}}{-2Abc+4Bac}\right.}{2}$$

$$+\frac{-2Aac+Bab+x^2(-Abc-2Bac+Bb^2)}{8a^2c^2-2ab^2c+x^4\cdot(8ac^3-2b^2c^2)+x^2\cdot(8abc^2-2b^3c)}$$

input
```
integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

output
```
-sqrt(-1/(4*a*c - b**2)**3)*(-A*b + 2*B*a)*log(x**2 + (-A*b**2 + 2*B*a*b -
 16*a**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*(-A*b + 2*B*a) + 8*a*b**2*c*sqrt(
-1/(4*a*c - b**2)**3)*(-A*b + 2*B*a) - b**4*sqrt(-1/(4*a*c - b**2)**3)*(-A
*b + 2*B*a))/(-2*A*b*c + 4*B*a*c))/2 + sqrt(-1/(4*a*c - b**2)**3)*(-A*b +
2*B*a)*log(x**2 + (-A*b**2 + 2*B*a*b + 16*a**2*c**2*sqrt(-1/(4*a*c - b**2)
**3)*(-A*b + 2*B*a) - 8*a*b**2*c*sqrt(-1/(4*a*c - b**2)**3)*(-A*b + 2*B*a)
 + b**4*sqrt(-1/(4*a*c - b**2)**3)*(-A*b + 2*B*a))/(-2*A*b*c + 4*B*a*c))/2
 + (-2*A*a*c + B*a*b + x**2*(-A*b*c - 2*B*a*c + B*b**2))/(8*a**2*c**2 - 2*
a*b**2*c + x**4*(8*a*c**3 - 2*b**2*c**2) + x**2*(8*a*b*c**2 - 2*b**3*c))
```

3.114.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.114.8 Giac [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.12

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = -\frac{(2\,Ba-Ab)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}} - \frac{Bb^2x^2-2\,Bacx^2-Abcx^2+Bab-2\,Aac}{2\,(cx^4+bx^2+a)(b^2c-4\,ac^2)}$$

input
```
integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
-(2*B*a - A*b)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqr
t(-b^2 + 4*a*c)) - 1/2*(B*b^2*x^2 - 2*B*a*c*x^2 - A*b*c*x^2 + B*a*b - 2*A*
a*c)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c^2))
```

3.114.9 Mupad [B] (verification not implemented)

Time = 7.61 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.64

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = -\frac{\frac{x^2\,\left(-B\,b^2+A\,c\,b+2\,B\,a\,c\right)}{2\,c\,\left(4\,a\,c-b^2\right)}+\frac{a\,\left(2\,A\,c-B\,b\right)}{2\,c\,\left(4\,a\,c-b^2\right)}}{c\,x^4+b\,x^2+a} - \frac{\mathrm{atan}\left(\frac{{\left(4\,a\,c-b^2\right)}^4\,\left(x^2\,\left(\frac{\left(A\,b-2\,B\,a\right)\,\left(A\,b\,c^2-2\,B\,a\,c^2\right)}{a\,{\left(4\,a\,c-b^2\right)}^{7/2}}+\frac{\left(2\,b^3\,c^2-8\,a\,b\,c^3\right)\,{\left(A\,b-2\,B\,a\right)}^2\,\left(b^3-4\,a\,b\,c\right)}{2\,a\,{\left(4\,a\,c-b^2\right)}^{13/2}}\right)-\frac{2\,c^2\,{\left(A\,b-2\,B\,a\right)}^2\,\left(b^3-4\,a\,b\,c\right)}{{\left(4\,a\,c-b^2\right)}^{11/2}}\right)}{2\,A^2\,b^2\,c^2-8\,A\,B\,a\,b\,c^2+8\,B^2\,a^2\,c^2}\right)\,(A\,b}{{\left(4\,a\,c-b^2\right)}^{3/2}}$$

input
```
int((x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

output
```
- ((x^2*(A*b*c - B*b^2 + 2*B*a*c))/(2*c*(4*a*c - b^2)) + (a*(2*A*c - B*b))
/(2*c*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) - (atan(((4*a*c - b^2)^4*(x^2*((
(A*b - 2*B*a)*(A*b*c^2 - 2*B*a*c^2))/(a*(4*a*c - b^2)^(7/2)) + ((2*b^3*c^2
 - 8*a*b*c^3)*(A*b - 2*B*a)^2*(b^3 - 4*a*b*c))/(2*a*(4*a*c - b^2)^(13/2)))
 - (2*c^2*(A*b - 2*B*a)^2*(b^3 - 4*a*b*c))/(4*a*c - b^2)^(11/2)))/(2*A^2*b
^2*c^2 + 8*B^2*a^2*c^2 - 8*A*B*a*b*c^2))*(A*b - 2*B*a))/(4*a*c - b^2)^(3/2
)
```

3.114. $\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$

3.115 $\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$

3.115.1 Optimal result

Integrand size = 23, antiderivative size = 94

$$\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = -\frac{Ab-2aB-(bB-2Ac)x^2}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{(bB-2Ac)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

output
```
1/2*(-A*b+2*B*a+(-2*A*c+B*b)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-(-2*A*c+B*b
)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

3.115.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.07

$$\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \frac{\frac{B(2a+bx^2)-A(b+2cx^2)}{a+bx^2+cx^4} + \frac{2(bB-2Ac)\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{2(b^2-4ac)}$$

input
```
Integrate[(x*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((B*(2*a + b*x^2) - A*(b + 2*c*x^2))/(a + b*x^2 + c*x^4) + (2*(b*B - 2*A*c
)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(2*(b^2 -
4*a*c))
```

3.115.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1576, 1159, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \text{ 1576}$$

$$\frac{1}{2}\int \frac{Bx^2+A}{(cx^4+bx^2+a)^2}dx^2$$

$$\Big\downarrow \text{ 1159}$$

$$\frac{1}{2}\left(\frac{(bB-2Ac)\int \frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}-\frac{-2aB-(x^2(bB-2Ac))+Ab}{(b^2-4ac)(a+bx^2+cx^4)}\right)$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{1}{2}\left(-\frac{2(bB-2Ac)\int \frac{1}{-x^4+b^2-4ac}d(2cx^2+b)}{b^2-4ac}-\frac{-2aB-(x^2(bB-2Ac))+Ab}{(b^2-4ac)(a+bx^2+cx^4)}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}\left(-\frac{2(bB-2Ac)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{-2aB-(x^2(bB-2Ac))+Ab}{(b^2-4ac)(a+bx^2+cx^4)}\right)$$

input
```
Int[(x*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(-((A*b - 2*a*B - (b*B - 2*A*c)*x^2)/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)))
- (2*(b*B - 2*A*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)
^(3/2))/2
```

3.115.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

3.115.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01

method	result
default	$\frac{(2Ac-Bb)x^2+Ab-2Ba}{2(4ac-b^2)(cx^4+bx^2+a)}+\frac{(2Ac-Bb)\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}}$
risch	$\frac{\frac{(2Ac-Bb)x^2}{8ac-2b^2}+\frac{Ab-2Ba}{8ac-2b^2}}{cx^4+bx^2+a}+\frac{\ln\left(\left(\left(-4ac+b^2\right)^{\frac{3}{2}}+4abc-b^3\right)x^2+8ca^2-2b^2a\right)Ac}{\left(-4ac+b^2\right)^{\frac{3}{2}}}-\frac{\ln\left(\left(\left(-4ac+b^2\right)^{\frac{3}{2}}+4abc-b^3\right)x^2+8ca^2-2b^2a\right)B}{2\left(-4ac+b^2\right)^{\frac{3}{2}}}$

input
```
int(x*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*((2*A*c-B*b)*x^2+A*b-2*B*a)/(4*a*c-b^2)/(c*x^4+b*x^2+a)+(2*A*c-B*b)/(4
*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

3.115.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. 2(88) = 176.

Time = 0.26 (sec) , antiderivative size = 474, normalized size of antiderivative = 5.04

$$\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$$
$$= \left[\frac{2\,Bab^2 - Ab^3 + (Bb^3 + 8\,Aac^2 - 2\,(2\,Bab + Ab^2)c)x^2 + ((Bbc - 2\,Ac^2)x^4 + Bab - 2\,Aac + (Bb^2 - 2\,A}{2\,(ab^4 - 8\,a^2b^2c + 16\,a^3c^2 + (b^4c - 8\,ab^2c^2 + 16\,a^2c^3)x^4}\right.$$

input
```
integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/2*(2*B*a*b^2 - A*b^3 + (B*b^3 + 8*A*a*c^2 - 2*(2*B*a*b + A*b^2)*c)*x^2
+ ((B*b*c - 2*A*c^2)*x^4 + B*a*b - 2*A*a*c + (B*b^2 - 2*A*b*c)*x^2)*sqrt(b
^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(
b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 4*(2*B*a^2 - A*a*b)*c)/(a*b^4 - 8*a^2
*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*
b^3*c + 16*a^2*b*c^2)*x^2), 1/2*(2*B*a*b^2 - A*b^3 + (B*b^3 + 8*A*a*c^2 -
2*(2*B*a*b + A*b^2)*c)*x^2 - 2*((B*b*c - 2*A*c^2)*x^4 + B*a*b - 2*A*a*c +
(B*b^2 - 2*A*b*c)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2
+ 4*a*c)/(b^2 - 4*a*c)) - 4*(2*B*a^2 - A*a*b)*c)/(a*b^4 - 8*a^2*b^2*c + 16
*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*
a^2*b*c^2)*x^2)]
```

3.115.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 374 vs. 2(83) = 166.

Time = 2.28 (sec) , antiderivative size = 374, normalized size of antiderivative = 3.98

$$\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \frac{\sqrt{-\frac{1}{(4ac-b^2)^3}}(-2Ac+Bb)\log\left(x^2+\frac{-2Abc+Bb^2-16a^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}(-2Ac+Bb)+8ab^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}(-2Ac+Bb)-b^4\sqrt{-\frac{1}{(4ac-b^2)^3}}(-2Ac+Bb)}{-4Ac^2+2Bbc}\right)}{2} - \frac{\sqrt{-\frac{1}{(4ac-b^2)^3}}(-2Ac+Bb)\log\left(x^2+\frac{-2Abc+Bb^2+16a^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}(-2Ac+Bb)-8ab^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}(-2Ac+Bb)+b^4\sqrt{-\frac{1}{(4ac-b^2)^3}}(-2Ac+Bb)}{-4Ac^2+2Bbc}\right)}{2} + \frac{Ab-2Ba+x^2\cdot(2Ac-Bb)}{8a^2c-2ab^2+x^4\cdot(8ac^2-2b^2c)+x^2\cdot(8abc-2b^3)}$$

input
```
integrate(x*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

output
```
sqrt(-1/(4*a*c - b**2)**3)*(-2*A*c + B*b)*log(x**2 + (-2*A*b*c + B*b**2 -
16*a**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*(-2*A*c + B*b) + 8*a*b**2*c*sqrt(-
1/(4*a*c - b**2)**3)*(-2*A*c + B*b) - b**4*sqrt(-1/(4*a*c - b**2)**3)*(-2*
A*c + B*b))/(-4*A*c**2 + 2*B*b*c))/2 - sqrt(-1/(4*a*c - b**2)**3)*(-2*A*c
+ B*b)*log(x**2 + (-2*A*b*c + B*b**2 + 16*a**2*c**2*sqrt(-1/(4*a*c - b**2)
**3)*(-2*A*c + B*b) - 8*a*b**2*c*sqrt(-1/(4*a*c - b**2)**3)*(-2*A*c + B*b)
 + b**4*sqrt(-1/(4*a*c - b**2)**3)*(-2*A*c + B*b))/(-4*A*c**2 + 2*B*b*c))/
2 + (A*b - 2*B*a + x**2*(2*A*c - B*b))/(8*a**2*c - 2*a*b**2 + x**4*(8*a*c*
*2 - 2*b**2*c) + x**2*(8*a*b*c - 2*b**3))
```

3.115.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.115.8 Giac [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.09

$$\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \frac{(Bb-2\,Ac)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}} + \frac{Bbx^2-2\,Acx^2+2\,Ba-Ab}{2\,(cx^4+bx^2+a)(b^2-4\,ac)}$$

input

```
integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output

```
(B*b - 2*A*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt
(-b^2 + 4*a*c)) + 1/2*(B*b*x^2 - 2*A*c*x^2 + 2*B*a - A*b)/((c*x^4 + b*x^2
+ a)*(b^2 - 4*a*c))
```

3.115.9 Mupad [B] (verification not implemented)

Time = 7.67 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.81

$$\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \frac{\frac{A\,b-2\,B\,a}{2\,(4\,a\,c-b^2)}+\frac{x^2\,(2\,A\,c-B\,b)}{2\,(4\,a\,c-b^2)}}{c\,x^4+b\,x^2+a} + \frac{\mathrm{atan}\left(\frac{\left(x^2\,\left(\frac{(2\,A\,c-B\,b)\,\left(2\,A\,c^3-B\,b\,c^2\right)}{a\,\left(4\,a\,c-b^2\right)^{7/2}}+\frac{\left(2\,b^3\,c^2-8\,a\,b\,c^3\right)\,(2\,A\,c-B\,b)^2\,\left(b^3-4\,a\,b\,c\right)}{2\,a\,\left(4\,a\,c-b^2\right)^{13/2}}\right)-\frac{2\,c^2\,(2\,A\,c-B\,b)^2\,\left(b^3-4\,a\,b\,c\right)}{\left(4\,a\,c-b^2\right)^{11/2}}\right)\,(4\,a\,c-b^2)^4}{8\,A^2\,c^4-8\,A\,B\,b\,c^3+2\,B^2\,b^2\,c^2}\right)\,(2\,A\,c}{(4\,a\,c-b^2)^{3/2}}$$

input

```
int((x*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

output

```
((A*b - 2*B*a)/(2*(4*a*c - b^2)) + (x^2*(2*A*c - B*b))/(2*(4*a*c - b^2)))/
(a + b*x^2 + c*x^4) + (atan(((x^2*(((2*A*c - B*b)*(2*A*c^3 - B*b*c^2))/(a*
(4*a*c - b^2)^(7/2)) + ((2*b^3*c^2 - 8*a*b*c^3)*(2*A*c - B*b)^2*(b^3 - 4*a
*b*c))/(2*a*(4*a*c - b^2)^(13/2))) - (2*c^2*(2*A*c - B*b)^2*(b^3 - 4*a*b*c
))/(4*a*c - b^2)^(11/2))*(4*a*c - b^2)^4)/(8*A^2*c^4 + 2*B^2*b^2*c^2 - 8*A
*B*b*c^3))*(2*A*c - B*b))/(4*a*c - b^2)^(3/2)
```

3.116 $\int \frac{A+Bx^2}{x(a+bx^2+cx^4)^2}\,dx$

3.116.1 Optimal result

Integrand size = 25, antiderivative size = 150

$$\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx = -\frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(4a^2Bc+A(b^3-6abc)\right)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}} + \frac{A\log(x)}{a^2} - \frac{A\log\left(a+bx^2+cx^4\right)}{4a^2}$$

output
```
1/2*(-a*b*B+A*(-2*a*c+b^2)+(A*b-2*B*a)*c*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+
a)+1/2*(4*a^2*B*c+A*(-6*a*b*c+b^3))*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2)
)/a^2/(-4*a*c+b^2)^(3/2)+A*ln(x)/a^2-1/4*A*ln(c*x^4+b*x^2+a)/a^2
```

3.116.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.62

$$\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx = \frac{-\frac{2a\left(aB\left(b+2cx^2\right)-A\left(b^2-2ac+bcx^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + 4A\log(x) - \frac{\left(4a^2Bc+A\left(b^3-6abc+b^2\sqrt{b^2-4ac}-4ac\sqrt{b^2-4ac}\right)\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}} + \cdots}{4a^2}$$

input
```
Integrate[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)^2),x]
```

output
```
((-2*a*(a*B*(b + 2*c*x^2) - A*(b^2 - 2*a*c + b*c*x^2)))/((b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)) + 4*A*Log[x] - ((4*a^2*B*c + A*(b^3 - 6*a*b*c + b^2*Sqrt
[b^2 - 4*a*c] - 4*a*c*Sqrt[b^2 - 4*a*c]))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*
x^2])/(b^2 - 4*a*c)^(3/2) + ((4*a^2*B*c + A*(b^3 - 6*a*b*c - b^2*Sqrt[b^2
- 4*a*c] + 4*a*c*Sqrt[b^2 - 4*a*c]))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])
/(b^2 - 4*a*c)^(3/2))/(4*a^2)
```

3.116.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.20, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1578, 1235, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)^2}dx$$

$$\Big\downarrow \text{ 1578}$$

$$\frac{1}{2}\int \frac{Bx^2+A}{x^2\left(cx^4+bx^2+a\right)^2}dx^2$$

$$\Big\downarrow \text{ 1235}$$

$$\frac{1}{2}\left(\frac{cx^2(Ab-2aB)-2aAc-abB+Ab^2}{a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{(Ab-2aB)cx^2+A\left(b^2-4ac\right)}{x^2\left(cx^4+bx^2+a\right)}dx^2}{a\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\left(\frac{\int \frac{(Ab-2aB)cx^2+A\left(b^2-4ac\right)}{x^2\left(cx^4+bx^2+a\right)}dx^2}{a\left(b^2-4ac\right)}+\frac{cx^2(Ab-2aB)-2aAc-abB+Ab^2}{a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \text{ 1200}$$

$$\frac{1}{2}\left(\frac{\int \left(\frac{-Ab^3+5aAcb-Ac\left(b^2-4ac\right)x^2-2a^2Bc}{a\left(cx^4+bx^2+a\right)}-\frac{A\left(4ac-b^2\right)}{ax^2}\right)dx^2}{a\left(b^2-4ac\right)}+\frac{cx^2(Ab-2aB)-2aAc-abB+Ab^2}{a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{\frac{(4a^2Bc+A(b^3-6abc))\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}+\frac{A\log(x^2)(b^2-4ac)}{a}-\frac{A(b^2-4ac)\log(a+bx^2+cx^4)}{2a}}{a(b^2-4ac)}+\frac{cx^2(Ab-2aB)-2aAc-}{a(b^2-4ac)(a+bx^2}\right.$$

input
```
Int[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)^2),x]
```

output
```
((A*b^2 - a*b*B - 2*a*A*c + (A*b - 2*a*B)*c*x^2)/(a*(b^2 - 4*a*c)*(a + b*x
^2 + c*x^4)) + (((4*a^2*B*c + A*(b^3 - 6*a*b*c))*ArcTanh[(b + 2*c*x^2)/Sqr
t[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]) + (A*(b^2 - 4*a*c)*Log[x^2])/a - (A
*(b^2 - 4*a*c)*Log[a + b*x^2 + c*x^4])/(2*a))/(a*(b^2 - 4*a*c)))/2
```

3.116.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.116.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.41

method	result
default	$\frac{A\ln(x)}{a^2}-\frac{\frac{\frac{ac(Ab-2Ba)x^2}{4ac-b^2}-\frac{a\left(2Aac-A\,b^2+abB\right)}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{\frac{\left(4Aa\,c^2-A\,b^2c\right)\ln\left(c\,x^4+b\,x^2+a\right)}{2c}+\frac{2\left(5Aabc-A\,b^3-2a^2Bc-\frac{\left(4Aa\,c^2-A\,b^2c\right)b}{2c}\right)\arctan\left(\right.}{\sqrt{4ac-b^2}}}{4ac-b^2}}{2a^2}$
risch	$\frac{-\frac{c(Ab-2Ba)x^2}{2a\left(4ac-b^2\right)}+\frac{2Aac-A\,b^2+abB}{2\left(4ac-b^2\right)a}}{c\,x^4+b\,x^2+a}+\frac{A\ln(x)}{a^2}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(64a^5c^3-48b^2a^4c^2+12b^4a^3c-a^2b^6\right)\_Z^2+\left(64c^3a^3A-48a^2b^2c^2A+12a\,b^4\right.\right.}\right.}{}$

input
```
int((B*x^2+A)/x/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
A*ln(x)/a^2-1/2/a^2*((a*c*(A*b-2*B*a)/(4*a*c-b^2)*x^2-a*(2*A*a*c-A*b^2+B*a
*b)/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/(4*a*c-b^2)*(1/2*(4*A*a*c^2-A*b^2*c)/c*
ln(c*x^4+b*x^2+a)+2*(5*A*a*b*c-A*b^3-2*a^2*B*c-1/2*(4*A*a*c^2-A*b^2*c)*b/c
)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))))
```

3.116.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(140) = 280$.

Time = 0.60 (sec) , antiderivative size = 1014, normalized size of antiderivative = 6.76

$$\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx$$
$$= \left[-\frac{2\,Ba^2b^3-2\,Aab^4-16\,Aa^3c^2-2\,(4\,(2\,Ba^3-Aa^2b)c^2-(2\,Ba^2b^2-Aab^3)c)x^2-(Aab^3+(Ab^3c+2\,(2\right.$$

$$-\frac{2\,Ba^2b^3-2\,Aab^4-16\,Aa^3c^2-2\,(4\,(2\,Ba^3-Aa^2b)c^2-(2\,Ba^2b^2-Aab^3)c)x^2-2\,(Aab^3+(Ab^3c+2\,(}{}$$

input
```
integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*B*a^2*b^3 - 2*A*a*b^4 - 16*A*a^3*c^2 - 2*(4*(2*B*a^3 - A*a^2*b)*c
^2 - (2*B*a^2*b^2 - A*a*b^3)*c)*x^2 - (A*a*b^3 + (A*b^3*c + 2*(2*B*a^2 - 3
*A*a*b)*c^2)*x^4 + (A*b^4 + 2*(2*B*a^2*b - 3*A*a*b^2)*c)*x^2 + 2*(2*B*a^3
- 3*A*a^2*b)*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c
 + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 4*(2*B*a^3*b -
3*A*a^2*b^2)*c + (A*a*b^4 - 8*A*a^2*b^2*c + 16*A*a^3*c^2 + (A*b^4*c - 8*A*
a*b^2*c^2 + 16*A*a^2*c^3)*x^4 + (A*b^5 - 8*A*a*b^3*c + 16*A*a^2*b*c^2)*x^2
)*log(c*x^4 + b*x^2 + a) - 4*(A*a*b^4 - 8*A*a^2*b^2*c + 16*A*a^3*c^2 + (A*
b^4*c - 8*A*a*b^2*c^2 + 16*A*a^2*c^3)*x^4 + (A*b^5 - 8*A*a*b^3*c + 16*A*a^
2*b*c^2)*x^2)*log(x))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8
*a^3*b^2*c^2 + 16*a^4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^
2), -1/4*(2*B*a^2*b^3 - 2*A*a*b^4 - 16*A*a^3*c^2 - 2*(4*(2*B*a^3 - A*a^2*b
)*c^2 - (2*B*a^2*b^2 - A*a*b^3)*c)*x^2 - 2*(A*a*b^3 + (A*b^3*c + 2*(2*B*a^
2 - 3*A*a*b)*c^2)*x^4 + (A*b^4 + 2*(2*B*a^2*b - 3*A*a*b^2)*c)*x^2 + 2*(2*B
*a^3 - 3*A*a^2*b)*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 +
4*a*c)/(b^2 - 4*a*c)) - 4*(2*B*a^3*b - 3*A*a^2*b^2)*c + (A*a*b^4 - 8*A*a^2
*b^2*c + 16*A*a^3*c^2 + (A*b^4*c - 8*A*a*b^2*c^2 + 16*A*a^2*c^3)*x^4 + (A*
b^5 - 8*A*a*b^3*c + 16*A*a^2*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a) - 4*(A*a*b
^4 - 8*A*a^2*b^2*c + 16*A*a^3*c^2 + (A*b^4*c - 8*A*a*b^2*c^2 + 16*A*a^2*c^
3)*x^4 + (A*b^5 - 8*A*a*b^3*c + 16*A*a^2*b*c^2)*x^2)*log(x))/(a^3*b^4 -...
```

3.116.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((B*x**2+A)/x/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.116.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.116.8 Giac [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.34

$$\begin{aligned}\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx &= -\frac{\left(Ab^3+4\,Ba^2c-6\,Aabc\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\left(a^2b^2-4\,a^3c\right)\sqrt{-b^2+4\,ac}}-\frac{A\log\left(cx^4+bx^2+a\right)}{4\,a^2}+\frac{A\log\left(x^2\right)}{2\,a^2}\\&\quad+\frac{Ab^2cx^4-4\,Aac^2x^4+Ab^3x^2-4\,Ba^2cx^2-2\,Aabcx^2-2\,Ba^2b+3\,Aab^2-8\,Aa^2c}{4\,\left(cx^4+bx^2+a\right)\left(a^2b^2-4\,a^3c\right)}\end{aligned}$$

input
```
integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*
c))/((a^2*b^2 - 4*a^3*c)*sqrt(-b^2 + 4*a*c)) - 1/4*A*log(c*x^4 + b*x^2 + a
)/a^2 + 1/2*A*log(x^2)/a^2 + 1/4*(A*b^2*c*x^4 - 4*A*a*c^2*x^4 + A*b^3*x^2
- 4*B*a^2*c*x^2 - 2*A*a*b*c*x^2 - 2*B*a^2*b + 3*A*a*b^2 - 8*A*a^2*c)/((c*x
^4 + b*x^2 + a)*(a^2*b^2 - 4*a^3*c))
```

3.116.9 Mupad [B] (verification not implemented)

Time = 12.30 (sec) , antiderivative size = 7119, normalized size of antiderivative = 47.46

$$\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^2)/(x*(a + b*x^2 + c*x^4)^2),x)
```

output
```
((2*A*a*c - A*b^2 + B*a*b)/(2*a*(4*a*c - b^2)) - (c*x^2*(A*b - 2*B*a))/(2*
a*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) + (A*log(x))/a^2 - (log((((A + a^2*(
-(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)^2/(a^4*(4*a*c - b^2)^3))^(1/2))*(((A + a^
2*(-(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)^2/(a^4*(4*a*c - b^2)^3))^(1/2))*((4*b*
c^2*(A*b^3 + 2*B*a^2*c - 5*A*a*b*c))/(a*(4*a*c - b^2)) - (b*c^2*(A + a^2*(
-(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)^2/(a^4*(4*a*c - b^2)^3))^(1/2))*(a*b + 3*
b^2*x^2 - 10*a*c*x^2))/a^2 + (2*c^3*x^2*(A*b^3 + 8*B*a*b^2 - 20*B*a^2*c -
10*A*a*b*c))/(a*(4*a*c - b^2))))/(4*a^2) + (c^3*(A*b - 2*B*a)*(4*A*b^3 + 2
*B*a^2*c - 17*A*a*b*c))/(a^2*(4*a*c - b^2)^2) - (2*c^4*x^2*(A*b - 2*B*a)*(
10*A*a*c - 3*A*b^2 + B*a*b))/(a^2*(4*a*c - b^2)^2)))/(4*a^2) + (c^5*x^2*(A
*b - 2*B*a)^3)/(a^3*(4*a*c - b^2)^3) - (A*c^4*(A*b - 2*B*a)^2)/(a^3*(4*a*c
 - b^2)^2))*(((A - a^2*(-(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)^2/(a^4*(4*a*c - b
^2)^3))^(1/2))*(((A - a^2*(-(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)^2/(a^4*(4*a*c
- b^2)^3))^(1/2))*((4*b*c^2*(A*b^3 + 2*B*a^2*c - 5*A*a*b*c))/(a*(4*a*c - b
^2)) - (b*c^2*(A - a^2*(-(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)^2/(a^4*(4*a*c - b
^2)^3))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^2 + (2*c^3*x^2*(A*b^3 + 8
*B*a*b^2 - 20*B*a^2*c - 10*A*a*b*c))/(a*(4*a*c - b^2))))/(4*a^2) + (c^3*(A
*b - 2*B*a)*(4*A*b^3 + 2*B*a^2*c - 17*A*a*b*c))/(a^2*(4*a*c - b^2)^2) - (2
*c^4*x^2*(A*b - 2*B*a)*(10*A*a*c - 3*A*b^2 + B*a*b))/(a^2*(4*a*c - b^2)^2)
))/(4*a^2) + (c^5*x^2*(A*b - 2*B*a)^3)/(a^3*(4*a*c - b^2)^3) - (A*c^4*(...
```

3.117 $\int \frac{A+Bx^2}{x^3(a+bx^2+cx^4)^2}\,dx$

3.117.1 Optimal result

Integrand size = 25, antiderivative size = 223

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx = -\frac{2Ab^2-abB-6aAc}{2a^2\left(b^2-4ac\right)x^2} - \frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} + \frac{\left(abB(b^2-6ac)-2A(b^4-6ab^2c+6a^2c^2)\right)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\left(b^2-4ac\right)^{3/2}} - \frac{(2Ab-aB)\log(x)}{a^3} + \frac{(2Ab-aB)\log\left(a+bx^2+cx^4\right)}{4a^3}$$

output
```
1/2*(6*A*a*c-2*A*b^2+B*a*b)/a^2/(-4*a*c+b^2)/x^2+1/2*(-a*b*B+A*(-2*a*c+b^2
)+(A*b-2*B*a)*c*x^2)/a/(-4*a*c+b^2)/x^2/(c*x^4+b*x^2+a)+1/2*(a*b*B*(-6*a*c
+b^2)-2*A*(6*a^2*c^2-6*a*b^2*c+b^4))*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2
))/a^3/(-4*a*c+b^2)^(3/2)-(2*A*b-B*a)*ln(x)/a^3+1/4*(2*A*b-B*a)*ln(c*x^4+b
*x^2+a)/a^3
```

3.117.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.70

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx$$
$$= \frac{-\frac{2aA}{x^2} - \frac{2a\left(aB\left(-b^2+2ac-bcx^2\right)+A\left(b^3-3abc+b^2cx^2-2ac^2x^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + 4(-2Ab+aB)\log(x) + \frac{\left(aB\left(-b^3+6abc-b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2 \cdots\right.\right.}{\cdots}}{\cdots}$$

input
```
Integrate[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

output
```
((-2*a*A)/x^2 - (2*a*(a*B*(-b^2 + 2*a*c - b*c*x^2) + A*(b^3 - 3*a*b*c + b^
2*c*x^2 - 2*a*c^2*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + 4*(-2*A*b +
 a*B)*Log[x] + ((a*B*(-b^3 + 6*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[
b^2 - 4*a*c]) + 2*A*(b^4 - 6*a*b^2*c + 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] -
 4*a*b*c*Sqrt[b^2 - 4*a*c]))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 -
4*a*c)^(3/2) + ((a*B*(b^3 - 6*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[b
^2 - 4*a*c]) + 2*A*(-b^4 + 6*a*b^2*c - 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] -
 4*a*b*c*Sqrt[b^2 - 4*a*c]))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 -
4*a*c)^(3/2))/(4*a^3)
```

3.117.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx$$

$$\Big\downarrow \ 1578$$

$$\frac{1}{2}\int \frac{Bx^2+A}{x^4\left(cx^4+bx^2+a\right)^2}dx^2$$

$$\Big\downarrow \ 1235$$

$$\frac{1}{2}\left(\frac{cx^2(Ab-2aB)-2aAc-abB+Ab^2}{ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int -\frac{2Ab^2-aBb+2(Ab-2aB)cx^2-6aAc}{x^4\left(cx^4+bx^2+a\right)}dx^2}{a\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \ 25$$

$$\frac{1}{2}\left(\frac{\int -\frac{-2(Ab-2aB)cx^2+abB-2A(b^2-3ac)}{x^4(cx^4+bx^2+a)}dx^2}{a(b^2-4ac)}+\frac{cx^2(Ab-2aB)-2aAc-abB+Ab^2}{ax^2(b^2-4ac)(a+bx^2+cx^4)}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\left(\frac{cx^2(Ab-2aB)-2aAc-abB+Ab^2}{ax^2(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int -\frac{2Ab^2-aBb+2(Ab-2aB)cx^2-6aAc}{x^4(cx^4+bx^2+a)}dx^2}{a(b^2-4ac)}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\left(\frac{\int -\frac{-2(Ab-2aB)cx^2+abB-2A(b^2-3ac)}{x^4(cx^4+bx^2+a)}dx^2}{a(b^2-4ac)}+\frac{cx^2(Ab-2aB)-2aAc-abB+Ab^2}{ax^2(b^2-4ac)(a+bx^2+cx^4)}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\left(\frac{cx^2(Ab-2aB)-2aAc-abB+Ab^2}{ax^2(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int -\frac{2Ab^2-aBb+2(Ab-2aB)cx^2-6aAc}{x^4(cx^4+bx^2+a)}dx^2}{a(b^2-4ac)}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\left(\frac{\int -\frac{-2(Ab-2aB)cx^2+abB-2A(b^2-3ac)}{x^4(cx^4+bx^2+a)}dx^2}{a(b^2-4ac)}+\frac{cx^2(Ab-2aB)-2aAc-abB+Ab^2}{ax^2(b^2-4ac)(a+bx^2+cx^4)}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\left(\frac{cx^2(Ab-2aB)-2aAc-abB+Ab^2}{ax^2(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int -\frac{2Ab^2-aBb+2(Ab-2aB)cx^2-6aAc}{x^4(cx^4+bx^2+a)}dx^2}{a(b^2-4ac)}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\left(\frac{\int -\frac{-2(Ab-2aB)cx^2+abB-2A(b^2-3ac)}{x^4(cx^4+bx^2+a)}dx^2}{a(b^2-4ac)}+\frac{cx^2(Ab-2aB)-2aAc-abB+Ab^2}{ax^2(b^2-4ac)(a+bx^2+cx^4)}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\left(\frac{cx^2(Ab-2aB)-2aAc-abB+Ab^2}{ax^2(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int -\frac{2Ab^2-aBb+2(Ab-2aB)cx^2-6aAc}{x^4(cx^4+bx^2+a)}dx^2}{a(b^2-4ac)}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\left(\frac{\int -\frac{-2(Ab-2aB)cx^2+abB-2A(b^2-3ac)}{x^4(cx^4+bx^2+a)}dx^2}{a(b^2-4ac)}+\frac{cx^2(Ab-2aB)-2aAc-abB+Ab^2}{ax^2(b^2-4ac)(a+bx^2+cx^4)}\right)$$

$$\Big\downarrow \text{ 25}$$

3.117. $\int \frac{A+Bx^2}{x^3(a+bx^2+cx^4)^2}\,dx$

$$\frac{1}{2}\left(\frac{cx^2(Ab-2aB)-2aAc-abB+Ab^2}{ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{2Ab^2-aBb+2(Ab-2aB)cx^2-6aAc}{x^4\left(cx^4+bx^2+a\right)}dx^2}{a\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{\int -\frac{-2(Ab-2aB)cx^2+abB-2A\left(b^2-3ac\right)}{x^4\left(cx^4+bx^2+a\right)}dx^2}{a\left(b^2-4ac\right)}+\frac{cx^2(Ab-2aB)-2aAc-abB+Ab^2}{ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{cx^2(Ab-2aB)-2aAc-abB+Ab^2}{ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{2Ab^2-aBb+2(Ab-2aB)cx^2-6aAc}{x^4\left(cx^4+bx^2+a\right)}dx^2}{a\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{\int -\frac{-2(Ab-2aB)cx^2+abB-2A\left(b^2-3ac\right)}{x^4\left(cx^4+bx^2+a\right)}dx^2}{a\left(b^2-4ac\right)}+\frac{cx^2(Ab-2aB)-2aAc-abB+Ab^2}{ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{cx^2(Ab-2aB)-2aAc-abB+Ab^2}{ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{2Ab^2-aBb+2(Ab-2aB)cx^2-6aAc}{x^4\left(cx^4+bx^2+a\right)}dx^2}{a\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{\int -\frac{-2(Ab-2aB)cx^2+abB-2A\left(b^2-3ac\right)}{x^4\left(cx^4+bx^2+a\right)}dx^2}{a\left(b^2-4ac\right)}+\frac{cx^2(Ab-2aB)-2aAc-abB+Ab^2}{ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{cx^2(Ab-2aB)-2aAc-abB+Ab^2}{ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{2Ab^2-aBb+2(Ab-2aB)cx^2-6aAc}{x^4\left(cx^4+bx^2+a\right)}dx^2}{a\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{\int -\frac{-2(Ab-2aB)cx^2+abB-2A\left(b^2-3ac\right)}{x^4\left(cx^4+bx^2+a\right)}dx^2}{a\left(b^2-4ac\right)}+\frac{cx^2(Ab-2aB)-2aAc-abB+Ab^2}{ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{cx^2(Ab-2aB)-2aAc-abB+Ab^2}{ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{2Ab^2-aBb+2(Ab-2aB)cx^2-6aAc}{x^4\left(cx^4+bx^2+a\right)}dx^2}{a\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \text{25}$$

3.117. $\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx$

$$\frac{1}{2}\left(\frac{\int -\frac{-2(Ab-2aB)cx^2+abB-2A(b^2-3ac)}{x^4(cx^4+bx^2+a)}dx^2}{a(b^2-4ac)}+\frac{cx^2(Ab-2aB)-2aAc-abB+Ab^2}{ax^2(b^2-4ac)(a+bx^2+cx^4)}\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\left(\frac{cx^2(Ab-2aB)-2aAc-abB+Ab^2}{ax^2(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int -\frac{2Ab^2-aBb+2(Ab-2aB)cx^2-6aAc}{x^4(cx^4+bx^2+a)}dx^2}{a(b^2-4ac)}\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\left(\frac{\int -\frac{-2(Ab-2aB)cx^2+abB-2A(b^2-3ac)}{x^4(cx^4+bx^2+a)}dx^2}{a(b^2-4ac)}+\frac{cx^2(Ab-2aB)-2aAc-abB+Ab^2}{ax^2(b^2-4ac)(a+bx^2+cx^4)}\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\left(\frac{cx^2(Ab-2aB)-2aAc-abB+Ab^2}{ax^2(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int -\frac{2Ab^2-aBb+2(Ab-2aB)cx^2-6aAc}{x^4(cx^4+bx^2+a)}dx^2}{a(b^2-4ac)}\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\left(\frac{\int -\frac{-2(Ab-2aB)cx^2+abB-2A(b^2-3ac)}{x^4(cx^4+bx^2+a)}dx^2}{a(b^2-4ac)}+\frac{cx^2(Ab-2aB)-2aAc-abB+Ab^2}{ax^2(b^2-4ac)(a+bx^2+cx^4)}\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\left(\frac{cx^2(Ab-2aB)-2aAc-abB+Ab^2}{ax^2(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int -\frac{2Ab^2-aBb+2(Ab-2aB)cx^2-6aAc}{x^4(cx^4+bx^2+a)}dx^2}{a(b^2-4ac)}\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\left(\frac{\int -\frac{-2(Ab-2aB)cx^2+abB-2A(b^2-3ac)}{x^4(cx^4+bx^2+a)}dx^2}{a(b^2-4ac)}+\frac{cx^2(Ab-2aB)-2aAc-abB+Ab^2}{ax^2(b^2-4ac)(a+bx^2+cx^4)}\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\left(\frac{cx^2(Ab-2aB)-2aAc-abB+Ab^2}{ax^2(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int -\frac{2Ab^2-aBb+2(Ab-2aB)cx^2-6aAc}{x^4(cx^4+bx^2+a)}dx^2}{a(b^2-4ac)}\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\left(\frac{\int -\frac{-2(Ab-2aB)cx^2+abB-2A(b^2-3ac)}{x^4(cx^4+bx^2+a)}dx^2}{a(b^2-4ac)}+\frac{cx^2(Ab-2aB)-2aAc-abB+Ab^2}{ax^2(b^2-4ac)(a+bx^2+cx^4)}\right)$$

$$\Big\downarrow \; 25$$

3.117. $\int \frac{A+Bx^2}{x^3(a+bx^2+cx^4)^2}\,dx$

$$\frac{1}{2}\left(\frac{cx^2(Ab-2aB)-2aAc-abB+Ab^2}{ax^2\,(b^2-4ac)\,(a+bx^2+cx^4)}-\frac{\int -\frac{2Ab^2-aBb+2(Ab-2aB)cx^2-6aAc}{x^4(cx^4+bx^2+a)}dx^2}{a\,(b^2-4ac)}\right)$$

input
```
Int[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

output
```
$Aborted
```

3.117.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.117.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.35

method	result
default	$-\frac{A}{2a^2x^2}+\frac{(-2Ab+Ba)\ln(x)}{a^3}-\frac{\frac{ac\left(2Aac-A\,b^2+abB\right)x^2}{4ac-b^2}+\frac{a\left(3Aabc-A\,b^3-2a^2Bc+Ba\,b^2\right)}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{\frac{\left(-8Aab\,c^2+2A\,b^3c+4B\,a^2c^2-Ba\,b^2c\right)\ln\left(c \cdots\right.}{2c}}{\cdots}$
risch	$\frac{-\frac{c\left(6Aac-2A\,b^2+abB\right)x^4}{2a^2\left(4ac-b^2\right)}-\frac{\left(7Aabc-2A\,b^3-2a^2Bc+Ba\,b^2\right)x^2}{2\left(4ac-b^2\right)a^2}-\frac{A}{2a}}{x^2(c\,x^4+b\,x^2+a)}-\frac{2\ln(x)Ab}{a^3}+\frac{\ln(x)B}{a^2}+\frac{\left(\_R=\text{RootOf}\left(\left(64a^6c^3-48b^2a^5c^2+12a^4b \cdots\right.\right.\right.}{\cdots}$

input
```
int((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*A/a^2/x^2+(-2*A*b+B*a)/a^3*ln(x)-1/2/a^3*((a*c*(2*A*a*c-A*b^2+B*a*b)/
(4*a*c-b^2)*x^2+a*(3*A*a*b*c-A*b^3-2*B*a^2*c+B*a*b^2)/(4*a*c-b^2))/(c*x^4+
b*x^2+a)+1/(4*a*c-b^2)*(1/2*(-8*A*a*b*c^2+2*A*b^3*c+4*B*a^2*c^2-B*a*b^2*c)
/c*ln(c*x^4+b*x^2+a)+2*(6*A*a^2*c^2-10*A*a*b^2*c+2*A*b^4+5*a^2*b*B*c-B*a*b
^3-1/2*(-8*A*a*b*c^2+2*A*b^3*c+4*B*a^2*c^2-B*a*b^2*c)*b/c)/(4*a*c-b^2)^(1/
2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))))
```

3.117.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 806 vs. 2(207) = 414.

Time = 1.15 (sec) , antiderivative size = 1635, normalized size of antiderivative = 7.33

$$\int \frac{A+Bx^2}{x^3\,(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*A*a^2*b^4 - 16*A*a^3*b^2*c + 32*A*a^4*c^2 + 2*(24*A*a^3*c^3 + 2*(
2*B*a^3*b - 7*A*a^2*b^2)*c^2 - (B*a^2*b^3 - 2*A*a*b^4)*c)*x^4 - 2*(B*a^2*b
^4 - 2*A*a*b^5 + 4*(2*B*a^4 - 7*A*a^3*b)*c^2 - 3*(2*B*a^3*b^2 - 5*A*a^2*b^
3)*c)*x^2 + ((12*A*a^2*c^3 + 6*(B*a^2*b - 2*A*a*b^2)*c^2 - (B*a*b^3 - 2*A*
b^4)*c)*x^6 - (B*a*b^4 - 2*A*b^5 - 12*A*a^2*b*c^2 - 6*(B*a^2*b^2 - 2*A*a*b
^3)*c)*x^4 - (B*a^2*b^3 - 2*A*a*b^4 - 12*A*a^3*c^2 - 6*(B*a^3*b - 2*A*a^2*
b^2)*c)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c +
(2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + ((16*(B*a^3 - 2*A*
a^2*b)*c^3 - 8*(B*a^2*b^2 - 2*A*a*b^3)*c^2 + (B*a*b^4 - 2*A*b^5)*c)*x^6 +
(B*a*b^5 - 2*A*b^6 + 16*(B*a^3*b - 2*A*a^2*b^2)*c^2 - 8*(B*a^2*b^3 - 2*A*a
*b^4)*c)*x^4 + (B*a^2*b^4 - 2*A*a*b^5 + 16*(B*a^4 - 2*A*a^3*b)*c^2 - 8*(B*
a^3*b^2 - 2*A*a^2*b^3)*c)*x^2)*log(c*x^4 + b*x^2 + a) - 4*((16*(B*a^3 - 2*
A*a^2*b)*c^3 - 8*(B*a^2*b^2 - 2*A*a*b^3)*c^2 + (B*a*b^4 - 2*A*b^5)*c)*x^6
+ (B*a*b^5 - 2*A*b^6 + 16*(B*a^3*b - 2*A*a^2*b^2)*c^2 - 8*(B*a^2*b^3 - 2*A
*a*b^4)*c)*x^4 + (B*a^2*b^4 - 2*A*a*b^5 + 16*(B*a^4 - 2*A*a^3*b)*c^2 - 8*(
B*a^3*b^2 - 2*A*a^2*b^3)*c)*x^2)*log(x))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*
a^5*c^3)*x^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^4 + (a^4*b^4 - 8*a
^5*b^2*c + 16*a^6*c^2)*x^2), -1/4*(2*A*a^2*b^4 - 16*A*a^3*b^2*c + 32*A*a^4
*c^2 + 2*(24*A*a^3*c^3 + 2*(2*B*a^3*b - 7*A*a^2*b^2)*c^2 - (B*a^2*b^3 - 2*
A*a*b^4)*c)*x^4 - 2*(B*a^2*b^4 - 2*A*a*b^5 + 4*(2*B*a^4 - 7*A*a^3*b)*c^...
```

3.117.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((B*x**2+A)/x**3/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.117.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A + Bx^2}{x^3 (a + bx^2 + cx^4)^2} dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.117.8 Giac [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.12

$$\begin{aligned}\int \frac{A + Bx^2}{x^3 (a + bx^2 + cx^4)^2} dx &= -\frac{(Bab^3 - 2Ab^4 - 6Ba^2bc + 12Aab^2c - 12Aa^2c^2)\arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{2(a^3b^2 - 4a^4c)\sqrt{-b^2+4ac}} \\ &\quad + \frac{Babcx^4 - 2Ab^2cx^4 + 6Aac^2x^4 + Bab^2x^2 - 2Ab^3x^2 - 2Ba^2cx^2 + 7Aabcx^2 - Aab^2 + 4Aa^2c}{2(cx^6 + bx^4 + ax^2)(a^2b^2 - 4a^3c)} \\ &\quad - \frac{(Ba - 2Ab)\log(cx^4 + bx^2 + a)}{4a^3} + \frac{(Ba - 2Ab)\log(x^2)}{2a^3}\end{aligned}$$

input
```
integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(B*a*b^3 - 2*A*b^4 - 6*B*a^2*b*c + 12*A*a*b^2*c - 12*A*a^2*c^2)*arcta
n((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a^3*b^2 - 4*a^4*c)*sqrt(-b^2 + 4*a*c
)) + 1/2*(B*a*b*c*x^4 - 2*A*b^2*c*x^4 + 6*A*a*c^2*x^4 + B*a*b^2*x^2 - 2*A*
b^3*x^2 - 2*B*a^2*c*x^2 + 7*A*a*b*c*x^2 - A*a*b^2 + 4*A*a^2*c)/((c*x^6 + b
*x^4 + a*x^2)*(a^2*b^2 - 4*a^3*c)) - 1/4*(B*a - 2*A*b)*log(c*x^4 + b*x^2 +
 a)/a^3 + 1/2*(B*a - 2*A*b)*log(x^2)/a^3
```

3.117.9 Mupad [B] (verification not implemented)

Time = 13.33 (sec) , antiderivative size = 10034, normalized size of antiderivative = 45.00

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^2),x)
```

output
```
(log(((c^4*(2*A*b - B*a)*(6*A*a*c - 2*A*b^2 + B*a*b)^2)/(a^6*(4*a*c - b^2)
^2) - ((((B*a - 2*A*b + a^3*(-(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b
^2*c + 6*B*a^2*b*c)^2/(a^6*(4*a*c - b^2)^3))^(1/2))*((b*c^2*(B*a - 2*A*b +
 a^3*(-(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c)^2/(
a^6*(4*a*c - b^2)^3))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^3 + (4*b*c^
2*(2*A*b^4 + 6*A*a^2*c^2 - B*a*b^3 - 10*A*a*b^2*c + 5*B*a^2*b*c))/(a^2*(4*
a*c - b^2)) + (2*c^3*x^2*(2*A*b^4 - 60*A*a^2*c^2 - B*a*b^3 + 4*A*a*b^2*c +
 10*B*a^2*b*c))/(a^2*(4*a*c - b^2))))/(4*a^3) + (c^3*(36*A^2*a^3*c^3 - 16*
A^2*b^6 - 4*B^2*a^2*b^4 + 16*A*B*a*b^5 - 216*A^2*a^2*b^2*c^2 + 116*A^2*a*b
^4*c + 17*B^2*a^3*b^2*c - 92*A*B*a^2*b^3*c + 108*A*B*a^3*b*c^2))/(a^4*(4*a
*c - b^2)^2) - (2*c^4*x^2*(12*A^2*b^5 + 3*B^2*a^2*b^3 - 12*A*B*a*b^4 - 60*
A*B*a^3*c^2 - 82*A^2*a*b^3*c - 10*B^2*a^3*b*c + 138*A^2*a^2*b*c^2 + 61*A*B
*a^2*b^2*c))/(a^4*(4*a*c - b^2)^2))*(B*a - 2*A*b + a^3*(-(2*A*b^4 + 12*A*a
^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c)^2/(a^6*(4*a*c - b^2)^3))^(1
/2)))/(4*a^3) + (c^5*x^2*(6*A*a*c - 2*A*b^2 + B*a*b)^3)/(a^6*(4*a*c - b^2)
^3))*((c^4*(2*A*b - B*a)*(6*A*a*c - 2*A*b^2 + B*a*b)^2)/(a^6*(4*a*c - b^2)
^2) - ((((2*A*b - B*a + a^3*(-(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b
^2*c + 6*B*a^2*b*c)^2/(a^6*(4*a*c - b^2)^3))^(1/2))*((4*b*c^2*(2*A*b^4 + 6
*A*a^2*c^2 - B*a*b^3 - 10*A*a*b^2*c + 5*B*a^2*b*c))/(a^2*(4*a*c - b^2)) -
(b*c^2*(2*A*b - B*a + a^3*(-(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*...
```

3.118 $\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$

3.118.1 Optimal result

Integrand size = 25, antiderivative size = 425

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \frac{(3b^2B-Abc-10aBc)x}{2c^2(b^2-4ac)}-\frac{(bB-2Ac)x^3}{2c(b^2-4ac)}-\frac{x^5(Ab-2aB-(bB-2Ac)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)}$$
$$-\frac{\left(3b^3B-Ab^2c-13abBc+6aAc^2-\frac{3b^4B-Ab^3c-19ab^2Bc+8aAbc^2+20a^2Bc^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{5/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}$$
$$-\frac{\left(3b^3B-Ab^2c-13abBc+6aAc^2+\frac{3b^4B-Ab^3c-19ab^2Bc+8aAbc^2+20a^2Bc^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{5/2}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
1/2*(-A*b*c-10*B*a*c+3*B*b^2)*x/c^2/(-4*a*c+b^2)-1/2*(-2*A*c+B*b)*x^3/c/(-
4*a*c+b^2)-1/2*x^5*(A*b-2*B*a-(-2*A*c+B*b)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+
a)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(3*B*b^3-A*b
^2*c-13*B*a*b*c+6*A*a*c^2+(-8*A*a*b*c^2+A*b^3*c-20*B*a^2*c^2+19*B*a*b^2*c-
3*B*b^4)/(-4*a*c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^2)*2^(1/2)/(b-(-4*a*c+b^2)^
(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(3
*B*b^3-A*b^2*c-13*B*a*b*c+6*A*a*c^2+(8*A*a*b*c^2-A*b^3*c+20*B*a^2*c^2-19*B
*a*b^2*c+3*B*b^4)/(-4*a*c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^2)*2^(1/2)/(b+(-4*
a*c+b^2)^(1/2))^(1/2)
```

3.118.2 Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.07

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{4B\sqrt{c}x + \frac{2\sqrt{c}x(-2a^2Bc+b^2(bB-Ac)x^2+a(b^2B+2Ac^2x^2-bc(A+3Bx^2)))}{(b^2-4ac)(a+bx^2+cx^4)} - \frac{\sqrt{2}\left(-3b^4B+b^2c\left(19aB-A\sqrt{b^2-4ac}\right)+2ac^2\left(-10aB+3A\sqrt{\right.\right.}}{}}{}$$

input
```
Integrate[(x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(4*B*Sqrt[c]*x + (2*Sqrt[c]*x*(-2*a^2*B*c + b^2*(b*B - A*c)*x^2 + a*(b^2*B
 + 2*A*c^2*x^2 - b*c*(A + 3*B*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4))
- (Sqrt[2]*(-3*b^4*B + b^2*c*(19*a*B - A*Sqrt[b^2 - 4*a*c]) + 2*a*c^2*(-10
*a*B + 3*A*Sqrt[b^2 - 4*a*c]) + b^3*(A*c + 3*B*Sqrt[b^2 - 4*a*c]) - a*b*c*
(8*A*c + 13*B*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt
[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt
[2]*(3*b^4*B - b^2*c*(19*a*B + A*Sqrt[b^2 - 4*a*c]) + 2*a*c^2*(10*a*B + 3*
A*Sqrt[b^2 - 4*a*c]) + a*b*c*(8*A*c - 13*B*Sqrt[b^2 - 4*a*c]) + b^3*(-(A*c
) + 3*B*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*c^(5/2))
```

3.118.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 400, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1598, 1602, 27, 1602, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$\Big\downarrow \ 1598$$

$$\frac{\int \frac{x^4\left(5(Ab-2aB)-3(bB-2Ac)x^2\right)}{cx^4+bx^2+a}dx}{2\left(b^2-4ac\right)} - \frac{x^5\left(-2aB-\left(x^2(bB-2Ac)\right)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \ 1602$$

$$\frac{-\frac{\int -\frac{3x^2\left(\left(3Bb^2-Acb-10aBc\right)x^2+3a(bB-2Ac)\right)}{cx^4+bx^2+a}dx}{3c}-\frac{x^3(bB-2Ac)}{c}}{2\left(b^2-4ac\right)}-\frac{x^5\left(-2aB-\left(x^2(bB-2Ac)\right)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{x^2\left(\left(3Bb^2-Acb-10aBc\right)x^2+3a(bB-2Ac)\right)}{cx^4+bx^2+a}dx}{c}-\frac{x^3(bB-2Ac)}{c}}{2\left(b^2-4ac\right)}-\frac{x^5\left(-2aB-\left(x^2(bB-2Ac)\right)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$\downarrow$ 1602

$$\frac{\frac{\frac{x\left(-10aBc-Abc+3b^2B\right)}{c}-\frac{\int \frac{\left(3Bb^3-Acb^2-13aBcb+6aAc^2\right)x^2+a\left(3Bb^2-Acb-10aBc\right)}{cx^4+bx^2+a}dx}{c}}{c}-\frac{x^3(bB-2Ac)}{c}}{2\left(b^2-4ac\right)}-\frac{x^5\left(-2aB-\left(x^2(bB-2Ac)\right)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$\downarrow$ 1480

$$\frac{\frac{\frac{x\left(-10aBc-Abc+3b^2B\right)}{c}-\frac{\frac{1}{2}\left(-\frac{20a^2Bc^2+8aAbc^2-19ab^2Bc-Ab^3c+3b^4B}{\sqrt{b^2-4ac}}+6aAc^2-13abBc-Ab^2c+3b^3B\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(\frac{20a^2Bc^2+8aAbc^2-1}{\sqrt{b^2}}\right.}{c}}{c}}{2\left(b^2-4ac\right)}$$

$$\frac{x^5\left(-2aB-\left(x^2(bB-2Ac)\right)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$\downarrow$ 218

$$\frac{\frac{\frac{x\left(-10aBc-Abc+3b^2B\right)}{c}-\frac{\frac{\left(-\frac{20a^2Bc^2+8aAbc^2-19ab^2Bc-Ab^3c+3b^4B}{\sqrt{b^2-4ac}}+6aAc^2-13abBc-Ab^2c+3b^3B\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{20a^2Bc^2+8aAbc^2-19ab^2}{\sqrt{b^2-4ac}}\right.}{}}{c}}{c}}{2\left(b^2-4ac\right)}$$

$$\frac{x^5\left(-2aB-\left(x^2(bB-2Ac)\right)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

input

```
Int[(x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

3.118. $\int \frac{x^6\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

output
```
-1/2*(x^5*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c
*x^4)) + (-(((b*B - 2*A*c)*x^3)/c) + (((3*b^2*B - A*b*c - 10*a*B*c)*x)/c -
 (((3*b^3*B - A*b^2*c - 13*a*b*B*c + 6*a*A*c^2 - (3*b^4*B - A*b^3*c - 19*a
*b^2*B*c + 8*a*A*b*c^2 + 20*a^2*B*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^
2 - 4*a*c]]) + ((3*b^3*B - A*b^2*c - 13*a*b*B*c + 6*a*A*c^2 + (3*b^4*B - A
*b^3*c - 19*a*b^2*B*c + 8*a*A*b*c^2 + 20*a^2*B*c^2)/Sqrt[b^2 - 4*a*c])*Arc
Tan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqr
t[b + Sqrt[b^2 - 4*a*c]]))/c)/c)/(2*(b^2 - 4*a*c))
```

3.118.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1598
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Simp[f
^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1
)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
 GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.118. $\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

3.118.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.51

method	result
risch	$\frac{Bx}{c^2}+\frac{-\frac{\left(2Aa\,c^2-A\,b^2c-3Babc+B\,b^3\right)x^3}{2\left(4ac-b^2\right)}+\frac{a\left(Abc+2Bac-B\,b^2\right)x}{8ac-2b^2}}{c^2(c\,x^4+b\,x^2+a)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(\frac{\left(6Aa\,c^2-A\,b^2c-13Babc+3B\,b^3\right)\_}{4ac-b^2}\right.}{2c\_}}{4c^2}$
default	$\frac{Bx}{c^2}+\frac{-\frac{\left(2Aa\,c^2-A\,b^2c-3Babc+B\,b^3\right)x^3}{2\left(4ac-b^2\right)}+\frac{a\left(Abc+2Bac-B\,b^2\right)x}{8ac-2b^2}}{c\,x^4+b\,x^2+a}+\frac{2c\left(\frac{\left(6Aa\,c^2\sqrt{-4ac+b^2}-A\,b^2c\sqrt{-4ac+b^2}+8Aab\,c^2-A\,b^3c-13Babc\sqrt{-4a\right.}{8c\sqrt{-4}}\right.}{}$

input
```
int(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
B*x/c^2+(-1/2*(2*A*a*c^2-A*b^2*c-3*B*a*b*c+B*b^3)/(4*a*c-b^2)*x^3+1/2*a*(A
*b*c+2*B*a*c-B*b^2)/(4*a*c-b^2)*x)/c^2/(c*x^4+b*x^2+a)+1/4/c^2*sum(((6*A*a
*c^2-A*b^2*c-13*B*a*b*c+3*B*b^3)/(4*a*c-b^2)*_R^2-a*(A*b*c+10*B*a*c-3*B*b^
2)/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

3.118.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7252 vs. 2(379) = 758.

Time = 7.53 (sec) , antiderivative size = 7252, normalized size of antiderivative = 17.06

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.118.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**6*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.118.7 Maxima [F]

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(Bx^2+A)x^6}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*((B*b^3 + 2*A*a*c^2 - (3*B*a*b + A*b^2)*c)*x^3 + (B*a*b^2 - (2*B*a^2 +
 A*a*b)*c)*x)/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2
- 4*a*b*c^3)*x^2) + B*x/c^2 - 1/2*integrate((3*B*a*b^2 + (3*B*b^3 + 6*A*a*
c^2 - (13*B*a*b + A*b^2)*c)*x^2 - (10*B*a^2 + A*a*b)*c)/(c*x^4 + b*x^2 + a
), x)/(b^2*c^2 - 4*a*c^3)
```

3.118.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5675 vs. 2(379) = 758.

Time = 1.51 (sec) , antiderivative size = 5675, normalized size of antiderivative = 13.35

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
B*x/c^2 + 1/2*(B*b^3*x^3 - 3*B*a*b*c*x^3 - A*b^2*c*x^3 + 2*A*a*c^2*x^3 + B
*a*b^2*x - 2*B*a^2*c*x - A*a*b*c*x)/((c*x^4 + b*x^2 + a)*(b^2*c^2 - 4*a*c^
3)) + 1/16*((2*b^4*c^3 - 20*a*b^2*c^4 + 48*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^2*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 12*(b^2 - 4*a*c)*a*c^4)*(b
^2*c^2 - 4*a*c^3)^2*A - (6*b^5*c^2 - 50*a*b^3*c^3 + 104*a^2*b*c^4 - 3*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 25*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 6*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 52*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 26*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 13*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 6*(b^2 - 4*a*c)*b^3*c^2 + 26*(b^
2 - 4*a*c)*a*b*c^3)*(b^2*c^2 - 4*a*c^3)^2*B + 2*(sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b^5*c^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^...
```

3.118.9 Mupad [B] (verification not implemented)

Time = 10.14 (sec) , antiderivative size = 16604, normalized size of antiderivative = 39.07

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

output
```
(B*x)/c^2 - atan(((((10240*B*a^5*c^7 - 16*A*a*b^7*c^4 + 1024*A*a^4*b*c^7 +
 48*B*a*b^8*c^3 + 192*A*a^2*b^5*c^5 - 768*A*a^3*b^3*c^6 - 736*B*a^2*b^6*c^
4 + 4224*B*a^3*b^4*c^5 - 10752*B*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 1
2*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*((9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) -
 A^2*b^11*c^2 - 9*B^2*b^13 + 6*A*B*b^12*c - 288*A^2*a^2*b^7*c^4 + 1504*A^2
*a^3*b^5*c^5 - 3840*A^2*a^4*b^3*c^6 - 2077*B^2*a^2*b^9*c^2 + 10656*B^2*a^3
*b^7*c^3 - 30240*B^2*a^4*b^5*c^4 + 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(
4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 15360*A*
B*a^6*c^7 + 213*B^2*a*b^11*c + 27*A^2*a*b^9*c^3 + 3840*A^2*a^5*b*c^7 - 9*A
^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) - 26880*B^2*a^6*b*c^6 + 1548*A*B*a^2*b^8
*c^3 - 8064*A*B*a^3*b^6*c^4 + 22400*A*B*a^4*b^4*c^5 - 30720*A*B*a^5*b^2*c^
6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a*b^10*c^2 - 6*A*B*b
^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(
32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*
b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*
a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 -
8*a*b^2*c^4)))*((9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^11*c^2 - 9*B^2
*b^13 + 6*A*B*b^12*c - 288*A^2*a^2*b^7*c^4 + 1504*A^2*a^3*b^5*c^5 - 3840*A
^2*a^4*b^3*c^6 - 2077*B^2*a^2*b^9*c^2 + 10656*B^2*a^3*b^7*c^3 - 30240*B^2*
a^4*b^5*c^4 + 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1...
```

3.118. $\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$

3.119 $\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$

3.119.1 Optimal result

Integrand size = 25, antiderivative size = 336

$$\begin{aligned}\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx &= -\frac{(bB-2Ac)x}{2c(b^2-4ac)}-\frac{x^3(Ab-2aB-(bB-2Ac)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)}\\ &\quad+\frac{\left(b^2B+Abc-6aBc-\frac{b^3B+Ab^2c-8abBc+4aAc^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}\\ &\quad+\frac{\left(b^2B+Abc-6aBc+\frac{b^3B+Ab^2c-8abBc+4aAc^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

output
```
-1/2*(-2*A*c+B*b)*x/c/(-4*a*c+b^2)-1/2*x^3*(A*b-2*B*a-(-2*A*c+B*b)*x^2)/(-
4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1
/2))^(1/2))*(B*b^2+A*b*c-6*B*a*c+(-4*A*a*c^2-A*b^2*c+8*B*a*b*c-B*b^3)/(-4*
a*c+b^2)^(1/2))/c^(3/2)/(-4*a*c+b^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+
1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(B*b^2+A*b*c-6*
B*a*c+(4*A*a*c^2+A*b^2*c-8*B*a*b*c+B*b^3)/(-4*a*c+b^2)^(1/2))/c^(3/2)/(-4*
a*c+b^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.119.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.08

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$$
$$= \frac{\frac{2\sqrt{c}(-abBx+b(-bB+Ac)x^3+2acx(A+Bx^2))}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\left(-b^3B+bc\left(8aB+A\sqrt{b^2-4ac}\right)+b^2\left(-Ac+B\sqrt{b^2-4ac}\right)-2ac\left(2Ac+3B\sqrt{b^2-4ac}\right)\right)\text{arctan}}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}}{4c^{3/2}}$$

input
```
Integrate[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((2*Sqrt[c]*(-(a*b*B*x) + b*(-(b*B) + A*c)*x^3 + 2*a*c*x*(A + B*x^2)))/((b
^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-(b^3*B) + b*c*(8*a*B + A*Sqr
t[b^2 - 4*a*c]) + b^2*(-(A*c) + B*Sqrt[b^2 - 4*a*c]) - 2*a*c*(2*A*c + 3*B*
Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]
])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^3*B + 2
*a*c*(2*A*c - 3*B*Sqrt[b^2 - 4*a*c]) + b^2*(A*c + B*Sqrt[b^2 - 4*a*c]) + b
*(-8*a*B*c + A*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + S
qrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*
c^(3/2))
```

3.119.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1598, 1602, 25, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^2}dx$$
$$\Big\downarrow \text{ 1598}$$
$$\frac{\int \frac{x^2\left(3(Ab-2aB)-(bB-2Ac)x^2\right)}{cx^4+bx^2+a}dx}{2\left(b^2-4ac\right)}-\frac{x^3\left(-2aB-\left(x^2(bB-2Ac)\right)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$
$$\Big\downarrow \text{ 1602}$$

$$-\frac{\frac{\int -\frac{\left(Bb^2+Acb-6aBc\right)x^2+a(bB-2Ac)}{cx^4+bx^2+a}dx}{c}-\frac{x(bB-2Ac)}{c}}{2\left(b^2-4ac\right)}-\frac{x^3\left(-2aB-\left(x^2(bB-2Ac)\right)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{\left(Bb^2+Acb-6aBc\right)x^2+a(bB-2Ac)}{cx^4+bx^2+a}dx}{c}-\frac{x(bB-2Ac)}{c}}{2\left(b^2-4ac\right)}-\frac{x^3\left(-2aB-\left(x^2(bB-2Ac)\right)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \; 1480$$

$$\frac{\frac{\frac{1}{2}\left(-\frac{4aAc^2-8abBc+Ab^2c+b^3B}{\sqrt{b^2-4ac}}-6aBc+Abc+b^2B\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(\frac{4aAc^2-8abBc+Ab^2c+b^3B}{\sqrt{b^2-4ac}}-6aBc+Abc+b^2B\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4\right.}}}{c}}{2\left(b^2-4ac\right)}$$
$$\frac{x^3\left(-2aB-\left(x^2(bB-2Ac)\right)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\frac{\left(-\frac{4aAc^2-8abBc+Ab^2c+b^3B}{\sqrt{b^2-4ac}}-6aBc+Abc+b^2B\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{4aAc^2-8abBc+Ab^2c+b^3B}{\sqrt{b^2-4ac}}-6aBc+Abc+b^2B\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{c}-\frac{x(}{}}{2\left(b^2-4ac\right)}$$
$$\frac{x^3\left(-2aB-\left(x^2(bB-2Ac)\right)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

input
```
Int[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
-1/2*(x^3*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c
*x^4)) + (-(((b*B - 2*A*c)*x)/c) + (((b^2*B + A*b*c - 6*a*B*c - (b^3*B + A
*b^2*c - 8*a*b*B*c + 4*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]
*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a
*c]]) + ((b^2*B + A*b*c - 6*a*B*c + (b^3*B + A*b^2*c - 8*a*b*B*c + 4*a*A*c
^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*
c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/c)/(2*(b^2 - 4*a*c))
```

3.119. $\int \frac{x^4\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

3.119.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1598
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Simp[f
^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1
)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
 GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

3.119.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.53

method	result
risch	$\frac{-\frac{\left(Abc+2Bac-B\,b^{2}\right)x^{3}}{2c\left(4ac-b^{2}\right)}-\frac{a(2Ac-Bb)x}{2\left(4ac-b^{2}\right)c}}{c\,x^{4}+b\,x^{2}+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{4}+\_Z^{2}b+a\right)}\frac{\left(-\frac{\left(Abc-6Bac+B\,b^{2}\right)\_R^{2}}{4ac-b^{2}}+\frac{a(2Ac-Bb)}{4ac-b^{2}}\right)\ln\left(x-\_R\right)}{2c\_R^{3}+\_Rb}}{4c}$
default	$\frac{-\frac{\left(Abc+2Bac-B\,b^{2}\right)x^{3}}{2c\left(4ac-b^{2}\right)}-\frac{a(2Ac-Bb)x}{2\left(4ac-b^{2}\right)c}}{c\,x^{4}+b\,x^{2}+a}+\frac{\left(-Abc\sqrt{-4ac+b^{2}}-4Aa\,c^{2}-A\,b^{2}c+6Bac\sqrt{-4ac+b^{2}}-B\,b^{2}\sqrt{-4ac+b^{2}}+8Babc-B\,b^{3}\right)\sqrt{2}\,\arctan\left(\frac{}{\sqrt{(}}\right.}{4c\sqrt{-4ac+b^{2}}\,\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}$

input
```
int(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*(A*b*c+2*B*a*c-B*b^2)/c/(4*a*c-b^2)*x^3-1/2*a*(2*A*c-B*b)/(4*a*c-b^2
)/c*x)/(c*x^4+b*x^2+a)+1/4/c*sum((-(A*b*c-6*B*a*c+B*b^2)/(4*a*c-b^2)*_R^2+
a*(2*A*c-B*b)/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*
b+a))
```

3.119.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4658 vs. $2(292) = 584$.

Time = 2.31 (sec) , antiderivative size = 4658, normalized size of antiderivative = 13.86

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
-1/4*(2*(B*b^2 - (2*B*a + A*b)*c)*x^3 + sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4
 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-(B^2*b^5 - 12*(4*A
*B*a^2 - A^2*a*b)*c^3 + (60*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c^2 - (15*
B^2*a*b^3 - 2*A*B*b^4)*c + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a
^3*c^6)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*
B^4*a^2 - 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3
)*c)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 1
2*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log(-(5*B^4*a*b^4 - 3*A*B^3*b^
5 - 4*A^4*a*c^4 + (20*A^3*B*a*b - 3*A^4*b^2)*c^3 + 3*(108*B^4*a^3 - 108*A*
B^3*a^2*b + 28*A^2*B^2*a*b^2 - 3*A^3*B*b^3)*c^2 - (81*B^4*a^2*b^2 - 65*A*B
^3*a*b^3 + 9*A^2*B^2*b^4)*c)*x + 1/2*sqrt(1/2)*(B^3*b^7 - 17*B^3*a*b^5*c -
 32*A^3*a^2*c^5 + 16*(18*A*B^2*a^3 - 3*A^2*B*a^2*b + A^3*a*b^2)*c^4 - 2*(7
2*B^3*a^3*b + 72*A*B^2*a^2*b^2 - 12*A^2*B*a*b^3 + A^3*b^4)*c^3 + (88*B^3*a
^2*b^3 + 18*A*B^2*a*b^4 - 3*A^2*B*b^5)*c^2 - (B*b^8*c^3 + 256*(3*B*a^4 - A
*a^3*b)*c^7 - 64*(10*B*a^3*b^2 - 3*A*a^2*b^3)*c^6 + 48*(4*B*a^2*b^4 - A*a*
b^5)*c^5 - 4*(6*B*a*b^6 - A*b^7)*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B
^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2
- 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^
8 - 64*a^3*c^9)))*sqrt(-(B^2*b^5 - 12*(4*A*B*a^2 - A^2*a*b)*c^3 + (60*B^2*
a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c^2 - (15*B^2*a*b^3 - 2*A*B*b^4)*c + (b...
```

3.119.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.119.7 Maxima [F]

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(Bx^2+A)x^4}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*((B*b^2 - (2*B*a + A*b)*c)*x^3 + (B*a*b - 2*A*a*c)*x)/((b^2*c^2 - 4*a
*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2) + 1/2*integrate
((B*a*b - 2*A*a*c + (B*b^2 - (6*B*a - A*b)*c)*x^2)/(c*x^4 + b*x^2 + a), x)
/(b^2*c - 4*a*c^2)
```

3.119.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4538 vs. 2(292) = 584.

Time = 1.40 (sec) , antiderivative size = 4538, normalized size of antiderivative = 13.51

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(B*b^2*x^3 - 2*B*a*c*x^3 - A*b*c*x^3 + B*a*b*x - 2*A*a*c*x)/((c*x^4 +
 b*x^2 + a)*(b^2*c - 4*a*c^2)) + 1/16*((2*b^3*c^3 - 8*a*b*c^4 - sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c + 4*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^3 - 2*(b^2 - 4*a*c)*b*c^3)*(b^2*c - 4*
a*c^2)^2*A + (2*b^4*c^2 - 20*a*b^2*c^3 + 48*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^2*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 12*(b^2 - 4*a*c)*a*c^3)*(b^2*c
- 4*a*c^2)^2*B - 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 8*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 2*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 2*a*b^4*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^3*c^5 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b
*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 + 16*a^2*b^2*c^5
- 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^6 - 32*a^3*c^6 + 2*(b...
```

3.119.9 Mupad [B] (verification not implemented)

Time = 10.98 (sec) , antiderivative size = 12396, normalized size of antiderivative = 36.89

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

output
```
- ((x^3*(A*b*c - B*b^2 + 2*B*a*c))/(2*c*(4*a*c - b^2)) + (x*(2*A*a*c - B*a
*b))/(2*c*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) - atan(((((2048*A*a^4*c^6 -
32*A*a*b^6*c^3 + 16*B*a*b^7*c^2 - 1024*B*a^4*b*c^5 + 384*A*a^2*b^4*c^4 - 1
536*A*a^3*b^2*c^5 - 192*B*a^2*b^5*c^3 + 768*B*a^3*b^3*c^4)/(8*(b^6*c - 64*
a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) - (x*(-(B^2*b^11 + A^2*b^9*c^2 +
 A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A
*B*b^10*c - 96*A^2*a^2*b^5*c^4 + 512*A^2*a^3*b^3*c^5 + 288*B^2*a^2*b^7*c^2
 - 1504*B^2*a^3*b^5*c^3 + 3840*B^2*a^4*b^3*c^4 + 3072*A*B*a^5*c^6 - 27*B^2
*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 768*A^2*a^4*b*c^6 - 3840*B
^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c^3 - 128*A*B*a^3*b^4*c^4 - 1536*A*B*a^4*b^
2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a*b^8*c^2)/(32*(4096*a
^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3
840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 -
1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))
*(-(B^2*b^11 + A^2*b^9*c^2 + A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*b^2*(-
(4*a*c - b^2)^9)^(1/2) + 2*A*B*b^10*c - 96*A^2*a^2*b^5*c^4 + 512*A^2*a^3*b
^3*c^5 + 288*B^2*a^2*b^7*c^2 - 1504*B^2*a^3*b^5*c^3 + 3840*B^2*a^4*b^3*c^4
 + 3072*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2)
- 768*A^2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c^3 - 128*A*B*a
^3*b^4*c^4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) ...
```

3.119. $\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$

3.120 $\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$

3.120.1 Optimal result

Integrand size = 25, antiderivative size = 276

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = -\frac{x(Ab-2aB-(bB-2Ac)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\left(bB-2Ac-\frac{b^2B-4Abc+4aBc}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(bB-2Ac+\frac{b^2B-4Abc+4aBc}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
-1/2*x*(A*b-2*B*a-(-2*A*c+B*b)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*arcta
n(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(B*b-2*A*c+(4*A*b*c-4*B*
a*c-B*b^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2
)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*
(B*b-2*A*c+(-4*A*b*c+4*B*a*c+B*b^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)*2^(1/
2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.120.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.08

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \frac{1}{4}\left(\frac{2x(B(2a+bx^2)-A(b+2cx^2))}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\left(-b^2B+4Abc-4aBc+bB\sqrt{b^2-4ac}-2Ac\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(b^2B-4Abc+4aBc+bB\sqrt{b^2-4ac}-2Ac\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}\right)$$

input
```
Integrate[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((2*x*(B*(2*a + b*x^2) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x
^4)) + (Sqrt[2]*(-(b^2*B) + 4*A*b*c - 4*a*B*c + b*B*Sqrt[b^2 - 4*a*c] - 2*
A*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*
c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]
*(b^2*B - 4*A*b*c + 4*a*B*c + b*B*Sqrt[b^2 - 4*a*c] - 2*A*c*Sqrt[b^2 - 4*a
*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^
2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/4
```

3.120.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1598, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \; 1598$$

$$\frac{\int \frac{(bB-2Ac)x^2+Ab-2aB}{cx^4+bx^2+a}dx}{2(b^2-4ac)}-\frac{x\left(-2aB-\left(x^2(bB-2Ac)\right)+Ab\right)}{2(b^2-4ac)(a+bx^2+cx^4)}$$

$$\Big\downarrow \; 1480$$

$$\frac{\frac{1}{2}\left(-\frac{4aBc-4Abc+b^2B}{\sqrt{b^2-4ac}}-2Ac+bB\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(\frac{4aBc-4Abc+b^2B}{\sqrt{b^2-4ac}}-2Ac+bB\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{2(b^2-4ac)}-\frac{x\left(-2aB-\left(x^2(bB-2Ac)\right)+Ab\right)}{2(b^2-4ac)(a+bx^2+cx^4)}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\left(-\frac{4aBc-4Abc+b^2B}{\sqrt{b^2-4ac}}-2Ac+bB\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{4aBc-4Abc+b^2B}{\sqrt{b^2-4ac}}-2Ac+bB\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{2(b^2-4ac)}-\frac{x\left(-2aB-\left(x^2(bB-2Ac)\right)+Ab\right)}{2(b^2-4ac)(a+bx^2+cx^4)}$$

input
```
Int[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
-1/2*(x*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x
^4)) + (((b*B - 2*A*c - (b^2*B - 4*A*b*c + 4*a*B*c)/Sqrt[b^2 - 4*a*c])*Arc
Tan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqr
t[b - Sqrt[b^2 - 4*a*c]]) + ((b*B - 2*A*c + (b^2*B - 4*A*b*c + 4*a*B*c)/Sq
rt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/
(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*(b^2 - 4*a*c))
```

3.120.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

3.120. $\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$

rule 1598
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Simp[f
^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1
)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
 GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.120.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.55

method	result
risch	$\frac{\frac{(2Ac-Bb)x^3}{8ac-2b^2}+\frac{(Ab-2Ba)x}{8ac-2b^2}}{c\,x^4+b\,x^2+a}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(\frac{(2Ac-Bb)\_R^2}{4ac-b^2}-\frac{Ab-2Ba}{4ac-b^2}\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}\right)}{4}$
default	$\frac{\frac{(2Ac-Bb)x^3}{8ac-2b^2}+\frac{(Ab-2Ba)x}{8ac-2b^2}}{c\,x^4+b\,x^2+a}+\frac{2c\left(\frac{\left(2Ac\sqrt{-4ac+b^2}+4Abc-Bb\sqrt{-4ac+b^2}-4Bac-B\,b^2\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8c\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(2Ac\sqrt{-4ac+b^2}-\right.}{}\right.}{4ac-b^2}$

input
```
int(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/2*(2*A*c-B*b)/(4*a*c-b^2)*x^3+1/2*(A*b-2*B*a)/(4*a*c-b^2)*x)/(c*x^4+b*x
^2+a)+1/4*sum(((2*A*c-B*b)/(4*a*c-b^2)*_R^2-(A*b-2*B*a)/(4*a*c-b^2))/(2*_R
^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

3.120.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3467 vs. 2(234) = 468.

Time = 1.16 (sec) , antiderivative size = 3467, normalized size of antiderivative = 12.56

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/4*(2*(B*b - 2*A*c)*x^3 - sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^
2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3*A^2*a*b)*c^
2 + (12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c + (a*b^6*c - 12*a^2*b^4*c^2
+ 48*a^3*b^2*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a
^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b^6*c - 12
*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))*log(-(3*B^4*a^2*b^2 - A*B^3*a
*b^3 - 4*A^4*a*c^3 + 3*(4*A^3*B*a*b - A^4*b^2)*c^2 + (4*B^4*a^3 - 12*A*B^3
*a^2*b + A^3*B*b^3)*c)*x + 1/2*sqrt(1/2)*(2*B^3*a^2*b^4 - A*B^2*a*b^5 - 16
*(2*A^2*B*a^3 - A^3*a^2*b)*c^3 + 8*(4*B^3*a^4 - 2*A*B^2*a^3*b + 2*A^2*B*a^
2*b^2 - A^3*a*b^3)*c^2 - (16*B^3*a^3*b^2 - 8*A*B^2*a^2*b^3 + 2*A^2*B*a*b^4
 - A^3*b^5)*c + (192*B*a^4*b^3*c^3 + 256*A*a^5*c^5 - 128*(2*B*a^5*b + A*a^
4*b^2)*c^4 - 8*(6*B*a^3*b^5 - A*a^2*b^6)*c^2 + (4*B*a^2*b^7 - A*a*b^8)*c)*
sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 4
8*a^4*b^2*c^4 - 64*a^5*c^5)))*sqrt(-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3*A^2*a*b)
*c^2 + (12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c + (a*b^6*c - 12*a^2*b^4*c
^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)
/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b^6*c -
 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))) + sqrt(1/2)*((b^2*c - 4*a
*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(B^2*a*b^3 - 4*(4
*A*B*a^2 - 3*A^2*a*b)*c^2 + (12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c +...
```

3.120.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.120.7 Maxima [F]

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(Bx^2+A)x^2}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*((B*b - 2*A*c)*x^3 + (2*B*a - A*b)*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 -
 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate(-((B*b - 2*A*c)*x^2 - 2*B*
a + A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

3.120.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3776 vs. 2(234) = 468.

Time = 1.14 (sec) , antiderivative size = 3776, normalized size of antiderivative = 13.68

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(B*b*x^3 - 2*A*c*x^3 + 2*B*a*x - A*b*x)/((c*x^4 + b*x^2 + a)*(b^2 - 4*
a*c)) + 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^2 - 8*a*b*c^3 -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(b^2 -
 4*a*c)^2*B + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c^3 - 4*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*(b^2 - 4*a*c)*b^3*
c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) - 4*(sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*a*b^4
*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*s...
```

3.120.9 Mupad [B] (verification not implemented)

Time = 10.41 (sec) , antiderivative size = 9444, normalized size of antiderivative = 34.22

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

output
```
((x*(A*b - 2*B*a))/(2*(4*a*c - b^2)) + (x^3*(2*A*c - B*b))/(2*(4*a*c - b^2
)))/(a + b*x^2 + c*x^4) - atan(((((16*A*b^7*c^2 + 2048*B*a^4*c^5 - 192*A*a
*b^5*c^3 - 1024*A*a^3*b*c^5 - 32*B*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*B*a
^2*b^4*c^3 - 1536*B*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 1
2*a*b^4*c)) - (x*(-(B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c
 + A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c
^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2
*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3
 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 -
 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2
)*(16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4
 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^9)^(1/2)
 + A^2*b^9*c + A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A
^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c
^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B
*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a
^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^
12*c)))^(1/2) - (x*(B^2*b^4*c - 8*A^2*a*c^4 + 10*A^2*b^2*c^3 + 8*B^2*a^2*c
^3 - 6*A*B*b^3*c^2 + 2*B^2*a*b^2*c^2 - 8*A*B*a*b*c^3))/(2*(b^4 + 16*a^2*c^
2 - 8*a*b^2*c)))*(-(B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^...
```

3.120. $\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$

3.121 $\int \frac{A+Bx^2}{(a+bx^2+cx^4)^2} dx$

3.121.1 Optimal result

Integrand size = 22, antiderivative size = 293

$$\int \frac{A+Bx^2}{(a+bx^2+cx^4)^2} dx = \frac{x(Ab^2-abB-2aAc+(Ab-2aB)cx^2)}{2a(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{c}\left(Ab-2aB+\frac{4abB+A(b^2-12ac)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{c}\left(Ab-2aB-\frac{Ab^2+4abB-12aAc}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
1/2*x*(A*b^2-a*b*B-2*A*a*c+(A*b-2*B*a)*c*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+
a)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(A*b
-2*B*a+(4*a*b*B+A*(-12*a*c+b^2))/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)*2^(1/2
)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2
)^(1/2))^(1/2))*c^(1/2)*(A*b-2*B*a+(12*A*a*c-A*b^2-4*B*a*b)/(-4*a*c+b^2)^(
1/2))/a/(-4*a*c+b^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.121.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^2}{(a+bx^2+cx^4)^2}\,dx$$
$$= \frac{\frac{2x(-aB(b+2cx^2)+A(b^2-2ac+bcx^2))}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\sqrt{c}\left(-2aB\left(-2b+\sqrt{b^2-4ac}\right)+A\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(-2a\right.}{}}{4a}$$

input
```
Integrate[(A + B*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((2*x*(-(a*B*(b + 2*c*x^2)) + A*(b^2 - 2*a*c + b*c*x^2)))/((b^2 - 4*a*c)*(
a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(-2*a*B*(-2*b + Sqrt[b^2 - 4*a*c])
+ A*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
 + (Sqrt[2]*Sqrt[c]*(-2*a*B*(2*b + Sqrt[b^2 - 4*a*c]) + A*(-b^2 + 12*a*c +
 b*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*
c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a)
```

3.121.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1492, 25, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{(a+bx^2+cx^4)^2}\,dx$$
$$\Big\downarrow \; 1492$$
$$\frac{x\left(cx^2(Ab-2aB)-2aAc-abB+Ab^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{Ab^2+aBb+(Ab-2aB)cx^2-6aAc}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}$$
$$\Big\downarrow \; 25$$
$$\frac{\int \frac{(Ab-2aB)cx^2+abB+A(b^2-6ac)}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}+\frac{x\left(cx^2(Ab-2aB)-2aAc-abB+Ab^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \text{1480}$$

$$\frac{\frac{1}{2}c\left(\frac{A(b^2-12ac)+4abB}{\sqrt{b^2-4ac}}-2aB+Ab\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}c\left(-\frac{-12aAc+4abB+Ab^2}{\sqrt{b^2-4ac}}-2aB+Ab\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{2a\left(b^2-4ac\right)}+$$
$$\frac{x\left(cx^2(Ab-2aB)-2aAc-abB+Ab^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\sqrt{c}\left(\frac{A\left(b^2-12ac\right)+4abB}{\sqrt{b^2-4ac}}-2aB+Ab\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{-12aAc+4abB+Ab^2}{\sqrt{b^2-4ac}}-2aB+Ab\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{2a\left(b^2-4ac\right)}+$$
$$\frac{x\left(cx^2(Ab-2aB)-2aAc-abB+Ab^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

input
```
Int[(A + B*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(x*(A*b^2 - a*b*B - 2*a*A*c + (A*b - 2*a*B)*c*x^2))/(2*a*(b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)) + ((Sqrt[c]*(A*b - 2*a*B + (4*a*b*B + A*(b^2 - 12*a*c))/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(A*b - 2*a*B - (A*b^2 +
 4*a*b*B - 12*a*A*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
+ Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*a*(b^2 -
4*a*c))
```

3.121.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

3.121.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.18 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.61

method	result
risch	$\frac{-\frac{c(Ab-2Ba)x^3}{2a\left(4ac-b^2\right)}+\frac{\left(2Aac-A\,b^2+abB\right)x}{2\left(4ac-b^2\right)a}}{c\,x^4+b\,x^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(-\frac{c(Ab-2Ba)\_R^2}{4ac-b^2}+\frac{6Aac-A\,b^2-abB}{4ac-b^2}\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}}{4a}$
default	$16c^2\left(-\frac{-\frac{\left(-A\sqrt{-4ac+b^2}-Ab+2Ba\right)\sqrt{-4ac+b^2}\,x}{16ac\left(x^2+\frac{b}{2c}-\frac{\sqrt{-4ac+b^2}}{2c}\right)}-\frac{\left(12A\sqrt{-4ac+b^2}\,ac-3A\sqrt{-4ac+b^2}\,b^2+28Aabc-3A\,b^3-8a^2Bc-6Ba\,b^2\right)\left(\sqrt{-4ac+b^2}-2\right.}{16a\left(4ac+3b^2\right)\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}}{4c(4ac-b^2)\sqrt{-4ac+b^2}}\right.$

input
```
int((B*x^2+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*c*(A*b-2*B*a)/a/(4*a*c-b^2)*x^3+1/2*(2*A*a*c-A*b^2+B*a*b)/(4*a*c-b^2
)/a*x)/(c*x^4+b*x^2+a)+1/4/a*sum((-c*(A*b-2*B*a)/(4*a*c-b^2)*_R^2+(6*A*a*c
-A*b^2-B*a*b)/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*
b+a))
```

3.121.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4885 vs. $2(254) = 508$.

Time = 3.28 (sec) , antiderivative size = 4885, normalized size of antiderivative = 16.67

$$\int \frac{A+Bx^2}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
-1/4*(2*(2*B*a - A*b)*c*x^3 - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b
^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(B^2*a^2*b^3 + 2*A*B*a*b^4 +
 A^2*b^5 - 12*(4*A*B*a^3 - 5*A^2*a^2*b)*c^2 + 3*(4*B^2*a^3*b - 4*A*B*a^2*b
^2 - 5*A^2*a*b^3)*c + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^
3)*sqrt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^4
*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a
^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b
^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((324*A^4*a^2*c^4 - 81*(4*A^3*B*a^
2*b + A^4*a*b^2)*c^3 - (4*B^4*a^4 - 20*A*B^3*a^3*b - 84*A^2*B^2*a^2*b^2 -
65*A^3*B*a*b^3 - 5*A^4*b^4)*c^2 - 3*(B^4*a^3*b^2 + 3*A*B^3*a^2*b^3 + 3*A^2
*B^2*a*b^4 + A^3*B*b^5)*c)*x + 1/2*sqrt(1/2)*(B^3*a^3*b^5 + 3*A*B^2*a^2*b^
6 + 3*A^2*B*a*b^7 + A^3*b^8 + 864*A^3*a^4*c^4 - 48*(2*A*B^2*a^5 + 7*A^2*B*
a^4*b + 14*A^3*a^3*b^2)*c^3 + 2*(8*B^3*a^5*b + 48*A*B^2*a^4*b^2 + 108*A^2*
B*a^3*b^3 + 95*A^3*a^2*b^4)*c^2 - (8*B^3*a^4*b^3 + 30*A*B^2*a^3*b^4 + 45*A
^2*B*a^2*b^5 + 23*A^3*a*b^6)*c - (B*a^4*b^8 + A*a^3*b^9 + 144*A*a^5*b^5*c^
2 - 256*(B*a^8 - 2*A*a^7*b)*c^4 + 64*(2*B*a^7*b^2 - 7*A*a^6*b^3)*c^3 - 4*(
2*B*a^5*b^6 + 5*A*a^4*b^7)*c)*sqrt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^
2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3
*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9
*c^3)))*sqrt(-(B^2*a^2*b^3 + 2*A*B*a*b^4 + A^2*b^5 - 12*(4*A*B*a^3 - 5*...
```

3.121.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.121.7 Maxima [F]

$$\int \frac{A+Bx^2}{(a+bx^2+cx^4)^2}\,dx=\int \frac{Bx^2+A}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate((B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*((2*B*a - A*b)*c*x^3 + (B*a*b - A*b^2 + 2*A*a*c)*x)/((a*b^2*c - 4*a^2
*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) + 1/2*integrate(-
((2*B*a - A*b)*c*x^2 - B*a*b - A*b^2 + 6*A*a*c)/(c*x^4 + b*x^2 + a), x)/(a
*b^2 - 4*a^2*c)
```

3.121.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4426 vs. 2(254) = 508.

Time = 1.38 (sec) , antiderivative size = 4426, normalized size of antiderivative = 15.11

$$\int \frac{A+Bx^2}{(a+bx^2+cx^4)^2}\,dx=\text{Too large to display}$$

input
```
integrate((B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(2*B*a*c*x^3 - A*b*c*x^3 + B*a*b*x - A*b^2*x + 2*A*a*c*x)/((c*x^4 + b
*x^2 + a)*(a*b^2 - 4*a^2*c)) + 1/16*((2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(a*b^2 - 4*a^2*c)^2
*A - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*
c^2 - 2*(b^2 - 4*a*c)*a*c^2)*(a*b^2 - 4*a^2*c)^2*B + 2*(sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*a*b^6*c +
 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b^4*c^2 + 28*a^2*b^4*c^2 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^4*c^3 - 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 10*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 128*a^3*b^2*c^3 + 24*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 192*a^4*c^4 + 2*(b^2 - 4
*a*c)*a*b^4*c - 20*(b^2 - 4*a*c)*a^2*b^2*c^2 + 48*(b^2 - 4*a*c)*a^3*c^3...
```

3.121.9 Mupad [B] (verification not implemented)

Time = 10.67 (sec) , antiderivative size = 12349, normalized size of antiderivative = 42.15

$$\int \frac{A+Bx^2}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^2)/(a + b*x^2 + c*x^4)^2,x)
```

output
```
atan(((((6144*A*a^5*c^6 + 16*A*a*b^8*c^2 - 1024*B*a^5*b*c^5 - 288*A*a^2*b^
6*c^3 + 1920*A*a^3*b^4*c^4 - 5632*A*a^4*b^2*c^5 + 16*B*a^2*b^7*c^2 - 192*B
*a^3*b^5*c^3 + 768*B*a^4*b^3*c^4)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c
+ 48*a^4*b^2*c^2)) - (x*(-(A^2*b^11 + B^2*a^2*b^9 + A^2*b^2*(-(4*a*c - b^2
)^9)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*a*b^10 + 288*A^2*a^2
*b^7*c^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2*a^4*b^3*c^4 - 96*B^2*a^4*b^5*c^
2 + 512*B^2*a^5*b^3*c^3 + 3072*A*B*a^6*c^5 - 27*A^2*a*b^9*c - 9*A^2*a*c*(-
(4*a*c - b^2)^9)^(1/2) - 3840*A^2*a^5*b*c^5 - 768*B^2*a^6*b*c^4 + 192*A*B*
a^3*b^6*c^2 - 128*A*B*a^4*b^4*c^3 - 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*
a*c - b^2)^9)^(1/2) - 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*
a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*
a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 -
768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(A^2*b^11 +
B^2*a^2*b^9 + A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^9
)^(1/2) + 2*A*B*a*b^10 + 288*A^2*a^2*b^7*c^2 - 1504*A^2*a^3*b^5*c^3 + 3840
*A^2*a^4*b^3*c^4 - 96*B^2*a^4*b^5*c^2 + 512*B^2*a^5*b^3*c^3 + 3072*A*B*a^6
*c^5 - 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 3840*A^2*a^5*
b*c^5 - 768*B^2*a^6*b*c^4 + 192*A*B*a^3*b^6*c^2 - 128*A*B*a^4*b^4*c^3 - 15
36*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a^2*b^8*c
)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280...
```

3.121. $\int \frac{A+Bx^2}{(a+bx^2+cx^4)^2}\,dx$

3.122 $\int \frac{A+Bx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx$

3.122.1 Optimal result

Integrand size = 25, antiderivative size = 389

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = -\frac{3Ab^2-abB-10aAc}{2a^2\left(b^2-4ac\right)x} - \frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)}$$
$$+\frac{\sqrt{c}\left(aB\left(b^2-12ac+b\sqrt{b^2-4ac}\right)-A\left(3b^3-16abc+3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{\cdots}}{\sqrt{b-\sqrt{b^2\cdots}}}\right)}{2\sqrt{2}a^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$-\frac{\sqrt{c}\left(3Ab^2-abB-10aAc+\frac{aB(b^2-12ac)-A(3b^3-16abc)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\left(b^2-4ac\right)\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
1/2*(10*A*a*c-3*A*b^2+B*a*b)/a^2/(-4*a*c+b^2)/x+1/2*(-a*b*B+A*(-2*a*c+b^2)
+(A*b-2*B*a)*c*x^2)/a/(-4*a*c+b^2)/x/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/2)*
c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(a*B*(b^2-12*a*c+b*(-4*a*c+b
^2)^(1/2))-A*(3*b^3-16*a*b*c+3*b^2*(-4*a*c+b^2)^(1/2)-10*a*c*(-4*a*c+b^2)^
(1/2)))/a^2/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*ar
ctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*A*b^2-a*b*
B-10*A*a*c+(a*B*(-12*a*c+b^2)-A*(-16*a*b*c+3*b^3))/(-4*a*c+b^2)^(1/2))/a^2
/(-4*a*c+b^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.122.2 Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 382, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx$$
$$= \frac{-\frac{4A}{x}+\frac{2x\left(aB\left(b^2-2ac+bcx^2\right)-A\left(b^3-3abc+b^2cx^2-2ac^2x^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\sqrt{c}\left(aB\left(b^2-12ac+b\sqrt{b^2-4ac}\right)+A\left(-3b^3+16abc-3b^2\sqrt{b^2-4ac}+10ac\sqrt{\right.\right.}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}}{}$$

input
```
Integrate[(A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

output
```
((-4*A)/x + (2*x*(a*B*(b^2 - 2*a*c + b*c*x^2) - A*(b^3 - 3*a*b*c + b^2*c*x
^2 - 2*a*c^2*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]
*(a*B*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c]) + A*(-3*b^3 + 16*a*b*c - 3*b^2*
Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a
*c]]) + (Sqrt[2]*Sqrt[c]*(a*B*(-b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c]) + A*(3
*b^3 - 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c]))*Arc
Tan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)
*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a^2)
```

3.122.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 379, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1600, 25, 1604, 25, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx$$
$$\Big\downarrow \text{1600}$$
$$-\frac{\int -\frac{3Ab^2-aBb+3(Ab-2aB)cx^2-10aAc}{x^2\left(cx^4+bx^2+a\right)}dx}{2a\left(b^2-4ac\right)}-\frac{-A\left(b^2-2ac\right)-\left(cx^2(Ab-2aB)\right)+abB}{2ax\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$
$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{3Ab^2-aBb+3(Ab-2aB)cx^2-10aAc}{x^2(cx^4+bx^2+a)}dx}{2a(b^2-4ac)}-\frac{-A(b^2-2ac)-(cx^2(Ab-2aB))+abB}{2ax(b^2-4ac)(a+bx^2+cx^4)}$$

$\downarrow$ 1604

$$\frac{-\frac{\int -\frac{-c(3Ab^2-aBb-10aAc)x^2+aB(b^2-6ac)-A(3b^3-13abc)}{cx^4+bx^2+a}dx}{a}-\frac{-10aAc-abB+3Ab^2}{ax}}{2a(b^2-4ac)}-\frac{-A(b^2-2ac)-(cx^2(Ab-2aB))+abB}{2ax(b^2-4ac)(a+bx^2+cx^4)}$$

$\downarrow$ 25

$$\frac{\frac{\int \frac{-c(3Ab^2-aBb-10aAc)x^2+aB(b^2-6ac)-A(3b^3-13abc)}{cx^4+bx^2+a}dx}{a}-\frac{-10aAc-abB+3Ab^2}{ax}}{2a(b^2-4ac)}-\frac{-A(b^2-2ac)-(cx^2(Ab-2aB))+abB}{2ax(b^2-4ac)(a+bx^2+cx^4)}$$

$\downarrow$ 1480

$$\frac{\frac{\frac{c\left(aB\left(b\sqrt{b^2-4ac}-12ac+b^2\right)-A\left(3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}-16abc+3b^3\right)\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx}{2\sqrt{b^2-4ac}}-\frac{1}{2}c\Big(\frac{aB(b^2-12ac)-A(3b^3-16abc)}{\sqrt{b^2-4ac}}-10aAc-abB+}{a}}{2a(b^2-4ac)}$$

$$-\frac{-A(b^2-2ac)-(cx^2(Ab-2aB))+abB}{2ax(b^2-4ac)(a+bx^2+cx^4)}$$

$\downarrow$ 218

$$\frac{\frac{\frac{\sqrt{c}\left(aB\left(b\sqrt{b^2-4ac}-12ac+b^2\right)-A\left(3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}-16abc+3b^3\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\Big(\frac{aB(b^2-12ac)-A(3b^3-16abc)}{\sqrt{b^2-4ac}}-10aAc-abB+3A}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{a}}{2a(b^2-4ac)}$$

$$-\frac{-A(b^2-2ac)-(cx^2(Ab-2aB))+abB}{2ax(b^2-4ac)(a+bx^2+cx^4)}$$

input
```
Int[(A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

3.122. $\int \frac{A+Bx^2}{x^2(a+bx^2+cx^4)^2}\,dx$

output
```
-1/2*(a*b*B - A*(b^2 - 2*a*c) - (A*b - 2*a*B)*c*x^2)/(a*(b^2 - 4*a*c)*x*(a
 + b*x^2 + c*x^4)) + (-((3*A*b^2 - a*b*B - 10*a*A*c)/(a*x)) + ((Sqrt[c]*(a
*B*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c]) - A*(3*b^3 - 16*a*b*c + 3*b^2*Sqrt
[b^2 - 4*a*c] - 10*a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4
*a*c]]) - (Sqrt[c]*(3*A*b^2 - a*b*B - 10*a*A*c + (a*B*(b^2 - 12*a*c) - A*(
3*b^3 - 16*a*b*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/a)/(2*a*(b^2 -
 4*a*c))
```

3.122.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1600
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^2 + c*x^4)^(p + 1)
*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2)/(2*a*f*(p + 1)*(b^2 - 4*a
*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^m*(a + b*x^2 + c
*x^4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1))
- a*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Int
egerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.122.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 379, normalized size of antiderivative = 0.97

method	result
default	$-\frac{A}{a^2 x}-\frac{\frac{c\left(2Aac-A\,b^2+abB\right)x^3}{8ac-2b^2}+\frac{\left(3Aabc-A\,b^3-2a^2Bc+Ba\,b^2\right)x}{8ac-2b^2}}{c\,x^4+b\,x^2+a}+\frac{2c\left(\frac{\left(10A\sqrt{-4ac+b^2}\,ac-3A\sqrt{-4ac+b^2}\,b^2-16Aabc+3A\,b^3+abB\sqrt{-4ac+\cdots}\right)}{8\sqrt{-4ac+b^2}\sqrt{\left(b+\sqrt{-\cdots}\right)}}\right)}{\cdots}$
risch	Expression too large to display

input
```
int((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-A/a^2/x-1/a^2*((1/2*c*(2*A*a*c-A*b^2+B*a*b)/(4*a*c-b^2)*x^3+1/2*(3*A*a*b*
c-A*b^3-2*B*a^2*c+B*a*b^2)/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*c*
(1/8*(10*A*(-4*a*c+b^2)^(1/2)*a*c-3*A*(-4*a*c+b^2)^(1/2)*b^2-16*A*a*b*c+3*
A*b^3+a*b*B*(-4*a*c+b^2)^(1/2)+12*a^2*B*c-B*a*b^2)/(-4*a*c+b^2)^(1/2)*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2))-1/8*(10*A*(-4*a*c+b^2)^(1/2)*a*c-3*A*(-4*a*c+b^2)^(1/2)*b^
2+16*A*a*b*c-3*A*b^3+a*b*B*(-4*a*c+b^2)^(1/2)-12*a^2*B*c+B*a*b^2)/(-4*a*c+
b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

3.122.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7583 vs. $2(338) = 676$.

Time = 8.89 (sec) , antiderivative size = 7583, normalized size of antiderivative = 19.49

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.122.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((B*x**2+A)/x**2/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.122.7 Maxima [F]

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = \int \frac{Bx^2+A}{\left(cx^4+bx^2+a\right)^2x^2}\,dx$$

input
```
integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*((10*A*a*c^2 + (B*a*b - 3*A*b^2)*c)*x^4 - 2*A*a*b^2 + 8*A*a^2*c + (B*a
*b^2 - 3*A*b^3 - (2*B*a^2 - 11*A*a*b)*c)*x^2)/((a^2*b^2*c - 4*a^3*c^2)*x^5
 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x) + 1/2*integrate((B*a
*b^2 - 3*A*b^3 + (10*A*a*c^2 + (B*a*b - 3*A*b^2)*c)*x^2 - (6*B*a^2 - 13*A*
a*b)*c)/(c*x^4 + b*x^2 + a), x)/(a^2*b^2 - 4*a^3*c)
```

3.122.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5408 vs. 2(338) = 676.

Time = 1.36 (sec) , antiderivative size = 5408, normalized size of antiderivative = 13.90

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(B*a*b*c*x^4 - 3*A*b^2*c*x^4 + 10*A*a*c^2*x^4 + B*a*b^2*x^2 - 3*A*b^3*
x^2 - 2*B*a^2*c*x^2 + 11*A*a*b*c*x^2 - 2*A*a*b^2 + 8*A*a^2*c)/((c*x^5 + b*
x^3 + a*x)*(a^2*b^2 - 4*a^3*c)) - 1/16*((6*b^4*c^2 - 44*a*b^2*c^3 + 80*a^2
*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 2
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 6*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 40*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 20*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 3*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 10*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 6*(b^2 - 4*a*c)*b^2*c^2 +
 20*(b^2 - 4*a*c)*a*c^3)*(a^2*b^2 - 4*a^3*c)^2*A - (2*a*b^3*c^2 - 8*a^2*b*
c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3 + 4*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c + 2*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c - sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*
a*b*c^2)*(a^2*b^2 - 4*a^3*c)^2*B + 2*(3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^2*b^7 - 37*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c - 6*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 6*a^2*b^7*c + 152*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 + 50*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*b^4*c^2 + 3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*...
```

3.122.9 Mupad [B] (verification not implemented)

Time = 11.09 (sec) , antiderivative size = 17591, normalized size of antiderivative = 45.22

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)^2),x)
```

output

```
- (A/a - (x^2*(3*A*b^3 - B*a*b^2 + 2*B*a^2*c - 11*A*a*b*c))/(2*a^2*(4*a*c
- b^2)) + (c*x^4*(10*A*a*c - 3*A*b^2 + B*a*b))/(2*a^2*(4*a*c - b^2)))/(a*x
 + b*x^3 + c*x^5) - atan((((-(9*A^2*b^13 + B^2*a^2*b^11 + 9*A^2*b^4*(-(4*a
*c - b^2)^9)^(1/2) - 6*A*B*a*b^12 + 2077*A^2*a^2*b^9*c^2 - 10656*A^2*a^3*b
^7*c^3 + 30240*A^2*a^4*b^5*c^4 - 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-
(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 288*B^2*a^
4*b^7*c^2 - 1504*B^2*a^5*b^5*c^3 + 3840*B^2*a^6*b^3*c^4 - 15360*A*B*a^7*c^
6 - 213*A^2*a*b^11*c + 26880*A^2*a^6*b*c^6 - 27*B^2*a^3*b^9*c - 3840*B^2*a
^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) - 1548*A*B*a^3*b^8*c^2 + 8
064*A*B*a^4*b^6*c^3 - 22400*A*B*a^5*b^4*c^4 + 30720*A*B*a^6*b^2*c^5 - 51*A
^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2)
 + 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*
b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3
+ 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(x*(-(9*A^2*b^13 + B^2*a^2
*b^11 + 9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^12 + 2077*A^2*a^2*b
^9*c^2 - 10656*A^2*a^3*b^7*c^3 + 30240*A^2*a^4*b^5*c^4 - 44800*A^2*a^5*b^3
*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^
2)^9)^(1/2) + 288*B^2*a^4*b^7*c^2 - 1504*B^2*a^5*b^5*c^3 + 3840*B^2*a^6*b^
3*c^4 - 15360*A*B*a^7*c^6 - 213*A^2*a*b^11*c + 26880*A^2*a^6*b*c^6 - 27*B^
2*a^3*b^9*c - 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2)...
```

3.122. $\int \frac{A+Bx^2}{x^2(a+bx^2+cx^4)^2}\,dx$

3.123 $\int \frac{A+Bx^2}{x^4(a+bx^2+cx^4)^2}\,dx$

3.123.1 Optimal result

Integrand size = 25, antiderivative size = 522

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2+cx^4\right)^2}\,dx=-\frac{5Ab^2-3abB-14aAc}{6a^2\left(b^2-4ac\right)x^3}$$
$$-\frac{aB(3b^2-10ac)-A(5b^3-19abc)}{2a^3\left(b^2-4ac\right)x}-\frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x^3\left(a+bx^2+cx^4\right)}$$
$$-\frac{\sqrt{c}\left(aB\left(3b^3-16abc+3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}\right)-A\left(5b^4-29ab^2c+28a^2c^2+5b^3\sqrt{b^2-4ac}-\right.\right.}{2\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt{c}\left(aB\left(3b^3-16abc-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}\right)-A\left(5b^4-29ab^2c+28a^2c^2-5b^3\sqrt{b^2-4ac}+\right.\right.}{2\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
1/6*(14*A*a*c-5*A*b^2+3*B*a*b)/a^2/(-4*a*c+b^2)/x^3+1/2*(-a*B*(-10*a*c+3*b
^2)+A*(-19*a*b*c+5*b^3))/a^3/(-4*a*c+b^2)/x+1/2*(-a*b*B+A*(-2*a*c+b^2)+(A*
b-2*B*a)*c*x^2)/a/(-4*a*c+b^2)/x^3/(c*x^4+b*x^2+a)-1/4*arctan(x*2^(1/2)*c^
(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(a*B*(3*b^3-16*a*b*c+3*b^2*(-4
*a*c+b^2)^(1/2)-10*a*c*(-4*a*c+b^2)^(1/2))-A*(5*b^4-29*a*b^2*c+28*a^2*c^2+
5*(-4*a*c+b^2)^(1/2)*b^3-19*(-4*a*c+b^2)^(1/2)*a*b*c))/a^3/(-4*a*c+b^2)^(3
/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(
-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(a*B*(3*b^3-16*a*b*c-3*b^2*(-4*a*c+b^2)^
(1/2)+10*a*c*(-4*a*c+b^2)^(1/2))-A*(5*b^4-29*a*b^2*c+28*a^2*c^2-5*(-4*a*c+
b^2)^(1/2)*b^3+19*(-4*a*c+b^2)^(1/2)*a*b*c))/a^3/(-4*a*c+b^2)^(3/2)*2^(1/2
)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.123.2 Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 487, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{-\frac{4aA}{x^3}+\frac{24Ab-12aB}{x}+\frac{6x\left(aB\left(-b^3+3abc-b^2cx^2+2ac^2x^2\right)+A\left(b^4-4ab^2c+2a^2c^2+b^3cx^2-3abc^2x^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{3\sqrt{2}\sqrt{c}\left(aB\left(-3b^3+16abc-3b^2\sqrt{b\right.\right.}{}}{}$$

input
```
Integrate[(A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)^2),x]
```

output
```
((-4*a*A)/x^3 + (24*A*b - 12*a*B)/x + (6*x*(a*B*(-b^3 + 3*a*b*c - b^2*c*x^
2 + 2*a*c^2*x^2) + A*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3*c*x^2 - 3*a*b*c^2*
x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (3*Sqrt[2]*Sqrt[c]*(a*B*(-3*b
^3 + 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c]) + A*(5
*b^4 - 29*a*b^2*c + 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b
^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b
^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (3*Sqrt[2]*Sqrt[c]*(a*B*(
-3*b^3 + 16*a*b*c + 3*b^2*Sqrt[b^2 - 4*a*c] - 10*a*c*Sqrt[b^2 - 4*a*c]) +
A*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 - 5*b^3*Sqrt[b^2 - 4*a*c] + 19*a*b*c*Sq
rt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])
/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(12*a^3)
```

3.123.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 518, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {1600, 25, 1604, 27, 25, 1604, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2+cx^4\right)^2}\,dx$$

$$\Big\downarrow \; 1600$$

$$-\frac{\int -\frac{5Ab^2-3aBb+5(Ab-2aB)cx^2-14aAc}{x^4\left(cx^4+bx^2+a\right)}dx}{2a\left(b^2-4ac\right)}-\frac{-A\left(b^2-2ac\right)-\left(cx^2(Ab-2aB)\right)+abB}{2ax^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{5Ab^2-3aBb+5(Ab-2aB)cx^2-14aAc}{x^4(cx^4+bx^2+a)}dx}{2a(b^2-4ac)}-\frac{-A(b^2-2ac)-(cx^2(Ab-2aB))+abB}{2ax^3(b^2-4ac)(a+bx^2+cx^4)}$$

$\Big\downarrow$ 1604

$$\frac{-\frac{\int \frac{3\left(5Ab^3-3aBb^2-19aAcb+c\left(5Ab^2-3aBb-14aAc\right)x^2+10a^2Bc\right)}{x^2\left(cx^4+bx^2+a\right)}dx}{3a}-\frac{-14aAc-3abB+5Ab^2}{3ax^3}}{2a(b^2-4ac)}-\frac{-A(b^2-2ac)-(cx^2(Ab-2aB))+abB}{2ax^3(b^2-4ac)(a+bx^2+cx^4)}$$

$\Big\downarrow$ 27

$$\frac{-\frac{\int -\frac{-c\left(5Ab^2-3aBb-14aAc\right)x^2+aB\left(3b^2-10ac\right)-A\left(5b^3-19abc\right)}{x^2\left(cx^4+bx^2+a\right)}dx}{a}-\frac{-14aAc-3abB+5Ab^2}{3ax^3}}{2a(b^2-4ac)}-\frac{-A(b^2-2ac)-(cx^2(Ab-2aB))+abB}{2ax^3(b^2-4ac)(a+bx^2+cx^4)}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{-c\left(5Ab^2-3aBb-14aAc\right)x^2+aB\left(3b^2-10ac\right)-A\left(5b^3-19abc\right)}{x^2\left(cx^4+bx^2+a\right)}dx}{a}-\frac{-14aAc-3abB+5Ab^2}{3ax^3}}{2a(b^2-4ac)}-\frac{-A(b^2-2ac)-(cx^2(Ab-2aB))+abB}{2ax^3(b^2-4ac)(a+bx^2+cx^4)}$$

$\Big\downarrow$ 1604

$$\frac{\frac{-\frac{\int \frac{c\left(aB\left(3b^2-10ac\right)-A\left(5b^3-19abc\right)\right)x^2+abB\left(3b^2-13ac\right)-A\left(5b^4-24acb^2+14a^2c^2\right)}{cx^4+bx^2+a}dx}{a}-\frac{aB\left(3b^2-10ac\right)-A\left(5b^3-19abc\right)}{ax}}{a}-\frac{-14aAc-3abB+5Ab^2}{3ax^3}}{2a(b^2-4ac)}-\frac{-A(b^2-2ac)-(cx^2(Ab-2aB))+abB}{2ax^3(b^2-4ac)(a+bx^2+cx^4)}$$

$\Big\downarrow$ 1480

$$-\frac{c\left(aB\left(3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}-16abc+3b^3\right)-A\left(28a^2c^2-29ab^2c-19abc\sqrt{b^2-4ac}+5b^3\sqrt{b^2-4ac}+5b^4\right)\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx}{2\sqrt{b^2-4ac}} - \ c\left(aB\left(-3b^2\sqrt{b^2-4a \right.\right.$$

$$a \qquad a$$

$$\frac{-A(b^2-2ac)-(cx^2(Ab-2aB))+abB}{2ax^3(b^2-4ac)(a+bx^2+cx^4)}$$

$\Big\downarrow$ 218

3.123. $\int \frac{A+Bx^2}{x^4(a+bx^2+cx^4)^2}\,dx$

$$-\frac{\frac{\sqrt{c}\left(aB\left(3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}-16abc+3b^3\right)-A\left(28a^2c^2-29ab^2c-19abc\sqrt{b^2-4ac}+5b^3\sqrt{b^2-4ac}+5b^4\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(aB\left(-3b^2\sqrt{b^2-4\right.\right.}{}}{a}}{a}$$

$$\frac{-A(b^2-2ac)-(cx^2(Ab-2aB))+abB}{2ax^3(b^2-4ac)(a+bx^2+cx^4)}$$

input
```
Int[(A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)^2),x]
```

output
```
-1/2*(a*b*B - A*(b^2 - 2*a*c) - (A*b - 2*a*B)*c*x^2)/(a*(b^2 - 4*a*c)*x^3*
(a + b*x^2 + c*x^4)) + (-1/3*(5*A*b^2 - 3*a*b*B - 14*a*A*c)/(a*x^3) + (-((
a*B*(3*b^2 - 10*a*c) - A*(5*b^3 - 19*a*b*c))/(a*x)) - ((Sqrt[c]*(a*B*(3*b^
3 - 16*a*b*c + 3*b^2*Sqrt[b^2 - 4*a*c] - 10*a*c*Sqrt[b^2 - 4*a*c]) - A*(5*
b^4 - 29*a*b^2*c + 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^
2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqr
t[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(a*B*(3*b^3
 - 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c]) - A*(5*b
^4 - 29*a*b^2*c + 28*a^2*c^2 - 5*b^3*Sqrt[b^2 - 4*a*c] + 19*a*b*c*Sqrt[b^2
 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt
[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/a)/a)/(2*a*(b^2 - 4*a*
c))
```

3.123.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1600
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^2 + c*x^4)^(p + 1)
*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2)/(2*a*f*(p + 1)*(b^2 - 4*a
*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^m*(a + b*x^2 + c
*x^4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1))
- a*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Int
egerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.123.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 484, normalized size of antiderivative = 0.93

method	result
default	$-\frac{A}{3a^2x^3}-\frac{-2Ab+Ba}{x\,a^3}-\frac{-\frac{c\left(3Aabc-A\,b^3-2a^2Bc+Ba\,b^2\right)x^3}{2\left(4ac-b^2\right)}+\frac{\left(2A\,a^2c^2-4Aa\,b^2c+A\,b^4+3a^2bBc-Ba\,b^3\right)x}{8ac-2b^2}}{c\,x^4+b\,x^2+a}+\frac{2c\left(\frac{\left(-19Aabc\sqrt{-4ac+b^2}+\right.}{}\right.}{}$
risch	Expression too large to display

input
```
int((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*A/a^2/x^3-(-2*A*b+B*a)/x/a^3-1/a^3*((-1/2*c*(3*A*a*b*c-A*b^3-2*B*a^2*
c+B*a*b^2)/(4*a*c-b^2)*x^3+1/2*(2*A*a^2*c^2-4*A*a*b^2*c+A*b^4+3*B*a^2*b*c-
B*a*b^3)/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*c*(1/8*(-19*A*a*b*c*
(-4*a*c+b^2)^(1/2)+5*A*b^3*(-4*a*c+b^2)^(1/2)-28*A*a^2*c^2+29*A*a*b^2*c-5*
A*b^4+10*a^2*B*c*(-4*a*c+b^2)^(1/2)-3*B*a*b^2*(-4*a*c+b^2)^(1/2)-16*a^2*b*
B*c+3*B*a*b^3)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/8*(-19*A*a*b*c*(-4
*a*c+b^2)^(1/2)+5*A*b^3*(-4*a*c+b^2)^(1/2)+28*A*a^2*c^2-29*A*a*b^2*c+5*A*b
^4+10*a^2*B*c*(-4*a*c+b^2)^(1/2)-3*B*a*b^2*(-4*a*c+b^2)^(1/2)+16*a^2*b*B*c
-3*B*a*b^3)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

3.123.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10190 vs. $2(460) = 920$.

Time = 21.40 (sec) , antiderivative size = 10190, normalized size of antiderivative = 19.52

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.123.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2+cx^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((B*x**2+A)/x**4/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.123.7 Maxima [F]

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2+cx^4\right)^2}\,dx=\int \frac{Bx^2+A}{\left(cx^4+bx^2+a\right)^2x^4}\,dx$$

input
```
integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/6*(3*((10*B*a^2 - 19*A*a*b)*c^2 - (3*B*a*b^2 - 5*A*b^3)*c)*x^6 - 2*A*a^2
*b^2 + 8*A*a^3*c - (9*B*a*b^3 - 15*A*b^4 - 14*A*a^2*c^2 - (33*B*a^2*b - 62
*A*a*b^2)*c)*x^4 - 2*(3*B*a^2*b^2 - 5*A*a*b^3 - 4*(3*B*a^3 - 5*A*a^2*b)*c)
*x^2)/((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2
- 4*a^5*c)*x^3) - 1/2*integrate((3*B*a*b^3 - 5*A*b^4 - 14*A*a^2*c^2 - ((10
*B*a^2 - 19*A*a*b)*c^2 - (3*B*a*b^2 - 5*A*b^3)*c)*x^2 - (13*B*a^2*b - 24*A
*a*b^2)*c)/(c*x^4 + b*x^2 + a), x)/(a^3*b^2 - 4*a^4*c)
```

3.123.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6327 vs. 2(460) = 920.

Time = 1.49 (sec) , antiderivative size = 6327, normalized size of antiderivative = 12.12

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2+cx^4\right)^2}\,dx=\text{Too large to display}$$

input
```
integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(B*a*b^2*c*x^3 - A*b^3*c*x^3 - 2*B*a^2*c^2*x^3 + 3*A*a*b*c^2*x^3 + B*
a*b^3*x - A*b^4*x - 3*B*a^2*b*c*x + 4*A*a*b^2*c*x - 2*A*a^2*c^2*x)/((a^3*b
^2 - 4*a^4*c)*(c*x^4 + b*x^2 + a)) + 1/16*((10*b^5*c^2 - 78*a*b^3*c^3 + 15
2*a^2*b*c^4 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
b^5 + 39*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c
 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 76
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 38*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 5*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 19*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 10*(b^2 - 4
*a*c)*b^3*c^2 + 38*(b^2 - 4*a*c)*a*b*c^3)*(a^3*b^2 - 4*a^4*c)^2*A - (6*a*b
^4*c^2 - 44*a^2*b^2*c^3 + 80*a^3*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b^3*c - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^3*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*c^3 - 6*(b^2 - 4*a*c)*a*b^2*c^2 + 20*(b^2 - 4*a*c)*a^2*c^3)*(
a^3*b^2 - 4*a^4*c)^2*B + 2*(5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a...
```

3.123.9 Mupad [B] (verification not implemented)

Time = 11.36 (sec) , antiderivative size = 21554, normalized size of antiderivative = 41.29

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)^2),x)
```

output
```
- atan((((-(25*A^2*b^15 + 9*B^2*a^2*b^13 - 25*A^2*b^6*(-(4*a*c - b^2)^9)^(
1/2) - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116
928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 + 49
*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(
1/2) + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^
4 - 44800*B^2*a^7*b^3*c^5 - 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 3584
0*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*
c + 26880*B^2*a^8*b*c^6 - 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7
278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 20
1600*A*B*a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 + 165*A^2*a*b^4*c*(-(4*a*c -
 b^2)^9)^(1/2) + 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) + 30*A*B*a*b^5*
(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2*b^12*c - 184*A*B*a^2*b^3*c*(-(4*a*c
 - b^2)^9)^(1/2) + 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^
12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 +
 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(917504*A*a^19*c^9 + x*(-(
25*A^2*b^15 + 9*B^2*a^2*b^13 - 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*
B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*
b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 + 49*A^2*a^3*c^3
*(-(4*a*c - b^2)^9)^(1/2) - 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*
B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800...
```

3.123. $\int \frac{A+Bx^2}{x^4(a+bx^2+cx^4)^2}\,dx$

3.124 $\int \frac{x^{11}(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

3.124.1 Optimal result

Integrand size = 25, antiderivative size = 365

$$\begin{aligned}\int \frac{x^{11}(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx &= \frac{(3b^4B-Ab^3c-21ab^2Bc+7aAbc^2+30a^2Bc^2)\,x^2}{2c^3(b^2-4ac)^2}\\
&-\frac{x^8(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2)}{4c(b^2-4ac)(a+bx^2+cx^4)^2}\\
&-\frac{x^4(a(3b^3B-Ab^2c-18abBc+16aAc^2)+(3b^4B-Ab^3c-20ab^2Bc+10aAbc^2+20a^2Bc^2)\,x^2)}{4c^2(b^2-4ac)^2(a+bx^2+cx^4)}\\
&-\frac{(3b^6B-Ab^5c-30ab^4Bc+10aAb^3c^2+90a^2b^2Bc^2-30a^2Abc^3-60a^3Bc^3)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^4(b^2-4ac)^{5/2}}\\
&-\frac{(3bB-Ac)\log(a+bx^2+cx^4)}{4c^4}\end{aligned}$$

output
```
1/2*(7*A*a*b*c^2-A*b^3*c+30*B*a^2*c^2-21*B*a*b^2*c+3*B*b^4)*x^2/c^3/(-4*a*
c+b^2)^2-1/4*x^8*(a*(-2*A*c+B*b)+(-A*b*c-2*B*a*c+B*b^2)*x^2)/c/(-4*a*c+b^2
)/(c*x^4+b*x^2+a)^2-1/4*x^4*(a*(16*A*a*c^2-A*b^2*c-18*B*a*b*c+3*B*b^3)+(10
*A*a*b*c^2-A*b^3*c+20*B*a^2*c^2-20*B*a*b^2*c+3*B*b^4)*x^2)/c^2/(-4*a*c+b^2
)^2/(c*x^4+b*x^2+a)-1/2*(-30*A*a^2*b*c^3+10*A*a*b^3*c^2-A*b^5*c-60*B*a^3*c
^3+90*B*a^2*b^2*c^2-30*B*a*b^4*c+3*B*b^6)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)
^(1/2))/c^4/(-4*a*c+b^2)^(5/2)-1/4*(-A*c+3*B*b)*ln(c*x^4+b*x^2+a)/c^4
```

3.124.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.19

$$\int \frac{x^{11}(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$$

$$= \frac{2Bc^2x^2 + \frac{b^7B-b^6c(A+6Bx^2)+4a^3c^4(8A+9Bx^2)-3a^2b^2c^3(13A+34Bx^2)+ab^4c^2(11A+48Bx^2)+ab^3c^2(61aB-30Acx^2)+2b^5c(-7aB+2A\cdots}{(b^2-4ac)^2(a+bx^2+cx^4)}}{\cdots}$$

input
```
Integrate[(x^11*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

output
```
(2*B*c^2*x^2 + (b^7*B - b^6*c*(A + 6*B*x^2) + 4*a^3*c^4*(8*A + 9*B*x^2) -
3*a^2*b^2*c^3*(13*A + 34*B*x^2) + a*b^4*c^2*(11*A + 48*B*x^2) + a*b^3*c^2*
(61*a*B - 30*A*c*x^2) + 2*b^5*c*(-7*a*B + 2*A*c*x^2) + 2*a^2*b*c^3*(-39*a*
B + 25*A*c*x^2))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (b^5*(-(b*B) + A*
c)*x^2 + a^3*c^2*(-5*b*B + 2*c*(A + B*x^2)) + a*b^3*(-(b^2*B) - 5*A*c^2*x^
2 + b*c*(A + 6*B*x^2)) + a^2*b*c*(5*b^2*B + 5*A*c^2*x^2 - b*c*(4*A + 9*B*x
^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (2*c*(-3*b^6*B + A*b^5*c + 3
0*a*b^4*B*c - 10*a*A*b^3*c^2 - 90*a^2*b^2*B*c^2 + 30*a^2*A*b*c^3 + 60*a^3*
B*c^3)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2) + c*
(-3*b*B + A*c)*Log[a + b*x^2 + c*x^4])/(4*c^5)
```

3.124.3 Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1578, 1233, 1233, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{x^{10}(Bx^2+A)}{(cx^4+bx^2+a)^3}dx^2$$

$$\Big\downarrow \text{1233}$$

$$\frac{1}{2}\left(\frac{\int \frac{x^6((3Bb^2-Acb-10aBc)x^2+4a(bB-2Ac))}{(cx^4+bx^2+a)^2}dx^2}{2c(b^2-4ac)}-\frac{x^8\left(x^2\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{2c(b^2-4ac)(a+bx^2+cx^4)^2}\right)$$

$\downarrow$ 1233

$$\frac{1}{2}\left(\frac{\frac{\int \frac{2x^2\left(\left(3Bb^4-Acb^3-21aBcb^2+7aAc^2b+30a^2Bc^2\right)x^2+a\left(3Bb^3-Acb^2-18aBcb+16aAc^2\right)\right)}{cx^4+bx^2+a}dx^2}{c(b^2-4ac)}-\frac{x^4(x^2(20a^2Bc^2+10aAbc^2-20ab^2Bc-Ab^3c+3b^4 B}{c(b^2-4ac)(a+bx^2}}{2c(b^2-4ac)}\right.$$

$\downarrow$ 27

$$\frac{1}{2}\left(\frac{\frac{2\int \frac{x^2\left(\left(3Bb^4-Acb^3-21aBcb^2+7aAc^2b+30a^2Bc^2\right)x^2+a\left(3Bb^3-Acb^2-18aBcb+16aAc^2\right)\right)}{cx^4+bx^2+a}dx^2}{c(b^2-4ac)}-\frac{x^4(x^2(20a^2Bc^2+10aAbc^2-20ab^2Bc-Ab^3c+3b^4}{c(b^2-4ac)(a+bx}}{2c(b^2-4ac)}\right.$$

$\downarrow$ 1200

$$\frac{1}{2}\left(\frac{\frac{2\int\left(-3B\left(-\frac{b^4}{c}+7ab^2-10a^2c\right)-A(b^3-7abc)-\frac{(b^2-4ac)^2(3bB-Ac)x^2+a\left(3Bb^4-Acb^3-21aBcb^2+7aAc^2b+30a^2Bc^2\right)}{c\left(cx^4+bx^2+a\right)}\right)dx^2}{c(b^2-4ac)}-\frac{x^4(x^2(20a^2Bc^2+1}{}}{2c(b^2-4ac)}\right.$$

$\downarrow$ 2009

$$\frac{1}{2}\left(\frac{\frac{2\left(-x^2\left(3B\left(-10a^2c+7ab^2-\frac{b^4}{c}\right)+A(b^3-7abc)\right)-\frac{\left(-60a^3Bc^3-30a^2Abc^3+90a^2b^2Bc^2+10aAb^3c^2-30ab^4Bc-Ab^5c+3b^6B\right)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\right.}{c(b^2-4ac)}}{2c(b^2-4a}\right.$$

input `Int[(x^11*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]`

3.124. $\int \frac{x^{11}(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

output
```
(-1/2*(x^8*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2))/(c*(b^2 - 4*
a*c)*(a + b*x^2 + c*x^4)^2) + (-((x^4*(a*(3*b^3*B - A*b^2*c - 18*a*b*B*c +
 16*a*A*c^2) + (3*b^4*B - A*b^3*c - 20*a*b^2*B*c + 10*a*A*b*c^2 + 20*a^2*B
*c^2)*x^2))/(c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4))) + (2*(-((3*B*(7*a*b^2 -
 b^4/c - 10*a^2*c) + A*(b^3 - 7*a*b*c))*x^2) - ((3*b^6*B - A*b^5*c - 30*a*
b^4*B*c + 10*a*A*b^3*c^2 + 90*a^2*b^2*B*c^2 - 30*a^2*A*b*c^3 - 60*a^3*B*c^
3)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]) - ((b
^2 - 4*a*c)^2*(3*b*B - A*c)*Log[a + b*x^2 + c*x^4])/(2*c^2)))/(c*(b^2 - 4*
a*c)))/(2*c*(b^2 - 4*a*c)))/2
```

3.124.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.124. $\int \frac{x^{11}(A+Bx^2)}{(a+bx^2+cx^4)^3} dx$

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.124.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 624, normalized size of antiderivative = 1.71

method	result
default	$\frac{B x^2}{2c^3} + \frac{\frac{\left(25A a^2 b c^3-15Aa b^3 c^2+2A b^5 c+18B a^3 c^3-51B a^2 b^2 c^2+24Ba b^4 c-3B b^6\right) x^6}{16a^2 c^2-8a b^2 c+b^4} + \frac{\left(32A a^3 c^4+11A a^2 b^2 c^3-19Aa b^4 c^2+3A b^6 c-42B a^3 b \cdots\right.}{2c\left(16a^2 c^2-8a b^2 c+b \cdots\right.}}{\cdots}$
risch	Expression too large to display

input
```
int(x^11*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*B*x^2/c^3+1/2/c^3*(((25*A*a^2*b*c^3-15*A*a*b^3*c^2+2*A*b^5*c+18*B*a^3*
c^3-51*B*a^2*b^2*c^2+24*B*a*b^4*c-3*B*b^6)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+
1/2*(32*A*a^3*c^4+11*A*a^2*b^2*c^3-19*A*a*b^4*c^2+3*A*b^6*c-42*B*a^3*b*c^3
-41*B*a^2*b^3*c^2+34*B*a*b^5*c-5*B*b^7)/c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4+a
*(31*A*a^2*b*c^3-22*A*a*b^3*c^2+3*A*b^5*c+14*B*a^3*c^3-71*B*a^2*b^2*c^2+38
*B*a*b^4*c-5*B*b^6)/c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+1/2*a^2*(24*A*a^2*c^3
-21*A*a*b^2*c^2+3*A*b^4*c-58*B*a^2*b*c^2+36*B*a*b^3*c-5*B*b^5)/c/(16*a^2*c
^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)^2+1/(16*a^2*c^2-8*a*b^2*c+b^4)*(1/2*(16
*A*a^2*c^3-8*A*a*b^2*c^2+A*b^4*c-48*B*a^2*b*c^2+24*B*a*b^3*c-3*B*b^5)/c*ln
(c*x^4+b*x^2+a)+2*(-7*A*a^2*b*c^2+A*a*b^3*c-30*a^3*B*c^2+21*B*a^2*b^2*c-3*
B*a*b^4-1/2*(16*A*a^2*c^3-8*A*a*b^2*c^2+A*b^4*c-48*B*a^2*b*c^2+24*B*a*b^3*
c-3*B*b^5)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))))
```

3.124.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1586 vs. 2(351) = 702.

Time = 0.53 (sec) , antiderivative size = 3196, normalized size of antiderivative = 8.76

$$\int \frac{x^{11}(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/4*(2*(B*b^6*c^3 - 12*B*a*b^4*c^4 + 48*B*a^2*b^2*c^5 - 64*B*a^3*c^6)*x^1
0 - 5*B*a^2*b^7 - 96*A*a^5*c^4 + 4*(B*b^7*c^2 - 12*B*a*b^5*c^3 + 48*B*a^2*
b^3*c^4 - 64*B*a^3*b*c^5)*x^8 - 2*(2*B*b^8*c + 100*(2*B*a^4 + A*a^3*b)*c^5
 - (254*B*a^3*b^2 + 85*A*a^2*b^3)*c^4 + (123*B*a^2*b^4 + 23*A*a*b^5)*c^3 -
 2*(13*B*a*b^6 + A*b^7)*c^2)*x^6 - (5*B*b^9 + 128*A*a^4*c^5 + 4*(22*B*a^4*
b + 3*A*a^3*b^2)*c^4 - (314*B*a^3*b^3 + 87*A*a^2*b^4)*c^3 + (225*B*a^2*b^5
 + 31*A*a*b^6)*c^2 - (58*B*a*b^7 + 3*A*b^8)*c)*x^4 + 4*(58*B*a^5*b + 27*A*
a^4*b^2)*c^3 - (202*B*a^4*b^3 + 33*A*a^3*b^4)*c^2 - 2*(5*B*a*b^8 + 4*(30*B
*a^5 + 31*A*a^4*b)*c^4 - (346*B*a^4*b^2 + 119*A*a^3*b^3)*c^3 + (235*B*a^3*
b^4 + 34*A*a^2*b^5)*c^2 - (59*B*a^2*b^6 + 3*A*a*b^7)*c)*x^2 - (3*B*a^2*b^6
 + (3*B*b^6*c^2 - 30*(2*B*a^3 + A*a^2*b)*c^5 + 10*(9*B*a^2*b^2 + A*a*b^3)*
c^4 - (30*B*a*b^4 + A*b^5)*c^3)*x^8 + 2*(3*B*b^7*c - 30*(2*B*a^3*b + A*a^2
*b^2)*c^4 + 10*(9*B*a^2*b^3 + A*a*b^4)*c^3 - (30*B*a*b^5 + A*b^6)*c^2)*x^6
 + (3*B*b^8 - 60*(2*B*a^4 + A*a^3*b)*c^4 + 10*(12*B*a^3*b^2 - A*a^2*b^3)*c
^3 + 2*(15*B*a^2*b^4 + 4*A*a*b^5)*c^2 - (24*B*a*b^6 + A*b^7)*c)*x^4 - 30*(
2*B*a^5 + A*a^4*b)*c^3 + 10*(9*B*a^4*b^2 + A*a^3*b^3)*c^2 + 2*(3*B*a*b^7 -
 30*(2*B*a^4*b + A*a^3*b^2)*c^3 + 10*(9*B*a^3*b^3 + A*a^2*b^4)*c^2 - (30*B
*a^2*b^5 + A*a*b^6)*c)*x^2 - (30*B*a^3*b^4 + A*a^2*b^5)*c)*sqrt(b^2 - 4*a*
c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a
*c))/(c*x^4 + b*x^2 + a)) + (56*B*a^3*b^5 + 3*A*a^2*b^6)*c - (3*B*a^2*b...
```

3.124.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11}(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**11*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

output
```
Timed out
```

3.124.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{11}(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.124.8 Giac [A] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 598, normalized size of antiderivative = 1.64

$$\int \frac{x^{11}(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$$
$$= \frac{(3\,Bb^6-30\,Bab^4c-Ab^5c+90\,Ba^2b^2c^2+10\,Aab^3c^2-60\,Ba^3c^3-30\,Aa^2bc^3)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(b^4c^4-8\,ab^2c^5+16\,a^2c^6)\sqrt{-b^2+4\,ac}}$$
$$+\frac{Bx^2}{2\,c^3}$$
$$+\frac{9\,Bb^5c^2x^8-72\,Bab^3c^3x^8-3\,Ab^4c^3x^8+144\,Ba^2bc^4x^8+24\,Aab^2c^4x^8-48\,Aa^2c^5x^8+6\,Bb^6cx^6-48\,Ba}{}$$
$$-\frac{(3\,Bb-Ac)\log(cx^4+bx^2+a)}{4\,c^4}$$

input
```
integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/2*(3*B*b^6 - 30*B*a*b^4*c - A*b^5*c + 90*B*a^2*b^2*c^2 + 10*A*a*b^3*c^2
- 60*B*a^3*c^3 - 30*A*a^2*b*c^3)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/
((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*sqrt(-b^2 + 4*a*c)) + 1/2*B*x^2/c^3
+ 1/8*(9*B*b^5*c^2*x^8 - 72*B*a*b^3*c^3*x^8 - 3*A*b^4*c^3*x^8 + 144*B*a^2*
b*c^4*x^8 + 24*A*a*b^2*c^4*x^8 - 48*A*a^2*c^5*x^8 + 6*B*b^6*c*x^6 - 48*B*a
*b^4*c^2*x^6 + 2*A*b^5*c^2*x^6 + 84*B*a^2*b^2*c^3*x^6 - 12*A*a*b^3*c^3*x^6
 + 72*B*a^3*c^4*x^6 + 4*A*a^2*b*c^4*x^6 - B*b^7*x^4 + 14*B*a*b^5*c*x^4 + 3
*A*b^6*c*x^4 - 82*B*a^2*b^3*c^2*x^4 - 20*A*a*b^4*c^2*x^4 + 204*B*a^3*b*c^3
*x^4 + 22*A*a^2*b^2*c^3*x^4 - 32*A*a^3*c^4*x^4 - 2*B*a*b^6*x^2 + 8*B*a^2*b
^4*c*x^2 + 6*A*a*b^5*c*x^2 + 4*B*a^3*b^2*c^2*x^2 - 40*A*a^2*b^3*c^2*x^2 +
56*B*a^4*c^3*x^2 + 28*A*a^3*b*c^3*x^2 - B*a^2*b^5 + 3*A*a^2*b^4*c + 28*B*a
^4*b*c^2 - 18*A*a^3*b^2*c^2)/((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*(c*x^4
+ b*x^2 + a)^2) - 1/4*(3*B*b - A*c)*log(c*x^4 + b*x^2 + a)/c^4
```

3.124.9 Mupad [B] (verification not implemented)

Time = 10.38 (sec) , antiderivative size = 4501, normalized size of antiderivative = 12.33

$$\int \frac{x^{11}(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int((x^11*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

output
```
((x^6*(18*B*a^3*c^3 - 3*B*b^6 + 2*A*b^5*c + 24*B*a*b^4*c - 15*A*a*b^3*c^2
+ 25*A*a^2*b*c^3 - 51*B*a^2*b^2*c^2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) +
 (a*(24*A*a^3*c^3 - 5*B*a*b^5 + 3*A*a*b^4*c + 36*B*a^2*b^3*c - 58*B*a^3*b*
c^2 - 21*A*a^2*b^2*c^2))/(4*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(14*B
*a^4*c^3 - 5*B*a*b^6 + 3*A*a*b^5*c + 31*A*a^3*b*c^3 + 38*B*a^2*b^4*c - 22*
A*a^2*b^3*c^2 - 71*B*a^3*b^2*c^2))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) -
(x^4*(5*B*b^7 - 32*A*a^3*c^4 - 3*A*b^6*c - 34*B*a*b^5*c + 19*A*a*b^4*c^2 +
 42*B*a^3*b*c^3 - 11*A*a^2*b^2*c^3 + 41*B*a^2*b^3*c^2))/(4*c*(b^4 + 16*a^2
*c^2 - 8*a*b^2*c)))/(a^2*c^3 + c^5*x^8 + x^4*(2*a*c^4 + b^2*c^3) + 2*b*c^4
*x^6 + 2*a*b*c^3*x^2) + (B*x^2)/(2*c^3) + (log(((a*(A*c - 3*B*b)^2)/c^6 -
(((8*a*(A*c - 3*B*b))/c^2 - (2*(2*a + b*x^2)*(A*c - 3*B*b + c^4*(-(60*B*a^
3*c^3 - 3*B*b^6 + A*b^5*c + 30*B*a*b^4*c - 10*A*a*b^3*c^2 + 30*A*a^2*b*c^3
 - 90*B*a^2*b^2*c^2)^2/(c^8*(4*a*c - b^2)^5))^(1/2)))/c^2 + (2*x^2*(60*B*a
^3*c^3 - 9*B*b^6 + 3*A*b^5*c + 78*B*a*b^4*c - 26*A*a*b^3*c^2 + 62*A*a^2*b*
c^3 - 186*B*a^2*b^2*c^2))/(c^2*(4*a*c - b^2)^2))*(A*c - 3*B*b + c^4*(-(60*
B*a^3*c^3 - 3*B*b^6 + A*b^5*c + 30*B*a*b^4*c - 10*A*a*b^3*c^2 + 30*A*a^2*b
*c^3 - 90*B*a^2*b^2*c^2)^2/(c^8*(4*a*c - b^2)^5))^(1/2)))/(4*c^4) + (x^2*(
A*c - 3*B*b)*(30*B*a^3*c^3 - 3*B*b^6 + A*b^5*c + 27*B*a*b^4*c - 9*A*a*b^3*
c^2 + 23*A*a^2*b*c^3 - 69*B*a^2*b^2*c^2))/(c^6*(4*a*c - b^2)^2))*((a*(A*c
- 3*B*b)^2)/c^6 + (((2*(2*a + b*x^2)*(3*B*b - A*c + c^4*(-(60*B*a^3*c^3...
```

3.124. $\int \frac{x^{11}(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

3.125 $\int \frac{x^9(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

3.125.1 Optimal result

Integrand size = 25, antiderivative size = 254

$$\begin{aligned}\int \frac{x^9(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx &= -\frac{x^6(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2)}{4c\,(b^2-4ac)\,(a+bx^2+cx^4)^2} \\ &\quad - \frac{x^2(2a(b^3B-7abBc+6aAc^2)+(2b^4B-15ab^2Bc+6aAbc^2+16a^2Bc^2)\,x^2)}{4c^2\,(b^2-4ac)^2\,(a+bx^2+cx^4)} \\ &\quad + \frac{(b^5B-10ab^3Bc+30a^2bBc^2-12a^2Ac^3)\,\mathrm{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\,(b^2-4ac)^{5/2}} + \frac{B\log\left(a+bx^2+cx^4\right)}{4c^3}\end{aligned}$$

output
```
-1/4*x^6*(a*(-2*A*c+B*b)+(-A*b*c-2*B*a*c+B*b^2)*x^2)/c/(-4*a*c+b^2)/(c*x^4
+b*x^2+a)^2-1/4*x^2*(2*a*(6*A*a*c^2-7*B*a*b*c+B*b^3)+(6*A*a*b*c^2+16*B*a^2
*c^2-15*B*a*b^2*c+2*B*b^4)*x^2)/c^2/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+1/2*(-1
2*A*a^2*c^3+30*B*a^2*b*c^2-10*B*a*b^3*c+B*b^5)*arctanh((2*c*x^2+b)/(-4*a*c
+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(5/2)+1/4*B*ln(c*x^4+b*x^2+a)/c^3
```

3.125.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.39

$$\int \frac{x^9(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$$

$$=\frac{\frac{-b^6B+b^5c(A+4Bx^2)-2ab^3c^2(4A+15Bx^2)+2a^2bc^3(11A+25Bx^2)+4a^2c^3(8aB-5Acx^2)+b^4c(11aB-2Acx^2)+ab^2c^2(-39aB+16Acx^2)}{(b^2-4ac)^2(a+bx^2+cx^4)}+\frac{2a}{}}{}$$

input
```
Integrate[(x^9*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

output
```
((-(b^6*B) + b^5*c*(A + 4*B*x^2) - 2*a*b^3*c^2*(4*A + 15*B*x^2) + 2*a^2*b*
c^3*(11*A + 25*B*x^2) + 4*a^2*c^3*(8*a*B - 5*A*c*x^2) + b^4*c*(11*a*B - 2*
A*c*x^2) + a*b^2*c^2*(-39*a*B + 16*A*c*x^2))/((b^2 - 4*a*c)^2*(a + b*x^2 +
 c*x^4)) + (2*a^3*B*c^2 + b^4*(b*B - A*c)*x^2 + a*b^2*(b^2*B + 4*A*c^2*x^2
 - b*c*(A + 5*B*x^2)) + a^2*c*(-4*b^2*B - 2*A*c^2*x^2 + b*c*(3*A + 5*B*x^2
)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (2*c*(b^5*B - 10*a*b^3*B*c + 3
0*a^2*b*B*c^2 - 12*a^2*A*c^3)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-
b^2 + 4*a*c)^(5/2) + B*c*Log[a + b*x^2 + c*x^4])/(4*c^4)
```

3.125.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {1578, 1233, 1233, 27, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$$

$$\Big\downarrow \text{ 1578}$$

$$\frac{1}{2}\int \frac{x^8(Bx^2+A)}{(cx^4+bx^2+a)^3}dx^2$$

$$\Big\downarrow \text{ 1233}$$

$$\frac{1}{2}\left(\frac{\int \frac{x^4\left(2B\left(b^2-4ac\right)x^2+3a(bB-2Ac)\right)}{\left(cx^4+bx^2+a\right)^2}dx^2}{2c\left(b^2-4ac\right)}-\frac{x^6\left(x^2\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}\right)$$

$\downarrow$ 1233

$$\frac{1}{2}\left(\frac{\frac{\int \frac{2\left(B\left(b^2-4ac\right)^2x^2+a\left(Bb^3-7aBcb+6aAc^2\right)\right)}{cx^4+bx^2+a}dx^2}{c(b^2-4ac)}-\frac{x^2\left(x^2\left(16a^2Bc^2+6aAbc^2-15ab^2Bc+2b^4B\right)+2a\left(6aAc^2-7abBc+b^3B\right)\right)}{c(b^2-4ac)(a+bx^2+cx^4)}}{2c\left(b^2-4ac\right)}-\frac{x^6\left(x^2\left(-2\ldots\right.\right.}{2c\ldots}\right.$$

$\downarrow$ 27

$$\frac{1}{2}\left(\frac{\frac{2\int \frac{B\left(b^2-4ac\right)^2x^2+a\left(Bb^3-7aBcb+6aAc^2\right)}{cx^4+bx^2+a}dx^2}{c(b^2-4ac)}-\frac{x^2\left(x^2\left(16a^2Bc^2+6aAbc^2-15ab^2Bc+2b^4B\right)+2a\left(6aAc^2-7abBc+b^3B\right)\right)}{c(b^2-4ac)(a+bx^2+cx^4)}}{2c\left(b^2-4ac\right)}-\frac{x^6\left(x^2\left(-2a\ldots\right.\right.}{2c\left(\ldots\right.}\right.$$

$\downarrow$ 1142

$$\frac{1}{2}\left(\frac{\frac{2\left(\frac{B\left(b^2-4ac\right)^2\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{2c}-\frac{\left(-12a^2Ac^3+30a^2bBc^2-10ab^3Bc+b^5B\right)\int \frac{1}{cx^4+bx^2+a}dx^2}{2c}\right)}{c(b^2-4ac)}-\frac{x^2\left(x^2\left(16a^2Bc^2+6aAbc^2-15ab^2Bc+2b^4B\right)+\ldots\right.}{c(b^2-4ac)(a+bx^2+cx\ldots}}{2c\left(b^2-4ac\right)}\right.$$

$\downarrow$ 1083

$$\frac{1}{2}\left(\frac{\frac{2\left(\frac{\left(-12a^2Ac^3+30a^2bBc^2-10ab^3Bc+b^5B\right)\int \frac{1}{-x^4+b^2-4ac}d\left(2cx^2+b\right)}{c}+\frac{B\left(b^2-4ac\right)^2\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{2c}\right)}{c(b^2-4ac)}-\frac{x^2\left(x^2\left(16a^2Bc^2+6aAbc^2-15ab^2Bc+\ldots\right.\right.}{c(b^2-4ac)(a+\ldots}}{2c\left(b^2-4ac\right)}\right.$$

$\downarrow$ 219

$$\frac{1}{2}\left(\frac{\frac{2\left(\frac{B\left(b^2-4ac\right)^2\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{2c}+\frac{\left(-12a^2Ac^3+30a^2bBc^2-10ab^3Bc+b^5B\right)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}\right)}{c(b^2-4ac)}-\frac{x^2\left(x^2\left(16a^2Bc^2+6aAbc^2-15ab^2Bc+2b^4\ldots\right.\right.}{c(b^2-4ac)(a+bx\ldots}}{2c\left(b^2-4ac\right)}\right.$$

3.125. $\int \frac{x^9\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^3}\,dx$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{\frac{2\left(\frac{\left(-12a^2Ac^3+30a^2bBc^2-10ab^3Bc+b^5B\right)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{B\left(b^2-4ac\right)^2\log\left(a+bx^2+cx^4\right)}{2c}\right)}{c(b^2-4ac)}-\frac{x^2\left(x^2\left(16a^2Bc^2+6aAbc^2-15ab^2Bc+2b^4\right.\right.}{c(b^2-4ac)(a+bx^2}}{2c\left(b^2-4ac\right)}\right.$$

input
```
Int[(x^9*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

output
```
(-1/2*(x^6*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2))/(c*(b^2 - 4*
a*c)*(a + b*x^2 + c*x^4)^2) + (-((x^2*(2*a*(b^3*B - 7*a*b*B*c + 6*a*A*c^2)
 + (2*b^4*B - 15*a*b^2*B*c + 6*a*A*b*c^2 + 16*a^2*B*c^2)*x^2))/(c*(b^2 - 4
*a*c)*(a + b*x^2 + c*x^4))) + (2*(((b^5*B - 10*a*b^3*B*c + 30*a^2*b*B*c^2
- 12*a^2*A*c^3)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*
a*c]) + (B*(b^2 - 4*a*c)^2*Log[a + b*x^2 + c*x^4])/(2*c)))/(c*(b^2 - 4*a*c
)))/(2*c*(b^2 - 4*a*c)))/2
```

3.125.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.125. $\int \frac{x^9(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.125.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 494 vs. $2(242) = 484$.

Time = 0.23 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.95

method	result
default	$\frac{-\frac{\left(10A\,a^2c^3-8Aa\,b^2c^2+A\,b^4c-25B\,a^2b\,c^2+15Ba\,b^3c-2B\,b^5\right)x^6}{c^2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{\left(2A\,a^2b\,c^3+8Aa\,b^3c^2-A\,b^5c+32B\,a^3c^3+11B\,a^2b^2c^2-19Ba\,b^4c+3B\,b^6\right)x^4}{2c^3\left(16a^2c^2-8a\,b^2c+b^4\right)}-\cdots}{2(c\,x^4+b\,x^2+a)^2}$
risch	Expression too large to display

input
```
int(x^9*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-1/c^2*(10*A*a^2*c^3-8*A*a*b^2*c^2+A*b^4*c-25*B*a^2*b*c^2+15*B*a*b^3*
c-2*B*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1/2*(2*A*a^2*b*c^3+8*A*a*b^3*c^2
-A*b^5*c+32*B*a^3*c^3+11*B*a^2*b^2*c^2-19*B*a*b^4*c+3*B*b^6)/c^3/(16*a^2*c
^2-8*a*b^2*c+b^4)*x^4-a*(6*A*a^2*c^3-10*A*a*b^2*c^2+A*b^4*c-31*B*a^2*b*c^2
+22*B*a*b^3*c-3*B*b^5)/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+1/2*a^2*(10*A*a*
b*c^2-A*b^3*c+24*B*a^2*c^2-21*B*a*b^2*c+3*B*b^4)/c^3/(16*a^2*c^2-8*a*b^2*c
+b^4))/(c*x^4+b*x^2+a)^2+1/2/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*(1/2*(16*B*a^2
*c^2-8*B*a*b^2*c+B*b^4)/c*ln(c*x^4+b*x^2+a)+2*(6*A*a^2*c^2-7*a^2*b*B*c+B*a
*b^3-1/2*(16*B*a^2*c^2-8*B*a*b^2*c+B*b^4)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2
*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

3.125.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1071 vs. $2(242) = 484$.

Time = 0.40 (sec) , antiderivative size = 2167, normalized size of antiderivative = 8.53

$$\int \frac{x^9(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/4*(3*B*a^2*b^6 + 2*(2*B*b^7*c + 40*A*a^3*c^5 - 2*(50*B*a^3*b + 21*A*a^2
*b^2)*c^4 + (85*B*a^2*b^3 + 12*A*a*b^4)*c^3 - (23*B*a*b^5 + A*b^6)*c^2)*x^
6 + (3*B*b^8 - 8*(16*B*a^4 + A*a^3*b)*c^4 - 6*(2*B*a^3*b^2 + 5*A*a^2*b^3)*
c^3 + 3*(29*B*a^2*b^4 + 4*A*a*b^5)*c^2 - (31*B*a*b^6 + A*b^7)*c)*x^4 - 8*(
12*B*a^5 + 5*A*a^4*b)*c^3 + 2*(54*B*a^4*b^2 + 7*A*a^3*b^3)*c^2 + 2*(3*B*a*
b^7 + 24*A*a^4*c^4 - 2*(62*B*a^4*b + 23*A*a^3*b^2)*c^3 + 7*(17*B*a^3*b^3 +
 2*A*a^2*b^4)*c^2 - (34*B*a^2*b^5 + A*a*b^6)*c)*x^2 - ((B*b^5*c^2 - 10*B*a
*b^3*c^3 + 30*B*a^2*b*c^4 - 12*A*a^2*c^5)*x^8 + B*a^2*b^5 - 10*B*a^3*b^3*c
 + 30*B*a^4*b*c^2 - 12*A*a^4*c^3 + 2*(B*b^6*c - 10*B*a*b^4*c^2 + 30*B*a^2*
b^2*c^3 - 12*A*a^2*b*c^4)*x^6 + (B*b^7 - 8*B*a*b^5*c + 10*B*a^2*b^3*c^2 -
24*A*a^3*c^4 + 12*(5*B*a^3*b - A*a^2*b^2)*c^3)*x^4 + 2*(B*a*b^6 - 10*B*a^2
*b^4*c + 30*B*a^3*b^2*c^2 - 12*A*a^3*b*c^3)*x^2)*sqrt(b^2 - 4*a*c)*log((2*
c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^
4 + b*x^2 + a)) - (33*B*a^3*b^4 + A*a^2*b^5)*c + (B*a^2*b^6 - 12*B*a^3*b^4
*c + 48*B*a^4*b^2*c^2 - 64*B*a^5*c^3 + (B*b^6*c^2 - 12*B*a*b^4*c^3 + 48*B*
a^2*b^2*c^4 - 64*B*a^3*c^5)*x^8 + 2*(B*b^7*c - 12*B*a*b^5*c^2 + 48*B*a^2*b
^3*c^3 - 64*B*a^3*b*c^4)*x^6 + (B*b^8 - 10*B*a*b^6*c + 24*B*a^2*b^4*c^2 +
32*B*a^3*b^2*c^3 - 128*B*a^4*c^4)*x^4 + 2*(B*a*b^7 - 12*B*a^2*b^5*c + 48*B
*a^3*b^3*c^2 - 64*B*a^4*b*c^3)*x^2)*log(c*x^4 + b*x^2 + a))/(a^2*b^6*c^3 -
 12*a^3*b^4*c^4 + 48*a^4*b^2*c^5 - 64*a^5*c^6 + (b^6*c^5 - 12*a*b^4*c^6...
```

3.125.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^9(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**9*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

output
```
Timed out
```

3.125.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^9(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.125.8 Giac [A] (verification not implemented)

Time = 1.52 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.83

$$\int \frac{x^9(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$$
$$= -\frac{(Bb^5-10\,Bab^3c+30\,Ba^2bc^2-12\,Aa^2c^3)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(b^4c^3-8\,ab^2c^4+16\,a^2c^5)\sqrt{-b^2+4\,ac}}+\frac{B\log\left(cx^4+bx^2+a\right)}{4\,c^3}$$
$$-\frac{3\,Bb^4c^2x^8-24\,Bab^2c^3x^8+48\,Ba^2c^4x^8-2\,Bb^5cx^6+12\,Bab^3c^2x^6+4\,Ab^4c^2x^6-4\,Ba^2bc^3x^6-32\,Aab^2}{}$$

input
```
integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
-1/2*(B*b^5 - 10*B*a*b^3*c + 30*B*a^2*b*c^2 - 12*A*a^2*c^3)*arctan((2*c*x^
2 + b)/sqrt(-b^2 + 4*a*c))/((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt(-b^2
 + 4*a*c)) + 1/4*B*log(c*x^4 + b*x^2 + a)/c^3 - 1/8*(3*B*b^4*c^2*x^8 - 24*
B*a*b^2*c^3*x^8 + 48*B*a^2*c^4*x^8 - 2*B*b^5*c*x^6 + 12*B*a*b^3*c^2*x^6 +
4*A*b^4*c^2*x^6 - 4*B*a^2*b*c^3*x^6 - 32*A*a*b^2*c^3*x^6 + 40*A*a^2*c^4*x^
6 - 3*B*b^6*x^4 + 20*B*a*b^4*c*x^4 + 2*A*b^5*c*x^4 - 22*B*a^2*b^2*c^2*x^4
- 16*A*a*b^3*c^2*x^4 + 32*B*a^3*c^3*x^4 - 4*A*a^2*b*c^3*x^4 - 6*B*a*b^5*x^
2 + 40*B*a^2*b^3*c*x^2 + 4*A*a*b^4*c*x^2 - 28*B*a^3*b*c^2*x^2 - 40*A*a^2*b
^2*c^2*x^2 + 24*A*a^3*c^3*x^2 - 3*B*a^2*b^4 + 18*B*a^3*b^2*c + 2*A*a^2*b^3
*c - 20*A*a^3*b*c^2)/((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*(c*x^4 + b*x^2
+ a)^2)
```

3.125.9 Mupad [B] (verification not implemented)

Time = 10.74 (sec) , antiderivative size = 3062, normalized size of antiderivative = 12.06

$$\int \frac{x^9(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int((x^9*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

output
```
((x^4*(3*B*b^6 + 32*B*a^3*c^3 - A*b^5*c - 19*B*a*b^4*c + 8*A*a*b^3*c^2 + 2
*A*a^2*b*c^3 + 11*B*a^2*b^2*c^2))/(4*c^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) +
 (x^6*(2*B*b^5 - 10*A*a^2*c^3 - A*b^4*c - 15*B*a*b^3*c + 8*A*a*b^2*c^2 + 2
5*B*a^2*b*c^2))/(2*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a*(24*B*a^3*c^2
+ 3*B*a*b^4 - A*a*b^3*c + 10*A*a^2*b*c^2 - 21*B*a^2*b^2*c))/(4*c^3*(b^4 +
16*a^2*c^2 - 8*a*b^2*c)) - (x^2*(6*A*a^3*c^3 - 3*B*a*b^5 + A*a*b^4*c + 22*
B*a^2*b^3*c - 31*B*a^3*b*c^2 - 10*A*a^2*b^2*c^2))/(2*c^3*(b^4 + 16*a^2*c^2
 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6
) - (log(((B^2*a)/c^4 - ((B + c^3*(-(B*b^5 - 12*A*a^2*c^3 - 10*B*a*b^3*c +
 30*B*a^2*b*c^2)^2/(c^6*(4*a*c - b^2)^5))^(1/2))*((8*B*a)/c - (2*(B + c^3*
(-(B*b^5 - 12*A*a^2*c^3 - 10*B*a*b^3*c + 30*B*a^2*b*c^2)^2/(c^6*(4*a*c - b
^2)^5))^(1/2))*(2*a + b*x^2))/c + (2*x^2*(3*B*b^5 - 12*A*a^2*c^3 - 26*B*a*
b^3*c + 62*B*a^2*b*c^2))/(c*(4*a*c - b^2)^2)))/(4*c^3) + (B*x^2*(B*b^5 - 6
*A*a^2*c^3 - 9*B*a*b^3*c + 23*B*a^2*b*c^2))/(c^4*(4*a*c - b^2)^2))*((B^2*a
)/c^4 - ((B - c^3*(-(B*b^5 - 12*A*a^2*c^3 - 10*B*a*b^3*c + 30*B*a^2*b*c^2)
^2/(c^6*(4*a*c - b^2)^5))^(1/2))*((8*B*a)/c - (2*(B - c^3*(-(B*b^5 - 12*A*
a^2*c^3 - 10*B*a*b^3*c + 30*B*a^2*b*c^2)^2/(c^6*(4*a*c - b^2)^5))^(1/2))*(
2*a + b*x^2))/c + (2*x^2*(3*B*b^5 - 12*A*a^2*c^3 - 26*B*a*b^3*c + 62*B*a^2
*b*c^2))/(c*(4*a*c - b^2)^2)))/(4*c^3) + (B*x^2*(B*b^5 - 6*A*a^2*c^3 - 9*B
*a*b^3*c + 23*B*a^2*b*c^2))/(c^4*(4*a*c - b^2)^2)))*(2*B*b^10 - 2048*B*...
```

3.126 $\int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

3.126.1 Optimal result

Integrand size = 25, antiderivative size = 146

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = -\frac{x^6(Ab-2aB-(bB-2Ac)x^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{3(Ab-2aB)x^2(2a+bx^2)}{4(b^2-4ac)^2(a+bx^2+cx^4)}+\frac{3a(Ab-2aB)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

output
```
-1/4*x^6*(A*b-2*B*a-(-2*A*c+B*b)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+3/4*(
A*b-2*B*a)*x^2*(b*x^2+2*a)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+3*a*(A*b-2*B*a)*
arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)
```

3.126.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.79

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$$
$$=\frac{1}{4}\left(\frac{b^5B-8ab^3Bc-b^4c(A+2Bx^2)-4a^2c^3(4A+5Bx^2)+ab^2c^2(5A+16Bx^2)+2abc^2(11aB-3Acx^2)}{c^3\,(b^2-4ac)^2\,(a+bx^2+cx^4)}\right.$$
$$+\frac{b^3(bB-Ac)x^2+a^2c(-3bB+2c(A+Bx^2))+ab(b^2B+3Ac^2x^2-bc(A+4Bx^2))}{c^3\,(-b^2+4ac)\,(a+bx^2+cx^4)^2}$$
$$\left.-\frac{12a(Ab-2aB)\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{5/2}}\right)$$

input
```
Integrate[(x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

output
```
((b^5*B - 8*a*b^3*B*c - b^4*c*(A + 2*B*x^2) - 4*a^2*c^3*(4*A + 5*B*x^2) +
a*b^2*c^2*(5*A + 16*B*x^2) + 2*a*b*c^2*(11*a*B - 3*A*c*x^2))/(c^3*(b^2 - 4
*a*c)^2*(a + b*x^2 + c*x^4)) + (b^3*(b*B - A*c)*x^2 + a^2*c*(-3*b*B + 2*c*
(A + B*x^2)) + a*b*(b^2*B + 3*A*c^2*x^2 - b*c*(A + 4*B*x^2)))/(c^3*(-b^2 +
 4*a*c)*(a + b*x^2 + c*x^4)^2) - (12*a*(A*b - 2*a*B)*ArcTan[(b + 2*c*x^2)/
Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/4
```

3.126.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1578, 1227, 1153, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^3}\,dx$$
$$\Big\downarrow \; 1578$$
$$\frac{1}{2}\int \frac{x^6\left(Bx^2+A\right)}{\left(cx^4+bx^2+a\right)^3}dx^2$$

$$\Big\downarrow \color{blue}{1227}$$

$$\frac{1}{2}\left(\frac{3(Ab-2aB)\int \frac{x^4}{(cx^4+bx^2+a)^2}dx^2}{2(b^2-4ac)}-\frac{x^6\left(-2aB-\left(x^2(bB-2Ac)\right)+Ab\right)}{2(b^2-4ac)\left(a+bx^2+cx^4\right)^2}\right)$$

$$\Big\downarrow \color{blue}{1153}$$

$$\frac{1}{2}\left(\frac{3(Ab-2aB)\left(\frac{x^2(2a+bx^2)}{(b^2-4ac)(a+bx^2+cx^4)}-\frac{2a\int \frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}\right)}{2(b^2-4ac)}-\frac{x^6\left(-2aB-\left(x^2(bB-2Ac)\right)+Ab\right)}{2(b^2-4ac)\left(a+bx^2+cx^4\right)^2}\right)$$

$$\Big\downarrow \color{blue}{1083}$$

$$\frac{1}{2}\left(\frac{3(Ab-2aB)\left(\frac{4a\int \frac{1}{-x^4+b^2-4ac}d(2cx^2+b)}{b^2-4ac}+\frac{x^2(2a+bx^2)}{(b^2-4ac)(a+bx^2+cx^4)}\right)}{2(b^2-4ac)}-\frac{x^6\left(-2aB-\left(x^2(bB-2Ac)\right)+Ab\right)}{2(b^2-4ac)\left(a+bx^2+cx^4\right)^2}\right)$$

$$\Big\downarrow \color{blue}{219}$$

$$\frac{1}{2}\left(\frac{3(Ab-2aB)\left(\frac{4a\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}+\frac{x^2(2a+bx^2)}{(b^2-4ac)(a+bx^2+cx^4)}\right)}{2(b^2-4ac)}-\frac{x^6\left(-2aB-\left(x^2(bB-2Ac)\right)+Ab\right)}{2(b^2-4ac)\left(a+bx^2+cx^4\right)^2}\right)$$

input
```
Int[(x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

output
```
(-1/2*(x^6*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/((b^2 - 4*a*c)*(a + b*x^2 +
c*x^4)^2) + (3*(A*b - 2*a*B)*((x^2*(2*a + b*x^2))/((b^2 - 4*a*c)*(a + b*x^
2 + c*x^4)) + (4*a*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)
^(3/2)))/(2*(b^2 - 4*a*c)))/2
```

3.126. $\int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

3.126.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1153
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*(2*p + 3)*((c*d^2 -
b*d*e + a*e^2)/((p + 1)*(b^2 - 4*a*c)))   Int[(d + e*x)^(m - 2)*(a + b*x +
c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
&& LtQ[p, -1]
```

rule 1227
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*(
(b*f - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[m*((b*(
e*f + d*g) - 2*(c*d*f + a*e*g))/((p + 1)*(b^2 - 4*a*c)))   Int[(d + e*x)^(m
 - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
&& EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.126.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(138) = 276$.

Time = 0.15 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.35

method	result
default	$\frac{-\frac{(3Aabc^2+10Ba^2c^2-8Bab^2c+Bb^4)x^6}{c(16a^2c^2-8ab^2c+b^4)}-\frac{(16Aa^2c^3+Aab^2c^2+Ab^4c-2Ba^2bc^2-8Bab^3c+Bb^5)x^4}{2(16a^2c^2-8ab^2c+b^4)c^2}-\frac{a(5Aabc^2+Ab^3c+6Ba^2c^2-10Bab^2c+}{(16a^2c^2-8ab^2c+b^4)c^2}}{2(cx^4+bx^2+a)^2}$
risch	$\frac{-\frac{(3Aabc^2+10Ba^2c^2-8Bab^2c+Bb^4)x^6}{2c(16a^2c^2-8ab^2c+b^4)}-\frac{(16Aa^2c^3+Aab^2c^2+Ab^4c-2Ba^2bc^2-8Bab^3c+Bb^5)x^4}{4(16a^2c^2-8ab^2c+b^4)c^2}-\frac{a(5Aabc^2+Ab^3c+6Ba^2c^2-10Bab^2c+}{2(16a^2c^2-8ab^2c+b^4)c^2}}{(cx^4+bx^2+a)^2}$

input
```
int(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-(3*A*a*b*c^2+10*B*a^2*c^2-8*B*a*b^2*c+B*b^4)/c/(16*a^2*c^2-8*a*b^2*c
+b^4)*x^6-1/2*(16*A*a^2*c^3+A*a*b^2*c^2+A*b^4*c-2*B*a^2*b*c^2-8*B*a*b^3*c+
B*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^2*x^4-a*(5*A*a*b*c^2+A*b^3*c+6*B*a^2*c
^2-10*B*a*b^2*c+B*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^2*x^2-1/2*a^2/c^2*(8*A
*a*c^2+A*b^2*c-10*B*a*b*c+B*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+
a)^2-3*a*(A*b-2*B*a)/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((
2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

3.126.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 677 vs. $2(140) = 280$.

Time = 0.30 (sec) , antiderivative size = 1378, normalized size of antiderivative = 9.44

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/4*(B*a^2*b^5 - 32*A*a^4*c^3 + 2*(B*b^6*c - 12*B*a*b^4*c^2 - 4*(10*B*a^
3 + 3*A*a^2*b)*c^4 + 3*(14*B*a^2*b^2 + A*a*b^3)*c^3)*x^6 + (B*b^7 - 64*A*a
^3*c^4 + 4*(2*B*a^3*b + 3*A*a^2*b^2)*c^3 + 3*(10*B*a^2*b^3 - A*a*b^4)*c^2
- (12*B*a*b^5 - A*b^6)*c)*x^4 + 4*(10*B*a^4*b + A*a^3*b^2)*c^2 + 2*(B*a*b^
6 - 4*(6*B*a^4 + 5*A*a^3*b)*c^3 + (46*B*a^3*b^2 + A*a^2*b^3)*c^2 - (14*B*a
^2*b^4 - A*a*b^5)*c)*x^2 + 6*((2*B*a^2 - A*a*b)*c^4*x^8 + 2*(2*B*a^2*b - A
*a*b^2)*c^3*x^6 + 2*(2*B*a^3*b - A*a^2*b^2)*c^2*x^2 + (2*(2*B*a^3 - A*a^2*
b)*c^3 + (2*B*a^2*b^2 - A*a*b^3)*c^2)*x^4 + (2*B*a^4 - A*a^3*b)*c^2)*sqrt(
b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt
(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (14*B*a^3*b^3 - A*a^2*b^4)*c)/(a^2*b
^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5 + (b^6*c^4 - 12*a*b^
4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)*x^8 + 2*(b^7*c^3 - 12*a*b^5*c^4 + 48*
a^2*b^3*c^5 - 64*a^3*b*c^6)*x^6 + (b^8*c^2 - 10*a*b^6*c^3 + 24*a^2*b^4*c^4
 + 32*a^3*b^2*c^5 - 128*a^4*c^6)*x^4 + 2*(a*b^7*c^2 - 12*a^2*b^5*c^3 + 48*
a^3*b^3*c^4 - 64*a^4*b*c^5)*x^2), -1/4*(B*a^2*b^5 - 32*A*a^4*c^3 + 2*(B*b^
6*c - 12*B*a*b^4*c^2 - 4*(10*B*a^3 + 3*A*a^2*b)*c^4 + 3*(14*B*a^2*b^2 + A*
a*b^3)*c^3)*x^6 + (B*b^7 - 64*A*a^3*c^4 + 4*(2*B*a^3*b + 3*A*a^2*b^2)*c^3
+ 3*(10*B*a^2*b^3 - A*a*b^4)*c^2 - (12*B*a*b^5 - A*b^6)*c)*x^4 + 4*(10*B*a
^4*b + A*a^3*b^2)*c^2 + 2*(B*a*b^6 - 4*(6*B*a^4 + 5*A*a^3*b)*c^3 + (46*B*a
^3*b^2 + A*a^2*b^3)*c^2 - (14*B*a^2*b^4 - A*a*b^5)*c)*x^2 + 12*((2*B*a^...
```

3.126.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**7*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

output
```
Timed out
```

3.126.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.126.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. 2(140) = 280.

Time = 1.45 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.18

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \frac{3\,(2\,Ba^2-Aab)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}} - \frac{2\,Bb^4cx^6-16\,Bab^2c^2x^6+20\,Ba^2c^3x^6+6\,Aabc^3x^6+Bb^5x^4-8\,Bab^3cx^4+Ab^4cx^4-2\,Ba^2bc^2x^4+Aa}{4\,(b^4c^2-}$$

input
```
integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
3*(2*B*a^2 - A*a*b)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b
^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) - 1/4*(2*B*b^4*c*x^6 - 16*B*a*b^2*c
^2*x^6 + 20*B*a^2*c^3*x^6 + 6*A*a*b*c^3*x^6 + B*b^5*x^4 - 8*B*a*b^3*c*x^4
+ A*b^4*c*x^4 - 2*B*a^2*b*c^2*x^4 + A*a*b^2*c^2*x^4 + 16*A*a^2*c^3*x^4 + 2
*B*a*b^4*x^2 - 20*B*a^2*b^2*c*x^2 + 2*A*a*b^3*c*x^2 + 12*B*a^3*c^2*x^2 + 1
0*A*a^2*b*c^2*x^2 + B*a^2*b^3 - 10*B*a^3*b*c + A*a^2*b^2*c + 8*A*a^3*c^2)/
((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*(c*x^4 + b*x^2 + a)^2)
```

3.126.9 Mupad [B] (verification not implemented)

Time = 7.85 (sec) , antiderivative size = 593, normalized size of antiderivative = 4.06

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$$
$$= \frac{3\,a\,\mathrm{atan}\left(\frac{\left(x^2\left(\frac{3\,(A\,b-2\,B\,a)\left(6\,B\,a^2\,c^2-3\,A\,a\,b\,c^2\right)}{\left(4\,a\,c-b^2\right)^{9/2}\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}-\frac{9\,a\,b\,(A\,b-2\,B\,a)^2\left(32\,a^2\,b\,c^4-16\,a\,b^3\,c^3+2\,b^5\,c^2\right)}{2\left(4\,a\,c-b^2\right)^{15/2}\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}\right)-\frac{18\,a^2\,b\,c^2\,(A\,b-2\,B\,a)^2}{\left(4\,a\,c-b^2\right)^{15/2}}\right)\left(b^4\left(4\,a\,c-b^2\right)^5\right.}{18\,A^2\,a^2\,b^2\,c^2-72\,A\,B\,a^3\,b\,c^2+72\,B^2\,a^4\,c^2}\right)}{\left(4\,a\,c-b^2\right)^{5/2}}$$
$$-\frac{\frac{x^4\left(-2\,B\,a^2\,b\,c^2+16\,A\,a^2\,c^3-8\,B\,a\,b^3\,c+A\,a\,b^2\,c^2+B\,b^5+A\,b^4\,c\right)}{4\,c^2\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{a^2\left(B\,b^3+A\,b^2\,c-10\,B\,a\,b\,c+8\,A\,a\,c^2\right)}{4\,c^2\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{x^6\left(10\,B\,a^2\,c^2-8\,B\,a\,b^2\,c+3\,A\right.}{2\,c\left(16\,a^2\,c^2-8\,a\,b^2\,c+\right.}}{x^4\left(b^2+2\,a\,c\right)+a^2+c^2\,x^8+2\,a\,b\,x^2+2\,b\,c\,x^6}$$

input
```
int((x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

output
```
(3*a*atan(((x^2*((3*(A*b - 2*B*a)*(6*B*a^2*c^2 - 3*A*a*b*c^2))/((4*a*c - b
^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (9*a*b*(A*b - 2*B*a)^2*(2*b^5*
c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4))/(2*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*
c^2 - 8*a*b^2*c))) - (18*a^2*b*c^2*(A*b - 2*B*a)^2)/(4*a*c - b^2)^(15/2))*
(b^4*(4*a*c - b^2)^5 + 16*a^2*c^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2
)^5))/(72*B^2*a^4*c^2 + 18*A^2*a^2*b^2*c^2 - 72*A*B*a^3*b*c^2))*(A*b - 2*B
*a))/(4*a*c - b^2)^(5/2) - ((x^4*(B*b^5 + 16*A*a^2*c^3 + A*b^4*c - 8*B*a*b
^3*c + A*a*b^2*c^2 - 2*B*a^2*b*c^2))/(4*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)
) + (a^2*(B*b^3 + 8*A*a*c^2 + A*b^2*c - 10*B*a*b*c))/(4*c^2*(b^4 + 16*a^2*
c^2 - 8*a*b^2*c)) + (x^6*(B*b^4 + 10*B*a^2*c^2 + 3*A*a*b*c^2 - 8*B*a*b^2*c
))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a*x^2*(B*b^4 + 6*B*a^2*c^2 + A*
b^3*c + 5*A*a*b*c^2 - 10*B*a*b^2*c))/(2*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)
))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6)
```

3.127 $\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

3.127.1 Optimal result

Integrand size = 25, antiderivative size = 185

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = -\frac{x^4(Ab-2aB-(bB-2Ac)x^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{a(b^2B-6Abc+8aBc)+(b^3B-4Ab^2c+2abBc+4aAc^2)x^2}{4c(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{(3abB-A(b^2+2ac))\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

output
```
-1/4*x^4*(A*b-2*B*a-(-2*A*c+B*b)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+1/4*(
-a*(-6*A*b*c+8*B*a*c+B*b^2)-(4*A*a*c^2-4*A*b^2*c+2*B*a*b*c+B*b^3)*x^2)/c/(
-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+(3*a*b*B-A*(2*a*c+b^2))*arctanh((2*c*x^2+b)/
(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)
```

3.127.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.26

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \frac{1}{4}\left(\frac{-b^4B+Ab^3c+2abc^2(A-3Bx^2)+4ac^2(-4aB+Acx^2)+b^2c(5aB+2Acx^2)}{c^2(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{2a^2Bc+b^2(-bB+Ac)x^2+a(-b^2B-2Ac^2x^2+bc(A+3Bx^2))}{c^2(-b^2+4ac)(a+bx^2+cx^4)^2} + \frac{4(-3abB+A(b^2+2ac))\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{5/2}}\right)$$

input
```
Integrate[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

output
```
((-(b^4*B) + A*b^3*c + 2*a*b*c^2*(A - 3*B*x^2) + 4*a*c^2*(-4*a*B + A*c*x^2
) + b^2*c*(5*a*B + 2*A*c*x^2))/(c^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) +
 (2*a^2*B*c + b^2*(-(b*B) + A*c)*x^2 + a*(-(b^2*B) - 2*A*c^2*x^2 + b*c*(A
+ 3*B*x^2)))/(c^2*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)^2) + (4*(-3*a*b*B + A
*(b^2 + 2*a*c))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(
5/2))/4
```

3.127.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1578, 1234, 25, 1224, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{x^4(Bx^2+A)}{(cx^4+bx^2+a)^3}dx^2$$

$$\Big\downarrow 1234$$

$$\frac{1}{2}\left(-\frac{\int -\frac{x^2((bB-2Ac)x^2+2(Ab-2aB))}{(cx^4+bx^2+a)^2}dx^2}{2(b^2-4ac)}-\frac{x^4(-2aB-(x^2(bB-2Ac))+Ab)}{2(b^2-4ac)(a+bx^2+cx^4)^2}\right)$$

$$\Big\downarrow 25$$

$$\frac{1}{2}\left(\frac{\int \frac{x^2((bB-2Ac)x^2+2(Ab-2aB))}{(cx^4+bx^2+a)^2}dx^2}{2(b^2-4ac)}-\frac{x^4(-2aB-(x^2(bB-2Ac))+Ab)}{2(b^2-4ac)(a+bx^2+cx^4)^2}\right)$$

$$\Big\downarrow 1224$$

$$\frac{1}{2}\left(\frac{-\frac{2(3abB-A(2ac+b^2))\int \frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}-\frac{a(8aBc-6Abc+b^2B)+x^2(4aAc^2+2abBc-4Ab^2c+b^3B)}{c(b^2-4ac)(a+bx^2+cx^4)}}{2(b^2-4ac)}-\frac{x^4(-2aB-(x^2(bB-2Ac}{2(b^2-4ac)(a+bx^2+}\right.$$

$$\Big\downarrow 1083$$

$$\frac{1}{2}\left(\frac{\frac{4(3abB-A(2ac+b^2))\int \frac{1}{-x^4+b^2-4ac}d(2cx^2+b)}{b^2-4ac}-\frac{a(8aBc-6Abc+b^2B)+x^2(4aAc^2+2abBc-4Ab^2c+b^3B)}{c(b^2-4ac)(a+bx^2+cx^4)}}{2(b^2-4ac)}-\frac{x^4(-2aB-(x^2(bB-}{2(b^2-4ac)(a+b}\right.$$

$$\Big\downarrow 219$$

$$\frac{1}{2}\left(\frac{\frac{4(3abB-A(2ac+b^2))\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{a(8aBc-6Abc+b^2B)+x^2(4aAc^2+2abBc-4Ab^2c+b^3B)}{c(b^2-4ac)(a+bx^2+cx^4)}}{2(b^2-4ac)}-\frac{x^4(-2aB-(x^2(bB-2}{2(b^2-4ac)(a+bx^2}\right.$$

input
```
Int[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

output
```
(-1/2*(x^4*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/((b^2 - 4*a*c)*(a + b*x^2 +
c*x^4)^2) + (-((a*(b^2*B - 6*A*b*c + 8*a*B*c) + (b^3*B - 4*A*b^2*c + 2*a*b
*B*c + 4*a*A*c^2)*x^2)/(c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4))) + (4*(3*a*b*
B - A*(b^2 + 2*a*c))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*
c)^(3/2))/(2*(b^2 - 4*a*c)))/2
```

3.127. $\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

3.127.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1224
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x))*((a + b*x + c*x^2)^(p
+ 1)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c
*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c))   Int[(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -
1] &&  !(IntegerQ[p] && NeQ[a, 0] && NiceSqrtQ[b^2 - 4*a*c])
```

rule 1234
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*(
(f*b - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p +
 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1)*Simp[g
*(2*a*e*m + b*d*(2*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*
(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1
] && GtQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.127. $\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

3.127.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.64

method	result
default	$\frac{\frac{c\left(2Aac+A\,b^2-3abB\right)x^6}{16a^2c^2-8a\,b^2c+b^4}+\frac{\left(6Aab\,c^2+3A\,b^3c-16B\,a^2c^2-Ba\,b^2c-B\,b^4\right)x^4}{2c\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{a\left(2Aa\,c^2-5A\,b^2c+5Babc+B\,b^3\right)x^2}{c\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{a^2\left(6Abc-8Bac-B\,b^2\right)}{2c\left(16a^2c^2-8a\,b^2c+b^4\right)}}{2(c\,x^4+b\,x^2+a)^2}+$
risch	$\frac{\frac{c\left(2Aac+A\,b^2-3abB\right)x^6}{32a^2c^2-16a\,b^2c+2b^4}+\frac{\left(6Aab\,c^2+3A\,b^3c-16B\,a^2c^2-Ba\,b^2c-B\,b^4\right)x^4}{4c\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{a\left(2Aa\,c^2-5A\,b^2c+5Babc+B\,b^3\right)x^2}{2c\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{a^2\left(6Abc-8Bac-B\,b^2\right)}{4c\left(16a^2c^2-8a\,b^2c+b^4\right)}}{(c\,x^4+b\,x^2+a)^2}-$

input
```
int(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(c*(2*A*a*c+A*b^2-3*B*a*b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1/2*(6*A*a*b
*c^2+3*A*b^3*c-16*B*a^2*c^2-B*a*b^2*c-B*b^4)/c/(16*a^2*c^2-8*a*b^2*c+b^4)*
x^4-1/c*a*(2*A*a*c^2-5*A*b^2*c+5*B*a*b*c+B*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)
*x^2+1/2*a^2*(6*A*b*c-8*B*a*c-B*b^2)/c/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+
b*x^2+a)^2+(2*A*a*c+A*b^2-3*B*a*b)/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^
(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

3.127.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 672 vs. $2(177) = 354$.

Time = 0.33 (sec) , antiderivative size = 1369, normalized size of antiderivative = 7.40

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/4*(B*a^2*b^4 + 2*(8*A*a^2*c^4 - 2*(6*B*a^2*b - A*a*b^2)*c^3 + (3*B*a*b
^3 - A*b^4)*c^2)*x^6 + (B*b^6 - 8*(8*B*a^3 - 3*A*a^2*b)*c^3 + 6*(2*B*a^2*b
^2 + A*a*b^3)*c^2 - 3*(B*a*b^4 + A*b^5)*c)*x^4 - 8*(4*B*a^4 - 3*A*a^3*b)*c
^2 + 2*(B*a*b^5 - 8*A*a^3*c^3 - 2*(10*B*a^3*b - 11*A*a^2*b^2)*c^2 + (B*a^2
*b^3 - 5*A*a*b^4)*c)*x^2 - 2*((2*A*a*c^4 - (3*B*a*b - A*b^2)*c^3)*x^8 + 2*
(2*A*a*b*c^3 - (3*B*a*b^2 - A*b^3)*c^2)*x^6 + 2*A*a^3*c^2 + (4*A*a^2*c^3 -
 2*(3*B*a^2*b - 2*A*a*b^2)*c^2 - (3*B*a*b^3 - A*b^4)*c)*x^4 + 2*(2*A*a^2*b
*c^2 - (3*B*a^2*b^2 - A*a*b^3)*c)*x^2 - (3*B*a^3*b - A*a^2*b^2)*c)*sqrt(b^
2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b
^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + 2*(2*B*a^3*b^2 - 3*A*a^2*b^3)*c)/(a^2*
b^6*c - 12*a^3*b^4*c^2 + 48*a^4*b^2*c^3 - 64*a^5*c^4 + (b^6*c^3 - 12*a*b^4
*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*x^8 + 2*(b^7*c^2 - 12*a*b^5*c^3 + 48*a
^2*b^3*c^4 - 64*a^3*b*c^5)*x^6 + (b^8*c - 10*a*b^6*c^2 + 24*a^2*b^4*c^3 +
32*a^3*b^2*c^4 - 128*a^4*c^5)*x^4 + 2*(a*b^7*c - 12*a^2*b^5*c^2 + 48*a^3*b
^3*c^3 - 64*a^4*b*c^4)*x^2), -1/4*(B*a^2*b^4 + 2*(8*A*a^2*c^4 - 2*(6*B*a^2
*b - A*a*b^2)*c^3 + (3*B*a*b^3 - A*b^4)*c^2)*x^6 + (B*b^6 - 8*(8*B*a^3 - 3
*A*a^2*b)*c^3 + 6*(2*B*a^2*b^2 + A*a*b^3)*c^2 - 3*(B*a*b^4 + A*b^5)*c)*x^4
 - 8*(4*B*a^4 - 3*A*a^3*b)*c^2 + 2*(B*a*b^5 - 8*A*a^3*c^3 - 2*(10*B*a^3*b
- 11*A*a^2*b^2)*c^2 + (B*a^2*b^3 - 5*A*a*b^4)*c)*x^2 + 4*((2*A*a*c^4 - (3*
B*a*b - A*b^2)*c^3)*x^8 + 2*(2*A*a*b*c^3 - (3*B*a*b^2 - A*b^3)*c^2)*x^6...
```

3.127.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

output
```
Timed out
```

3.127.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.127.8 Giac [A] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.45

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = -\frac{(3\,Bab-Ab^2-2\,Aac)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}} - \frac{6\,Babc^2x^6-2\,Ab^2c^2x^6-4\,Aac^3x^6+Bb^4x^4+Bab^2cx^4-3\,Ab^3cx^4+16\,Ba^2c^2x^4-6\,Aabc^2x^4+2\,Bab^3}{4\,(b^4c-8\,ab^2c^2+16\,a^2c^3)(cx^4+bx^2}$$

input
```
integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
-(3*B*a*b - A*b^2 - 2*A*a*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^
4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) - 1/4*(6*B*a*b*c^2*x^6 - 2
*A*b^2*c^2*x^6 - 4*A*a*c^3*x^6 + B*b^4*x^4 + B*a*b^2*c*x^4 - 3*A*b^3*c*x^4
 + 16*B*a^2*c^2*x^4 - 6*A*a*b*c^2*x^4 + 2*B*a*b^3*x^2 + 10*B*a^2*b*c*x^2 -
 10*A*a*b^2*c*x^2 + 4*A*a^2*c^2*x^2 + B*a^2*b^2 + 8*B*a^3*c - 6*A*a^2*b*c)
/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*(c*x^4 + b*x^2 + a)^2)
```

3.127.9 Mupad [B] (verification not implemented)

Time = 7.85 (sec) , antiderivative size = 625, normalized size of antiderivative = 3.38

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$$

$$= \frac{\operatorname{atan}\left(\frac{\left(x^2\left(\frac{\left(A\,b^2\,c^2-3\,B\,a\,b\,c^2+2\,A\,a\,c^3\right)\left(A\,b^2-3\,B\,a\,b+2\,A\,a\,c\right)}{a\,\left(4\,a\,c-b^2\right)^{9/2}\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{b\,\left(A\,b^2-3\,B\,a\,b+2\,A\,a\,c\right)^2\left(32\,a^2\,b\,c^4-16\,a\,b^3\,c^3+2\,b^5\,c^2\right)}{2\,a\,\left(4\,a\,c-b^2\right)^{15/2}\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}\right)+\frac{2\,b\,c^2\,\left(A\,b^2-3\,B\,a\,b+2\right.}{\left(4\,a\,c-b^2\right)^{15/2}}}{8\,A^2\,a^2\,c^4+8\,A^2\,a\,b^2\,c^3+2\,A^2\,b^4\,c^2-24\,A\,B\,a^2\,b\,c^3-12\,A\,B\,a\,b^3\,c^2+18\,B^2\,a}\right.}{\left(4\,a\,c-b^2\right)^{5/2}}$$

$$-\frac{\frac{x^4\,\left(16\,B\,a^2\,c^2+B\,a\,b^2\,c-6\,A\,a\,b\,c^2+B\,b^4-3\,A\,b^3\,c\right)}{4\,c\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}-\frac{c\,x^6\,\left(A\,b^2-3\,B\,a\,b+2\,A\,a\,c\right)}{2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{a\,\left(8\,B\,c\,a^2+B\,a\,b^2-6\,A\,c\,a\,b\right)}{4\,c\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{x^2\,\left(5\,B\,a^2\,b\,c+2\,A\,a^2\right.}{2\,c\,\left(16\,a^2\,c^2\right.}}{x^4\,\left(b^2+2\,a\,c\right)+a^2+c^2\,x^8+2\,a\,b\,x^2+2\,b\,c\,x^6}$$

input
```
int((x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

output
```
(atan(((x^2*(((A*b^2*c^2 + 2*A*a*c^3 - 3*B*a*b*c^2)*(A*b^2 + 2*A*a*c - 3*B
*a*b))/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (b*(A*b^2
+ 2*A*a*c - 3*B*a*b)^2*(2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4))/(2*a*(4*
a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + (2*b*c^2*(A*b^2 + 2*A
*a*c - 3*B*a*b)^2)/(4*a*c - b^2)^(15/2))*(b^4*(4*a*c - b^2)^5 + 16*a^2*c^2
*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5))/(8*A^2*a^2*c^4 + 2*A^2*b^4*
c^2 + 18*B^2*a^2*b^2*c^2 + 8*A^2*a*b^2*c^3 - 12*A*B*a*b^3*c^2 - 24*A*B*a^2
*b*c^3))*(A*b^2 + 2*A*a*c - 3*B*a*b))/(4*a*c - b^2)^(5/2) - ((x^4*(B*b^4 +
 16*B*a^2*c^2 - 3*A*b^3*c - 6*A*a*b*c^2 + B*a*b^2*c))/(4*c*(b^4 + 16*a^2*c
^2 - 8*a*b^2*c)) - (c*x^6*(A*b^2 + 2*A*a*c - 3*B*a*b))/(2*(b^4 + 16*a^2*c^
2 - 8*a*b^2*c)) + (a*(B*a*b^2 + 8*B*a^2*c - 6*A*a*b*c))/(4*c*(b^4 + 16*a^2
*c^2 - 8*a*b^2*c)) + (x^2*(2*A*a^2*c^2 + B*a*b^3 - 5*A*a*b^2*c + 5*B*a^2*b
*c))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*
x^8 + 2*a*b*x^2 + 2*b*c*x^6)
```

3.128 $\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^3} dx$

3.128.1 Optimal result

Integrand size = 25, antiderivative size = 170

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^3} dx = -\frac{a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2}{4c\,(b^2-4ac)\,(a+bx^2+cx^4)^2} + \frac{(b^2B-3Abc+2aBc)\,(b+2cx^2)}{4c\,(b^2-4ac)^2\,(a+bx^2+cx^4)} - \frac{(b^2B-3Abc+2aBc)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

output
```
1/4*(-a*(-2*A*c+B*b)-(-A*b*c-2*B*a*c+B*b^2)*x^2)/c/(-4*a*c+b^2)/(c*x^4+b*x
^2+a)^2+1/4*(-3*A*b*c+2*B*a*c+B*b^2)*(2*c*x^2+b)/c/(-4*a*c+b^2)^2/(c*x^4+b
*x^2+a)-(-3*A*b*c+2*B*a*c+B*b^2)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(
-4*a*c+b^2)^(5/2)
```

3.128.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.01

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \frac{1}{4}\left(\frac{(b^2B-3Abc+2aBc)\,(b+2cx^2)}{c\,(b^2-4ac)^2\,(a+bx^2+cx^4)} + \frac{abB+b(bB-Ac)x^2-2ac(A+Bx^2)}{c\,(-b^2+4ac)\,(a+bx^2+cx^4)^2} + \frac{4(b^2B-3Abc+2aBc)\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{5/2}}\right)$$

input
```
Integrate[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

output
```
(((b^2*B - 3*A*b*c + 2*a*B*c)*(b + 2*c*x^2))/(c*(b^2 - 4*a*c)^2*(a + b*x^2
 + c*x^4)) + (a*b*B + b*(b*B - A*c)*x^2 - 2*a*c*(A + B*x^2))/(c*(-b^2 + 4*
a*c)*(a + b*x^2 + c*x^4)^2) + (4*(b^2*B - 3*A*b*c + 2*a*B*c)*ArcTan[(b + 2
*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/4
```

3.128.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1578, 1224, 1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^3}dx$$

$$\Big\downarrow \text{ 1578}$$

$$\frac{1}{2}\int \frac{x^2\left(Bx^2+A\right)}{\left(cx^4+bx^2+a\right)^3}dx^2$$

$$\Big\downarrow \text{ 1224}$$

$$\frac{1}{2}\left(-\frac{\left(2aBc-3Abc+b^2B\right)\int \frac{1}{\left(cx^4+bx^2+a\right)^2}dx^2}{2c\left(b^2-4ac\right)}-\frac{x^2\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}\right)$$

$\downarrow$ 1086

$$\frac{1}{2}\left(-\frac{(2aBc-3Abc+b^2B)\left(-\frac{2c\int\frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)}{2c(b^2-4ac)}-\frac{x^2(-2aBc-Abc+b^2B)+a(bB-2Ac}{2c(b^2-4ac)(a+bx^2+cx^4)^2}\right.$$

$\downarrow$ 1083

$$\frac{1}{2}\left(-\frac{(2aBc-3Abc+b^2B)\left(\frac{4c\int\frac{1}{-x^4+b^2-4ac}d(2cx^2+b)}{b^2-4ac}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)}{2c(b^2-4ac)}-\frac{x^2(-2aBc-Abc+b^2B)+a(bB-}{2c(b^2-4ac)(a+bx^2+cx^4)^2}\right.$$

$\downarrow$ 219

$$\frac{1}{2}\left(-\frac{(2aBc-3Abc+b^2B)\left(\frac{4c\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)}{2c(b^2-4ac)}-\frac{x^2(-2aBc-Abc+b^2B)+a(bB-2}{2c(b^2-4ac)(a+bx^2+cx^4)^2}\right.$$

input
```
Int[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

output
```
(-1/2*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2)/(c*(b^2 - 4*a*c)*(
a + b*x^2 + c*x^4)^2) - ((b^2*B - 3*A*b*c + 2*a*B*c)*(-((b + 2*c*x^2)/((b^
2 - 4*a*c)*(a + b*x^2 + c*x^4))) + (4*c*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4
*a*c]])/(b^2 - 4*a*c)^(3/2)))/(2*c*(b^2 - 4*a*c)))/2
```

3.128.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.128. $\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1224
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x))*((a + b*x + c*x^2)^(p
+ 1)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c
*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c))   Int[(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -
1] &&  !(IntegerQ[p] && NeQ[a, 0] && NiceSqrtQ[b^2 - 4*a*c])
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.128.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.61

method	result
default	$\frac{-\frac{c\left(3Abc-2Bac-B\,b^{2}\right)x^{6}}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}-\frac{3b\left(3Abc-2Bac-B\,b^{2}\right)x^{4}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{\left(5Aabc+A\,b^{3}+2a^{2}Bc-5Ba\,b^{2}\right)x^{2}}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}-\frac{a\left(8Aac+A\,b^{2}-6abB\right)}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{2(c\,x^{4}+b\,x^{2}+a)^{2}}-\frac{(3Abc-2Bac-B\,b^{2})\,\ldots}{(16a^{2}c^{2}-8a\,b^{2}c\ldots}$
risch	$\frac{-\frac{c\left(3Abc-2Bac-B\,b^{2}\right)x^{6}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{3b\left(3Abc-2Bac-B\,b^{2}\right)x^{4}}{4\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{\left(5Aabc+A\,b^{3}+2a^{2}Bc-5Ba\,b^{2}\right)x^{2}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{a\left(8Aac+A\,b^{2}-6abB\right)}{4\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{(c\,x^{4}+b\,x^{2}+a)^{2}}-\frac{3\ln\left(\left(-(-4ac+b^{2})\ldots\right.\right.}{\ldots}$

input
```
int(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-c*(3*A*b*c-2*B*a*c-B*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6-3/2*b*(3*A*
b*c-2*B*a*c-B*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-(5*A*a*b*c+A*b^3+2*B*a^2
*c-5*B*a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-1/2*a*(8*A*a*c+A*b^2-6*B*a*b)
/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)^2-(3*A*b*c-2*B*a*c-B*b^2)/(16
*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(
1/2))
```

3.128.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(162) = 324$.

Time = 0.31 (sec) , antiderivative size = 1226, normalized size of antiderivative = 7.21

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/4*(2*(B*b^4*c - 4*(2*B*a^2 - 3*A*a*b)*c^3 - (2*B*a*b^2 + 3*A*b^3)*c^2)*
x^6 + 6*B*a^2*b^3 - A*a*b^4 + 32*A*a^3*c^2 + 3*(B*b^5 - 4*(2*B*a^2*b - 3*A
*a*b^2)*c^2 - (2*B*a*b^3 + 3*A*b^4)*c)*x^4 + 2*(5*B*a*b^4 - A*b^5 + 4*(2*B
*a^3 + 5*A*a^2*b)*c^2 - (22*B*a^2*b^2 + A*a*b^3)*c)*x^2 - 2*((B*b^2*c^2 +
(2*B*a - 3*A*b)*c^3)*x^8 + 2*(B*b^3*c + (2*B*a*b - 3*A*b^2)*c^2)*x^6 + B*a
^2*b^2 + (B*b^4 + 2*(2*B*a^2 - 3*A*a*b)*c^2 + (4*B*a*b^2 - 3*A*b^3)*c)*x^4
 + 2*(B*a*b^3 + (2*B*a^2*b - 3*A*a*b^2)*c)*x^2 + (2*B*a^3 - 3*A*a^2*b)*c)*
sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)
*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 4*(6*B*a^3*b + A*a^2*b^2)*c)/((
b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a
^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*
b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*
b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64
*a^4*b*c^3)*x^2), 1/4*(2*(B*b^4*c - 4*(2*B*a^2 - 3*A*a*b)*c^3 - (2*B*a*b^2
 + 3*A*b^3)*c^2)*x^6 + 6*B*a^2*b^3 - A*a*b^4 + 32*A*a^3*c^2 + 3*(B*b^5 - 4
*(2*B*a^2*b - 3*A*a*b^2)*c^2 - (2*B*a*b^3 + 3*A*b^4)*c)*x^4 + 2*(5*B*a*b^4
 - A*b^5 + 4*(2*B*a^3 + 5*A*a^2*b)*c^2 - (22*B*a^2*b^2 + A*a*b^3)*c)*x^2 -
 4*((B*b^2*c^2 + (2*B*a - 3*A*b)*c^3)*x^8 + 2*(B*b^3*c + (2*B*a*b - 3*A*b^
2)*c^2)*x^6 + B*a^2*b^2 + (B*b^4 + 2*(2*B*a^2 - 3*A*a*b)*c^2 + (4*B*a*b^2
- 3*A*b^3)*c)*x^4 + 2*(B*a*b^3 + (2*B*a^2*b - 3*A*a*b^2)*c)*x^2 + (2*B*...
```

3.128.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

output
```
Timed out
```

3.128.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.128.8 Giac [A] (verification not implemented)

Time = 1.55 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.34

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \frac{(Bb^2+2\,Bac-3\,Abc)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}} + \frac{2\,Bb^2cx^6+4\,Bac^2x^6-6\,Abc^2x^6+3\,Bb^3x^4+6\,Babcx^4-9\,Ab^2cx^4+10\,Bab^2x^2-2\,Ab^3x^2-4\,Ba^2cx^2}{4\,(cx^4+bx^2+a)^2(b^4-8\,ab^2c+16\,a^2c^2)}$$

input
```
integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
(B*b^2 + 2*B*a*c - 3*A*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4
 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) + 1/4*(2*B*b^2*c*x^6 + 4*B*
a*c^2*x^6 - 6*A*b*c^2*x^6 + 3*B*b^3*x^4 + 6*B*a*b*c*x^4 - 9*A*b^2*c*x^4 +
10*B*a*b^2*x^2 - 2*A*b^3*x^2 - 4*B*a^2*c*x^2 - 10*A*a*b*c*x^2 + 6*B*a^2*b
- A*a*b^2 - 8*A*a^2*c)/((c*x^4 + b*x^2 + a)^2*(b^4 - 8*a*b^2*c + 16*a^2*c^
2))
```

3.128.9 Mupad [B] (verification not implemented)

Time = 7.82 (sec) , antiderivative size = 587, normalized size of antiderivative = 3.45

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$$

$$= \frac{\operatorname{atan}\left(\frac{\left(x^2\left(\frac{(B\,b^2\,c^2-3\,A\,b\,c^3+2\,B\,a\,c^3)\,(B\,b^2-3\,A\,c\,b+2\,B\,a\,c)}{a\,(4\,a\,c-b^2)^{9/2}\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{b\,(B\,b^2-3\,A\,c\,b+2\,B\,a\,c)^2\,(32\,a^2\,b\,c^4-16\,a\,b^3\,c^3+2\,b^5\,c^2)}{2\,a\,(4\,a\,c-b^2)^{15/2}\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}\right)+\frac{2\,b\,c^2\,(B\,b^2-3\,A\,c\,b+2\,B\,\cdots}{(4\,a\,c-b^2)^{15/2}}\cdots}{18\,A^2\,b^2\,c^4-24\,A\,B\,a\,b\,c^4-12\,A\,B\,b^3\,c^3+8\,B^2\,a^2\,c^4+8\,B^2\,a\,b^2\,c^3+2\,B^2\,b^4\cdots}\right.\cdots}{(4\,a\,c-b^2)^{5/2}}$$

$$-\frac{\frac{-6\,B\,a^2\,b+8\,A\,c\,a^2+A\,a\,b^2}{4\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{x^2\,(2\,B\,c\,a^2-5\,B\,a\,b^2+5\,A\,c\,a\,b+A\,b^3)}{2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}-\frac{3\,b\,x^4\,(B\,b^2-3\,A\,c\,b+2\,B\,a\,c)}{4\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}-\frac{c\,x^6\,(B\,b^2-3\,A\,c\,b+2\,B\,a\,c)}{2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}}{x^4\,(b^2+2\,a\,c)+a^2+c^2\,x^8+2\,a\,b\,x^2+2\,b\,c\,x^6}$$

input
```
int((x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

output
```
(atan(((x^2*(((B*b^2*c^2 - 3*A*b*c^3 + 2*B*a*c^3)*(B*b^2 - 3*A*b*c + 2*B*a
*c))/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (b*(B*b^2 -
3*A*b*c + 2*B*a*c)^2*(2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4))/(2*a*(4*a*
c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + (2*b*c^2*(B*b^2 - 3*A*b
*c + 2*B*a*c)^2)/(4*a*c - b^2)^(15/2))*(b^4*(4*a*c - b^2)^5 + 16*a^2*c^2*(
4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5))/(18*A^2*b^2*c^4 + 8*B^2*a^2*c
^4 + 2*B^2*b^4*c^2 - 12*A*B*b^3*c^3 + 8*B^2*a*b^2*c^3 - 24*A*B*a*b*c^4))*(
B*b^2 - 3*A*b*c + 2*B*a*c))/(4*a*c - b^2)^(5/2) - ((A*a*b^2 + 8*A*a^2*c -
6*B*a^2*b)/(4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(A*b^3 - 5*B*a*b^2 +
2*B*a^2*c + 5*A*a*b*c))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (3*b*x^4*(B*b
^2 - 3*A*b*c + 2*B*a*c))/(4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (c*x^6*(B*b^
2 - 3*A*b*c + 2*B*a*c))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c +
b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6)
```

3.129 $\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^3} dx$

3.129.1 Optimal result

Integrand size = 23, antiderivative size = 139

$$\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^3} dx = -\frac{Ab-2aB-(bB-2Ac)x^2}{4(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{3(bB-2Ac)(b+2cx^2)}{4(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{3c(bB-2Ac)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

output
```
1/4*(-A*b+2*B*a+(-2*A*c+B*b)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2-3/4*(-2*A
*c+B*b)*(2*c*x^2+b)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+3*c*(-2*A*c+B*b)*arctan
h((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)
```

3.129.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.02

$$\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^3} dx = \frac{-\frac{3(bB-2Ac)(b+2cx^2)}{a+bx^2+cx^4} + \frac{(b^2-4ac)(B(2a+bx^2)-A(b+2cx^2))}{(a+bx^2+cx^4)^2} - \frac{12c(bB-2Ac)\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{4(b^2-4ac)^2}$$

input
```
Integrate[(x*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

output
```
((-3*(b*B - 2*A*c)*(b + 2*c*x^2))/(a + b*x^2 + c*x^4) + ((b^2 - 4*a*c)*(B*
(2*a + b*x^2) - A*(b + 2*c*x^2)))/(a + b*x^2 + c*x^4)^2 - (12*c*(b*B - 2*A
*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(4*(b^2
- 4*a*c)^2)
```

3.129.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1576, 1159, 1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$$

$$\Big\downarrow \text{1576}$$

$$\frac{1}{2}\int \frac{Bx^2+A}{(cx^4+bx^2+a)^3}dx^2$$

$$\Big\downarrow \text{1159}$$

$$\frac{1}{2}\left(\frac{3(bB-2Ac)\int \frac{1}{(cx^4+bx^2+a)^2}dx^2}{2(b^2-4ac)}-\frac{-2aB-(x^2(bB-2Ac))+Ab}{2(b^2-4ac)(a+bx^2+cx^4)^2}\right)$$

$$\Big\downarrow \text{1086}$$

$$\frac{1}{2}\left(\frac{3(bB-2Ac)\left(-\frac{2c\int \frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)}{2(b^2-4ac)}-\frac{-2aB-(x^2(bB-2Ac))+Ab}{2(b^2-4ac)(a+bx^2+cx^4)^2}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{3(bB-2Ac)\left(\frac{4c\int \frac{1}{-x^4+b^2-4ac}d(2cx^2+b)}{b^2-4ac}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)}{2(b^2-4ac)}-\frac{-2aB-(x^2(bB-2Ac))+Ab}{2(b^2-4ac)(a+bx^2+cx^4)^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{3(bB-2Ac)\left(\frac{4c\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)}{2(b^2-4ac)}-\frac{-2aB-(x^2(bB-2Ac))+Ab}{2(b^2-4ac)(a+bx^2+cx^4)^2}\right)$$

input
```
Int[(x*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

output
```
(-1/2*(A*b - 2*a*B - (b*B - 2*A*c)*x^2)/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)
^2) + (3*(b*B - 2*A*c)*(-((b + 2*c*x^2)/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)
)) + (4*c*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)))/
(2*(b^2 - 4*a*c)))/2
```

3.129.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

3.129.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.06

method	result
default	$\frac{(2Ac-Bb)x^2+Ab-2Ba}{4(4ac-b^2)(c\,x^4+b\,x^2+a)^2}+\frac{3(2Ac-Bb)\left(\frac{2c\,x^2+b}{\left(4ac-b^2\right)\left(c\,x^4+b\,x^2+a\right)}+\frac{4c\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}\right)}{4(4ac-b^2)}$
risch	$\frac{\frac{3c^2(2Ac-Bb)x^6}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{9bc(2Ac-Bb)x^4}{4\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{\left(5ac+b^2\right)(2Ac-Bb)x^2}{32a^2c^2-16a\,b^2c+2b^4}+\frac{10Aabc-A\,b^3-8a^2Bc-Ba\,b^2}{64a^2c^2-32a\,b^2c+4b^4}}{(c\,x^4+b\,x^2+a)^2}-\frac{3c^2\ln\left(\left(\left(-4ac+b^2\right)^{\frac{5}{2}}-16a^2b\,c\right.\right.}{}$

input
```
int(x*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*((2*A*c-B*b)*x^2+A*b-2*B*a)/(4*a*c-b^2)/(c*x^4+b*x^2+a)^2+3/4*(2*A*c-B
*b)/(4*a*c-b^2)*((2*c*x^2+b)/(4*a*c-b^2)/(c*x^4+b*x^2+a)+4*c/(4*a*c-b^2)^(
3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

3.129.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 542 vs. 2(131) = 262.

Time = 0.30 (sec) , antiderivative size = 1109, normalized size of antiderivative = 7.98

$$\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/4*(6*(B*b^3*c^2 + 8*A*a*c^4 - 2*(2*B*a*b + A*b^2)*c^3)*x^6 + B*a*b^4 +
 A*b^5 + 9*(B*b^4*c + 8*A*a*b*c^3 - 2*(2*B*a*b^2 + A*b^3)*c^2)*x^4 - 8*(4*
B*a^3 - 5*A*a^2*b)*c^2 + 2*(B*b^5 + 40*A*a^2*c^3 - 2*(10*B*a^2*b + A*a*b^2
)*c^2 + (B*a*b^3 - 2*A*b^4)*c)*x^2 + 6*((B*b*c^3 - 2*A*c^4)*x^8 + 2*(B*b^2
*c^2 - 2*A*b*c^3)*x^6 + B*a^2*b*c - 2*A*a^2*c^2 + (B*b^3*c - 4*A*a*c^3 + 2
*(B*a*b - A*b^2)*c^2)*x^4 + 2*(B*a*b^2*c - 2*A*a*b*c^2)*x^2)*sqrt(b^2 - 4*
a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4
*a*c))/(c*x^4 + b*x^2 + a)) + 2*(2*B*a^2*b^2 - 7*A*a*b^3)*c)/((b^6*c^2 - 1
2*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c +
48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 6
4*a^3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 1
28*a^4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)
*x^2), -1/4*(6*(B*b^3*c^2 + 8*A*a*c^4 - 2*(2*B*a*b + A*b^2)*c^3)*x^6 + B*a
*b^4 + A*b^5 + 9*(B*b^4*c + 8*A*a*b*c^3 - 2*(2*B*a*b^2 + A*b^3)*c^2)*x^4 -
 8*(4*B*a^3 - 5*A*a^2*b)*c^2 + 2*(B*b^5 + 40*A*a^2*c^3 - 2*(10*B*a^2*b + A
*a*b^2)*c^2 + (B*a*b^3 - 2*A*b^4)*c)*x^2 - 12*((B*b*c^3 - 2*A*c^4)*x^8 + 2
*(B*b^2*c^2 - 2*A*b*c^3)*x^6 + B*a^2*b*c - 2*A*a^2*c^2 + (B*b^3*c - 4*A*a*
c^3 + 2*(B*a*b - A*b^2)*c^2)*x^4 + 2*(B*a*b^2*c - 2*A*a*b*c^2)*x^2)*sqrt(-
b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + 2*(
2*B*a^2*b^2 - 7*A*a*b^3)*c)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -...
```

3.129.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 661 vs. $2(134) = 268$.

Time = 165.16 (sec) , antiderivative size = 661, normalized size of antiderivative = 4.76

$$\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$$

$$= \frac{3c\sqrt{-\frac{1}{(4ac-b^2)^5}}(-2Ac+Bb)\log\left(x^2+\frac{-6Abc^2+3Bb^2c-192a^3c^4\sqrt{-\frac{1}{(4ac-b^2)^5}}(-2Ac+Bb)+144a^2b^2c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}(-2Ac+B\cdots}{-12Ac^3+6Bbc^2\cdots}\right.}{2}$$

$$-\frac{3c\sqrt{-\frac{1}{(4ac-b^2)^5}}(-2Ac+Bb)\log\left(x^2+\frac{-6Abc^2+3Bb^2c+192a^3c^4\sqrt{-\frac{1}{(4ac-b^2)^5}}(-2Ac+Bb)-144a^2b^2c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}(-2Ac\cdots}{-12Ac^3+6Bb\cdots}\right.}{2}$$

$$+\frac{10Aabc-Ab^3-8Ba^2c-Bab^2+x^6\cdot(12Ac^3-6Bbc^2)+x^4\cdot(18Abc^2-9Bb^2c)+x\cdots}{64a^4c^2-32a^3b^2c+4a^2b^4+x^8\cdot(64a^2c^4-32ab^2c^3+4b^4c^2)+x^6\cdot(128a^2bc^3-64ab^3c^2+8b^5c)+x^4\cdot(1\cdots}$$

input
```
integrate(x*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

output
```
3*c*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b)*log(x**2 + (-6*A*b*c**2 + 3*
B*b**2*c - 192*a**3*c**4*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b) + 144*a
**2*b**2*c**3*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b) - 36*a*b**4*c**2*s
qrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b) + 3*b**6*c*sqrt(-1/(4*a*c - b**2)
**5)*(-2*A*c + B*b))/(-12*A*c**3 + 6*B*b*c**2))/2 - 3*c*sqrt(-1/(4*a*c - b
**2)**5)*(-2*A*c + B*b)*log(x**2 + (-6*A*b*c**2 + 3*B*b**2*c + 192*a**3*c*
*4*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b) - 144*a**2*b**2*c**3*sqrt(-1/
(4*a*c - b**2)**5)*(-2*A*c + B*b) + 36*a*b**4*c**2*sqrt(-1/(4*a*c - b**2)*
*5)*(-2*A*c + B*b) - 3*b**6*c*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b))/(
-12*A*c**3 + 6*B*b*c**2))/2 + (10*A*a*b*c - A*b**3 - 8*B*a**2*c - B*a*b**2
 + x**6*(12*A*c**3 - 6*B*b*c**2) + x**4*(18*A*b*c**2 - 9*B*b**2*c) + x**2*
(20*A*a*c**2 + 4*A*b**2*c - 10*B*a*b*c - 2*B*b**3))/(64*a**4*c**2 - 32*a**
3*b**2*c + 4*a**2*b**4 + x**8*(64*a**2*c**4 - 32*a*b**2*c**3 + 4*b**4*c**2
) + x**6*(128*a**2*b*c**3 - 64*a*b**3*c**2 + 8*b**5*c) + x**4*(128*a**3*c*
*3 - 24*a*b**4*c + 4*b**6) + x**2*(128*a**3*b*c**2 - 64*a**2*b**3*c + 8*a*
b**5))
```

3.129.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.129.8 Giac [A] (verification not implemented)

Time = 1.45 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.50

$$\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = -\frac{3\,(Bbc-2\,Ac^2)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}}$$
$$-\frac{6\,Bbc^2x^6-12\,Ac^3x^6+9\,Bb^2cx^4-18\,Abc^2x^4+2\,Bb^3x^2+10\,Babcx^2-4\,Ab^2cx^2-20\,Aac^2x^2+Bab^2-}{4\,(cx^4+bx^2+a)^2(b^4-8\,ab^2c+16\,a^2c^2)}$$

input
```
integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
-3*(B*b*c - 2*A*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*
b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) - 1/4*(6*B*b*c^2*x^6 - 12*A*c^3*x^
6 + 9*B*b^2*c*x^4 - 18*A*b*c^2*x^4 + 2*B*b^3*x^2 + 10*B*a*b*c*x^2 - 4*A*b^
2*c*x^2 - 20*A*a*c^2*x^2 + B*a*b^2 + A*b^3 + 8*B*a^2*c - 10*A*a*b*c)/((c*x
^4 + b*x^2 + a)^2*(b^4 - 8*a*b^2*c + 16*a^2*c^2))
```

3.129.9 Mupad [B] (verification not implemented)

Time = 7.74 (sec) , antiderivative size = 517, normalized size of antiderivative = 3.72

$$\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$$
$$= \frac{3\,c\,\mathrm{atan}\left(\frac{\left(x^2\left(\frac{3\,c\,(2\,A\,c-B\,b)\,\left(6\,A\,c^4-3\,B\,b\,c^3\right)}{a\,\left(4\,a\,c-b^2\right)^{9/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{9\,b\,c^2\,(2\,A\,c-B\,b)^2\,\left(32\,a^2\,b\,c^4-16\,a\,b^3\,c^3+2\,b^5\,c^2\right)}{2\,a\,\left(4\,a\,c-b^2\right)^{15/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}\right)+\frac{18\,b\,c^4\,(2\,A\,c-B\,b)^2}{\left(4\,a\,c-b^2\right)^{15/2}}\right)\,\left(b^4\,\left(4\,a\,c-b^2\right)^5\cdots}{72\,A^2\,c^6-72\,A\,B\,b\,c^5+18\,B^2\,b^2\,c^4}\cdots}{\left(4\,a\,c-b^2\right)^{5/2}\cdots}$$
$$-\frac{\frac{8\,B\,c\,a^2+B\,a\,b^2-10\,A\,c\,a\,b+A\,b^3}{4\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}-\frac{9\,x^4\,\left(2\,A\,b\,c^2-B\,b^2\,c\right)}{4\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{x^2\,\left(B\,b^3-2\,A\,b^2\,c+5\,B\,a\,b\,c-10\,A\,a\,c^2\right)}{2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}-\frac{3\,c^2\,x^6\,(2\,A\,c-B\,b)}{2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}}{x^4\,\left(b^2+2\,a\,c\right)+a^2+c^2\,x^8+2\,a\,b\,x^2+2\,b\,c\,x^6}$$

input
```
int((x*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

output
```
(3*c*atan(((x^2*((3*c*(2*A*c - B*b)*(6*A*c^4 - 3*B*b*c^3))/(a*(4*a*c - b^2
)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b*c^2*(2*A*c - B*b)^2*(2*b^5*
c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4))/(2*a*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^
2*c^2 - 8*a*b^2*c))) + (18*b*c^4*(2*A*c - B*b)^2)/(4*a*c - b^2)^(15/2))*(b
^4*(4*a*c - b^2)^5 + 16*a^2*c^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^
5))/(72*A^2*c^6 + 18*B^2*b^2*c^4 - 72*A*B*b*c^5))*(2*A*c - B*b))/(4*a*c -
b^2)^(5/2) - ((A*b^3 + B*a*b^2 + 8*B*a^2*c - 10*A*a*b*c)/(4*(b^4 + 16*a^2*
c^2 - 8*a*b^2*c)) - (9*x^4*(2*A*b*c^2 - B*b^2*c))/(4*(b^4 + 16*a^2*c^2 - 8
*a*b^2*c)) + (x^2*(B*b^3 - 10*A*a*c^2 - 2*A*b^2*c + 5*B*a*b*c))/(2*(b^4 +
16*a^2*c^2 - 8*a*b^2*c)) - (3*c^2*x^6*(2*A*c - B*b))/(2*(b^4 + 16*a^2*c^2
- 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6)
```

3.129. $\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

3.130 $\int \frac{A+Bx^2}{x(a+bx^2+cx^4)^3} dx$

3.130.1 Optimal result

Integrand size = 25, antiderivative size = 252

$$
\begin{aligned}
\int \frac{A+Bx^2}{x(a+bx^2+cx^4)^3}\,dx
&= -\frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{4a(b^2-4ac)(a+bx^2+cx^4)^2} \\
&\quad+\frac{6a^2bBc+A(2b^4-15ab^2c+16a^2c^2)+2c(6a^2Bc+A(b^3-7abc))x^2}{4a^2(b^2-4ac)^2(a+bx^2+cx^4)} \\
&\quad-\frac{(12a^3Bc^2-A(b^5-10ab^3c+30a^2bc^2))\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3(b^2-4ac)^{5/2}} \\
&\quad+\frac{A\log(x)}{a^3}-\frac{A\log(a+bx^2+cx^4)}{4a^3}
\end{aligned}
$$

output
```
1/4*(-a*b*B+A*(-2*a*c+b^2)+(A*b-2*B*a)*c*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+
a)^2+1/4*(6*a^2*b*B*c+A*(16*a^2*c^2-15*a*b^2*c+2*b^4)+2*c*(6*a^2*B*c+A*(-7
*a*b*c+b^3))*x^2)/a^2/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)-1/2*(12*a^3*B*c^2-A*(
30*a^2*b*c^2-10*a*b^3*c+b^5))*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^3/
(-4*a*c+b^2)^(5/2)+A*ln(x)/a^3-1/4*A*ln(c*x^4+b*x^2+a)/a^3
```

3.130.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.57

$$\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)^3}\,dx$$
$$= \frac{\frac{a^2\left(-aB\left(b+2cx^2\right)+A\left(b^2-2ac+bcx^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{a\left(2Ab^3\left(b+cx^2\right)-aAbc\left(15b+14cx^2\right)+2a^2c\left(3bB+8Ac+6Bcx^2\right)\right)}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+4A\log(x)-\frac{\left(-12a^3Bc^2+A\right.}{}}{}$$

input
```
Integrate[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)^3),x]
```

output
```
((a^2*(-(a*B*(b + 2*c*x^2)) + A*(b^2 - 2*a*c + b*c*x^2)))/((b^2 - 4*a*c)*(
a + b*x^2 + c*x^4)^2) + (a*(2*A*b^3*(b + c*x^2) - a*A*b*c*(15*b + 14*c*x^2
) + 2*a^2*c*(3*b*B + 8*A*c + 6*B*c*x^2)))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*
x^4)) + 4*A*Log[x] - ((-12*a^3*B*c^2 + A*(b^5 - 10*a*b^3*c + 30*a^2*b*c^2
+ b^4*Sqrt[b^2 - 4*a*c] - 8*a*b^2*c*Sqrt[b^2 - 4*a*c] + 16*a^2*c^2*Sqrt[b^
2 - 4*a*c]))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2) - (
(12*a^3*B*c^2 + A*(-b^5 + 10*a*b^3*c - 30*a^2*b*c^2 + b^4*Sqrt[b^2 - 4*a*c
] - 8*a*b^2*c*Sqrt[b^2 - 4*a*c] + 16*a^2*c^2*Sqrt[b^2 - 4*a*c]))*Log[b + S
qrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2))/(4*a^3)
```

3.130.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1578, 1235, 25, 1235, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)^3}\,dx$$
$$\Big\downarrow \ 1578$$
$$\frac{1}{2}\int \frac{Bx^2+A}{x^2\left(cx^4+bx^2+a\right)^3}dx^2$$
$$\Big\downarrow \ 1235$$

$$\frac{1}{2}\left(\frac{cx^2(Ab-2aB)-2aAc-abB+Ab^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{\int -\frac{3(Ab-2aB)cx^2+2A\left(b^2-4ac\right)}{x^2\left(cx^4+bx^2+a\right)^2}dx^2}{2a\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\left(\frac{\int \frac{3(Ab-2aB)cx^2+2A\left(b^2-4ac\right)}{x^2\left(cx^4+bx^2+a\right)^2}dx^2}{2a\left(b^2-4ac\right)}+\frac{cx^2(Ab-2aB)-2aAc-abB+Ab^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}\right)$$

$$\Big\downarrow \; 1235$$

$$\frac{1}{2}\left(\frac{\frac{2cx^2\left(6a^2Bc-7aAbc+Ab^3\right)+A\left(16a^2c^2-15ab^2c+2b^4\right)+6a^2bBc}{a(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int -\frac{2\left(A\left(b^2-4ac\right)^2+c\left(6Bca^2+A\left(b^3-7abc\right)\right)x^2\right)}{x^2\left(cx^4+bx^2+a\right)}dx^2}{a(b^2-4ac)}}{2a\left(b^2-4ac\right)}+\frac{cx^2(Ab-2aB)}{2a\left(b^2-4ac\right.}\right.$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\left(\frac{\frac{2\int \frac{A\left(b^2-4ac\right)^2+c\left(6Bca^2+A\left(b^3-7abc\right)\right)x^2}{x^2\left(cx^4+bx^2+a\right)}dx^2}{a(b^2-4ac)}+\frac{2cx^2\left(6a^2Bc-7aAbc+Ab^3\right)+A\left(16a^2c^2-15ab^2c+2b^4\right)+6a^2bBc}{a(b^2-4ac)(a+bx^2+cx^4)}}{2a\left(b^2-4ac\right)}+\frac{cx^2(Ab-2aB)-}{2a\left(b^2-4ac\right)(}\right.$$

$$\Big\downarrow \; 1200$$

$$\frac{1}{2}\left(\frac{\frac{2\int \left(\frac{A\left(4ac-b^2\right)^2}{ax^2}+\frac{6Bc^2a^3-Ac\left(b^2-4ac\right)^2x^2-A\left(b^5-9acb^3+23a^2c^2b\right)}{a\left(cx^4+bx^2+a\right)}\right)dx^2}{a(b^2-4ac)}+\frac{2cx^2\left(6a^2Bc-7aAbc+Ab^3\right)+A\left(16a^2c^2-15ab^2c+2b^4\right)+6a^2bBc}{a(b^2-4ac)(a+bx^2+cx^4)}}{2a\left(b^2-4ac\right)}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{\frac{2cx^2\left(6a^2Bc-7aAbc+Ab^3\right)+A\left(16a^2c^2-15ab^2c+2b^4\right)+6a^2bBc}{a(b^2-4ac)(a+bx^2+cx^4)}+\frac{2\left(-\frac{\left(12a^3Bc^2-A\left(30a^2bc^2-10ab^3c+b^5\right)\right)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}+\frac{A\log\left(x^2\right)}{}\right.}{a(b^2-4ac)}}{2a\left(b^2-4ac\right)}\right.$$

3.130. $\int \frac{A+Bx^2}{x(a+bx^2+cx^4)^3}\,dx$

input
```
Int[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)^3),x]
```

output
```
((A*b^2 - a*b*B - 2*a*A*c + (A*b - 2*a*B)*c*x^2)/(2*a*(b^2 - 4*a*c)*(a + b
*x^2 + c*x^4)^2) + ((6*a^2*b*B*c + A*(2*b^4 - 15*a*b^2*c + 16*a^2*c^2) + 2
*c*(A*b^3 - 7*a*A*b*c + 6*a^2*B*c)*x^2)/(a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)) + (2*(-(((12*a^3*B*c^2 - A*(b^5 - 10*a*b^3*c + 30*a^2*b*c^2))*ArcTanh[
(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c])) + (A*(b^2 - 4*a*c
)^2*Log[x^2])/a - (A*(b^2 - 4*a*c)^2*Log[a + b*x^2 + c*x^4])/(2*a)))/(a*(b
^2 - 4*a*c)))/(2*a*(b^2 - 4*a*c)))/2
```

3.130.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.130.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.75

method	result
default	$\frac{A\ln(x)}{a^3}-\frac{\frac{\frac{a c^2\left(7Aabc-A b^3-6a^2Bc\right)x^6}{16a^2c^2-8a b^2c+b^4}-\frac{ac\left(16A a^2c^2-29Aa b^2c+4A b^4+18a^2bBc\right)x^4}{2\left(16a^2c^2-8a b^2c+b^4\right)}+\frac{a\left(A a^2b c^2+6Aa b^3c-A b^5-10a^3B c^2-2B a^2b^2c\right)x^2}{16a^2c^2-8a b^2c+b^4}}{\left(c x^4+b x^2+a\right)^2}}{}$
risch	$\frac{-\frac{c^2\left(7Aabc-A b^3-6a^2Bc\right)x^6}{2a^2\left(16a^2c^2-8a b^2c+b^4\right)}+\frac{c\left(16A a^2c^2-29Aa b^2c+4A b^4+18a^2bBc\right)x^4}{4\left(16a^2c^2-8a b^2c+b^4\right)a^2}-\frac{\left(A a^2b c^2+6Aa b^3c-A b^5-10a^3B c^2-2B a^2b^2c\right)x^2}{2a^2\left(16a^2c^2-8a b^2c+b^4\right)}+\frac{24A a^2c^2-21}{4a}}{(c x^4+b x^2+a)^2}$

input
```
int((B*x^2+A)/x/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
A*ln(x)/a^3-1/2/a^3*((a*c^2*(7*A*a*b*c-A*b^3-6*B*a^2*c)/(16*a^2*c^2-8*a*b^
2*c+b^4)*x^6-1/2*a*c*(16*A*a^2*c^2-29*A*a*b^2*c+4*A*b^4+18*B*a^2*b*c)/(16*
a^2*c^2-8*a*b^2*c+b^4)*x^4+a*(A*a^2*b*c^2+6*A*a*b^3*c-A*b^5-10*B*a^3*c^2-2
*B*a^2*b^2*c)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-1/2*a^2*(24*A*a^2*c^2-21*A*a*
b^2*c+3*A*b^4+10*B*a^2*b*c-B*a*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x
^2+a)^2+1/(16*a^2*c^2-8*a*b^2*c+b^4)*(1/2*(16*A*a^2*c^3-8*A*a*b^2*c^2+A*b^
4*c)/c*ln(c*x^4+b*x^2+a)+2*(23*A*a^2*b*c^2-9*A*a*b^3*c+A*b^5-6*a^3*B*c^2-1
/2*(16*A*a^2*c^3-8*A*a*b^2*c^2+A*b^4*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c
*x^2+b)/(4*a*c-b^2)^(1/2))))
```

3.130.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1235 vs. $2(240) = 480$.

Time = 1.95 (sec) , antiderivative size = 2494, normalized size of antiderivative = 9.90

$$\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/4*(B*a^3*b^5 - 3*A*a^2*b^6 + 96*A*a^5*c^3 - 2*(A*a*b^5*c^2 - 4*(6*B*a^
4 - 7*A*a^3*b)*c^4 + (6*B*a^3*b^2 - 11*A*a^2*b^3)*c^3)*x^6 - (4*A*a*b^6*c
- 64*A*a^4*c^4 - 12*(6*B*a^4*b - 11*A*a^3*b^2)*c^3 + 9*(2*B*a^3*b^3 - 5*A*
a^2*b^4)*c^2)*x^4 + 4*(10*B*a^5*b - 27*A*a^4*b^2)*c^2 - 2*(A*a*b^7 - 4*(10
*B*a^5 - A*a^4*b)*c^3 + (2*B*a^4*b^2 + 23*A*a^3*b^3)*c^2 + 2*(B*a^3*b^4 -
5*A*a^2*b^5)*c)*x^2 - ((A*b^5*c^2 - 10*A*a*b^3*c^3 - 6*(2*B*a^3 - 5*A*a^2*
b)*c^4)*x^8 + A*a^2*b^5 - 10*A*a^3*b^3*c + 2*(A*b^6*c - 10*A*a*b^4*c^2 - 6
*(2*B*a^3*b - 5*A*a^2*b^2)*c^3)*x^6 + (A*b^7 - 8*A*a*b^5*c - 12*(2*B*a^4 -
 5*A*a^3*b)*c^3 - 2*(6*B*a^3*b^2 - 5*A*a^2*b^3)*c^2)*x^4 - 6*(2*B*a^5 - 5*
A*a^4*b)*c^2 + 2*(A*a*b^6 - 10*A*a^2*b^4*c - 6*(2*B*a^4*b - 5*A*a^3*b^2)*c
^2)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c
*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (14*B*a^4*b^3 - 33*A*a
^3*b^4)*c + (A*a^2*b^6 - 12*A*a^3*b^4*c + 48*A*a^4*b^2*c^2 - 64*A*a^5*c^3
+ (A*b^6*c^2 - 12*A*a*b^4*c^3 + 48*A*a^2*b^2*c^4 - 64*A*a^3*c^5)*x^8 + 2*(
A*b^7*c - 12*A*a*b^5*c^2 + 48*A*a^2*b^3*c^3 - 64*A*a^3*b*c^4)*x^6 + (A*b^8
 - 10*A*a*b^6*c + 24*A*a^2*b^4*c^2 + 32*A*a^3*b^2*c^3 - 128*A*a^4*c^4)*x^4
 + 2*(A*a*b^7 - 12*A*a^2*b^5*c + 48*A*a^3*b^3*c^2 - 64*A*a^4*b*c^3)*x^2)*l
og(c*x^4 + b*x^2 + a) - 4*(A*a^2*b^6 - 12*A*a^3*b^4*c + 48*A*a^4*b^2*c^2 -
 64*A*a^5*c^3 + (A*b^6*c^2 - 12*A*a*b^4*c^3 + 48*A*a^2*b^2*c^4 - 64*A*a^3*
c^5)*x^8 + 2*(A*b^7*c - 12*A*a*b^5*c^2 + 48*A*a^2*b^3*c^3 - 64*A*a^3*b*...
```

3.130.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((B*x**2+A)/x/(c*x**4+b*x**2+a)**3,x)
```

output
```
Timed out
```

3.130.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx^2}{x(a+bx^2+cx^4)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.130.8 Giac [A] (verification not implemented)

Time = 1.51 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.67

$$\int \frac{A+Bx^2}{x(a+bx^2+cx^4)^3}\,dx = -\frac{(Ab^5-10\,Aab^3c-12\,Ba^3c^2+30\,Aa^2bc^2)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(a^3b^4-8\,a^4b^2c+16\,a^5c^2)\sqrt{-b^2+4\,ac}} - \frac{A\log(cx^4+bx^2+a)}{4\,a^3} + \frac{A\log(x^2)}{2\,a^3} + \frac{3\,Ab^4c^2x^8-24\,Aab^2c^3x^8+48\,Aa^2c^4x^8+6\,Ab^5cx^6-44\,Aab^3c^2x^6+24\,Ba^3c^3x^6+68\,Aa^2bc^3x^6+3\,Ab^6x}{}$$

input
```
integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
-1/2*(A*b^5 - 10*A*a*b^3*c - 12*B*a^3*c^2 + 30*A*a^2*b*c^2)*arctan((2*c*x^
2 + b)/sqrt(-b^2 + 4*a*c))/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt(-b^2
 + 4*a*c)) - 1/4*A*log(c*x^4 + b*x^2 + a)/a^3 + 1/2*A*log(x^2)/a^3 + 1/8*(
3*A*b^4*c^2*x^8 - 24*A*a*b^2*c^3*x^8 + 48*A*a^2*c^4*x^8 + 6*A*b^5*c*x^6 -
44*A*a*b^3*c^2*x^6 + 24*B*a^3*c^3*x^6 + 68*A*a^2*b*c^3*x^6 + 3*A*b^6*x^4 -
 10*A*a*b^4*c*x^4 + 36*B*a^3*b*c^2*x^4 - 58*A*a^2*b^2*c^2*x^4 + 128*A*a^3*
c^3*x^4 + 10*A*a*b^5*x^2 + 8*B*a^3*b^2*c*x^2 - 72*A*a^2*b^3*c*x^2 + 40*B*a
^4*c^2*x^2 + 92*A*a^3*b*c^2*x^2 - 2*B*a^3*b^3 + 9*A*a^2*b^4 + 20*B*a^4*b*c
 - 66*A*a^3*b^2*c + 96*A*a^4*c^2)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(c
*x^4 + b*x^2 + a)^2)
```

3.130.9 Mupad [B] (verification not implemented)

Time = 14.34 (sec) , antiderivative size = 11674, normalized size of antiderivative = 46.33

$$\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^2)/(x*(a + b*x^2 + c*x^4)^3),x)
```

output
```
((3*A*b^4 + 24*A*a^2*c^2 - B*a*b^3 - 21*A*a*b^2*c + 10*B*a^2*b*c)/(4*a*(b^
4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(A*b^5 + 10*B*a^3*c^2 - 6*A*a*b^3*c -
A*a^2*b*c^2 + 2*B*a^2*b^2*c))/(2*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^
4*(16*A*a^2*c^3 + 4*A*b^4*c - 29*A*a*b^2*c^2 + 18*B*a^2*b*c^2))/(4*a^2*(b^
4 + 16*a^2*c^2 - 8*a*b^2*c)) + (c^2*x^6*(A*b^3 + 6*B*a^2*c - 7*A*a*b*c))/(
2*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8
+ 2*a*b*x^2 + 2*b*c*x^6) + (A*log(x))/a^3 - (log(((c^5*x^2*(A*b^3 + 6*B*a^
2*c - 7*A*a*b*c)^3)/(a^6*(4*a*c - b^2)^6) - ((A + a^3*(-(A*b^5 - 12*B*a^3*
c^2 - 10*A*a*b^3*c + 30*A*a^2*b*c^2)^2/(a^6*(4*a*c - b^2)^5))^(1/2))*((c^3
*(4*A^2*b^8 - 36*B^2*a^5*c^3 + 302*A^2*a^2*b^4*c^2 - 497*A^2*a^3*b^2*c^3 -
 61*A^2*a*b^6*c - 204*A*B*a^3*b^3*c^2 + 24*A*B*a^2*b^5*c + 468*A*B*a^4*b*c
^3))/(a^4*(4*a*c - b^2)^4) - ((A + a^3*(-(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^
3*c + 30*A*a^2*b*c^2)^2/(a^6*(4*a*c - b^2)^5))^(1/2))*((2*c^3*x^2*(A*b^5 +
 60*B*a^3*c^2 - 2*A*a*b^3*c + 10*A*a^2*b*c^2 - 24*B*a^2*b^2*c))/(a^2*(4*a*
c - b^2)^2) + (4*b*c^2*(A*b^5 - 6*B*a^3*c^2 - 9*A*a*b^3*c + 23*A*a^2*b*c^2
))/(a^2*(4*a*c - b^2)^2) + (b*c^2*(A + a^3*(-(A*b^5 - 12*B*a^3*c^2 - 10*A*
a*b^3*c + 30*A*a^2*b*c^2)^2/(a^6*(4*a*c - b^2)^5))^(1/2))*(a*b + 3*b^2*x^2
 - 10*a*c*x^2))/a^3))/(4*a^3) + (c^4*x^2*(6*A^2*b^7 + 409*A^2*a^2*b^3*c^2
+ 480*A*B*a^4*c^3 - 89*A^2*a*b^5*c - 560*A^2*a^3*b*c^3 + 36*B^2*a^4*b*c^2
- 324*A*B*a^3*b^2*c^2 + 42*A*B*a^2*b^4*c))/(a^4*(4*a*c - b^2)^4)))/(4*a...
```

3.131 $\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)^3}\,dx$

3.131.1 Optimal result

Integrand size = 25, antiderivative size = 363

$$\begin{aligned}
&\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)^3}\,dx \\
&= \frac{abB(b^2-7ac)-3A(b^4-7ab^2c+10a^2c^2)}{2a^3\left(b^2-4ac\right)^2x^2} - \frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2} \\
&\quad - \frac{abB(b^2-10ac)-A(3b^4-20ab^2c+20a^2c^2)+c(aB(b^2-16ac)-3A(b^3-6abc))x^2}{4a^2\left(b^2-4ac\right)^2x^2\left(a+bx^2+cx^4\right)} \\
&\quad + \frac{(abB(b^4-10ab^2c+30a^2c^2)-3A(b^6-10ab^4c+30a^2b^2c^2-20a^3c^3))\,\mathrm{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^4\left(b^2-4ac\right)^{5/2}} \\
&\quad - \frac{(3Ab-aB)\log(x)}{a^4} + \frac{(3Ab-aB)\log\left(a+bx^2+cx^4\right)}{4a^4}
\end{aligned}$$

output
```
1/2*(a*b*B*(-7*a*c+b^2)-3*A*(10*a^2*c^2-7*a*b^2*c+b^4))/a^3/(-4*a*c+b^2)^2
/x^2+1/4*(-a*b*B+A*(-2*a*c+b^2)+(A*b-2*B*a)*c*x^2)/a/(-4*a*c+b^2)/x^2/(c*x
^4+b*x^2+a)^2+1/4*(-a*b*B*(-10*a*c+b^2)+A*(20*a^2*c^2-20*a*b^2*c+3*b^4)-c*
(a*B*(-16*a*c+b^2)-3*A*(-6*a*b*c+b^3))*x^2)/a^2/(-4*a*c+b^2)^2/x^2/(c*x^4+
b*x^2+a)+1/2*(a*b*B*(30*a^2*c^2-10*a*b^2*c+b^4)-3*A*(-20*a^3*c^3+30*a^2*b^
2*c^2-10*a*b^4*c+b^6))*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^4/(-4*a*c
+b^2)^(5/2)-(3*A*b-B*a)*ln(x)/a^4+1/4*(3*A*b-B*a)*ln(c*x^4+b*x^2+a)/a^4
```

3.131.2 Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 642, normalized size of antiderivative = 1.77

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)^3}\,dx$$
$$= \frac{-\frac{2aA}{x^2} - \frac{a^2\left(aB\left(-b^2+2ac-bcx^2\right)+A\left(b^3-3abc+b^2cx^2-2ac^2x^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{a\left(aB\left(2b^4-15ab^2c+16a^2c^2+2b^3cx^2-14abc^2x^2\right)-A\left(4b^5-29ab^3c+46a^2b\right.\right.}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}}{}$$

input
```
Integrate[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^3),x]
```

output
```
((-2*a*A)/x^2 - (a^2*(a*B*(-b^2 + 2*a*c - b*c*x^2) + A*(b^3 - 3*a*b*c + b^
2*c*x^2 - 2*a*c^2*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (a*(a*B*(
2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b^3*c*x^2 - 14*a*b*c^2*x^2) - A*(4*b^5
 - 29*a*b^3*c + 46*a^2*b*c^2 + 4*b^4*c*x^2 - 26*a*b^2*c^2*x^2 + 28*a^2*c^3
*x^2)))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + 4*(-3*A*b + a*B)*Log[x] +
((-(a*B*(b^5 - 10*a*b^3*c + 30*a^2*b*c^2 + b^4*Sqrt[b^2 - 4*a*c] - 8*a*b^2
*c*Sqrt[b^2 - 4*a*c] + 16*a^2*c^2*Sqrt[b^2 - 4*a*c])) + 3*A*(b^6 - 10*a*b^
4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*a*c] - 8*a*b^3*c*Sqrt
[b^2 - 4*a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c]))*Log[b - Sqrt[b^2 - 4*a*c]
 + 2*c*x^2])/(b^2 - 4*a*c)^(5/2) + ((a*B*(b^5 - 10*a*b^3*c + 30*a^2*b*c^2
- b^4*Sqrt[b^2 - 4*a*c] + 8*a*b^2*c*Sqrt[b^2 - 4*a*c] - 16*a^2*c^2*Sqrt[b^
2 - 4*a*c]) + 3*A*(-b^6 + 10*a*b^4*c - 30*a^2*b^2*c^2 + 20*a^3*c^3 + b^5*S
qrt[b^2 - 4*a*c] - 8*a*b^3*c*Sqrt[b^2 - 4*a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4
*a*c]))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2))/(4*a^4)
```

3.131.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1578, 1235, 25, 1235, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)^3}\,dx$$

$$\Big\downarrow \; 1578$$

$$\frac{1}{2}\int \frac{Bx^2+A}{x^4\left(cx^4+bx^2+a\right)^3}dx^2$$

$$\Big\downarrow \text{1235}$$

$$\frac{1}{2}\left(\frac{cx^2(Ab-2aB)-2aAc-abB+Ab^2}{2ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{\int -\frac{3Ab^2-aBb+4(Ab-2aB)cx^2-10aAc}{x^4\left(cx^4+bx^2+a\right)^2}dx^2}{2a\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{\int \frac{3Ab^2-aBb+4(Ab-2aB)cx^2-10aAc}{x^4\left(cx^4+bx^2+a\right)^2}dx^2}{2a\left(b^2-4ac\right)}+\frac{cx^2(Ab-2aB)-2aAc-abB+Ab^2}{2ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}\right)$$

$$\Big\downarrow \text{1235}$$

$$\frac{1}{2}\left(\frac{-\frac{\int \frac{2\left(c\left(aB\left(b^2-16ac\right)-3A\left(b^3-6abc\right)\right)x^2+abB\left(b^2-7ac\right)-3A\left(b^4-7acb^2+10a^2c^2\right)\right)}{x^4\left(cx^4+bx^2+a\right)}dx^2}{a(b^2-4ac)}-\frac{-A(20a^2c^2-20ab^2c+3b^4)+cx^2(aB(b^2-16ac)-3A(b^3}{ax^2(b^2-4ac)(a+bx^2+cx^4)}}{2a\left(b^2-4ac\right)}\right.$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{-\frac{2\int \frac{c\left(aB\left(b^2-16ac\right)-3A\left(b^3-6abc\right)\right)x^2+abB\left(b^2-7ac\right)-3A\left(b^4-7acb^2+10a^2c^2\right)}{x^4\left(cx^4+bx^2+a\right)}dx^2}{a(b^2-4ac)}-\frac{-A(20a^2c^2-20ab^2c+3b^4)+cx^2(aB(b^2-16ac)-3A(b^3-}{ax^2(b^2-4ac)(a+bx^2+cx^4)}}{2a\left(b^2-4ac\right)}\right.$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{2}\left(\frac{-\frac{2\int \left(-\frac{(aB-3Ab)\left(4ac-b^2\right)^2}{a^2x^2}+\frac{-\left((3Ab-aB)c\left(b^2-4ac\right)^2x^2\right)+abB\left(b^4-9acb^2+23a^2c^2\right)-3A\left(b^6-9acb^4+23a^2c^2b^2-10a^3c^3\right)}{a^2\left(cx^4+bx^2+a\right)}+\frac{abB\left(b^2-7ac\right)-3A\left(b^4-\right.}{ax^4}\right.}{a(b^2-4ac)}}{2a\left(b^2-4ac\right)}\right.$$

$$\Big\downarrow \text{2009}$$

3.131. $\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)^3}\,dx$

$$\frac{1}{2}\left(\frac{-\frac{-A(20a^2c^2-20ab^2c+3b^4)+cx^2\left(aB\left(b^2-16ac\right)-3A\left(b^3-6abc\right)\right)+abB\left(b^2-10ac\right)}{ax^2(b^2-4ac)(a+bx^2+cx^4)}-\frac{2\left(\frac{\log\left(x^2\right)\left(b^2-4ac\right)^2(3Ab-aB)}{a^2}-\frac{\left(b^2-4ac\right)^2(3Ab-aB)\ \mathrm{lo}}{2a^2}\right.}{}}{2a\,(b^2}\right.$$

input
```
Int[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^3),x]
```

output
```
((A*b^2 - a*b*B - 2*a*A*c + (A*b - 2*a*B)*c*x^2)/(2*a*(b^2 - 4*a*c)*x^2*(a
 + b*x^2 + c*x^4)^2) + (-((a*b*B*(b^2 - 10*a*c) - A*(3*b^4 - 20*a*b^2*c +
20*a^2*c^2) + c*(a*B*(b^2 - 16*a*c) - 3*A*(b^3 - 6*a*b*c))*x^2)/(a*(b^2 -
4*a*c)*x^2*(a + b*x^2 + c*x^4))) - (2*(-((a*b*B*(b^2 - 7*a*c) - 3*A*(b^4 -
 7*a*b^2*c + 10*a^2*c^2))/(a*x^2)) - ((a*b*B*(b^4 - 10*a*b^2*c + 30*a^2*c^
2) - 3*A*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3))*ArcTanh[(b + 2*
c*x^2)/Sqrt[b^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]) + ((3*A*b - a*B)*(b^2 -
 4*a*c)^2*Log[x^2])/a^2 - ((3*A*b - a*B)*(b^2 - 4*a*c)^2*Log[a + b*x^2 + c
*x^4])/(2*a^2)))/(a*(b^2 - 4*a*c)))/(2*a*(b^2 - 4*a*c)))/2
```

3.131.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.131.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 616, normalized size of antiderivative = 1.70

method	result
default	$-\frac{A}{2a^3x^2}+\frac{(-3Ab+Ba)\ln(x)}{a^4}-\frac{\frac{a\,c^2\left(14A\,a^2c^2-13Aa\,b^2c+2A\,b^4+7a^2bBc-Ba\,b^3\right)x^6}{16a^2c^2-8a\,b^2c+b^4}+\frac{ac\left(74A\,a^2b\,c^2-55Aa\,b^3c+8A\,b^5-16a^3B\,c^2+29B\,a\right.}{32a^2c^2-16a\,b^2c+2b^4}}{}$
risch	Expression too large to display

input
```
int((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*A/a^3/x^2+(-3*A*b+B*a)/a^4*ln(x)-1/2/a^4*((a*c^2*(14*A*a^2*c^2-13*A*a
*b^2*c+2*A*b^4+7*B*a^2*b*c-B*a*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1/2*a*c
*(74*A*a^2*b*c^2-55*A*a*b^3*c+8*A*b^5-16*B*a^3*c^2+29*B*a^2*b^2*c-4*B*a*b^
4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4+a*(18*A*a^3*c^3+7*A*a^2*b^2*c^2-12*A*a*b
^4*c+2*A*b^6+B*a^3*b*c^2+6*B*a^2*b^3*c-B*a*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)
*x^2+1/2*a^2*(58*A*a^2*b*c^2-36*A*a*b^3*c+5*A*b^5-24*B*a^3*c^2+21*B*a^2*b^
2*c-3*B*a*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)^2+1/(16*a^2*c^2
-8*a*b^2*c+b^4)*(1/2*(-48*A*a^2*b*c^3+24*A*a*b^3*c^2-3*A*b^5*c+16*B*a^3*c^
3-8*B*a^2*b^2*c^2+B*a*b^4*c)/c*ln(c*x^4+b*x^2+a)+2*(30*A*a^3*c^3-69*A*a^2*
b^2*c^2+27*A*a*b^4*c-3*b^6*A+23*B*a^3*b*c^2-9*B*a^2*b^3*c+B*a*b^5-1/2*(-48
*A*a^2*b*c^3+24*A*a*b^3*c^2-3*A*b^5*c+16*B*a^3*c^3-8*B*a^2*b^2*c^2+B*a*b^4
*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))))
```

3.131.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1966 vs. $2(346) = 692$.

Time = 3.98 (sec) , antiderivative size = 3956, normalized size of antiderivative = 10.90

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/4*(2*A*a^3*b^6 - 24*A*a^4*b^4*c + 96*A*a^5*b^2*c^2 - 128*A*a^6*c^3 - 2
*(120*A*a^4*c^5 + 2*(14*B*a^4*b - 57*A*a^3*b^2)*c^4 - 11*(B*a^3*b^3 - 3*A*
a^2*b^4)*c^3 + (B*a^2*b^5 - 3*A*a*b^6)*c^2)*x^8 + (8*(8*B*a^5 - 69*A*a^4*b
)*c^4 - 6*(22*B*a^4*b^2 - 81*A*a^3*b^3)*c^3 + 45*(B*a^3*b^4 - 3*A*a^2*b^5)
*c^2 - 4*(B*a^2*b^6 - 3*A*a*b^7)*c)*x^6 - 2*(B*a^2*b^7 - 3*A*a*b^8 + 200*A
*a^5*c^4 + 2*(2*B*a^5*b - 11*A*a^4*b^2)*c^3 + (23*B*a^4*b^3 - 79*A*a^3*b^4
)*c^2 - 10*(B*a^3*b^5 - 3*A*a^2*b^6)*c)*x^4 - (3*B*a^3*b^6 - 9*A*a^2*b^7 -
 8*(12*B*a^6 - 61*A*a^5*b)*c^3 + 2*(54*B*a^5*b^2 - 197*A*a^4*b^3)*c^2 - (3
3*B*a^4*b^4 - 104*A*a^3*b^5)*c)*x^2 - ((60*A*a^3*c^5 + 30*(B*a^3*b - 3*A*a
^2*b^2)*c^4 - 10*(B*a^2*b^3 - 3*A*a*b^4)*c^3 + (B*a*b^5 - 3*A*b^6)*c^2)*x^
10 + 2*(60*A*a^3*b*c^4 + 30*(B*a^3*b^2 - 3*A*a^2*b^3)*c^3 - 10*(B*a^2*b^4
- 3*A*a*b^5)*c^2 + (B*a*b^6 - 3*A*b^7)*c)*x^8 + (B*a*b^7 - 3*A*b^8 + 120*A
*a^4*c^4 + 60*(B*a^4*b - 2*A*a^3*b^2)*c^3 + 10*(B*a^3*b^3 - 3*A*a^2*b^4)*c
^2 - 8*(B*a^2*b^5 - 3*A*a*b^6)*c)*x^6 + 2*(B*a^2*b^6 - 3*A*a*b^7 + 60*A*a^
4*b*c^3 + 30*(B*a^4*b^2 - 3*A*a^3*b^3)*c^2 - 10*(B*a^3*b^4 - 3*A*a^2*b^5)*
c)*x^4 + (B*a^3*b^5 - 3*A*a^2*b^6 + 60*A*a^5*c^3 + 30*(B*a^5*b - 3*A*a^4*b
^2)*c^2 - 10*(B*a^4*b^3 - 3*A*a^3*b^4)*c)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^
2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4
+ b*x^2 + a)) - ((64*(B*a^4 - 3*A*a^3*b)*c^5 - 48*(B*a^3*b^2 - 3*A*a^2*b^3
)*c^4 + 12*(B*a^2*b^4 - 3*A*a*b^5)*c^3 - (B*a*b^6 - 3*A*b^7)*c^2)*x^10 ...
```

3.131.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)^3}\,dx=\text{Timed out}$$

input
```
integrate((B*x**2+A)/x**3/(c*x**4+b*x**2+a)**3,x)
```

output
```
Timed out
```

3.131.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx^2}{x^3(a+bx^2+cx^4)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.131.8 Giac [A] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 648, normalized size of antiderivative = 1.79

$$\int \frac{A+Bx^2}{x^3(a+bx^2+cx^4)^3}\,dx =$$

$$-\frac{(Bab^5 - 3\,Ab^6 - 10\,Ba^2b^3c + 30\,Aab^4c + 30\,Ba^3bc^2 - 90\,Aa^2b^2c^2 + 60\,Aa^3c^3)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(a^4b^4 - 8\,a^5b^2c + 16\,a^6c^2)\sqrt{-b^2+4\,ac}}$$

$$+\frac{3\,Bab^4c^2x^8 - 9\,Ab^5c^2x^8 - 24\,Ba^2b^2c^3x^8 + 72\,Aab^3c^3x^8 + 48\,Ba^3c^4x^8 - 144\,Aa^2bc^4x^8 + 6\,Bab^5cx^6 - 18}{}$$

$$-\frac{(Ba-3\,Ab)\log(cx^4+bx^2+a)}{4\,a^4} + \frac{(Ba-3\,Ab)\log(x^2)}{2\,a^4} - \frac{Bax^2 - 3\,Abx^2 + Aa}{2\,a^4x^2}$$

input
```
integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
-1/2*(B*a*b^5 - 3*A*b^6 - 10*B*a^2*b^3*c + 30*A*a*b^4*c + 30*B*a^3*b*c^2 -
 90*A*a^2*b^2*c^2 + 60*A*a^3*c^3)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))
/((a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*sqrt(-b^2 + 4*a*c)) + 1/8*(3*B*a*b^
4*c^2*x^8 - 9*A*b^5*c^2*x^8 - 24*B*a^2*b^2*c^3*x^8 + 72*A*a*b^3*c^3*x^8 +
48*B*a^3*c^4*x^8 - 144*A*a^2*b*c^4*x^8 + 6*B*a*b^5*c*x^6 - 18*A*b^6*c*x^6
- 44*B*a^2*b^3*c^2*x^6 + 136*A*a*b^4*c^2*x^6 + 68*B*a^3*b*c^3*x^6 - 236*A*
a^2*b^2*c^3*x^6 - 56*A*a^3*c^4*x^6 + 3*B*a*b^6*x^4 - 9*A*b^7*x^4 - 10*B*a^
2*b^4*c*x^4 + 38*A*a*b^5*c*x^4 - 58*B*a^3*b^2*c^2*x^4 + 110*A*a^2*b^3*c^2*
x^4 + 128*B*a^4*c^3*x^4 - 436*A*a^3*b*c^3*x^4 + 10*B*a^2*b^5*x^2 - 26*A*a*
b^6*x^2 - 72*B*a^3*b^3*c*x^2 + 192*A*a^2*b^4*c*x^2 + 92*B*a^4*b*c^2*x^2 -
316*A*a^3*b^2*c^2*x^2 - 72*A*a^4*c^3*x^2 + 9*B*a^3*b^4 - 19*A*a^2*b^5 - 66
*B*a^4*b^2*c + 144*A*a^3*b^3*c + 96*B*a^5*c^2 - 260*A*a^4*b*c^2)/((a^4*b^4
 - 8*a^5*b^2*c + 16*a^6*c^2)*(c*x^4 + b*x^2 + a)^2) - 1/4*(B*a - 3*A*b)*lo
g(c*x^4 + b*x^2 + a)/a^4 + 1/2*(B*a - 3*A*b)*log(x^2)/a^4 - 1/2*(B*a*x^2 -
 3*A*b*x^2 + A*a)/(a^4*x^2)
```

3.131.9 Mupad [B] (verification not implemented)

Time = 18.35 (sec) , antiderivative size = 16265, normalized size of antiderivative = 44.81

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^3),x)
```

output
```
(log(((c^5*x^2*(3*A*b^4 + 30*A*a^2*c^2 - B*a*b^3 - 21*A*a*b^2*c + 7*B*a^2*
b*c)^3)/(a^9*(4*a*c - b^2)^6) - ((B*a - 3*A*b + a^4*(-(60*A*a^3*c^3 - 3*A*
b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a^2*
b^2*c^2)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*(((B*a - 3*A*b + a^4*(-(60*A*a^3*
c^3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 -
 90*A*a^2*b^2*c^2)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*((4*b*c^2*(30*A*a^3*c^3
 - 3*A*b^6 + B*a*b^5 + 27*A*a*b^4*c - 9*B*a^2*b^3*c + 23*B*a^3*b*c^2 - 69*
A*a^2*b^2*c^2))/(a^3*(4*a*c - b^2)^2) + (2*c^3*x^2*(B*a*b^5 - 300*A*a^3*c^
3 - 3*A*b^6 + 6*A*a*b^4*c - 2*B*a^2*b^3*c + 10*B*a^3*b*c^2 + 90*A*a^2*b^2*
c^2))/(a^3*(4*a*c - b^2)^2) + (b*c^2*(B*a - 3*A*b + a^4*(-(60*A*a^3*c^3 -
3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A*
a^2*b^2*c^2)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2
))/a^4))/(4*a^4) + (c^3*(900*A^2*a^5*c^5 - 36*A^2*b^10 - 4*B^2*a^2*b^8 + 2
4*A*B*a*b^9 - 3078*A^2*a^2*b^6*c^2 + 7533*A^2*a^3*b^4*c^3 - 7020*A^2*a^4*b
^2*c^4 - 302*B^2*a^4*b^4*c^2 + 497*B^2*a^5*b^2*c^3 + 549*A^2*a*b^8*c + 61*
B^2*a^3*b^6*c + 1932*A*B*a^3*b^5*c^2 - 4002*A*B*a^4*b^3*c^3 - 366*A*B*a^2*
b^7*c + 2340*A*B*a^5*b*c^4))/(a^6*(4*a*c - b^2)^4) - (c^4*x^2*(54*A^2*b^9
+ 6*B^2*a^2*b^7 - 36*A*B*a*b^8 + 4311*A^2*a^2*b^5*c^2 - 9900*A^2*a^3*b^3*c
^3 + 409*B^2*a^4*b^3*c^2 - 2400*A*B*a^5*c^4 - 801*A^2*a*b^7*c + 8100*A^2*a
^4*b*c^4 - 89*B^2*a^3*b^5*c - 560*B^2*a^5*b*c^3 - 2664*A*B*a^3*b^4*c^2 ...
```

3.131. $\int \frac{A+Bx^2}{x^3(a+bx^2+cx^4)^3}\,dx$

3.132 $\int \frac{x^8(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

3.132.1 Optimal result

Integrand size = 25, antiderivative size = 554

$$\begin{aligned}
\int \frac{x^8(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx =& -\frac{(3b^3B+Ab^2c-24abBc+20aAc^2)\,x}{8c^2\,(b^2-4ac)^2} \\
&+\frac{(b^2B+12Abc-28aBc)\,x^3}{8c\,(b^2-4ac)^2}-\frac{x^7(Ab-2aB-(bB-2Ac)x^2)}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2} \\
&-\frac{x^5(7Ab^2-12abB-4aAc+(b^2B+12Abc-28aBc)\,x^2)}{8\,(b^2-4ac)^2\,(a+bx^2+cx^4)} \\
&+\frac{\left(3b^4B+Ab^3c-27ab^2Bc-16aAbc^2+84a^2Bc^2-\frac{3b^5B+Ab^4c-33ab^3Bc-18aAb^2c^2+132a^2bBc^2-40a^2Ac^3}{\sqrt{b^2-4ac}}\right)\arctan\left(\cdots\right.}{8\sqrt{2}c^{5/2}\,(b^2-4ac)^2\,\sqrt{b-\sqrt{b^2-4ac}}} \\
&+\frac{\left(3b^4B+Ab^3c-27ab^2Bc-16aAbc^2+84a^2Bc^2+\frac{3b^5B+Ab^4c-33ab^3Bc-18aAb^2c^2+132a^2bBc^2-40a^2Ac^3}{\sqrt{b^2-4ac}}\right)\arctan\left(\cdots\right.}{8\sqrt{2}c^{5/2}\,(b^2-4ac)^2\,\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}$$

output
```
-1/8*(20*A*a*c^2+A*b^2*c-24*B*a*b*c+3*B*b^3)*x/c^2/(-4*a*c+b^2)^2+1/8*(12*
A*b*c-28*B*a*c+B*b^2)*x^3/c/(-4*a*c+b^2)^2-1/4*x^7*(A*b-2*B*a-(-2*A*c+B*b)
*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2-1/8*x^5*(7*A*b^2-12*a*b*B-4*A*a*c+(12
*A*b*c-28*B*a*c+B*b^2)*x^2)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+1/16*arctan(x*2
^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(3*B*b^4+A*b^3*c-27*B*a*b^2*c
-16*A*a*b*c^2+84*B*a^2*c^2+(40*A*a^2*c^3+18*A*a*b^2*c^2-A*b^4*c-132*B*a^2*
b*c^2+33*B*a*b^3*c-3*B*b^5)/(-4*a*c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^2)^2*2^(
1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/16*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c
+b^2)^(1/2))^(1/2))*(3*B*b^4+A*b^3*c-27*B*a*b^2*c-16*A*a*b*c^2+84*B*a^2*c^
2+(-40*A*a^2*c^3-18*A*a*b^2*c^2+A*b^4*c+132*B*a^2*b*c^2-33*B*a*b^3*c+3*B*b
^5)/(-4*a*c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^2)^2*2^(1/2)/(b+(-4*a*c+b^2)^(1/
2))^(1/2)
```

3.132.2 Mathematica [A] (verified)

Time = 1.52 (sec) , antiderivative size = 644, normalized size of antiderivative = 1.16

$$\int \frac{x^8(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$$

$$= \frac{\frac{2x(2b^5B-b^4c(2A+5Bx^2)-4a^2c^3(9A+11Bx^2)+ab^2c^2(11A+37Bx^2)+16abc^2(3aB-Acx^2)+b^3c(-17aB+Acx^2))}{(b^2-4ac)^2(a+bx^2+cx^4)} - \frac{4x(b^3(bB-Ac)x^2+a^2c(-}{}}{}$$

input
```
Integrate[(x^8*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

output
```
((2*x*(2*b^5*B - b^4*c*(2*A + 5*B*x^2) - 4*a^2*c^3*(9*A + 11*B*x^2) + a*b^
2*c^2*(11*A + 37*B*x^2) + 16*a*b*c^2*(3*a*B - A*c*x^2) + b^3*c*(-17*a*B +
A*c*x^2)))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - (4*x*(b^3*(b*B - A*c)*x
^2 + a^2*c*(-3*b*B + 2*c*(A + B*x^2)) + a*b*(b^2*B + 3*A*c^2*x^2 - b*c*(A
+ 4*B*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (Sqrt[2]*Sqrt[c]*(-3
*b^5*B + b^3*c*(33*a*B + A*Sqrt[b^2 - 4*a*c]) - 4*a*b*c^2*(33*a*B + 4*A*Sq
rt[b^2 - 4*a*c]) + 9*a*b^2*c*(2*A*c - 3*B*Sqrt[b^2 - 4*a*c]) + b^4*(-(A*c)
 + 3*B*Sqrt[b^2 - 4*a*c]) + 4*a^2*c^2*(10*A*c + 21*B*Sqrt[b^2 - 4*a*c]))*A
rcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/
2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(3*b^5*B + 4*a*b*c^2*(3
3*a*B - 4*A*Sqrt[b^2 - 4*a*c]) + b^4*(A*c + 3*B*Sqrt[b^2 - 4*a*c]) - 9*a*b
^2*c*(2*A*c + 3*B*Sqrt[b^2 - 4*a*c]) + 4*a^2*c^2*(-10*A*c + 21*B*Sqrt[b^2
- 4*a*c]) + b^3*(-33*a*B*c + A*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2
 - 4*a*c]]))/(16*c^3)
```

3.132.3 Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 544, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1598, 1598, 1602, 27, 1602, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8 (A+Bx^2)}{(a+bx^2+cx^4)^3} \, dx$$

$$\Big\downarrow \; 1598$$

$$\frac{\int \frac{x^6(7(Ab-2aB)-(bB-2Ac)x^2)}{(cx^4+bx^2+a)^2}dx}{4(b^2-4ac)} - \frac{x^7\left(-2aB-\left(x^2(bB-2Ac)\right)+Ab\right)}{4(b^2-4ac)(a+bx^2+cx^4)^2}$$

$$\Big\downarrow \; 1598$$

$$\frac{\frac{\int \frac{x^4\left(3\left(Bb^2+12Acb-28aBc\right)x^2+5\left(7Ab^2-12aBb-4aAc\right)\right)}{cx^4+bx^2+a}dx}{2(b^2-4ac)} - \frac{x^5\left(x^2\left(-28aBc+12Abc+b^2B\right)-4aAc-12abB+7Ab^2\right)}{2(b^2-4ac)(a+bx^2+cx^4)}}{4(b^2-4ac)} - \frac{x^7\left(-2aB-\left(x^2(bB-2Ac)\right)+Ab\right)}{4(b^2-4ac)(a+bx^2+cx^4)^2}$$

$$\Big\downarrow \; 1602$$

$$\frac{\frac{\frac{x^3\left(-28aBc+12Abc+b^2B\right)}{c} - \frac{\int \frac{3x^2\left(\left(3Bb^3+Acb^2-24aBcb+20aAc^2\right)x^2+3a\left(Bb^2+12Acb-28aBc\right)\right)}{cx^4+bx^2+a}dx}{3c}}{2(b^2-4ac)} - \frac{x^5\left(x^2\left(-28aBc+12Abc+b^2B\right)-4aAc-12abB+7\right.}{2(b^2-4ac)(a+bx^2+cx^4)}}{4(b^2-4ac)}$$
$$\frac{x^7\left(-2aB-\left(x^2(bB-2Ac)\right)+Ab\right)}{4(b^2-4ac)(a+bx^2+cx^4)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{x^3\left(-28aBc+12Abc+b^2B\right)}{c} - \frac{\int \frac{x^2\left(\left(3Bb^3+Acb^2-24aBcb+20aAc^2\right)x^2+3a\left(Bb^2+12Acb-28aBc\right)\right)}{cx^4+bx^2+a}dx}{c}}{2(b^2-4ac)} - \frac{x^5\left(x^2\left(-28aBc+12Abc+b^2B\right)-4aAc-12abB+7\right.}{2(b^2-4ac)(a+bx^2+cx^4)}}{4(b^2-4ac)}$$
$$\frac{x^7\left(-2aB-\left(x^2(bB-2Ac)\right)+Ab\right)}{4(b^2-4ac)(a+bx^2+cx^4)^2}$$

$$\Big\downarrow \; 1602$$

$$\frac{\frac{\frac{x^3\left(-28aBc+12Abc+b^2B\right)}{c}-\frac{x\left(20aAc^2-24abBc+Ab^2c+3b^3B\right)}{c}-\frac{\int\frac{\left(3Bb^4+Acb^3-27aBcb^2-16aAc^2b+84a^2Bc^2\right)x^2+a\left(3Bb^3+Acb^2-24aBcb+20aAc^2\right)}{cx^4+bx^2+a}dx}{c}}{2(b^2-4ac)}}{4\left(b^2-4ac\right)}-$$

$$\frac{x^7\left(-2aB-\left(x^2(bB-2Ac)\right)+Ab\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

$$\Big\downarrow \text{1480}$$

$$\frac{x^3\left(-28aBc+12Abc+b^2B\right)}{c}-\frac{x\left(20aAc^2-24abBc+Ab^2c+3b^3B\right)}{c}-\frac{\frac{1}{2}\left(-\frac{-40a^2Ac^3+132a^2bBc^2-18aAb^2c^2-33ab^3Bc+Ab^4c+3b^5B}{\sqrt{b^2-4ac}}+84a^2Bc^2-16aAbc^2-27ab^2B\right.}{}$$

$$\frac{x^7\left(-2aB-\left(x^2(bB-2Ac)\right)+Ab\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{x^3\left(-28aBc+12Abc+b^2B\right)}{c}-\frac{x\left(20aAc^2-24abBc+Ab^2c+3b^3B\right)}{c}-\frac{\frac{\left(-\frac{-40a^2Ac^3+132a^2bBc^2-18aAb^2c^2-33ab^3Bc+Ab^4c+3b^5B}{\sqrt{b^2-4ac}}+84a^2Bc^2-16aAbc^2-27ab^2Bc\right.}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}}{}$$

$$\frac{x^7\left(-2aB-\left(x^2(bB-2Ac)\right)+Ab\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

input
```
Int[(x^8*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

output
```
-1/4*(x^7*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c
*x^4)^2) + (-1/2*(x^5*(7*A*b^2 - 12*a*b*B - 4*a*A*c + (b^2*B + 12*A*b*c -
28*a*B*c)*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (((b^2*B + 12*A*b*c
- 28*a*B*c)*x^3)/c - (((3*b^3*B + A*b^2*c - 24*a*b*B*c + 20*a*A*c^2)*x)/c
- (((3*b^4*B + A*b^3*c - 27*a*b^2*B*c - 16*a*A*b*c^2 + 84*a^2*B*c^2 - (3*b
^5*B + A*b^4*c - 33*a*b^3*B*c - 18*a*A*b^2*c^2 + 132*a^2*b*B*c^2 - 40*a^2*
A*c^3)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4
*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((3*b^4*B + A*b^3
*c - 27*a*b^2*B*c - 16*a*A*b*c^2 + 84*a^2*B*c^2 + (3*b^5*B + A*b^4*c - 33*
a*b^3*B*c - 18*a*A*b^2*c^2 + 132*a^2*b*B*c^2 - 40*a^2*A*c^3)/Sqrt[b^2 - 4*
a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sq
rt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/c)/c)/(2*(b^2 - 4*a*c)))/(4*(b^2 - 4*a
*c))
```

3.132. $\int \frac{x^8\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^3}\,dx$

3.132.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1598
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Simp[f
^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1
)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
 GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

3.132. $\int \frac{x^8(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

3.132.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.17 (sec) , antiderivative size = 445, normalized size of antiderivative = 0.80

method	result
risch	$\frac{-\frac{\left(16Aabc^2-Ab^3c+44Ba^2c^2-37Bab^2c+5Bb^4\right)x^7}{8\left(16a^2c^2-8ab^2c+b^4\right)c}-\frac{\left(36Aa^2c^3+5Aab^2c^2+Ab^4c-4Ba^2bc^2-20Bab^3c+3Bb^5\right)x^5}{8c^2\left(16a^2c^2-8ab^2c+b^4\right)}-\frac{a\left(28Aabc^2+2Ab^3c+28B\right.}{8c^2\left(16a^2c^2-\right.}}{\left(cx^4+bx^2+a\right)^2}$
default	$\frac{-\frac{\left(16Aabc^2-Ab^3c+44Ba^2c^2-37Bab^2c+5Bb^4\right)x^7}{8\left(16a^2c^2-8ab^2c+b^4\right)c}-\frac{\left(36Aa^2c^3+5Aab^2c^2+Ab^4c-4Ba^2bc^2-20Bab^3c+3Bb^5\right)x^5}{8c^2\left(16a^2c^2-8ab^2c+b^4\right)}-\frac{a\left(28Aabc^2+2Ab^3c+28B\right.}{8c^2\left(16a^2c^2-\right.}}{\left(cx^4+bx^2+a\right)^2}$

input
```
int(x^8*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/8*(16*A*a*b*c^2-A*b^3*c+44*B*a^2*c^2-37*B*a*b^2*c+5*B*b^4)/(16*a^2*c^2
-8*a*b^2*c+b^4)/c*x^7-1/8*(36*A*a^2*c^3+5*A*a*b^2*c^2+A*b^4*c-4*B*a^2*b*c^
2-20*B*a*b^3*c+3*B*b^5)/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-1/8*a/c^2*(28*A
*a*b*c^2+2*A*b^3*c+28*B*a^2*c^2-49*B*a*b^2*c+6*B*b^4)/(16*a^2*c^2-8*a*b^2*
c+b^4)*x^3-1/8*a^2*(20*A*a*c^2+A*b^2*c-24*B*a*b*c+3*B*b^3)/c^2/(16*a^2*c^2
-8*a*b^2*c+b^4)*x)/(c*x^4+b*x^2+a)^2+1/16/c^2*sum((-(16*A*a*b*c^2-A*b^3*c-
84*B*a^2*c^2+27*B*a*b^2*c-3*B*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*_R^2+a*(20*A
*a*c^2+A*b^2*c-24*B*a*b*c+3*B*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4))/(2*_R^3*c+_
R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

3.132.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9636 vs. 2(507) = 1014.

Time = 18.80 (sec) , antiderivative size = 9636, normalized size of antiderivative = 17.39

$$\int \frac{x^8(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.132.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^8(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**8*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

output
```
Timed out
```

3.132.7 Maxima [F]

$$\int \frac{x^8(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \int \frac{(Bx^2+A)x^8}{(cx^4+bx^2+a)^3}\,dx$$

input
```
integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/8*((5*B*b^4*c + 4*(11*B*a^2 + 4*A*a*b)*c^3 - (37*B*a*b^2 + A*b^3)*c^2)*
x^7 + (3*B*b^5 + 36*A*a^2*c^3 - (4*B*a^2*b - 5*A*a*b^2)*c^2 - (20*B*a*b^3
- A*b^4)*c)*x^5 + (6*B*a*b^4 + 28*(B*a^3 + A*a^2*b)*c^2 - (49*B*a^2*b^2 -
2*A*a*b^3)*c)*x^3 + (3*B*a^2*b^3 + 20*A*a^3*c^2 - (24*B*a^3*b - A*a^2*b^2)
*c)*x)/((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2
*c^3 + 16*a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^
2 - 6*a*b^4*c^3 + 32*a^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*
b*c^4)*x^2) - 1/8*integrate(-(3*B*a*b^3 + 20*A*a^2*c^2 + (3*B*b^4 + 4*(21*
B*a^2 - 4*A*a*b)*c^2 - (27*B*a*b^2 - A*b^3)*c)*x^2 - (24*B*a^2*b - A*a*b^2
)*c)/(c*x^4 + b*x^2 + a), x)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)
```

3.132.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3987 vs. $2(507) = 1014$.

Time = 2.02 (sec) , antiderivative size = 3987, normalized size of antiderivative = 7.20

$$\int \frac{x^8(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/32*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 12*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*b^5*c^2 - 2*b^6*c^2 - 144*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*
b^2*c^3 - 32*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 24*a*b^4*c^3 - 2*b^5*c^3 + 320*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 160*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^2*b*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^2*c^4 + 288*a^2*b^2*c^4 + 112*a*b^3*c^4 - 80*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*c^5 - 640*a^3*c^5 - 416*a^2*b*c^5 + sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 56*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 208*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 104*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + 2*(b^2 - 4*a*c)*b^4*c^2 + 32*(b^2 - 4*a*c)
*a*b^2*c^3 + 2*(b^2 - 4*a*c)*b^3*c^3 - 160*(b^2 - 4*a*c)*a^2*c^4 - 104*(b^
2 - 4*a*c)*a*b*c^4)*A + 3*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 - 1
6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^6*c - 2*b^7*c + 80*sqrt(2)*sqrt(b*c + sqrt(b^2 - ...
```

3.132.9 Mupad [B] (verification not implemented)

Time = 10.32 (sec) , antiderivative size = 22911, normalized size of antiderivative = 41.36

$$\int \frac{x^8(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int((x^8*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

output

```
- ((x^5*(3*B*b^5 + 36*A*a^2*c^3 + A*b^4*c - 20*B*a*b^3*c + 5*A*a*b^2*c^2 -
 4*B*a^2*b*c^2))/(8*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^7*(5*B*b^4 +
44*B*a^2*c^2 - A*b^3*c + 16*A*a*b*c^2 - 37*B*a*b^2*c))/(8*c*(b^4 + 16*a^2*
c^2 - 8*a*b^2*c)) + (x^3*(28*B*a^3*c^2 + 6*B*a*b^4 + 2*A*a*b^3*c + 28*A*a^
2*b*c^2 - 49*B*a^2*b^2*c))/(8*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a^2*x
*(3*B*b^3 + 20*A*a*c^2 + A*b^2*c - 24*B*a*b*c))/(8*c^2*(b^4 + 16*a^2*c^2 -
 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6)
- atan(((((256*A*a*b^12*c^4 - 5242880*A*a^7*c^10 + 768*B*a*b^13*c^3 + 6291
456*B*a^7*b*c^9 - 61440*A*a^3*b^8*c^6 + 655360*A*a^4*b^6*c^7 - 2949120*A*a
^5*b^4*c^8 + 6291456*A*a^6*b^2*c^9 - 21504*B*a^2*b^11*c^4 + 245760*B*a^3*b
^9*c^5 - 1474560*B*a^4*b^7*c^6 + 4915200*B*a^5*b^5*c^7 - 8650752*B*a^6*b^3
*c^8)/(512*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 12
80*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) - (x*(-(9*B^2*b^19
+ A^2*b^17*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140
*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776
*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921
*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 285
1776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7
+ 27095040*B^2*a^8*b^3*c^8 + A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 441*B
^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a...
```

3.132. $\int \frac{x^8(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

3.133 $\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

3.133.1 Optimal result

Integrand size = 25, antiderivative size = 461

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = -\frac{(b^2B-12Abc+20aBc)\,x}{8c\,(b^2-4ac)^2}-\frac{x^5(Ab-2aB-(bB-2Ac)x^2)}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2}$$
$$-\frac{x^3(5Ab^2-12abB+4aAc-(b^2B-12Abc+20aBc)\,x^2)}{8\,(b^2-4ac)^2\,(a+bx^2+cx^4)}$$
$$+\frac{\left(b^3B+3Ab^2c-16abBc+12aAc^2-\frac{b^4B+3Ab^3c-18ab^2Bc+36aAbc^2-40a^2Bc^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}c^{3/2}\,(b^2-4ac)^2\,\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\left(b^3B+3Ab^2c-16abBc+12aAc^2+\frac{b^4B+3Ab^3c-18ab^2Bc+36aAbc^2-40a^2Bc^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}c^{3/2}\,(b^2-4ac)^2\,\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
-1/8*(-12*A*b*c+20*B*a*c+B*b^2)*x/c/(-4*a*c+b^2)^2-1/4*x^5*(A*b-2*B*a-(-2*
A*c+B*b)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2-1/8*x^3*(5*A*b^2-12*a*b*B+4*A
*a*c-(-12*A*b*c+20*B*a*c+B*b^2)*x^2)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+1/16*a
rctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(B*b^3+3*A*b^2*c-16*
B*a*b*c+12*A*a*c^2+(-36*A*a*b*c^2-3*A*b^3*c+40*B*a^2*c^2+18*B*a*b^2*c-B*b^
4)/(-4*a*c+b^2)^(1/2))/c^(3/2)/(-4*a*c+b^2)^2*2^(1/2)/(b-(-4*a*c+b^2)^(1/2
))^(1/2)+1/16*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(B*b^
3+3*A*b^2*c-16*B*a*b*c+12*A*a*c^2+(36*A*a*b*c^2+3*A*b^3*c-40*B*a^2*c^2-18*
B*a*b^2*c+B*b^4)/(-4*a*c+b^2)^(1/2))/c^(3/2)/(-4*a*c+b^2)^2*2^(1/2)/(b+(-4
*a*c+b^2)^(1/2))^(1/2)
```

3.133.2 Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.18

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$$

$$= \frac{\frac{2x(-2b^4B+4abc^2(A-4Bx^2)+b^3c(2A+Bx^2)+12ac^2(-3aB+Acx^2)+b^2c(11aB+3Acx^2))}{(b^2-4ac)^2(a+bx^2+cx^4)} - \frac{4x(2a^2Bc+b^2(-bB+Ac)x^2+a(-b^2B-2Ac^2x^2+}{(b^2-4ac)(a+bx^2+cx^4)^2}}{}$$

input
```
Integrate[(x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

output
```
((2*x*(-2*b^4*B + 4*a*b*c^2*(A - 4*B*x^2) + b^3*c*(2*A + B*x^2) + 12*a*c^2
*(-3*a*B + A*c*x^2) + b^2*c*(11*a*B + 3*A*c*x^2)))/((b^2 - 4*a*c)^2*(a + b
*x^2 + c*x^4)) - (4*x*(2*a^2*B*c + b^2*(-(b*B) + A*c)*x^2 + a*(-(b^2*B) -
2*A*c^2*x^2 + b*c*(A + 3*B*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) +
 (Sqrt[2]*Sqrt[c]*(-(b^4*B) + 3*b^2*c*(6*a*B + A*Sqrt[b^2 - 4*a*c]) + 4*a*
c^2*(10*a*B + 3*A*Sqrt[b^2 - 4*a*c]) + b^3*(-3*A*c + B*Sqrt[b^2 - 4*a*c])
- 4*a*b*c*(9*A*c + 4*B*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]
) + (Sqrt[2]*Sqrt[c]*(b^4*B + 3*b^2*c*(-6*a*B + A*Sqrt[b^2 - 4*a*c]) + 4*a
*c^2*(-10*a*B + 3*A*Sqrt[b^2 - 4*a*c]) + 4*a*b*c*(9*A*c - 4*B*Sqrt[b^2 - 4
*a*c]) + b^3*(3*A*c + B*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]
]))/(16*c^2)
```

3.133.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 458, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1598, 1598, 1602, 25, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$$

$$\Big\downarrow \; 1598$$

$$\frac{\int \frac{x^4((bB-2Ac)x^2+5(Ab-2aB))}{(cx^4+bx^2+a)^2}dx}{4(b^2-4ac)}-\frac{x^5(-2aB-(x^2(bB-2Ac))+Ab)}{4(b^2-4ac)(a+bx^2+cx^4)^2}$$

$\big\downarrow$ 1598

$$\frac{\frac{\int \frac{x^2\left(3\left(5Ab^2-12aBb+4aAc\right)-\left(Bb^2-12Acb+20aBc\right)x^2\right)}{cx^4+bx^2+a}dx}{2(b^2-4ac)}-\frac{x^3(-x^2(20aBc-12Abc+b^2B)+4aAc-12abB+5Ab^2)}{2(b^2-4ac)(a+bx^2+cx^4)}}{4(b^2-4ac)}-\frac{x^5(-2aB-(x^2(bB-2Ac))+Ab)}{4(b^2-4ac)(a+bx^2+cx^4)^2}$$

$\big\downarrow$ 1602

$$\frac{-\frac{\frac{\int -\frac{\left(Bb^3+3Acb^2-16aBcb+12aAc^2\right)x^2+a\left(Bb^2-12Acb+20aBc\right)}{cx^4+bx^2+a}dx}{c}-\frac{x\left(20aBc-12Abc+b^2B\right)}{c}}{2(b^2-4ac)}-\frac{x^3(-x^2(20aBc-12Abc+b^2B)+4aAc-12abB+5Ab^2)}{2(b^2-4ac)(a+bx^2+cx^4)}}{4(b^2-4ac)}-\frac{x^5(-2aB-(x^2(bB-2Ac))+Ab)}{4(b^2-4ac)(a+bx^2+cx^4)^2}$$

$\big\downarrow$ 25

$$\frac{\frac{\frac{\int \frac{\left(Bb^3+3Acb^2-16aBcb+12aAc^2\right)x^2+a\left(Bb^2-12Acb+20aBc\right)}{cx^4+bx^2+a}dx}{c}-\frac{x\left(20aBc-12Abc+b^2B\right)}{c}}{2(b^2-4ac)}-\frac{x^3(-x^2(20aBc-12Abc+b^2B)+4aAc-12abB+5Ab^2)}{2(b^2-4ac)(a+bx^2+cx^4)}}{4(b^2-4ac)}-\frac{x^5(-2aB-(x^2(bB-2Ac))+Ab)}{4(b^2-4ac)(a+bx^2+cx^4)^2}$$

$\big\downarrow$ 1480

$$\frac{\frac{\frac{\frac{1}{2}\left(-\frac{-40a^2Bc^2+36aAbc^2-18ab^2Bc+3Ab^3c+b^4B}{\sqrt{b^2-4ac}}+12aAc^2-16abBc+3Ab^2c+b^3B\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(\frac{-40a^2Bc^2+36aAbc^2-18ab^2Bc+3Ab^3c+b^4}{\sqrt{b^2-4ac}}\right.}{c}}{2(b^2-4ac)}}{4(b^2-4}$$

$$\frac{x^5(-2aB-(x^2(bB-2Ac))+Ab)}{4(b^2-4ac)(a+bx^2+cx^4)^2}$$

$\big\downarrow$ 218

$$\frac{\frac{\frac{\left(-\frac{-40a^2Bc^2+36aAbc^2-18ab^2Bc+3Ab^3c+b^4B}{\sqrt{b^2-4ac}}+12aAc^2-16abBc+3Ab^2c+b^3B\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{-40a^2Bc^2+36aAbc^2-18ab^2Bc+3Ab^3c+b^4B}{\sqrt{b^2-4ac}}+1\right.}{\sqrt{2}\sqrt{c}\sqrt{}}}{c}}{2(b^2-4ac)}}{4(b^2-4ac}$$

$$\frac{x^5(-2aB-(x^2(bB-2Ac))+Ab)}{4(b^2-4ac)(a+bx^2+cx^4)^2}$$

3.133. $\displaystyle\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

input
```
Int[(x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

output
```
-1/4*(x^5*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c
*x^4)^2) + (-1/2*(x^3*(5*A*b^2 - 12*a*b*B + 4*a*A*c - (b^2*B - 12*A*b*c +
20*a*B*c)*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (-(((b^2*B - 12*A*b*
c + 20*a*B*c)*x)/c) + (((b^3*B + 3*A*b^2*c - 16*a*b*B*c + 12*a*A*c^2 - (b^
4*B + 3*A*b^3*c - 18*a*b^2*B*c + 36*a*A*b*c^2 - 40*a^2*B*c^2)/Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*S
qrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^3*B + 3*A*b^2*c - 16*a*b*B*c + 1
2*a*A*c^2 + (b^4*B + 3*A*b^3*c - 18*a*b^2*B*c + 36*a*A*b*c^2 - 40*a^2*B*c^
2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c
]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/c)/(2*(b^2 - 4*a*c)))/
(4*(b^2 - 4*a*c))
```

3.133.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1598
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Simp[f
^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1
)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
 GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

3.133.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 375, normalized size of antiderivative = 0.81

method	result
risch	$\dfrac{\frac{\left(12 A a c^{2}+3 A b^{2} c-16 B a b c+B b^{3}\right) x^{7}}{128 a^{2} c^{2}-64 a b^{2} c+8 b^{4}}+\frac{\left(16 A a b c^{2}+5 A b^{3} c-36 B a^{2} c^{2}-5 B a b^{2} c-B b^{4}\right) x^{5}}{8 c \left(16 a^{2} c^{2}-8 a b^{2} c+b^{4}\right)}-\frac{a \left(4 A a c^{2}-19 A b^{2} c+28 B a b c+2 B b^{3}\right) x^{3}}{8 c \left(16 a^{2} c^{2}-8 a b^{2} c+b^{4}\right)}+\frac{a^{2} (12 A b c-2}{8 c (16 a^{2} c^{2}-}}{\left(c x^{4}+b x^{2}+a\right)^{2}}$
default	$\dfrac{\frac{\left(12 A a c^{2}+3 A b^{2} c-16 B a b c+B b^{3}\right) x^{7}}{128 a^{2} c^{2}-64 a b^{2} c+8 b^{4}}+\frac{\left(16 A a b c^{2}+5 A b^{3} c-36 B a^{2} c^{2}-5 B a b^{2} c-B b^{4}\right) x^{5}}{8 c \left(16 a^{2} c^{2}-8 a b^{2} c+b^{4}\right)}-\frac{a \left(4 A a c^{2}-19 A b^{2} c+28 B a b c+2 B b^{3}\right) x^{3}}{8 c \left(16 a^{2} c^{2}-8 a b^{2} c+b^{4}\right)}+\frac{a^{2} (12 A b c-2}{8 c (16 a^{2} c^{2}-}}{\left(c x^{4}+b x^{2}+a\right)^{2}}$

input
```
int(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8*(12*A*a*c^2+3*A*b^2*c-16*B*a*b*c+B*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^
7+1/8*(16*A*a*b*c^2+5*A*b^3*c-36*B*a^2*c^2-5*B*a*b^2*c-B*b^4)/c/(16*a^2*c^
2-8*a*b^2*c+b^4)*x^5-1/8/c*a*(4*A*a*c^2-19*A*b^2*c+28*B*a*b*c+2*B*b^3)/(16
*a^2*c^2-8*a*b^2*c+b^4)*x^3+1/8*a^2*(12*A*b*c-20*B*a*c-B*b^2)/c/(16*a^2*c^
2-8*a*b^2*c+b^4)*x)/(c*x^4+b*x^2+a)^2+1/16/c*sum(((12*A*a*c^2+3*A*b^2*c-16
*B*a*b*c+B*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*_R^2-a*(12*A*b*c-20*B*a*c-B*b^2
)/(16*a^2*c^2-8*a*b^2*c+b^4))/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z
^2*b+a))
```

3.133.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7060 vs. $2(414) = 828$.

Time = 6.01 (sec) , antiderivative size = 7060, normalized size of antiderivative = 15.31

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.133.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**6*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

output
```
Timed out
```

3.133.7 Maxima [F]

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \int \frac{(Bx^2+A)x^6}{(cx^4+bx^2+a)^3}\,dx$$

input
```
integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*((B*b^3*c + 12*A*a*c^3 - (16*B*a*b - 3*A*b^2)*c^2)*x^7 - (B*b^4 + 4*(9
*B*a^2 - 4*A*a*b)*c^2 + 5*(B*a*b^2 - A*b^3)*c)*x^5 - (2*B*a*b^3 + 4*A*a^2*
c^2 + (28*B*a^2*b - 19*A*a*b^2)*c)*x^3 - (B*a^2*b^2 + 4*(5*B*a^3 - 3*A*a^2
*b)*c)*x)/((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^
2*c^2 + 16*a^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c
 - 6*a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c
^3)*x^2) + 1/8*integrate((B*a*b^2 + (B*b^3 + 12*A*a*c^2 - (16*B*a*b - 3*A*
b^2)*c)*x^2 + 4*(5*B*a^2 - 3*A*a*b)*c)/(c*x^4 + b*x^2 + a), x)/(b^4*c - 8*
a*b^2*c^2 + 16*a^2*c^3)
```

3.133.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7578 vs. $2(414) = 828$.

Time = 2.35 (sec) , antiderivative size = 7578, normalized size of antiderivative = 16.44

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
-1/64*(3*(2*b^4*c^3 - 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^3*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^
4 - 2*(b^2 - 4*a*c)*b^2*c^3 - 8*(b^2 - 4*a*c)*a*c^4)*(b^4*c - 8*a*b^2*c^2
+ 16*a^2*c^3)^2*A + (2*b^5*c^2 - 40*a*b^3*c^3 + 128*a^2*b*c^4 - sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 20*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 32*(b^2 - 4*a*
c)*a*b*c^3)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)^2*B + 24*(sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^7*c^3 - 12*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*b^5*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^4 - 2*
a*b^7*c^4 + 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^5 + 16*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^5 + sqrt(2)*sqrt(b*c + ...
```

3.133.9 Mupad [B] (verification not implemented)

Time = 9.74 (sec) , antiderivative size = 19041, normalized size of antiderivative = 41.30

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int((x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

output
```
atan(((((5242880*B*a^7*c^8 + 3072*A*a*b^11*c^3 - 3145728*A*a^6*b*c^8 - 256
*B*a*b^12*c^2 - 61440*A*a^2*b^9*c^4 + 491520*A*a^3*b^7*c^5 - 1966080*A*a^4
*b^5*c^6 + 3932160*A*a^5*b^3*c^7 + 61440*B*a^3*b^8*c^4 - 655360*B*a^4*b^6*
c^5 + 2949120*B*a^5*b^4*c^6 - 6291456*B*a^6*b^2*c^7)/(512*(b^12*c + 4096*a
^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4
*c^5 - 6144*a^5*b^2*c^6)) - (x*(-(B^2*b^17 + 9*A^2*b^15*c^2 + 9*A^2*c^2*(-
(4*a*c - b^2)^15)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c
 - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6
- 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 -
10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 6
80960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*
B^2*a*b^15*c - 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 -
 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 240
00*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 17
81760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4*a*c - b^2)
^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a
*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 25
8048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^
8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*(256*b^11*c^3 - 5120*a*b^9*c^4
- 262144*a^5*b*c^8 + 40960*a^2*b^7*c^5 - 163840*a^3*b^5*c^6 + 327680*a^...
```

3.134 $\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

3.134.1 Optimal result

Integrand size = 25, antiderivative size = 380

$$\begin{aligned}\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx &= -\frac{x^3(Ab-2aB-(bB-2Ac)x^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{3x(4abB-A(b^2+4ac)+(b^2B-4Abc+4aBc)x^2)}{8(b^2-4ac)^2(a+bx^2+cx^4)}\\ &\quad+\frac{3\left(b^2B-4Abc+4aBc-\frac{b^3B-6Ab^2c+12abBc-8aAc^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\sqrt{c}(b^2-4ac)^2\sqrt{b-\sqrt{b^2-4ac}}}\\ &\quad+\frac{3\left(b^2B-4Abc+4aBc+\frac{b^3B-6Ab^2c+12abBc-8aAc^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\sqrt{c}(b^2-4ac)^2\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

output
```
-1/4*x^3*(A*b-2*B*a-(-2*A*c+B*b)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+3/8*x
*(4*a*b*B-A*(4*a*c+b^2)+(-4*A*b*c+4*B*a*c+B*b^2)*x^2)/(-4*a*c+b^2)^2/(c*x^
4+b*x^2+a)+3/16*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(B*
b^2-4*A*b*c+4*B*a*c+(8*A*a*c^2+6*A*b^2*c-12*B*a*b*c-B*b^3)/(-4*a*c+b^2)^(1
/2))/(-4*a*c+b^2)^2*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+3/16*arct
an(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(B*b^2-4*A*b*c+4*B*a*c+
(-8*A*a*c^2-6*A*b^2*c+12*B*a*b*c+B*b^3)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^2
*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.134.2 Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.18

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$$

$$= \frac{\frac{-4abBx+4b(-bB+Ac)x^3+8acx(A+Bx^2)}{(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{2x(2b^3B+4ac^2(A+3Bx^2)+4bc(aB-3Acx^2)+b^2(-7Ac+3Bcx^2))}{(b^2-4ac)^2(a+bx^2+cx^4)}+\frac{3\sqrt{2}\sqrt{c}\left(-b^3B-4bc\left(3aB\right.\right.}{}}{}$$

input
```
Integrate[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

output
```
((-4*a*b*B*x + 4*b*(-(b*B) + A*c)*x^3 + 8*a*c*x*(A + B*x^2))/((b^2 - 4*a*c
)*(a + b*x^2 + c*x^4)^2) + (2*x*(2*b^3*B + 4*a*c^2*(A + 3*B*x^2) + 4*b*c*(
a*B - 3*A*c*x^2) + b^2*(-7*A*c + 3*B*c*x^2)))/((b^2 - 4*a*c)^2*(a + b*x^2
+ c*x^4)) + (3*Sqrt[2]*Sqrt[c]*(-(b^3*B) - 4*b*c*(3*a*B + A*Sqrt[b^2 - 4*a
*c]) + 4*a*c*(2*A*c + B*Sqrt[b^2 - 4*a*c]) + b^2*(6*A*c + B*Sqrt[b^2 - 4*a
*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a
*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(b^3*B + 4*b*c
*(3*a*B - A*Sqrt[b^2 - 4*a*c]) + b^2*(-6*A*c + B*Sqrt[b^2 - 4*a*c]) + 4*a*
c*(-2*A*c + B*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt
[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(16*c)
```

3.134.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1598, 27, 1598, 25, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$$

$$\Big\downarrow \ 1598$$

$$\frac{\int \frac{3x^2((bB-2Ac)x^2+Ab-2aB)}{(cx^4+bx^2+a)^2}dx}{4(b^2-4ac)}-\frac{x^3(-2aB-(x^2(bB-2Ac))+Ab)}{4(b^2-4ac)(a+bx^2+cx^4)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{3\int \frac{x^2((bB-2Ac)x^2+Ab-2aB)}{(cx^4+bx^2+a)^2}dx}{4(b^2-4ac)}-\frac{x^3\left(-2aB-\left(x^2(bB-2Ac)\right)+Ab\right)}{4(b^2-4ac)(a+bx^2+cx^4)^2}$$

$\downarrow$ 1598

$$\frac{3\left(\frac{\int -\frac{-\left(\left(Bb^2-4Acb+4aBc\right)x^2\right)+4abB-A\left(b^2+4ac\right)}{cx^4+bx^2+a}dx}{2(b^2-4ac)}+\frac{x(x^2(4aBc-4Abc+b^2B)-A(4ac+b^2)+4abB)}{2(b^2-4ac)(a+bx^2+cx^4)}\right)}{4(b^2-4ac)}-\frac{x^3\left(-2aB-\left(x^2(bB-2Ac)\right)+Ab\right)}{4(b^2-4ac)(a+bx^2+cx^4)^2}$$

$\downarrow$ 25

$$\frac{3\left(\frac{x(x^2(4aBc-4Abc+b^2B)-A(4ac+b^2)+4abB)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int \frac{-\left(\left(Bb^2-4Acb+4aBc\right)x^2\right)+4abB-A\left(b^2+4ac\right)}{cx^4+bx^2+a}dx}{2(b^2-4ac)}\right)}{4(b^2-4ac)}-\frac{x^3\left(-2aB-\left(x^2(bB-2Ac)\right)+Ab\right)}{4(b^2-4ac)(a+bx^2+cx^4)^2}$$

$\downarrow$ 1480

$$\frac{3\left(\frac{x(x^2(4aBc-4Abc+b^2B)-A(4ac+b^2)+4abB)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{-\frac{1}{2}\left(-\frac{-8aAc^2+12abBc-6Ab^2c+b^3B}{\sqrt{b^2-4ac}}+4aBc-4Abc+b^2B\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx-\frac{1}{2}\left(\frac{-8}{}\right.}{2(b^2-4ac)}\right.}{4(b^2-4ac)}$$

$$\frac{x^3\left(-2aB-\left(x^2(bB-2Ac)\right)+Ab\right)}{4(b^2-4ac)(a+bx^2+cx^4)^2}$$

$\downarrow$ 218

$$\frac{3\left(\frac{x(x^2(4aBc-4Abc+b^2B)-A(4ac+b^2)+4abB)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{-\frac{\left(-\frac{-8aAc^2+12abBc-6Ab^2c+b^3B}{\sqrt{b^2-4ac}}+4aBc-4Abc+b^2B\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{-8aAc^2+12}{\sqrt{}}\right.}{}}{2(b^2-4ac)}\right.}{4(b^2-4ac)}$$

$$\frac{x^3\left(-2aB-\left(x^2(bB-2Ac)\right)+Ab\right)}{4(b^2-4ac)(a+bx^2+cx^4)^2}$$

input

```
Int[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

3.134. $\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

output
```
-1/4*(x^3*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c
*x^4)^2) + (3*((x*(4*a*b*B - A*(b^2 + 4*a*c) + (b^2*B - 4*A*b*c + 4*a*B*c)
*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (-(((b^2*B - 4*A*b*c + 4*a*
B*c - (b^3*B - 6*A*b^2*c + 12*a*b*B*c - 8*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcT
an[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt
[b - Sqrt[b^2 - 4*a*c]])) - ((b^2*B - 4*A*b*c + 4*a*B*c + (b^3*B - 6*A*b^2
*c + 12*a*b*B*c - 8*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]
]))/(2*(b^2 - 4*a*c))))/(4*(b^2 - 4*a*c))
```

3.134.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1598
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Simp[f
^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1
)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
 GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.134.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.86

method	result
risch	$\frac{-\frac{3c\left(4Abc-4Bac-B\,b^{2}\right)x^{7}}{8\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{\left(4Aa\,c^{2}-19A\,b^{2}c+16Babc+5B\,b^{3}\right)x^{5}}{128a^{2}c^{2}-64a\,b^{2}c+8b^{4}}-\frac{\left(16Aabc+5A\,b^{3}+4a^{2}Bc-19Ba\,b^{2}\right)x^{3}}{8\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{3a\left(4Aac+A\,b^{2}-4abB\right)x}{8\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}+ \frac{3\left(-\cdots\right.}{\cdots}$
default	$\frac{-\frac{3c\left(4Abc-4Bac-B\,b^{2}\right)x^{7}}{8\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{\left(4Aa\,c^{2}-19A\,b^{2}c+16Babc+5B\,b^{3}\right)x^{5}}{128a^{2}c^{2}-64a\,b^{2}c+8b^{4}}-\frac{\left(16Aabc+5A\,b^{3}+4a^{2}Bc-19Ba\,b^{2}\right)x^{3}}{8\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{3a\left(4Aac+A\,b^{2}-4abB\right)x}{8\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}+ \frac{3c\left(\cdots\right.}{\cdots}$

input
```
int(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-3/8*c*(4*A*b*c-4*B*a*c-B*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7+1/8*(4*A*a*
c^2-19*A*b^2*c+16*B*a*b*c+5*B*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-1/8*(16*
A*a*b*c+5*A*b^3+4*B*a^2*c-19*B*a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-3/8*a
*(4*A*a*c+A*b^2-4*B*a*b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x)/(c*x^4+b*x^2+a)^2+3
/16*sum((-(4*A*b*c-4*B*a*c-B*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*_R^2+(4*A*a*c
+A*b^2-4*B*a*b)/(16*a^2*c^2-8*a*b^2*c+b^4))/(2*_R^3*c+_R*b)*ln(x-_R),_R=Ro
otOf(_Z^4*c+_Z^2*b+a))
```

3.134.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5650 vs. 2(336) = 672.

Time = 4.21 (sec) , antiderivative size = 5650, normalized size of antiderivative = 14.87

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.134.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

output
```
Timed out
```

3.134.7 Maxima [F]

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \int \frac{(Bx^2+A)x^4}{(cx^4+bx^2+a)^3}\,dx$$

input
```
integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*(3*(B*b^2*c + 4*(B*a - A*b)*c^2)*x^7 + (5*B*b^3 + 4*A*a*c^2 + (16*B*a*
b - 19*A*b^2)*c)*x^5 + (19*B*a*b^2 - 5*A*b^3 - 4*(B*a^2 + 4*A*a*b)*c)*x^3
+ 3*(4*B*a^2*b - A*a*b^2 - 4*A*a^2*c)*x)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*
c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^
2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b
^3*c + 16*a^3*b*c^2)*x^2) - 3/8*integrate((4*B*a*b - A*b^2 - 4*A*a*c - (B*
b^2 + 4*(B*a - A*b)*c)*x^2)/(c*x^4 + b*x^2 + a), x)/(b^4 - 8*a*b^2*c + 16*
a^2*c^2)
```

3.134.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3164 vs. $2(336) = 672$.

Time = 1.74 (sec) , antiderivative size = 3164, normalized size of antiderivative = 8.33

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
3/32*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 4*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
^5*c - 2*b^6*c - 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 +
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 8*a*b^4*c^2 + 2*b^5*c^2
+ 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 32*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 32*a^2*b^2*c^3 + 16*a*b^3*c^3 - 16*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 128*a^3*c^4 - 96*a^2*b*c^4
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 48*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 24*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*(b^2 - 4*a*c)*b^4*c - 2*(b^
2 - 4*a*c)*b^3*c^2 - 32*(b^2 - 4*a*c)*a^2*c^3 - 24*(b^2 - 4*a*c)*a*b*c^3)*
A - 2*(2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 - 16*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b^4*c - 4*a*b^5*c + 32*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3
*b*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 2*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 32*a^2*b^3*c^2 + 6*a*b^4*...
```

3.134.9 Mupad [B] (verification not implemented)

Time = 9.43 (sec) , antiderivative size = 16688, normalized size of antiderivative = 43.92

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int((x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

output
```
atan(((((3*(1048576*A*a^6*c^8 - 256*A*b^12*c^2 + 4096*A*a*b^10*c^3 + 1024*
B*a*b^11*c^2 - 1048576*B*a^6*b*c^7 - 20480*A*a^2*b^8*c^4 + 327680*A*a^4*b^
4*c^6 - 1048576*A*a^5*b^2*c^7 - 20480*B*a^2*b^9*c^3 + 163840*B*a^3*b^7*c^4
 - 655360*B*a^4*b^5*c^5 + 1310720*B*a^5*b^3*c^6))/(512*(b^12 + 4096*a^6*c^
6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c
^5 - 24*a*b^10*c)) - (x*(-(9*(B^2*a*b^15 + B^2*a*(-(4*a*c - b^2)^15)^(1/2)
 + A^2*b^15*c - A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4
160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440
*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2
*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^
8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 8192
0*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a
^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^
7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 +
720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^1
0*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2
621440*a^10*b^2*c^10 + a*b^20*c)))^(1/2)*(256*b^11*c^2 - 5120*a*b^9*c^3 -
262144*a^5*b*c^7 + 40960*a^2*b^7*c^4 - 163840*a^3*b^5*c^5 + 327680*a^4*b^3
*c^6))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^
6*c)))*(-(9*(B^2*a*b^15 + B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c ...
```

3.134. $\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

3.135 $\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

3.135.1 Optimal result

Integrand size = 25, antiderivative size = 438

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = -\frac{x(Ab-2aB-(bB-2Ac)x^2)}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2}$$
$$-\frac{x(aB(7b^2-4ac)-A(b^3+8abc)+c(12abB-A(b^2+20ac))\,x^2)}{8a\,(b^2-4ac)^2\,(a+bx^2+cx^4)}$$
$$+\frac{\sqrt{c}\left(6aB\left(3b^2+4ac-2b\sqrt{b^2-4ac}\right)+A\left(b^3-52abc+b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{\cdots}}{\sqrt{b-\sqrt{b^2\cdots}}}\right.}{8\sqrt{2}a\,(b^2-4ac)^{5/2}\,\sqrt{b-\sqrt{b^2-4ac}}}$$
$$-\frac{\sqrt{c}\left(6aB\left(3b^2+4ac+2b\sqrt{b^2-4ac}\right)+A\left(b^3-52abc-b^2\sqrt{b^2-4ac}-20ac\sqrt{b^2-4ac}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{\cdots}}{\sqrt{b+\sqrt{b^2\cdots}}}\right.}{8\sqrt{2}a\,(b^2-4ac)^{5/2}\,\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
-1/4*x*(A*b-2*B*a-(-2*A*c+B*b)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2-1/8*x*(
a*B*(-4*a*c+7*b^2)-A*(8*a*b*c+b^3)+c*(12*a*b*B-A*(20*a*c+b^2))*x^2)/a/(-4*
a*c+b^2)^2/(c*x^4+b*x^2+a)+1/16*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(
1/2))^(1/2))*c^(1/2)*(6*a*B*(3*b^2+4*a*c-2*b*(-4*a*c+b^2)^(1/2))+A*(b^3-52
*a*b*c+b^2*(-4*a*c+b^2)^(1/2)+20*a*c*(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)^(
5/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/16*arctan(x*2^(1/2)*c^(1/2)/(b
+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(6*a*B*(3*b^2+4*a*c+2*b*(-4*a*c+b^2)^(
1/2))+A*(b^3-52*a*b*c-b^2*(-4*a*c+b^2)^(1/2)-20*a*c*(-4*a*c+b^2)^(1/2)))/a
/(-4*a*c+b^2)^(5/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.135.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.00

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \frac{1}{16}\left(\frac{4x(B(2a+bx^2)-A(b+2cx^2))}{(b^2-4ac)\,(a+bx^2+cx^4)^2}\right.$$
$$+\frac{2x(aB(-7b^2+4ac-12bcx^2)+A(b^3+8abc+b^2cx^2+20ac^2x^2))}{a\,(b^2-4ac)^2\,(a+bx^2+cx^4)}$$
$$+\frac{\sqrt{2}\sqrt{c}\big(6aB\big(3b^2+4ac-2b\sqrt{b^2-4ac}\big)+A\big(b^3-52abc+b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}\big)\big)\arctan\Big(\frac{}{\sqrt{b-}}}{a\,(b^2-4ac)^{5/2}\,\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt{2}\sqrt{c}\big(-6aB\big(3b^2+4ac+2b\sqrt{b^2-4ac}\big)+A\big(-b^3+52abc+b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}\big)\big)\arctan\Big(}{a\,(b^2-4ac)^{5/2}\,\sqrt{b+\sqrt{b^2-4ac}}}$$

input
```
Integrate[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

output
```
((4*x*(B*(2*a + b*x^2) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x
^4)^2) + (2*x*(a*B*(-7*b^2 + 4*a*c - 12*b*c*x^2) + A*(b^3 + 8*a*b*c + b^2*
c*x^2 + 20*a*c^2*x^2)))/(a*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sqrt[2]
*Sqrt[c]*(6*a*B*(3*b^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c]) + A*(b^3 - 52*a*b*
c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt
[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-6*a*B*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 -
 4*a*c]) + A*(-b^3 + 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 -
4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a*(b^2
- 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/16
```

3.135.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 403, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1598, 1492, 25, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^3}\,dx$$

$$\Big\downarrow \text{1598}$$

$$\frac{\int \frac{5(bB-2Ac)x^2+Ab-2aB}{(cx^4+bx^2+a)^2}dx}{4(b^2-4ac)}-\frac{x\left(-2aB-\left(x^2(bB-2Ac)\right)+Ab\right)}{4(b^2-4ac)\left(a+bx^2+cx^4\right)^2}$$

$$\Big\downarrow \text{1492}$$

$$\frac{-\frac{\int -\frac{-c\left(12abB-A\left(b^2+20ac\right)\right)x^2+3aB\left(b^2+4ac\right)+A\left(b^3-16abc\right)}{cx^4+bx^2+a}dx}{2a(b^2-4ac)}-\frac{x(-A(8abc+b^3)+cx^2(12abB-A(20ac+b^2))+aB(7b^2-4ac))}{2a(b^2-4ac)(a+bx^2+cx^4)}}{4(b^2-4ac)}-\frac{x\left(-2aB-\left(x^2(bB-2Ac)\right)+Ab\right)}{4(b^2-4ac)\left(a+bx^2+cx^4\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{-c\left(12abB-A\left(b^2+20ac\right)\right)x^2+3aB\left(b^2+4ac\right)+A\left(b^3-16abc\right)}{cx^4+bx^2+a}dx}{2a(b^2-4ac)}-\frac{x(-A(8abc+b^3)+cx^2(12abB-A(20ac+b^2))+aB(7b^2-4ac))}{2a(b^2-4ac)(a+bx^2+cx^4)}}{4(b^2-4ac)}-\frac{x\left(-2aB-\left(x^2(bB-2Ac)\right)+Ab\right)}{4(b^2-4ac)\left(a+bx^2+cx^4\right)^2}$$

$$\Big\downarrow \text{1480}$$

$$\frac{\frac{-\frac{1}{2}c\left(-A(20ac+b^2)-\frac{A\left(b^3-52abc\right)+6aB\left(4ac+3b^2\right)}{\sqrt{b^2-4ac}}+12abB\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx-\frac{1}{2}c\left(-A(20ac+b^2)+\frac{A\left(b^3-52abc\right)+6aB\left(4ac+3b^2\right)}{\sqrt{b^2-4ac}}+12abB\right.}{2a(b^2-4ac)}}{4(b^2-4ac)}$$

$$\frac{x\left(-2aB-\left(x^2(bB-2Ac)\right)+Ab\right)}{4(b^2-4ac)\left(a+bx^2+cx^4\right)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{-\frac{\sqrt{c}\left(-A\left(20ac+b^2\right)-\frac{A\left(b^3-52abc\right)+6aB\left(4ac+3b^2\right)}{\sqrt{b^2-4ac}}+12abB\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-A\left(20ac+b^2\right)+\frac{A\left(b^3-52abc\right)+6aB\left(4ac+3b^2\right)}{\sqrt{b^2-4ac}}+12abB\right)\text{arctan}}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{2a(b^2-4ac)}}{4(b^2-4ac)}$$

$$\frac{x\left(-2aB-\left(x^2(bB-2Ac)\right)+Ab\right)}{4(b^2-4ac)\left(a+bx^2+cx^4\right)^2}$$

input
```
Int[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

3.135. $\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

output
```
-1/4*(x*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x
^4)^2) + (-1/2*(x*(a*B*(7*b^2 - 4*a*c) - A*(b^3 + 8*a*b*c) + c*(12*a*b*B -
 A*(b^2 + 20*a*c))*x^2))/(a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (-((Sqrt[
c]*(12*a*b*B - A*(b^2 + 20*a*c) - (6*a*B*(3*b^2 + 4*a*c) + A*(b^3 - 52*a*b
*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a
*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]])) - (Sqrt[c]*(12*a*b*B - A*(b^
2 + 20*a*c) + (6*a*B*(3*b^2 + 4*a*c) + A*(b^3 - 52*a*b*c))/Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt
[b + Sqrt[b^2 - 4*a*c]]))/(2*a*(b^2 - 4*a*c)))/(4*(b^2 - 4*a*c))
```

3.135.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

3.135. $\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

rule 1598
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Simp[f
^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1
)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
 GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.135.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.85

method	result
risch	$\frac{\frac{c^2\left(20Aac+A\,b^2-12abB\right)x^7}{8a\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{c\left(28Aabc+2A\,b^3+4a^2Bc-19Ba\,b^2\right)x^5}{8a\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{\left(36A\,a^2c^2+5Aa\,b^2c+A\,b^4-16a^2bBc-5Ba\,b^3\right)x^3}{8a\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{\left(16Aabc-A\,b^3-12a^2B \cdots\right.}{128a^2c^2-64a\,b^2 \cdots}}{\left(c\,x^4+b\,x^2+a\right)^2}$
default	$\frac{\frac{c^2\left(20Aac+A\,b^2-12abB\right)x^7}{8a\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{c\left(28Aabc+2A\,b^3+4a^2Bc-19Ba\,b^2\right)x^5}{8a\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{\left(36A\,a^2c^2+5Aa\,b^2c+A\,b^4-16a^2bBc-5Ba\,b^3\right)x^3}{8a\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{\left(16Aabc-A\,b^3-12a^2B \cdots\right.}{128a^2c^2-64a\,b^2 \cdots}}{\left(c\,x^4+b\,x^2+a\right)^2}$

input
```
int(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8*c^2*(20*A*a*c+A*b^2-12*B*a*b)/a/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7+1/8/a*
c*(28*A*a*b*c+2*A*b^3+4*B*a^2*c-19*B*a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5
+1/8*(36*A*a^2*c^2+5*A*a*b^2*c+A*b^4-16*B*a^2*b*c-5*B*a*b^3)/a/(16*a^2*c^2
-8*a*b^2*c+b^4)*x^3+1/8*(16*A*a*b*c-A*b^3-12*B*a^2*c-3*B*a*b^2)/(16*a^2*c^
2-8*a*b^2*c+b^4)*x)/(c*x^4+b*x^2+a)^2+1/16/a*sum((c*(20*A*a*c+A*b^2-12*B*a
*b)/(16*a^2*c^2-8*a*b^2*c+b^4)*_R^2-(16*A*a*b*c-A*b^3-12*B*a^2*c-3*B*a*b^2
)/(16*a^2*c^2-8*a*b^2*c+b^4))/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z
^2*b+a))
```

3.135.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7270 vs. $2(382) = 764$.

Time = 8.33 (sec) , antiderivative size = 7270, normalized size of antiderivative = 16.60

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.135.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

output
```
Timed out
```

3.135.7 Maxima [F]

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \int \frac{(Bx^2+A)x^2}{(cx^4+bx^2+a)^3}\,dx$$

input
```
integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*((20*A*a*c^3 - (12*B*a*b - A*b^2)*c^2)*x^7 + (4*(B*a^2 + 7*A*a*b)*c^2
- (19*B*a*b^2 - 2*A*b^3)*c)*x^5 - (5*B*a*b^3 - A*b^4 - 36*A*a^2*c^2 + (16*
B*a^2*b - 5*A*a*b^2)*c)*x^3 - (3*B*a^2*b^2 + A*a*b^3 + 4*(3*B*a^3 - 4*A*a^
2*b)*c)*x)/((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4
*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*
b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*
c^2)*x^2) + 1/8*integrate((3*B*a*b^2 + A*b^3 + (20*A*a*c^2 - (12*B*a*b - A
*b^2)*c)*x^2 + 4*(3*B*a^2 - 4*A*a*b)*c)/(c*x^4 + b*x^2 + a), x)/(a*b^4 - 8
*a^2*b^2*c + 16*a^3*c^2)
```

3.135.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7267 vs. $2(382) = 764$.

Time = 2.18 (sec) , antiderivative size = 7267, normalized size of antiderivative = 16.59

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
-1/64*((2*b^4*c^2 + 32*a*b^2*c^3 - 160*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*b^3*c + 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*c^2 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^2*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 - 40*(b^2 - 4*a*c)*a*c^3)*(a*b^4 - 8*
a^2*b^2*c + 16*a^3*c^2)^2*A - 12*(2*a*b^3*c^2 - 8*a^2*b*c^3 - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3 + 4*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*a*b*c^2)*(a*b^4 -
8*a^2*b^2*c + 16*a^3*c^2)^2*B - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b^9 - 28*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c - 2*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^8*c - 2*a*b^9*c + 240*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 + 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b^6*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^2 + 5
6*a^2*b^7*c^2 - 832*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 -
288*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 - 24*sqrt(2)*sq...
```

3.135.9 Mupad [B] (verification not implemented)

Time = 9.69 (sec) , antiderivative size = 18992, normalized size of antiderivative = 43.36

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int((x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

output
```
((x^3*(A*b^4 + 36*A*a^2*c^2 - 5*B*a*b^3 + 5*A*a*b^2*c - 16*B*a^2*b*c))/(8*
a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (x*(A*b^3 + 3*B*a*b^2 + 12*B*a^2*c - 1
6*A*a*b*c))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^5*(4*B*a^2*c^2 + 2*A*b
^3*c + 28*A*a*b*c^2 - 19*B*a*b^2*c))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))
+ (c*x^7*(20*A*a*c^2 + A*b^2*c - 12*B*a*b*c))/(8*a*(b^4 + 16*a^2*c^2 - 8*a
*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) + at
an(((((256*A*a*b^13*c^2 - 3145728*B*a^8*c^8 + 4194304*A*a^7*b*c^8 - 9216*A
*a^2*b^11*c^3 + 122880*A*a^3*b^9*c^4 - 819200*A*a^4*b^7*c^5 + 2949120*A*a^
5*b^5*c^6 - 5505024*A*a^6*b^3*c^7 + 768*B*a^2*b^12*c^2 - 12288*B*a^3*b^10*
c^3 + 61440*B*a^4*b^8*c^4 - 983040*B*a^6*b^4*c^6 + 3145728*B*a^7*b^2*c^7)/
(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5
*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - (x*(-(A^2*b^17 + 9*B^2*
a^2*b^15 + A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 9*B^2*a^2*(-(4*a*c - b^2)^1
5)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 +
 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 +
1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 -
103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 9
83040*A*B*a^9*c^8 - 55*A^2*a*b^15*c - 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2)
 - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240
*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 9...
```

3.136 $\int \frac{A+Bx^2}{(a+bx^2+cx^4)^3}\,dx$

3.136.1 Optimal result

Integrand size = 22, antiderivative size = 460

$$\begin{aligned}\int \frac{A+Bx^2}{(a+bx^2+cx^4)^3}\,dx &= \frac{x(Ab^2-abB-2aAc+(Ab-2aB)cx^2)}{4a(b^2-4ac)(a+bx^2+cx^4)^2}\\ &+\frac{x(abB(b^2+8ac)+A(3b^4-25ab^2c+28a^2c^2)+c(aB(b^2+20ac)+3A(b^3-8abc))x^2)}{8a^2(b^2-4ac)^2(a+bx^2+cx^4)}\\ &+\frac{\sqrt{c}\left(aB(b^2+20ac)+3A(b^3-8abc)+\frac{abB(b^2-52ac)+3A(b^4-10ab^2c+56a^2c^2)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^2(b^2-4ac)^2\sqrt{b-\sqrt{b^2-4ac}}}\\ &+\frac{\sqrt{c}\left(aB(b^2+20ac)+3A(b^3-8abc)-\frac{abB(b^2-52ac)+3A(b^4-10ab^2c+56a^2c^2)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^2(b^2-4ac)^2\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

output
```
1/4*x*(A*b^2-a*b*B-2*A*a*c+(A*b-2*B*a)*c*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+
a)^2+1/8*x*(a*b*B*(8*a*c+b^2)+A*(28*a^2*c^2-25*a*b^2*c+3*b^4)+c*(a*B*(20*a
*c+b^2)+3*A*(-8*a*b*c+b^3))*x^2)/a^2/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+1/16*a
rctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(a*B*(20*a*c
+b^2)+3*A*(-8*a*b*c+b^3)+(a*b*B*(-52*a*c+b^2)+3*A*(56*a^2*c^2-10*a*b^2*c+b
^4))/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^2*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))
^(1/2)+1/16*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)
*(a*B*(20*a*c+b^2)+3*A*(-8*a*b*c+b^3)+(-a*b*B*(-52*a*c+b^2)-3*A*(56*a^2*c^
2-10*a*b^2*c+b^4))/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^2*2^(1/2)/(b+(-4*a
*c+b^2)^(1/2))^(1/2)
```

3.136.2 Mathematica [A] (verified)

Time = 1.30 (sec) , antiderivative size = 516, normalized size of antiderivative = 1.12

$$\int \frac{A+Bx^2}{(a+bx^2+cx^4)^3}\,dx$$

$$= \frac{-\frac{4ax(aB(b+2cx^2)-A(b^2-2ac+bcx^2))}{(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{2x(aB(b^3+8abc+b^2cx^2+20ac^2x^2)+A(3b^4-25ab^2c+28a^2c^2+3b^3cx^2-24abc^2x^2))}{(b^2-4ac)^2(a+bx^2+cx^4)}+\frac{\sqrt{2}\sqrt{c}\Big(aB\Big(}{}}{}$$

input
```
Integrate[(A + B*x^2)/(a + b*x^2 + c*x^4)^3,x]
```

output
```
((-4*a*x*(a*B*(b + 2*c*x^2) - A*(b^2 - 2*a*c + b*c*x^2)))/((b^2 - 4*a*c)*(
a + b*x^2 + c*x^4)^2) + (2*x*(a*B*(b^3 + 8*a*b*c + b^2*c*x^2 + 20*a*c^2*x^
2) + A*(3*b^4 - 25*a*b^2*c + 28*a^2*c^2 + 3*b^3*c*x^2 - 24*a*b*c^2*x^2)))/
((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(a*B*(b^3 - 52*a*
b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c]) + 3*A*(b^4 - 10*a*
b^2*c + 56*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c]))*A
rcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/
2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(a*B*(-b^3 + 52*a*b*c +
 b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c]) + 3*A*(-b^4 + 10*a*b^2*
c - 56*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c]))*ArcTa
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*S
qrt[b + Sqrt[b^2 - 4*a*c]]))/(16*a^2)
```

3.136.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1492, 25, 1492, 25, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{(a+bx^2+cx^4)^3}\,dx$$

$$\Big\downarrow \text{1492}$$

$$\frac{x(cx^2(Ab-2aB)-2aAc-abB+Ab^2)}{4a(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{\int -\frac{3Ab^2+aBb+5(Ab-2aB)cx^2-14aAc}{(cx^4+bx^2+a)^2}dx}{4a(b^2-4ac)}$$

$\downarrow$ 25

$$\frac{\int \frac{3Ab^2+aBb+5(Ab-2aB)cx^2-14aAc}{(cx^4+bx^2+a)^2}dx}{4a\left(b^2-4ac\right)}+\frac{x\left(cx^2(Ab-2aB)-2aAc-abB+Ab^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

$\downarrow$ 1492

$$\frac{\frac{x\left(A\left(28a^2c^2-25ab^2c+3b^4\right)+cx^2\left(3A\left(b^3-8abc\right)+aB\left(20ac+b^2\right)\right)+abB\left(8ac+b^2\right)\right)}{2a(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int -\frac{c\left(aB\left(b^2+20ac\right)+3A\left(b^3-8abc\right)\right)x^2+abB\left(b^2-16ac\right)+3A\left(b^4-9\right.}{cx^4+bx^2+a}}{2a(b^2-4ac)}}{4a\left(b^2-4ac\right)}$$

$$\frac{x\left(cx^2(Ab-2aB)-2aAc-abB+Ab^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

$\downarrow$ 25

$$\frac{\frac{\int \frac{c\left(aB\left(b^2+20ac\right)+3A\left(b^3-8abc\right)\right)x^2+abB\left(b^2-16ac\right)+3A\left(b^4-9acb^2+28a^2c^2\right)}{cx^4+bx^2+a}dx}{2a(b^2-4ac)}+\frac{x\left(A\left(28a^2c^2-25ab^2c+3b^4\right)+cx^2\left(3A\left(b^3-8abc\right)+aB\left(20ac+b^2\right)\right)+\right.}{2a(b^2-4ac)(a+bx^2+cx^4)}}{4a\left(b^2-4ac\right)}$$

$$\frac{x\left(cx^2(Ab-2aB)-2aAc-abB+Ab^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

$\downarrow$ 1480

$$\frac{\frac{\frac{1}{2}c\left(\frac{3A\left(56a^2c^2-10ab^2c+b^4\right)+abB\left(b^2-52ac\right)}{\sqrt{b^2-4ac}}+3A\left(b^3-8abc\right)+aB\left(20ac+b^2\right)\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}c\left(-\frac{3A\left(56a^2c^2-10ab^2c+b^4\right)+abB\left(b^2-52a\right.}{\sqrt{b^2-4ac}}\right.}{2a(b^2-4ac)}}{4a\left(b^2\right.}$$

$$\frac{x\left(cx^2(Ab-2aB)-2aAc-abB+Ab^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

$\downarrow$ 218

$$\frac{\frac{\frac{\sqrt{c}\left(\frac{3A\left(56a^2c^2-10ab^2c+b^4\right)+abB\left(b^2-52ac\right)}{\sqrt{b^2-4ac}}+3A\left(b^3-8abc\right)+aB\left(20ac+b^2\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{3A\left(56a^2c^2-10ab^2c+b^4\right)+abB\left(b^2-52ac\right)}{\sqrt{b^2-4ac}}+3\right.}{\sqrt{2}\sqrt{\sqrt{}}}}{2a(b^2-4ac)}}{4a\left(b^2-4a\right.}$$

$$\frac{x\left(cx^2(Ab-2aB)-2aAc-abB+Ab^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

input
```
Int[(A + B*x^2)/(a + b*x^2 + c*x^4)^3,x]
```

3.136. $\int \frac{A+Bx^2}{(a+bx^2+cx^4)^3}\,dx$

output
```
(x*(A*b^2 - a*b*B - 2*a*A*c + (A*b - 2*a*B)*c*x^2))/(4*a*(b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)^2) + ((x*(a*b*B*(b^2 + 8*a*c) + A*(3*b^4 - 25*a*b^2*c + 2
8*a^2*c^2) + c*(a*B*(b^2 + 20*a*c) + 3*A*(b^3 - 8*a*b*c))*x^2))/(2*a*(b^2
- 4*a*c)*(a + b*x^2 + c*x^4)) + ((Sqrt[c]*(a*B*(b^2 + 20*a*c) + 3*A*(b^3 -
 8*a*b*c) + (a*b*B*(b^2 - 52*a*c) + 3*A*(b^4 - 10*a*b^2*c + 56*a^2*c^2))/S
qrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(a*B*(b^2 + 20*a*c) + 3*
A*(b^3 - 8*a*b*c) - (a*b*B*(b^2 - 52*a*c) + 3*A*(b^4 - 10*a*b^2*c + 56*a^2
*c^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4
*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*a*(b^2 - 4*a*c)))/(4*a*
(b^2 - 4*a*c))
```

3.136.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

3.136.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.39 (sec) , antiderivative size = 447, normalized size of antiderivative = 0.97

method	result
risch	$\frac{-\frac{c^{2}\left(24Aabc-3A\,b^{3}-20a^{2}Bc-Ba\,b^{2}\right)x^{7}}{8a^{2}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{c\left(28A\,a^{2}c^{2}-49Aa\,b^{2}c+6A\,b^{4}+28a^{2}bBc+2Ba\,b^{3}\right)x^{5}}{8a^{2}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{\left(4A\,a^{2}b\,c^{2}+20Aa\,b^{3}c-3A\,b^{5}-36a^{3}B\,c^{2}-5B\,a^{2}\right.}{8a^{2}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}$
default	Expression too large to display

input
```
int((B*x^2+A)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/8*c^2*(24*A*a*b*c-3*A*b^3-20*B*a^2*c-B*a*b^2)/a^2/(16*a^2*c^2-8*a*b^2*
c+b^4)*x^7+1/8/a^2*c*(28*A*a^2*c^2-49*A*a*b^2*c+6*A*b^4+28*B*a^2*b*c+2*B*a
*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-1/8*(4*A*a^2*b*c^2+20*A*a*b^3*c-3*A*b
^5-36*B*a^3*c^2-5*B*a^2*b^2*c-B*a*b^4)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+
1/8*(44*A*a^2*c^2-37*A*a*b^2*c+5*A*b^4+16*B*a^2*b*c-B*a*b^3)/(16*a^2*c^2-8
*a*b^2*c+b^4)/a*x)/(c*x^4+b*x^2+a)^2+1/16/a^2*sum((-c*(24*A*a*b*c-3*A*b^3-
20*B*a^2*c-B*a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*_R^2+(84*A*a^2*c^2-27*A*a*b
^2*c+3*A*b^4-16*B*a^2*b*c+B*a*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4))/(2*_R^3*c+_
R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

3.136.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9909 vs. 2(417) = 834.

Time = 21.04 (sec) , antiderivative size = 9909, normalized size of antiderivative = 21.54

$$\int \frac{A+Bx^2}{\left(a+bx^2+cx^4\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.136.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\left(a+bx^2+cx^4\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

output
```
Timed out
```

3.136.7 Maxima [F]

$$\int \frac{A+Bx^2}{\left(a+bx^2+cx^4\right)^3}\,dx = \int \frac{Bx^2+A}{\left(cx^4+bx^2+a\right)^3}\,dx$$

input
```
integrate((B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*((4*(5*B*a^2 - 6*A*a*b)*c^3 + (B*a*b^2 + 3*A*b^3)*c^2)*x^7 + (28*A*a^2
*c^3 + 7*(4*B*a^2*b - 7*A*a*b^2)*c^2 + 2*(B*a*b^3 + 3*A*b^4)*c)*x^5 + (B*a
*b^4 + 3*A*b^5 + 4*(9*B*a^3 - A*a^2*b)*c^2 + 5*(B*a^2*b^2 - 4*A*a*b^3)*c)*
x^3 - (B*a^2*b^3 - 5*A*a*b^4 - 44*A*a^3*c^2 - (16*B*a^3*b - 37*A*a^2*b^2)*
c)*x)/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^5*b^
2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (a^2
*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b
*c^2)*x^2) - 1/8*integrate(-(B*a*b^3 + 3*A*b^4 + 84*A*a^2*c^2 + (4*(5*B*a^
2 - 6*A*a*b)*c^2 + (B*a*b^2 + 3*A*b^3)*c)*x^2 - (16*B*a^2*b + 27*A*a*b^2)*
c)/(c*x^4 + b*x^2 + a), x)/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)
```

3.136.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4605 vs. $2(417) = 834$.

Time = 1.79 (sec) , antiderivative size = 4605, normalized size of antiderivative = 10.01

$$\int \frac{A+Bx^2}{\left(a+bx^2+cx^4\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/32*(3*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8 - 17*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*b^7*c - 2*b^8*c + 116*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^
2 + 26*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 + sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 34*a*b^6*c^2 - 2*b^7*c^2 - 368*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 128*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 13*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^4*c^3 - 232*a^2*b^4*c^3 + 30*a*b^5*c^3 + 448*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^4*c^4 + 224*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*
c^4 + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + 736*a^3*b^2
*c^4 - 176*a^2*b^3*c^4 - 112*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c
^5 - 896*a^4*c^5 + 352*a^3*b*c^5 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^7 - 15*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^5*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^6*c + 88*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^2*b^3*c^2 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b^4*c^2 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
b^5*c^2 - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
3*b*c^3 - 88*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2
*b^2*c^3 - 11*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)...
```

3.136.9 Mupad [B] (verification not implemented)

Time = 10.18 (sec) , antiderivative size = 22914, normalized size of antiderivative = 49.81

$$\int \frac{A+Bx^2}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^2)/(a + b*x^2 + c*x^4)^3,x)
```

output
```
((x^3*(3*A*b^5 + 36*B*a^3*c^2 + B*a*b^4 - 20*A*a*b^3*c - 4*A*a^2*b*c^2 + 5
*B*a^2*b^2*c))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^5*(28*A*a^2*c^3
 + 6*A*b^4*c + 2*B*a*b^3*c - 49*A*a*b^2*c^2 + 28*B*a^2*b*c^2))/(8*a^2*(b^4
 + 16*a^2*c^2 - 8*a*b^2*c)) + (x*(5*A*b^4 + 44*A*a^2*c^2 - B*a*b^3 - 37*A*
a*b^2*c + 16*B*a^2*b*c))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (c*x^7*(20
*B*a^2*c^2 + 3*A*b^3*c - 24*A*a*b*c^2 + B*a*b^2*c))/(8*a^2*(b^4 + 16*a^2*c
^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x
^6) + atan((((4194304*B*a^9*b*c^8 - 22020096*A*a^9*c^9 + 768*A*a^2*b^14*c
^2 - 22272*A*a^3*b^12*c^3 + 282624*A*a^4*b^10*c^4 - 2027520*A*a^5*b^8*c^5
+ 8847360*A*a^6*b^6*c^6 - 23396352*A*a^7*b^4*c^7 + 34603008*A*a^8*b^2*c^8
+ 256*B*a^3*b^13*c^2 - 9216*B*a^4*b^11*c^3 + 122880*B*a^5*b^9*c^4 - 819200
*B*a^6*b^7*c^5 + 2949120*B*a^7*b^5*c^6 - 5505024*B*a^8*b^3*c^7)/(512*(a^4*
b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3
+ 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) - (x*(-(9*A^2*b^19 + B^2*a^2*b^17
+ 9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c
^2 - 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^
9*c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*
a^8*b^3*c^8 + 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*a^2*b^2*(-(4
*a*c - b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 3
4880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 +...
```

3.136. $\int \frac{A+Bx^2}{(a+bx^2+cx^4)^3}\,dx$

3.137 $\int \frac{x(-7+4x^2)}{4-5x^2+x^4}\,dx$

3.137.1 Optimal result

Integrand size = 21, antiderivative size = 25

$$\int \frac{x(-7+4x^2)}{4-5x^2+x^4}\,dx = \frac{1}{2}\log\left(1-x^2\right)+\frac{3}{2}\log\left(4-x^2\right)$$

output
```
1/2*ln(-x^2+1)+3/2*ln(-x^2+4)
```

3.137.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x(-7+4x^2)}{4-5x^2+x^4}\,dx = \frac{1}{2}\log\left(1-x^2\right)+\frac{3}{2}\log\left(4-x^2\right)$$

input
```
Integrate[(x*(-7 + 4*x^2))/(4 - 5*x^2 + x^4),x]
```

output
```
Log[1 - x^2]/2 + (3*Log[4 - x^2])/2
```

3.137.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1576, 25, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(4x^2-7)}{x^4-5x^2+4}dx$$

$$\Big\downarrow \text{1576}$$

$$\frac{1}{2}\int -\frac{7-4x^2}{x^4-5x^2+4}dx^2$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \frac{7-4x^2}{x^4-5x^2+4}dx^2$$

$$\Big\downarrow \text{1141}$$

$$-\frac{1}{2}\int \left(\frac{3}{4-x^2}+\frac{1}{1-x^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\log\left(1-x^2\right)+3\log\left(4-x^2\right)\right)$$

input

```
Int[(x*(-7 + 4*x^2))/(4 - 5*x^2 + x^4),x]
```

output

```
(Log[1 - x^2] + 3*Log[4 - x^2])/2
```

3.137.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.137.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

method	result	size
default	$\frac{\ln(x^2-1)}{2}+\frac{3\ln(x^2-4)}{2}$	18
risch	$\frac{\ln(x^2-1)}{2}+\frac{3\ln(x^2-4)}{2}$	18
norman	$\frac{3\ln(x-2)}{2}+\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}+\frac{3\ln(x+2)}{2}$	26
parallelrisch	$\frac{3\ln(x-2)}{2}+\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}+\frac{3\ln(x+2)}{2}$	26

input
```
int(x*(4*x^2-7)/(x^4-5*x^2+4),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x^2-1)+3/2*ln(x^2-4)
```

3.137.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{x(-7+4x^2)}{4-5x^2+x^4}\,dx = \frac{1}{2}\log\left(x^2-1\right) + \frac{3}{2}\log\left(x^2-4\right)$$

input
```
integrate(x*(4*x^2-7)/(x^4-5*x^2+4),x, algorithm="fricas")
```

output
```
1/2*log(x^2 - 1) + 3/2*log(x^2 - 4)
```

3.137.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{x(-7+4x^2)}{4-5x^2+x^4}\,dx = \frac{3\log\left(x^2-4\right)}{2} + \frac{\log\left(x^2-1\right)}{2}$$

input
```
integrate(x*(4*x**2-7)/(x**4-5*x**2+4),x)
```

output
```
3*log(x**2 - 4)/2 + log(x**2 - 1)/2
```

3.137.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{x(-7+4x^2)}{4-5x^2+x^4}\,dx = \frac{1}{2}\log\left(x^2-1\right) + \frac{3}{2}\log\left(x^2-4\right)$$

input
```
integrate(x*(4*x^2-7)/(x^4-5*x^2+4),x, algorithm="maxima")
```

output
```
1/2*log(x^2 - 1) + 3/2*log(x^2 - 4)
```

3.137.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{x(-7+4x^2)}{4-5x^2+x^4}\,dx = \frac{1}{2}\,\log\left(\left|x^2-1\right|\right)+\frac{3}{2}\,\log\left(\left|x^2-4\right|\right)$$

input
```
integrate(x*(4*x^2-7)/(x^4-5*x^2+4),x, algorithm="giac")
```

output
```
1/2*log(abs(x^2 - 1)) + 3/2*log(abs(x^2 - 4))
```

3.137.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{x(-7+4x^2)}{4-5x^2+x^4}\,dx = \frac{\ln\left(x^2-1\right)}{2}+\frac{3\,\ln\left(x^2-4\right)}{2}$$

input
```
int((x*(4*x^2 - 7))/(x^4 - 5*x^2 + 4),x)
```

output
```
log(x^2 - 1)/2 + (3*log(x^2 - 4))/2
```

3.138 $\int \frac{-7x+4x^3}{4-5x^2+x^4}\,dx$

3.138.1 Optimal result

Integrand size = 22, antiderivative size = 25

$$\int \frac{-7x+4x^3}{4-5x^2+x^4}\,dx=\frac{1}{2}\log\left(1-x^2\right)+\frac{3}{2}\log\left(4-x^2\right)$$

output
```
1/2*ln(-x^2+1)+3/2*ln(-x^2+4)
```

3.138.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{-7x+4x^3}{4-5x^2+x^4}\,dx=\frac{1}{2}\log\left(1-x^2\right)+\frac{3}{2}\log\left(4-x^2\right)$$

input
```
Integrate[(-7*x + 4*x^3)/(4 - 5*x^2 + x^4),x]
```

output
```
Log[1 - x^2]/2 + (3*Log[4 - x^2])/2
```

3.138.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2027, 1576, 25, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^3-7x}{x^4-5x^2+4} \, dx$$
$$\Big\downarrow \text{2027}$$
$$\int \frac{x(4x^2-7)}{x^4-5x^2+4}dx$$
$$\Big\downarrow \text{1576}$$
$$\frac{1}{2}\int -\frac{7-4x^2}{x^4-5x^2+4}dx^2$$
$$\Big\downarrow \text{25}$$
$$-\frac{1}{2}\int \frac{7-4x^2}{x^4-5x^2+4}dx^2$$
$$\Big\downarrow \text{1141}$$
$$-\frac{1}{2}\int \left(\frac{3}{4-x^2}+\frac{1}{1-x^2}\right)dx^2$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{2}\left(\log\left(1-x^2\right)+3\log\left(4-x^2\right)\right)$$

input
```
Int[(-7*x + 4*x^3)/(4 - 5*x^2 + x^4),x]
```

output
```
(Log[1 - x^2] + 3*Log[4 - x^2])/2
```

3.138.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.138.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

method	result	size
default	$\frac{\ln(x^2-1)}{2}+\frac{3\ln(x^2-4)}{2}$	18
risch	$\frac{\ln(x^2-1)}{2}+\frac{3\ln(x^2-4)}{2}$	18
norman	$\frac{3\ln(x-2)}{2}+\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}+\frac{3\ln(x+2)}{2}$	26
parallelrisch	$\frac{3\ln(x-2)}{2}+\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}+\frac{3\ln(x+2)}{2}$	26

input
```
int((4*x^3-7*x)/(x^4-5*x^2+4),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x^2-1)+3/2*ln(x^2-4)
```

3.138.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{-7x+4x^3}{4-5x^2+x^4}\,dx = \frac{1}{2}\log\left(x^2-1\right)+\frac{3}{2}\log\left(x^2-4\right)$$

input
```
integrate((4*x^3-7*x)/(x^4-5*x^2+4),x, algorithm="fricas")
```

output
```
1/2*log(x^2 - 1) + 3/2*log(x^2 - 4)
```

3.138.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{-7x+4x^3}{4-5x^2+x^4}\,dx = \frac{3\log\left(x^2-4\right)}{2}+\frac{\log\left(x^2-1\right)}{2}$$

input
```
integrate((4*x**3-7*x)/(x**4-5*x**2+4),x)
```

output
```
3*log(x**2 - 4)/2 + log(x**2 - 1)/2
```

3.138.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{-7x+4x^3}{4-5x^2+x^4}\,dx = \frac{3}{2}\log\left(x+2\right)+\frac{1}{2}\log\left(x+1\right)+\frac{1}{2}\log\left(x-1\right)+\frac{3}{2}\log\left(x-2\right)$$

input
```
integrate((4*x^3-7*x)/(x^4-5*x^2+4),x, algorithm="maxima")
```

output
```
3/2*log(x + 2) + 1/2*log(x + 1) + 1/2*log(x - 1) + 3/2*log(x - 2)
```

3.138.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{-7x + 4x^3}{4 - 5x^2 + x^4}\, dx = \frac{1}{2}\,\log\left(\left|x^2 - 1\right|\right) + \frac{3}{2}\,\log\left(\left|x^2 - 4\right|\right)$$

input
```
integrate((4*x^3-7*x)/(x^4-5*x^2+4),x, algorithm="giac")
```

output
```
1/2*log(abs(x^2 - 1)) + 3/2*log(abs(x^2 - 4))
```

3.138.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{-7x + 4x^3}{4 - 5x^2 + x^4}\, dx = \frac{\ln\left(x^2 - 1\right)}{2} + \frac{3\,\ln\left(x^2 - 4\right)}{2}$$

input
```
int(-(7*x - 4*x^3)/(x^4 - 5*x^2 + 4),x)
```

output
```
log(x^2 - 1)/2 + (3*log(x^2 - 4))/2
```

3.139 $\int \frac{x(2+x^2)}{1+x^2+x^4} \, dx$

3.139.1 Optimal result

Integrand size = 17, antiderivative size = 37

$$\int \frac{x(2+x^2)}{1+x^2+x^4} \, dx = \frac{1}{2}\sqrt{3}\arctan\left(\frac{1+2x^2}{\sqrt{3}}\right)+\frac{1}{4}\log\left(1+x^2+x^4\right)$$

output
```
1/4*ln(x^4+x^2+1)+1/2*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

3.139.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{x(2+x^2)}{1+x^2+x^4} \, dx = \frac{1}{2}\sqrt{3}\arctan\left(\frac{1+2x^2}{\sqrt{3}}\right)+\frac{1}{4}\log\left(1+x^2+x^4\right)$$

input
```
Integrate[(x*(2 + x^2))/(1 + x^2 + x^4),x]
```

output
```
(Sqrt[3]*ArcTan[(1 + 2*x^2)/Sqrt[3]])/2 + Log[1 + x^2 + x^4]/4
```

3.139.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {1576, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(x^2+2)}{x^4+x^2+1} dx$$

$$\Big\downarrow \text{1576}$$

$$\frac{1}{2}\int \frac{x^2+2}{x^4+x^2+1}dx^2$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{3}{2}\int \frac{1}{x^4+x^2+1}dx^2+\frac{1}{2}\int \frac{2x^2+1}{x^4+x^2+1}dx^2\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{2x^2+1}{x^4+x^2+1}dx^2-3\int \frac{1}{-x^4-3}d(2x^2+1)\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{2x^2+1}{x^4+x^2+1}dx^2+\sqrt{3}\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\sqrt{3}\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)+\frac{1}{2}\log\left(x^4+x^2+1\right)\right)$$

input
```
Int[(x*(2 + x^2))/(1 + x^2 + x^4),x]
```

output
```
(Sqrt[3]*ArcTan[(1 + 2*x^2)/Sqrt[3]] + Log[1 + x^2 + x^4]/2)/2
```

3.139.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

3.139.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\ln\left(x^{4}+x^{2}+1\right)}{4}+\frac{\arctan\left(\frac{\left(2x^{2}+1\right)\sqrt{3}}{3}\right)\sqrt{3}}{2}$	31
risch	$\frac{\ln\left(4x^{4}+4x^{2}+4\right)}{4}+\frac{\arctan\left(\frac{\left(2x^{2}+1\right)\sqrt{3}}{3}\right)\sqrt{3}}{2}$	35

input
```
int(x*(x^2+2)/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(x^4+x^2+1)+1/2*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

3.139.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{x(2+x^2)}{1+x^2+x^4}\,dx = \frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right) + \frac{1}{4}\log\left(x^4+x^2+1\right)$$

input
```
integrate(x*(x^2+2)/(x^4+x^2+1),x, algorithm="fricas")
```

output
```
1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*log(x^4 + x^2 + 1)
```

3.139.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{x(2+x^2)}{1+x^2+x^4}\,dx = \frac{\log\left(x^4+x^2+1\right)}{4} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3}+\frac{\sqrt{3}}{3}\right)}{2}$$

input
```
integrate(x*(x**2+2)/(x**4+x**2+1),x)
```

output
```
log(x**4 + x**2 + 1)/4 + sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/2
```

3.139.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{x(2+x^2)}{1+x^2+x^4}\,dx = \frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right) + \frac{1}{4}\log\left(x^4+x^2+1\right)$$

input
```
integrate(x*(x^2+2)/(x^4+x^2+1),x, algorithm="maxima")
```

output
```
1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*log(x^4 + x^2 + 1)
```

3.139.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{x(2+x^2)}{1+x^2+x^4}\,dx = \frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right) + \frac{1}{4}\log\left(x^4+x^2+1\right)$$

input
```
integrate(x*(x^2+2)/(x^4+x^2+1),x, algorithm="giac")
```

output
```
1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*log(x^4 + x^2 + 1)
```

3.139.9 Mupad [B] (verification not implemented)

Time = 7.56 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{x(2+x^2)}{1+x^2+x^4}\,dx = \frac{\ln\left(x^4+x^2+1\right)}{4} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x^2}{3}+\frac{\sqrt{3}}{3}\right)}{2}$$

input
```
int((x*(x^2 + 2))/(x^2 + x^4 + 1),x)
```

output
```
log(x^2 + x^4 + 1)/4 + (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^2)/3))/2
```

3.140 $\int \frac{2x+x^3}{1+x^2+x^4}\,dx$

3.140.1 Optimal result

Integrand size = 18, antiderivative size = 37

$$\int \frac{2x+x^3}{1+x^2+x^4}\,dx=\frac{1}{2}\sqrt{3}\arctan\left(\frac{1+2x^2}{\sqrt{3}}\right)+\frac{1}{4}\log\left(1+x^2+x^4\right)$$

output
```
1/4*ln(x^4+x^2+1)+1/2*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

3.140.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{2x+x^3}{1+x^2+x^4}\,dx=\frac{1}{2}\sqrt{3}\arctan\left(\frac{1+2x^2}{\sqrt{3}}\right)+\frac{1}{4}\log\left(1+x^2+x^4\right)$$

input
```
Integrate[(2*x + x^3)/(1 + x^2 + x^4),x]
```

output
```
(Sqrt[3]*ArcTan[(1 + 2*x^2)/Sqrt[3]])/2 + Log[1 + x^2 + x^4]/4
```

3.140.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2027, 1576, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+2x}{x^4+x^2+1}dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{x(x^2+2)}{x^4+x^2+1}dx$$

$$\Big\downarrow \text{1576}$$

$$\frac{1}{2}\int \frac{x^2+2}{x^4+x^2+1}dx^2$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{3}{2}\int \frac{1}{x^4+x^2+1}dx^2+\frac{1}{2}\int \frac{2x^2+1}{x^4+x^2+1}dx^2\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{2x^2+1}{x^4+x^2+1}dx^2-3\int \frac{1}{-x^4-3}d(2x^2+1)\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{2x^2+1}{x^4+x^2+1}dx^2+\sqrt{3}\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\sqrt{3}\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)+\frac{1}{2}\log\left(x^4+x^2+1\right)\right)$$

input
```
Int[(2*x + x^3)/(1 + x^2 + x^4),x]
```

output
```
(Sqrt[3]*ArcTan[(1 + 2*x^2)/Sqrt[3]] + Log[1 + x^2 + x^4]/2)/2
```

3.140.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.140.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\ln(x^4+x^2+1)}{4}+\frac{\arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)\sqrt{3}}{2}$	31
risch	$\frac{\ln(4x^4+4x^2+4)}{4}+\frac{\arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)\sqrt{3}}{2}$	35

input
```
int((x^3+2*x)/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(x^4+x^2+1)+1/2*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

3.140.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{2x+x^3}{1+x^2+x^4}\,dx = \frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right)+\frac{1}{4}\log\left(x^4+x^2+1\right)$$

input
```
integrate((x^3+2*x)/(x^4+x^2+1),x, algorithm="fricas")
```

output
```
1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*log(x^4 + x^2 + 1)
```

3.140.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{2x+x^3}{1+x^2+x^4}\,dx = \frac{\log\left(x^4+x^2+1\right)}{4}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3}+\frac{\sqrt{3}}{3}\right)}{2}$$

input
```
integrate((x**3+2*x)/(x**4+x**2+1),x)
```

output
```
log(x**4 + x**2 + 1)/4 + sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/2
```

3.140.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.43

$$\int \frac{2x+x^3}{1+x^2+x^4}\,dx = -\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}\log\left(x^2+x+1\right)+\frac{1}{4}\log\left(x^2-x+1\right)$$

input
```
integrate((x^3+2*x)/(x^4+x^2+1),x, algorithm="maxima")
```

output
```
-1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x - 1)) + 1/4*log(x^2 + x + 1) + 1/4*log(x^2 - x + 1)
```

3.140.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{2x+x^3}{1+x^2+x^4}\,dx = \frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x^2+1)\right)+\frac{1}{4}\log\left(x^4+x^2+1\right)$$

input
```
integrate((x^3+2*x)/(x^4+x^2+1),x, algorithm="giac")
```

output
```
1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*log(x^4 + x^2 + 1)
```

3.140.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{2x+x^3}{1+x^2+x^4}\,dx = \frac{\ln\left(x^4+x^2+1\right)}{4}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x^2}{3}+\frac{\sqrt{3}}{3}\right)}{2}$$

input
```
int((2*x + x^3)/(x^2 + x^4 + 1),x)
```

output
```
log(x^2 + x^4 + 1)/4 + (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^2)/3))/2
```

3.141 $\int \frac{11x+2x^3}{(3+2x^2+x^4)^2} \, dx$

3.141.1 Optimal result

Integrand size = 22, antiderivative size = 45

$$\int \frac{11x+2x^3}{(3+2x^2+x^4)^2} \, dx = \frac{5+9x^2}{8(3+2x^2+x^4)} + \frac{9\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}$$

output
```
1/8*(9*x^2+5)/(x^4+2*x^2+3)+9/16*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

3.141.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{11x+2x^3}{(3+2x^2+x^4)^2} \, dx = \frac{5+9x^2}{8(3+2x^2+x^4)} + \frac{9\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}$$

input
```
Integrate[(11*x + 2*x^3)/(3 + 2*x^2 + x^4)^2,x]
```

output
```
(5 + 9*x^2)/(8*(3 + 2*x^2 + x^4)) + (9*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[
2])
```

3.141.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.20, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {2027, 1576, 1159, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^3+11x}{(x^4+2x^2+3)^2}dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{x(2x^2+11)}{(x^4+2x^2+3)^2}dx$$

$$\Big\downarrow \text{1576}$$

$$\frac{1}{2}\int \frac{2x^2+11}{(x^4+2x^2+3)^2}dx^2$$

$$\Big\downarrow \text{1159}$$

$$\frac{1}{2}\left(\frac{9}{4}\int \frac{1}{x^4+2x^2+3}dx^2+\frac{9x^2+5}{4(x^4+2x^2+3)}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{9x^2+5}{4(x^4+2x^2+3)}-\frac{9}{2}\int \frac{1}{-x^4-8}d(2x^2+2)\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{9\arctan\left(\frac{2x^2+2}{2\sqrt{2}}\right)}{4\sqrt{2}}+\frac{9x^2+5}{4(x^4+2x^2+3)}\right)$$

input
```
Int[(11*x + 2*x^3)/(3 + 2*x^2 + x^4)^2,x]
```

output
```
((5 + 9*x^2)/(4*(3 + 2*x^2 + x^4)) + (9*ArcTan[(2 + 2*x^2)/(2*Sqrt[2])])/(
4*Sqrt[2]))/2
```

3.141.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.141.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

method	result	size
risch	$\frac{\frac{9x^2}{8}+\frac{5}{8}}{x^4+2x^2+3}+\frac{9\arctan\left(\frac{(x^2+1)\sqrt{2}}{2}\right)\sqrt{2}}{16}$	38
default	$\frac{18x^2+10}{16x^4+32x^2+48}+\frac{9\sqrt{2}\arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)}{16}$	41

input
```
int((2*x^3+11*x)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
(9/8*x^2+5/8)/(x^4+2*x^2+3)+9/16*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

3.141.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int \frac{11x+2x^3}{(3+2x^2+x^4)^2}\,dx = \frac{9\sqrt{2}(x^4+2x^2+3)\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right)+18x^2+10}{16\,(x^4+2x^2+3)}$$

input
```
integrate((2*x^3+11*x)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

output
```
1/16*(9*sqrt(2)*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 18*x^2 +
 10)/(x^4 + 2*x^2 + 3)
```

3.141.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{11x+2x^3}{(3+2x^2+x^4)^2}\,dx = \frac{9x^2+5}{8x^4+16x^2+24}+\frac{9\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

input
```
integrate((2*x**3+11*x)/(x**4+2*x**2+3)**2,x)
```

output
```
(9*x**2 + 5)/(8*x**4 + 16*x**2 + 24) + 9*sqrt(2)*atan(sqrt(2)*x**2/2 + sqr
t(2)/2)/16
```

3.141.7 Maxima [F]

$$\int \frac{11x+2x^3}{(3+2x^2+x^4)^2}\,dx = \int \frac{2x^3+11x}{(x^4+2x^2+3)^2}\,dx$$

input
```
integrate((2*x^3+11*x)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

output
```
1/8*(9*x^2 + 5)/(x^4 + 2*x^2 + 3) + 9/4*integrate(x/(x^4 + 2*x^2 + 3), x)
```

3.141.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int \frac{11x+2x^3}{(3+2x^2+x^4)^2}\,dx = \frac{9}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right) + \frac{9\,x^2+5}{8\,(x^4+2\,x^2+3)}$$

input
```
integrate((2*x^3+11*x)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

output
```
9/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1/8*(9*x^2 + 5)/(x^4 + 2*x^2
+ 3)
```

3.141.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{11x+2x^3}{(3+2x^2+x^4)^2}\,dx = \frac{\frac{9\,x^2}{8}+\frac{5}{8}}{x^4+2\,x^2+3} + \frac{9\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

input
```
int((11*x + 2*x^3)/(2*x^2 + x^4 + 3)^2,x)
```

output
```
((9*x^2)/8 + 5/8)/(2*x^2 + x^4 + 3) + (9*2^(1/2)*atan(2^(1/2)/2 + (2^(1/2)
*x^2)/2))/16
```

3.142 $\int x^5(2+3x^2)\sqrt{3+5x^2+x^4}\,dx$

3.142.1 Optimal result

Integrand size = 25, antiderivative size = 102

$$\int x^5(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = -\frac{1633}{256}(5+2x^2)\sqrt{3+5x^2+x^4}+\frac{3}{10}x^4(3+5x^2+x^4)^{3/2} + \frac{1}{480}(1837-510x^2)(3+5x^2+x^4)^{3/2} + \frac{21229}{512}\operatorname{arctanh}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)$$

output
```
3/10*x^4*(x^4+5*x^2+3)^(3/2)+1/480*(-510*x^2+1837)*(x^4+5*x^2+3)^(3/2)+212
29/512*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))-1633/256*(2*x^2+5)*(x^4+
5*x^2+3)^(1/2)
```

3.142.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.68

$$\int x^5(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \frac{\sqrt{3+5x^2+x^4}(-78387+12250x^2-2248x^4+1680x^6+1152x^8)}{3840} - \frac{21229}{512}\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)$$

input
```
Integrate[x^5*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

output
```
(Sqrt[3 + 5*x^2 + x^4]*(-78387 + 12250*x^2 - 2248*x^4 + 1680*x^6 + 1152*x^
8))/3840 - (21229*Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/512
```

3.142.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1578, 1236, 27, 1225, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 (3x^2+2) \sqrt{x^4+5x^2+3} \, dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int x^4 (3x^2+2) \sqrt{x^4+5x^2+3}dx^2$$

$$\Big\downarrow \text{1236}$$

$$\frac{1}{2}\left(\frac{1}{5}\int -\frac{1}{2}x^2 (85x^2+36) \sqrt{x^4+5x^2+3}dx^2+\frac{3}{5}\left(x^4+5x^2+3\right)^{3/2} x^4\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{3}{5}x^4 \left(x^4+5x^2+3\right)^{3/2}-\frac{1}{10}\int x^2 (85x^2+36) \sqrt{x^4+5x^2+3}dx^2\right)$$

$$\Big\downarrow \text{1225}$$

$$\frac{1}{2}\left(\frac{1}{10}\left(\frac{1}{24}(1837-510x^2) \left(x^4+5x^2+3\right)^{3/2}-\frac{8165}{16}\int \sqrt{x^4+5x^2+3}dx^2\right)+\frac{3}{5}\left(x^4+5x^2+3\right)^{3/2} x^4\right)$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{2}\left(\frac{1}{10}\left(\frac{1}{24}(1837-510x^2) \left(x^4+5x^2+3\right)^{3/2}-\frac{8165}{16}\left(\frac{1}{4}(2x^2+5) \sqrt{x^4+5x^2+3}-\frac{13}{8}\int \frac{1}{\sqrt{x^4+5x^2+3}}dx^2\right)\right)\right.$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}\left(\frac{1}{10}\left(\frac{1}{24}(1837-510x^2) \left(x^4+5x^2+3\right)^{3/2}-\frac{8165}{16}\left(\frac{1}{4}(2x^2+5) \sqrt{x^4+5x^2+3}-\frac{13}{4}\int \frac{1}{4-x^4}d\frac{2x^2+5}{\sqrt{x^4+5x^2+3}}\right.\right.\right.$$

$$\downarrow 219$$

$$\frac{1}{2}\left(\frac{1}{10}\left(\frac{1}{24}(1837-510x^2)\left(x^4+5x^2+3\right)^{3/2}-\frac{8165}{16}\left(\frac{1}{4}(2x^2+5)\sqrt{x^4+5x^2+3}-\frac{13}{8}\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right.\right.\right.\right.$$

input
```
Int[x^5*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

output
```
((3*x^4*(3 + 5*x^2 + x^4)^(3/2))/5 + (((1837 - 510*x^2)*(3 + 5*x^2 + x^4)^
(3/2))/24 - (8165*(((5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/4 - (13*ArcTanh[(5
+ 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/8))/16)/10)/2
```

3.142.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.142.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.57

method	result
risch	$\frac{(1152x^8+1680x^6-2248x^4+12250x^2-78387)\sqrt{x^4+5x^2+3}}{3840}+\frac{21229\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{512}$
trager	$\left(\frac{3}{10}x^8+\frac{7}{16}x^6-\frac{281}{480}x^4+\frac{1225}{384}x^2-\frac{26129}{1280}\right)\sqrt{x^4+5x^2+3}-\frac{21229\ln\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)}{512}$
pseudoelliptic	$\frac{21229\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)}{512}+\frac{(1152x^8+1680x^6-2248x^4+12250x^2-78387)\sqrt{x^4+5x^2+3}}{3840}$
default	$\frac{3x^4(x^4+5x^2+3)^{\frac{3}{2}}}{10}-\frac{17x^2(x^4+5x^2+3)^{\frac{3}{2}}}{16}+\frac{1837(x^4+5x^2+3)^{\frac{3}{2}}}{480}-\frac{1633(2x^2+5)\sqrt{x^4+5x^2+3}}{256}+\frac{21229\ln\left(\frac{5}{2}+x^2+\sqrt{\right.}{512}$
elliptic	$\frac{3x^8\sqrt{x^4+5x^2+3}}{10}+\frac{7x^6\sqrt{x^4+5x^2+3}}{16}-\frac{281x^4\sqrt{x^4+5x^2+3}}{480}+\frac{1225x^2\sqrt{x^4+5x^2+3}}{384}-\frac{26129\sqrt{x^4+5x^2+3}}{1280}+\frac{21229\ln}{}$

input
```
int(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3840*(1152*x^8+1680*x^6-2248*x^4+12250*x^2-78387)*(x^4+5*x^2+3)^(1/2)+21
229/512*ln(5/2+x^2+(x^4+5*x^2+3)^(1/2))
```

3.142.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.60

$$\int x^5(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \frac{1}{3840}\left(1152\,x^8+1680\,x^6-2248\,x^4+12250\,x^2-78387\right)\sqrt{x^4+5\,x^2+3} - \frac{21229}{512}\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)$$

input
```
integrate(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
1/3840*(1152*x^8 + 1680*x^6 - 2248*x^4 + 12250*x^2 - 78387)*sqrt(x^4 + 5*x
^2 + 3) - 21229/512*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

3.142.6 Sympy [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.03

$$\int x^5(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \sqrt{x^4+5x^2+3}\left(\frac{x^6}{4}+\frac{5x^4}{24}-\frac{89x^2}{96}+\frac{365}{64}\right) + \frac{3\sqrt{x^4+5x^2+3}\left(\frac{x^8}{5}+\frac{x^6}{8}-\frac{127x^4}{240}+\frac{527x^2}{192}-\frac{11143}{640}\right)}{2} + \frac{21229\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)}{512}$$

input
```
integrate(x**5*(3*x**2+2)*(x**4+5*x**2+3)**(1/2),x)
```

output
```
sqrt(x**4 + 5*x**2 + 3)*(x**6/4 + 5*x**4/24 - 89*x**2/96 + 365/64) + 3*sqr
t(x**4 + 5*x**2 + 3)*(x**8/5 + x**6/8 - 127*x**4/240 + 527*x**2/192 - 1114
3/640)/2 + 21229*log(2*x**2 + 2*sqrt(x**4 + 5*x**2 + 3) + 5)/512
```

3.142.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.02

$$\int x^5(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \frac{3}{10}\left(x^4+5x^2+3\right)^{\frac{3}{2}}x^4 - \frac{17}{16}\left(x^4+5x^2+3\right)^{\frac{3}{2}}x^2 - \frac{1633}{128}\sqrt{x^4+5x^2+3}x^2 + \frac{1837}{480}\left(x^4+5x^2+3\right)^{\frac{3}{2}} - \frac{8165}{256}\sqrt{x^4+5x^2+3} + \frac{21229}{512}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

input
```
integrate(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
3/10*(x^4 + 5*x^2 + 3)^(3/2)*x^4 - 17/16*(x^4 + 5*x^2 + 3)^(3/2)*x^2 - 163
3/128*sqrt(x^4 + 5*x^2 + 3)*x^2 + 1837/480*(x^4 + 5*x^2 + 3)^(3/2) - 8165/
256*sqrt(x^4 + 5*x^2 + 3) + 21229/512*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3)
+ 5)
```

3.142.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00

$$\int x^5(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \frac{1}{1280}\sqrt{x^4+5x^2+3}\left(2\left(4\left(6\left(8x^2+5\right)x^2-127\right)x^2+2635\right)x^2-33429\right) + \frac{1}{192}\sqrt{x^4+5x^2+3}\left(2\left(4\left(6x^2+5\right)x^2-89\right)x^2+1095\right) - \frac{21229}{512}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

input
```
integrate(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
1/1280*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*(8*x^2 + 5)*x^2 - 127)*x^2 + 2635)*x
^2 - 33429) + 1/192*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*x^2 + 5)*x^2 - 89)*x^2
+ 1095) - 21229/512*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

3.142.9 Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00

$$\int x^5 (2+3x^2) \sqrt{3+5x^2+x^4}\, dx = \frac{21229 \ln \left(\sqrt{x^4+5\,x^2+3}+x^2+\frac{5}{2}\right)}{512} - \frac{17\,x^2\,\left(x^4+5\,x^2+3\right)^{3/2}}{16} + \frac{3\,x^4\,\left(x^4+5\,x^2+3\right)^{3/2}}{10} + \frac{51\,\left(\frac{x^2}{2}+\frac{5}{4}\right)\,\sqrt{x^4+5\,x^2+3}}{16} + \frac{1837\,\sqrt{x^4+5\,x^2+3}\,\left(8\,x^4+10\,x^2-51\right)}{3840}$$

input
```
int(x^5*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2),x)
```

output
```
(21229*log((5*x^2 + x^4 + 3)^(1/2) + x^2 + 5/2))/512 - (17*x^2*(5*x^2 + x^
4 + 3)^(3/2))/16 + (3*x^4*(5*x^2 + x^4 + 3)^(3/2))/10 + (51*(x^2/2 + 5/4)*
(5*x^2 + x^4 + 3)^(1/2))/16 + (1837*(5*x^2 + x^4 + 3)^(1/2)*(10*x^2 + 8*x^
4 - 51))/3840
```

3.143 $\int x^3(2+3x^2)\sqrt{3+5x^2+x^4}\,dx$

3.143.1 Optimal result

Integrand size = 25, antiderivative size = 81

$$\int x^3(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \frac{259}{128}(5+2x^2)\sqrt{3+5x^2+x^4} - \frac{1}{48}(59-18x^2)(3+5x^2+x^4)^{3/2} - \frac{3367}{256}\operatorname{arctanh}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)$$

output
```
-1/48*(-18*x^2+59)*(x^4+5*x^2+3)^(3/2)-3367/256*arctanh(1/2*(2*x^2+5)/(x^4
+5*x^2+3)^(1/2))+259/128*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)
```

3.143.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.79

$$\int x^3(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \frac{1}{384}\sqrt{3+5x^2+x^4}(2469-374x^2+248x^4+144x^6) + \frac{3367}{256}\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)$$

input
```
Integrate[x^3*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

output
```
(Sqrt[3 + 5*x^2 + x^4]*(2469 - 374*x^2 + 248*x^4 + 144*x^6))/384 + (3367*L
og[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/256
```

3.143.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1578, 1225, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (3x^2+2) \sqrt{x^4+5x^2+3} \, dx$$

$$\Big\downarrow \text{ 1578}$$

$$\frac{1}{2} \int x^2 (3x^2+2) \sqrt{x^4+5x^2+3} dx^2$$

$$\Big\downarrow \text{ 1225}$$

$$\frac{1}{2} \left(\frac{259}{16} \int \sqrt{x^4+5x^2+3} dx^2 - \frac{1}{24} (59-18x^2) (x^4+5x^2+3)^{3/2} \right)$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{1}{2} \left(\frac{259}{16} \left(\frac{1}{4} (2x^2+5) \sqrt{x^4+5x^2+3} - \frac{13}{8} \int \frac{1}{\sqrt{x^4+5x^2+3}} dx^2 \right) - \frac{1}{24} (59-18x^2) (x^4+5x^2+3)^{3/2} \right)$$

$$\Big\downarrow \text{ 1092}$$

$$\frac{1}{2} \left(\frac{259}{16} \left(\frac{1}{4} (2x^2+5) \sqrt{x^4+5x^2+3} - \frac{13}{4} \int \frac{1}{4-x^4} d\frac{2x^2+5}{\sqrt{x^4+5x^2+3}} \right) - \frac{1}{24} (59-18x^2) (x^4+5x^2+3)^{3/2} \right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2} \left(\frac{259}{16} \left(\frac{1}{4} (2x^2+5) \sqrt{x^4+5x^2+3} - \frac{13}{8} \text{arctanh} \left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}} \right) \right) - \frac{1}{24} (59-18x^2) (x^4+5x^2+3)^{3/2} \right)$$

input
```
Int[x^3*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

output
```
(-1/24*((59 - 18*x^2)*(3 + 5*x^2 + x^4)^(3/2)) + (259*(((5 + 2*x^2)*Sqrt[3
 + 5*x^2 + x^4])/4 - (13*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/8
))/16)/2
```

3.143.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.143.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.65

method	result
risch	$\frac{(144x^6+248x^4-374x^2+2469)\sqrt{x^4+5x^2+3}}{384}-\frac{3367\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{256}$
trager	$\left(\frac{3}{8}x^6+\frac{31}{48}x^4-\frac{187}{192}x^2+\frac{823}{128}\right)\sqrt{x^4+5x^2+3}-\frac{3367\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)}{256}$
pseudoelliptic	$-\frac{3367\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)}{256}+\frac{(288x^6+496x^4-748x^2+4938)\sqrt{x^4+5x^2+3}}{768}$
default	$\frac{3x^2(x^4+5x^2+3)^{\frac{3}{2}}}{8}-\frac{59(x^4+5x^2+3)^{\frac{3}{2}}}{48}+\frac{259(2x^2+5)\sqrt{x^4+5x^2+3}}{128}-\frac{3367\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{256}$
elliptic	$\frac{823\sqrt{x^4+5x^2+3}}{128}-\frac{3367\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{256}+\frac{3x^6\sqrt{x^4+5x^2+3}}{8}+\frac{31x^4\sqrt{x^4+5x^2+3}}{48}-\frac{187x^2\sqrt{x^4+5x^2+3}}{192}$

input
```
int(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/384*(144*x^6+248*x^4-374*x^2+2469)*(x^4+5*x^2+3)^(1/2)-3367/256*ln(5/2+x
^2+(x^4+5*x^2+3)^(1/2))
```

3.143.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.69

$$\int x^3(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \frac{1}{384}\left(144\,x^6+248\,x^4-374\,x^2+2469\right)\sqrt{x^4+5\,x^2+3} + \frac{3367}{256}\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)$$

input
```
integrate(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
1/384*(144*x^6 + 248*x^4 - 374*x^2 + 2469)*sqrt(x^4 + 5*x^2 + 3) + 3367/25
6*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

3.143.6 Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.16

$$\int x^3(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \left(\frac{x^4}{3}+\frac{5x^2}{12}-\frac{17}{8}\right)\sqrt{x^4+5x^2+3} + \frac{3\sqrt{x^4+5x^2+3}\left(\frac{x^6}{4}+\frac{5x^4}{24}-\frac{89x^2}{96}+\frac{365}{64}\right)}{2} - \frac{3367\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)}{256}$$

input
```
integrate(x**3*(3*x**2+2)*(x**4+5*x**2+3)**(1/2),x)
```

output
```
(x**4/3 + 5*x**2/12 - 17/8)*sqrt(x**4 + 5*x**2 + 3) + 3*sqrt(x**4 + 5*x**2
 + 3)*(x**6/4 + 5*x**4/24 - 89*x**2/96 + 365/64)/2 - 3367*log(2*x**2 + 2*s
qrt(x**4 + 5*x**2 + 3) + 5)/256
```

3.143.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.07

$$\int x^3(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \frac{3}{8}\left(x^4+5x^2+3\right)^{\frac{3}{2}}x^2 + \frac{259}{64}\sqrt{x^4+5x^2+3}x^2 - \frac{59}{48}\left(x^4+5x^2+3\right)^{\frac{3}{2}} + \frac{1295}{128}\sqrt{x^4+5x^2+3} - \frac{3367}{256}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

input
```
integrate(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
3/8*(x^4 + 5*x^2 + 3)^(3/2)*x^2 + 259/64*sqrt(x^4 + 5*x^2 + 3)*x^2 - 59/48
*(x^4 + 5*x^2 + 3)^(3/2) + 1295/128*sqrt(x^4 + 5*x^2 + 3) - 3367/256*log(2
*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

3.143.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.09

$$\int x^3(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \frac{1}{128}\sqrt{x^4+5x^2+3}\left(2\left(4\left(6x^2+5\right)x^2-89\right)x^2+1095\right) + \frac{1}{24}\sqrt{x^4+5x^2+3}\left(2\left(4x^2+5\right)x^2-51\right) + \frac{3367}{256}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

input
```
integrate(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
1/128*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*x^2 + 5)*x^2 - 89)*x^2 + 1095) + 1/24
*sqrt(x^4 + 5*x^2 + 3)*(2*(4*x^2 + 5)*x^2 - 51) + 3367/256*log(2*x^2 - 2*s
qrt(x^4 + 5*x^2 + 3) + 5)
```

3.143.9 Mupad [B] (verification not implemented)

Time = 7.68 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05

$$\int x^3(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \frac{3x^2\left(x^4+5x^2+3\right)^{3/2}}{8} - \frac{3367\ln\left(\sqrt{x^4+5x^2+3}+x^2+\frac{5}{2}\right)}{256} - \frac{9\left(\frac{x^2}{2}+\frac{5}{4}\right)\sqrt{x^4+5x^2+3}}{8} - \frac{59\sqrt{x^4+5x^2+3}\left(8x^4+10x^2-51\right)}{384}$$

input
```
int(x^3*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2),x)
```

output
```
(3*x^2*(5*x^2 + x^4 + 3)^(3/2))/8 - (3367*log((5*x^2 + x^4 + 3)^(1/2) + x^
2 + 5/2))/256 - (9*(x^2/2 + 5/4)*(5*x^2 + x^4 + 3)^(1/2))/8 - (59*(5*x^2 +
 x^4 + 3)^(1/2)*(10*x^2 + 8*x^4 - 51))/384
```

3.144 $\int x(2+3x^2)\sqrt{3+5x^2+x^4}\,dx$

3.144.1 Optimal result

Integrand size = 23, antiderivative size = 74

$$\int x(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = -\frac{11}{16}(5+2x^2)\sqrt{3+5x^2+x^4}+\frac{1}{2}(3+5x^2+x^4)^{3/2} + \frac{143}{32}\operatorname{arctanh}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)$$

output
```
1/2*(x^4+5*x^2+3)^(3/2)+143/32*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))-
11/16*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)
```

3.144.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.80

$$\int x(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \frac{1}{16}\sqrt{3+5x^2+x^4}(-31+18x^2+8x^4) - \frac{143}{32}\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)$$

input
```
Integrate[x*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

output
```
(Sqrt[3 + 5*x^2 + x^4]*(-31 + 18*x^2 + 8*x^4))/16 - (143*Log[-5 - 2*x^2 +
2*Sqrt[3 + 5*x^2 + x^4]])/32
```

3.144.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1576, 1160, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(3x^2+2\right)\sqrt{x^4+5x^2+3}\,dx$$

$$\Big\downarrow \text{1576}$$

$$\frac{1}{2}\int \left(3x^2+2\right)\sqrt{x^4+5x^2+3}dx^2$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{2}\left(\left(x^4+5x^2+3\right)^{3/2}-\frac{11}{2}\int \sqrt{x^4+5x^2+3}dx^2\right)$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{2}\left(\left(x^4+5x^2+3\right)^{3/2}-\frac{11}{2}\left(\frac{1}{4}\left(2x^2+5\right)\sqrt{x^4+5x^2+3}-\frac{13}{8}\int \frac{1}{\sqrt{x^4+5x^2+3}}dx^2\right)\right)$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}\left(\left(x^4+5x^2+3\right)^{3/2}-\frac{11}{2}\left(\frac{1}{4}\left(2x^2+5\right)\sqrt{x^4+5x^2+3}-\frac{13}{4}\int \frac{1}{4-x^4}d\frac{2x^2+5}{\sqrt{x^4+5x^2+3}}\right)\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\left(x^4+5x^2+3\right)^{3/2}-\frac{11}{2}\left(\frac{1}{4}\left(2x^2+5\right)\sqrt{x^4+5x^2+3}-\frac{13}{8}\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)\right)\right)$$

input
```
Int[x*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

output
```
((3 + 5*x^2 + x^4)^(3/2) - (11*(((5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/4 - (1
3*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/8))/2)/2
```

3.144.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

3.144.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.65

method	result	size
risch	$\frac{(8x^4+18x^2-31)\sqrt{x^4+5x^2+3}}{16}+\frac{143\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{32}$	48
trager	$\left(\frac{1}{2}x^4+\frac{9}{8}x^2-\frac{31}{16}\right)\sqrt{x^4+5x^2+3}+\frac{143\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)}{32}$	51
pseudoelliptic	$\frac{143\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)}{32}+\frac{(8x^4+18x^2-31)\sqrt{x^4+5x^2+3}}{16}$	52
default	$-\frac{11(2x^2+5)\sqrt{x^4+5x^2+3}}{16}+\frac{143\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{32}+\frac{(x^4+5x^2+3)^{\frac{3}{2}}}{2}$	57
elliptic	$\frac{143\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{32}-\frac{31\sqrt{x^4+5x^2+3}}{16}+\frac{x^4\sqrt{x^4+5x^2+3}}{2}+\frac{9x^2\sqrt{x^4+5x^2+3}}{8}$	70

input
```
int(x*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/16*(8*x^4+18*x^2-31)*(x^4+5*x^2+3)^(1/2)+143/32*ln(5/2+x^2+(x^4+5*x^2+3)
^(1/2))
```

3.144.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.69

$$\int x(2+3x^2)\sqrt{3+5x^2+x^4}\,dx=\frac{1}{16}\left(8\,x^4+18\,x^2-31\right)\sqrt{x^4+5\,x^2+3}-\frac{143}{32}\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)$$

input
```
integrate(x*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
1/16*(8*x^4 + 18*x^2 - 31)*sqrt(x^4 + 5*x^2 + 3) - 143/32*log(-2*x^2 + 2*s
qrt(x^4 + 5*x^2 + 3) - 5)
```

3.144.6 Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08

$$\int x(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \left(\frac{x^2}{2}+\frac{5}{4}\right)\sqrt{x^4+5x^2+3} + \frac{3\left(\frac{x^4}{3}+\frac{5x^2}{12}-\frac{17}{8}\right)\sqrt{x^4+5x^2+3}}{2} + \frac{143\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)}{32}$$

input
```
integrate(x*(3*x**2+2)*(x**4+5*x**2+3)**(1/2),x)
```

output
```
(x**2/2 + 5/4)*sqrt(x**4 + 5*x**2 + 3) + 3*(x**4/3 + 5*x**2/12 - 17/8)*sqr
t(x**4 + 5*x**2 + 3)/2 + 143*log(2*x**2 + 2*sqrt(x**4 + 5*x**2 + 3) + 5)/3
2
```

3.144.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

$$\int x(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = -\frac{11}{8}\sqrt{x^4+5\,x^2+3}x^2 + \frac{1}{2}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}} - \frac{55}{16}\sqrt{x^4+5\,x^2+3} + \frac{143}{32}\log\left(2\,x^2+2\sqrt{x^4+5\,x^2+3}+5\right)$$

input
```
integrate(x*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
-11/8*sqrt(x^4 + 5*x^2 + 3)*x^2 + 1/2*(x^4 + 5*x^2 + 3)^(3/2) - 55/16*sqrt
(x^4 + 5*x^2 + 3) + 143/32*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

3.144.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int x(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \frac{1}{16}\sqrt{x^4+5x^2+3}\left(2\left(4x^2+5\right)x^2-51\right) + \frac{1}{4}\sqrt{x^4+5x^2+3}\left(2x^2+5\right) - \frac{143}{32}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

input
```
integrate(x*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
1/16*sqrt(x^4 + 5*x^2 + 3)*(2*(4*x^2 + 5)*x^2 - 51) + 1/4*sqrt(x^4 + 5*x^2
 + 3)*(2*x^2 + 5) - 143/32*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

3.144.9 Mupad [B] (verification not implemented)

Time = 7.67 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.91

$$\int x(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \frac{143\ln\left(\sqrt{x^4+5x^2+3}+x^2+\frac{5}{2}\right)}{32} + \left(\frac{x^2}{2}+\frac{5}{4}\right)\sqrt{x^4+5x^2+3} + \frac{\sqrt{x^4+5x^2+3}\left(8x^4+10x^2-51\right)}{16}$$

input
```
int(x*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2),x)
```

output
```
(143*log((5*x^2 + x^4 + 3)^(1/2) + x^2 + 5/2))/32 + (x^2/2 + 5/4)*(5*x^2 +
 x^4 + 3)^(1/2) + ((5*x^2 + x^4 + 3)^(1/2)*(10*x^2 + 8*x^4 - 51))/16
```

3.145 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x} \, dx$

3.145.1 Optimal result

Integrand size = 25, antiderivative size = 94

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x} \, dx = \frac{1}{8}(23+6x^2)\sqrt{3+5x^2+x^4} + \frac{1}{16}\text{arctanh}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right) - \sqrt{3}\text{arctanh}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)$$

output
```
1/16*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))-arctanh(1/6*(5*x^2+6)*3^(1
/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)+1/8*(6*x^2+23)*(x^4+5*x^2+3)^(1/2)
```

3.145.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.94

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x} \, dx = \frac{1}{8}(23+6x^2)\sqrt{3+5x^2+x^4} + 2\sqrt{3}\text{arctanh}\left(\frac{x^2-\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right) - \frac{1}{16}\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)$$

input
```
Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x,x]
```

output
```
((23 + 6*x^2)*Sqrt[3 + 5*x^2 + x^4])/8 + 2*Sqrt[3]*ArcTanh[(x^2 - Sqrt[3 +
 5*x^2 + x^4])/Sqrt[3]] - Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]]/16
```

3.145.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {1578, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x^2+2)\sqrt{x^4+5x^2+3}}{x}\,dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{(3x^2+2)\sqrt{x^4+5x^2+3}}{x^2}dx^2$$

$$\Big\downarrow \text{1231}$$

$$\frac{1}{2}\left(\frac{1}{4}(6x^2+23)\sqrt{x^4+5x^2+3}-\frac{1}{4}\int -\frac{x^2+48}{2x^2\sqrt{x^4+5x^2+3}}dx^2\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{1}{8}\int \frac{x^2+48}{x^2\sqrt{x^4+5x^2+3}}dx^2+\frac{1}{4}\sqrt{x^4+5x^2+3}(6x^2+23)\right)$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{2}\left(\frac{1}{8}\left(\int \frac{1}{\sqrt{x^4+5x^2+3}}dx^2+48\int \frac{1}{x^2\sqrt{x^4+5x^2+3}}dx^2\right)+\frac{1}{4}\sqrt{x^4+5x^2+3}(6x^2+23)\right)$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}\left(\frac{1}{8}\left(2\int \frac{1}{4-x^4}d\frac{2x^2+5}{\sqrt{x^4+5x^2+3}}+48\int \frac{1}{x^2\sqrt{x^4+5x^2+3}}dx^2\right)+\frac{1}{4}\sqrt{x^4+5x^2+3}(6x^2+23)\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{1}{8}\left(48\int \frac{1}{x^2\sqrt{x^4+5x^2+3}}dx^2+\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)\right)+\frac{1}{4}\sqrt{x^4+5x^2+3}(6x^2+23)\right)$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{2}\left(\frac{1}{8}\left(\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-96\int\frac{1}{12-x^4}d\frac{5x^2+6}{\sqrt{x^4+5x^2+3}}\right)+\frac{1}{4}\sqrt{x^4+5x^2+3}(6x^2+23)\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}\left(\frac{1}{8}\left(\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-16\sqrt{3}\text{arctanh}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)\right)+\frac{1}{4}\sqrt{x^4+5x^2+3}(6x^2+23)\right)$$

input
```
Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x,x]
```

output
```
(((23 + 6*x^2)*Sqrt[3 + 5*x^2 + x^4])/4 + (ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 +
 5*x^2 + x^4])] - 16*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2
 + x^4])])/8)/2
```

3.145.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.145.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.88

method	result
elliptic	$\frac{\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{16}+\frac{23\sqrt{x^4+5x^2+3}}{8}+\frac{3x^2\sqrt{x^4+5x^2+3}}{4}-\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}$
default	$\frac{3(2x^2+5)\sqrt{x^4+5x^2+3}}{8}+\frac{\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{16}+\sqrt{x^4+5x^2+3}-\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}$
pseudoelliptic	$\frac{3x^2\sqrt{x^4+5x^2+3}}{4}+\frac{23\sqrt{x^4+5x^2+3}}{8}+\frac{\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)}{16}-\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}$
trager	$\left(\frac{3x^2}{4}+\frac{23}{8}\right)\sqrt{x^4+5x^2+3}+\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{-5\operatorname{RootOf}\left(\_Z^2-3\right)x^2+6\sqrt{x^4+5x^2+3}-6\operatorname{Roo}}{x^2}\right.$

input
```
int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/16*ln(5/2+x^2+(x^4+5*x^2+3)^(1/2))+23/8*(x^4+5*x^2+3)^(1/2)+3/4*x^2*(x^4
+5*x^2+3)^(1/2)-arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)
```

3.145.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x}\,dx = \frac{1}{8}\sqrt{x^4+5x^2+3}(6x^2+23) + \sqrt{3}\log\left(\frac{25x^2-2\sqrt{3}(5x^2+6)-2\sqrt{x^4+5x^2+3}(5\sqrt{3}-6)+30}{x^2}\right) - \frac{1}{16}\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x,x, algorithm="fricas")
```

output
```
1/8*sqrt(x^4 + 5*x^2 + 3)*(6*x^2 + 23) + sqrt(3)*log((25*x^2 - 2*sqrt(3)*(
5*x^2 + 6) - 2*sqrt(x^4 + 5*x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 1/16*log
(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

3.145.6 Sympy [F]

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x}\,dx = \int \frac{(3x^2+2)\sqrt{x^4+5x^2+3}}{x}\,dx$$

input
```
integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x,x)
```

output
```
Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x, x)
```

3.145.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.95

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x}\,dx = \frac{3}{4}\sqrt{x^4+5x^2+3}x^2 - \sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right) + \frac{23}{8}\sqrt{x^4+5x^2+3}+\frac{1}{16}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x,x, algorithm="maxima")
```

output
```
3/4*sqrt(x^4 + 5*x^2 + 3)*x^2 - sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3
)/x^2 + 6/x^2 + 5) + 23/8*sqrt(x^4 + 5*x^2 + 3) + 1/16*log(2*x^2 + 2*sqrt(
x^4 + 5*x^2 + 3) + 5)
```

3.145.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.04

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x}\,dx = \frac{1}{8}\sqrt{x^4+5x^2+3}(6x^2+23) + \sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right) - \frac{1}{16}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x,x, algorithm="giac")
```

output
```
1/8*sqrt(x^4 + 5*x^2 + 3)*(6*x^2 + 23) + sqrt(3)*log((x^2 + sqrt(3) - sqrt
(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - sqrt(x^4 + 5*x^2 + 3))) - 1/16*log(2*x
^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

3.145.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.91

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x}\,dx = \frac{\ln\left(\sqrt{x^4+5\,x^2+3}+x^2+\frac{5}{2}\right)}{16} - \sqrt{3}\,\ln\left(\frac{3}{x^2}+\frac{\sqrt{3}\,\sqrt{x^4+5\,x^2+3}}{x^2}+\frac{5}{2}\right) + \frac{3\left(\frac{x^2}{2}+\frac{5}{4}\right)\sqrt{x^4+5\,x^2+3}}{2} + \sqrt{x^4+5\,x^2+3}$$

input
```
int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x,x)
```

output
```
log((5*x^2 + x^4 + 3)^(1/2) + x^2 + 5/2)/16 - 3^(1/2)*log(3/x^2 + (3^(1/2)
*(5*x^2 + x^4 + 3)^(1/2))/x^2 + 5/2) + (3*(x^2/2 + 5/4)*(5*x^2 + x^4 + 3)^
(1/2))/2 + (5*x^2 + x^4 + 3)^(1/2)
```

3.146 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^3}\,dx$

3.146.1 Optimal result

Integrand size = 25, antiderivative size = 97

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^3}\,dx = -\frac{(2-3x^2)\sqrt{3+5x^2+x^4}}{2x^2} + \frac{19}{4}\operatorname{arctanh}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right) - \frac{7\operatorname{arctanh}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{\sqrt{3}}$$

output
```
19/4*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))-7/3*arctanh(1/6*(5*x^2+6)*
3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-1/2*(-3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^
2
```

3.146.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.94

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^3}\,dx = \frac{(-2+3x^2)\sqrt{3+5x^2+x^4}}{2x^2} + \frac{14\operatorname{arctanh}\left(\frac{x^2-\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{19}{4}\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)$$

input
```
Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^3,x]
```

output
```
((-2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/(2*x^2) + (14*ArcTanh[(x^2 - Sqrt[3 +
 5*x^2 + x^4])/Sqrt[3]])/Sqrt[3] - (19*Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 +
 x^4]])/4
```

3.146.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {1578, 1230, 25, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x^2+2)\sqrt{x^4+5x^2+3}}{x^3}dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{(3x^2+2)\sqrt{x^4+5x^2+3}}{x^4}dx^2$$

$$\Big\downarrow \text{1230}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int -\frac{19x^2+28}{x^2\sqrt{x^4+5x^2+3}}dx^2-\frac{\sqrt{x^4+5x^2+3}(2-3x^2)}{x^2}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{19x^2+28}{x^2\sqrt{x^4+5x^2+3}}dx^2-\frac{(2-3x^2)\sqrt{x^4+5x^2+3}}{x^2}\right)$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(19\int \frac{1}{\sqrt{x^4+5x^2+3}}dx^2+28\int \frac{1}{x^2\sqrt{x^4+5x^2+3}}dx^2\right)-\frac{(2-3x^2)\sqrt{x^4+5x^2+3}}{x^2}\right)$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(38\int \frac{1}{4-x^4}d\frac{2x^2+5}{\sqrt{x^4+5x^2+3}}+28\int \frac{1}{x^2\sqrt{x^4+5x^2+3}}dx^2\right)-\frac{(2-3x^2)\sqrt{x^4+5x^2+3}}{x^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(28\int \frac{1}{x^2\sqrt{x^4+5x^2+3}}dx^2+19\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)\right)-\frac{(2-3x^2)\sqrt{x^4+5x^2+3}}{x^2}\right)$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(19\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-56\int \frac{1}{12-x^4}d\frac{5x^2+6}{\sqrt{x^4+5x^2+3}}\right)-\frac{(2-3x^2)\sqrt{x^4+5x^2+3}}{x^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(19\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{28\text{arctanh}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{\sqrt{3}}\right)-\frac{(2-3x^2)\sqrt{x^4+5x^2+3}}{x^2}\right)$$

input
```
Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^3,x]
```

output
```
(-(((2 - 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^2) + (19*ArcTanh[(5 + 2*x^2)/(2*S
qrt[3 + 5*x^2 + x^4])] - (28*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2
 + x^4])])/Sqrt[3])/2)/2
```

3.146.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

3.146. $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^3}\,dx$

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.146.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.86

method	result
risch	$-\frac{\sqrt{x^4+5x^2+3}}{x^2}+\frac{19\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{4}+\frac{3\sqrt{x^4+5x^2+3}}{2}-\frac{7\,\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{3}$
elliptic	$-\frac{\sqrt{x^4+5x^2+3}}{x^2}+\frac{19\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{4}+\frac{3\sqrt{x^4+5x^2+3}}{2}-\frac{7\,\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{3}$
trager	$\frac{(3x^2-2)\sqrt{x^4+5x^2+3}}{2x^2}+\frac{7\,\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{-5\,\operatorname{RootOf}\left(\_Z^2-3\right)x^2+6\sqrt{x^4+5x^2+3}-6\,\operatorname{RootOf}\left(\_Z^2-3\right)}{x^2}\right)}{3}+\frac{19\ln}{}$
pseudoelliptic	$\frac{-28\,\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}\,x^2+18x^2\sqrt{x^4+5x^2+3}+57\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)x^2-12\sqrt{x^4+5x^2+3}}{12x^2}$
default	$\frac{7\sqrt{x^4+5x^2+3}}{3}+\frac{19\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{4}-\frac{7\,\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{3}-\frac{(x^4+5x^2+3)^{\frac{3}{2}}}{3x^2}+\frac{(2x^2+5)\sqrt{x^4+5x}}{6}$

input
```
int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-(x^4+5*x^2+3)^(1/2)/x^2+19/4*ln(5/2+x^2+(x^4+5*x^2+3)^(1/2))+3/2*(x^4+5*x
^2+3)^(1/2)-7/3*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)
```

3.146.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.15

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^3}\,dx$$
$$= \frac{56\sqrt{3}x^2 \log\left(\frac{25\,x^2-2\sqrt{3}(5\,x^2+6)-2\sqrt{x^4+5\,x^2+3}\left(5\sqrt{3}-6\right)+30}{x^2}\right) - 114\,x^2 \log\left(-2\,x^2+2\sqrt{x^4+5\,x^2+3}-5\right)+21\,x}{24\,x^2}$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
1/24*(56*sqrt(3)*x^2*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*
x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 114*x^2*log(-2*x^2 + 2*sqrt(x^4 + 5*
x^2 + 3) - 5) + 21*x^2 + 12*sqrt(x^4 + 5*x^2 + 3)*(3*x^2 - 2))/x^2
```

3.146.6 Sympy [F]

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^3}\,dx = \int \frac{(3x^2+2)\sqrt{x^4+5x^2+3}}{x^3}\,dx$$

input
```
integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**3,x)
```

output
```
Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**3, x)
```

3.146.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.92

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^3}\,dx = -\frac{7}{3}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right) + \frac{3}{2}\sqrt{x^4+5x^2+3} - \frac{\sqrt{x^4+5x^2+3}}{x^2} + \frac{19}{4}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
-7/3*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 3/2*sq
rt(x^4 + 5*x^2 + 3) - sqrt(x^4 + 5*x^2 + 3)/x^2 + 19/4*log(2*x^2 + 2*sqrt(
x^4 + 5*x^2 + 3) + 5)
```

3.146.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.42

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^3}\,dx = \frac{7}{3}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right) + \frac{3}{2}\sqrt{x^4+5x^2+3} + \frac{5x^2-5\sqrt{x^4+5x^2+3}+6}{\left(x^2-\sqrt{x^4+5x^2+3}\right)^2-3} - \frac{19}{4}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^3,x, algorithm="giac")
```

output
```
7/3*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - s
qrt(x^4 + 5*x^2 + 3))) + 3/2*sqrt(x^4 + 5*x^2 + 3) + (5*x^2 - 5*sqrt(x^4 +
 5*x^2 + 3) + 6)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 3) - 19/4*log(2*x^2 -
2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

3.146.9 Mupad [B] (verification not implemented)

Time = 8.01 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.87

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^3}\,dx = \frac{19\ln\left(\sqrt{x^4+5x^2+3}+x^2+\frac{5}{2}\right)}{4} - \frac{\sqrt{x^4+5x^2+3}}{x^2} - \frac{7\sqrt{3}\ln\left(\frac{3}{x^2}+\frac{\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{5}{2}\right)}{3} + \frac{3\sqrt{x^4+5x^2+3}}{2}$$

input
```
int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^3,x)
```

output
```
(19*log((5*x^2 + x^4 + 3)^(1/2) + x^2 + 5/2))/4 - (5*x^2 + x^4 + 3)^(1/2)/
x^2 - (7*3^(1/2)*log(3/x^2 + (3^(1/2)*(5*x^2 + x^4 + 3)^(1/2))/x^2 + 5/2))
/3 + (3*(5*x^2 + x^4 + 3)^(1/2))/2
```

3.147 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^5}\,dx$

3.147.1 Optimal result

Integrand size = 25, antiderivative size = 99

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^5}\,dx = -\frac{(6+23x^2)\sqrt{3+5x^2+x^4}}{12x^4} + \frac{3}{2}\text{arctanh}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right) - \frac{77\text{arctanh}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{24\sqrt{3}}$$

output
```
3/2*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))-77/72*arctanh(1/6*(5*x^2+6)
*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-1/12*(23*x^2+6)*(x^4+5*x^2+3)^(1/2)/
x^4
```

3.147.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.92

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^5}\,dx = \frac{1}{36}\left(-\frac{3(6+23x^2)\sqrt{3+5x^2+x^4}}{x^4} + 77\sqrt{3}\text{arctanh}\left(\frac{x^2-\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right) - 54\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)\right)$$

input
```
Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^5,x]
```

output
```
((-3*(6 + 23*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^4 + 77*Sqrt[3]*ArcTanh[(x^2 - S
qrt[3 + 5*x^2 + x^4])/Sqrt[3]] - 54*Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^
4]])/36
```

3.147.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {1578, 1229, 25, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x^2+2)\sqrt{x^4+5x^2+3}}{x^5}dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{(3x^2+2)\sqrt{x^4+5x^2+3}}{x^6}dx^2$$

$$\Big\downarrow \text{1229}$$

$$\frac{1}{2}\left(-\frac{1}{12}\int -\frac{36x^2+77}{x^2\sqrt{x^4+5x^2+3}}dx^2-\frac{\sqrt{x^4+5x^2+3}(23x^2+6)}{6x^4}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{12}\int \frac{36x^2+77}{x^2\sqrt{x^4+5x^2+3}}dx^2-\frac{(23x^2+6)\sqrt{x^4+5x^2+3}}{6x^4}\right)$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{2}\left(\frac{1}{12}\left(36\int \frac{1}{\sqrt{x^4+5x^2+3}}dx^2+77\int \frac{1}{x^2\sqrt{x^4+5x^2+3}}dx^2\right)-\frac{(23x^2+6)\sqrt{x^4+5x^2+3}}{6x^4}\right)$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}\left(\frac{1}{12}\left(72\int \frac{1}{4-x^4}d\frac{2x^2+5}{\sqrt{x^4+5x^2+3}}+77\int \frac{1}{x^2\sqrt{x^4+5x^2+3}}dx^2\right)-\frac{(23x^2+6)\sqrt{x^4+5x^2+3}}{6x^4}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{1}{12}\left(77\int \frac{1}{x^2\sqrt{x^4+5x^2+3}}dx^2+36\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)\right)-\frac{(23x^2+6)\sqrt{x^4+5x^2+3}}{6x^4}\right)$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{2}\left(\frac{1}{12}\left(36\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-154\int \frac{1}{12-x^4}d\frac{5x^2+6}{\sqrt{x^4+5x^2+3}}\right)-\frac{(23x^2+6)\sqrt{x^4+5x^2+3}}{6x^4}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{1}{12}\left(36\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{77\text{arctanh}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{\sqrt{3}}\right)-\frac{(23x^2+6)\sqrt{x^4+5x^2+3}}{6x^4}\right)$$

input
```
Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^5,x]
```

output
```
(-1/6*((6 + 23*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^4 + (36*ArcTanh[(5 + 2*x^2)/(
2*Sqrt[3 + 5*x^2 + x^4])] - (77*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*
x^2 + x^4])])/Sqrt[3])/12)/2
```

3.147.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.147.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.87

method	result
risch	$-\frac{23x^6+121x^4+99x^2+18}{12x^4\sqrt{x^4+5x^2+3}}+\frac{3\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{2}-\frac{77\,\mathrm{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{72}$
elliptic	$\frac{3\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{2}-\frac{\sqrt{x^4+5x^2+3}}{2x^4}-\frac{23\sqrt{x^4+5x^2+3}}{12x^2}-\frac{77\,\mathrm{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{72}$
trager	$-\frac{(23x^2+6)\sqrt{x^4+5x^2+3}}{12x^4}+\frac{3\ln\left(-2x^2-2\sqrt{x^4+5x^2+3}-5\right)}{2}+\frac{77\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{-5\,\mathrm{RootOf}\left(\_Z^2-3\right)x^2+6\sqrt{x}}{x}\right.}{72}$
pseudoelliptic	$\frac{-77\,\mathrm{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}\,x^4+108\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)x^4-138x^2\sqrt{x^4+5x^2+3}-36\sqrt{x^4+5x^2+3}}{72x^4}$
default	$-\frac{13(x^4+5x^2+3)^{\frac{3}{2}}}{36x^2}+\frac{77\sqrt{x^4+5x^2+3}}{72}+\frac{3\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{2}-\frac{77\,\mathrm{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{72}+\frac{13(2x^2+5)}{}$

input
```
int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/12*(23*x^6+121*x^4+99*x^2+18)/x^4/(x^4+5*x^2+3)^(1/2)+3/2*ln(5/2+x^2+(x
^4+5*x^2+3)^(1/2))-77/72*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2)
)*3^(1/2)
```

3.147.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.13

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^5}\,dx$$
$$=\frac{77\sqrt{3}x^4\log\left(\frac{25\,x^2-2\sqrt{3}(5\,x^2+6)-2\sqrt{x^4+5\,x^2+3}\left(5\sqrt{3}-6\right)+30}{x^2}\right)-108\,x^4\log\left(-2\,x^2+2\sqrt{x^4+5\,x^2+3}-5\right)-138}{72\,x^4}$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
1/72*(77*sqrt(3)*x^4*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*
x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 108*x^4*log(-2*x^2 + 2*sqrt(x^4 + 5*
x^2 + 3) - 5) - 138*x^4 - 6*sqrt(x^4 + 5*x^2 + 3)*(23*x^2 + 6))/x^4
```

3.147.6 Sympy [F]

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^5}\,dx = \int \frac{(3x^2+2)\sqrt{x^4+5x^2+3}}{x^5}\,dx$$

input
```
integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**5,x)
```

output
```
Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**5, x)
```

3.147.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.07

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^5}\,dx = -\frac{77}{72}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right) + \frac{1}{6}\sqrt{x^4+5x^2+3} - \frac{13\sqrt{x^4+5x^2+3}}{12x^2} - \frac{(x^4+5x^2+3)^{\frac{3}{2}}}{6x^4} + \frac{3}{2}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
-77/72*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 1/6*
sqrt(x^4 + 5*x^2 + 3) - 13/12*sqrt(x^4 + 5*x^2 + 3)/x^2 - 1/6*(x^4 + 5*x^2
 + 3)^(3/2)/x^4 + 3/2*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

3.147.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(79) = 158$.

Time = 0.32 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.71

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^5}\,dx = \frac{77}{72}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right) + \frac{127\left(x^2-\sqrt{x^4+5x^2+3}\right)^3+228\left(x^2-\sqrt{x^4+5x^2+3}\right)^2-159x^2+159\sqrt{x^4+5x^2+3}-324}{12\left(\left(x^2-\sqrt{x^4+5x^2+3}\right)^2-3\right)^2} - \frac{3}{2}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^5,x, algorithm="giac")
```

output
```
77/72*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) -
 sqrt(x^4 + 5*x^2 + 3))) + 1/12*(127*(x^2 - sqrt(x^4 + 5*x^2 + 3))^3 + 228
*(x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 159*x^2 + 159*sqrt(x^4 + 5*x^2 + 3) - 3
24)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 3)^2 - 3/2*log(2*x^2 - 2*sqrt(x^4 +
 5*x^2 + 3) + 5)
```

3.147.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^5}\,dx=\int \frac{(3\,x^2+2)\,\sqrt{x^4+5\,x^2+3}}{x^5}\,dx$$

input
```
int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^5,x)
```

output
```
int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^5, x)
```

3.148 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^7}\,dx$

3.148.1 Optimal result

Integrand size = 25, antiderivative size = 90

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^7}\,dx = -\frac{(6+5x^2)\sqrt{3+5x^2+x^4}}{18x^4} - \frac{(3+5x^2+x^4)^{3/2}}{9x^6} + \frac{13\text{arctanh}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{36\sqrt{3}}$$

output
```
-1/9*(x^4+5*x^2+3)^(3/2)/x^6+13/108*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x
^2+3)^(1/2))*3^(1/2)-1/18*(5*x^2+6)*(x^4+5*x^2+3)^(1/2)/x^4
```

3.148.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.78

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^7}\,dx = \frac{1}{54}\left(-\frac{3\sqrt{3+5x^2+x^4}(6+16x^2+7x^4)}{x^6} - 13\sqrt{3}\text{arctanh}\left(\frac{x^2-\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right)\right)$$

input
```
Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^7,x]
```

output
```
((-3*Sqrt[3 + 5*x^2 + x^4]*(6 + 16*x^2 + 7*x^4))/x^6 - 13*Sqrt[3]*ArcTanh[
(x^2 - Sqrt[3 + 5*x^2 + x^4])/Sqrt[3]])/54
```

3.148.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1578, 1228, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x^2+2)\sqrt{x^4+5x^2+3}}{x^7}dx$$

$$\Big\downarrow \text{ 1578}$$

$$\frac{1}{2}\int \frac{(3x^2+2)\sqrt{x^4+5x^2+3}}{x^8}dx^2$$

$$\Big\downarrow \text{ 1228}$$

$$\frac{1}{2}\left(\frac{4}{3}\int \frac{\sqrt{x^4+5x^2+3}}{x^6}dx^2-\frac{2(x^4+5x^2+3)^{3/2}}{9x^6}\right)$$

$$\Big\downarrow \text{ 1152}$$

$$\frac{1}{2}\left(\frac{4}{3}\left(-\frac{13}{24}\int \frac{1}{x^2\sqrt{x^4+5x^2+3}}dx^2-\frac{\sqrt{x^4+5x^2+3}(5x^2+6)}{12x^4}\right)-\frac{2(x^4+5x^2+3)^{3/2}}{9x^6}\right)$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{1}{2}\left(\frac{4}{3}\left(\frac{13}{12}\int \frac{1}{12-x^4}d\frac{5x^2+6}{\sqrt{x^4+5x^2+3}}-\frac{(5x^2+6)\sqrt{x^4+5x^2+3}}{12x^4}\right)-\frac{2(x^4+5x^2+3)^{3/2}}{9x^6}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}\left(\frac{4}{3}\left(\frac{13\text{arctanh}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{24\sqrt{3}}-\frac{(5x^2+6)\sqrt{x^4+5x^2+3}}{12x^4}\right)-\frac{2(x^4+5x^2+3)^{3/2}}{9x^6}\right)$$

input
```
Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^7,x]
```

output
```
((-2*(3 + 5*x^2 + x^4)^(3/2))/(9*x^6) + (4*(-1/12*((6 + 5*x^2)*Sqrt[3 + 5*
x^2 + x^4])/x^4 + (13*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4]
)])/(24*Sqrt[3])))/3)/2
```

3.148.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.148. $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^7}\,dx$

3.148.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.73

method	result
pseudoelliptic	$\frac{13 \operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}\,x^6-6\sqrt{x^4+5x^2+3}\,(7x^4+16x^2+6)}{108x^6}$
risch	$-\frac{7x^8+51x^6+107x^4+78x^2+18}{18x^6\sqrt{x^4+5x^2+3}}+\frac{13 \operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{108}$
trager	$-\frac{(7x^4+16x^2+6)\sqrt{x^4+5x^2+3}}{18x^6}+\frac{13 \operatorname{RootOf}(\_Z^2-3) \ln\left(-\frac{5 \operatorname{RootOf}(\_Z^2-3)x^2+6\sqrt{x^4+5x^2+3}+6 \operatorname{RootOf}(\_Z^2-3)}{x^2}\right)}{108}$
elliptic	$\frac{13 \operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{108}-\frac{\sqrt{x^4+5x^2+3}}{3x^6}-\frac{8\sqrt{x^4+5x^2+3}}{9x^4}-\frac{7\sqrt{x^4+5x^2+3}}{18x^2}$
default	$-\frac{(x^4+5x^2+3)^{\frac{3}{2}}}{9x^6}-\frac{(x^4+5x^2+3)^{\frac{3}{2}}}{9x^4}+\frac{5(x^4+5x^2+3)^{\frac{3}{2}}}{54x^2}-\frac{13\sqrt{x^4+5x^2+3}}{108}+\frac{13 \operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{108}-\frac{5(2x}{}$

input
```
int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
1/108*(13*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)*x^6-6
*(x^4+5*x^2+3)^(1/2)*(7*x^4+16*x^2+6))/x^6
```

3.148.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^7}\,dx = \frac{13\sqrt{3}x^6 \log\left(\frac{25\,x^2+2\,\sqrt{3}(5\,x^2+6)+2\,\sqrt{x^4+5\,x^2+3}\left(5\,\sqrt{3}+6\right)+30}{x^2}\right)-42\,x^6-6\,(7\,x^4+16\,x^2+6)\sqrt{x^4+5\,x^2+3}}{108\,x^6}$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^7,x, algorithm="fricas")
```

output
```
1/108*(13*sqrt(3)*x^6*log((25*x^2 + 2*sqrt(3)*(5*x^2 + 6) + 2*sqrt(x^4 + 5
*x^2 + 3)*(5*sqrt(3) + 6) + 30)/x^2) - 42*x^6 - 6*(7*x^4 + 16*x^2 + 6)*sqr
t(x^4 + 5*x^2 + 3))/x^6
```

3.148.6 Sympy [F]

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^7}\,dx = \int \frac{(3x^2+2)\sqrt{x^4+5x^2+3}}{x^7}\,dx$$

input
```
integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**7,x)
```

output
```
Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**7, x)
```

3.148.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.10

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^7}\,dx = \frac{13}{108}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right) + \frac{1}{9}\sqrt{x^4+5x^2+3} + \frac{5\sqrt{x^4+5x^2+3}}{18x^2} - \frac{(x^4+5x^2+3)^{\frac{3}{2}}}{9x^4} - \frac{(x^4+5x^2+3)^{\frac{3}{2}}}{9x^6}$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^7,x, algorithm="maxima")
```

output
```
13/108*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 1/9*
sqrt(x^4 + 5*x^2 + 3) + 5/18*sqrt(x^4 + 5*x^2 + 3)/x^2 - 1/9*(x^4 + 5*x^2
+ 3)^(3/2)/x^4 - 1/9*(x^4 + 5*x^2 + 3)^(3/2)/x^6
```

3.148.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(72) = 144$.

Time = 0.31 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.10

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^7}\,dx = -\frac{13}{108}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right) + \frac{67\left(x^2-\sqrt{x^4+5x^2+3}\right)^5+306\left(x^2-\sqrt{x^4+5x^2+3}\right)^4+430\left(x^2-\sqrt{x^4+5x^2+3}\right)^3+90\left(x^2-\sqrt{x\right.}{18\left(\left(x^2-\sqrt{x^4+5x^2+3}\right)^2-3\right)^3}$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^7,x, algorithm="giac")
```

output
```
-13/108*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3)
 - sqrt(x^4 + 5*x^2 + 3))) + 1/18*(67*(x^2 - sqrt(x^4 + 5*x^2 + 3))^5 + 30
6*(x^2 - sqrt(x^4 + 5*x^2 + 3))^4 + 430*(x^2 - sqrt(x^4 + 5*x^2 + 3))^3 +
90*(x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 63*x^2 + 63*sqrt(x^4 + 5*x^2 + 3) + 1
08)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 3)^3
```

3.148.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^7}\,dx = \int \frac{(3\,x^2+2)\,\sqrt{x^4+5\,x^2+3}}{x^7}\,dx$$

input
```
int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^7,x)
```

output
```
int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^7, x)
```

3.149 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^9}\,dx$

3.149.1 Optimal result

Integrand size = 25, antiderivative size = 111

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^9}\,dx = \frac{67(6+5x^2)\sqrt{3+5x^2+x^4}}{1728x^4} - \frac{(3+5x^2+x^4)^{3/2}}{12x^8} - \frac{11(3+5x^2+x^4)^{3/2}}{216x^6} - \frac{871\operatorname{arctanh}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{3456\sqrt{3}}$$

output
```
-1/12*(x^4+5*x^2+3)^(3/2)/x^8-11/216*(x^4+5*x^2+3)^(3/2)/x^6-871/10368*arc
tanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)+67/1728*(5*x^2+6)*
(x^4+5*x^2+3)^(1/2)/x^4
```

3.149.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.72

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^9}\,dx = \frac{\sqrt{3+5x^2+x^4}(-432-984x^2-182x^4+247x^6)}{1728x^8} + \frac{871\operatorname{arctanh}\left(\frac{x^2}{\sqrt{3}} - \frac{\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right)}{1728\sqrt{3}}$$

input
```
Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^9,x]
```

output
```
(Sqrt[3 + 5*x^2 + x^4]*(-432 - 984*x^2 - 182*x^4 + 247*x^6))/(1728*x^8) +
(871*ArcTanh[x^2/Sqrt[3] - Sqrt[3 + 5*x^2 + x^4]/Sqrt[3]])/(1728*Sqrt[3])
```

3.149.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1578, 1237, 25, 1228, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x^2+2)\sqrt{x^4+5x^2+3}}{x^9}dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{(3x^2+2)\sqrt{x^4+5x^2+3}}{x^{10}}dx^2$$

$$\Big\downarrow \text{1237}$$

$$\frac{1}{2}\left(-\frac{1}{12}\int -\frac{(11-2x^2)\sqrt{x^4+5x^2+3}}{x^8}dx^2-\frac{(x^4+5x^2+3)^{3/2}}{6x^8}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{12}\int \frac{(11-2x^2)\sqrt{x^4+5x^2+3}}{x^8}dx^2-\frac{(x^4+5x^2+3)^{3/2}}{6x^8}\right)$$

$$\Big\downarrow \text{1228}$$

$$\frac{1}{2}\left(\frac{1}{12}\left(-\frac{67}{6}\int \frac{\sqrt{x^4+5x^2+3}}{x^6}dx^2-\frac{11(x^4+5x^2+3)^{3/2}}{9x^6}\right)-\frac{(x^4+5x^2+3)^{3/2}}{6x^8}\right)$$

$$\Big\downarrow \text{1152}$$

$$\frac{1}{2}\left(\frac{1}{12}\left(-\frac{67}{6}\left(-\frac{13}{24}\int \frac{1}{x^2\sqrt{x^4+5x^2+3}}dx^2-\frac{\sqrt{x^4+5x^2+3}(5x^2+6)}{12x^4}\right)-\frac{11(x^4+5x^2+3)^{3/2}}{9x^6}\right)-\frac{(x^4+5x^2+}{6x^8}\right.$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{2}\left(\frac{1}{12}\left(-\frac{67}{6}\left(\frac{13}{12}\int \frac{1}{12-x^4}d\frac{5x^2+6}{\sqrt{x^4+5x^2+3}}-\frac{(5x^2+6)\sqrt{x^4+5x^2+3}}{12x^4}\right)-\frac{11(x^4+5x^2+3)^{3/2}}{9x^6}\right)-\frac{(x^4+5x^2}{6x}\right.$$

$$\Big\downarrow 219$$

$$\frac{1}{2}\left(\frac{1}{12}\left(-\frac{67}{6}\left(\frac{13\text{arctanh}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{24\sqrt{3}}-\frac{\left(5x^2+6\right)\sqrt{x^4+5x^2+3}}{12x^4}\right)-\frac{11\left(x^4+5x^2+3\right)^{3/2}}{9x^6}\right)-\frac{\left(x^4+5x^2+\right.}{6x^8}\right.$$

input
```
Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^9,x]
```

output
```
(-1/6*(3 + 5*x^2 + x^4)^(3/2)/x^8 + ((-11*(3 + 5*x^2 + x^4)^(3/2))/(9*x^6)
 - (67*(-1/12*((6 + 5*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^4 + (13*ArcTanh[(6 + 5
*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(24*Sqrt[3])))/6)/12)/2
```

3.149.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.149.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.64

method	result
pseudoelliptic	$\frac{-871 \operatorname{arctanh}\left(\frac{\left(5 x^{2}+6\right) \sqrt{3}}{6 \sqrt{x^{4}+5 x^{2}+3}}\right) \sqrt{3}\, x^{8}+6 \sqrt{x^{4}+5 x^{2}+3}\, \left(247 x^{6}-182 x^{4}-984 x^{2}-432\right)}{10368 x^{8}}$
risch	$\frac{247 x^{10}+1053 x^{8}-1153 x^{6}-5898 x^{4}-5112 x^{2}-1296}{1728 x^{8} \sqrt{x^{4}+5 x^{2}+3}}-\frac{871 \operatorname{arctanh}\left(\frac{\left(5 x^{2}+6\right) \sqrt{3}}{6 \sqrt{x^{4}+5 x^{2}+3}}\right) \sqrt{3}}{10368}$
trager	$\frac{\left(247 x^{6}-182 x^{4}-984 x^{2}-432\right) \sqrt{x^{4}+5 x^{2}+3}}{1728 x^{8}}-\frac{871 \operatorname{RootOf}\left(\_Z^{2}-3\right) \ln\left(-\frac{5 \operatorname{RootOf}\left(\_Z^{2}-3\right) x^{2}+6 \sqrt{x^{4}+5 x^{2}+3}+6 \operatorname{RootOf}\left(\right.}{x^{2}}\right.}{10368}$
elliptic	$\frac{247 \sqrt{x^{4}+5 x^{2}+3}}{1728 x^{2}}-\frac{871 \operatorname{arctanh}\left(\frac{\left(5 x^{2}+6\right) \sqrt{3}}{6 \sqrt{x^{4}+5 x^{2}+3}}\right) \sqrt{3}}{10368}-\frac{\sqrt{x^{4}+5 x^{2}+3}}{4 x^{8}}-\frac{41 \sqrt{x^{4}+5 x^{2}+3}}{72 x^{6}}-\frac{91 \sqrt{x^{4}+5 x^{2}+3}}{864 x^{4}}$
default	$-\frac{\left(x^{4}+5 x^{2}+3\right)^{\frac{3}{2}}}{12 x^{8}}-\frac{11 \left(x^{4}+5 x^{2}+3\right)^{\frac{3}{2}}}{216 x^{6}}+\frac{67 \left(x^{4}+5 x^{2}+3\right)^{\frac{3}{2}}}{864 x^{4}}-\frac{335 \left(x^{4}+5 x^{2}+3\right)^{\frac{3}{2}}}{5184 x^{2}}+\frac{871 \sqrt{x^{4}+5 x^{2}+3}}{10368}-\frac{871 \operatorname{arctanh}}{}$

input
```
int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
1/10368*(-871*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)*x
^8+6*(x^4+5*x^2+3)^(1/2)*(247*x^6-182*x^4-984*x^2-432))/x^8
```

3.149.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.86

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^9}\,dx$$
$$= \frac{871\sqrt{3}x^8\log\left(\frac{25\,x^2-2\sqrt{3}(5\,x^2+6)-2\sqrt{x^4+5\,x^2+3}\left(5\sqrt{3}-6\right)+30}{x^2}\right)+1482\,x^8+6\,(247\,x^6-182\,x^4-984\,x^2-432)\sqrt{}}{10368\,x^8}$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^9,x, algorithm="fricas")
```

output
```
1/10368*(871*sqrt(3)*x^8*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4
+ 5*x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) + 1482*x^8 + 6*(247*x^6 - 182*x^4
- 984*x^2 - 432)*sqrt(x^4 + 5*x^2 + 3))/x^8
```

3.149.6 Sympy [F]

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^9}\,dx = \int \frac{(3x^2+2)\sqrt{x^4+5x^2+3}}{x^9}\,dx$$

input
```
integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**9,x)
```

output
```
Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**9, x)
```

3.149.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^9}\,dx = -\frac{871}{10368}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right) - \frac{67}{864}\sqrt{x^4+5x^2+3} - \frac{335\sqrt{x^4+5x^2+3}}{1728x^2} + \frac{67(x^4+5x^2+3)^{\frac{3}{2}}}{864x^4} - \frac{11(x^4+5x^2+3)^{\frac{3}{2}}}{216x^6} - \frac{(x^4+5x^2+3)^{\frac{3}{2}}}{12x^8}$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^9,x, algorithm="maxima")
```

output
```
-871/10368*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) -
67/864*sqrt(x^4 + 5*x^2 + 3) - 335/1728*sqrt(x^4 + 5*x^2 + 3)/x^2 + 67/864
*(x^4 + 5*x^2 + 3)^(3/2)/x^4 - 11/216*(x^4 + 5*x^2 + 3)^(3/2)/x^6 - 1/12*(
x^4 + 5*x^2 + 3)^(3/2)/x^8
```

3.149.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(89) = 178.

Time = 0.29 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.10

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^9}\,dx = \frac{871}{10368}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right) - \frac{871\left(x^2-\sqrt{x^4+5x^2+3}\right)^7 - 5184\left(x^2-\sqrt{x^4+5x^2+3}\right)^6 - 57389\left(x^2-\sqrt{x^4+5x^2+3}\right)^5 - 165888}{1728}$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^9,x, algorithm="giac")
```

output
```
871/10368*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(
3) - sqrt(x^4 + 5*x^2 + 3))) - 1/1728*(871*(x^2 - sqrt(x^4 + 5*x^2 + 3))^7
 - 5184*(x^2 - sqrt(x^4 + 5*x^2 + 3))^6 - 57389*(x^2 - sqrt(x^4 + 5*x^2 +
3))^5 - 165888*(x^2 - sqrt(x^4 + 5*x^2 + 3))^4 - 204807*(x^2 - sqrt(x^4 +
5*x^2 + 3))^3 - 93312*(x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 2403*x^2 + 2403*sq
rt(x^4 + 5*x^2 + 3) - 5184)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 3)^4
```

3.149.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^9}\,dx = \int \frac{(3\,x^2+2)\,\sqrt{x^4+5\,x^2+3}}{x^9}\,dx$$

input
```
int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^9,x)
```

output
```
int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^9, x)
```

3.150 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^{11}}\,dx$

3.150.1 Optimal result

Integrand size = 25, antiderivative size = 132

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^{11}}\,dx = -\frac{161(6+5x^2)\sqrt{3+5x^2+x^4}}{5184x^4} - \frac{(3+5x^2+x^4)^{3/2}}{15x^{10}} - \frac{(3+5x^2+x^4)^{3/2}}{36x^8} + \frac{173(3+5x^2+x^4)^{3/2}}{3240x^6} + \frac{2093\operatorname{arctanh}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{10368\sqrt{3}}$$

output
```
-1/15*(x^4+5*x^2+3)^(3/2)/x^10-1/36*(x^4+5*x^2+3)^(3/2)/x^8+173/3240*(x^4+
5*x^2+3)^(3/2)/x^6+2093/31104*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^
(1/2))*3^(1/2)-161/5184*(5*x^2+6)*(x^4+5*x^2+3)^(1/2)/x^4
```

3.150.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.61

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^{11}}\,dx = \frac{-\frac{3\sqrt{3+5x^2+x^4}(5184+10800x^2+1176x^4-1370x^6+2641x^8)}{x^{10}} - 10465\sqrt{3}\operatorname{arctanh}\left(\frac{x^2-\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right)}{77760}$$

input
```
Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^11,x]
```

output
```
((-3*Sqrt[3 + 5*x^2 + x^4]*(5184 + 10800*x^2 + 1176*x^4 - 1370*x^6 + 2641*
x^8))/x^10 - 10465*Sqrt[3]*ArcTanh[(x^2 - Sqrt[3 + 5*x^2 + x^4])/Sqrt[3]])
/77760
```

3.150.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.14, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {1578, 1237, 27, 1237, 27, 1228, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x^2+2)\sqrt{x^4+5x^2+3}}{x^{11}}dx$$

$$\Big\downarrow \text{ 1578}$$

$$\frac{1}{2}\int \frac{(3x^2+2)\sqrt{x^4+5x^2+3}}{x^{12}}dx^2$$

$$\Big\downarrow \text{ 1237}$$

$$\frac{1}{2}\left(-\frac{1}{15}\int -\frac{2(5-2x^2)\sqrt{x^4+5x^2+3}}{x^{10}}dx^2-\frac{2(x^4+5x^2+3)^{3/2}}{15x^{10}}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(\frac{2}{15}\int \frac{(5-2x^2)\sqrt{x^4+5x^2+3}}{x^{10}}dx^2-\frac{2(x^4+5x^2+3)^{3/2}}{15x^{10}}\right)$$

$$\Big\downarrow \text{ 1237}$$

$$\frac{1}{2}\left(\frac{2}{15}\left(-\frac{1}{12}\int \frac{(10x^2+173)\sqrt{x^4+5x^2+3}}{2x^8}dx^2-\frac{5(x^4+5x^2+3)^{3/2}}{12x^8}\right)-\frac{2(x^4+5x^2+3)^{3/2}}{15x^{10}}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(\frac{2}{15}\left(-\frac{1}{24}\int \frac{(10x^2+173)\sqrt{x^4+5x^2+3}}{x^8}dx^2-\frac{5(x^4+5x^2+3)^{3/2}}{12x^8}\right)-\frac{2(x^4+5x^2+3)^{3/2}}{15x^{10}}\right)$$

$$\Big\downarrow \text{ 1228}$$

$$\frac{1}{2}\left(\frac{2}{15}\left(\frac{1}{24}\left(\frac{805}{6}\int\frac{\sqrt{x^4+5x^2+3}}{x^6}dx^2+\frac{173\left(x^4+5x^2+3\right)^{3/2}}{9x^6}\right)-\frac{5\left(x^4+5x^2+3\right)^{3/2}}{12x^8}\right)-\frac{2\left(x^4+5x^2+3\right)^{3/2}}{15x^{10}}\right)$$

$$\Big\downarrow 1152$$

$$\frac{1}{2}\left(\frac{2}{15}\left(\frac{1}{24}\left(\frac{805}{6}\left(-\frac{13}{24}\int\frac{1}{x^2\sqrt{x^4+5x^2+3}}dx^2-\frac{\sqrt{x^4+5x^2+3}\left(5x^2+6\right)}{12x^4}\right)+\frac{173\left(x^4+5x^2+3\right)^{3/2}}{9x^6}\right)-\frac{5(x^4+$$

$$\Big\downarrow 1154$$

$$\frac{1}{2}\left(\frac{2}{15}\left(\frac{1}{24}\left(\frac{805}{6}\left(\frac{13}{12}\int\frac{1}{12-x^4}d\frac{5x^2+6}{\sqrt{x^4+5x^2+3}}-\frac{\left(5x^2+6\right)\sqrt{x^4+5x^2+3}}{12x^4}\right)+\frac{173\left(x^4+5x^2+3\right)^{3/2}}{9x^6}\right)-\frac{5(x^4$$

$$\Big\downarrow 219$$

$$\frac{1}{2}\left(\frac{2}{15}\left(\frac{1}{24}\left(\frac{805}{6}\left(\frac{13\text{arctanh}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{24\sqrt{3}}-\frac{\left(5x^2+6\right)\sqrt{x^4+5x^2+3}}{12x^4}\right)+\frac{173\left(x^4+5x^2+3\right)^{3/2}}{9x^6}\right)-\frac{5(x^4+$$

input
```
Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^11,x]
```

output
```
((-2*(3 + 5*x^2 + x^4)^(3/2))/(15*x^10) + (2*((-5*(3 + 5*x^2 + x^4)^(3/2))
/(12*x^8) + ((173*(3 + 5*x^2 + x^4)^(3/2))/(9*x^6) + (805*(-1/12*((6 + 5*x
^2)*Sqrt[3 + 5*x^2 + x^4])/x^4 + (13*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3
 + 5*x^2 + x^4])])/(24*Sqrt[3])))/6)/24))/15)/2
```

3.150.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.150. $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^{11}}\,dx$

3.150.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.58

method	result
pseudoelliptic	$\frac{10465 \operatorname{arctanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}\,x^{10}-6\sqrt{x^4+5x^2+3}\,(2641x^8-1370x^6+1176x^4+10800x^2+5184)}{155520x^{10}}$
risch	$-\frac{2641x^{12}+11835x^{10}+2249x^8+12570x^6+62712x^4+58320x^2+15552}{25920x^{10}\sqrt{x^4+5x^2+3}}+\frac{2093 \operatorname{arctanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{31104}$
trager	$-\frac{(2641x^8-1370x^6+1176x^4+10800x^2+5184)\sqrt{x^4+5x^2+3}}{25920x^{10}}-\frac{2093 \operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{-5 \operatorname{RootOf}\left(\_Z^2-3\right)x^2+6\sqrt{\ldots}}{\ldots}\right.}{31104}$
elliptic	$-\frac{\sqrt{x^4+5x^2+3}}{5x^{10}}-\frac{5\sqrt{x^4+5x^2+3}}{12x^8}-\frac{49\sqrt{x^4+5x^2+3}}{1080x^6}+\frac{137\sqrt{x^4+5x^2+3}}{2592x^4}-\frac{2641\sqrt{x^4+5x^2+3}}{25920x^2}+\frac{2093 \operatorname{arctanh}\left(\frac{(5x^2\ldots}{6\sqrt{x^4\ldots}}\right.}{31104}$
default	$-\frac{(x^4+5x^2+3)^{\frac{3}{2}}}{36x^8}+\frac{173(x^4+5x^2+3)^{\frac{3}{2}}}{3240x^6}-\frac{161(x^4+5x^2+3)^{\frac{3}{2}}}{2592x^4}+\frac{805(x^4+5x^2+3)^{\frac{3}{2}}}{15552x^2}-\frac{2093\sqrt{x^4+5x^2+3}}{31104}+\frac{2093 \operatorname{arct}\ldots}{\ldots}$

input
```
int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^11,x,method=_RETURNVERBOSE)
```

output
```
1/155520*(10465*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)
*x^10-6*(x^4+5*x^2+3)^(1/2)*(2641*x^8-1370*x^6+1176*x^4+10800*x^2+5184))/x
^10
```

3.150.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.76

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^{11}}\,dx$$
$$= \frac{10465\sqrt{3}x^{10}\log\left(\frac{25\,x^2+2\sqrt{3}(5\,x^2+6)+2\sqrt{x^4+5\,x^2+3}\left(5\sqrt{3}+6\right)+30}{x^2}\right)-15846\,x^{10}-6\,(2641\,x^8-1370\,x^6+1176\,x^4\ldots}{155520\,x^{10}}$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^11,x, algorithm="fricas")
```

output
```
1/155520*(10465*sqrt(3)*x^10*log((25*x^2 + 2*sqrt(3)*(5*x^2 + 6) + 2*sqrt(
x^4 + 5*x^2 + 3)*(5*sqrt(3) + 6) + 30)/x^2) - 15846*x^10 - 6*(2641*x^8 - 1
370*x^6 + 1176*x^4 + 10800*x^2 + 5184)*sqrt(x^4 + 5*x^2 + 3))/x^10
```

3.150.6 Sympy [F]

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^{11}}\,dx = \int \frac{(3x^2+2)\sqrt{x^4+5x^2+3}}{x^{11}}\,dx$$

input
```
integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**11,x)
```

output
```
Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**11, x)
```

3.150.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.01

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^{11}}\,dx = \frac{2093}{31104}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right) + \frac{161}{2592}\sqrt{x^4+5x^2+3} + \frac{805\sqrt{x^4+5x^2+3}}{5184x^2} - \frac{161(x^4+5x^2+3)^{\frac{3}{2}}}{2592x^4} + \frac{173(x^4+5x^2+3)^{\frac{3}{2}}}{3240x^6} - \frac{(x^4+5x^2+3)^{\frac{3}{2}}}{36x^8} - \frac{(x^4+5x^2+3)^{\frac{3}{2}}}{15x^{10}}$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^11,x, algorithm="maxima")
```

output
```
2093/31104*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) +
161/2592*sqrt(x^4 + 5*x^2 + 3) + 805/5184*sqrt(x^4 + 5*x^2 + 3)/x^2 - 161/
2592*(x^4 + 5*x^2 + 3)^(3/2)/x^4 + 173/3240*(x^4 + 5*x^2 + 3)^(3/2)/x^6 -
1/36*(x^4 + 5*x^2 + 3)^(3/2)/x^8 - 1/15*(x^4 + 5*x^2 + 3)^(3/2)/x^10
```

3.150.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(106) = 212$.

Time = 0.31 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.93

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^{11}}\,dx = -\frac{2093}{31104}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right) + \frac{10465\left(x^2-\sqrt{x^4+5x^2+3}\right)^9 - 42830\left(x^2-\sqrt{x^4+5x^2+3}\right)^7 + 1270080\left(x^2-\sqrt{x^4+5x^2+3}\right)^6 + 70}{}$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^11,x, algorithm="giac")
```

output
```
-2093/31104*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqr
t(3) - sqrt(x^4 + 5*x^2 + 3))) + 1/25920*(10465*(x^2 - sqrt(x^4 + 5*x^2 +
3))^9 - 42830*(x^2 - sqrt(x^4 + 5*x^2 + 3))^7 + 1270080*(x^2 - sqrt(x^4 +
5*x^2 + 3))^6 + 7060800*(x^2 - sqrt(x^4 + 5*x^2 + 3))^5 + 15310080*(x^2 -
sqrt(x^4 + 5*x^2 + 3))^4 + 16095870*(x^2 - sqrt(x^4 + 5*x^2 + 3))^3 + 7568
640*(x^2 - sqrt(x^4 + 5*x^2 + 3))^2 + 1096335*x^2 - 1096335*sqrt(x^4 + 5*x
^2 + 3) + 202176)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 3)^5
```

3.150.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^{11}}\,dx = \int \frac{(3x^2+2)\sqrt{x^4+5x^2+3}}{x^{11}}\,dx$$

input
```
int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^11,x)
```

output
```
int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^11, x)
```

3.151 $\int x^4(2+3x^2)\sqrt{3+5x^2+x^4}\,dx$

3.151.1 Optimal result

Integrand size = 25, antiderivative size = 322

$$\int x^4(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = -\frac{1924x(5+\sqrt{13}+2x^2)}{105\sqrt{3+5x^2+x^4}} + \frac{13}{3}x\sqrt{3+5x^2+x^4} - \frac{26}{35}x^3\sqrt{3+5x^2+x^4} + \frac{1}{21}x^5(11+7x^2)\sqrt{3+5x^2+x^4} + \frac{962\sqrt{\frac{2}{3}(5+\sqrt{13})}\sqrt{\frac{6+(5-\sqrt{13})x^2}{6+(5+\sqrt{13})x^2}}(6+(5+\sqrt{13})x^2)\,E\left(\arctan\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right)|\frac{1}{6}(-13+5\sqrt{13})\right)}{105\sqrt{3+5x^2+x^4}} - \frac{13\sqrt{\frac{6+(5-\sqrt{13})x^2}{6+(5+\sqrt{13})x^2}}(6+(5+\sqrt{13})x^2)\,\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right),\frac{1}{6}(-13+5\sqrt{13})\right)}{\sqrt{6(5+\sqrt{13})}\sqrt{3+5x^2+x^4}}$$

output
```
-1924/105*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+3)^(1/2)+13/3*x*(x^4+5*x^2+3)^(1
/2)-26/35*x^3*(x^4+5*x^2+3)^(1/2)+1/21*x^5*(7*x^2+11)*(x^4+5*x^2+3)^(1/2)+
962/315*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*
EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+
30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(1/2))^(1/2)*((6+x^2*(5-
13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)-13*(1/(36+x^2*(
30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13
^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*
(6+x^2*(5+13^(1/2)))*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^
4+5*x^2+3)^(1/2)/(30+6*13^(1/2))^(1/2)
```

3.151.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.06 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.74

$$\int x^4(2+3x^2)\sqrt{3+5x^2+x^4}\,dx$$
$$= \frac{2730x+4082x^3+460x^5+604x^7+460x^9+70x^{11}-1924i\sqrt{2}(-5+\sqrt{13})\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x}}{}$$

input
```
Integrate[x^4*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

output
```
(2730*x + 4082*x^3 + 460*x^5 + 604*x^7 + 460*x^9 + 70*x^11 - (1924*I)*Sqrt
[2]*(-5 + Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 +
 Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (
5*Sqrt[13])/6] + (13*I)*Sqrt[2]*(-635 + 148*Sqrt[13])*Sqrt[(-5 + Sqrt[13]
- 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[S
qrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(210*Sqrt[3 + 5*x^2 + x^
4])
```

3.151.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1596, 27, 1602, 1602, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(3x^2+2)\sqrt{x^4+5x^2+3}\,dx$$
$$\Big\downarrow \text{1596}$$
$$\frac{1}{63}\int -\frac{117x^4(2x^2+1)}{\sqrt{x^4+5x^2+3}}dx+\frac{1}{21}(7x^2+11)\sqrt{x^4+5x^2+3}x^5$$
$$\Big\downarrow \text{27}$$
$$\frac{1}{21}x^5(7x^2+11)\sqrt{x^4+5x^2+3}-\frac{13}{7}\int \frac{x^4(2x^2+1)}{\sqrt{x^4+5x^2+3}}dx$$

$$\Big\downarrow \text{1602}$$

$$\frac{1}{21}x^5\left(7x^2+11\right)\sqrt{x^4+5x^2+3}-\frac{13}{7}\left(\frac{2}{5}x^3\sqrt{x^4+5x^2+3}-\frac{1}{5}\int\frac{x^2\left(35x^2+18\right)}{\sqrt{x^4+5x^2+3}}dx\right)$$

$$\Big\downarrow \text{1602}$$

$$\frac{1}{21}x^5\left(7x^2+11\right)\sqrt{x^4+5x^2+3}-\frac{13}{7}\left(\frac{1}{5}\left(\frac{1}{3}\int\frac{296x^2+105}{\sqrt{x^4+5x^2+3}}dx-\frac{35}{3}x\sqrt{x^4+5x^2+3}\right)+\frac{2}{5}\sqrt{x^4+5x^2+3}x^3\right)$$

$$\Big\downarrow \text{1503}$$

$$\frac{1}{21}x^5\left(7x^2+11\right)\sqrt{x^4+5x^2+3}-\frac{13}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(105\int\frac{1}{\sqrt{x^4+5x^2+3}}dx+296\int\frac{x^2}{\sqrt{x^4+5x^2+3}}dx\right)-\frac{35}{3}x\sqrt{x^4+5x^2+3}\right)+\frac{2}{5}\sqrt{x^4+5x^2+3}x^3\right)$$

$$\Big\downarrow \text{1412}$$

$$\frac{1}{21}x^5\left(7x^2+11\right)\sqrt{x^4+5x^2+3}-\frac{13}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(296\int\frac{x^2}{\sqrt{x^4+5x^2+3}}dx+\frac{35\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(\right.}\right.\right.}{\sqrt{x^4+5x^2+3}}\right.\right.\right.$$

$$\Big\downarrow \text{1455}$$

$$\frac{1}{21}x^5\left(7x^2+11\right)\sqrt{x^4+5x^2+3}-\frac{13}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{35\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{\right.}\right.}{\sqrt{x^4+5x^2+3}}\right.\right.\right.$$

input
```
Int[x^4*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

3.151. $\int x^4(2+3x^2)\sqrt{3+5x^2+x^4}\,dx$

output
```
(x^5*(11 + 7*x^2)*Sqrt[3 + 5*x^2 + x^4])/21 - (13*((2*x^3*Sqrt[3 + 5*x^2 +
 x^4])/5 + ((-35*x*Sqrt[3 + 5*x^2 + x^4])/3 + (296*((x*(5 + Sqrt[13] + 2*x
^2))/(2*Sqrt[3 + 5*x^2 + x^4]) - (Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sq
rt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[
ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(2*Sqrt[3 + 5*x^2
 + x^4])) + (35*Sqrt[3/(2*(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(
6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5
 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/Sqrt[3 + 5*x^2 + x^4])/3)/5))/7
```

3.151.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1596
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((b*e*2
*p + c*d*(m + 4*p + 3) + c*e*(4*p + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p
 + 3))), x] + Simp[2*(p/(c*(4*p + m + 1)*(m + 4*p + 3)))   Int[(f*x)^m*(a +
 b*x^2 + c*x^4)^(p - 1)*Simp[2*a*c*d*(m + 4*p + 3) - a*b*e*(m + 1) + (2*a*c
*e*(4*p + m + 1) + b*c*d*(m + 4*p + 3) - b^2*e*(m + 2*p + 1))*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] &
& NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[
p] || IntegerQ[m])
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

3.151.4 Maple [A] (verified)

Time = 5.58 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.70

method	result
risch	$\frac{x(35x^6+55x^4-78x^2+455)\sqrt{x^4+5x^2+3}}{105} - \frac{78\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6}, \frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}} + \frac{46176\sqrt{}}{}$
default	$\frac{x^7\sqrt{x^4+5x^2+3}}{3} + \frac{11x^5\sqrt{x^4+5x^2+3}}{21} - \frac{26x^3\sqrt{x^4+5x^2+3}}{35} + \frac{13x\sqrt{x^4+5x^2+3}}{3} - \frac{78\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x}}$
elliptic	$\frac{x^7\sqrt{x^4+5x^2+3}}{3} + \frac{11x^5\sqrt{x^4+5x^2+3}}{21} - \frac{26x^3\sqrt{x^4+5x^2+3}}{35} + \frac{13x\sqrt{x^4+5x^2+3}}{3} - \frac{78\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x}}$

input
```
int(x^4*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/105*x*(35*x^6+55*x^4-78*x^2+455)*(x^4+5*x^2+3)^(1/2)-78/(-30+6*13^(1/2))
^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)
/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/
6*39^(1/2))+46176/35/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1
/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^(1/2))*(El
lipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1
/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))
```

3.151.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.43

$$\int x^4\left(2+3x^2\right)\sqrt{3+5x^2+x^4}\,dx =$$
$$-\frac{3848\left(\sqrt{13}\sqrt{2}x-5\sqrt{2}x\right)\sqrt{\sqrt{13}-5}\,E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{13}-5}}{2x}\right)|\,\frac{5}{6}\sqrt{13}+\frac{19}{6}\right)-13\left(261\sqrt{13}\sqrt{2}x-1655\sqrt{2}\right.}{}$$

input
```
integrate(x^4*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
-1/420*(3848*(sqrt(13)*sqrt(2)*x - 5*sqrt(2)*x)*sqrt(sqrt(13) - 5)*ellipti
c_e(arcsin(1/2*sqrt(2)*sqrt(sqrt(13) - 5)/x), 5/6*sqrt(13) + 19/6) - 13*(2
61*sqrt(13)*sqrt(2)*x - 1655*sqrt(2)*x)*sqrt(sqrt(13) - 5)*elliptic_f(arcs
in(1/2*sqrt(2)*sqrt(sqrt(13) - 5)/x), 5/6*sqrt(13) + 19/6) - 4*(35*x^8 + 5
5*x^6 - 78*x^4 + 455*x^2 - 3848)*sqrt(x^4 + 5*x^2 + 3))/x
```

3.151.6 Sympy [F]

$$\int x^4\left(2+3x^2\right)\sqrt{3+5x^2+x^4}\,dx = \int x^4\cdot\left(3x^2+2\right)\sqrt{x^4+5x^2+3}\,dx$$

input
```
integrate(x**4*(3*x**2+2)*(x**4+5*x**2+3)**(1/2),x)
```

output
```
Integral(x**4*(3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3), x)
```

3.151.7 Maxima [F]

$$\int x^4(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \int \sqrt{x^4+5\,x^2+3}(3\,x^2+2)x^4\,dx$$

input
```
integrate(x^4*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)*x^4, x)
```

3.151.8 Giac [F]

$$\int x^4(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \int \sqrt{x^4+5\,x^2+3}(3\,x^2+2)x^4\,dx$$

input
```
integrate(x^4*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)*x^4, x)
```

3.151.9 Mupad [F(-1)]

Timed out.

$$\int x^4(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \int x^4\,(3\,x^2+2)\,\sqrt{x^4+5\,x^2+3}\,dx$$

input
```
int(x^4*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2),x)
```

output
```
int(x^4*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2), x)
```

3.152 $\int x^2(2+3x^2)\sqrt{3+5x^2+x^4}\,dx$

3.152.1 Optimal result

Integrand size = 25, antiderivative size = 305

$$\begin{aligned}\int x^2(2+3x^2)\sqrt{3+5x^2+x^4}\,dx &= \frac{1247x(5+\sqrt{13}+2x^2)}{210\sqrt{3+5x^2+x^4}}-\frac{4}{3}x\sqrt{3+5x^2+x^4}+\frac{1}{35}x^3(29+15x^2)\sqrt{3+5x^2+x^4}\\ &\quad -\frac{1247\sqrt{\frac{1}{6}(5+\sqrt{13})}\sqrt{\frac{6+(5-\sqrt{13})x^2}{6+(5+\sqrt{13})x^2}}\,(6+(5+\sqrt{13})x^2)\,E\left(\arctan\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right)|\frac{1}{6}(-13+5\sqrt{13})\right)}{210\sqrt{3+5x^2+x^4}}\\ &\quad +\frac{2\sqrt{\frac{2}{3(5+\sqrt{13})}}\sqrt{\frac{6+(5-\sqrt{13})x^2}{6+(5+\sqrt{13})x^2}}\,(6+(5+\sqrt{13})x^2)\,\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right),\frac{1}{6}(-13+5\sqrt{13})\right)}{\sqrt{3+5x^2+x^4}}\end{aligned}$$

output
```
1247/210*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+3)^(1/2)-4/3*x*(x^4+5*x^2+3)^(1/2
)+1/35*x^3*(15*x^2+29)*(x^4+5*x^2+3)^(1/2)+2/3*(1/(36+x^2*(30+6*13^(1/2)))
)^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13^(1/2))^(1/2)/(
36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1
/2)))*6^(1/2)/(5+13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))
))^(1/2)/(x^4+5*x^2+3)^(1/2)-1247/1260*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*
(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(
30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(3
0+6*13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4
+5*x^2+3)^(1/2)
```

3.152.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.09 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.77

$$\int x^2(2+3x^2)\sqrt{3+5x^2+x^4}\,dx$$
$$= \frac{4x(-420-439x^2+430x^4+312x^6+45x^8)+1247i\sqrt{2}(-5+\sqrt{13})\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}E\Big(i\text{ar}}{}$$

input
```
Integrate[x^2*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

output
```
(4*x*(-420 - 439*x^2 + 430*x^4 + 312*x^6 + 45*x^8) + (1247*I)*Sqrt[2]*(-5
+ Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13
] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[1
3])/6] - I*Sqrt[2]*(-5395 + 1247*Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-
5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 +
Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(420*Sqrt[3 + 5*x^2 + x^4])
```

3.152.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1596, 25, 1602, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(3x^2+2)\sqrt{x^4+5x^2+3}\,dx$$
$$\Big\downarrow \text{1596}$$
$$\frac{1}{35}\int -\frac{x^2(140x^2+51)}{\sqrt{x^4+5x^2+3}}dx+\frac{1}{35}(15x^2+29)\sqrt{x^4+5x^2+3}x^3$$
$$\Big\downarrow \text{25}$$
$$\frac{1}{35}x^3(15x^2+29)\sqrt{x^4+5x^2+3}-\frac{1}{35}\int \frac{x^2(140x^2+51)}{\sqrt{x^4+5x^2+3}}dx$$
$$\Big\downarrow \text{1602}$$

$$\frac{1}{35}\left(\frac{1}{3}\int \frac{1247x^2+420}{\sqrt{x^4+5x^2+3}}dx-\frac{140}{3}x\sqrt{x^4+5x^2+3}\right)+\frac{1}{35}\left(15x^2+29\right)\sqrt{x^4+5x^2+3}x^3$$

$\downarrow$ 1503

$$\frac{1}{35}\left(\frac{1}{3}\left(420\int \frac{1}{\sqrt{x^4+5x^2+3}}dx+1247\int \frac{x^2}{\sqrt{x^4+5x^2+3}}dx\right)-\frac{140}{3}x\sqrt{x^4+5x^2+3}\right)+$$
$$\frac{1}{35}\left(15x^2+29\right)\sqrt{x^4+5x^2+3}x^3$$

$\downarrow$ 1412

$$\frac{1}{35}\left(\frac{1}{3}\left(1247\int \frac{x^2}{\sqrt{x^4+5x^2+3}}dx+\frac{70\sqrt{\frac{6}{5+\sqrt{13}}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{\ldots}\right.}\right.\right.}{\sqrt{x^4+5x^2+3}}\right.\right.$$
$$\frac{1}{35}\left(15x^2+29\right)\sqrt{x^4+5x^2+3}x^3$$

$\downarrow$ 1455

$$\frac{1}{35}\left(\frac{1}{3}\left(\frac{70\sqrt{\frac{6}{5+\sqrt{13}}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}+\right.\right.$$
$$\frac{1}{35}\left(15x^2+29\right)\sqrt{x^4+5x^2+3}x^3$$

input
```
Int[x^2*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

output
```
(x^3*(29 + 15*x^2)*Sqrt[3 + 5*x^2 + x^4])/35 + ((-140*x*Sqrt[3 + 5*x^2 + x
^4])/3 + (1247*((x*(5 + Sqrt[13] + 2*x^2))/(2*Sqrt[3 + 5*x^2 + x^4]) - (Sq
rt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2
)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-
13 + 5*Sqrt[13])/6])/(2*Sqrt[3 + 5*x^2 + x^4])) + (70*Sqrt[6/(5 + Sqrt[13]
)]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[
13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6
])/Sqrt[3 + 5*x^2 + x^4])/3)/35
```

3.152. $\int x^2(2+3x^2)\sqrt{3+5x^2+x^4}\,dx$

3.152.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1596
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((b*e*2
*p + c*d*(m + 4*p + 3) + c*e*(4*p + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p
 + 3))), x] + Simp[2*(p/(c*(4*p + m + 1)*(m + 4*p + 3)))   Int[(f*x)^m*(a +
 b*x^2 + c*x^4)^(p - 1)*Simp[2*a*c*d*(m + 4*p + 3) - a*b*e*(m + 1) + (2*a*c
*e*(4*p + m + 1) + b*c*d*(m + 4*p + 3) - b^2*e*(m + 2*p + 1))*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] &
& NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[
p] || IntegerQ[m])
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

3.152.4 Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.72

method	result
risch	$\frac{x(45x^4+87x^2-140)\sqrt{x^4+5x^2+3}}{105}+\frac{24\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}-\frac{14964\sqrt{1-\left(-\right.}}{}$
default	$\frac{3x^5\sqrt{x^4+5x^2+3}}{7}+\frac{29x^3\sqrt{x^4+5x^2+3}}{35}-\frac{4x\sqrt{x^4+5x^2+3}}{3}+\frac{24\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\right.}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}$
elliptic	$\frac{3x^5\sqrt{x^4+5x^2+3}}{7}+\frac{29x^3\sqrt{x^4+5x^2+3}}{35}-\frac{4x\sqrt{x^4+5x^2+3}}{3}+\frac{24\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\right.}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}$

input
```
int(x^2*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/105*x*(45*x^4+87*x^2-140)*(x^4+5*x^2+3)^(1/2)+24/(-30+6*13^(1/2))^(1/2)*
(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5
*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1
/2))-14964/35/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-
(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF
(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-
30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))
```

3.152.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.44

$$\int x^2(2+3x^2)\sqrt{3+5x^2+x^4}\,dx$$
$$= \frac{1247\left(\sqrt{13}\sqrt{2}x-5\sqrt{2}x\right)\sqrt{\sqrt{13}-5}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{13}-5}}{2x}\right)\mid\frac{5}{6}\sqrt{13}+\frac{19}{6}\right)-\left(1107\sqrt{13}\sqrt{2}x-6935\sqrt{2}x\right)\sqrt{}}{420\,x}$$

input
```
integrate(x^2*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
1/420*(1247*(sqrt(13)*sqrt(2)*x - 5*sqrt(2)*x)*sqrt(sqrt(13) - 5)*elliptic
_e(arcsin(1/2*sqrt(2)*sqrt(sqrt(13) - 5)/x), 5/6*sqrt(13) + 19/6) - (1107*
sqrt(13)*sqrt(2)*x - 6935*sqrt(2)*x)*sqrt(sqrt(13) - 5)*elliptic_f(arcsin(
1/2*sqrt(2)*sqrt(sqrt(13) - 5)/x), 5/6*sqrt(13) + 19/6) + 4*(45*x^6 + 87*x
^4 - 140*x^2 + 1247)*sqrt(x^4 + 5*x^2 + 3))/x
```

3.152.6 Sympy [F]

$$\int x^2(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \int x^2\cdot(3x^2+2)\sqrt{x^4+5x^2+3}\,dx$$

input
```
integrate(x**2*(3*x**2+2)*(x**4+5*x**2+3)**(1/2),x)
```

output
```
Integral(x**2*(3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3), x)
```

3.152.7 Maxima [F]

$$\int x^2(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \int \sqrt{x^4+5x^2+3}(3x^2+2)x^2\,dx$$

input
```
integrate(x^2*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)*x^2, x)
```

3.152.8 Giac [F]

$$\int x^2(2+3x^2)\sqrt{3+5x^2+x^4}\,dx=\int \sqrt{x^4+5\,x^2+3}\,(3\,x^2+2)x^2\,dx$$

input
```
integrate(x^2*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)*x^2, x)
```

3.152.9 Mupad [F(-1)]

Timed out.

$$\int x^2(2+3x^2)\sqrt{3+5x^2+x^4}\,dx=\int x^2\left(3\,x^2+2\right)\sqrt{x^4+5\,x^2+3}\,dx$$

input
```
int(x^2*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2),x)
```

output
```
int(x^2*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2), x)
```

3.153 $\int (2+3x^2)\sqrt{3+5x^2+x^4}\,dx$

3.153.1 Optimal result

Integrand size = 22, antiderivative size = 279

$$\int (2+3x^2)\sqrt{3+5x^2+x^4}\,dx=-\frac{23x(5+\sqrt{13}+2x^2)}{15\sqrt{3+5x^2+x^4}}+\frac{1}{15}x(25+9x^2)\sqrt{3+5x^2+x^4}$$
$$+\frac{23\sqrt{\frac{1}{6}(5+\sqrt{13})}\sqrt{\frac{6+(5-\sqrt{13})x^2}{6+(5+\sqrt{13})x^2}}\,(6+(5+\sqrt{13})x^2)\,E\left(\arctan\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right)|\frac{1}{6}(-13+5\sqrt{13})\right)}{15\sqrt{3+5x^2+x^4}}$$
$$+\frac{\sqrt{\frac{6+(5-\sqrt{13})x^2}{6+(5+\sqrt{13})x^2}}\,(6+(5+\sqrt{13})x^2)\,\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right),\frac{1}{6}(-13+5\sqrt{13})\right)}{\sqrt{6(5+\sqrt{13})}\sqrt{3+5x^2+x^4}}$$

output
```
-23/15*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+3)^(1/2)+1/15*x*(9*x^2+25)*(x^4+5*x
^2+3)^(1/2)+23/90*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2
)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2)
,1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(1/2))^(1/2)*(
(6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)+(1/(3
6+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(
30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^
(1/2))*(6+x^2*(5+13^(1/2)))*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1
/2)/(x^4+5*x^2+3)^(1/2)/(30+6*13^(1/2))^(1/2)
```

3.153.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.06 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.82

$$\int (2+3x^2)\sqrt{3+5x^2+x^4}\,dx$$
$$= \frac{2x(75+152x^2+70x^4+9x^6)-23i\sqrt{2}(-5+\sqrt{13})\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right.\right.}{30\sqrt{}}$$

input
```
Integrate[(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

output
```
(2*x*(75 + 152*x^2 + 70*x^4 + 9*x^6) - (23*I)*Sqrt[2]*(-5 + Sqrt[13])*Sqrt
[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*Ellip
ticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] + I*Sqrt[
2]*(-130 + 23*Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt
[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6
 + (5*Sqrt[13])/6])/(30*Sqrt[3 + 5*x^2 + x^4])
```

3.153.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1490, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3x^2+2)\sqrt{x^4+5x^2+3}\,dx$$
$$\Big\downarrow \text{1490}$$
$$\frac{1}{15}\int \frac{15-46x^2}{\sqrt{x^4+5x^2+3}}dx+\frac{1}{15}x\sqrt{x^4+5x^2+3}(9x^2+25)$$
$$\Big\downarrow \text{1503}$$
$$\frac{1}{15}\left(15\int \frac{1}{\sqrt{x^4+5x^2+3}}dx-46\int \frac{x^2}{\sqrt{x^4+5x^2+3}}dx\right)+\frac{1}{15}x\sqrt{x^4+5x^2+3}(9x^2+25)$$
$$\Big\downarrow \text{1412}$$

$$\frac{1}{15}\left(\frac{5\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}-4\right.$$

$$\frac{1}{15}x\sqrt{x^4+5x^2+3}\left(9x^2+25\right)$$

$$\Big\downarrow \text{1455}$$

$$\frac{1}{15}\left(\frac{5\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}-4\right.$$

$$\frac{1}{15}x\sqrt{x^4+5x^2+3}\left(9x^2+25\right)$$

input
```
Int[(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

output
```
(x*(25 + 9*x^2)*Sqrt[3 + 5*x^2 + x^4])/15 + (-46*((x*(5 + Sqrt[13] + 2*x^2
))/(2*Sqrt[3 + 5*x^2 + x^4]) - (Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt
[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[Ar
cTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(2*Sqrt[3 + 5*x^2 +
 x^4])) + (5*Sqrt[3/(2*(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 +
 (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 +
Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/Sqrt[3 + 5*x^2 + x^4])/15
```

3.153.3.1 Defintions of rubi rules used

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

3.153.4 Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.77

method	result
risch	$\frac{x(9x^2+25)\sqrt{x^4+5x^2+3}}{15}+\frac{6\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}+\frac{552\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\cdots}{}$
default	$\frac{5x\sqrt{x^4+5x^2+3}}{3}+\frac{6\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}+\frac{552\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}\cdots\right.}}{}$
elliptic	$\frac{5x\sqrt{x^4+5x^2+3}}{3}+\frac{6\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}+\frac{552\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}\cdots\right.}}{}$

input
```
int((3*x^2+2)*(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*x*(9*x^2+25)*(x^4+5*x^2+3)^(1/2)+6/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/
6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/
2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))+552/5/
(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(
1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-30+6*
13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))
^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))
```

3.153.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.46

$$\int (2+3x^2)\sqrt{3+5x^2+x^4}\,dx = -\frac{46\left(\sqrt{13}\sqrt{2}x-5\sqrt{2}x\right)\sqrt{\sqrt{13}-5}E(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{13}-5}}{2x}\right)\mid\frac{5}{6}\sqrt{13}+\frac{19}{6})-\left(51\sqrt{13}\sqrt{2}x-205\sqrt{2}x\right)\sqrt{\sqrt{}}}{60\,x}$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
-1/60*(46*(sqrt(13)*sqrt(2)*x - 5*sqrt(2)*x)*sqrt(sqrt(13) - 5)*elliptic_e
(arcsin(1/2*sqrt(2)*sqrt(sqrt(13) - 5)/x), 5/6*sqrt(13) + 19/6) - (51*sqrt
(13)*sqrt(2)*x - 205*sqrt(2)*x)*sqrt(sqrt(13) - 5)*elliptic_f(arcsin(1/2*s
qrt(2)*sqrt(sqrt(13) - 5)/x), 5/6*sqrt(13) + 19/6) - 4*(9*x^4 + 25*x^2 - 4
6)*sqrt(x^4 + 5*x^2 + 3))/x
```

3.153.6 Sympy [F]

$$\int (2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \int (3x^2+2)\sqrt{x^4+5x^2+3}\,dx$$

input
```
integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2),x)
```

output
```
Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3), x)
```

3.153.7 Maxima [F]

$$\int (2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \int \sqrt{x^4+5\,x^2+3}\,(3\,x^2+2)\,dx$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2), x)
```

3.153.8 Giac [F]

$$\int (2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \int \sqrt{x^4+5\,x^2+3}\,(3\,x^2+2)\,dx$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2), x)
```

3.153.9 Mupad [F(-1)]

Timed out.

$$\int (2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \int (3\,x^2+2)\,\sqrt{x^4+5\,x^2+3}\,dx$$

input
```
int((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2),x)
```

output
```
int((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2), x)
```

3.154 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^2}\,dx$

3.154.1 Optimal result

Integrand size = 25, antiderivative size = 284

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^2}\,dx = \frac{9x(5+\sqrt{13}+2x^2)}{2\sqrt{3+5x^2+x^4}} - \frac{(2-x^2)\sqrt{3+5x^2+x^4}}{x} - \frac{3\sqrt{\frac{3}{2}(5+\sqrt{13})}\sqrt{\frac{6+(5-\sqrt{13})x^2}{6+(5+\sqrt{13})x^2}}\left(6+(5+\sqrt{13})x^2\right)E\left(\arctan\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right)|\frac{1}{6}(-13+5\sqrt{13})\right)}{2\sqrt{3+5x^2+x^4}} + \frac{8\sqrt{\frac{2}{3(5+\sqrt{13})}}\sqrt{\frac{6+(5-\sqrt{13})x^2}{6+(5+\sqrt{13})x^2}}\left(6+(5+\sqrt{13})x^2\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right),\frac{1}{6}(-13+5\sqrt{13})\right)}{\sqrt{3+5x^2+x^4}}$$

output
```
9/2*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+3)^(1/2)-(-x^2+2)*(x^4+5*x^2+3)^(1/2)/
x+8/3*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*El
lipticF(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30
*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*6^(1/2)/(5+13^(1/2))^(1/2)*((6+x^2*
(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)-3/4*(1/(36+x
^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+
6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/
2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^
2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)
```

3.154.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.81 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.81

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^2}\,dx$$
$$= \frac{4(-6-7x^2+3x^4+x^6)+9i\sqrt{2}(-5+\sqrt{13})\,x\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}E\left(i\text{arcsinh}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)|\frac{19}{6}\right.}{4x\sqrt{3+}}$$

input
```
Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^2,x]
```

output
```
(4*(-6 - 7*x^2 + 3*x^4 + x^6) + (9*I)*Sqrt[2]*(-5 + Sqrt[13])*x*Sqrt[(-5 +
 Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I
*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqrt[2]*(-1
3 + 9*Sqrt[13])*x*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + S
qrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*
Sqrt[13])/6])/(4*x*Sqrt[3 + 5*x^2 + x^4])
```

3.154.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1594, 27, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x^2+2)\sqrt{x^4+5x^2+3}}{x^2}\,dx$$
$$\Big\downarrow \text{1594}$$
$$-\frac{1}{3}\int -\frac{3(9x^2+16)}{\sqrt{x^4+5x^2+3}}dx-\frac{\sqrt{x^4+5x^2+3}(2-x^2)}{x}$$
$$\Big\downarrow \text{27}$$
$$\int \frac{9x^2+16}{\sqrt{x^4+5x^2+3}}dx-\frac{(2-x^2)\sqrt{x^4+5x^2+3}}{x}$$
$$\Big\downarrow \text{1503}$$

$$16\int \frac{1}{\sqrt{x^4+5x^2+3}}dx+9\int \frac{x^2}{\sqrt{x^4+5x^2+3}}dx-\frac{\sqrt{x^4+5x^2+3}(2-x^2)}{x}$$

$$\Big\downarrow \text{1412}$$

$$9\int \frac{x^2}{\sqrt{x^4+5x^2+3}}dx+\frac{8\sqrt{\frac{2}{3(5+\sqrt{13})}}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}-\frac{\sqrt{x^4+5x^2+3}(2-x^2)}{x}$$

$$\Big\downarrow \text{1455}$$

$$\frac{8\sqrt{\frac{2}{3(5+\sqrt{13})}}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}+9\left(\frac{x(2x^2+\sqrt{13}+5)}{2\sqrt{x^4+5x^2+3}}-\frac{\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}(-13+\cdots\right)}{2\sqrt{x^4+5x^2+3}}\right.$$

$$\frac{\sqrt{x^4+5x^2+3}(2-x^2)}{x}$$

input
```
Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^2,x]
```

output
```
-(((2 - x^2)*Sqrt[3 + 5*x^2 + x^4])/x) + 9*((x*(5 + Sqrt[13] + 2*x^2))/(2*
Sqrt[3 + 5*x^2 + x^4]) - (Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*
x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[S
qrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(2*Sqrt[3 + 5*x^2 + x^4])
) + (8*Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 +
Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[1
3])/6]*x], (-13 + 5*Sqrt[13])/6])/Sqrt[3 + 5*x^2 + x^4]
```

3.154. $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^2}\,dx$

3.154.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1594
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((d*(m
+ 4*p + 3) + e*(m + 1)*x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Simp[2*(p/(f^2
*(m + 1)*(m + 4*p + 3)))   Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Si
mp[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*
x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && G
tQ[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
 || IntegerQ[m])
```

3.154. $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^2}\,dx$

3.154.4 Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.79

method	result
default	$x\sqrt{x^4+5x^2+3}+\frac{96\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}-\frac{324\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{}}{}$
risch	$\frac{x^6+3x^4-7x^2-6}{x\sqrt{x^4+5x^2+3}}+\frac{96\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}-\frac{324\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-(}}{}$
elliptic	$x\sqrt{x^4+5x^2+3}+\frac{96\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}-\frac{324\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{}}{}$

input
```
int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
x*(x^4+5*x^2+3)^(1/2)+96/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2
)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/
6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-324/(-30+6*13^(1/2))^
(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/
(x^4+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5
/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2
)+1/6*39^(1/2)))-2*(x^4+5*x^2+3)^(1/2)/x
```

3.154.5 Fricas [F]

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^2}\,dx=\int \frac{\sqrt{x^4+5\,x^2+3}(3\,x^2+2)}{x^2}\,dx$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
integral(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^2, x)
```

3.154.6 Sympy [F]

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^2}\,dx = \int \frac{(3x^2+2)\sqrt{x^4+5x^2+3}}{x^2}\,dx$$

input
```
integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**2,x)
```

output
```
Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**2, x)
```

3.154.7 Maxima [F]

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^2}\,dx = \int \frac{\sqrt{x^4+5\,x^2+3}(3\,x^2+2)}{x^2}\,dx$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^2, x)
```

3.154.8 Giac [F]

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^2}\,dx = \int \frac{\sqrt{x^4+5\,x^2+3}(3\,x^2+2)}{x^2}\,dx$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^2, x)
```

3.154.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^2}\,dx = \int \frac{(3\,x^2+2)\,\sqrt{x^4+5\,x^2+3}}{x^2}\,dx$$

input
```
int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^2,x)
```

output
```
int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^2, x)
```

3.155 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^4}\,dx$

3.155.1 Optimal result

Integrand size = 25, antiderivative size = 305

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^4}\,dx = \frac{32x\left(5+\sqrt{13}+2x^2\right)}{9\sqrt{3+5x^2+x^4}} - \frac{64\sqrt{3+5x^2+x^4}}{9x} - \frac{(2-9x^2)\sqrt{3+5x^2+x^4}}{3x^3} - \frac{16\sqrt{\frac{2}{3}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{9\sqrt{3+5x^2+x^4}} + \frac{49\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{3\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{3+5x^2+x^4}}$$

output
```
32/9*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+3)^(1/2)-64/9*(x^4+5*x^2+3)^(1/2)/x-1
/3*(-9*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^3-16/27*(1/(36+x^2*(30+6*13^(1/2))))^(
1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+
x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)
))*(30+6*13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)
/(x^4+5*x^2+3)^(1/2)+49/3*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6
*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)
))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*((6+x^2*(5-13^(
1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)/(30+6*13^(1/2))^(1/
2)
```

3.155.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.68 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.78

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^4}\,dx$$
$$= \frac{-2(18+141x^2+191x^4+37x^6)+32i\sqrt{2}(-5+\sqrt{13})\,x^3\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}E\left(i\text{arcsinh}\left(\sqrt{\frac{2}{5+}}\right.\right.}{18}$$

input
```
Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^4,x]
```

output
```
(-2*(18 + 141*x^2 + 191*x^4 + 37*x^6) + (32*I)*Sqrt[2]*(-5 + Sqrt[13])*x^3
*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*
EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*
Sqrt[2]*(-13 + 32*Sqrt[13])*x^3*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13
])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*
x], 19/6 + (5*Sqrt[13])/6])/(18*x^3*Sqrt[3 + 5*x^2 + x^4])
```

3.155.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1594, 25, 1604, 25, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x^2+2)\sqrt{x^4+5x^2+3}}{x^4}dx$$
$$\Big\downarrow \text{1594}$$
$$-\frac{1}{3}\int -\frac{49x^2+64}{x^2\sqrt{x^4+5x^2+3}}dx-\frac{\sqrt{x^4+5x^2+3}(2-9x^2)}{3x^3}$$
$$\Big\downarrow \text{25}$$
$$\frac{1}{3}\int \frac{49x^2+64}{x^2\sqrt{x^4+5x^2+3}}dx-\frac{(2-9x^2)\sqrt{x^4+5x^2+3}}{3x^3}$$
$$\Big\downarrow \text{1604}$$

$$\frac{1}{3}\left(-\frac{1}{3}\int -\frac{64x^2+147}{\sqrt{x^4+5x^2+3}}dx-\frac{64\sqrt{x^4+5x^2+3}}{3x}\right)-\frac{(2-9x^2)\sqrt{x^4+5x^2+3}}{3x^3}$$

$\downarrow$ 25

$$\frac{1}{3}\left(\frac{1}{3}\int \frac{64x^2+147}{\sqrt{x^4+5x^2+3}}dx-\frac{64\sqrt{x^4+5x^2+3}}{3x}\right)-\frac{(2-9x^2)\sqrt{x^4+5x^2+3}}{3x^3}$$

$\downarrow$ 1503

$$\frac{1}{3}\left(\frac{1}{3}\left(147\int \frac{1}{\sqrt{x^4+5x^2+3}}dx+64\int \frac{x^2}{\sqrt{x^4+5x^2+3}}dx\right)-\frac{64\sqrt{x^4+5x^2+3}}{3x}\right)-\frac{(2-9x^2)\sqrt{x^4+5x^2+3}}{3x^3}$$

$\downarrow$ 1412

$$\frac{1}{3}\left(\frac{1}{3}\left(64\int \frac{x^2}{\sqrt{x^4+5x^2+3}}dx+\frac{49\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{\right.}\right.}\right.\right.}{\sqrt{x^4+5x^2+3}}\right.\right.$$

$$\frac{(2-9x^2)\sqrt{x^4+5x^2+3}}{3x^3}$$

$\downarrow$ 1455

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{49\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}\right.\right.$$

$$\frac{(2-9x^2)\sqrt{x^4+5x^2+3}}{3x^3}$$

input
```
Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^4,x]
```

3.155. $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^4}\,dx$

output
```
-1/3*((2 - 9*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^3 + ((-64*Sqrt[3 + 5*x^2 + x^4]
)/(3*x) + (64*((x*(5 + Sqrt[13] + 2*x^2))/(2*Sqrt[3 + 5*x^2 + x^4]) - (Sqr
t[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)
]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-1
3 + 5*Sqrt[13])/6])/(2*Sqrt[3 + 5*x^2 + x^4])) + (49*Sqrt[3/(2*(5 + Sqrt[1
3]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sq
rt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13]
)/6])/Sqrt[3 + 5*x^2 + x^4])/3)/3
```

3.155.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

3.155. $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^4}\,dx$

rule 1594
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((d*(m
+ 4*p + 3) + e*(m + 1)*x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Simp[2*(p/(f^2
*(m + 1)*(m + 4*p + 3)))   Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Si
mp[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*
x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && G
tQ[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
 || IntegerQ[m])
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.155.4 Maple [A] (verified)

Time = 2.15 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.75

method	result
default	$-\frac{37\sqrt{x^4+5x^2+3}}{9x}+\frac{98\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}-\frac{256\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-(}{}$
risch	$-\frac{37x^6+191x^4+141x^2+18}{9x^3\sqrt{x^4+5x^2+3}}+\frac{98\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}-\frac{256\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)}{}$
elliptic	$-\frac{37\sqrt{x^4+5x^2+3}}{9x}+\frac{98\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}-\frac{256\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-(}{}$

input
```
int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-37/9*(x^4+5*x^2+3)^(1/2)/x+98/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2
))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*Ellipt
icF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-256/(-30+6*13^(
1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^
(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(
1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*
3^(1/2)+1/6*39^(1/2)))-2/3*(x^4+5*x^2+3)^(1/2)/x^3
```

3.155.5 Fricas [F]

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^4}\,dx = \int \frac{\sqrt{x^4+5\,x^2+3}(3\,x^2+2)}{x^4}\,dx$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
integral(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^4, x)
```

3.155.6 Sympy [F]

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^4}\,dx = \int \frac{(3x^2+2)\sqrt{x^4+5x^2+3}}{x^4}\,dx$$

input
```
integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**4,x)
```

output
```
Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**4, x)
```

3.155.7 Maxima [F]

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^4}\,dx = \int \frac{\sqrt{x^4+5\,x^2+3}(3\,x^2+2)}{x^4}\,dx$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^4, x)
```

3.155.8 Giac [F]

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^4}\,dx = \int \frac{\sqrt{x^4+5\,x^2+3}(3\,x^2+2)}{x^4}\,dx$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^4,x, algorithm="giac")
```

output
```
integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^4, x)
```

3.155.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^4}\,dx = \int \frac{(3\,x^2+2)\,\sqrt{x^4+5\,x^2+3}}{x^4}\,dx$$

input
```
int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^4,x)
```

output
```
int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^4, x)
```

3.156 $\int x^5(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx$

3.156.1 Optimal result

Integrand size = 25, antiderivative size = 127

$$\int x^5(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx=\frac{28379(5+2x^2)\sqrt{3+5x^2+x^4}}{2048}-\frac{2183}{768}(5+2x^2)\left(3+5x^2+x^4\right)^{3/2}+\frac{3}{14}x^4\left(3+5x^2+x^4\right)^{5/2}+\frac{(3313-1070x^2)\left(3+5x^2+x^4\right)^{5/2}}{1680}-\frac{368927\operatorname{arctanh}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)}{4096}$$

output
```
-2183/768*(2*x^2+5)*(x^4+5*x^2+3)^(3/2)+3/14*x^4*(x^4+5*x^2+3)^(5/2)+1/168
0*(-1070*x^2+3313)*(x^4+5*x^2+3)^(5/2)-368927/4096*arctanh(1/2*(2*x^2+5)/(
x^4+5*x^2+3)^(1/2))+28379/2048*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)
```

3.156.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.62

$$\int x^5(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx=\frac{\sqrt{3+5x^2+x^4}(9546951-1499570x^2+283304x^4+154800x^6+482944x^8+323840x^{10}+460}{215040}+\frac{368927\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)}{4096}$$

input
```
Integrate[x^5*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

output
```
(Sqrt[3 + 5*x^2 + x^4]*(9546951 - 1499570*x^2 + 283304*x^4 + 154800*x^6 +
482944*x^8 + 323840*x^10 + 46080*x^12))/215040 + (368927*Log[-5 - 2*x^2 +
2*Sqrt[3 + 5*x^2 + x^4]])/4096
```

3.156.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {1578, 1236, 27, 1225, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 (3x^2+2) (x^4+5x^2+3)^{3/2} dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int x^4 (3x^2+2) (x^4+5x^2+3)^{3/2} dx^2$$

$$\Big\downarrow \text{1236}$$

$$\frac{1}{2}\left(\frac{1}{7}\int -\frac{1}{2}x^2 (107x^2+36) (x^4+5x^2+3)^{3/2} dx^2+\frac{3}{7}(x^4+5x^2+3)^{5/2} x^4\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{3}{7}x^4 (x^4+5x^2+3)^{5/2}-\frac{1}{14}\int x^2 (107x^2+36) (x^4+5x^2+3)^{3/2} dx^2\right)$$

$$\Big\downarrow \text{1225}$$

$$\frac{1}{2}\left(\frac{1}{14}\left(\frac{1}{60}(3313-1070x^2) (x^4+5x^2+3)^{5/2}-\frac{15281}{24}\int (x^4+5x^2+3)^{3/2} dx^2\right)+\frac{3}{7}(x^4+5x^2+3)^{5/2} x^4\right)$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{2}\left(\frac{1}{14}\left(\frac{1}{60}(3313-1070x^2) (x^4+5x^2+3)^{5/2}-\frac{15281}{24}\left(\frac{1}{8}(2x^2+5) (x^4+5x^2+3)^{3/2}-\frac{39}{16}\int \sqrt{x^4+5x^2+3}dx^2\right.\right.\right.$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{2}\left(\frac{1}{14}\left(\frac{1}{60}(3313-1070x^2)\left(x^4+5x^2+3\right)^{5/2}-\frac{15281}{24}\left(\frac{1}{8}(2x^2+5)\left(x^4+5x^2+3\right)^{3/2}-\frac{39}{16}\left(\frac{1}{4}(2x^2+5)\sqrt{x^4+\right.\right.\right.\right.$$

$$\Big\downarrow \; 1092$$

$$\frac{1}{2}\left(\frac{1}{14}\left(\frac{1}{60}(3313-1070x^2)\left(x^4+5x^2+3\right)^{5/2}-\frac{15281}{24}\left(\frac{1}{8}(2x^2+5)\left(x^4+5x^2+3\right)^{3/2}-\frac{39}{16}\left(\frac{1}{4}(2x^2+5)\sqrt{x^4+\right.\right.\right.\right.$$

$$\Big\downarrow \; 219$$

$$\frac{1}{2}\left(\frac{1}{14}\left(\frac{1}{60}(3313-1070x^2)\left(x^4+5x^2+3\right)^{5/2}-\frac{15281}{24}\left(\frac{1}{8}(2x^2+5)\left(x^4+5x^2+3\right)^{3/2}-\frac{39}{16}\left(\frac{1}{4}(2x^2+5)\sqrt{x^4+\right.\right.\right.\right.$$

input
```
Int[x^5*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

output
```
((3*x^4*(3 + 5*x^2 + x^4)^(5/2))/7 + (((3313 - 1070*x^2)*(3 + 5*x^2 + x^4)
^(5/2))/60 - (15281*(((5 + 2*x^2)*(3 + 5*x^2 + x^4)^(3/2))/8 - (39*(((5 +
2*x^2)*Sqrt[3 + 5*x^2 + x^4])/4 - (13*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^
2 + x^4])])/8))/16))/24)/14)/2
```

3.156.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.156.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.54

method	result
risch	$\frac{\left(46080 x^{12}+323840 x^{10}+482944 x^{8}+154800 x^{6}+283304 x^{4}-1499570 x^{2}+9546951\right) \sqrt{x^{4}+5 x^{2}+3}}{215040}-\frac{368927 \ln \left(\frac{5}{2}+x^{2}+\sqrt{x^{4}+\cdots}\right.}{4096}$
trager	$\left(\frac{3}{14} x^{12}+\frac{253}{168} x^{10}+\frac{539}{240} x^{8}+\frac{645}{896} x^{6}+\frac{5059}{3840} x^{4}-\frac{149957}{21504} x^{2}+\frac{3182317}{71680}\right) \sqrt{x^{4}+5 x^{2}+3}-\frac{368927 \ln \left(2 \cdots\right.}{\cdots}$
pseudoelliptic	$-\frac{368927 \ln \left(2 x^{2}+5+2 \sqrt{x^{4}+5 x^{2}+3}\right)}{4096}+\frac{\left(46080 x^{12}+323840 x^{10}+482944 x^{8}+154800 x^{6}+283304 x^{4}-1499570 x^{2}+9546951\right) \cdots}{215040}$
default	$\frac{3 x^{12} \sqrt{x^{4}+5 x^{2}+3}}{14}+\frac{253 x^{10} \sqrt{x^{4}+5 x^{2}+3}}{168}+\frac{539 x^{8} \sqrt{x^{4}+5 x^{2}+3}}{240}+\frac{645 x^{6} \sqrt{x^{4}+5 x^{2}+3}}{896}+\frac{5059 x^{4} \sqrt{x^{4}+5 x^{2}+3}}{3840}-\frac{1499\cdots}{\cdots}$
elliptic	$\frac{3 x^{12} \sqrt{x^{4}+5 x^{2}+3}}{14}+\frac{253 x^{10} \sqrt{x^{4}+5 x^{2}+3}}{168}+\frac{539 x^{8} \sqrt{x^{4}+5 x^{2}+3}}{240}+\frac{645 x^{6} \sqrt{x^{4}+5 x^{2}+3}}{896}+\frac{5059 x^{4} \sqrt{x^{4}+5 x^{2}+3}}{3840}-\frac{1499\cdots}{\cdots}$

input
```
int(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/215040*(46080*x^12+323840*x^10+482944*x^8+154800*x^6+283304*x^4-1499570*
x^2+9546951)*(x^4+5*x^2+3)^(1/2)-368927/4096*ln(5/2+x^2+(x^4+5*x^2+3)^(1/2
))
```

3.156.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.56

$$\int x^5(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx = \frac{1}{215040}\left(46080\,x^{12}+323840\,x^{10}+482944\,x^8+154800\,x^6+283304\,x^4-1499570\,x^2+954695\right. + \frac{368927}{4096}\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)$$

input
```
integrate(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
1/215040*(46080*x^12 + 323840*x^10 + 482944*x^8 + 154800*x^6 + 283304*x^4
- 1499570*x^2 + 9546951)*sqrt(x^4 + 5*x^2 + 3) + 368927/4096*log(-2*x^2 +
2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

3.156.6 Sympy [A] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.72

$$\int x^5(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx = 3\sqrt{x^4+5x^2+3}\left(\frac{x^6}{4}+\frac{5x^4}{24}-\frac{89x^2}{96}+\frac{365}{64}\right) + \frac{19\sqrt{x^4+5x^2+3}\left(\frac{x^8}{5}+\frac{x^6}{8}-\frac{127x^4}{240}+\frac{527x^2}{192}-\frac{11143}{640}\right)}{2} + \frac{17\sqrt{x^4+5x^2+3}\left(\frac{x^{10}}{6}+\frac{x^8}{12}-\frac{11x^6}{32}+\frac{107x^4}{64}-\frac{2279x^2}{256}+\frac{29049}{512}\right)}{2} + \frac{3\sqrt{x^4+5x^2+3}\left(\frac{x^{12}}{7}+\frac{5x^{10}}{84}-\frac{29x^8}{120}+\frac{509x^6}{448}-\frac{3623x^4}{640}+\frac{108481x^2}{3584}-\frac{6918747}{35840}\right)}{2} - \frac{368927\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)}{4096}$$

input
```
integrate(x**5*(3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)
```

output
```
3*sqrt(x**4 + 5*x**2 + 3)*(x**6/4 + 5*x**4/24 - 89*x**2/96 + 365/64) + 19*
sqrt(x**4 + 5*x**2 + 3)*(x**8/5 + x**6/8 - 127*x**4/240 + 527*x**2/192 - 1
1143/640)/2 + 17*sqrt(x**4 + 5*x**2 + 3)*(x**10/6 + x**8/12 - 11*x**6/32 +
 107*x**4/64 - 2279*x**2/256 + 29049/512)/2 + 3*sqrt(x**4 + 5*x**2 + 3)*(x
**12/7 + 5*x**10/84 - 29*x**8/120 + 509*x**6/448 - 3623*x**4/640 + 108481*
x**2/3584 - 6918747/35840)/2 - 368927*log(2*x**2 + 2*sqrt(x**4 + 5*x**2 +
3) + 5)/4096
```

3.156.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.06

$$\int x^5(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx = \frac{3}{14}\left(x^4+5\,x^2+3\right)^{\frac{5}{2}}x^4 - \frac{107}{168}\left(x^4+5\,x^2+3\right)^{\frac{5}{2}}x^2 - \frac{2183}{384}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}x^2 + \frac{3313}{1680}\left(x^4+5\,x^2+3\right)^{\frac{5}{2}} + \frac{28379}{1024}\sqrt{x^4+5\,x^2+3}x^2 - \frac{10915}{768}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}} + \frac{141895}{2048}\sqrt{x^4+5\,x^2+3} - \frac{368927}{4096}\log\left(2\,x^2+2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

input
```
integrate(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
3/14*(x^4 + 5*x^2 + 3)^(5/2)*x^4 - 107/168*(x^4 + 5*x^2 + 3)^(5/2)*x^2 - 2
183/384*(x^4 + 5*x^2 + 3)^(3/2)*x^2 + 3313/1680*(x^4 + 5*x^2 + 3)^(5/2) +
28379/1024*sqrt(x^4 + 5*x^2 + 3)*x^2 - 10915/768*(x^4 + 5*x^2 + 3)^(3/2) +
 141895/2048*sqrt(x^4 + 5*x^2 + 3) - 368927/4096*log(2*x^2 + 2*sqrt(x^4 +
5*x^2 + 3) + 5)
```

3.156.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.63

$$\int x^5 (2+3x^2) (3+5x^2+x^4)^{3/2} \, dx = \frac{1}{71680} \sqrt{x^4+5x^2+3} (2 (4 (2 (8 (10 (12x^2+5)x^2-203)x^2+7635)x^2-76083)x^2+1627215 + \frac{17}{3072} \sqrt{x^4+5x^2+3} (2 (4 (2 (8 (2x^2+1)x^2-33)x^2+321)x^2-6837)x^2+87147) + \frac{19}{3840} \sqrt{x^4+5x^2+3} (2 (4 (6 (8x^2+5)x^2-127)x^2+2635)x^2-33429) + \frac{1}{64} \sqrt{x^4+5x^2+3} (2 (4 (6x^2+5)x^2-89)x^2+1095) + \frac{368927}{4096} \log \left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

input
```
integrate(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

output
```
1/71680*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(2*(8*(10*(12*x^2 + 5)*x^2 - 203)*x^2
+ 7635)*x^2 - 76083)*x^2 + 1627215)*x^2 - 20756241) + 17/3072*sqrt(x^4 + 5
*x^2 + 3)*(2*(4*(2*(8*(2*x^2 + 1)*x^2 - 33)*x^2 + 321)*x^2 - 6837)*x^2 + 8
7147) + 19/3840*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*(8*x^2 + 5)*x^2 - 127)*x^2
+ 2635)*x^2 - 33429) + 1/64*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*x^2 + 5)*x^2 -
89)*x^2 + 1095) + 368927/4096*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

3.156.9 Mupad [F(-1)]

Timed out.

$$\int x^5 (2+3x^2) (3+5x^2+x^4)^{3/2} \, dx = \int x^5 (3x^2+2) (x^4+5x^2+3)^{3/2} \, dx$$

input
```
int(x^5*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2),x)
```

output
```
int(x^5*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2), x)
```

3.157 $\int x^3(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx$

3.157.1 Optimal result

Integrand size = 25, antiderivative size = 106

$$\int x^3(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx = -\frac{4797(5+2x^2)\sqrt{3+5x^2+x^4}}{1024}+\frac{123}{128}(5+2x^2)\left(3+5x^2+x^4\right)^{3/2} -\frac{1}{40}(27-10x^2)\left(3+5x^2+x^4\right)^{5/2}+\frac{62361\text{arctanh}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)}{2048}$$

output
```
123/128*(2*x^2+5)*(x^4+5*x^2+3)^(3/2)-1/40*(-10*x^2+27)*(x^4+5*x^2+3)^(5/2
)+62361/2048*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))-4797/1024*(2*x^2+5
)*(x^4+5*x^2+3)^(1/2)
```

3.157.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.70

$$\int x^3(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx = \frac{\sqrt{3+5x^2+x^4}(-77229+12390x^2+5064x^4+14960x^6+9344x^8+1280x^{10})}{5120} -\frac{62361\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)}{2048}$$

input
```
Integrate[x^3*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

output
```
(Sqrt[3 + 5*x^2 + x^4]*(-77229 + 12390*x^2 + 5064*x^4 + 14960*x^6 + 9344*x
^8 + 1280*x^10))/5120 - (62361*Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/
2048
```

3.157.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1578, 1225, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (3x^2+2) (x^4+5x^2+3)^{3/2} dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int x^2 (3x^2+2) (x^4+5x^2+3)^{3/2} dx^2$$

$$\Big\downarrow \text{1225}$$

$$\frac{1}{2}\left(\frac{123}{8}\int (x^4+5x^2+3)^{3/2} dx^2-\frac{1}{20}(27-10x^2) (x^4+5x^2+3)^{5/2}\right)$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{2}\left(\frac{123}{8}\left(\frac{1}{8}(2x^2+5) (x^4+5x^2+3)^{3/2}-\frac{39}{16}\int \sqrt{x^4+5x^2+3}dx^2\right)-\frac{1}{20}(27-10x^2) (x^4+5x^2+3)^{5/2}\right)$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{2}\left(\frac{123}{8}\left(\frac{1}{8}(2x^2+5) (x^4+5x^2+3)^{3/2}-\frac{39}{16}\left(\frac{1}{4}(2x^2+5) \sqrt{x^4+5x^2+3}-\frac{13}{8}\int \frac{1}{\sqrt{x^4+5x^2+3}}dx^2\right)\right)-\frac{1}{20}(27$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}\left(\frac{123}{8}\left(\frac{1}{8}(2x^2+5) (x^4+5x^2+3)^{3/2}-\frac{39}{16}\left(\frac{1}{4}(2x^2+5) \sqrt{x^4+5x^2+3}-\frac{13}{4}\int \frac{1}{4-x^4}d\frac{2x^2+5}{\sqrt{x^4+5x^2+3}}\right)\right)-\frac{1}{2}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{123}{8}\left(\frac{1}{8}(2x^2+5)\left(x^4+5x^2+3\right)^{3/2}-\frac{39}{16}\left(\frac{1}{4}(2x^2+5)\sqrt{x^4+5x^2+3}-\frac{13}{8}\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)\right)\right)-$$

input
```
Int[x^3*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

output
```
(-1/20*((27 - 10*x^2)*(3 + 5*x^2 + x^4)^(5/2)) + (123*(((5 + 2*x^2)*(3 + 5
*x^2 + x^4)^(3/2))/8 - (39*(((5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/4 - (13*Ar
cTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/8))/16))/8)/2
```

3.157.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.157.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.59

method	result
risch	$\frac{(1280x^{10}+9344x^8+14960x^6+5064x^4+12390x^2-77229)\sqrt{x^4+5x^2+3}}{5120}+\frac{62361\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{2048}$
trager	$\left(\frac{1}{4}x^{10}+\frac{73}{40}x^8+\frac{187}{64}x^6+\frac{633}{640}x^4+\frac{1239}{512}x^2-\frac{77229}{5120}\right)\sqrt{x^4+5x^2+3}+\frac{62361\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)}{2048}$
pseudoelliptic	$\frac{62361\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)}{2048}+\frac{(1280x^{10}+9344x^8+14960x^6+5064x^4+12390x^2-77229)\sqrt{x^4+5x^2+3}}{5120}$
default	$\frac{x^{10}\sqrt{x^4+5x^2+3}}{4}+\frac{73x^8\sqrt{x^4+5x^2+3}}{40}+\frac{187x^6\sqrt{x^4+5x^2+3}}{64}+\frac{633x^4\sqrt{x^4+5x^2+3}}{640}+\frac{1239x^2\sqrt{x^4+5x^2+3}}{512}-\frac{77229\sqrt{}}{5}$
elliptic	$\frac{x^{10}\sqrt{x^4+5x^2+3}}{4}+\frac{73x^8\sqrt{x^4+5x^2+3}}{40}+\frac{187x^6\sqrt{x^4+5x^2+3}}{64}+\frac{633x^4\sqrt{x^4+5x^2+3}}{640}+\frac{1239x^2\sqrt{x^4+5x^2+3}}{512}-\frac{77229\sqrt{}}{5}$

input
```
int(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/5120*(1280*x^10+9344*x^8+14960*x^6+5064*x^4+12390*x^2-77229)*(x^4+5*x^2+
3)^(1/2)+62361/2048*ln(5/2+x^2+(x^4+5*x^2+3)^(1/2))
```

3.157.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.62

$$\int x^3(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx=\frac{1}{5120}\left(1280\,x^{10}+9344\,x^8+14960\,x^6+5064\,x^4+12390\,x^2-77229\right)\sqrt{x^4+5\,x^2+3}-\frac{62361}{2048}\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)$$

input
```
integrate(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
1/5120*(1280*x^10 + 9344*x^8 + 14960*x^6 + 5064*x^4 + 12390*x^2 - 77229)*s
qrt(x^4 + 5*x^2 + 3) - 62361/2048*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5
)
```

3.157.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(94) = 188$.

Time = 1.16 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.79

$$\int x^3(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx = 3\left(\frac{x^4}{3}+\frac{5x^2}{12}-\frac{17}{8}\right)\sqrt{x^4+5x^2+3} + \frac{19\sqrt{x^4+5x^2+3}\left(\frac{x^6}{4}+\frac{5x^4}{24}-\frac{89x^2}{96}+\frac{365}{64}\right)}{2} + \frac{17\sqrt{x^4+5x^2+3}\left(\frac{x^8}{5}+\frac{x^6}{8}-\frac{127x^4}{240}+\frac{527x^2}{192}-\frac{11143}{640}\right)}{2} + \frac{3\sqrt{x^4+5x^2+3}\left(\frac{x^{10}}{6}+\frac{x^8}{12}-\frac{11x^6}{32}+\frac{107x^4}{64}-\frac{2279x^2}{256}+\frac{29049}{512}\right)}{2} + \frac{62361\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)}{2048}$$

input
```
integrate(x**3*(3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)
```

output
```
3*(x**4/3 + 5*x**2/12 - 17/8)*sqrt(x**4 + 5*x**2 + 3) + 19*sqrt(x**4 + 5*x
**2 + 3)*(x**6/4 + 5*x**4/24 - 89*x**2/96 + 365/64)/2 + 17*sqrt(x**4 + 5*x
**2 + 3)*(x**8/5 + x**6/8 - 127*x**4/240 + 527*x**2/192 - 11143/640)/2 + 3
*sqrt(x**4 + 5*x**2 + 3)*(x**10/6 + x**8/12 - 11*x**6/32 + 107*x**4/64 - 2
279*x**2/256 + 29049/512)/2 + 62361*log(2*x**2 + 2*sqrt(x**4 + 5*x**2 + 3)
 + 5)/2048
```

3.157.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.11

$$\int x^3(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx = \frac{1}{4}\left(x^4+5\,x^2+3\right)^{\frac{5}{2}}x^2 + \frac{123}{64}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}x^2 - \frac{27}{40}\left(x^4+5\,x^2+3\right)^{\frac{5}{2}} - \frac{4797}{512}\sqrt{x^4+5\,x^2+3}x^2 + \frac{615}{128}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}} - \frac{23985}{1024}\sqrt{x^4+5\,x^2+3} + \frac{62361}{2048}\log\left(2\,x^2+2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

input
```
integrate(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(x^4 + 5*x^2 + 3)^(5/2)*x^2 + 123/64*(x^4 + 5*x^2 + 3)^(3/2)*x^2 - 27/
40*(x^4 + 5*x^2 + 3)^(5/2) - 4797/512*sqrt(x^4 + 5*x^2 + 3)*x^2 + 615/128*
(x^4 + 5*x^2 + 3)^(3/2) - 23985/1024*sqrt(x^4 + 5*x^2 + 3) + 62361/2048*lo
g(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

3.157.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(88) = 176$.

Time = 0.32 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.69

$$\int x^3(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx = \frac{1}{1024}\sqrt{x^4+5x^2+3}\left(2\left(4\left(2\left(8\left(2x^2+1\right)x^2-33\right)x^2+321\right)x^2-6837\right)x^2+87147\right) + \frac{17}{3840}\sqrt{x^4+5x^2+3}\left(2\left(4\left(6\left(8x^2+5\right)x^2-127\right)x^2+2635\right)x^2-33429\right) + \frac{19}{384}\sqrt{x^4+5x^2+3}\left(2\left(4\left(6x^2+5\right)x^2-89\right)x^2+1095\right) + \frac{1}{8}\sqrt{x^4+5x^2+3}\left(2\left(4x^2+5\right)x^2-51\right) - \frac{62361}{2048}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

input
```
integrate(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

output
```
1/1024*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(2*(8*(2*x^2 + 1)*x^2 - 33)*x^2 + 321)*
x^2 - 6837)*x^2 + 87147) + 17/3840*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*(8*x^2 +
 5)*x^2 - 127)*x^2 + 2635)*x^2 - 33429) + 19/384*sqrt(x^4 + 5*x^2 + 3)*(2*
(4*(6*x^2 + 5)*x^2 - 89)*x^2 + 1095) + 1/8*sqrt(x^4 + 5*x^2 + 3)*(2*(4*x^2
 + 5)*x^2 - 51) - 62361/2048*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

3.157.9 Mupad [F(-1)]

Timed out.

$$\int x^3(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx = \int x^3\left(3x^2+2\right)\left(x^4+5x^2+3\right)^{3/2}\,dx$$

input
```
int(x^3*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2),x)
```

output
```
int(x^3*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2), x)
```

3.158 $\int x(2+3x^2)(3+5x^2+x^4)^{3/2}\,dx$

3.158.1 Optimal result

Integrand size = 23, antiderivative size = 99

$$\int x(2+3x^2)(3+5x^2+x^4)^{3/2}\,dx = \frac{429}{256}(5+2x^2)\sqrt{3+5x^2+x^4} - \frac{11}{32}(5+2x^2)(3+5x^2+x^4)^{3/2} + \frac{3}{10}(3+5x^2+x^4)^{5/2} - \frac{5577}{512}\operatorname{arctanh}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)$$

output
```
-11/32*(2*x^2+5)*(x^4+5*x^2+3)^(3/2)+3/10*(x^4+5*x^2+3)^(5/2)-5577/512*arc
tanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))+429/256*(2*x^2+5)*(x^4+5*x^2+3)^(1
/2)
```

3.158.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.70

$$\int x(2+3x^2)(3+5x^2+x^4)^{3/2}\,dx = \frac{\sqrt{3+5x^2+x^4}(7581+2170x^2+5304x^4+2960x^6+384x^8)}{1280} + \frac{5577}{512}\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)$$

input
```
Integrate[x*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

output
```
(Sqrt[3 + 5*x^2 + x^4]*(7581 + 2170*x^2 + 5304*x^4 + 2960*x^6 + 384*x^8))/
1280 + (5577*Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/512
```

3.158.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {1576, 1160, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(3x^2+2\right)\left(x^4+5x^2+3\right)^{3/2}dx$$

$$\Big\downarrow \text{1576}$$

$$\frac{1}{2}\int \left(3x^2+2\right)\left(x^4+5x^2+3\right)^{3/2}dx^2$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{2}\left(\frac{3}{5}\left(x^4+5x^2+3\right)^{5/2}-\frac{11}{2}\int \left(x^4+5x^2+3\right)^{3/2}dx^2\right)$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{2}\left(\frac{3}{5}\left(x^4+5x^2+3\right)^{5/2}-\frac{11}{2}\left(\frac{1}{8}\left(2x^2+5\right)\left(x^4+5x^2+3\right)^{3/2}-\frac{39}{16}\int \sqrt{x^4+5x^2+3}dx^2\right)\right)$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{2}\left(\frac{3}{5}\left(x^4+5x^2+3\right)^{5/2}-\frac{11}{2}\left(\frac{1}{8}\left(2x^2+5\right)\left(x^4+5x^2+3\right)^{3/2}-\frac{39}{16}\left(\frac{1}{4}\left(2x^2+5\right)\sqrt{x^4+5x^2+3}-\frac{13}{8}\int \frac{1}{\sqrt{x^4+5x\cdots}}\right.\right.\right.$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}\left(\frac{3}{5}\left(x^4+5x^2+3\right)^{5/2}-\frac{11}{2}\left(\frac{1}{8}\left(2x^2+5\right)\left(x^4+5x^2+3\right)^{3/2}-\frac{39}{16}\left(\frac{1}{4}\left(2x^2+5\right)\sqrt{x^4+5x^2+3}-\frac{13}{4}\int \frac{1}{4-x^4}d\frac{\cdots}{\sqrt{\cdots}}\right.\right.\right.$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{3}{5}\left(x^4+5x^2+3\right)^{5/2}-\frac{11}{2}\left(\frac{1}{8}\left(2x^2+5\right)\left(x^4+5x^2+3\right)^{3/2}-\frac{39}{16}\left(\frac{1}{4}\left(2x^2+5\right)\sqrt{x^4+5x^2+3}-\frac{13}{8}\text{arctanh}\left(\frac{\cdots}{2\sqrt{\cdots}}\right.\right.\right.\right.$$

input
```
Int[x*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

output
```
((3*(3 + 5*x^2 + x^4)^(5/2))/5 - (11*(((5 + 2*x^2)*(3 + 5*x^2 + x^4)^(3/2)
)/8 - (39*(((5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/4 - (13*ArcTanh[(5 + 2*x^2)
/(2*Sqrt[3 + 5*x^2 + x^4])])/8))/16))/2)/2
```

3.158.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

3.158.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.59

method	result
risch	$\frac{(384x^8+2960x^6+5304x^4+2170x^2+7581)\sqrt{x^4+5x^2+3}}{1280}-\frac{5577\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{512}$
trager	$\left(\frac{3}{10}x^8+\frac{37}{16}x^6+\frac{663}{160}x^4+\frac{217}{128}x^2+\frac{7581}{1280}\right)\sqrt{x^4+5x^2+3}-\frac{5577\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)}{512}$
pseudoelliptic	$-\frac{5577\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)}{512}+\frac{(384x^8+2960x^6+5304x^4+2170x^2+7581)\sqrt{x^4+5x^2+3}}{1280}$
default	$-\frac{5577\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{512}+\frac{37x^6\sqrt{x^4+5x^2+3}}{16}+\frac{663x^4\sqrt{x^4+5x^2+3}}{160}+\frac{217x^2\sqrt{x^4+5x^2+3}}{128}+\frac{7581\sqrt{x^4+5x^2+}}{1280}$
elliptic	$-\frac{5577\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{512}+\frac{37x^6\sqrt{x^4+5x^2+3}}{16}+\frac{663x^4\sqrt{x^4+5x^2+3}}{160}+\frac{217x^2\sqrt{x^4+5x^2+3}}{128}+\frac{7581\sqrt{x^4+5x^2+}}{1280}$

input
```
int(x*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/1280*(384*x^8+2960*x^6+5304*x^4+2170*x^2+7581)*(x^4+5*x^2+3)^(1/2)-5577/
512*ln(5/2+x^2+(x^4+5*x^2+3)^(1/2))
```

3.158.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.62

$$\int x(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}dx=\frac{1}{1280}\left(384\,x^8+2960\,x^6+5304\,x^4+2170\,x^2+7581\right)\sqrt{x^4+5\,x^2+3}+\frac{5577}{512}\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)$$

input
```
integrate(x*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
1/1280*(384*x^8 + 2960*x^6 + 5304*x^4 + 2170*x^2 + 7581)*sqrt(x^4 + 5*x^2
+ 3) + 5577/512*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

3.158.6 Sympy [A] (verification not implemented)

Time = 1.10 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.67

$$\int x(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx = 3\left(\frac{x^2}{2}+\frac{5}{4}\right)\sqrt{x^4+5x^2+3} + \frac{19\left(\frac{x^4}{3}+\frac{5x^2}{12}-\frac{17}{8}\right)\sqrt{x^4+5x^2+3}}{2} + \frac{17\sqrt{x^4+5x^2+3}\left(\frac{x^6}{4}+\frac{5x^4}{24}-\frac{89x^2}{96}+\frac{365}{64}\right)}{2} + \frac{3\sqrt{x^4+5x^2+3}\left(\frac{x^8}{5}+\frac{x^6}{8}-\frac{127x^4}{240}+\frac{527x^2}{192}-\frac{11143}{640}\right)}{2} - \frac{5577\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)}{512}$$

input
```
integrate(x*(3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)
```

output
```
3*(x**2/2 + 5/4)*sqrt(x**4 + 5*x**2 + 3) + 19*(x**4/3 + 5*x**2/12 - 17/8)*
sqrt(x**4 + 5*x**2 + 3)/2 + 17*sqrt(x**4 + 5*x**2 + 3)*(x**6/4 + 5*x**4/24
 - 89*x**2/96 + 365/64)/2 + 3*sqrt(x**4 + 5*x**2 + 3)*(x**8/5 + x**6/8 - 1
27*x**4/240 + 527*x**2/192 - 11143/640)/2 - 5577*log(2*x**2 + 2*sqrt(x**4
+ 5*x**2 + 3) + 5)/512
```

3.158.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.02

$$\int x(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx = -\frac{11}{16}\left(x^4+5x^2+3\right)^{\frac{3}{2}}x^2 + \frac{3}{10}\left(x^4+5x^2+3\right)^{\frac{5}{2}} + \frac{429}{128}\sqrt{x^4+5x^2+3}\,x^2 - \frac{55}{32}\left(x^4+5x^2+3\right)^{\frac{3}{2}} + \frac{2145}{256}\sqrt{x^4+5x^2+3} - \frac{5577}{512}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

input
```
integrate(x*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
-11/16*(x^4 + 5*x^2 + 3)^(3/2)*x^2 + 3/10*(x^4 + 5*x^2 + 3)^(5/2) + 429/12
8*sqrt(x^4 + 5*x^2 + 3)*x^2 - 55/32*(x^4 + 5*x^2 + 3)^(3/2) + 2145/256*sqr
t(x^4 + 5*x^2 + 3) - 5577/512*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

3.158.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.53

$$\int x(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx = \frac{1}{1280}\sqrt{x^4+5\,x^2+3}\left(2\left(4\left(6\left(8\,x^2+5\right)x^2-127\right)x^2+2635\right)x^2-33429\right) + \frac{17}{384}\sqrt{x^4+5\,x^2+3}\left(2\left(4\left(6\,x^2+5\right)x^2-89\right)x^2+1095\right) + \frac{19}{48}\sqrt{x^4+5\,x^2+3}\left(2\left(4\,x^2+5\right)x^2-51\right) + \frac{3}{4}\sqrt{x^4+5\,x^2+3}\left(2\,x^2+5\right) + \frac{5577}{512}\log\left(2\,x^2-2\sqrt{x^4+5\,x^2+3}+5\right)$$

input
```
integrate(x*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

output
```
1/1280*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*(8*x^2 + 5)*x^2 - 127)*x^2 + 2635)*x
^2 - 33429) + 17/384*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*x^2 + 5)*x^2 - 89)*x^2
 + 1095) + 19/48*sqrt(x^4 + 5*x^2 + 3)*(2*(4*x^2 + 5)*x^2 - 51) + 3/4*sqrt
(x^4 + 5*x^2 + 3)*(2*x^2 + 5) + 5577/512*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 +
3) + 5)
```

3.158.9 Mupad [B] (verification not implemented)

Time = 7.79 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.28

$$\int x(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx = \frac{\left(x^2+\frac{5}{2}\right)\left(x^4+5\,x^2+3\right)^{3/2}}{4} - \frac{15\,x^2\left(x^4+5\,x^2+3\right)^{3/2}}{16} - \frac{5577\,\ln\left(\sqrt{x^4+5\,x^2+3}+x^2+\frac{5}{2}\right)}{512} + \frac{585\left(2\,x^2+5\right)\sqrt{x^4+5\,x^2+3}}{256} - \frac{39\left(\frac{x^2}{2}+\frac{5}{4}\right)\sqrt{x^4+5\,x^2+3}}{16} - \frac{75\left(x^4+5\,x^2+3\right)^{3/2}}{32} + \frac{3\left(x^4+5\,x^2+3\right)^{5/2}}{10}$$

input
```
int(x*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2),x)
```

output
```
((x^2 + 5/2)*(5*x^2 + x^4 + 3)^(3/2))/4 - (15*x^2*(5*x^2 + x^4 + 3)^(3/2))
/16 - (5577*log((5*x^2 + x^4 + 3)^(1/2) + x^2 + 5/2))/512 + (585*(2*x^2 +
5)*(5*x^2 + x^4 + 3)^(1/2))/256 - (39*(x^2/2 + 5/4)*(5*x^2 + x^4 + 3)^(1/2
))/16 - (75*(5*x^2 + x^4 + 3)^(3/2))/32 + (3*(5*x^2 + x^4 + 3)^(5/2))/10
```

3.158. $\int x(2+3x^2)\left(3+5x^2+x^4\right)^{3/2} dx$

3.159 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x}\,dx$

3.159.1 Optimal result

Integrand size = 25, antiderivative size = 119

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x}\,dx = \frac{1}{128}(199-74x^2)\sqrt{3+5x^2+x^4} + \frac{1}{48}(61+18x^2)(3+5x^2+x^4)^{3/2} + \frac{2401}{256}\operatorname{arctanh}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right) - 3\sqrt{3}\operatorname{arctanh}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)$$

output
```
1/48*(18*x^2+61)*(x^4+5*x^2+3)^(3/2)+2401/256*arctanh(1/2*(2*x^2+5)/(x^4+5
*x^2+3)^(1/2))-3*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2
)+1/128*(-74*x^2+199)*(x^4+5*x^2+3)^(1/2)
```

3.159.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.83

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x}\,dx = 6\sqrt{3}\operatorname{arctanh}\left(\frac{x^2-\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right) + \frac{1}{768}\left(2\sqrt{3+5x^2+x^4}(2061+2650x^2+1208x^4+144x^6) - 7203\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)\right)$$

input
```
Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x,x]
```

output
```
6*Sqrt[3]*ArcTanh[(x^2 - Sqrt[3 + 5*x^2 + x^4])/Sqrt[3]] + (2*Sqrt[3 + 5*x
^2 + x^4]*(2061 + 2650*x^2 + 1208*x^4 + 144*x^6) - 7203*Log[-5 - 2*x^2 + 2
*Sqrt[3 + 5*x^2 + x^4]])/768
```

3.159.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1578, 1231, 27, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x^2+2)\,(x^4+5x^2+3)^{3/2}}{x}dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{(3x^2+2)\,(x^4+5x^2+3)^{3/2}}{x^2}dx^2$$

$$\Big\downarrow \text{1231}$$

$$\frac{1}{2}\left(\frac{1}{24}(18x^2+61)\,(x^4+5x^2+3)^{3/2}-\frac{1}{8}\int -\frac{(96-37x^2)\sqrt{x^4+5x^2+3}}{2x^2}dx^2\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{1}{16}\int \frac{(96-37x^2)\sqrt{x^4+5x^2+3}}{x^2}dx^2+\frac{1}{24}(18x^2+61)\,(x^4+5x^2+3)^{3/2}\right)$$

$$\Big\downarrow \text{1231}$$

$$\frac{1}{2}\left(\frac{1}{16}\left(\frac{1}{4}(199-74x^2)\sqrt{x^4+5x^2+3}-\frac{1}{4}\int -\frac{2401x^2+2304}{2x^2\sqrt{x^4+5x^2+3}}dx^2\right)+\frac{1}{24}(18x^2+61)\,(x^4+5x^2+3)^{3/2}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{1}{16}\left(\frac{1}{8}\int \frac{2401x^2+2304}{x^2\sqrt{x^4+5x^2+3}}dx^2+\frac{1}{4}\sqrt{x^4+5x^2+3}(199-74x^2)\right)+\frac{1}{24}(18x^2+61)\,(x^4+5x^2+3)^{3/2}\right)$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{2}\left(\frac{1}{16}\left(\frac{1}{8}\left(2401\int\frac{1}{\sqrt{x^4+5x^2+3}}dx^2+2304\int\frac{1}{x^2\sqrt{x^4+5x^2+3}}dx^2\right)+\frac{1}{4}\sqrt{x^4+5x^2+3}(199-74x^2)\right)+\frac{1}{24}(\right.$$

$\big\downarrow$ 1092

$$\frac{1}{2}\left(\frac{1}{16}\left(\frac{1}{8}\left(4802\int\frac{1}{4-x^4}d\frac{2x^2+5}{\sqrt{x^4+5x^2+3}}+2304\int\frac{1}{x^2\sqrt{x^4+5x^2+3}}dx^2\right)+\frac{1}{4}\sqrt{x^4+5x^2+3}(199-74x^2)\right)+\right.$$

$\big\downarrow$ 219

$$\frac{1}{2}\left(\frac{1}{16}\left(\frac{1}{8}\left(2304\int\frac{1}{x^2\sqrt{x^4+5x^2+3}}dx^2+2401\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)\right)+\frac{1}{4}\sqrt{x^4+5x^2+3}(199-74x^2)\right)\right.$$

$\big\downarrow$ 1154

$$\frac{1}{2}\left(\frac{1}{16}\left(\frac{1}{8}\left(2401\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-4608\int\frac{1}{12-x^4}d\frac{5x^2+6}{\sqrt{x^4+5x^2+3}}\right)+\frac{1}{4}\sqrt{x^4+5x^2+3}(199-74x^2\right.\right.$$

$\big\downarrow$ 219

$$\frac{1}{2}\left(\frac{1}{16}\left(\frac{1}{8}\left(2401\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-768\sqrt{3}\text{arctanh}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)\right)+\frac{1}{4}\sqrt{x^4+5x^2+3}(199-\right.\right.$$

input
```
Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x,x]
```

output
```
(((61 + 18*x^2)*(3 + 5*x^2 + x^4)^(3/2))/24 + (((199 - 74*x^2)*Sqrt[3 + 5*
x^2 + x^4])/4 + (2401*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])] - 768
*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/8)/16)/2
```

3.159.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.159. $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x} \, dx$

3.159.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.73

method	result
pseudoelliptic	$-3\,\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}+\frac{2401\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)}{256}+\frac{(288x^6+2416x^4+5300x^2+4122)\sqrt{x^4+5x}}{768}$
trager	$\left(\frac{3}{8}x^6+\frac{151}{48}x^4+\frac{1325}{192}x^2+\frac{687}{128}\right)\sqrt{x^4+5x^2+3}+3\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{-5\,\mathrm{RootOf}\left(\_Z^2-3\right.}{}\right.\right.$
default	$\frac{2401\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{256}+\frac{3x^6\sqrt{x^4+5x^2+3}}{8}+\frac{151x^4\sqrt{x^4+5x^2+3}}{48}+\frac{1325x^2\sqrt{x^4+5x^2+3}}{192}+\frac{687\sqrt{x^4+5x^2+3}}{128}$
elliptic	$\frac{2401\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{256}+\frac{3x^6\sqrt{x^4+5x^2+3}}{8}+\frac{151x^4\sqrt{x^4+5x^2+3}}{48}+\frac{1325x^2\sqrt{x^4+5x^2+3}}{192}+\frac{687\sqrt{x^4+5x^2+3}}{128}$

input
```
int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-3*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)+2401/256*ln(
2*x^2+5+2*(x^4+5*x^2+3)^(1/2))+1/768*(288*x^6+2416*x^4+5300*x^2+4122)*(x^4
+5*x^2+3)^(1/2)
```

3.159.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.89

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x}\,dx=\frac{1}{384}\left(144\,x^6+1208\,x^4+2650\,x^2+2061\right)\sqrt{x^4+5\,x^2+3}$$
$$+3\sqrt{3}\log\left(\frac{25\,x^2-2\sqrt{3}(5\,x^2+6)-2\sqrt{x^4+5\,x^2+3}(5\sqrt{3}-6)+30}{x^2}\right)$$
$$-\frac{2401}{256}\log\left(-2\,x^2+2\sqrt{x^4+5\,x^2+3}-5\right)$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x,x, algorithm="fricas")
```

output
```
1/384*(144*x^6 + 1208*x^4 + 2650*x^2 + 2061)*sqrt(x^4 + 5*x^2 + 3) + 3*sqr
t(3)*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*x^2 + 3)*(5*sqrt
(3) - 6) + 30)/x^2) - 2401/256*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

3.159.6 Sympy [F]

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x}\,dx = \int \frac{(3x^2+2)(x^4+5x^2+3)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x,x)
```

output
```
Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x, x)
```

3.159.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.01

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x}\,dx = \frac{3}{8}\,(x^4+5\,x^2+3)^{\frac{3}{2}}x^2 - \frac{37}{64}\,\sqrt{x^4+5\,x^2+3}x^2 + \frac{61}{48}\,(x^4+5\,x^2+3)^{\frac{3}{2}} - 3\,\sqrt{3}\log\left(\frac{2\,\sqrt{3}\sqrt{x^4+5\,x^2+3}}{x^2}+\frac{6}{x^2}+5\right) + \frac{199}{128}\,\sqrt{x^4+5\,x^2+3} + \frac{2401}{256}\,\log\left(2\,x^2+2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x,x, algorithm="maxima")
```

output
```
3/8*(x^4 + 5*x^2 + 3)^(3/2)*x^2 - 37/64*sqrt(x^4 + 5*x^2 + 3)*x^2 + 61/48*
(x^4 + 5*x^2 + 3)^(3/2) - 3*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^
2 + 6/x^2 + 5) + 199/128*sqrt(x^4 + 5*x^2 + 3) + 2401/256*log(2*x^2 + 2*sq
rt(x^4 + 5*x^2 + 3) + 5)
```

3.159.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.95

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x}\,dx = \frac{1}{384}\,\sqrt{x^4+5\,x^2+3}\left(2\,\left(4\,\left(18\,x^2+151\right)x^2+1325\right)x^2+2061\right) + 3\,\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5\,x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5\,x^2+3}}\right) - \frac{2401}{256}\,\log\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x,x, algorithm="giac")
```

output
```
1/384*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(18*x^2 + 151)*x^2 + 1325)*x^2 + 2061) +
 3*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - sq
rt(x^4 + 5*x^2 + 3))) - 2401/256*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

3.159.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x}\,dx = \int \frac{(3\,x^2+2)\,(x^4+5\,x^2+3)^{3/2}}{x}\,dx$$

input
```
int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x,x)
```

output
```
int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x, x)
```

3.160 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^3} dx$

3.160.1 Optimal result

Integrand size = 25, antiderivative size = 122

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^3} dx = \frac{3}{16}(109+18x^2)\sqrt{3+5x^2+x^4} - \frac{(2-x^2)(3+5x^2+x^4)^{3/2}}{2x^2} + \frac{609}{32}\text{arctanh}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right) - 12\sqrt{3}\text{arctanh}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)$$

output
```
-1/2*(-x^2+2)*(x^4+5*x^2+3)^(3/2)/x^2+609/32*arctanh(1/2*(2*x^2+5)/(x^4+5*
x^2+3)^(1/2))-12*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2
)+3/16*(18*x^2+109)*(x^4+5*x^2+3)^(1/2)
```

3.160.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.84

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^3} dx = 24\sqrt{3}\text{arctanh}\left(\frac{x^2-\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right) + \frac{1}{32}\left(\frac{2\sqrt{3+5x^2+x^4}(-48+271x^2+78x^4+8x^6)}{x^2} - 609\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)\right)$$

input
```
Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^3,x]
```

output
```
24*Sqrt[3]*ArcTanh[(x^2 - Sqrt[3 + 5*x^2 + x^4])/Sqrt[3]] + ((2*Sqrt[3 + 5
*x^2 + x^4]*(-48 + 271*x^2 + 78*x^4 + 8*x^6))/x^2 - 609*Log[-5 - 2*x^2 + 2
*Sqrt[3 + 5*x^2 + x^4]])/32
```

3.160.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1578, 1230, 27, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x^2+2)(x^4+5x^2+3)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{(3x^2+2)(x^4+5x^2+3)^{3/2}}{x^4}dx^2$$

$$\Big\downarrow \text{1230}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int -\frac{3(9x^2+16)\sqrt{x^4+5x^2+3}}{x^2}dx^2-\frac{(2-x^2)(x^4+5x^2+3)^{3/2}}{x^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{3}{2}\int \frac{(9x^2+16)\sqrt{x^4+5x^2+3}}{x^2}dx^2-\frac{(2-x^2)(x^4+5x^2+3)^{3/2}}{x^2}\right)$$

$$\Big\downarrow \text{1231}$$

$$\frac{1}{2}\left(\frac{3}{2}\left(\frac{1}{4}(18x^2+109)\sqrt{x^4+5x^2+3}-\frac{1}{4}\int -\frac{203x^2+384}{2x^2\sqrt{x^4+5x^2+3}}dx^2\right)-\frac{(2-x^2)(x^4+5x^2+3)^{3/2}}{x^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{3}{2}\left(\frac{1}{8}\int \frac{203x^2+384}{x^2\sqrt{x^4+5x^2+3}}dx^2+\frac{1}{4}\sqrt{x^4+5x^2+3}(18x^2+109)\right)-\frac{(2-x^2)(x^4+5x^2+3)^{3/2}}{x^2}\right)$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{2}\left(\frac{3}{2}\left(\frac{1}{8}\left(203\int \frac{1}{\sqrt{x^4+5x^2+3}}dx^2+384\int \frac{1}{x^2\sqrt{x^4+5x^2+3}}dx^2\right)+\frac{1}{4}\sqrt{x^4+5x^2+3}(18x^2+109)\right)-\frac{(2-x^2}{}\right.$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}\left(\frac{3}{2}\left(\frac{1}{8}\left(406\int \frac{1}{4-x^4}d\frac{2x^2+5}{\sqrt{x^4+5x^2+3}}+384\int \frac{1}{x^2\sqrt{x^4+5x^2+3}}dx^2\right)+\frac{1}{4}\sqrt{x^4+5x^2+3}(18x^2+109)\right)-\frac{(2}{}\right.$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{3}{2}\left(\frac{1}{8}\left(384\int \frac{1}{x^2\sqrt{x^4+5x^2+3}}dx^2+203\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)\right)+\frac{1}{4}\sqrt{x^4+5x^2+3}(18x^2+109)\right)-\frac{(2}{}\right.$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{2}\left(\frac{3}{2}\left(\frac{1}{8}\left(203\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-768\int \frac{1}{12-x^4}d\frac{5x^2+6}{\sqrt{x^4+5x^2+3}}\right)+\frac{1}{4}\sqrt{x^4+5x^2+3}(18x^2+109)\right)-\right.$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{3}{2}\left(\frac{1}{8}\left(203\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-128\sqrt{3}\text{arctanh}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)\right)+\frac{1}{4}\sqrt{x^4+5x^2+3}(18x^2+1\right.\right.$$

input
```
Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^3,x]
```

output
```
(-(((2 - x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^2) + (3*(((109 + 18*x^2)*Sqrt[3 +
 5*x^2 + x^4])/4 + (203*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])] - 1
28*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/8))/2)/
2
```

3.160. $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^3}\,dx$

3.160.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

3.160. $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^3} \, dx$

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.160.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$\frac{-384\,\operatorname{arctanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}\,x^2+609\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)x^2+16\sqrt{x^4+5x^2+3}\,\left(x^6+\frac{39}{4}x^4+\frac{271}{8}x^2-6\right)}{32x^2}$
trager	$\frac{\left(8x^6+78x^4+271x^2-48\right)\sqrt{x^4+5x^2+3}}{16x^2}-12\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{5\operatorname{RootOf}\left(\_Z^2-3\right)x^2+6\sqrt{x^4+5x^2+3}+6}{x^2}\right.$
default	$\frac{609\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{32}+\frac{x^4\sqrt{x^4+5x^2+3}}{2}+\frac{39x^2\sqrt{x^4+5x^2+3}}{8}+\frac{271\sqrt{x^4+5x^2+3}}{16}-12\operatorname{arctanh}\left(\frac{(5x^2+}{6\sqrt{x^4+}}\right.$
risch	$\frac{609\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{32}+\frac{x^4\sqrt{x^4+5x^2+3}}{2}+\frac{39x^2\sqrt{x^4+5x^2+3}}{8}+\frac{271\sqrt{x^4+5x^2+3}}{16}-12\operatorname{arctanh}\left(\frac{(5x^2+}{6\sqrt{x^4+}}\right.$
elliptic	$\frac{609\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{32}+\frac{x^4\sqrt{x^4+5x^2+3}}{2}+\frac{39x^2\sqrt{x^4+5x^2+3}}{8}+\frac{271\sqrt{x^4+5x^2+3}}{16}-12\operatorname{arctanh}\left(\frac{(5x^2+}{6\sqrt{x^4+}}\right.$

input
```
int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/32*(-384*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)*x^2+
609*ln(2*x^2+5+2*(x^4+5*x^2+3)^(1/2))*x^2+16*(x^4+5*x^2+3)^(1/2)*(x^6+39/4
*x^4+271/8*x^2-6))/x^2
```

3.160.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^3}\,dx = \frac{1536\sqrt{3}x^2\log\left(\frac{25\,x^2-2\,\sqrt{3}(5\,x^2+6)-2\,\sqrt{x^4+5\,x^2+3}\left(5\,\sqrt{3}-6\right)+30}{x^2}\right)-2436\,x^2\log\left(\right.}{}$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
1/128*(1536*sqrt(3)*x^2*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 +
 5*x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 2436*x^2*log(-2*x^2 + 2*sqrt(x^4
+ 5*x^2 + 3) - 5) + 1541*x^2 + 8*(8*x^6 + 78*x^4 + 271*x^2 - 48)*sqrt(x^4
+ 5*x^2 + 3))/x^2
```

3.160.6 Sympy [F]

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^3}\,dx = \int \frac{(3x^2+2)(x^4+5x^2+3)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**3,x)
```

output
```
Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x**3, x)
```

3.160.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.98

$$\int \frac{(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}}{x^3}\,dx = \frac{27}{8}\sqrt{x^4+5\,x^2+3}x^2 + \frac{1}{2}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}} - 12\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5\,x^2+3}}{x^2}+\frac{6}{x^2}+5\right) + \frac{327}{16}\sqrt{x^4+5\,x^2+3} - \frac{\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}}{x^2} + \frac{609}{32}\log\left(2\,x^2+2\sqrt{x^4+5\,x^2+3}+5\right)$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
27/8*sqrt(x^4 + 5*x^2 + 3)*x^2 + 1/2*(x^4 + 5*x^2 + 3)^(3/2) - 12*sqrt(3)*
log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 327/16*sqrt(x^4 + 5
*x^2 + 3) - (x^4 + 5*x^2 + 3)^(3/2)/x^2 + 609/32*log(2*x^2 + 2*sqrt(x^4 +
5*x^2 + 3) + 5)
```

3.160.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.25

$$\int \frac{(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}}{x^3}\,dx = \frac{1}{16}\sqrt{x^4+5\,x^2+3}\left(2\left(4\,x^2+39\right)x^2+271\right) + 12\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5\,x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5\,x^2+3}}\right) + \frac{3\left(5\,x^2-5\sqrt{x^4+5\,x^2+3}+6\right)}{\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^2-3} - \frac{609}{32}\log\left(2\,x^2-2\sqrt{x^4+5\,x^2+3}+5\right)$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^3,x, algorithm="giac")
```

output
```
1/16*sqrt(x^4 + 5*x^2 + 3)*(2*(4*x^2 + 39)*x^2 + 271) + 12*sqrt(3)*log((x^
2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - sqrt(x^4 + 5*x^2 + 3
))) + 3*(5*x^2 - 5*sqrt(x^4 + 5*x^2 + 3) + 6)/((x^2 - sqrt(x^4 + 5*x^2 + 3
))^2 - 3) - 609/32*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

3.160.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}}{x^3}\,dx=\int \frac{\left(3\,x^2+2\right)\,{\left(x^4+5\,x^2+3\right)}^{3/2}}{x^3}\,dx$$

input
```
int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^3,x)
```

output
```
int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^3, x)
```

3.161 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^5}\,dx$

3.161.1 Optimal result

Integrand size = 25, antiderivative size = 127

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^5}\,dx = -\frac{3(28-19x^2)\sqrt{3+5x^2+x^4}}{8x^2} - \frac{(2-3x^2)(3+5x^2+x^4)^{3/2}}{4x^4} + \frac{453}{16}\operatorname{arctanh}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right) - \frac{127}{8}\sqrt{3}\operatorname{arctanh}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)$$

output
```
-1/4*(-3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^4+453/16*arctanh(1/2*(2*x^2+5)/(x^4+
5*x^2+3)^(1/2))-127/8*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3
^(1/2)-3/8*(-19*x^2+28)*(x^4+5*x^2+3)^(1/2)/x^2
```

3.161.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.80

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^5}\,dx = \frac{1}{16}\left(\frac{2\sqrt{3+5x^2+x^4}(-12-86x^2+83x^4+6x^6)}{x^4} + 508\sqrt{3}\operatorname{arctanh}\left(\frac{x^2-\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right) - 453\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)\right)$$

input
```
Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^5,x]
```

output
```
((2*Sqrt[3 + 5*x^2 + x^4]*(-12 - 86*x^2 + 83*x^4 + 6*x^6))/x^4 + 508*Sqrt[
3]*ArcTanh[(x^2 - Sqrt[3 + 5*x^2 + x^4])/Sqrt[3]] - 453*Log[-5 - 2*x^2 + 2
*Sqrt[3 + 5*x^2 + x^4]])/16
```

3.161.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1578, 1230, 27, 1230, 25, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x^2+2)(x^4+5x^2+3)^{3/2}}{x^5}dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{(3x^2+2)(x^4+5x^2+3)^{3/2}}{x^6}dx^2$$

$$\Big\downarrow \text{1230}$$

$$\frac{1}{2}\left(-\frac{3}{8}\int -\frac{2(19x^2+28)\sqrt{x^4+5x^2+3}}{x^4}dx^2-\frac{(2-3x^2)(x^4+5x^2+3)^{3/2}}{2x^4}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{3}{4}\int \frac{(19x^2+28)\sqrt{x^4+5x^2+3}}{x^4}dx^2-\frac{(2-3x^2)(x^4+5x^2+3)^{3/2}}{2x^4}\right)$$

$$\Big\downarrow \text{1230}$$

$$\frac{1}{2}\left(\frac{3}{4}\left(-\frac{1}{2}\int -\frac{151x^2+254}{x^2\sqrt{x^4+5x^2+3}}dx^2-\frac{\sqrt{x^4+5x^2+3}(28-19x^2)}{x^2}\right)-\frac{(2-3x^2)(x^4+5x^2+3)^{3/2}}{2x^4}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{151x^2+254}{x^2\sqrt{x^4+5x^2+3}}dx^2-\frac{(28-19x^2)\sqrt{x^4+5x^2+3}}{x^2}\right)-\frac{(2-3x^2)(x^4+5x^2+3)^{3/2}}{2x^4}\right)$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{2}\left(\frac{3}{4}\left(\frac{1}{2}\left(151\int\frac{1}{\sqrt{x^4+5x^2+3}}dx^2+254\int\frac{1}{x^2\sqrt{x^4+5x^2+3}}dx^2\right)-\frac{(28-19x^2)\sqrt{x^4+5x^2+3}}{x^2}\right)-\frac{(2-3x^2)}{}\right.$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}\left(\frac{3}{4}\left(\frac{1}{2}\left(302\int\frac{1}{4-x^4}d\frac{2x^2+5}{\sqrt{x^4+5x^2+3}}+254\int\frac{1}{x^2\sqrt{x^4+5x^2+3}}dx^2\right)-\frac{(28-19x^2)\sqrt{x^4+5x^2+3}}{x^2}\right)-\frac{(2-}{}\right.$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{3}{4}\left(\frac{1}{2}\left(254\int\frac{1}{x^2\sqrt{x^4+5x^2+3}}dx^2+151\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)\right)-\frac{(28-19x^2)\sqrt{x^4+5x^2+3}}{x^2}\right)-\frac{(2-}{}\right.$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{2}\left(\frac{3}{4}\left(\frac{1}{2}\left(151\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-508\int\frac{1}{12-x^4}d\frac{5x^2+6}{\sqrt{x^4+5x^2+3}}\right)-\frac{(28-19x^2)\sqrt{x^4+5x^2+3}}{x^2}\right)-\right.$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{3}{4}\left(\frac{1}{2}\left(151\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{254\text{arctanh}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{\sqrt{3}}\right)-\frac{(28-19x^2)\sqrt{x^4+5x^2+3}}{x^2}\right)-\frac{(2}{}\right.$$

input
```
Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^5,x]
```

output
```
(-1/2*((2 - 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^4 + (3*(-(((28 - 19*x^2)*Sqr
t[3 + 5*x^2 + x^4])/x^2) + (151*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^
4])] - (254*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/Sqrt[3
])/2))/4)/2
```

3.161. $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^5}\,dx$

3.161.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.161. $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^5}\,dx$

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.161.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$\frac{-254\,\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}\,x^4+453\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)x^4+12\sqrt{x^4+5x^2+3}\,\left(x^6+\frac{83}{6}x^4-\frac{43}{3}x^2-2\right)}{16x^4}$
trager	$\frac{(6x^6+83x^4-86x^2-12)\sqrt{x^4+5x^2+3}}{8x^4}+\frac{453\ln\left(-2x^2-2\sqrt{x^4+5x^2+3}-5\right)}{16}+\frac{127\,\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{-5\,\operatorname{RootOf}\left(\_Z\right.}{}\right.}{}$
default	$\frac{453\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{16}+\frac{3x^2\sqrt{x^4+5x^2+3}}{4}+\frac{83\sqrt{x^4+5x^2+3}}{8}-\frac{43\sqrt{x^4+5x^2+3}}{4x^2}-\frac{127\,\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right.}{8}$
risch	$-\frac{43x^6+221x^4+159x^2+18}{4x^4\sqrt{x^4+5x^2+3}}+\frac{453\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{16}+\frac{3x^2\sqrt{x^4+5x^2+3}}{4}+\frac{83\sqrt{x^4+5x^2+3}}{8}-\frac{127\,\operatorname{arctanh}\left(\right.}{}$
elliptic	$\frac{453\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{16}+\frac{3x^2\sqrt{x^4+5x^2+3}}{4}+\frac{83\sqrt{x^4+5x^2+3}}{8}-\frac{43\sqrt{x^4+5x^2+3}}{4x^2}-\frac{127\,\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right.}{8}$

input
```
int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/16*(-254*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)*x^4+
453*ln(2*x^2+5+2*(x^4+5*x^2+3)^(1/2))*x^4+12*(x^4+5*x^2+3)^(1/2)*(x^6+83/6
*x^4-43/3*x^2-2))/x^4
```

3.161.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.96

$$\int \frac{(2+3x^2)\,(3+5x^2+x^4)^{3/2}}{x^5}\,dx = \frac{1016\sqrt{3}x^4\log\left(\frac{25\,x^2-2\sqrt{3}(5\,x^2+6)-2\sqrt{x^4+5\,x^2+3}\left(5\sqrt{3}-6\right)+30}{x^2}\right)-1812\,x^4\log\left(\right.}{}$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^5,x, algorithm="fricas")
```

output
```
1/64*(1016*sqrt(3)*x^4*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 +
5*x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 1812*x^4*log(-2*x^2 + 2*sqrt(x^4 +
 5*x^2 + 3) - 5) + 67*x^4 + 8*(6*x^6 + 83*x^4 - 86*x^2 - 12)*sqrt(x^4 + 5*
x^2 + 3))/x^4
```

3.161.6 Sympy [F]

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^5}\,dx = \int \frac{(3x^2+2)(x^4+5x^2+3)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**5,x)
```

output
```
Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x**5, x)
```

3.161.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.08

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^5}\,dx = \frac{7}{2}\sqrt{x^4+5x^2+3}x^2 + \frac{1}{6}(x^4+5x^2+3)^{\frac{3}{2}} - \frac{127}{8}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2} + \frac{6}{x^2} + 5\right) + \frac{197}{8}\sqrt{x^4+5x^2+3} - \frac{23(x^4+5x^2+3)^{\frac{3}{2}}}{12x^2} - \frac{(x^4+5x^2+3)^{\frac{5}{2}}}{6x^4} + \frac{453}{16}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^5,x, algorithm="maxima")
```

output
```
7/2*sqrt(x^4 + 5*x^2 + 3)*x^2 + 1/6*(x^4 + 5*x^2 + 3)^(3/2) - 127/8*sqrt(3
)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 197/8*sqrt(x^4 +
5*x^2 + 3) - 23/12*(x^4 + 5*x^2 + 3)^(3/2)/x^2 - 1/6*(x^4 + 5*x^2 + 3)^(5/
2)/x^4 + 453/16*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

3.161.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.50

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^5}\,dx = \frac{1}{8}\sqrt{x^4+5x^2+3}(6x^2+83)$$
$$+\frac{127}{8}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right)$$
$$+\frac{227\left(x^2-\sqrt{x^4+5x^2+3}\right)^3+348\left(x^2-\sqrt{x^4+5x^2+3}\right)^2-459x^2+459\sqrt{x^4+5x^2+3}-684}{4\left(\left(x^2-\sqrt{x^4+5x^2+3}\right)^2-3\right)^2}$$
$$-\frac{453}{16}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^5,x, algorithm="giac")
```

output
```
1/8*sqrt(x^4 + 5*x^2 + 3)*(6*x^2 + 83) + 127/8*sqrt(3)*log((x^2 + sqrt(3)
- sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - sqrt(x^4 + 5*x^2 + 3))) + 1/4*(2
27*(x^2 - sqrt(x^4 + 5*x^2 + 3))^3 + 348*(x^2 - sqrt(x^4 + 5*x^2 + 3))^2 -
 459*x^2 + 459*sqrt(x^4 + 5*x^2 + 3) - 684)/((x^2 - sqrt(x^4 + 5*x^2 + 3))
^2 - 3)^2 - 453/16*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

3.161.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^5}\,dx = \int \frac{(3x^2+2)(x^4+5x^2+3)^{3/2}}{x^5}\,dx$$

input
```
int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^5,x)
```

output
```
int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^5, x)
```

3.162 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^7}\,dx$

3.162.1 Optimal result

Integrand size = 25, antiderivative size = 127

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^7}\,dx = -\frac{(67-32x^2)\sqrt{3+5x^2+x^4}}{12x^2} - \frac{(2+7x^2)(3+5x^2+x^4)^{3/2}}{6x^6} + \frac{49}{4}\operatorname{arctanh}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right) - \frac{527\operatorname{arctanh}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{24\sqrt{3}}$$

output
```
-1/6*(7*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^6+49/4*arctanh(1/2*(2*x^2+5)/(x^4+5*x
^2+3)^(1/2))-527/72*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(
1/2)-1/12*(-32*x^2+67)*(x^4+5*x^2+3)^(1/2)/x^2
```

3.162.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.80

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^7}\,dx = \frac{1}{36}\left(\frac{3\sqrt{3+5x^2+x^4}(-12-62x^2-141x^4+18x^6)}{x^6} + 527\sqrt{3}\operatorname{arctanh}\left(\frac{x^2-\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right) - 441\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)\right)$$

input
```
Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^7,x]
```

output
```
((3*Sqrt[3 + 5*x^2 + x^4]*(-12 - 62*x^2 - 141*x^4 + 18*x^6))/x^6 + 527*Sqr
t[3]*ArcTanh[(x^2 - Sqrt[3 + 5*x^2 + x^4])/Sqrt[3]] - 441*Log[-5 - 2*x^2 +
 2*Sqrt[3 + 5*x^2 + x^4]])/36
```

3.162.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1578, 1229, 27, 1230, 25, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x^2+2)(x^4+5x^2+3)^{3/2}}{x^7}dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{(3x^2+2)(x^4+5x^2+3)^{3/2}}{x^8}dx^2$$

$$\Big\downarrow \text{1229}$$

$$\frac{1}{2}\left(-\frac{1}{12}\int -\frac{2(32x^2+67)\sqrt{x^4+5x^2+3}}{x^4}dx^2-\frac{(7x^2+2)(x^4+5x^2+3)^{3/2}}{3x^6}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{1}{6}\int \frac{(32x^2+67)\sqrt{x^4+5x^2+3}}{x^4}dx^2-\frac{(7x^2+2)(x^4+5x^2+3)^{3/2}}{3x^6}\right)$$

$$\Big\downarrow \text{1230}$$

$$\frac{1}{2}\left(\frac{1}{6}\left(-\frac{1}{2}\int -\frac{294x^2+527}{x^2\sqrt{x^4+5x^2+3}}dx^2-\frac{\sqrt{x^4+5x^2+3}(67-32x^2)}{x^2}\right)-\frac{(7x^2+2)(x^4+5x^2+3)^{3/2}}{3x^6}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{294x^2+527}{x^2\sqrt{x^4+5x^2+3}}dx^2-\frac{(67-32x^2)\sqrt{x^4+5x^2+3}}{x^2}\right)-\frac{(7x^2+2)(x^4+5x^2+3)^{3/2}}{3x^6}\right)$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{2}\left(\frac{1}{6}\left(\frac{1}{2}\left(294\int \frac{1}{\sqrt{x^4+5x^2+3}}dx^2+527\int \frac{1}{x^2\sqrt{x^4+5x^2+3}}dx^2\right)-\frac{(67-32x^2)\sqrt{x^4+5x^2+3}}{x^2}\right)-\frac{(7x^2+2)}{}\right.$$

$\Big\downarrow$ 1092

$$\frac{1}{2}\left(\frac{1}{6}\left(\frac{1}{2}\left(588\int \frac{1}{4-x^4}d\frac{2x^2+5}{\sqrt{x^4+5x^2+3}}+527\int \frac{1}{x^2\sqrt{x^4+5x^2+3}}dx^2\right)-\frac{(67-32x^2)\sqrt{x^4+5x^2+3}}{x^2}\right)-\frac{(7x^2}{}\right.$$

$\Big\downarrow$ 219

$$\frac{1}{2}\left(\frac{1}{6}\left(\frac{1}{2}\left(527\int \frac{1}{x^2\sqrt{x^4+5x^2+3}}dx^2+294\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)\right)-\frac{(67-32x^2)\sqrt{x^4+5x^2+3}}{x^2}\right)-\frac{(7x}{}\right.$$

$\Big\downarrow$ 1154

$$\frac{1}{2}\left(\frac{1}{6}\left(\frac{1}{2}\left(294\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-1054\int \frac{1}{12-x^4}d\frac{5x^2+6}{\sqrt{x^4+5x^2+3}}\right)-\frac{(67-32x^2)\sqrt{x^4+5x^2+3}}{x^2}\right)-\right.$$

$\Big\downarrow$ 219

$$\frac{1}{2}\left(\frac{1}{6}\left(\frac{1}{2}\left(294\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{527\text{arctanh}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{\sqrt{3}}\right)-\frac{(67-32x^2)\sqrt{x^4+5x^2+3}}{x^2}\right)-\frac{(7}{}\right.$$

input
```
Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^7,x]
```

output
```
(-1/3*((2 + 7*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^6 + (-(((67 - 32*x^2)*Sqrt[3
 + 5*x^2 + x^4])/x^2) + (294*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])
] - (527*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/Sqrt[3])/
2)/6)/2
```

3.162. $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^7}\,dx$

3.162.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

3.162. $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^7}\,dx$

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.162.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$\frac{-527\,\operatorname{arctanh}\left(\frac{\left(5x^{2}+6\right)\sqrt{3}}{6\sqrt{x^{4}+5x^{2}+3}}\right)\sqrt{3}\,x^{6}+882\ln\left(2x^{2}+5+2\sqrt{x^{4}+5x^{2}+3}\right)x^{6}+108\left(x^{6}-\frac{47}{6}x^{4}-\frac{31}{9}x^{2}-\frac{2}{3}\right)\sqrt{x^{4}+5x^{2}+3}}{72x^{6}}$
risch	$-\frac{141x^{8}+767x^{6}+745x^{4}+246x^{2}+36}{12x^{6}\sqrt{x^{4}+5x^{2}+3}}+\frac{49\ln\left(\frac{5}{2}+x^{2}+\sqrt{x^{4}+5x^{2}+3}\right)}{4}+\frac{3\sqrt{x^{4}+5x^{2}+3}}{2}-\frac{527\,\operatorname{arctanh}\left(\frac{\left(5x^{2}+6\right)\sqrt{3}}{6\sqrt{x^{4}+5x^{2}+3}}\right)\sqrt{}}{72}$
trager	$\frac{\left(18x^{6}-141x^{4}-62x^{2}-12\right)\sqrt{x^{4}+5x^{2}+3}}{12x^{6}}-\frac{49\ln\left(2x^{2}-2\sqrt{x^{4}+5x^{2}+3}+5\right)}{4}+\frac{527\,\operatorname{RootOf}\left(\_Z^{2}-3\right)\ln\left(-\frac{-5\,\operatorname{RootOf}\left(\_Z\right.}{}\right.}{7}$
default	$\frac{49\ln\left(\frac{5}{2}+x^{2}+\sqrt{x^{4}+5x^{2}+3}\right)}{4}-\frac{\sqrt{x^{4}+5x^{2}+3}}{x^{6}}-\frac{31\sqrt{x^{4}+5x^{2}+3}}{6x^{4}}-\frac{47\sqrt{x^{4}+5x^{2}+3}}{4x^{2}}-\frac{527\,\operatorname{arctanh}\left(\frac{\left(5x^{2}+6\right)\sqrt{3}}{6\sqrt{x^{4}+5x^{2}+3}}\right)\sqrt{3}}{72}$
elliptic	$\frac{49\ln\left(\frac{5}{2}+x^{2}+\sqrt{x^{4}+5x^{2}+3}\right)}{4}-\frac{\sqrt{x^{4}+5x^{2}+3}}{x^{6}}-\frac{31\sqrt{x^{4}+5x^{2}+3}}{6x^{4}}-\frac{47\sqrt{x^{4}+5x^{2}+3}}{4x^{2}}-\frac{527\,\operatorname{arctanh}\left(\frac{\left(5x^{2}+6\right)\sqrt{3}}{6\sqrt{x^{4}+5x^{2}+3}}\right)\sqrt{3}}{72}$

input
```
int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
1/72*(-527*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)*x^6+
882*ln(2*x^2+5+2*(x^4+5*x^2+3)^(1/2))*x^6+108*(x^6-47/6*x^4-31/9*x^2-2/3)*
(x^4+5*x^2+3)^(1/2))/x^6
```

3.162.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.96

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^7}\,dx = \frac{527\sqrt{3}x^6\log\left(\frac{25x^2-2\sqrt{3}(5x^2+6)-2\sqrt{x^4+5x^2+3}\left(5\sqrt{3}-6\right)+30}{x^2}\right)-882x^6\log\left(-\right.}{}$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^7,x, algorithm="fricas")
```

output
```
1/72*(527*sqrt(3)*x^6*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5
*x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 882*x^6*log(-2*x^2 + 2*sqrt(x^4 + 5
*x^2 + 3) - 5) - 711*x^6 + 6*(18*x^6 - 141*x^4 - 62*x^2 - 12)*sqrt(x^4 + 5
*x^2 + 3))/x^6
```

3.162.6 Sympy [F]

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^7}\,dx = \int \frac{(3x^2+2)(x^4+5x^2+3)^{\frac{3}{2}}}{x^7}\,dx$$

input
```
integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**7,x)
```

output
```
Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x**7, x)
```

3.162.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.21

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^7}\,dx = \frac{67}{36}\sqrt{x^4+5\,x^2+3}x^2 + \frac{11}{54}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}} - \frac{527}{72}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5\,x^2+3}}{x^2}+\frac{6}{x^2}+5\right) + \frac{431}{36}\sqrt{x^4+5\,x^2+3} - \frac{79\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}}{108\,x^2} - \frac{11\left(x^4+5\,x^2+3\right)^{\frac{5}{2}}}{54\,x^4} - \frac{\left(x^4+5\,x^2+3\right)^{\frac{5}{2}}}{9\,x^6} + \frac{49}{4}\log\left(2\,x^2+2\sqrt{x^4+5\,x^2+3}+5\right)$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^7,x, algorithm="maxima")
```

output
```
67/36*sqrt(x^4 + 5*x^2 + 3)*x^2 + 11/54*(x^4 + 5*x^2 + 3)^(3/2) - 527/72*s
qrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 431/36*sqrt(
x^4 + 5*x^2 + 3) - 79/108*(x^4 + 5*x^2 + 3)^(3/2)/x^2 - 11/54*(x^4 + 5*x^2
 + 3)^(5/2)/x^4 - 1/9*(x^4 + 5*x^2 + 3)^(5/2)/x^6 + 49/4*log(2*x^2 + 2*sqr
t(x^4 + 5*x^2 + 3) + 5)
```

3.162.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(103) = 206$.

Time = 0.32 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.79

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^7}\,dx = \frac{527}{72}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5\,x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5\,x^2+3}}\right) + \frac{3}{2}\sqrt{x^4+5\,x^2+3} + \frac{829\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^5+1824\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^4-2200\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^3-5292\left(x^2-\right.}{12\left(\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^2-3\right)^3} - \frac{49}{4}\log\left(2\,x^2-2\sqrt{x^4+5\,x^2+3}+5\right)$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^7,x, algorithm="giac")
```

output
```
527/72*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3)
- sqrt(x^4 + 5*x^2 + 3))) + 3/2*sqrt(x^4 + 5*x^2 + 3) + 1/12*(829*(x^2 - s
qrt(x^4 + 5*x^2 + 3))^5 + 1824*(x^2 - sqrt(x^4 + 5*x^2 + 3))^4 - 2200*(x^2
 - sqrt(x^4 + 5*x^2 + 3))^3 - 5292*(x^2 - sqrt(x^4 + 5*x^2 + 3))^2 + 2799*
x^2 - 2799*sqrt(x^4 + 5*x^2 + 3) + 5724)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2
- 3)^3 - 49/4*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

3.162.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^7}\,dx = \int \frac{(3\,x^2+2)\,(x^4+5\,x^2+3)^{3/2}}{x^7}\,dx$$

input
```
int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^7,x)
```

output
```
int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^7, x)
```

3.163 $\int x^4(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx$

3.163.1 Optimal result

Integrand size = 25, antiderivative size = 356

$$\int x^4\left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2}\,dx=\frac{176723x\left(5+\sqrt{13}+2x^2\right)}{4290\sqrt{3+5x^2+x^4}}-\frac{4210}{429}x\sqrt{3+5x^2+x^4}+\frac{1251}{715}x^3\sqrt{3+5x^2+x^4}-\frac{1}{429}x^5\left(283+272x^2\right)\sqrt{3+5x^2+x^4}+\frac{1}{143}x^5\left(71+33x^2\right)\left(3+5x^2+x^4\right)^{3/2}-\frac{176723\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{4290\sqrt{3+5x^2+x^4}}+\frac{2105\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{143\sqrt{3+5x^2+x^4}}$$

output
```
1/143*x^5*(33*x^2+71)*(x^4+5*x^2+3)^(3/2)+176723/4290*x*(5+2*x^2+13^(1/2))
/(x^4+5*x^2+3)^(1/2)-4210/429*x*(x^4+5*x^2+3)^(1/2)+1251/715*x^3*(x^4+5*x^
2+3)^(1/2)-1/429*x^5*(272*x^2+283)*(x^4+5*x^2+3)^(1/2)+2105/429*(1/(36+x^2
*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*
13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2)
)*(6+x^2*(5+13^(1/2)))*6^(1/2)/(5+13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6
+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)-176723/25740*(1/(36+x^2*(30+
6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1
/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+
x^2*(5+13^(1/2)))*(30+6*13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13
^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)
```

3.163.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.17 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.70

$$\int x^4(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx = \frac{4x(-63150-93991x^2+3055x^4+29003x^6+39650x^8+24635x^{10}+6015x^{12}+495x^{14})+176}{}$$

input
```
Integrate[x^4*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

output
```
(4*x*(-63150 - 93991*x^2 + 3055*x^4 + 29003*x^6 + 39650*x^8 + 24635*x^10 +
 6015*x^12 + 495*x^14) + (176723*I)*Sqrt[2]*(-5 + Sqrt[13])*Sqrt[(-5 + Sqr
t[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*Arc
Sinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqrt[2]*(-75731
5 + 176723*Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5
+ Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 +
(5*Sqrt[13])/6])/(8580*Sqrt[3 + 5*x^2 + x^4])
```

3.163.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {1596, 25, 1596, 27, 1602, 1602, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 (3x^2+2) (x^4+5x^2+3)^{3/2} \, dx$$

$$\Big\downarrow \text{ 1596}$$

$$\frac{3}{143}\int -x^4 (272x^2+69) \sqrt{x^4+5x^2+3}dx+\frac{1}{143}(33x^2+71) (x^4+5x^2+3)^{3/2} x^5$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{143}x^5 (33x^2+71) (x^4+5x^2+3)^{3/2}-\frac{3}{143}\int x^4 (272x^2+69) \sqrt{x^4+5x^2+3}dx$$

$$\Big\downarrow \text{ 1596}$$

$$\frac{1}{143}x^5 (33x^2+71) (x^4+5x^2+3)^{3/2}- \frac{3}{143}\left(\frac{1}{63}\int -\frac{21x^4 (1251x^2+794)}{\sqrt{x^4+5x^2+3}}dx+\frac{1}{9}(272x^2+283) \sqrt{x^4+5x^2+3}x^5\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{143}x^5 (33x^2+71) (x^4+5x^2+3)^{3/2}- \frac{3}{143}\left(\frac{1}{9}x^5 (272x^2+283) \sqrt{x^4+5x^2+3}-\frac{1}{3}\int \frac{x^4 (1251x^2+794)}{\sqrt{x^4+5x^2+3}}dx\right)$$

$$\Big\downarrow \text{ 1602}$$

$$\frac{1}{143}x^5 (33x^2+71) (x^4+5x^2+3)^{3/2}- \frac{3}{143}\left(\frac{1}{3}\left(\frac{1}{5}\int \frac{x^2 (21050x^2+11259)}{\sqrt{x^4+5x^2+3}}dx-\frac{1251}{5}x^3\sqrt{x^4+5x^2+3}\right)+\frac{1}{9}(272x^2+283) \sqrt{x^4+5x^2+3}x^5\right)$$

$$\Big\downarrow \text{ 1602}$$

$$\frac{1}{143}x^5 (33x^2+71) (x^4+5x^2+3)^{3/2}- \frac{3}{143}\left(\frac{1}{3}\left(\frac{1}{5}\left(\frac{21050}{3}x\sqrt{x^4+5x^2+3}-\frac{1}{3}\int \frac{176723x^2+63150}{\sqrt{x^4+5x^2+3}}dx\right)-\frac{1251}{5}x^3\sqrt{x^4+5x^2+3}\right)+\frac{1}{9}(272x^2+283)\right.$$

$$\Big\downarrow \text{ 1503}$$

$$\frac{1}{143}x^5\left(33x^2+71\right)\left(x^4+5x^2+3\right)^{3/2}-$$
$$\frac{3}{143}\left(\frac{1}{3}\left(\frac{1}{5}\left(\frac{1}{3}\left(-63150\int\frac{1}{\sqrt{x^4+5x^2+3}}dx-176723\int\frac{x^2}{\sqrt{x^4+5x^2+3}}dx\right)+\frac{21050}{3}\sqrt{x^4+5x^2+3}x\right)-\frac{1251}{5}x\right.\right.\right.$$

$\Big\downarrow$ 1412

$$\frac{1}{143}x^5\left(33x^2+71\right)\left(x^4+5x^2+3\right)^{3/2}-$$
$$\frac{3}{143}\left(\frac{1}{3}\left(\frac{1}{5}\left(\frac{1}{3}\left(-176723\int\frac{x^2}{\sqrt{x^4+5x^2+3}}dx-\frac{10525\sqrt{\frac{6}{5+\sqrt{13}}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\text{arc}\right.}{\sqrt{x^4+5x^2+3}}\right.\right.\right.\right.$$

$\Big\downarrow$ 1455

$$\frac{1}{143}x^5\left(33x^2+71\right)\left(x^4+5x^2+3\right)^{3/2}-$$
$$\frac{3}{143}\left(\frac{1}{3}\left(\frac{1}{5}\left(\frac{1}{3}\left(-\frac{10525\sqrt{\frac{6}{5+\sqrt{13}}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13\right.\right.}{\sqrt{x^4+5x^2+3}}\right.\right.\right.\right.$$

input
```
Int[x^4*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

output
```
(x^5*(71 + 33*x^2)*(3 + 5*x^2 + x^4)^(3/2))/143 - (3*((x^5*(283 + 272*x^2)
*Sqrt[3 + 5*x^2 + x^4])/9 + ((-1251*x^3*Sqrt[3 + 5*x^2 + x^4])/5 + ((21050
*x*Sqrt[3 + 5*x^2 + x^4])/3 + (-176723*((x*(5 + Sqrt[13] + 2*x^2))/(2*Sqrt
[3 + 5*x^2 + x^4]) - (Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)
/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[
(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(2*Sqrt[3 + 5*x^2 + x^4])) -
(10525*Sqrt[6/(5 + Sqrt[13])]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt
[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/
6]*x], (-13 + 5*Sqrt[13])/6])/Sqrt[3 + 5*x^2 + x^4])/3)/5)/3))/143
```

3.163. $\int x^4(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx$

3.163.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1596
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((b*e*2
*p + c*d*(m + 4*p + 3) + c*e*(4*p + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p
 + 3))), x] + Simp[2*(p/(c*(4*p + m + 1)*(m + 4*p + 3)))   Int[(f*x)^m*(a +
 b*x^2 + c*x^4)^(p - 1)*Simp[2*a*c*d*(m + 4*p + 3) - a*b*e*(m + 1) + (2*a*c
*e*(4*p + m + 1) + b*c*d*(m + 4*p + 3) - b^2*e*(m + 2*p + 1))*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] &
& NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[
p] || IntegerQ[m])
```

3.163. $\int x^4(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx$

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

3.163.4 Maple [A] (verified)

Time = 4.21 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.66

method	result
risch	$\frac{x(495x^{10}+3540x^8+5450x^6+1780x^4+3753x^2-21050)\sqrt{x^4+5x^2+3}}{2145}+\frac{25260\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6}}{6}\right.}{143\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}$
default	$\frac{3x^{11}\sqrt{x^4+5x^2+3}}{13}+\frac{236x^9\sqrt{x^4+5x^2+3}}{143}+\frac{1090x^7\sqrt{x^4+5x^2+3}}{429}+\frac{356x^5\sqrt{x^4+5x^2+3}}{429}+\frac{1251x^3\sqrt{x^4+5x^2+3}}{715}-\frac{4210x\sqrt{x^4+}}{429}$
elliptic	$\frac{3x^{11}\sqrt{x^4+5x^2+3}}{13}+\frac{236x^9\sqrt{x^4+5x^2+3}}{143}+\frac{1090x^7\sqrt{x^4+5x^2+3}}{429}+\frac{356x^5\sqrt{x^4+5x^2+3}}{429}+\frac{1251x^3\sqrt{x^4+5x^2+3}}{715}-\frac{4210x\sqrt{x^4+}}{429}$

input
```
int(x^4*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2145*x*(495*x^10+3540*x^8+5450*x^6+1780*x^4+3753*x^2-21050)*(x^4+5*x^2+3
)^(1/2)+25260/143/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)
*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-3
0+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-2120676/715/(-30+6*13^(1/2))
^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)
/(x^4+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),
5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/
2)+1/6*39^(1/2)))
```

3.163.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.42

$$\int x^4(2+3x^2)(3+5x^2+x^4)^{3/2}\,dx = \frac{176723\left(\sqrt{13}\sqrt{2}x-5\sqrt{2}x\right)\sqrt{\sqrt{13}-5}E(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{13}-5}}{2x}\right)\mid\frac{5}{6}\sqrt{13}+\frac{19}{6})-\left(155673\sqrt{13}\sqrt{2}\right.}{}$$

input
```
integrate(x^4*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
1/8580*(176723*(sqrt(13)*sqrt(2)*x - 5*sqrt(2)*x)*sqrt(sqrt(13) - 5)*ellip
tic_e(arcsin(1/2*sqrt(2)*sqrt(sqrt(13) - 5)/x), 5/6*sqrt(13) + 19/6) - (15
5673*sqrt(13)*sqrt(2)*x - 988865*sqrt(2)*x)*sqrt(sqrt(13) - 5)*elliptic_f(
arcsin(1/2*sqrt(2)*sqrt(sqrt(13) - 5)/x), 5/6*sqrt(13) + 19/6) + 4*(495*x^
12 + 3540*x^10 + 5450*x^8 + 1780*x^6 + 3753*x^4 - 21050*x^2 + 176723)*sqrt
(x^4 + 5*x^2 + 3))/x
```

3.163.6 Sympy [F]

$$\int x^4(2+3x^2)(3+5x^2+x^4)^{3/2}\,dx = \int x^4\cdot(3x^2+2)\left(x^4+5x^2+3\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**4*(3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)
```

output
```
Integral(x**4*(3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2), x)
```

3.163.7 Maxima [F]

$$\int x^4(2+3x^2)(3+5x^2+x^4)^{3/2}\,dx = \int (x^4+5\,x^2+3)^{\frac{3}{2}}(3\,x^2+2)x^4\,dx$$

input
```
integrate(x^4*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)*x^4, x)
```

3.163.8 Giac [F]

$$\int x^4 (2+3x^2) \left(3+5x^2+x^4\right)^{3/2} \, dx = \int \left(x^4+5\,x^2+3\right)^{\frac{3}{2}} (3\,x^2+2) x^4 \, dx$$

input
```
integrate(x^4*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

output
```
integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)*x^4, x)
```

3.163.9 Mupad [F(-1)]

Timed out.

$$\int x^4 (2+3x^2) \left(3+5x^2+x^4\right)^{3/2} \, dx = \int x^4 \left(3\,x^2+2\right) \left(x^4+5\,x^2+3\right)^{3/2} \, dx$$

input
```
int(x^4*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2),x)
```

output
```
int(x^4*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2), x)
```

3.164 $\int x^2(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx$

3.164.1 Optimal result

Integrand size = 25, antiderivative size = 331

$$\int x^2(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx = -\frac{49949x(5+\sqrt{13}+2x^2)}{3465\sqrt{3+5x^2+x^4}}+\frac{353}{99}x\sqrt{3+5x^2+x^4}-\frac{x^3(911+890x^2)\sqrt{3+5x^2+x^4}}{1155}+\frac{1}{99}x^3(67+27x^2)\left(3+5x^2+x^4\right)^{3/2}+\frac{49949\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{3465\sqrt{3+5x^2+x^4}}-\frac{353\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{33\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{3+5x^2+x^4}}$$

output
```
1/99*x^3*(27*x^2+67)*(x^4+5*x^2+3)^(3/2)-49949/3465*x*(5+2*x^2+13^(1/2))/(
x^4+5*x^2+3)^(1/2)+353/99*x*(x^4+5*x^2+3)^(1/2)-1/1155*x^3*(890*x^2+911)*(
x^4+5*x^2+3)^(1/2)+49949/20790*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*
(30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^
(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(
1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3
)^(1/2)-353/33*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))
^(1/2)*EllipticF(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/
6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*((6+x^2*(5-13^(1/2)))/(6+x
^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)/(30+6*13^(1/2))^(1/2)
```

3.164.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.62 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.74

$$\int x^2(2+3x^2)\left(3+5x^2+x^4\right)^{3/2} dx = \frac{2x(37065+74681x^2+69535x^4+84962x^6+50075x^8+11795x^{10}+945x^{12})-49949i\sqrt{2}(-5}{}$$

input
```
Integrate[x^2*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

output
```
(2*x*(37065 + 74681*x^2 + 69535*x^4 + 84962*x^6 + 50075*x^8 + 11795*x^10 +
 945*x^12) - (49949*I)*Sqrt[2]*(-5 + Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2
)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(
5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] + I*Sqrt[2]*(-212680 + 49949*Sqr
t[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2
*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6
])/(6930*Sqrt[3 + 5*x^2 + x^4])
```

3.164.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {1596, 25, 1596, 25, 1602, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(3x^2+2)\left(x^4+5x^2+3\right)^{3/2} dx$$

$$\Big\downarrow \text{1596}$$

$$\frac{1}{33}\int -x^2(178x^2+3)\sqrt{x^4+5x^2+3}dx+\frac{1}{99}(27x^2+67)\left(x^4+5x^2+3\right)^{3/2}x^3$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{99}x^3(27x^2+67)\left(x^4+5x^2+3\right)^{3/2}-\frac{1}{33}\int x^2(178x^2+3)\sqrt{x^4+5x^2+3}dx$$

$$\Big\downarrow \text{1596}$$

$$\frac{1}{33}\left(-\frac{1}{35}\int -\frac{x^2\left(12355x^2+7884\right)}{\sqrt{x^4+5x^2+3}}dx-\frac{1}{35}\left(890x^2+911\right)\sqrt{x^4+5x^2+3}x^3\right)+$$
$$\frac{1}{99}\left(27x^2+67\right)\left(x^4+5x^2+3\right)^{3/2}x^3$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{33}\left(\frac{1}{35}\int \frac{x^2\left(12355x^2+7884\right)}{\sqrt{x^4+5x^2+3}}dx-\frac{1}{35}x^3\left(890x^2+911\right)\sqrt{x^4+5x^2+3}\right)+$$
$$\frac{1}{99}\left(27x^2+67\right)\left(x^4+5x^2+3\right)^{3/2}x^3$$

$$\Big\downarrow \text{ 1602}$$

$$\frac{1}{33}\left(\frac{1}{35}\left(\frac{12355}{3}x\sqrt{x^4+5x^2+3}-\frac{1}{3}\int \frac{99898x^2+37065}{\sqrt{x^4+5x^2+3}}dx\right)-\frac{1}{35}x^3\left(890x^2+911\right)\sqrt{x^4+5x^2+3}\right)+$$
$$\frac{1}{99}\left(27x^2+67\right)\left(x^4+5x^2+3\right)^{3/2}x^3$$

$$\Big\downarrow \text{ 1503}$$

$$\frac{1}{33}\left(\frac{1}{35}\left(\frac{1}{3}\left(-37065\int \frac{1}{\sqrt{x^4+5x^2+3}}dx-99898\int \frac{x^2}{\sqrt{x^4+5x^2+3}}dx\right)+\frac{12355}{3}\sqrt{x^4+5x^2+3}x\right)-\frac{1}{35}x^3(890x$$
$$\frac{1}{99}\left(27x^2+67\right)\left(x^4+5x^2+3\right)^{3/2}x^3$$

$$\Big\downarrow \text{ 1412}$$

$$\frac{1}{33}\left(\frac{1}{35}\left(\frac{1}{3}\left(-99898\int \frac{x^2}{\sqrt{x^4+5x^2+3}}dx-\frac{12355\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\text{arcta}\right.}{\sqrt{x^4+5x^2+3}}\right.\right.\right.$$
$$\frac{1}{99}\left(27x^2+67\right)\left(x^4+5x^2+3\right)^{3/2}x^3$$

$$\Big\downarrow \text{ 1455}$$

$$\frac{1}{33}\left(\frac{1}{35}\left(\frac{1}{3}\left(-\frac{12355\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}(-13\right.}{\sqrt{x^4+5x^2+3}}\right.\right.\right.$$
$$\frac{1}{99}\left(27x^2+67\right)\left(x^4+5x^2+3\right)^{3/2}x^3$$

3.164. $\int x^2(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx$

input
```
Int[x^2*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

output
```
(x^3*(67 + 27*x^2)*(3 + 5*x^2 + x^4)^(3/2))/99 + (-1/35*(x^3*(911 + 890*x^
2)*Sqrt[3 + 5*x^2 + x^4]) + ((12355*x*Sqrt[3 + 5*x^2 + x^4])/3 + (-99898*(
(x*(5 + Sqrt[13] + 2*x^2))/(2*Sqrt[3 + 5*x^2 + x^4]) - (Sqrt[(5 + Sqrt[13]
)/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqr
t[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])
/6])/(2*Sqrt[3 + 5*x^2 + x^4])) - (12355*Sqrt[3/(2*(5 + Sqrt[13]))]*Sqrt[(
6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)
*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/Sqrt[3
 + 5*x^2 + x^4])/3)/35)/33
```

3.164.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1596
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((b*e*2
*p + c*d*(m + 4*p + 3) + c*e*(4*p + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p
 + 3))), x] + Simp[2*(p/(c*(4*p + m + 1)*(m + 4*p + 3)))   Int[(f*x)^m*(a +
 b*x^2 + c*x^4)^(p - 1)*Simp[2*a*c*d*(m + 4*p + 3) - a*b*e*(m + 1) + (2*a*c
*e*(4*p + m + 1) + b*c*d*(m + 4*p + 3) - b^2*e*(m + 2*p + 1))*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] &
& NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[
p] || IntegerQ[m])
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

3.164.4 Maple [A] (verified)

Time = 2.78 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.70

method	result
risch	$\frac{x(945x^8+7070x^6+11890x^4+4302x^2+12355)\sqrt{x^4+5x^2+3}}{3465}-\frac{706\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\right.}{11\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}$
default	$\frac{3x^9\sqrt{x^4+5x^2+3}}{11}+\frac{202x^7\sqrt{x^4+5x^2+3}}{99}+\frac{2378x^5\sqrt{x^4+5x^2+3}}{693}+\frac{478x^3\sqrt{x^4+5x^2+3}}{385}+\frac{353x\sqrt{x^4+5x^2+3}}{99}-\frac{706\sqrt{1-\left(-\frac{5}{6}+\right.}}{}$
elliptic	$\frac{3x^9\sqrt{x^4+5x^2+3}}{11}+\frac{202x^7\sqrt{x^4+5x^2+3}}{99}+\frac{2378x^5\sqrt{x^4+5x^2+3}}{693}+\frac{478x^3\sqrt{x^4+5x^2+3}}{385}+\frac{353x\sqrt{x^4+5x^2+3}}{99}-\frac{706\sqrt{1-\left(-\frac{5}{6}+\right.}}{}$

input
```
int(x^2*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3465*x*(945*x^8+7070*x^6+11890*x^4+4302*x^2+12355)*(x^4+5*x^2+3)^(1/2)-7
06/11/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/
6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2)
)^(1/2),5/6*3^(1/2)+1/6*39^(1/2))+399592/385/(-30+6*13^(1/2))^(1/2)*(1-(-5
/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3
)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1
/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/
2)))
```

3.164.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.44

$$\int x^2(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx =$$

$$-\frac{99898\left(\sqrt{13}\sqrt{2}x-5\sqrt{2}x\right)\sqrt{\sqrt{13}-5}E(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{13}-5}}{2x}\right)\mid\frac{5}{6}\sqrt{13}+\frac{19}{6})-\left(87543\sqrt{13}\sqrt{2}x-561265\sqrt{2}\right.}{}$$

input
```
integrate(x^2*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
-1/13860*(99898*(sqrt(13)*sqrt(2)*x - 5*sqrt(2)*x)*sqrt(sqrt(13) - 5)*elli
ptic_e(arcsin(1/2*sqrt(2)*sqrt(sqrt(13) - 5)/x), 5/6*sqrt(13) + 19/6) - (8
7543*sqrt(13)*sqrt(2)*x - 561265*sqrt(2)*x)*sqrt(sqrt(13) - 5)*elliptic_f(
arcsin(1/2*sqrt(2)*sqrt(sqrt(13) - 5)/x), 5/6*sqrt(13) + 19/6) - 4*(945*x^
10 + 7070*x^8 + 11890*x^6 + 4302*x^4 + 12355*x^2 - 99898)*sqrt(x^4 + 5*x^2
 + 3))/x
```

3.164.6 Sympy [F]

$$\int x^2(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx = \int x^2\cdot\left(3x^2+2\right)\left(x^4+5x^2+3\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**2*(3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)
```

output
```
Integral(x**2*(3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2), x)
```

3.164.7 Maxima [F]

$$\int x^2(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx=\int (x^4+5\,x^2+3)^{\frac{3}{2}}(3\,x^2+2)x^2\,dx$$

input
```
integrate(x^2*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)*x^2, x)
```

3.164.8 Giac [F]

$$\int x^2(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx=\int (x^4+5\,x^2+3)^{\frac{3}{2}}(3\,x^2+2)x^2\,dx$$

input
```
integrate(x^2*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

output
```
integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)*x^2, x)
```

3.164.9 Mupad [F(-1)]

Timed out.

$$\int x^2(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx=\int x^2\left(3\,x^2+2\right)\left(x^4+5\,x^2+3\right)^{3/2}\,dx$$

input
```
int(x^2*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2),x)
```

output
```
int(x^2*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2), x)
```

3.165 $\int (2+3x^2)(3+5x^2+x^4)^{3/2}\,dx$

3.165.1 Optimal result

Integrand size = 22, antiderivative size = 308

$$\int (2+3x^2)(3+5x^2+x^4)^{3/2}\,dx=\frac{203x(5+\sqrt{13}+2x^2)}{30\sqrt{3+5x^2+x^4}}-\frac{1}{15}x(5+12x^2)\sqrt{3+5x^2+x^4}+\frac{1}{3}x(3+x^2)(3+5x^2+x^4)^{3/2}-\frac{203\sqrt{\frac{1}{6}(5+\sqrt{13})}\sqrt{\frac{6+(5-\sqrt{13})x^2}{6+(5+\sqrt{13})x^2}}(6+(5+\sqrt{13})x^2)E\left(\arctan\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right)|\frac{1}{6}(-13+5\sqrt{13})\right)}{30\sqrt{3+5x^2+x^4}}+\frac{5\sqrt{\frac{2}{3(5+\sqrt{13})}}\sqrt{\frac{6+(5-\sqrt{13})x^2}{6+(5+\sqrt{13})x^2}}(6+(5+\sqrt{13})x^2)\operatorname{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right),\frac{1}{6}(-13+5\sqrt{13})\right)}{\sqrt{3+5x^2+x^4}}$$

output
```
1/3*x*(x^2+3)*(x^4+5*x^2+3)^(3/2)+203/30*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+3
)^(1/2)-1/15*x*(12*x^2+5)*(x^4+5*x^2+3)^(1/2)+5/3*(1/(36+x^2*(30+6*13^(1/2
))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13^(1/2))^(1/2
)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13
^(1/2)))*6^(1/2)/(5+13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/
2))))^(1/2)/(x^4+5*x^2+3)^(1/2)-203/180*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)
*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*
(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(
30+6*13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^
4+5*x^2+3)^(1/2)
```

3.165.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.66 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.78

$$\int (2+3x^2) \left(3+5x^2+x^4\right)^{3/2} dx = \frac{4x(120+434x^2+550x^4+293x^6+65x^8+5x^{10})+203i\sqrt{2}(-5+\sqrt{13})\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{}}}{}$$

input

```
Integrate[(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

output

```
(4*x*(120 + 434*x^2 + 550*x^4 + 293*x^6 + 65*x^8 + 5*x^10) + (203*I)*Sqrt[
2]*(-5 + Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 +
Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5
*Sqrt[13])/6] - I*Sqrt[2]*(-715 + 203*Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^
2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/
(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(60*Sqrt[3 + 5*x^2 + x^4])
```

3.165.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1490, 27, 1490, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3x^2+2) \left(x^4+5x^2+3\right)^{3/2} dx$$

$$\Big\downarrow \text{1490}$$

$$\frac{1}{21}\int 21(3-4x^2)\sqrt{x^4+5x^2+3}dx+\frac{1}{3}x(x^2+3)\left(x^4+5x^2+3\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\int (3-4x^2)\sqrt{x^4+5x^2+3}dx+\frac{1}{3}x(x^2+3)\left(x^4+5x^2+3\right)^{3/2}$$

$$\Big\downarrow \text{1490}$$

$$\frac{1}{15}\int \frac{203x^2+150}{\sqrt{x^4+5x^2+3}}dx+\frac{1}{3}x\left(x^2+3\right)\left(x^4+5x^2+3\right)^{3/2}-\frac{1}{15}x\left(12x^2+5\right)\sqrt{x^4+5x^2+3}$$

$$\Big\downarrow \text{ 1503}$$

$$\frac{1}{15}\left(150\int \frac{1}{\sqrt{x^4+5x^2+3}}dx+203\int \frac{x^2}{\sqrt{x^4+5x^2+3}}dx\right)+\frac{1}{3}x\left(x^2+3\right)\left(x^4+5x^2+3\right)^{3/2}-\frac{1}{15}x\left(12x^2+5\right)\sqrt{x^4+5x^2+3}$$

$$\Big\downarrow \text{ 1412}$$

$$\frac{1}{15}\left(203\int \frac{x^2}{\sqrt{x^4+5x^2+3}}dx+\frac{25\sqrt{\frac{6}{5+\sqrt{13}}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right.\right.}{\sqrt{x^4+5x^2+3}}\right.$$
$$\frac{1}{3}x\left(x^2+3\right)\left(x^4+5x^2+3\right)^{3/2}-\frac{1}{15}x\left(12x^2+5\right)\sqrt{x^4+5x^2+3}$$

$$\Big\downarrow \text{ 1455}$$

$$\frac{1}{15}\left(\frac{25\sqrt{\frac{6}{5+\sqrt{13}}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}+203\right.$$
$$\frac{1}{3}x\left(x^2+3\right)\left(x^4+5x^2+3\right)^{3/2}-\frac{1}{15}x\left(12x^2+5\right)\sqrt{x^4+5x^2+3}$$

input
```
Int[(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

output
```
-1/15*(x*(5 + 12*x^2)*Sqrt[3 + 5*x^2 + x^4]) + (x*(3 + x^2)*(3 + 5*x^2 + x
^4)^(3/2))/3 + (203*((x*(5 + Sqrt[13] + 2*x^2))/(2*Sqrt[3 + 5*x^2 + x^4])
- (Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13]
)*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x
], (-13 + 5*Sqrt[13])/6])/(2*Sqrt[3 + 5*x^2 + x^4])) + (25*Sqrt[6/(5 + Sqr
t[13])]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 +
Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[1
3])/6])/Sqrt[3 + 5*x^2 + x^4])/15
```

3.165. $\int \left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2}\,dx$

3.165.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

3.165. $\int (2+3x^2)(3+5x^2+x^4)^{3/2}\,dx$

3.165.4 Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.73

method	result
risch	$\frac{x(5x^6+40x^4+78x^2+40)\sqrt{x^4+5x^2+3}}{15}+\frac{60\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}-\frac{2436\sqrt{1-(}}{}$
default	$\frac{8x^5\sqrt{x^4+5x^2+3}}{3}+\frac{26x^3\sqrt{x^4+5x^2+3}}{5}+\frac{8x\sqrt{x^4+5x^2+3}}{3}+\frac{60\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\right.}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}$
elliptic	$\frac{8x^5\sqrt{x^4+5x^2+3}}{3}+\frac{26x^3\sqrt{x^4+5x^2+3}}{5}+\frac{8x\sqrt{x^4+5x^2+3}}{3}+\frac{60\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\right.}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}$

input
```
int((3*x^2+2)*(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*x*(5*x^6+40*x^4+78*x^2+40)*(x^4+5*x^2+3)^(1/2)+60/(-30+6*13^(1/2))^(1
/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x
^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*3
9^(1/2))-2436/5/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(
1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^(1/2))*(Ellipti
cF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*
(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))
```

3.165.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.45

$$\int (2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx=\frac{203\left(\sqrt{13}\sqrt{2}x-5\sqrt{2}x\right)\sqrt{\sqrt{13}-5}\,E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{13}-5}}{2x}\right)\mid\frac{5}{6}\sqrt{13}+\frac{19}{6}\right)-\left(153\sqrt{13}\sqrt{2}x-126\right.}{}$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
1/60*(203*(sqrt(13)*sqrt(2)*x - 5*sqrt(2)*x)*sqrt(sqrt(13) - 5)*elliptic_e
(arcsin(1/2*sqrt(2)*sqrt(sqrt(13) - 5)/x), 5/6*sqrt(13) + 19/6) - (153*sqr
t(13)*sqrt(2)*x - 1265*sqrt(2)*x)*sqrt(sqrt(13) - 5)*elliptic_f(arcsin(1/2
*sqrt(2)*sqrt(sqrt(13) - 5)/x), 5/6*sqrt(13) + 19/6) + 4*(5*x^8 + 40*x^6 +
 78*x^4 + 40*x^2 + 203)*sqrt(x^4 + 5*x^2 + 3))/x
```

3.165.6 Sympy [F]

$$\int (2+3x^2) (3+5x^2+x^4)^{3/2}\, dx = \int (3x^2+2) (x^4+5x^2+3)^{\frac{3}{2}}\, dx$$

input
```
integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)
```

output
```
Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2), x)
```

3.165.7 Maxima [F]

$$\int (2+3x^2) (3+5x^2+x^4)^{3/2}\, dx = \int (x^4+5\,x^2+3)^{\frac{3}{2}} (3\,x^2+2)\, dx$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2), x)
```

3.165.8 Giac [F]

$$\int (2+3x^2) (3+5x^2+x^4)^{3/2}\, dx = \int (x^4+5\,x^2+3)^{\frac{3}{2}} (3\,x^2+2)\, dx$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

output
```
integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2), x)
```

3.165.9 Mupad [F(-1)]

Timed out.

$$\int (2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx=\int \left(3\,x^2+2\right)\,\left(x^4+5\,x^2+3\right)^{3/2}\,dx$$

input
```
int((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2),x)
```

output
```
int((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2), x)
```

3.166 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^2}\,dx$

3.166.1 Optimal result

Integrand size = 25, antiderivative size = 312

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^2}\,dx = \frac{412x(5+\sqrt{13}+2x^2)}{35\sqrt{3+5x^2+x^4}} + \frac{1}{35}x(655+129x^2)\sqrt{3+5x^2+x^4} - \frac{(14-3x^2)(3+5x^2+x^4)^{3/2}}{7x} - \frac{206\sqrt{\frac{2}{3}(5+\sqrt{13})}\sqrt{\frac{6+(5-\sqrt{13})x^2}{6+(5+\sqrt{13})x^2}}(6+(5+\sqrt{13})x^2)E\left(\arctan\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right)|\frac{1}{6}(-13+5\sqrt{13})\right)}{35\sqrt{3+5x^2+x^4}} + \frac{19\sqrt{\frac{3}{2(5+\sqrt{13})}}\sqrt{\frac{6+(5-\sqrt{13})x^2}{6+(5+\sqrt{13})x^2}}(6+(5+\sqrt{13})x^2)\operatorname{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right),\frac{1}{6}(-13+5\sqrt{13})\right)}{\sqrt{3+5x^2+x^4}}$$

output
```
-1/7*(-3*x^2+14)*(x^4+5*x^2+3)^(3/2)/x+412/35*x*(5+2*x^2+13^(1/2))/(x^4+5*
x^2+3)^(1/2)+1/35*x*(129*x^2+655)*(x^4+5*x^2+3)^(1/2)+19/2*(1/(36+x^2*(30+
6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13^(1
/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+
x^2*(5+13^(1/2)))*6^(1/2)/(5+13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*
(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)-206/105*(1/(36+x^2*(30+6*13^(1/2)
)))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)
/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^
(1/2)))*(30+6*13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^
(1/2)/(x^4+5*x^2+3)^(1/2)
```

3.166.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.51 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.75

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^2}\,dx = \frac{-1260+3884x^4+2130x^6+418x^8+30x^{10}+412i\sqrt{2}(-5+\sqrt{13})\,x\sqrt{}}{}$$

input
```
Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^2,x]
```

output
```
(-1260 + 3884*x^4 + 2130*x^6 + 418*x^8 + 30*x^10 + (412*I)*Sqrt[2]*(-5 + S
qrt[13])*x*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13]
 + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13
])/6] - I*Sqrt[2]*(-65 + 412*Sqrt[13])*x*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5
+ Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sq
rt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(70*x*Sqrt[3 + 5*x^2 + x^4])
```

3.166.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1594, 25, 1490, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x^2+2)(x^4+5x^2+3)^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{1594}$$

$$-\frac{3}{7}\int -\left((43x^2+88)\sqrt{x^4+5x^2+3}\right)dx-\frac{(14-3x^2)(x^4+5x^2+3)^{3/2}}{7x}$$

$$\Big\downarrow \text{25}$$

$$\frac{3}{7}\int (43x^2+88)\sqrt{x^4+5x^2+3}dx-\frac{(14-3x^2)(x^4+5x^2+3)^{3/2}}{7x}$$

$$\Big\downarrow \text{1490}$$

$$\frac{3}{7}\left(\frac{1}{15}\int \frac{824x^2+1995}{\sqrt{x^4+5x^2+3}}dx+\frac{1}{15}x\sqrt{x^4+5x^2+3}(129x^2+655)\right)-\frac{(14-3x^2)\,(x^4+5x^2+3)^{3/2}}{7x}$$

$\big\downarrow$ 1503

$$\frac{3}{7}\left(\frac{1}{15}\left(1995\int \frac{1}{\sqrt{x^4+5x^2+3}}dx+824\int \frac{x^2}{\sqrt{x^4+5x^2+3}}dx\right)+\frac{1}{15}x\sqrt{x^4+5x^2+3}(129x^2+655)\right)-\frac{(14-3x^2)\,(x^4+5x^2+3)^{3/2}}{7x}$$

$\big\downarrow$ 1412

$$\frac{3}{7}\left(\frac{1}{15}\left(824\int \frac{x^2}{\sqrt{x^4+5x^2+3}}dx+\frac{665\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\right.}\right.\right.}{\sqrt{x^4+5x^2+3}}\right.\right.$$

$$\frac{(14-3x^2)\,(x^4+5x^2+3)^{3/2}}{7x}$$

$\big\downarrow$ 1455

$$\frac{3}{7}\left(\frac{1}{15}\left(\frac{665\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}}\left(5+\sqrt{13}\right)x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right.}{\sqrt{x^4+5x^2+3}}\right.\right.$$

$$\frac{(14-3x^2)\,(x^4+5x^2+3)^{3/2}}{7x}$$

input
```
Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^2,x]
```

output
```
-1/7*((14 - 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x + (3*((x*(655 + 129*x^2)*Sqr
t[3 + 5*x^2 + x^4])/15 + (824*((x*(5 + Sqrt[13] + 2*x^2))/(2*Sqrt[3 + 5*x^
2 + x^4]) - (Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5
+ Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt
[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(2*Sqrt[3 + 5*x^2 + x^4])) + (665*Sqrt
[3/(2*(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x
^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x],
(-13 + 5*Sqrt[13])/6])/Sqrt[3 + 5*x^2 + x^4])/15))/7
```

3.166. $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^2}\,dx$

3.166.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

3.166. $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^2} \, dx$

rule 1594
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((d*(m
+ 4*p + 3) + e*(m + 1)*x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Simp[2*(p/(f^2
*(m + 1)*(m + 4*p + 3)))   Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Si
mp[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*
x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && G
tQ[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
 || IntegerQ[m])
```

3.166.4 Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.75

method	result
risch	$\frac{15x^{10}+209x^8+1065x^6+1942x^4-630}{35x\sqrt{x^4+5x^2+3}}+\frac{342\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}-\frac{29664\sqrt{1-}}{}$
default	$\frac{3x^5\sqrt{x^4+5x^2+3}}{7}+\frac{134x^3\sqrt{x^4+5x^2+3}}{35}+10x\sqrt{x^4+5x^2+3}+\frac{342\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+}}{6}\right.}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}$
elliptic	$\frac{3x^5\sqrt{x^4+5x^2+3}}{7}+\frac{134x^3\sqrt{x^4+5x^2+3}}{35}+10x\sqrt{x^4+5x^2+3}+\frac{342\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+}}{6}\right.}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}$

input
```
int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/35*(15*x^10+209*x^8+1065*x^6+1942*x^4-630)/x/(x^4+5*x^2+3)^(1/2)+342/(-3
0+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2
))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5
/6*3^(1/2)+1/6*39^(1/2))-29664/35/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(
1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+
13^(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)
)-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))
```

3.166.5 Fricas [F]

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^2}\,dx = \int \frac{(x^4+5\,x^2+3)^{\frac{3}{2}}(3\,x^2+2)}{x^2}\,dx$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
integral((3*x^6 + 17*x^4 + 19*x^2 + 6)*sqrt(x^4 + 5*x^2 + 3)/x^2, x)
```

3.166.6 Sympy [F]

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^2}\,dx = \int \frac{(3x^2+2)(x^4+5x^2+3)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**2,x)
```

output
```
Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x**2, x)
```

3.166.7 Maxima [F]

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^2}\,dx = \int \frac{(x^4+5\,x^2+3)^{\frac{3}{2}}(3\,x^2+2)}{x^2}\,dx$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^2, x)
```

3.166.8 Giac [F]

$$\int \frac{(2+3x^2)\,(3+5x^2+x^4)^{3/2}}{x^2}\,dx = \int \frac{(x^4+5\,x^2+3)^{\frac{3}{2}}(3\,x^2+2)}{x^2}\,dx$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^2,x, algorithm="giac")
```

output
```
integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^2, x)
```

3.166.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x^2)\,(3+5x^2+x^4)^{3/2}}{x^2}\,dx = \int \frac{(3\,x^2+2)\,\left(x^4+5\,x^2+3\right)^{3/2}}{x^2}\,dx$$

input
```
int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^2,x)
```

output
```
int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^2, x)
```

3.167 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^4}\,dx$

3.167.1 Optimal result

Integrand size = 25, antiderivative size = 314

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^4}\,dx = \frac{949x(5+\sqrt{13}+2x^2)}{30\sqrt{3+5x^2+x^4}} - \frac{13(24-5x^2)\sqrt{3+5x^2+x^4}}{15x} - \frac{(10-9x^2)(3+5x^2+x^4)^{3/2}}{15x^3} - \frac{949\sqrt{\frac{1}{6}(5+\sqrt{13})}\sqrt{\frac{6+(5-\sqrt{13})x^2}{6+(5+\sqrt{13})x^2}}(6+(5+\sqrt{13})x^2)E\left(\arctan\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right)|\frac{1}{6}(-13+5\sqrt{13})\right)}{30\sqrt{3+5x^2+x^4}} + \frac{65\sqrt{\frac{2}{3(5+\sqrt{13})}}\sqrt{\frac{6+(5-\sqrt{13})x^2}{6+(5+\sqrt{13})x^2}}(6+(5+\sqrt{13})x^2)\,\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right),\frac{1}{6}(-13+5\sqrt{13})\right)}{\sqrt{3+5x^2+x^4}}$$

output
```
-1/15*(-9*x^2+10)*(x^4+5*x^2+3)^(3/2)/x^3+949/30*x*(5+2*x^2+13^(1/2))/(x^4
+5*x^2+3)^(1/2)-13/15*(-5*x^2+24)*(x^4+5*x^2+3)^(1/2)/x+65/3*(1/(36+x^2*(3
0+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13^
(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(
6+x^2*(5+13^(1/2)))*6^(1/2)/(5+13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^
2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)-949/180*(1/(36+x^2*(30+6*13^(1/
2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/
2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+1
3^(1/2)))*(30+6*13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2)))
)^(1/2)/(x^4+5*x^2+3)^(1/2)
```

3.167.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.25 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.79

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^4}\,dx = \frac{4(-90-1155x^2-1405x^4+192x^6+145x^8+9x^{10})+949i\sqrt{2}(-5+\sqrt{}}{}$$

input
```
Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^4,x]
```

output
```
(4*(-90 - 1155*x^2 - 1405*x^4 + 192*x^6 + 145*x^8 + 9*x^10) + (949*I)*Sqrt
[2]*(-5 + Sqrt[13])*x^3*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt
[5 + Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6
 + (5*Sqrt[13])/6] - (13*I)*Sqrt[2]*(-65 + 73*Sqrt[13])*x^3*Sqrt[(-5 + Sqr
t[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*Arc
Sinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(60*x^3*Sqrt[3 + 5
*x^2 + x^4])
```

3.167.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1594, 27, 1594, 25, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x^2+2)(x^4+5x^2+3)^{3/2}}{x^4}\,dx$$

$$\Big\downarrow \text{1594}$$

$$-\frac{1}{5}\int -\frac{13(5x^2+8)\sqrt{x^4+5x^2+3}}{x^2}dx-\frac{(10-9x^2)(x^4+5x^2+3)^{3/2}}{15x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{13}{5}\int \frac{(5x^2+8)\sqrt{x^4+5x^2+3}}{x^2}dx-\frac{(10-9x^2)(x^4+5x^2+3)^{3/2}}{15x^3}$$

$$\Big\downarrow \text{1594}$$

$$\frac{13}{5}\left(-\frac{1}{3}\int -\frac{73x^2+150}{\sqrt{x^4+5x^2+3}}dx-\frac{\sqrt{x^4+5x^2+3}(24-5x^2)}{3x}\right)-\frac{(10-9x^2)\left(x^4+5x^2+3\right)^{3/2}}{15x^3}$$

$\Big\downarrow$ 25

$$\frac{13}{5}\left(\frac{1}{3}\int \frac{73x^2+150}{\sqrt{x^4+5x^2+3}}dx-\frac{(24-5x^2)\sqrt{x^4+5x^2+3}}{3x}\right)-\frac{(10-9x^2)\left(x^4+5x^2+3\right)^{3/2}}{15x^3}$$

$\Big\downarrow$ 1503

$$\frac{13}{5}\left(\frac{1}{3}\left(150\int \frac{1}{\sqrt{x^4+5x^2+3}}dx+73\int \frac{x^2}{\sqrt{x^4+5x^2+3}}dx\right)-\frac{(24-5x^2)\sqrt{x^4+5x^2+3}}{3x}\right)-\frac{(10-9x^2)\left(x^4+5x^2+3\right)^{3/2}}{15x^3}$$

$\Big\downarrow$ 1412

$$\frac{13}{5}\left(\frac{1}{3}\left(73\int \frac{x^2}{\sqrt{x^4+5x^2+3}}dx+\frac{25\sqrt{\frac{6}{5+\sqrt{13}}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\right.\right.}{\sqrt{x^4+5x^2+3}}\right.\right.$$

$$\frac{(10-9x^2)\left(x^4+5x^2+3\right)^{3/2}}{15x^3}$$

$\Big\downarrow$ 1455

$$\frac{13}{5}\left(\frac{1}{3}\left(\frac{25\sqrt{\frac{6}{5+\sqrt{13}}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}+\right.\right.$$

$$\frac{(10-9x^2)\left(x^4+5x^2+3\right)^{3/2}}{15x^3}$$

input
```
Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^4,x]
```

3.167. $\int \frac{(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}}{x^4}\,dx$

output
```
-1/15*((10 - 9*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^3 + (13*(-1/3*((24 - 5*x^2)
*Sqrt[3 + 5*x^2 + x^4])/x + (73*((x*(5 + Sqrt[13] + 2*x^2))/(2*Sqrt[3 + 5*
x^2 + x^4]) - (Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (
5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sq
rt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(2*Sqrt[3 + 5*x^2 + x^4])) + (25*Sqr
t[6/(5 + Sqrt[13])]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)
]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-1
3 + 5*Sqrt[13])/6])/Sqrt[3 + 5*x^2 + x^4])/3))/5
```

3.167.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

3.167. $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^4} dx$

rule 1594
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((d*(m
 + 4*p + 3) + e*(m + 1)*x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Simp[2*(p/(f^2
*(m + 1)*(m + 4*p + 3)))   Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Si
mp[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*
x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && G
tQ[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
 || IntegerQ[m])
```

3.167.4 Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.76

method	result
risch	$\frac{9x^{10}+145x^8+192x^6-1405x^4-1155x^2-90}{15x^3\sqrt{x^4+5x^2+3}}+\frac{780\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}-\frac{1138}{}$
default	$-\frac{67\sqrt{x^4+5x^2+3}}{3x}+\frac{3x^3\sqrt{x^4+5x^2+3}}{5}+\frac{20x\sqrt{x^4+5x^2+3}}{3}+\frac{780\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}\right.}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}$
elliptic	$-\frac{67\sqrt{x^4+5x^2+3}}{3x}+\frac{3x^3\sqrt{x^4+5x^2+3}}{5}+\frac{20x\sqrt{x^4+5x^2+3}}{3}+\frac{780\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}\right.}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}$

input
```
int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/15*(9*x^10+145*x^8+192*x^6-1405*x^4-1155*x^2-90)/x^3/(x^4+5*x^2+3)^(1/2)
+780/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6
*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))
^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-11388/5/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1
/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1
/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*3
9^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))
```

3.167.5 Fricas [F]

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^4}\,dx = \int \frac{(x^4+5\,x^2+3)^{\frac{3}{2}}(3\,x^2+2)}{x^4}\,dx$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
integral((3*x^6 + 17*x^4 + 19*x^2 + 6)*sqrt(x^4 + 5*x^2 + 3)/x^4, x)
```

3.167.6 Sympy [F]

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^4}\,dx = \int \frac{(3x^2+2)(x^4+5x^2+3)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**4,x)
```

output
```
Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x**4, x)
```

3.167.7 Maxima [F]

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^4}\,dx = \int \frac{(x^4+5\,x^2+3)^{\frac{3}{2}}(3\,x^2+2)}{x^4}\,dx$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^4, x)
```

3.167.8 Giac [F]

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^4}\,dx = \int \frac{(x^4+5x^2+3)^{\frac{3}{2}}(3x^2+2)}{x^4}\,dx$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^4,x, algorithm="giac")
```

output
```
integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^4, x)
```

3.167.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^4}\,dx = \int \frac{(3x^2+2)\,(x^4+5x^2+3)^{3/2}}{x^4}\,dx$$

input
```
int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^4,x)
```

output
```
int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^4, x)
```

3.168 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^6}\,dx$

3.168.1 Optimal result

Integrand size = 25, antiderivative size = 331

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^6}\,dx = \frac{361x\left(5+\sqrt{13}+2x^2\right)}{15\sqrt{3+5x^2+x^4}} - \frac{722\sqrt{3+5x^2+x^4}}{15x} - \frac{(40-87x^2)\sqrt{3+5x^2+x^4}}{5x^3} - \frac{(2-5x^2)(3+5x^2+x^4)^{3/2}}{5x^5} - \frac{361\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)\middle|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{15\sqrt{3+5x^2+x^4}} + \frac{103\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)\operatorname{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{3+5x^2+x^4}}$$

output
```
-1/5*(-5*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^5+361/15*x*(5+2*x^2+13^(1/2))/(x^4+5
*x^2+3)^(1/2)-722/15*(x^4+5*x^2+3)^(1/2)/x-1/5*(-87*x^2+40)*(x^4+5*x^2+3)^
(1/2)/x^3-361/90*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)
))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),
1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(1/2))^(1/2)*((
6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)+103*(1
/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(
x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2
))^(1/2))*(6+x^2*(5+13^(1/2)))*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))
^(1/2)/(x^4+5*x^2+3)^(1/2)/(30+6*13^(1/2))^(1/2)
```

3.168.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.27 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.74

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^6}\,dx = \frac{-108-810x^2-3438x^4-4040x^6-634x^8+30x^{10}+361i\sqrt{2}(-5+\sqrt{}}{}$$

input
```
Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^6,x]
```

output
```
(-108 - 810*x^2 - 3438*x^4 - 4040*x^6 - 634*x^8 + 30*x^10 + (361*I)*Sqrt[2
]*(-5 + Sqrt[13])*x^5*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5
 + Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 +
 (5*Sqrt[13])/6] - I*Sqrt[2]*(-260 + 361*Sqrt[13])*x^5*Sqrt[(-5 + Sqrt[13]
 - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[
Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(30*x^5*Sqrt[3 + 5*x^2
+ x^4])
```

3.168.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {1594, 27, 1594, 25, 1604, 25, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x^2+2)(x^4+5x^2+3)^{3/2}}{x^6}\,dx$$

$$\Big\downarrow \text{1594}$$

$$-\frac{1}{5}\int -\frac{3(29x^2+40)\sqrt{x^4+5x^2+3}}{x^4}dx-\frac{(2-5x^2)(x^4+5x^2+3)^{3/2}}{5x^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{5}\int \frac{(29x^2+40)\sqrt{x^4+5x^2+3}}{x^4}dx-\frac{(2-5x^2)(x^4+5x^2+3)^{3/2}}{5x^5}$$

$$\Big\downarrow \text{1594}$$

$$\frac{3}{5}\left(-\frac{1}{3}\int -\frac{515x^2+722}{x^2\sqrt{x^4+5x^2+3}}dx-\frac{\sqrt{x^4+5x^2+3}(40-87x^2)}{3x^3}\right)-\frac{(2-5x^2)(x^4+5x^2+3)^{3/2}}{5x^5}$$

$\downarrow$ 25

$$\frac{3}{5}\left(\frac{1}{3}\int \frac{515x^2+722}{x^2\sqrt{x^4+5x^2+3}}dx-\frac{(40-87x^2)\sqrt{x^4+5x^2+3}}{3x^3}\right)-\frac{(2-5x^2)(x^4+5x^2+3)^{3/2}}{5x^5}$$

$\downarrow$ 1604

$$\frac{3}{5}\left(\frac{1}{3}\left(-\frac{1}{3}\int -\frac{722x^2+1545}{\sqrt{x^4+5x^2+3}}dx-\frac{722\sqrt{x^4+5x^2+3}}{3x}\right)-\frac{(40-87x^2)\sqrt{x^4+5x^2+3}}{3x^3}\right)-\frac{(2-5x^2)(x^4+5x^2+3)^{3/2}}{5x^5}$$

$\downarrow$ 25

$$\frac{3}{5}\left(\frac{1}{3}\left(\frac{1}{3}\int \frac{722x^2+1545}{\sqrt{x^4+5x^2+3}}dx-\frac{722\sqrt{x^4+5x^2+3}}{3x}\right)-\frac{(40-87x^2)\sqrt{x^4+5x^2+3}}{3x^3}\right)-\frac{(2-5x^2)(x^4+5x^2+3)^{3/2}}{5x^5}$$

$\downarrow$ 1503

$$\frac{3}{5}\left(\frac{1}{3}\left(\frac{1}{3}\left(1545\int \frac{1}{\sqrt{x^4+5x^2+3}}dx+722\int \frac{x^2}{\sqrt{x^4+5x^2+3}}dx\right)-\frac{722\sqrt{x^4+5x^2+3}}{3x}\right)-\frac{(40-87x^2)\sqrt{x^4+5x}}{3x^3}\right.$$
$$\frac{(2-5x^2)(x^4+5x^2+3)^{3/2}}{5x^5}$$

$\downarrow$ 1412

$$\frac{3}{5}\left(\frac{1}{3}\left(\frac{1}{3}\left(722\int \frac{x^2}{\sqrt{x^4+5x^2+3}}dx+\frac{515\sqrt{\frac{3}{2(5+\sqrt{13})}}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}((5+\sqrt{13})x^2+6)\,\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}}(\right.\right.}{\sqrt{x^4+5x^2+3}}\right.\right.\right.$$
$$\frac{(2-5x^2)(x^4+5x^2+3)^{3/2}}{5x^5}$$

$\downarrow$ 1455

3.168. $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^6}\,dx$

$$\frac{3}{5}\left(\frac{1}{3}\left(\frac{1}{3}\left(\frac{515\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{\right.\right.}{\sqrt{x^4+5x^2+3}}\right.\right.\right.$$

$$\frac{\left(2-5x^2\right)\left(x^4+5x^2+3\right)^{3/2}}{5x^5}$$

input
```
Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^6,x]
```

output
```
-1/5*((2 - 5*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^5 + (3*(-1/3*((40 - 87*x^2)*S
qrt[3 + 5*x^2 + x^4])/x^3 + ((-722*Sqrt[3 + 5*x^2 + x^4])/(3*x) + (722*((x
*(5 + Sqrt[13] + 2*x^2))/(2*Sqrt[3 + 5*x^2 + x^4]) - (Sqrt[(5 + Sqrt[13])/
6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[
13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6
])/(2*Sqrt[3 + 5*x^2 + x^4])) + (515*Sqrt[3/(2*(5 + Sqrt[13]))]*Sqrt[(6 +
(5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*Ell
ipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/Sqrt[3 + 5
*x^2 + x^4])/3)/3))/5
```

3.168.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1594
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((d*(m
+ 4*p + 3) + e*(m + 1)*x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Simp[2*(p/(f^2
*(m + 1)*(m + 4*p + 3)))   Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Si
mp[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*
x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && G
tQ[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
 || IntegerQ[m])
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.168. $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^6} \, dx$

3.168.4 Maple [A] (verified)

Time = 4.09 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.72

method	result
risch	$\frac{15x^{10}-317x^{8}-2020x^{6}-1719x^{4}-405x^{2}-54}{15x^{5}\sqrt{x^{4}+5x^{2}+3}}+\frac{618\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^{2}}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^{2}}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^{4}+5x^{2}+3}}-866$
default	$-\frac{6\sqrt{x^{4}+5x^{2}+3}}{5x^{5}}-\frac{7\sqrt{x^{4}+5x^{2}+3}}{x^{3}}-\frac{392\sqrt{x^{4}+5x^{2}+3}}{15x}+\frac{618\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^{2}}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^{2}}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\right.}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^{4}+5x^{2}+3}}$
elliptic	$-\frac{6\sqrt{x^{4}+5x^{2}+3}}{5x^{5}}-\frac{7\sqrt{x^{4}+5x^{2}+3}}{x^{3}}-\frac{392\sqrt{x^{4}+5x^{2}+3}}{15x}+\frac{618\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^{2}}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^{2}}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\right.}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^{4}+5x^{2}+3}}$

input
```
int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
1/15*(15*x^10-317*x^8-2020*x^6-1719*x^4-405*x^2-54)/x^5/(x^4+5*x^2+3)^(1/2
)+618/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/
6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2)
)^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-8664/5/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1
/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1
/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*3
9^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))
```

3.168.5 Fricas [F]

$$\int \frac{(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}}{x^6}\,dx=\int \frac{\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}\left(3\,x^2+2\right)}{x^6}\,dx$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^6,x, algorithm="fricas")
```

output
```
integral((3*x^6 + 17*x^4 + 19*x^2 + 6)*sqrt(x^4 + 5*x^2 + 3)/x^6, x)
```

3.168.6 Sympy [F]

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^6}\,dx = \int \frac{(3x^2+2)(x^4+5x^2+3)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**6,x)
```

output
```
Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x**6, x)
```

3.168.7 Maxima [F]

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^6}\,dx = \int \frac{(x^4+5x^2+3)^{\frac{3}{2}}(3x^2+2)}{x^6}\,dx$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^6,x, algorithm="maxima")
```

output
```
integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^6, x)
```

3.168.8 Giac [F]

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^6}\,dx = \int \frac{(x^4+5x^2+3)^{\frac{3}{2}}(3x^2+2)}{x^6}\,dx$$

input
```
integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^6,x, algorithm="giac")
```

output
```
integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^6, x)
```

3.168.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x^2)\,(3+5x^2+x^4)^{3/2}}{x^6}\,dx = \int \frac{(3\,x^2+2)\,\left(x^4+5\,x^2+3\right)^{3/2}}{x^6}\,dx$$

input
```
int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^6,x)
```

output
```
int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^6, x)
```

3.169 $\int \frac{x^5(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

3.169.1 Optimal result

Integrand size = 27, antiderivative size = 153

$$\int \frac{x^5(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{Bx^4\sqrt{a+bx^2+cx^4}}{6c} + \frac{(15b^2B-18Abc-16aBc-2c(5bB-6Ac)x^2)\sqrt{a+bx^2+cx^4}}{48c^3} - \frac{(5b^3B-6Ab^2c-12abBc+8aAc^2)\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{7/2}}$$

output
```
-1/32*(8*A*a*c^2-6*A*b^2*c-12*B*a*b*c+5*B*b^3)*arctanh(1/2*(2*c*x^2+b)/c^(
1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(7/2)+1/6*B*x^4*(c*x^4+b*x^2+a)^(1/2)/c+1/48
*(15*B*b^2-18*A*b*c-16*B*a*c-2*c*(-6*A*c+5*B*b)*x^2)*(c*x^4+b*x^2+a)^(1/2)
/c^3
```

3.169.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.88

$$\int \frac{x^5(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{\sqrt{a+bx^2+cx^4}(15b^2B-18Abc-16aBc-10bBcx^2+12Ac^2x^2+8Bc^2x^4)}{48c^3} + \frac{(5b^3B-6Ab^2c-12abBc+8aAc^2)\log\left(b+2cx^2-2\sqrt{c}\sqrt{a+bx^2+cx^4}\right)}{32c^{7/2}}$$

input
```
Integrate[(x^5*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(Sqrt[a + b*x^2 + c*x^4]*(15*b^2*B - 18*A*b*c - 16*a*B*c - 10*b*B*c*x^2 +
12*A*c^2*x^2 + 8*B*c^2*x^4))/(48*c^3) + ((5*b^3*B - 6*A*b^2*c - 12*a*b*B*c
 + 8*a*A*c^2)*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/(32*c^
(7/2))
```

3.169.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1578, 1236, 27, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\left(A+Bx^2\right)}{\sqrt{a+bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{x^4\left(Bx^2+A\right)}{\sqrt{cx^4+bx^2+a}}dx^2$$

$$\Big\downarrow \text{1236}$$

$$\frac{1}{2}\left(\frac{\int -\frac{x^2\left((5bB-6Ac)x^2+4aB\right)}{2\sqrt{cx^4+bx^2+a}}dx^2}{3c}+\frac{Bx^4\sqrt{a+bx^2+cx^4}}{3c}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{Bx^4\sqrt{a+bx^2+cx^4}}{3c}-\frac{\int \frac{x^2\left((5bB-6Ac)x^2+4aB\right)}{\sqrt{cx^4+bx^2+a}}dx^2}{6c}\right)$$

$$\Big\downarrow \text{1225}$$

$$\frac{1}{2}\left(\frac{Bx^4\sqrt{a+bx^2+cx^4}}{3c}-\frac{\frac{3\left(8aAc^2-12abBc-6Ab^2c+5b^3B\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx^2}{8c^2}-\frac{\sqrt{a+bx^2+cx^4}\left(-16aBc-2cx^2(5bB-6Ac)-18Abc+15\right.}{4c^2}}{6c}\right.$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}\left(\frac{Bx^4\sqrt{a+bx^2+cx^4}}{3c}-\frac{\frac{3(8aAc^2-12abBc-6Ab^2c+5b^3B)\int\frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}}{4c^2}-\frac{\sqrt{a+bx^2+cx^4}(-16aBc-2cx^2(5bB-6Ac)-18Abc}{4c^2}}{6c}\right.$$

$$\Big\downarrow 219$$

$$\frac{1}{2}\left(\frac{Bx^4\sqrt{a+bx^2+cx^4}}{3c}-\frac{\frac{3(8aAc^2-12abBc-6Ab^2c+5b^3B)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8c^{5/2}}-\frac{\sqrt{a+bx^2+cx^4}(-16aBc-2cx^2(5bB-6Ac)-}{4c^2}}{6c}\right.$$

input
```
Int[(x^5*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
((B*x^4*Sqrt[a + b*x^2 + c*x^4])/(3*c) - (-1/4*((15*b^2*B - 18*A*b*c - 16*
a*B*c - 2*c*(5*b*B - 6*A*c)*x^2)*Sqrt[a + b*x^2 + c*x^4])/c^2 + (3*(5*b^3*
B - 6*A*b^2*c - 12*a*b*B*c + 8*a*A*c^2)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*S
qrt[a + b*x^2 + c*x^4])])/(8*c^(5/2)))/(6*c))/2
```

3.169.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.169.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.82

method	result
risch	$-\frac{\left(-8B\,c^{2}x^{4}-12A\,c^{2}x^{2}+10Bbc\,x^{2}+18Abc+16Bac-15B\,b^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}{48c^{3}}-\frac{\left(8Aa\,c^{2}-6A\,b^{2}c-12Babc+5B\,b^{3}\right)\ln\left(\frac{\frac{b}{2}+c}{\sqrt{}}\right.}{32c^{\frac{7}{2}}}$
pseudoelliptic	$-\frac{3\left(\sqrt{c\,x^{4}+b\,x^{2}+a}\left(\left(\frac{5B\,x^{2}}{9}+A\right)b+\frac{8Ba}{9}\right)c^{\frac{3}{2}}-\frac{2\sqrt{c\,x^{4}+b\,x^{2}+a}\left(\frac{2B\,x^{2}}{3}+A\right)x^{2}c^{\frac{5}{2}}}{3}-\frac{5B\,b^{2}\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{c}}{6}+\frac{2\left(Aa\,c^{2}-\frac{3}{4}A\,b^{2}c-\right.}{}\right.}{8c^{\frac{7}{2}}}$
default	$B\left(\frac{x^{4}\sqrt{c\,x^{4}+b\,x^{2}+a}}{6c}-\frac{5b\,x^{2}\sqrt{c\,x^{4}+b\,x^{2}+a}}{24c^{2}}+\frac{5b^{2}\sqrt{c\,x^{4}+b\,x^{2}+a}}{16c^{3}}-\frac{5b^{3}\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{32c^{\frac{7}{2}}}+\frac{3ba\ln\left(\frac{\frac{b}{2}+}{\sqrt{}}\right.}{}\right.$
elliptic	$\frac{B\,x^{4}\sqrt{c\,x^{4}+b\,x^{2}+a}}{6c}-\frac{5Bb\,x^{2}\sqrt{c\,x^{4}+b\,x^{2}+a}}{24c^{2}}+\frac{5B\,b^{2}\sqrt{c\,x^{4}+b\,x^{2}+a}}{16c^{3}}-\frac{5B\,b^{3}\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{32c^{\frac{7}{2}}}+\frac{3Bba\ln}{}$

input
```
int(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/48*(-8*B*c^2*x^4-12*A*c^2*x^2+10*B*b*c*x^2+18*A*b*c+16*B*a*c-15*B*b^2)*
(c*x^4+b*x^2+a)^(1/2)/c^3-1/32*(8*A*a*c^2-6*A*b^2*c-12*B*a*b*c+5*B*b^3)/c^
(7/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))
```

3.169.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.06

$$\int \frac{x^5(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \left[\frac{3\,(5\,Bb^3+8\,Aac^2-6\,(2\,Bab+Ab^2)c)\sqrt{c}\log\left(-8\,c^2x^4-8\,bcx^2-b^2+4\,\sqrt{cx^4+bx^2+a}(2\,cx^2+b)\sqrt{c}-\right.}{192\,c^4}\right.$$

input
```
integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/192*(3*(5*B*b^3 + 8*A*a*c^2 - 6*(2*B*a*b + A*b^2)*c)*sqrt(c)*log(-8*c^2
*x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) -
 4*a*c) + 4*(8*B*c^3*x^4 + 15*B*b^2*c - 2*(8*B*a + 9*A*b)*c^2 - 2*(5*B*b*c
^2 - 6*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/c^4, 1/96*(3*(5*B*b^3 + 8*A*a*
c^2 - 6*(2*B*a*b + A*b^2)*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(
2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(8*B*c^3*x^4 + 15*B*b
^2*c - 2*(8*B*a + 9*A*b)*c^2 - 2*(5*B*b*c^2 - 6*A*c^3)*x^2)*sqrt(c*x^4 + b
*x^2 + a))/c^4]
```

3.169.6 Sympy [A] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.12

$$\int \frac{x^5(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{\begin{cases} \left(-\frac{a\left(A-\frac{5Bb}{6c}\right)}{2c}-\frac{b\left(-\frac{2Ba}{3c}-\frac{3b\left(A-\frac{5Bb}{6c}\right)}{4c}\right)}{2c}\right)\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{a+bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } a-\frac{b^2}{4c}\neq 0\\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise}\end{cases}\right)+\sqrt{a+bx^2+cx^4} \\ \frac{\frac{2A\left(a^2\sqrt{a+bx^2}-\frac{2a(a+bx^2)^{\frac{3}{2}}}{3}+\frac{(a+bx^2)^{\frac{5}{2}}}{5}\right)}{b^2}+\frac{2B\left(-a^3\sqrt{a+bx^2}+a^2(a+bx^2)^{\frac{3}{2}}-\frac{3a(a+bx^2)^{\frac{5}{2}}}{5}+\frac{(a+bx^2)^{\frac{7}{2}}}{7}\right)}{b^3}}{b} \\ \frac{\frac{Ax^6}{3}+\frac{Bx^8}{4}}{\sqrt{a}} \end{cases}}{2}$$

input
```
integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Piecewise(((-a*(A - 5*B*b/(6*c))/(2*c) - b*(-2*B*a/(3*c) - 3*b*(A - 5*B*b/
(6*c))/(4*c))/(2*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(a + b*x**2 + c*x**4
) + 2*c*x**2)/sqrt(c), Ne(a - b**2/(4*c), 0)), ((b/(2*c) + x**2)*log(b/(2*
c) + x**2)/sqrt(c*(b/(2*c) + x**2)**2), True)) + sqrt(a + b*x**2 + c*x**4)
*(B*x**4/(3*c) + x**2*(A - 5*B*b/(6*c))/(2*c) + (-2*B*a/(3*c) - 3*b*(A - 5
*B*b/(6*c))/(4*c))/c), Ne(c, 0)), ((2*A*(a**2*sqrt(a + b*x**2) - 2*a*(a +
b*x**2)**(3/2)/3 + (a + b*x**2)**(5/2)/5)/b**2 + 2*B*(-a**3*sqrt(a + b*x**
2) + a**2*(a + b*x**2)**(3/2) - 3*a*(a + b*x**2)**(5/2)/5 + (a + b*x**2)**
(7/2)/7)/b**3)/b, Ne(b, 0)), ((A*x**6/3 + B*x**8/4)/sqrt(a), True))/2
```

3.169.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.169.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.89

$$\int \frac{x^5(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{48}\sqrt{cx^4+bx^2+a}\left(2\left(\frac{4\,Bx^2}{c}-\frac{5\,Bbc-6\,Ac^2}{c^3}\right)x^2+\frac{15\,Bb^2-16\,Bac-18\,Abc}{c^3}\right) + \frac{(5\,Bb^3-12\,Babc-6\,Ab^2c+8\,Aac^2)\log\left(\left|2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}+b\right|\right)}{32\,c^{\frac{7}{2}}}$$

input
```
integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/48*sqrt(c*x^4 + b*x^2 + a)*(2*(4*B*x^2/c - (5*B*b*c - 6*A*c^2)/c^3)*x^2
+ (15*B*b^2 - 16*B*a*c - 18*A*b*c)/c^3) + 1/32*(5*B*b^3 - 12*B*a*b*c - 6*A
*b^2*c + 8*A*a*c^2)*log(abs(2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt
(c) + b))/c^(7/2)
```

3.169.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^5\,(B\,x^2+A)}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.170 $\int \frac{x^3(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

3.170.1 Optimal result

Integrand size = 27, antiderivative size = 100

$$\int \frac{x^3(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = -\frac{(3bB-4Ac-2Bcx^2)\sqrt{a+bx^2+cx^4}}{8c^2} + \frac{(3b^2B-4Abc-4aBc)\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{5/2}}$$

output
```
1/16*(-4*A*b*c-4*B*a*c+3*B*b^2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x
^2+a)^(1/2))/c^(5/2)-1/8*(-2*B*c*x^2-4*A*c+3*B*b)*(c*x^4+b*x^2+a)^(1/2)/c^
2
```

3.170.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04

$$\int \frac{x^3(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{(-3bB+4Ac+2Bcx^2)\sqrt{a+bx^2+cx^4}}{8c^2} + \frac{(-3b^2B+4Abc+4aBc)\log\left(bc^2+2c^3x^2-2c^{5/2}\sqrt{a+bx^2+cx^4}\right)}{16c^{5/2}}$$

input
```
Integrate[(x^3*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
((-3*b*B + 4*A*c + 2*B*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c^2) + ((-3*b^2*
B + 4*A*b*c + 4*a*B*c)*Log[b*c^2 + 2*c^3*x^2 - 2*c^(5/2)*Sqrt[a + b*x^2 +
c*x^4]])/(16*c^(5/2))
```

3.170.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1578, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{x^2(Bx^2+A)}{\sqrt{cx^4+bx^2+a}}dx^2$$

$$\Big\downarrow \text{1225}$$

$$\frac{1}{2}\left(\frac{(-4aBc-4Abc+3b^2B)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx^2}{8c^2}-\frac{\sqrt{a+bx^2+cx^4}(-4Ac+3bB-2Bcx^2)}{4c^2}\right)$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}\left(\frac{(-4aBc-4Abc+3b^2B)\int \frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}}{4c^2}-\frac{\sqrt{a+bx^2+cx^4}(-4Ac+3bB-2Bcx^2)}{4c^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{(-4aBc-4Abc+3b^2B)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8c^{5/2}}-\frac{\sqrt{a+bx^2+cx^4}(-4Ac+3bB-2Bcx^2)}{4c^2}\right)$$

input
```
Int[(x^3*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(-1/4*((3*b*B - 4*A*c - 2*B*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/c^2 + ((3*b^2*
B - 4*A*b*c - 4*a*B*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c
*x^4])])/(8*c^(5/2)))/2
```

3.170.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.170.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.88

method	result
risch	$\frac{(2B x^2 c+4Ac-3Bb)\sqrt{c x^4+b x^2+a}}{8c^2}-\frac{(4Abc+4Bac-3B b^2)\ln\left(\frac{\frac{b}{2}+c x^2}{\sqrt{c}}+\sqrt{c x^4+b x^2+a}\right)}{16c^{\frac{5}{2}}}$
default	$B\left(\frac{x^2\sqrt{c x^4+b x^2+a}}{4c}-\frac{3b\sqrt{c x^4+b x^2+a}}{8c^2}+\frac{3b^2\ln\left(\frac{\frac{b}{2}+c x^2}{\sqrt{c}}+\sqrt{c x^4+b x^2+a}\right)}{16c^{\frac{5}{2}}}-\frac{a\ln\left(\frac{\frac{b}{2}+c x^2}{\sqrt{c}}+\sqrt{c x^4+b x^2+a}\right)}{4c^{\frac{3}{2}}}\right)+$
elliptic	$\frac{B x^2\sqrt{c x^4+b x^2+a}}{4c}-\frac{3Bb\sqrt{c x^4+b x^2+a}}{8c^2}+\frac{3B b^2\ln\left(\frac{\frac{b}{2}+c x^2}{\sqrt{c}}+\sqrt{c x^4+b x^2+a}\right)}{16c^{\frac{5}{2}}}-\frac{Ba\ln\left(\frac{\frac{b}{2}+c x^2}{\sqrt{c}}+\sqrt{c x^4+b x^2+a}\right)}{4c^{\frac{3}{2}}}+$
pseudoelliptic	$\frac{4B c^{\frac{3}{2}} x^2\sqrt{c x^4+b x^2+a}+8A c^{\frac{3}{2}}\sqrt{c x^4+b x^2+a}-4A\ln\left(\frac{2c x^2+2\sqrt{c x^4+b x^2+a}\sqrt{c}+b}{\sqrt{c}}\right)bc+4A\ln(2)bc-6Bb\sqrt{c x^4+b x^2+a}\sqrt{c}-}{16c^{\frac{5}{2}}}$

input
```
int(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(2*B*c*x^2+4*A*c-3*B*b)*(c*x^4+b*x^2+a)^(1/2)/c^2-1/16*(4*A*b*c+4*B*a*
c-3*B*b^2)/c^(5/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))
```

3.170.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.33

$$\int \frac{x^3(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \left[-\frac{(3\,Bb^2-4\,(Ba+Ab)c)\sqrt{c}\log\left(-8\,c^2x^4-8\,bcx^2-b^2+4\,\sqrt{cx^4+bx^2+a}(2\,cx^2+b)\sqrt{c}-4\,ac\right)-4\,(2\right.}{32\,c^3}$$
$$-\frac{(3\,Bb^2-4\,(Ba+Ab)c)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}(2\,cx^2+b)\sqrt{-c}}{2\,(c^2x^4+bcx^2+ac)}\right)-2\,(2\,Bc^2x^2-3\,Bbc+4\,Ac^2)\sqrt{cx^4+bx^2}}{16\,c^3}$$

input
```
integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/32*((3*B*b^2 - 4*(B*a + A*b)*c)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b
^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 4*(2*B*c^2
*x^2 - 3*B*b*c + 4*A*c^2)*sqrt(c*x^4 + b*x^2 + a))/c^3, -1/16*((3*B*b^2 -
4*(B*a + A*b)*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)
*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - 2*(2*B*c^2*x^2 - 3*B*b*c + 4*A*c^2)
*sqrt(c*x^4 + b*x^2 + a))/c^3]
```

3.170.6 Sympy [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.33

$$\int \frac{x^3(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{\begin{cases} \left(-\frac{Ba}{2c} - \frac{b\left(A-\frac{3Bb}{4c}\right)}{2c}\right)\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{a+bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } a-\frac{b^2}{4c}\neq 0\\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise}\end{cases}\right) + \left(\frac{Bx^2}{2c}+\frac{A-\frac{3Bb}{4c}}{c}\right)\sqrt{a+bx^2+cx^4} \\ \frac{\frac{2A\left(-a\sqrt{a+bx^2}+\frac{(a+bx^2)^{\frac{3}{2}}}{3}\right)}{b} + \frac{2B\left(a^2\sqrt{a+bx^2}-\frac{2a(a+bx^2)^{\frac{3}{2}}}{3}+\frac{(a+bx^2)^{\frac{5}{2}}}{5}\right)}{b^2}}{b} \\ \frac{\frac{Ax^4}{2}+\frac{Bx^6}{3}}{\sqrt{a}} \end{cases}}{2}$$

input
```
integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Piecewise(((-B*a/(2*c) - b*(A - 3*B*b/(4*c))/(2*c))*Piecewise((log(b + 2*s
qrt(c)*sqrt(a + b*x**2 + c*x**4) + 2*c*x**2)/sqrt(c), Ne(a - b**2/(4*c), 0
)), ((b/(2*c) + x**2)*log(b/(2*c) + x**2)/sqrt(c*(b/(2*c) + x**2)**2), Tru
e)) + (B*x**2/(2*c) + (A - 3*B*b/(4*c))/c)*sqrt(a + b*x**2 + c*x**4), Ne(c
, 0)), ((2*A*(-a*sqrt(a + b*x**2) + (a + b*x**2)**(3/2)/3)/b + 2*B*(a**2*s
qrt(a + b*x**2) - 2*a*(a + b*x**2)**(3/2)/3 + (a + b*x**2)**(5/2)/5)/b**2)
/b, Ne(b, 0)), ((A*x**4/2 + B*x**6/3)/sqrt(a), True))/2
```

3.170.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.170.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.96

$$\int \frac{x^3(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{8}\sqrt{cx^4+bx^2+a}\left(\frac{2\,Bx^2}{c}-\frac{3\,Bb-4\,Ac}{c^2}\right) - \frac{(3\,Bb^2-4\,Bac-4\,Abc)\log\left(\left|2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}+b\right|\right)}{16\,c^{\frac{5}{2}}}$$

input
```
integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/8*sqrt(c*x^4 + b*x^2 + a)*(2*B*x^2/c - (3*B*b - 4*A*c)/c^2) - 1/16*(3*B*
b^2 - 4*B*a*c - 4*A*b*c)*log(abs(2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))
*sqrt(c) + b))/c^(5/2)
```

3.170.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^3\,(B\,x^2+A)}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.171 $\int \frac{x(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

3.171.1 Optimal result

Integrand size = 25, antiderivative size = 76

$$\int \frac{x(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{B\sqrt{a+bx^2+cx^4}}{2c} - \frac{(bB-2Ac)\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}$$

output
```
-1/4*(-2*A*c+B*b)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c
^(3/2)+1/2*B*(c*x^4+b*x^2+a)^(1/2)/c
```

3.171.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.01

$$\int \frac{x(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{B\sqrt{a+bx^2+cx^4}}{2c} + \frac{(-bB+2Ac)\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}$$

input
```
Integrate[(x*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(B*Sqrt[a + b*x^2 + c*x^4])/(2*c) + ((-(b*B) + 2*A*c)*ArcTanh[(b + 2*c*x^2
)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*c^(3/2))
```

3.171.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1576, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$\Big\downarrow \text{1576}$$

$$\frac{1}{2}\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2+a}}dx^2$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{2}\left(\frac{B\sqrt{a+bx^2+cx^4}}{c}-\frac{(bB-2Ac)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx^2}{2c}\right)$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}\left(\frac{B\sqrt{a+bx^2+cx^4}}{c}-\frac{(bB-2Ac)\int \frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}}{c}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{B\sqrt{a+bx^2+cx^4}}{c}-\frac{(bB-2Ac)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2c^{3/2}}\right)$$

input
```
Int[(x*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
((B*Sqrt[a + b*x^2 + c*x^4])/c - ((b*B - 2*A*c)*ArcTanh[(b + 2*c*x^2)/(2*S
qrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(2*c^(3/2)))/2
```

3.171.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

3.171.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.86

method	result	si
risch	$\frac{B\sqrt{c\,x^{4}+b\,x^{2}+a}}{2c}+\frac{(2Ac-Bb)\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{4c^{\frac{3}{2}}}$	6
elliptic	$\frac{A\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{2\sqrt{c}}+\frac{B\sqrt{c\,x^{4}+b\,x^{2}+a}}{2c}-\frac{Bb\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{4c^{\frac{3}{2}}}$	9
default	$B\left(\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}}{2c}-\frac{b\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{4c^{\frac{3}{2}}}\right)+\frac{A\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{2\sqrt{c}}$	9
pseudoelliptic	$\frac{2A\ln\left(\frac{2c\,x^{2}+2\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{c}+b}{\sqrt{c}}\right)c-2A\ln(2)c-B\ln\left(\frac{2c\,x^{2}+2\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{c}+b}{\sqrt{c}}\right)b+B\ln(2)b+2B\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{c}}{4c^{\frac{3}{2}}}$	1

input
```
int(x*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*B*(c*x^4+b*x^2+a)^(1/2)/c+1/4*(2*A*c-B*b)/c^(3/2)*ln((1/2*b+c*x^2)/c^(
1/2)+(c*x^4+b*x^2+a)^(1/2))
```

3.171.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.34

$$\int \frac{x(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \left[\frac{4\sqrt{cx^4+bx^2+a}Bc-(Bb-2\,Ac)\sqrt{c}\log\left(-8\,c^2x^4-8\,bcx^2-b^2-4\sqrt{cx^4+bx^2+a}(2\,cx^2+b)\sqrt{c}-4\,c\right.}{8\,c^2}\right.$$

input
```
integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(4*sqrt(c*x^4 + b*x^2 + a)*B*c - (B*b - 2*A*c)*sqrt(c)*log(-8*c^2*x^4
 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a
*c))/c^2, 1/4*(2*sqrt(c*x^4 + b*x^2 + a)*B*c + (B*b - 2*A*c)*sqrt(-c)*arct
an(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 +
 a*c)))/c^2]
```

3.171.6 Sympy [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.17

$$\int \frac{x(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \frac{\begin{cases} \frac{B\sqrt{a+bx^2+cx^4}}{c}+\left(A-\frac{Bb}{2c}\right)\left(\begin{cases} \frac{\log\left(b+2\sqrt{c}\sqrt{a+bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } a-\frac{b^2}{4c}\neq 0 \\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise} \end{cases}\right) & \text{for } c\neq 0 \\ \frac{2A\sqrt{a+bx^2}+\frac{2B\left(-a\sqrt{a+bx^2}+\frac{(a+bx^2)^{\frac{3}{2}}}{3}\right)}{b}}{b} & \text{for } b\neq 0 \\ \frac{Ax^2+\frac{Bx^4}{2}}{\sqrt{a}} & \text{otherwise} \end{cases}}{2}$$

input
```
integrate(x*(B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Piecewise((B*sqrt(a + b*x**2 + c*x**4)/c + (A - B*b/(2*c))*Piecewise((log(
b + 2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + 2*c*x**2)/sqrt(c), Ne(a - b**2/(
4*c), 0)), ((b/(2*c) + x**2)*log(b/(2*c) + x**2)/sqrt(c*(b/(2*c) + x**2)**
2), True)), Ne(c, 0)), ((2*A*sqrt(a + b*x**2) + 2*B*(-a*sqrt(a + b*x**2) +
 (a + b*x**2)**(3/2)/3)/b)/b, Ne(b, 0)), ((A*x**2 + B*x**4/2)/sqrt(a), Tru
e))/2
```

3.171.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.171.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.88

$$\int \frac{x(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{\sqrt{cx^4+bx^2+a}B}{2\,c} + \frac{(Bb-2\,Ac)\log\left(\left|2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}+b\right|\right)}{4\,c^{\frac{3}{2}}}$$

input
```
integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(c*x^4 + b*x^2 + a)*B/c + 1/4*(B*b - 2*A*c)*log(abs(2*(sqrt(c)*x^2
 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) + b))/c^(3/2)
```

3.171.9 Mupad [B] (verification not implemented)

Time = 8.17 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.21

$$\int \frac{x(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{A\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)}{2\sqrt{c}}+\frac{B\sqrt{c\,x^4+b\,x^2+a}}{2\,c} - \frac{B\,b\,\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)}{4\,c^{3/2}}$$

input
```
int((x*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
(A*log((a + b*x^2 + c*x^4)^(1/2) + (b/2 + c*x^2)/c^(1/2)))/(2*c^(1/2)) + (
B*(a + b*x^2 + c*x^4)^(1/2))/(2*c) - (B*b*log((a + b*x^2 + c*x^4)^(1/2) +
(b/2 + c*x^2)/c^(1/2)))/(4*c^(3/2))
```

3.172 $\int \frac{A+Bx^2}{x\sqrt{a+bx^2+cx^4}}\,dx$

3.172.1 Optimal result

Integrand size = 27, antiderivative size = 90

$$\int \frac{A+Bx^2}{x\sqrt{a+bx^2+cx^4}}\,dx=-\frac{A\operatorname{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}+\frac{B\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}}$$

output
```
-1/2*A*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(1/2)+1/2*
B*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(1/2)
```

3.172.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^2}{x\sqrt{a+bx^2+cx^4}}\,dx=\frac{A\operatorname{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{B\log\left(b+2cx^2-2\sqrt{c}\sqrt{a+bx^2+cx^4}\right)}{2\sqrt{c}}$$

input
```
Integrate[(A + B*x^2)/(x*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
(A*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/Sqrt[a] - (B*
Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/(2*Sqrt[c])
```

3.172.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1578, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x\sqrt{a+bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{Bx^2+A}{x^2\sqrt{cx^4+bx^2+a}}dx^2$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{2}\left(A\int \frac{1}{x^2\sqrt{cx^4+bx^2+a}}dx^2+B\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx^2\right)$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}\left(A\int \frac{1}{x^2\sqrt{cx^4+bx^2+a}}dx^2+2B\int \frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(A\int \frac{1}{x^2\sqrt{cx^4+bx^2+a}}dx^2+\frac{B\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{c}}\right)$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{2}\left(\frac{B\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{c}}-2A\int \frac{1}{4a-x^4}d\frac{bx^2+2a}{\sqrt{cx^4+bx^2+a}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{B\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{c}}-\frac{A\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{a}}\right)$$

input
```
Int[(A + B*x^2)/(x*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
(-((A*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/Sqrt[a])
 + (B*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/Sqrt[c])
/2
```

3.172.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.172.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{B\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{2\sqrt{c}}-\frac{A\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{2\sqrt{a}}$	76
elliptic	$\frac{B\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{2\sqrt{c}}-\frac{A\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{2\sqrt{a}}$	76
pseudoelliptic	$-\frac{A\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)\sqrt{c}-B\sqrt{a}\left(-\ln\left(2\right)+\ln\left(\frac{2c\,x^{2}+2\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{c}+b}{\sqrt{c}}\right)\right)}{2\sqrt{a}\,\sqrt{c}}$	91

input
```
int((B*x^2+A)/x/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*B*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)-1/2*A/a^(1/2
)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

3.172.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 517, normalized size of antiderivative = 5.74

$$\int \frac{A+Bx^2}{x\sqrt{a+bx^2+cx^4}}\,dx$$
$$=\left[\frac{Ba\sqrt{c}\log\left(-8\,c^2x^4-8\,bcx^2-b^2-4\,\sqrt{cx^4+bx^2+a}(2\,cx^2+b)\sqrt{c}-4\,ac\right)+A\sqrt{a}c\log\left(-\frac{(b^2+4\,ac)x^4+8\,ab}{\ldots}\right.}{4\,ac}\right.$$
$$-\frac{2\,Ba\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}(2\,cx^2+b)\sqrt{-c}}{2\,(c^2x^4+bcx^2+ac)}\right)-A\sqrt{a}c\log\left(-\frac{(b^2+4\,ac)x^4+8\,abx^2-4\,\sqrt{cx^4+bx^2+a}(bx^2+2\,a)\sqrt{a}+8\,a^2}{x^4}\right)}{4\,ac},$$

input
```
integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(B*a*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2
+ a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + A*sqrt(a)*c*log(-((b^2 + 4*a*c)*x^4
+ 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4
))/(a*c), -1/4*(2*B*a*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2
 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - A*sqrt(a)*c*log(-((b^2 + 4*a*c
)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^
2)/x^4))/(a*c), 1/4*(2*A*sqrt(-a)*c*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*
x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + B*a*sqrt(c)*log(-8*c^2*x^
4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*
a*c))/(a*c), 1/2*(A*sqrt(-a)*c*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 +
 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - B*a*sqrt(-c)*arctan(1/2*sqrt(c
*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)))/(a*c)
]
```

3.172.6 Sympy [F]

$$\int \frac{A+Bx^2}{x\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{A+Bx^2}{x\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((B*x**2+A)/x/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x**2)/(x*sqrt(a + b*x**2 + c*x**4)), x)
```

3.172.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx^2}{x\sqrt{a+bx^2+cx^4}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.172.8 Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx^2}{x\sqrt{a+bx^2+cx^4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.172.9 Mupad [B] (verification not implemented)

Time = 8.00 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx^2}{x\sqrt{a+bx^2+cx^4}}\,dx = \frac{B\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)}{2\sqrt{c}} - \frac{A\ln\left(\frac{1}{x^2}\right)}{2\sqrt{a}} - \frac{A\ln\left(2\,a+2\sqrt{a}\sqrt{c\,x^4+b\,x^2+a}+b\,x^2\right)}{2\sqrt{a}}$$

input
```
int((A + B*x^2)/(x*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
(B*log((a + b*x^2 + c*x^4)^(1/2) + (b/2 + c*x^2)/c^(1/2)))/(2*c^(1/2)) - (
A*log(1/x^2))/(2*a^(1/2)) - (A*log(2*a + 2*a^(1/2)*(a + b*x^2 + c*x^4)^(1/
2) + b*x^2))/(2*a^(1/2))
```

3.173 $\int \frac{A+Bx^2}{x^3\sqrt{a+bx^2+cx^4}}\,dx$

3.173.1 Optimal result

Integrand size = 27, antiderivative size = 80

$$\int \frac{A+Bx^2}{x^3\sqrt{a+bx^2+cx^4}}\,dx=-\frac{A\sqrt{a+bx^2+cx^4}}{2ax^2}+\frac{(Ab-2aB)\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}}$$

output
```
1/4*(A*b-2*B*a)*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(
3/2)-1/2*A*(c*x^4+b*x^2+a)^(1/2)/a/x^2
```

3.173.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx^2}{x^3\sqrt{a+bx^2+cx^4}}\,dx=-\frac{A\sqrt{a+bx^2+cx^4}}{2ax^2}+\frac{(-Ab+2aB)\text{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{2a^{3/2}}$$

input
```
Integrate[(A + B*x^2)/(x^3*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
-1/2*(A*Sqrt[a + b*x^2 + c*x^4])/(a*x^2) + ((-(A*b) + 2*a*B)*ArcTanh[(Sqrt
[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/(2*a^(3/2))
```

3.173.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1578, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^3\sqrt{a+bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{Bx^2+A}{x^4\sqrt{cx^4+bx^2+a}}dx^2$$

$$\Big\downarrow \text{1228}$$

$$\frac{1}{2}\left(-\frac{(Ab-2aB)\int \frac{1}{x^2\sqrt{cx^4+bx^2+a}}dx^2}{2a}-\frac{A\sqrt{a+bx^2+cx^4}}{ax^2}\right)$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{2}\left(\frac{(Ab-2aB)\int \frac{1}{4a-x^4}d\frac{bx^2+2a}{\sqrt{cx^4+bx^2+a}}}{a}-\frac{A\sqrt{a+bx^2+cx^4}}{ax^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{(Ab-2aB)\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}}-\frac{A\sqrt{a+bx^2+cx^4}}{ax^2}\right)$$

input
```
Int[(A + B*x^2)/(x^3*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
(-((A*Sqrt[a + b*x^2 + c*x^4])/(a*x^2)) + ((A*b - 2*a*B)*ArcTanh[(2*a + b*
x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*a^(3/2)))/2
```

3.173.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.173.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89

method	result	size
risch	$-\frac{A\sqrt{c\,x^{4}+b\,x^{2}+a}}{2a\,x^{2}}+\frac{(Ab-2Ba)\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{4a^{\frac{3}{2}}}$	71
pseudoelliptic	$\frac{x^{2}(Ab-2Ba)\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)-2A\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{4a^{\frac{3}{2}}\,x^{2}}$	75
elliptic	$-\frac{A\sqrt{c\,x^{4}+b\,x^{2}+a}}{2a\,x^{2}}+\frac{Ab\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{4a^{\frac{3}{2}}}-\frac{B\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{2\sqrt{a}}$	104
default	$-\frac{B\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{2\sqrt{a}}+A\left(-\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}}{2a\,x^{2}}+\frac{b\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{4a^{\frac{3}{2}}}\right)$	105

input
```
int((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*A*(c*x^4+b*x^2+a)^(1/2)/a/x^2+1/4*(A*b-2*B*a)/a^(3/2)*ln((2*a+b*x^2+2
*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

3.173.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.46

$$\int \frac{A+Bx^2}{x^3\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \left[-\frac{(2\,Ba-Ab)\sqrt{a}x^2\log\left(-\frac{(b^2+4\,ac)x^4+8\,abx^2+4\,\sqrt{cx^4+bx^2+a}(bx^2+2\,a)\sqrt{a}+8\,a^2}{x^4}\right)+4\,\sqrt{cx^4+bx^2+a}Aa}{8\,a^2x^2}, \frac{(2\,Ba-\right.$$

input
```
integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*((2*B*a - A*b)*sqrt(a)*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*s
qrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 4*sqrt(c*x^4
+ b*x^2 + a)*A*a)/(a^2*x^2), 1/4*((2*B*a - A*b)*sqrt(-a)*x^2*arctan(1/2*sq
rt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) -
2*sqrt(c*x^4 + b*x^2 + a)*A*a)/(a^2*x^2)]
```

3.173.6 Sympy [F]

$$\int \frac{A+Bx^2}{x^3\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{A+Bx^2}{x^3\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((B*x**2+A)/x**3/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x**2)/(x**3*sqrt(a + b*x**2 + c*x**4)), x)
```

3.173.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A + Bx^2}{x^3\sqrt{a + bx^2 + cx^4}}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.173.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.55

$$\int \frac{A + Bx^2}{x^3\sqrt{a + bx^2 + cx^4}}\, dx = \frac{(2\,Ba - Ab)\arctan\left(-\frac{\sqrt{c}x^2 - \sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{2\,\sqrt{-a}a} + \frac{\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}\right)Ab + 2\,Aa\sqrt{c}}{2\left(\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}\right)^2 - a\right)a}$$

input
```
integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/2*(2*B*a - A*b)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a)
)/(sqrt(-a)*a) + 1/2*((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*A*b + 2*A*a*
sqrt(c))/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)*a)
```

3.173.9 Mupad [B] (verification not implemented)

Time = 8.15 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.29

$$\int \frac{A+Bx^2}{x^3\sqrt{a+bx^2+cx^4}}\,dx = \frac{A\,b\,\mathrm{atanh}\left(\frac{\frac{b\,x^2}{2}+a}{\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}\right)}{4\,a^{3/2}} - \frac{B\,\ln\left(2\,a+2\,\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}+b\,x^2\right)}{2\,\sqrt{a}} - \frac{A\,\sqrt{c\,x^4+b\,x^2+a}}{2\,a\,x^2} - \frac{B\,\ln\left(\frac{1}{x^2}\right)}{2\,\sqrt{a}}$$

input
```
int((A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
(A*b*atanh((a + (b*x^2)/2)/(a^(1/2)*(a + b*x^2 + c*x^4)^(1/2))))/(4*a^(3/2
)) - (B*log(2*a + 2*a^(1/2)*(a + b*x^2 + c*x^4)^(1/2) + b*x^2))/(2*a^(1/2)
) - (A*(a + b*x^2 + c*x^4)^(1/2))/(2*a*x^2) - (B*log(1/x^2))/(2*a^(1/2))
```

3.174 $\int \frac{A+Bx^2}{x^5\sqrt{a+bx^2+cx^4}}\,dx$

3.174.1 Optimal result

Integrand size = 27, antiderivative size = 124

$$\int \frac{A+Bx^2}{x^5\sqrt{a+bx^2+cx^4}}\,dx = -\frac{A\sqrt{a+bx^2+cx^4}}{4ax^4}+\frac{(3Ab-4aB)\sqrt{a+bx^2+cx^4}}{8a^2x^2} - \frac{(3Ab^2-4abB-4aAc)\,\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{5/2}}$$

output
```
-1/16*(-4*A*a*c+3*A*b^2-4*B*a*b)*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*
x^2+a)^(1/2))/a^(5/2)-1/4*A*(c*x^4+b*x^2+a)^(1/2)/a/x^4+1/8*(3*A*b-4*B*a)*
(c*x^4+b*x^2+a)^(1/2)/a^2/x^2
```

3.174.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.17

$$\int \frac{A+Bx^2}{x^5\sqrt{a+bx^2+cx^4}}\,dx = \frac{\sqrt{a}\sqrt{a+bx^2+cx^4}(3Abx^2-2a(A+2Bx^2))+3Ab^2x^4\text{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)+4a(bB+Ac)x^4\text{arctan}}{8a^{5/2}x^4}$$

input
```
Integrate[(A + B*x^2)/(x^5*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
(Sqrt[a]*Sqrt[a + b*x^2 + c*x^4]*(3*A*b*x^2 - 2*a*(A + 2*B*x^2)) + 3*A*b^2
*x^4*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]] + 4*a*(b*B +
 A*c)*x^4*ArcTanh[(-(Sqrt[c]*x^2) + Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/(8*
a^(5/2)*x^4)
```

3.174.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1578, 1237, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^5\sqrt{a+bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{Bx^2+A}{x^6\sqrt{cx^4+bx^2+a}}dx^2$$

$$\Big\downarrow \text{1237}$$

$$\frac{1}{2}\left(-\frac{\int \frac{2Acx^2+3Ab-4aB}{2x^4\sqrt{cx^4+bx^2+a}}dx^2}{2a}-\frac{A\sqrt{a+bx^2+cx^4}}{2ax^4}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{\int \frac{2Acx^2+3Ab-4aB}{x^4\sqrt{cx^4+bx^2+a}}dx^2}{4a}-\frac{A\sqrt{a+bx^2+cx^4}}{2ax^4}\right)$$

$$\Big\downarrow \text{1228}$$

$$\frac{1}{2}\left(-\frac{-\frac{(-4aAc-4abB+3Ab^2)\int \frac{1}{x^2\sqrt{cx^4+bx^2+a}}dx^2}{2a}-\frac{(3Ab-4aB)\sqrt{a+bx^2+cx^4}}{ax^2}}{4a}-\frac{A\sqrt{a+bx^2+cx^4}}{2ax^4}\right)$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{2}\left(-\frac{\frac{(-4aAc-4abB+3Ab^2)\int \frac{1}{4a-x^4}d\frac{bx^2+2a}{\sqrt{cx^4+bx^2+a}}}{a}-\frac{(3Ab-4aB)\sqrt{a+bx^2+cx^4}}{ax^2}}{4a}-\frac{A\sqrt{a+bx^2+cx^4}}{2ax^4}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(-\frac{\frac{(-4aAc-4abB+3Ab^2)\operatorname{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}}-\frac{(3Ab-4aB)\sqrt{a+bx^2+cx^4}}{ax^2}}{4a}-\frac{A\sqrt{a+bx^2+cx^4}}{2ax^4}\right)$$

input
```
Int[(A + B*x^2)/(x^5*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
(-1/2*(A*Sqrt[a + b*x^2 + c*x^4])/(a*x^4) - (-(((3*A*b - 4*a*B)*Sqrt[a + b
*x^2 + c*x^4])/(a*x^2)) + ((3*A*b^2 - 4*a*b*B - 4*a*A*c)*ArcTanh[(2*a + b*
x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*a^(3/2)))/(4*a))/2
```

3.174.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.174.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.79

method	result
risch	$-\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}\,\left(-3Ab\,x^{2}+4Ba\,x^{2}+2Aa\right)}{8a^{2}x^{4}}+\frac{\left(4Aac-3A\,b^{2}+4abB\right)\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{16a^{\frac{5}{2}}}$
pseudoelliptic	$\frac{\frac{\left(\left(ac-\frac{3b^{2}}{4}\right)A+abB\right)x^{4}\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{4}+\frac{3\left(\frac{2\left(-2B\,x^{2}-A\right)a^{\frac{3}{2}}}{3}+A\sqrt{a}\,b\,x^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}{8}}{a^{\frac{5}{2}}x^{4}}$
default	$B\left(-\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}}{2a\,x^{2}}+\frac{b\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{4a^{\frac{3}{2}}}\right)+A\left(-\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}}{4a\,x^{4}}+\frac{3b\sqrt{c\,x^{4}+b\,x^{2}+a}}{8a^{2}x^{2}}-\frac{3b^{2}\ln(}{}\right.$
elliptic	$-\frac{B\sqrt{c\,x^{4}+b\,x^{2}+a}}{2a\,x^{2}}+\frac{Bb\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{4a^{\frac{3}{2}}}-\frac{A\sqrt{c\,x^{4}+b\,x^{2}+a}}{4a\,x^{4}}+\frac{3Ab\sqrt{c\,x^{4}+b\,x^{2}+a}}{8a^{2}x^{2}}-\frac{3A\,b^{2}\ln\left(2a+b\right.}{}$

input
```
int((B*x^2+A)/x^5/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(c*x^4+b*x^2+a)^(1/2)*(-3*A*b*x^2+4*B*a*x^2+2*A*a)/a^2/x^4+1/16*(4*A*
a*c-3*A*b^2+4*B*a*b)/a^(5/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2)
)/x^2)
```

3.174.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.06

$$\int \frac{A+Bx^2}{x^5\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \left[\frac{(4\,Bab-3\,Ab^2+4\,Aac)\sqrt{a}x^4\log\left(-\frac{(b^2+4\,ac)x^4+8\,abx^2+4\,\sqrt{cx^4+bx^2+a}(bx^2+2\,a)\sqrt{a}+8\,a^2}{x^4}\right)-4\,\sqrt{cx^4+bx^2+a}(2\,\ldots}{32\,a^3x^4}\right.$$
$$-\frac{(4\,Bab-3\,Ab^2+4\,Aac)\sqrt{-a}x^4\arctan\left(\frac{\sqrt{cx^4+bx^2+a}(bx^2+2\,a)\sqrt{-a}}{2\,(acx^4+abx^2+a^2)}\right)+2\,\sqrt{cx^4+bx^2+a}(2\,Aa^2+(4\,Ba^2-\ldots}{16\,a^3x^4}$$

input
```
integrate((B*x^2+A)/x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/32*((4*B*a*b - 3*A*b^2 + 4*A*a*c)*sqrt(a)*x^4*log(-((b^2 + 4*a*c)*x^4 +
 8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4)
 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*A*a^2 + (4*B*a^2 - 3*A*a*b)*x^2))/(a^3*x^4
), -1/16*((4*B*a*b - 3*A*b^2 + 4*A*a*c)*sqrt(-a)*x^4*arctan(1/2*sqrt(c*x^4
 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*sqrt(c
*x^4 + b*x^2 + a)*(2*A*a^2 + (4*B*a^2 - 3*A*a*b)*x^2))/(a^3*x^4)]
```

3.174.6 Sympy [F]

$$\int \frac{A+Bx^2}{x^5\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{A+Bx^2}{x^5\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((B*x**2+A)/x**5/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x**2)/(x**5*sqrt(a + b*x**2 + c*x**4)), x)
```

3.174.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx^2}{x^5\sqrt{a+bx^2+cx^4}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x^2+A)/x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.174.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(106) = 212$.

Time = 0.31 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.73

$$\int \frac{A+Bx^2}{x^5\sqrt{a+bx^2+cx^4}}\,dx = -\frac{(4\,Bab-3\,Ab^2+4\,Aac)\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}a^2} + \frac{4\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^3 Bab-3\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^3 Ab^2+4\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^3 Aa}{}$$

input
```
integrate((B*x^2+A)/x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-1/8*(4*B*a*b - 3*A*b^2 + 4*A*a*c)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x
^2 + a))/sqrt(-a))/(sqrt(-a)*a^2) + 1/8*(4*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x
^2 + a))^3*B*a*b - 3*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*A*b^2 + 4*(
sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*A*a*c + 8*(sqrt(c)*x^2 - sqrt(c*x
^4 + b*x^2 + a))^2*B*a^2*sqrt(c) - 4*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a
))*B*a^2*b + 5*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*A*a*b^2 + 4*(sqrt(c
)*x^2 - sqrt(c*x^4 + b*x^2 + a))*A*a^2*c - 8*B*a^3*sqrt(c) + 8*A*a^2*b*sqr
t(c))/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)^2*a^2)
```

3.174.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx^2}{x^5\sqrt{a + bx^2 + cx^4}}\,dx = \int \frac{B\,x^2 + A}{x^5\,\sqrt{c\,x^4 + b\,x^2 + a}}\,dx$$

input
```
int((A + B*x^2)/(x^5*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int((A + B*x^2)/(x^5*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.175 $\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2+cx^4}}\,dx$

3.175.1 Optimal result

Integrand size = 27, antiderivative size = 177

$$\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2+cx^4}}\,dx = -\frac{A\sqrt{a+bx^2+cx^4}}{6ax^6}+\frac{(5Ab-6aB)\sqrt{a+bx^2+cx^4}}{24a^2x^4} - \frac{(15Ab^2-18abB-16aAc)\sqrt{a+bx^2+cx^4}}{48a^3x^2} + \frac{(5Ab^3-6ab^2B-12aAbc+8a^2Bc)\,\mathrm{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{7/2}}$$

output
```
1/32*(-12*A*a*b*c+5*A*b^3+8*B*a^2*c-6*B*a*b^2)*arctanh(1/2*(b*x^2+2*a)/a^(
1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(7/2)-1/6*A*(c*x^4+b*x^2+a)^(1/2)/a/x^6+1/24
*(5*A*b-6*B*a)*(c*x^4+b*x^2+a)^(1/2)/a^2/x^4-1/48*(-16*A*a*c+15*A*b^2-18*B
*a*b)*(c*x^4+b*x^2+a)^(1/2)/a^3/x^2
```

3.175.2 Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2+cx^4}}\,dx = \frac{\sqrt{a+bx^2+cx^4}(-8a^2A+10aAbx^2-12a^2Bx^2-15Ab^2x^4+18abBx^4+16aAcx^4)}{48a^3x^6} + \frac{(-5Ab^3-8a^2Bc)\,\text{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{16a^{7/2}} - \frac{3b(bB+2Ac)\text{arctanh}\left(\frac{-\sqrt{c}x^2+\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{8a^{5/2}}$$

input
```
Integrate[(A + B*x^2)/(x^7*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
(Sqrt[a + b*x^2 + c*x^4]*(-8*a^2*A + 10*a*A*b*x^2 - 12*a^2*B*x^2 - 15*A*b^
2*x^4 + 18*a*b*B*x^4 + 16*a*A*c*x^4))/(48*a^3*x^6) + ((-5*A*b^3 - 8*a^2*B*
c)*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/(16*a^(7/2))
- (3*b*(b*B + 2*A*c)*ArcTanh[(-(Sqrt[c]*x^2) + Sqrt[a + b*x^2 + c*x^4])/Sq
rt[a]])/(8*a^(5/2))
```

3.175.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {1578, 1237, 27, 1237, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2+cx^4}}\,dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{Bx^2+A}{x^8\sqrt{cx^4+bx^2+a}}dx^2$$

$$\Big\downarrow \text{1237}$$

$$\frac{1}{2}\left(-\frac{\int \frac{4Acx^2+5Ab-6aB}{2x^6\sqrt{cx^4+bx^2+a}}dx^2}{3a}-\frac{A\sqrt{a+bx^2+cx^4}}{3ax^6}\right)$$

$\downarrow$ 27

$$\frac{1}{2}\left(-\frac{\int \frac{4Acx^2+5Ab-6aB}{x^6\sqrt{cx^4+bx^2+a}}dx^2}{6a}-\frac{A\sqrt{a+bx^2+cx^4}}{3ax^6}\right)$$

$\downarrow$ 1237

$$\frac{1}{2}\left(-\frac{-\frac{\int \frac{15Ab^2-18aBb+2(5Ab-6aB)cx^2-16aAc}{2x^4\sqrt{cx^4+bx^2+a}}dx^2}{2a}-\frac{(5Ab-6aB)\sqrt{a+bx^2+cx^4}}{2ax^4}}{6a}-\frac{A\sqrt{a+bx^2+cx^4}}{3ax^6}\right)$$

$\downarrow$ 27

$$\frac{1}{2}\left(-\frac{-\frac{\int \frac{15Ab^2-18aBb+2(5Ab-6aB)cx^2-16aAc}{x^4\sqrt{cx^4+bx^2+a}}dx^2}{4a}-\frac{(5Ab-6aB)\sqrt{a+bx^2+cx^4}}{2ax^4}}{6a}-\frac{A\sqrt{a+bx^2+cx^4}}{3ax^6}\right)$$

$\downarrow$ 1228

$$\frac{1}{2}\left(-\frac{-\frac{-\frac{3\left(8a^2Bc-12aAbc-6ab^2B+5Ab^3\right)\int \frac{1}{x^2\sqrt{cx^4+bx^2+a}}dx^2}{2a}-\frac{\sqrt{a+bx^2+cx^4}\left(-16aAc-18abB+15Ab^2\right)}{ax^2}}{4a}-\frac{(5Ab-6aB)\sqrt{a+bx^2+cx^4}}{2ax^4}}{6a}-A\sqrt{a-}\right.$$

$\downarrow$ 1154

$$\frac{1}{2}\left(-\frac{-\frac{3\left(8a^2Bc-12aAbc-6ab^2B+5Ab^3\right)\int \frac{1}{4a-x^4}d\frac{bx^2+2a}{\sqrt{cx^4+bx^2+a}}}{a}-\frac{\sqrt{a+bx^2+cx^4}\left(-16aAc-18abB+15Ab^2\right)}{ax^2}}{4a}-\frac{(5Ab-6aB)\sqrt{a+bx^2+cx^4}}{2ax^4}}{6a}-A\sqrt{a-}\right.$$

$\downarrow$ 219

$$\frac{1}{2}\left(-\frac{-\frac{3\left(8a^2Bc-12aAbc-6ab^2B+5Ab^3\right)\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}}-\frac{\sqrt{a+bx^2+cx^4}\left(-16aAc-18abB+15Ab^2\right)}{ax^2}}{4a}-\frac{(5Ab-6aB)\sqrt{a+bx^2+cx^4}}{2ax^4}}{6a}-\right.$$

input
```
Int[(A + B*x^2)/(x^7*Sqrt[a + b*x^2 + c*x^4]),x]
```

3.175. $\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2+cx^4}}\,dx$

output
```
(-1/3*(A*Sqrt[a + b*x^2 + c*x^4])/(a*x^6) - (-1/2*((5*A*b - 6*a*B)*Sqrt[a
+ b*x^2 + c*x^4])/(a*x^4) - (-(((15*A*b^2 - 18*a*b*B - 16*a*A*c)*Sqrt[a +
b*x^2 + c*x^4])/(a*x^2)) + (3*(5*A*b^3 - 6*a*b^2*B - 12*a*A*b*c + 8*a^2*B*
c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*a^(3/2))
)/(4*a))/(6*a))/2
```

3.175.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.175.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.78

method	result
risch	$-\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}\,\left(-16 A a c\,x^{4}+15 A\,b^{2} x^{4}-18 B a b\,x^{4}-10 A a b\,x^{2}+12 B\,a^{2} x^{2}+8 A\,a^{2}\right)}{48 a^{3} x^{6}}-\frac{\left(12 A a b c-5 A\,b^{3}-8 a^{2} B c+6 B a\,b^{2}\right) \ln}{32 a^{\frac{7}{2}}}$
pseudoelliptic	$-\frac{\frac{9\left(\left(a b c-\frac{5}{12} b^{3}\right) A-\frac{2\left(a c-\frac{3 b^{2}}{4}\right) a B}{3}\right) x^{6} \ln\left(\frac{2 a+b\,x^{2}+2 \sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{4}+\left(\left(\left(-2 A c-\frac{9 B b}{4}\right) x^{4}-\frac{5 A b\,x^{2}}{4}\right) a^{\frac{3}{2}}+\left(\frac{3 B\,x^{2}}{2}+A\right) a^{\frac{5}{2}}\right.}{6 a^{\frac{7}{2}} x^{6}}$
default	$A\left(-\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}}{6 a\,x^{6}}+\frac{5 b\sqrt{c\,x^{4}+b\,x^{2}+a}}{24 a^{2} x^{4}}-\frac{5 b^{2} \sqrt{c\,x^{4}+b\,x^{2}+a}}{16 a^{3} x^{2}}+\frac{5 b^{3} \ln\left(\frac{2 a+b\,x^{2}+2 \sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{32 a^{\frac{7}{2}}}-\frac{3 b c \ln\left(\frac{2 a+}{}\right.}{}\right.$
elliptic	$-\frac{B\sqrt{c\,x^{4}+b\,x^{2}+a}}{4 a\,x^{4}}+\frac{3 B b\sqrt{c\,x^{4}+b\,x^{2}+a}}{8 a^{2} x^{2}}-\frac{3 B\,b^{2} \ln\left(\frac{2 a+b\,x^{2}+2 \sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{16 a^{\frac{5}{2}}}+\frac{B c \ln\left(\frac{2 a+b\,x^{2}+2 \sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{4 a^{\frac{3}{2}}}$

input
```
int((B*x^2+A)/x^7/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/48*(c*x^4+b*x^2+a)^(1/2)*(-16*A*a*c*x^4+15*A*b^2*x^4-18*B*a*b*x^4-10*A*
a*b*x^2+12*B*a^2*x^2+8*A*a^2)/a^3/x^6-1/32/a^(7/2)*(12*A*a*b*c-5*A*b^3-8*B
*a^2*c+6*B*a*b^2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

3.175.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.92

$$\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2+cx^4}}\,dx$$

$$=\left[\frac{3\,(6\,Bab^2-5\,Ab^3-4\,(2\,Ba^2-3\,Aab)c)\sqrt{a}x^6\log\left(-\frac{(b^2+4\,ac)x^4+8\,abx^2-4\,\sqrt{cx^4+bx^2+a}(bx^2+2\,a)\sqrt{a}+8\,a^2}{x^4}\right)+4\,((1}{192\,a^4x^6}\right.$$

input
```
integrate((B*x^2+A)/x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/192*(3*(6*B*a*b^2 - 5*A*b^3 - 4*(2*B*a^2 - 3*A*a*b)*c)*sqrt(a)*x^6*log(
-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*
sqrt(a) + 8*a^2)/x^4) + 4*((18*B*a^2*b - 15*A*a*b^2 + 16*A*a^2*c)*x^4 - 8*
A*a^3 - 2*(6*B*a^3 - 5*A*a^2*b)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(a^4*x^6), 1
/96*(3*(6*B*a*b^2 - 5*A*b^3 - 4*(2*B*a^2 - 3*A*a*b)*c)*sqrt(-a)*x^6*arctan
(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a
^2)) + 2*((18*B*a^2*b - 15*A*a*b^2 + 16*A*a^2*c)*x^4 - 8*A*a^3 - 2*(6*B*a^
3 - 5*A*a^2*b)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(a^4*x^6)]
```

3.175.6 Sympy [F]

$$\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((B*x**2+A)/x**7/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x**2)/(x**7*sqrt(a + b*x**2 + c*x**4)), x)
```

3.175.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2+cx^4}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((B*x^2+A)/x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.175.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. $2(155) = 310$.

Time = 0.31 (sec) , antiderivative size = 571, normalized size of antiderivative = 3.23

$$\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2+cx^4}}\,dx = \frac{(6\,Bab^2-5\,Ab^3-8\,Ba^2c+12\,Aabc)\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{16\sqrt{-a}a^3} - \frac{18\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^5Bab^2-15\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^5Ab^3-24\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right.}{}$$

input
```
integrate((B*x^2+A)/x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/16*(6*B*a*b^2 - 5*A*b^3 - 8*B*a^2*c + 12*A*a*b*c)*arctan(-(sqrt(c)*x^2 -
 sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^3) - 1/48*(18*(sqrt(c)*x^2
 - sqrt(c*x^4 + b*x^2 + a))^5*B*a*b^2 - 15*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x
^2 + a))^5*A*b^3 - 24*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*B*a^2*c +
36*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*A*a*b*c - 48*(sqrt(c)*x^2 - s
qrt(c*x^4 + b*x^2 + a))^3*B*a^2*b^2 + 40*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2
 + a))^3*A*a*b^3 - 96*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*A*a^2*b*c
- 48*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*B*a^3*b*sqrt(c) - 96*(sqrt(
c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*A*a^3*c^(3/2) + 30*(sqrt(c)*x^2 - sqrt
(c*x^4 + b*x^2 + a))*B*a^3*b^2 - 33*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a)
)*A*a^2*b^3 + 24*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*B*a^4*c - 36*(sqr
t(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*A*a^3*b*c + 48*B*a^4*b*sqrt(c) - 48*A*
a^3*b^2*sqrt(c) + 32*A*a^4*c^(3/2))/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 +
a))^2 - a)^3*a^3)
```

3.175.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{B\,x^2+A}{x^7\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((A + B*x^2)/(x^7*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int((A + B*x^2)/(x^7*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.176 $\int \frac{x^4(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

3.176.1 Optimal result

Integrand size = 27, antiderivative size = 403

$$\int \frac{x^4(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = -\frac{(4bB-5Ac)x\sqrt{a+bx^2+cx^4}}{15c^2} + \frac{Bx^3\sqrt{a+bx^2+cx^4}}{5c} + \frac{(8b^2B-10Abc-9aBc)\,x\sqrt{a+bx^2+cx^4}}{15c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}(8b^2B-10Abc-9aBc)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{15c^{11/4}\sqrt{a+bx^2+cx^4}} + \frac{\sqrt[4]{a}\left(8b^2B-10Abc-9aBc+\sqrt{a}\sqrt{c}(4bB-5Ac)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\ldots}{\sqrt[4]{a}\ldots}\right.\right.}{30c^{11/4}\sqrt{a+bx^2+cx^4}}$$

output
```
-1/15*(-5*A*c+4*B*b)*x*(c*x^4+b*x^2+a)^(1/2)/c^2+1/5*B*x^3*(c*x^4+b*x^2+a)
^(1/2)/c+1/15*(-10*A*b*c-9*B*a*c+8*B*b^2)*x*(c*x^4+b*x^2+a)^(1/2)/c^(5/2)/
(a^(1/2)+x^2*c^(1/2))-1/15*a^(1/4)*(-10*A*b*c-9*B*a*c+8*B*b^2)*(cos(2*arct
an(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE
(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2
)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(11/4)/(c
*x^4+b*x^2+a)^(1/2)+1/30*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2
)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4
))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(8*B*b^2-10*A*b
*c-9*B*a*c+(-5*A*c+4*B*b)*a^(1/2)*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c
^(1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2+a)^(1/2)
```

3.176.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.51 (sec) , antiderivative size = 532, normalized size of antiderivative = 1.32

$$\int \frac{x^4(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \frac{4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x(-4bB+5Ac+3Bcx^2)\left(a+bx^2+cx^4\right)+i(8b^2B-10Abc-9aBc)\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\cdots}}{\cdots}$$

input
```
Integrate[(x^4*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(-4*b*B + 5*A*c + 3*B*c*x^2)*(a + b
*x^2 + c*x^4) + I*(8*b^2*B - 10*A*b*c - 9*a*B*c)*(-b + Sqrt[b^2 - 4*a*c])*
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b
 - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcS
inh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b
 - Sqrt[b^2 - 4*a*c])] - I*(-8*b^3*B + b*c*(17*a*B - 10*A*Sqrt[b^2 - 4*a*c
]) + 2*b^2*(5*A*c + 4*B*Sqrt[b^2 - 4*a*c]) - a*c*(10*A*c + 9*B*Sqrt[b^2 -
4*a*c]))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*S
qrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Ellipti
cF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4
*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(60*c^3*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*S
qrt[a + b*x^2 + c*x^4])
```

3.176.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 400, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1602, 1602, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$\Big\downarrow \text{1602}$$

$$\frac{Bx^3\sqrt{a+bx^2+cx^4}}{5c}-\frac{\int \frac{x^2\left((4bB-5Ac)x^2+3aB\right)}{\sqrt{cx^4+bx^2+a}}dx}{5c}$$

$\Big\downarrow$ 1602

$$\frac{Bx^3\sqrt{a+bx^2+cx^4}}{5c}-\frac{\frac{x\sqrt{a+bx^2+cx^4}(4bB-5Ac)}{3c}-\frac{\int \frac{\left(8Bb^2-10Acb-9aBc\right)x^2+a(4bB-5Ac)}{\sqrt{cx^4+bx^2+a}}dx}{3c}}{5c}$$

$\Big\downarrow$ 1511

$$\frac{Bx^3\sqrt{a+bx^2+cx^4}}{5c}-\frac{\frac{x\sqrt{a+bx^2+cx^4}(4bB-5Ac)}{3c}-\frac{\frac{\sqrt{a}\left(\sqrt{a}\sqrt{c}(4bB-5Ac)-9aBc-10Abc+8b^2B\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\sqrt{a}\left(-9aBc-10Abc+8b^2B\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{3c}}{5c}$$

$\Big\downarrow$ 27

$$\frac{Bx^3\sqrt{a+bx^2+cx^4}}{5c}-\frac{\frac{x\sqrt{a+bx^2+cx^4}(4bB-5Ac)}{3c}-\frac{\frac{\sqrt{a}\left(\sqrt{a}\sqrt{c}(4bB-5Ac)-9aBc-10Abc+8b^2B\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\left(-9aBc-10Abc+8b^2B\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{3c}}{5c}$$

$\Big\downarrow$ 1416

$$\frac{Bx^3\sqrt{a+bx^2+cx^4}}{5c}-\frac{\frac{x\sqrt{a+bx^2+cx^4}(4bB-5Ac)}{3c}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}\sqrt{c}(4bB-5Ac)-9aBc-10Abc+8b^2B\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\cdots}{3c}}{5c}$$

$\Big\downarrow$ 1509

$$\frac{Bx^3\sqrt{a+bx^2+cx^4}}{5c}-\frac{\frac{x\sqrt{a+bx^2+cx^4}(4bB-5Ac)}{3c}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}\sqrt{c}(4bB-5Ac)-9aBc-10Abc+8b^2B\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\cdots}{3c}}{5c}$$

input
```
Int[(x^4*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

3.176. $\int \frac{x^4\left(A+Bx^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx$

output
```
(B*x^3*Sqrt[a + b*x^2 + c*x^4])/(5*c) - (((4*b*B - 5*A*c)*x*Sqrt[a + b*x^2
 + c*x^4])/(3*c) - (-(((8*b^2*B - 10*A*b*c - 9*a*B*c)*(-((x*Sqrt[a + b*x^2
 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqr
t[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/
4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*
x^4])))/Sqrt[c]) + (a^(1/4)*(8*b^2*B - 10*A*b*c - 9*a*B*c + Sqrt[a]*Sqrt[c
]*(4*b*B - 5*A*c))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[
a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt
[a]*Sqrt[c]))/4])/(2*c^(3/4)*Sqrt[a + b*x^2 + c*x^4]))/(3*c))/(5*c)
```

3.176.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

3.176.4 Maple [A] (verified)

Time = 5.78 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.11

method	result
elliptic	$\frac{B\,x^{3}\sqrt{c\,x^{4}+b\,x^{2}+a}}{5c}+\frac{\left(A-\frac{4Bb}{5c}\right)x\sqrt{c\,x^{4}+b\,x^{2}+a}}{3c}-\frac{a\left(A-\frac{4Bb}{5c}\right)\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,F\left(\frac{x\sqrt{2}\,\sqrt{\cdots}}{\cdots}\right.}{12c\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+\cdots}}$
risch	$\frac{x\left(3B\,x^{2}c+5Ac-4Bb\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}{15c^{2}}-\frac{\frac{5Aac\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,F\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}{2},\frac{\sqrt{-4+\cdots}}{\cdots}\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}\cdots}{\cdots}$
default	$B\left(\frac{x^{3}\sqrt{c\,x^{4}+b\,x^{2}+a}}{5c}-\frac{4bx\sqrt{c\,x^{4}+b\,x^{2}+a}}{15c^{2}}+\frac{ab\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,F\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}{2}\cdots\right.}{15c^{2}\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}\cdots\right.$

input
```
int(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*B*x^3*(c*x^4+b*x^2+a)^(1/2)/c+1/3*(A-4/5*B*b/c)/c*x*(c*x^4+b*x^2+a)^(1
/2)-1/12*a/c*(A-4/5*B*b/c)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*
(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1
/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))
/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(-3/5*a/c*B-2
/3*b/c*(A-4/5*B*b/c))*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b
+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)
/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-
b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/
2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*
(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

3.176.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.07

$$\int \frac{x^4(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$= \frac{\sqrt{\frac{1}{2}}\left((8\,Bb^2c-(9\,Ba+10\,Ab)c^2)x\sqrt{\frac{b^2-4\,ac}{c^2}}-(8\,Bb^3-(9\,Bab+10\,Ab^2)c)x\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}E(\arcsin\Bigg(}{}$$

input
```
integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/30*(sqrt(1/2)*((8*B*b^2*c - (9*B*a + 10*A*b)*c^2)*x*sqrt((b^2 - 4*a*c)/c
^2) - (8*B*b^3 - (9*B*a*b + 10*A*b^2)*c)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*
a*c)/c^2) - b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c
^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - s
qrt(1/2)*((8*B*b^2*c + 5*A*c^3 - (9*B*a + 2*(5*A + 2*B)*b)*c^2)*x*sqrt((b^
2 - 4*a*c)/c^2) - (8*B*b^3 - 5*A*b*c^2 - (9*B*a*b + 2*(5*A - 2*B)*b^2)*c)*
x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_f(arcsin(sqrt(
1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*
c)/c^2) + b^2 - 2*a*c)/(a*c)) + 2*(3*B*c^3*x^4 + 8*B*b^2*c - (9*B*a + 10*A
*b)*c^2 - (4*B*b*c^2 - 5*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(c^4*x)
```

3.176.6 Sympy [F]

$$\int \frac{x^4(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^4(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(x**4*(A + B*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

3.176.7 Maxima [F]

$$\int \frac{x^4(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)x^4}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*x^4/sqrt(c*x^4 + b*x^2 + a), x)
```

3.176.8 Giac [F]

$$\int \frac{x^4(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)x^4}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*x^4/sqrt(c*x^4 + b*x^2 + a), x)
```

3.176.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^4\,(B\,x^2+A)}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.177 $\int \frac{x^2(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

3.177.1 Optimal result

Integrand size = 27, antiderivative size = 336

$$\int \frac{x^2(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{Bx\sqrt{a+bx^2+cx^4}}{3c} - \frac{(2bB-3Ac)x\sqrt{a+bx^2+cx^4}}{3c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{\sqrt[4]{a}(2bB-3Ac)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3c^{7/4}\sqrt{a+bx^2+cx^4}} - \frac{\sqrt[4]{a}\left(2bB+\sqrt{a}B\sqrt{c}-3Ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{7/4}\sqrt{a+bx^2+cx^4}}$$

output
```
1/3*B*x*(c*x^4+b*x^2+a)^(1/2)/c-1/3*(-3*A*c+2*B*b)*x*(c*x^4+b*x^2+a)^(1/2)
/c^(3/2)/(a^(1/2)+x^2*c^(1/2))+1/3*a^(1/4)*(-3*A*c+2*B*b)*(cos(2*arctan(c^
(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(
2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2
*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+b
*x^2+a)^(1/2)-1/6*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2
*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2
*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(2*B*b-3*A*c+B*a^(1/2)
*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+b
*x^2+a)^(1/2)
```

3.177.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.01 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.43

$$\int \frac{x^2(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \frac{4Bc\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x(a+bx^2+cx^4)-i(2bB-3Ac)\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}E}{}$$

input
```
Integrate[(x^2*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(4*B*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(a + b*x^2 + c*x^4) - I*(2*b*B -
3*A*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt
[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])
]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-2*b^2*B + 3*A
*b*c + 2*a*B*c + 2*b*B*Sqrt[b^2 - 4*a*c] - 3*A*c*Sqrt[b^2 - 4*a*c])*Sqrt[(
b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sq
rt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sq
rt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqr
t[b^2 - 4*a*c])])/(12*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 +
 c*x^4])
```

3.177.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1602, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$\Big\downarrow \; 1602$$

$$\frac{Bx\sqrt{a+bx^2+cx^4}}{3c}-\frac{\int \frac{(2bB-3Ac)x^2+aB}{\sqrt{cx^4+bx^2+a}}dx}{3c}$$

$$\Big\downarrow \text{1511}$$

$$\frac{Bx\sqrt{a+bx^2+cx^4}}{3c}-\frac{\frac{\sqrt{a}\left(\sqrt{a}B\sqrt{c}-3Ac+2bB\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\sqrt{a}(2bB-3Ac)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{3c}$$

$$\Big\downarrow \text{27}$$

$$\frac{Bx\sqrt{a+bx^2+cx^4}}{3c}-\frac{\frac{\sqrt{a}\left(\sqrt{a}B\sqrt{c}-3Ac+2bB\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{(2bB-3Ac)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{3c}$$

$$\Big\downarrow \text{1416}$$

$$\frac{Bx\sqrt{a+bx^2+cx^4}}{3c}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}B\sqrt{c}-3Ac+2bB\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{(2bB-3Ac)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{3c}$$

$$\Big\downarrow \text{1509}$$

$$\frac{Bx\sqrt{a+bx^2+cx^4}}{3c}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}B\sqrt{c}-3Ac+2bB\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{(2bB-3Ac)\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+c}{\left(\sqrt{a}+\sqrt{c}\right.}}}{\sqrt[4]{}}\right.}{}}{3c}$$

input
```
Int[(x^2*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(B*x*Sqrt[a + b*x^2 + c*x^4])/(3*c) - (-(((2*b*B - 3*A*c)*(-((x*Sqrt[a + b
*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)
*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c
^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2
+ c*x^4])))/Sqrt[c]) + (a^(1/4)*(2*b*B + Sqrt[a]*B*Sqrt[c] - 3*A*c)*(Sqrt[
a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elli
pticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3
/4)*Sqrt[a + b*x^2 + c*x^4]))/(3*c)
```

3.177. $\int \frac{x^2\left(A+Bx^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx$

3.177.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

3.177. $\int \frac{x^2(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

3.177.4 Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.18

method	result
elliptic	$\frac{Bx\sqrt{c\,x^4+b\,x^2+a}}{3c}-\frac{aB\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{12c\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}$
risch	$\frac{Bx\sqrt{c\,x^4+b\,x^2+a}}{3c}+\frac{-\frac{Ba\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}}{}$
default	$B\left(\frac{x\sqrt{c\,x^4+b\,x^2+a}}{3c}-\frac{a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right.}{12c\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}\right.$

input
```
int(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*B*x*(c*x^4+b*x^2+a)^(1/2)/c-1/12*a/c*B*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^
(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4
*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1
/2*(A-2/3*B*b/c)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*
a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x
^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4
*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-E
llipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-
4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

3.177.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.03

$$\int \frac{x^2(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx =$$

$$-\frac{\sqrt{\frac{1}{2}}\left((2\,Bbc-3\,Ac^2)x\sqrt{\frac{b^2-4\,ac}{c^2}}-(2\,Bb^2-3\,Abc)x\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}E\left(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}}{x}\right)\mid \frac{bc\sqrt{}}{}\right.}{}$$

input
```
integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(sqrt(1/2)*((2*B*b*c - 3*A*c^2)*x*sqrt((b^2 - 4*a*c)/c^2) - (2*B*b^2
- 3*A*b*c)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_e(a
rcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt(
(b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*((2*B*b*c - (3*A + B)
*c^2)*x*sqrt((b^2 - 4*a*c)/c^2) - (2*B*b^2 - (3*A - B)*b*c)*x)*sqrt(c)*sqr
t((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*s
qrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2
- 2*a*c)/(a*c)) - 2*(B*c^2*x^2 - 2*B*b*c + 3*A*c^2)*sqrt(c*x^4 + b*x^2 + a
))/(c^3*x)
```

3.177.6 Sympy [F]

$$\int \frac{x^2(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^2(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(x**2*(A + B*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

3.177.7 Maxima [F]

$$\int \frac{x^2(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)x^2}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*x^2/sqrt(c*x^4 + b*x^2 + a), x)
```

3.177.8 Giac [F]

$$\int \frac{x^2(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)x^2}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*x^2/sqrt(c*x^4 + b*x^2 + a), x)
```

3.177.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^2\,(B\,x^2+A)}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.178 $\int \frac{A+Bx^2}{\sqrt{a+bx^2+cx^4}}\,dx$

3.178.1 Optimal result

Integrand size = 24, antiderivative size = 283

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{Bx\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}B\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{a+bx^2+cx^4}} + \frac{\sqrt[4]{a}\left(B+\frac{A\sqrt{c}}{\sqrt{a}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}$$

output
```
B*x*(c*x^4+b*x^2+a)^(1/2)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))-a^(1/4)*B*(cos(2*a
rctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Ellipt
icE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(
1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/
(c*x^4+b*x^2+a)^(1/2)+1/2*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/
2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/
4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(B+A*c^(1/2)/a
^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+b*x
^2+a)^(1/2)
```

3.178.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.18 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{i\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\left(B\left(-b+\sqrt{b^2-4ac}\right)E\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+(b}{2\sqrt{2}c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{a+bx^2+}}$$

input
```
Integrate[(A + B*x^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
((I/2)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqr
t[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(B*(-b + Sqrt[b^2 - 4*a*c])*Ellip
ticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 -
 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + (b*B - 2*A*c - B*Sqrt[b^2 - 4*a*c])*El
lipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^
2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(Sqrt[2]*c*Sqrt[c/(b + Sqrt[b^2 - 4
*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

3.178.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$\Big\downarrow \text{1511}$$

$$\left(\frac{\sqrt{a}B}{\sqrt{c}}+A\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{\sqrt{a}B\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}$$

$$\Big\downarrow \text{27}$$

$$\left(\frac{\sqrt{a}B}{\sqrt{c}}+A\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{B\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}$$

$$\Big\downarrow \text{1416}$$

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\frac{\sqrt{a}B}{\sqrt{c}}+A\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{B\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}$$

$$\Big\downarrow \text{1509}$$

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\frac{\sqrt{a}B}{\sqrt{c}}+A\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{B\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}x^2}\right)}{\sqrt{c}}$$

input
```
Int[(A + B*x^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
-((B*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(S
qrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*
EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^
(1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[c]) + ((A + (Sqrt[a]*B)/Sqrt[c])*(Sq
rt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*E
llipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a
^(1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

3.178. $\int \frac{A+Bx^2}{\sqrt{a+bx^2+cx^4}}\,dx$

3.178.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

3.178.4 Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.28

method	result
default	$\frac{A\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}F\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{cx^4+bx^2+a}}-\frac{Ba\sqrt{2}\sqrt{4-\frac{2(-b+}{}}}{}$
elliptic	$\frac{A\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}F\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{cx^4+bx^2+a}}-\frac{Ba\sqrt{2}\sqrt{4-\frac{2(-b+}{}}}{}$

input
```
int((B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*A*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2
))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(
1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b
*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*B*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)
^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(Ellipti
cF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+
b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

3.178.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{2\sqrt{cx^4+bx^2+a}Bac+\sqrt{\frac{1}{2}}\left(Bacx\sqrt{\frac{b^2-4ac}{c^2}}-Babx\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2-4ac}{c^2}}-b}{c}}E\left(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4ac}{c^2}}-b}{c}}}{x}\right)\mid \frac{bc\sqrt{b}\cdots}{}\right)}{}$$

input
```
integrate((B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(2*sqrt(c*x^4 + b*x^2 + a)*B*a*c + sqrt(1/2)*(B*a*c*x*sqrt((b^2 - 4*a*
c)/c^2) - B*a*b*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*ellipti
c_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*
sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*((B*a*c - A*c^2)
*x*sqrt((b^2 - 4*a*c)/c^2) - (B*a*b + A*b*c)*x)*sqrt(c)*sqrt((c*sqrt((b^2
- 4*a*c)/c^2) - b)/c)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*
c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c))
)/(a*c^2*x)
```

3.178.6 Sympy [F]

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{A+Bx^2}{\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

3.178.7 Maxima [F]

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/sqrt(c*x^4 + b*x^2 + a), x)
```

3.178.8 Giac [F]

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/sqrt(c*x^4 + b*x^2 + a), x)
```

3.178.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{B\,x^2+A}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((A + B*x^2)/(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((A + B*x^2)/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.179 $\int \frac{A+Bx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx$

3.179.1 Optimal result

Integrand size = 27, antiderivative size = 312

$$\int \frac{A+Bx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx=-\frac{A\sqrt{a+bx^2+cx^4}}{ax}+\frac{A\sqrt{c}x\sqrt{a+bx^2+cx^4}}{a\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{A\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}\sqrt{a+bx^2+cx^4}}+\frac{\left(\sqrt{a}B+A\sqrt{c}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}$$

output
```
-A*(c*x^4+b*x^2+a)^(1/2)/a/x+A*x*c^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a/(a^(1/2)+
x^2*c^(1/2))-A*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*ar
ctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2
-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)
+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/(c*x^4+b*x^2+a)^(1/2)+1/2*(cos(2*arctan(c^(
1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2
*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(B*a^(1/2)+A*
c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(
1/2)/a^(3/4)/c^(1/4)/(c*x^4+b*x^2+a)^(1/2)
```

3.179.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.71 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.44

$$\int \frac{A+Bx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \frac{-4A\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}(a+bx^2+cx^4)+iA\left(-b+\sqrt{b^2-4ac}\right)x\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}E\left(i\operatorname{arcsinh}\left(\right.\right.}{}$$

input
```
Integrate[(A + B*x^2)/(x^2*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
(-4*A*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(a + b*x^2 + c*x^4) + I*A*(-b + Sqrt
[b^2 - 4*a*c])*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]
*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt
[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(2*a*B + A*(-b + Sqrt[b^2 - 4*
a*c]))*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*S
qrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Ellipti
cF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4
*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(4*a*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*Sq
rt[a + b*x^2 + c*x^4])
```

3.179.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1604, 25, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx$$
$$\Big\downarrow \; 1604$$
$$-\frac{\int -\frac{Acx^2+aB}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{A\sqrt{a+bx^2+cx^4}}{ax}$$

$$\downarrow 25$$

$$\frac{\int \frac{Acx^2+aB}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{A\sqrt{a+bx^2+cx^4}}{ax}$$

$$\downarrow 1511$$

$$\frac{\sqrt{a}\left(\sqrt{a}B+A\sqrt{c}\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\sqrt{a}A\sqrt{c}\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{A\sqrt{a+bx^2+cx^4}}{ax}$$

$$\downarrow 27$$

$$\frac{\sqrt{a}\left(\sqrt{a}B+A\sqrt{c}\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-A\sqrt{c}\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{A\sqrt{a+bx^2+cx^4}}{ax}$$

$$\downarrow 1416$$

$$\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}B+A\sqrt{c}\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-A\sqrt{c}\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{A\sqrt{a+bx^2+cx^4}}{ax}$$

$$\downarrow 1509$$

$$\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}B+A\sqrt{c}\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-A\sqrt{c}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\text{ arc}\right.}{\sqrt[4]{c}\sqrt{a+bx^2+}}\right.}{a}$$

$$\frac{A\sqrt{a+bx^2+cx^4}}{ax}$$

input
```
Int[(A + B*x^2)/(x^2*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
-((A*Sqrt[a + b*x^2 + c*x^4])/(a*x)) + (-(A*Sqrt[c]*(-((x*Sqrt[a + b*x^2 +
 c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[
(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)
*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^
4]))) + (a^(1/4)*(Sqrt[a]*B + A*Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a +
 b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/
a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(1/4)*Sqrt[a + b*x^2 + c*x^4]
))/a
```

3.179. $\int \frac{A+Bx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx$

3.179.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.179. $\int \frac{A+Bx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx$

3.179.4 Maple [A] (verified)

Time = 1.94 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.23

method	result
elliptic	$-\frac{A\sqrt{c\,x^4+b\,x^2+a}}{ax}+\frac{B\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}$
default	$\frac{B\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}+A\left(-\frac{\sqrt{c\,x^4+b\,x}}{ax}\right.$
risch	$-\frac{A\sqrt{c\,x^4+b\,x^2+a}}{ax}+\frac{\frac{Ba\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}-}{}$

input
```
int((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-A*(c*x^4+b*x^2+a)^(1/2)/a/x+1/4*B*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/
2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a
*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2
)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*c*A*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^
2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b
+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^
(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/
2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/
c)^(1/2)))
```

3.179.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx = -\frac{2\sqrt{cx^4+bx^2+a}Aac+\sqrt{\frac{1}{2}}\left(Aacx\sqrt{\frac{b^2-4\,ac}{a^2}}-Abcx\right)\sqrt{a}\sqrt{\frac{a\sqrt{\frac{b^2-4\,ac}{a^2}}-b}{a}}E\left(\arcsin\left(\sqrt{\frac{1}{2}}x\sqrt{\frac{a\sqrt{\frac{b^2-4\,ac}{a^2}}-b}{a}}\right)\right.}{}$$

input
```
integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(2*sqrt(c*x^4 + b*x^2 + a)*A*a*c + sqrt(1/2)*(A*a*c*x*sqrt((b^2 - 4*a
*c)/a^2) - A*b*c*x)*sqrt(a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)*ellipt
ic_e(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)), 1/2*(a*b
*sqrt((b^2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(a*c)) + sqrt(1/2)*((B*a^2 - A*a*c
)*x*sqrt((b^2 - 4*a*c)/a^2) + (B*a*b + A*b*c)*x)*sqrt(a)*sqrt((a*sqrt((b^2
 - 4*a*c)/a^2) - b)/a)*elliptic_f(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4
*a*c)/a^2) - b)/a)), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(a*c)
))/(a^2*c*x)
```

3.179.6 Sympy [F]

$$\int \frac{A+Bx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{A+Bx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((B*x**2+A)/x**2/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x**2)/(x**2*sqrt(a + b*x**2 + c*x**4)), x)
```

3.179.7 Maxima [F]

$$\int \frac{A+Bx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{cx^4+bx^2+a}x^2}\,dx$$

input
```
integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^2), x)
```

3.179.8 Giac [F]

$$\int \frac{A+Bx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{cx^4+bx^2+a}x^2}\,dx$$

input
```
integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^2), x)
```

3.179.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{B\,x^2+A}{x^2\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int((A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.180 $\int \frac{A+Bx^2}{x^4\sqrt{a+bx^2+cx^4}}\,dx$

3.180.1 Optimal result

Integrand size = 27, antiderivative size = 376

$$\begin{aligned}\int \frac{A+Bx^2}{x^4\sqrt{a+bx^2+cx^4}}\,dx &= -\frac{A\sqrt{a+bx^2+cx^4}}{3ax^3}+\frac{(2Ab-3aB)\sqrt{a+bx^2+cx^4}}{3a^2x}-\frac{(2Ab-3aB)\sqrt{c}x\sqrt{a+bx^2+cx^4}}{3a^2\left(\sqrt{a}+\sqrt{c}x^2\right)}\\
&+\frac{(2Ab-3aB)\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3a^{7/4}\sqrt{a+bx^2+cx^4}}\\
&-\frac{\left(2Ab-3aB+\sqrt{a}A\sqrt{c}\right)\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6a^{7/4}\sqrt{a+bx^2+cx^4}}\end{aligned}$$

output
```
-1/3*A*(c*x^4+b*x^2+a)^(1/2)/a/x^3+1/3*(2*A*b-3*B*a)*(c*x^4+b*x^2+a)^(1/2)
/a^2/x-1/3*(2*A*b-3*B*a)*x*c^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a^2/(a^(1/2)+x^2*
c^(1/2))+1/3*(2*A*b-3*B*a)*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1
/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1
/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2
+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(7/4)/(c*x^4+b*x^2+a)^(1/2)-1/6*c^(1/
4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/
4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^
(1/2))*(a^(1/2)+x^2*c^(1/2))*(2*A*b-3*B*a+A*a^(1/2)*c^(1/2))*((c*x^4+b*x^2
+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(7/4)/(c*x^4+b*x^2+a)^(1/2)
```

3.180.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.47 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx^2}{x^4\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \frac{-\frac{4(a+bx^2+cx^4)(-2Abx^2+a(A+3Bx^2))}{x^3}+\frac{i\sqrt{2}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\left(-\left((2Ab-3aB)\left(-b+\sqrt{b^2-4ac}\right)E\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\right.\right.\right.\right.}}{12a^2\sqrt{a+}}$$

input
```
Integrate[(A + B*x^2)/(x^4*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
((-4*(a + b*x^2 + c*x^4)*(-2*A*b*x^2 + a*(A + 3*B*x^2)))/x^3 + (I*Sqrt[2]*
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(-((2*A*b - 3*a*B)*(-b + Sqrt[b^2 - 4*a*
c])*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + S
qrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]) + (3*a*B*(b - Sqrt[b^2 - 4*a*c
]) + 2*A*(-b^2 + a*c + b*Sqrt[b^2 - 4*a*c]))*EllipticF[I*ArcSinh[Sqrt[2]*S
qrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 -
 4*a*c])]))/Sqrt[c/(b + Sqrt[b^2 - 4*a*c])])/(12*a^2*Sqrt[a + b*x^2 + c*x^
4])
```

3.180.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {1604, 1604, 25, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^4\sqrt{a+bx^2+cx^4}}\,dx$$
$$\Big\downarrow \text{1604}$$
$$-\frac{\int \frac{Acx^2+2Ab-3aB}{x^2\sqrt{cx^4+bx^2+a}}dx}{3a}-\frac{A\sqrt{a+bx^2+cx^4}}{3ax^3}$$
$$\Big\downarrow \text{1604}$$

$$-\frac{-\frac{\int -\frac{c\left((2Ab-3aB)x^2+aA\right)}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{(2Ab-3aB)\sqrt{a+bx^2+cx^4}}{ax}}{3a}-\frac{A\sqrt{a+bx^2+cx^4}}{3ax^3}$$

$\downarrow$ 25

$$-\frac{\frac{\int \frac{c\left((2Ab-3aB)x^2+aA\right)}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{(2Ab-3aB)\sqrt{a+bx^2+cx^4}}{ax}}{3a}-\frac{A\sqrt{a+bx^2+cx^4}}{3ax^3}$$

$\downarrow$ 27

$$-\frac{\frac{c\int \frac{(2Ab-3aB)x^2+aA}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{(2Ab-3aB)\sqrt{a+bx^2+cx^4}}{ax}}{3a}-\frac{A\sqrt{a+bx^2+cx^4}}{3ax^3}$$

$\downarrow$ 1511

$$-\frac{\frac{c\left(\frac{\sqrt{a}(\sqrt{a}A\sqrt{c}-3aB+2Ab)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\sqrt{a}(2Ab-3aB)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{a}-\frac{(2Ab-3aB)\sqrt{a+bx^2+cx^4}}{ax}}{3a}-\frac{A\sqrt{a+bx^2+cx^4}}{3ax^3}$$

$\downarrow$ 27

$$-\frac{\frac{c\left(\frac{\sqrt{a}(\sqrt{a}A\sqrt{c}-3aB+2Ab)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{(2Ab-3aB)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{a}-\frac{(2Ab-3aB)\sqrt{a+bx^2+cx^4}}{ax}}{3a}-\frac{A\sqrt{a+bx^2+cx^4}}{3ax^3}$$

$\downarrow$ 1416

$$-\frac{\frac{c\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(\sqrt{a}A\sqrt{c}-3aB+2Ab)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{(2Ab-3aB)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{a}-\frac{(2Ab-3aB}{}}{3a}-\frac{A\sqrt{a+bx^2+cx^4}}{3ax^3}$$

$\downarrow$ 1509

3.180. $\int \frac{A+Bx^2}{x^4\sqrt{a+bx^2+cx^4}}\,dx$

$$-\frac{c\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}A\sqrt{c}-3aB+2Ab\right)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{(2Ab-3aB)\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E}{\sqrt[4]{c}\sqrt{a+}}\right.}{}\right.}{a} \quad 3a$$

$$\frac{A\sqrt{a+bx^2+cx^4}}{3ax^3}$$

input
```
Int[(A + B*x^2)/(x^4*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
-1/3*(A*Sqrt[a + b*x^2 + c*x^4])/(a*x^3) - (-(((2*A*b - 3*a*B)*Sqrt[a + b*
x^2 + c*x^4])/(a*x)) + (c*(-(((2*A*b - 3*a*B)*(-((x*Sqrt[a + b*x^2 + c*x^4
])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b
*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^
(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/
Sqrt[c]) + (a^(1/4)*(2*A*b - 3*a*B + Sqrt[a]*A*Sqrt[c])*(Sqrt[a] + Sqrt[c]
*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcT
an[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*Sqrt[a +
 b*x^2 + c*x^4])))/a)/(3*a)
```

3.180.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.180.4 Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.11

method	result
risch	$-\frac{\sqrt{c\,x^4+b\,x^2+a}\,(-2Ab\,x^2+3Ba\,x^2+Aa)}{3a^2x^3}-\frac{c\left(\frac{Aa\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\ \cdots\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{c\,x^4+b\,x^2+a}}\right.}{\cdots}$
elliptic	$-\frac{A\sqrt{c\,x^4+b\,x^2+a}}{3a\,x^3}+\frac{(2Ab-3Ba)\sqrt{c\,x^4+b\,x^2+a}}{3a^2x}-\frac{cA\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4a\cdots}}{a}}}{2}\cdots\right.}{12a\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{c\,x^4+b\,x^2+a}}$
default	$B\left(-\frac{\sqrt{c\,x^4+b\,x^2+a}}{ax}-\frac{c\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\left(F\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+\cdots}\right)}{ac}}}{2}\cdots\right.\right.}{2\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{c\,x^4+b\,x^2+a}\left(b+\sqrt{-4ac+\cdots}\right.}\right.$

input
```
int((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(c*x^4+b*x^2+a)^(1/2)*(-2*A*b*x^2+3*B*a*x^2+A*a)/a^2/x^3-1/3*c/a^2*(1
/4*A*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/
2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^
(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*
b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(2*A*b-3*B*a)*a*2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b
+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1
/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*
b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+
b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))
```

3.180.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^2}{x^4\sqrt{a+bx^2+cx^4}}\,dx =$$

$$-\frac{\sqrt{\frac{1}{2}}\left(\left(3\,Ba^2-2\,Aab\right)x^3\sqrt{\frac{b^2-4\,ac}{a^2}}-\left(3\,Bab-2\,Ab^2\right)x^3\right)\sqrt{a}\sqrt{\frac{a\sqrt{\frac{b^2-4\,ac}{a^2}}-b}{a}}E(\arcsin\left(\sqrt{\frac{1}{2}}x\sqrt{\frac{a\sqrt{\frac{b^2-4\,ac}{a^2}}-b}{a}}\right)\cdots}{\cdots}$$

input
```
integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(sqrt(1/2)*((3*B*a^2 - 2*A*a*b)*x^3*sqrt((b^2 - 4*a*c)/a^2) - (3*B*a*
b - 2*A*b^2)*x^3)*sqrt(a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)*elliptic
_e(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)), 1/2*(a*b*s
qrt((b^2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*(((A + 3*B)*a^2 -
 2*A*a*b)*x^3*sqrt((b^2 - 4*a*c)/a^2) + ((A - 3*B)*a*b + 2*A*b^2)*x^3)*sqr
t(a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)*elliptic_f(arcsin(sqrt(1/2)*x
*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2
) + b^2 - 2*a*c)/(a*c)) + 2*sqrt(c*x^4 + b*x^2 + a)*(A*a^2 + (3*B*a^2 - 2*
A*a*b)*x^2))/(a^3*x^3)
```

3.180.6 Sympy [F]

$$\int \frac{A+Bx^2}{x^4\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{A+Bx^2}{x^4\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((B*x**2+A)/x**4/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x**2)/(x**4*sqrt(a + b*x**2 + c*x**4)), x)
```

3.180.7 Maxima [F]

$$\int \frac{A+Bx^2}{x^4\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{cx^4+bx^2+a}x^4}\,dx$$

input
```
integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^4), x)
```

3.180.8 Giac [F]

$$\int \frac{A+Bx^2}{x^4\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{cx^4+bx^2+a}x^4}\,dx$$

input
```
integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^4), x)
```

3.180.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^4\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{B\,x^2+A}{x^4\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int((A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.181 $\int \frac{x^7(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$

3.181.1 Optimal result

Integrand size = 25, antiderivative size = 98

$$\int \frac{x^7(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = -\frac{89}{48}x^4\sqrt{3+5x^2+x^4}+\frac{3}{8}x^6\sqrt{3+5x^2+x^4} - \frac{1}{384}(24243-3802x^2)\sqrt{3+5x^2+x^4} + \frac{32801}{256}\operatorname{arctanh}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)$$

output
```
32801/256*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))-89/48*x^4*(x^4+5*x^2+
3)^(1/2)+3/8*x^6*(x^4+5*x^2+3)^(1/2)-1/384*(-3802*x^2+24243)*(x^4+5*x^2+3)
^(1/2)
```

3.181.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.65

$$\int \frac{x^7(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{1}{384}\sqrt{3+5x^2+x^4}(-24243+3802x^2-712x^4+144x^6) - \frac{32801}{256}\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)$$

input
```
Integrate[(x^7*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

output
```
(Sqrt[3 + 5*x^2 + x^4]*(-24243 + 3802*x^2 - 712*x^4 + 144*x^6))/384 - (328
01*Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/256
```

3.181.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {1578, 1236, 27, 1236, 27, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7(3x^2+2)}{\sqrt{x^4+5x^2+3}}dx$$

$$\Big\downarrow \text{ 1578}$$

$$\frac{1}{2}\int \frac{x^6(3x^2+2)}{\sqrt{x^4+5x^2+3}}dx^2$$

$$\Big\downarrow \text{ 1236}$$

$$\frac{1}{2}\left(\frac{1}{4}\int -\frac{x^4(89x^2+54)}{2\sqrt{x^4+5x^2+3}}dx^2+\frac{3}{4}\sqrt{x^4+5x^2+3}x^6\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(\frac{3}{4}x^6\sqrt{x^4+5x^2+3}-\frac{1}{8}\int \frac{x^4(89x^2+54)}{\sqrt{x^4+5x^2+3}}dx^2\right)$$

$$\Big\downarrow \text{ 1236}$$

$$\frac{1}{2}\left(\frac{1}{8}\left(-\frac{1}{3}\int -\frac{x^2(1901x^2+1068)}{2\sqrt{x^4+5x^2+3}}dx^2-\frac{89}{3}\sqrt{x^4+5x^2+3}x^4\right)+\frac{3}{4}\sqrt{x^4+5x^2+3}x^6\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(\frac{1}{8}\left(\frac{1}{6}\int \frac{x^2(1901x^2+1068)}{\sqrt{x^4+5x^2+3}}dx^2-\frac{89}{3}x^4\sqrt{x^4+5x^2+3}\right)+\frac{3}{4}\sqrt{x^4+5x^2+3}x^6\right)$$

$$\Big\downarrow \text{ 1225}$$

$$\frac{1}{2}\left(\frac{1}{8}\left(\frac{1}{6}\left(\frac{98403}{8}\int \frac{1}{\sqrt{x^4+5x^2+3}}dx^2-\frac{1}{4}(24243-3802x^2)\sqrt{x^4+5x^2+3}\right)-\frac{89}{3}x^4\sqrt{x^4+5x^2+3}\right)+\frac{3}{4}\sqrt{x^4 \cdots$$

$$\Big\downarrow \text{ 1092}$$

$$\frac{1}{2}\left(\frac{1}{8}\left(\frac{1}{6}\left(\frac{98403}{4}\int \frac{1}{4-x^4}d\frac{2x^2+5}{\sqrt{x^4+5x^2+3}}-\frac{1}{4}\left(24243-3802x^2\right)\sqrt{x^4+5x^2+3}\right)-\frac{89}{3}x^4\sqrt{x^4+5x^2+3}\right)+\frac{3}{4}\sqrt{}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{1}{8}\left(\frac{1}{6}\left(\frac{98403}{8}\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{1}{4}\left(24243-3802x^2\right)\sqrt{x^4+5x^2+3}\right)-\frac{89}{3}x^4\sqrt{x^4+5x^2+3}\right)+\frac{3}{4}$$

input
```
Int[(x^7*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

output
```
((3*x^6*Sqrt[3 + 5*x^2 + x^4])/4 + ((-89*x^4*Sqrt[3 + 5*x^2 + x^4])/3 + (-
1/4*((24243 - 3802*x^2)*Sqrt[3 + 5*x^2 + x^4]) + (98403*ArcTanh[(5 + 2*x^2
)/(2*Sqrt[3 + 5*x^2 + x^4])])/8)/6)/8)/2
```

3.181.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

3.181. $\int \frac{x^7(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.181.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.54

method	result
risch	$\frac{\left(144 x^{6}-712 x^{4}+3802 x^{2}-24243\right) \sqrt{x^{4}+5 x^{2}+3}}{384}+\frac{32801 \ln \left(\frac{5}{2}+x^{2}+\sqrt{x^{4}+5 x^{2}+3}\right)}{256}$
trager	$\left(\frac{3}{8} x^{6}-\frac{89}{48} x^{4}+\frac{1901}{192} x^{2}-\frac{8081}{128}\right) \sqrt{x^{4}+5 x^{2}+3}-\frac{32801 \ln \left(-2 x^{2}+2 \sqrt{x^{4}+5 x^{2}+3}-5\right)}{256}$
pseudoelliptic	$\frac{32801 \ln \left(2 x^{2}+5+2 \sqrt{x^{4}+5 x^{2}+3}\right)}{256}+\frac{\left(144 x^{6}-712 x^{4}+3802 x^{2}-24243\right) \sqrt{x^{4}+5 x^{2}+3}}{384}$
default	$\frac{3 x^{6} \sqrt{x^{4}+5 x^{2}+3}}{8}-\frac{89 x^{4} \sqrt{x^{4}+5 x^{2}+3}}{48}+\frac{1901 x^{2} \sqrt{x^{4}+5 x^{2}+3}}{192}-\frac{8081 \sqrt{x^{4}+5 x^{2}+3}}{128}+\frac{32801 \ln \left(\frac{5}{2}+x^{2}+\sqrt{x^{4}+5 x^{2}+3}\right)}{256}$
elliptic	$\frac{3 x^{6} \sqrt{x^{4}+5 x^{2}+3}}{8}-\frac{89 x^{4} \sqrt{x^{4}+5 x^{2}+3}}{48}+\frac{1901 x^{2} \sqrt{x^{4}+5 x^{2}+3}}{192}-\frac{8081 \sqrt{x^{4}+5 x^{2}+3}}{128}+\frac{32801 \ln \left(\frac{5}{2}+x^{2}+\sqrt{x^{4}+5 x^{2}+3}\right)}{256}$

input
```
int(x^7*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/384*(144*x^6-712*x^4+3802*x^2-24243)*(x^4+5*x^2+3)^(1/2)+32801/256*ln(5/
2+x^2+(x^4+5*x^2+3)^(1/2))
```

3.181.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.57

$$\int \frac{x^7(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{1}{384}\left(144\,x^6-712\,x^4+3802\,x^2-24243\right)\sqrt{x^4+5\,x^2+3} - \frac{32801}{256}\,\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)$$

input
```
integrate(x^7*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
1/384*(144*x^6 - 712*x^4 + 3802*x^2 - 24243)*sqrt(x^4 + 5*x^2 + 3) - 32801
/256*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

3.181.6 Sympy [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.66

$$\int \frac{x^7(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{\sqrt{x^4+5x^2+3}\cdot\left(\frac{3x^6}{4}-\frac{89x^4}{24}+\frac{1901x^2}{96}-\frac{8081}{64}\right)}{2} + \frac{32801\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)}{256}$$

input
```
integrate(x**7*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

output
```
sqrt(x**4 + 5*x**2 + 3)*(3*x**6/4 - 89*x**4/24 + 1901*x**2/96 - 8081/64)/2
 + 32801*log(2*x**2 + 2*sqrt(x**4 + 5*x**2 + 3) + 5)/256
```

3.181.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.92

$$\int \frac{x^7(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{3}{8}\,\sqrt{x^4+5\,x^2+3}x^6 - \frac{89}{48}\,\sqrt{x^4+5\,x^2+3}x^4 + \frac{1901}{192}\,\sqrt{x^4+5\,x^2+3}x^2 - \frac{8081}{128}\,\sqrt{x^4+5\,x^2+3} + \frac{32801}{256}\,\log\left(2\,x^2+2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

input
```
integrate(x^7*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
3/8*sqrt(x^4 + 5*x^2 + 3)*x^6 - 89/48*sqrt(x^4 + 5*x^2 + 3)*x^4 + 1901/192
*sqrt(x^4 + 5*x^2 + 3)*x^2 - 8081/128*sqrt(x^4 + 5*x^2 + 3) + 32801/256*lo
g(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

3.181.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.61

$$\int \frac{x^7(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{1}{384}\sqrt{x^4+5\,x^2+3}\left(2\left(4\left(18\,x^2-89\right)x^2+1901\right)x^2-24243\right) - \frac{32801}{256}\log\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

input
```
integrate(x^7*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
1/384*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(18*x^2 - 89)*x^2 + 1901)*x^2 - 24243) -
 32801/256*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

3.181.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^7(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \int \frac{x^7\,\left(3\,x^2+2\right)}{\sqrt{x^4+5\,x^2+3}}\,dx$$

input
```
int((x^7*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2),x)
```

output
```
int((x^7*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2), x)
```

3.182 $\int \frac{x^5(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$

3.182.1 Optimal result

Integrand size = 25, antiderivative size = 77

$$\int \frac{x^5(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{1}{2}x^4\sqrt{3+5x^2+x^4}+\frac{3}{16}\left(89-14x^2\right)\sqrt{3+5x^2+x^4} - \frac{1083}{32}\text{arctanh}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)$$

output
```
-1083/32*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))+1/2*x^4*(x^4+5*x^2+3)^
(1/2)+3/16*(-14*x^2+89)*(x^4+5*x^2+3)^(1/2)
```

3.182.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.77

$$\int \frac{x^5(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{1}{16}\sqrt{3+5x^2+x^4}\left(267-42x^2+8x^4\right) + \frac{1083}{32}\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)$$

input
```
Integrate[(x^5*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

output
```
(Sqrt[3 + 5*x^2 + x^4]*(267 - 42*x^2 + 8*x^4))/16 + (1083*Log[-5 - 2*x^2 +
 2*Sqrt[3 + 5*x^2 + x^4]])/32
```

3.182.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1578, 1236, 27, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(3x^2+2)}{\sqrt{x^4+5x^2+3}}dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{x^4(3x^2+2)}{\sqrt{x^4+5x^2+3}}dx^2$$

$$\Big\downarrow \text{1236}$$

$$\frac{1}{2}\left(\frac{1}{3}\int -\frac{9x^2(7x^2+4)}{2\sqrt{x^4+5x^2+3}}dx^2+\sqrt{x^4+5x^2+3}x^4\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(x^4\sqrt{x^4+5x^2+3}-\frac{3}{2}\int \frac{x^2(7x^2+4)}{\sqrt{x^4+5x^2+3}}dx^2\right)$$

$$\Big\downarrow \text{1225}$$

$$\frac{1}{2}\left(x^4\sqrt{x^4+5x^2+3}-\frac{3}{2}\left(\frac{361}{8}\int \frac{1}{\sqrt{x^4+5x^2+3}}dx^2-\frac{1}{4}(89-14x^2)\sqrt{x^4+5x^2+3}\right)\right)$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}\left(x^4\sqrt{x^4+5x^2+3}-\frac{3}{2}\left(\frac{361}{4}\int \frac{1}{4-x^4}d\frac{2x^2+5}{\sqrt{x^4+5x^2+3}}-\frac{1}{4}(89-14x^2)\sqrt{x^4+5x^2+3}\right)\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(x^4\sqrt{x^4+5x^2+3}-\frac{3}{2}\left(\frac{361}{8}\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{1}{4}(89-14x^2)\sqrt{x^4+5x^2+3}\right)\right)$$

input
```
Int[(x^5*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

output
```
(x^4*Sqrt[3 + 5*x^2 + x^4] - (3*(-1/4*((89 - 14*x^2)*Sqrt[3 + 5*x^2 + x^4]
) + (361*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/8))/2)/2
```

3.182.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.182. $\int \frac{x^5(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$

3.182.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.62

method	result	size
risch	$\frac{(8x^4-42x^2+267)\sqrt{x^4+5x^2+3}}{16}-\frac{1083\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{32}$	48
trager	$\left(\frac{1}{2}x^4-\frac{21}{8}x^2+\frac{267}{16}\right)\sqrt{x^4+5x^2+3}-\frac{1083\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)}{32}$	51
pseudoelliptic	$-\frac{1083\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)}{32}+\frac{(8x^4-42x^2+267)\sqrt{x^4+5x^2+3}}{16}$	52
default	$\frac{x^4\sqrt{x^4+5x^2+3}}{2}-\frac{21x^2\sqrt{x^4+5x^2+3}}{8}+\frac{267\sqrt{x^4+5x^2+3}}{16}-\frac{1083\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{32}$	70
elliptic	$\frac{x^4\sqrt{x^4+5x^2+3}}{2}-\frac{21x^2\sqrt{x^4+5x^2+3}}{8}+\frac{267\sqrt{x^4+5x^2+3}}{16}-\frac{1083\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{32}$	70

input
```
int(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/16*(8*x^4-42*x^2+267)*(x^4+5*x^2+3)^(1/2)-1083/32*ln(5/2+x^2+(x^4+5*x^2+
3)^(1/2))
```

3.182.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.66

$$\int \frac{x^5(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{1}{16}\left(8\,x^4-42\,x^2+267\right)\sqrt{x^4+5\,x^2+3} + \frac{1083}{32}\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)$$

input
```
integrate(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
1/16*(8*x^4 - 42*x^2 + 267)*sqrt(x^4 + 5*x^2 + 3) + 1083/32*log(-2*x^2 + 2
*sqrt(x^4 + 5*x^2 + 3) - 5)
```

3.182.6 Sympy [A] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.70

$$\int \frac{x^5(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{\left(x^4-\frac{21x^2}{4}+\frac{267}{8}\right)\sqrt{x^4+5x^2+3}}{2} - \frac{1083\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)}{32}$$

input
```
integrate(x**5*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

output
```
(x**4 - 21*x**2/4 + 267/8)*sqrt(x**4 + 5*x**2 + 3)/2 - 1083*log(2*x**2 + 2
*sqrt(x**4 + 5*x**2 + 3) + 5)/32
```

3.182.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.95

$$\int \frac{x^5(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{1}{2}\sqrt{x^4+5x^2+3}x^4 - \frac{21}{8}\sqrt{x^4+5x^2+3}x^2 + \frac{267}{16}\sqrt{x^4+5x^2+3} - \frac{1083}{32}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

input
```
integrate(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(x^4 + 5*x^2 + 3)*x^4 - 21/8*sqrt(x^4 + 5*x^2 + 3)*x^2 + 267/16*sq
rt(x^4 + 5*x^2 + 3) - 1083/32*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

3.182.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.69

$$\int \frac{x^5(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{1}{16}\sqrt{x^4+5x^2+3}\left(2\left(4x^2-21\right)x^2+267\right) + \frac{1083}{32}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

input
```
integrate(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
1/16*sqrt(x^4 + 5*x^2 + 3)*(2*(4*x^2 - 21)*x^2 + 267) + 1083/32*log(2*x^2
- 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

3.182.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \int \frac{x^5\,(3\,x^2+2)}{\sqrt{x^4+5\,x^2+3}}\,dx$$

input
```
int((x^5*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2),x)
```

output
```
int((x^5*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2), x)
```

3.183 $\int \frac{x^3(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$

3.183.1 Optimal result

Integrand size = 25, antiderivative size = 56

$$\int \frac{x^3(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = -\frac{1}{8}\left(37-6x^2\right)\sqrt{3+5x^2+x^4}+\frac{149}{16}\operatorname{arctanh}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)$$

output
```
149/16*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))-1/8*(-6*x^2+37)*(x^4+5*x
^2+3)^(1/2)
```

3.183.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \frac{x^3(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{1}{8}\left(-37+6x^2\right)\sqrt{3+5x^2+x^4}-\frac{149}{16}\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)$$

input
```
Integrate[(x^3*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

output
```
((-37 + 6*x^2)*Sqrt[3 + 5*x^2 + x^4])/8 - (149*Log[-5 - 2*x^2 + 2*Sqrt[3 +
 5*x^2 + x^4]])/16
```

3.183.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1578, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(3x^2+2)}{\sqrt{x^4+5x^2+3}}dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{x^2(3x^2+2)}{\sqrt{x^4+5x^2+3}}dx^2$$

$$\Big\downarrow \text{1225}$$

$$\frac{1}{2}\left(\frac{149}{8}\int \frac{1}{\sqrt{x^4+5x^2+3}}dx^2-\frac{1}{4}(37-6x^2)\sqrt{x^4+5x^2+3}\right)$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}\left(\frac{149}{4}\int \frac{1}{4-x^4}d\frac{2x^2+5}{\sqrt{x^4+5x^2+3}}-\frac{1}{4}(37-6x^2)\sqrt{x^4+5x^2+3}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{149}{8}\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{1}{4}(37-6x^2)\sqrt{x^4+5x^2+3}\right)$$

input
```
Int[(x^3*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

output
```
(-1/4*((37 - 6*x^2)*Sqrt[3 + 5*x^2 + x^4]) + (149*ArcTanh[(5 + 2*x^2)/(2*S
qrt[3 + 5*x^2 + x^4])])/8)/2
```

3.183.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.183.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

method	result	size
risch	$\frac{(6x^2-37)\sqrt{x^4+5x^2+3}}{8}+\frac{149\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{16}$	43
trager	$\left(\frac{3x^2}{4}-\frac{37}{8}\right)\sqrt{x^4+5x^2+3}+\frac{149\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)}{16}$	46
default	$\frac{3x^2\sqrt{x^4+5x^2+3}}{4}-\frac{37\sqrt{x^4+5x^2+3}}{8}+\frac{149\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{16}$	53
elliptic	$\frac{3x^2\sqrt{x^4+5x^2+3}}{4}-\frac{37\sqrt{x^4+5x^2+3}}{8}+\frac{149\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{16}$	53
pseudoelliptic	$\frac{149\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)}{16}-\frac{37\sqrt{x^4+5x^2+3}}{8}+\frac{3x^2\sqrt{x^4+5x^2+3}}{4}$	57

input
```
int(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(6*x^2-37)*(x^4+5*x^2+3)^(1/2)+149/16*ln(5/2+x^2+(x^4+5*x^2+3)^(1/2))
```

3.183.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \frac{x^3(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{1}{8}\sqrt{x^4+5\,x^2+3}\left(6\,x^2-37\right) - \frac{149}{16}\,\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)$$

input
```
integrate(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
1/8*sqrt(x^4 + 5*x^2 + 3)*(6*x^2 - 37) - 149/16*log(-2*x^2 + 2*sqrt(x^4 +
5*x^2 + 3) - 5)
```

3.183.6 Sympy [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

$$\int \frac{x^3(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{\left(\frac{3x^2}{2}-\frac{37}{4}\right)\sqrt{x^4+5x^2+3}}{2} + \frac{149\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)}{16}$$

input
```
integrate(x**3*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

output
```
(3*x**2/2 - 37/4)*sqrt(x**4 + 5*x**2 + 3)/2 + 149*log(2*x**2 + 2*sqrt(x**4
 + 5*x**2 + 3) + 5)/16
```

3.183.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \frac{x^3(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{3}{4}\sqrt{x^4+5x^2+3}x^2 - \frac{37}{8}\sqrt{x^4+5x^2+3} + \frac{149}{16}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

input
```
integrate(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
3/4*sqrt(x^4 + 5*x^2 + 3)*x^2 - 37/8*sqrt(x^4 + 5*x^2 + 3) + 149/16*log(2*
x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

3.183.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \frac{x^3(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{1}{8}\sqrt{x^4+5x^2+3}(6x^2-37) - \frac{149}{16}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

input
```
integrate(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
1/8*sqrt(x^4 + 5*x^2 + 3)*(6*x^2 - 37) - 149/16*log(2*x^2 - 2*sqrt(x^4 + 5
*x^2 + 3) + 5)
```

3.183.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \int \frac{x^3\,(3x^2+2)}{\sqrt{x^4+5x^2+3}}\,dx$$

input
```
int((x^3*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2),x)
```

output
```
int((x^3*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2), x)
```

3.184 $\int \frac{x(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$

3.184.1 Optimal result

Integrand size = 23, antiderivative size = 49

$$\int \frac{x(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{3}{2}\sqrt{3+5x^2+x^4}-\frac{11}{4}\text{arctanh}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)$$

output
```
-11/4*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))+3/2*(x^4+5*x^2+3)^(1/2)
```

3.184.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

$$\int \frac{x(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{3}{2}\sqrt{3+5x^2+x^4}+\frac{11}{4}\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)$$

input
```
Integrate[(x*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

output
```
(3*Sqrt[3 + 5*x^2 + x^4])/2 + (11*Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]
])/4
```

3.184.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1576, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(3x^2+2)}{\sqrt{x^4+5x^2+3}}dx$$

$$\Big\downarrow \text{ 1576}$$

$$\frac{1}{2}\int \frac{3x^2+2}{\sqrt{x^4+5x^2+3}}dx^2$$

$$\Big\downarrow \text{ 1160}$$

$$\frac{1}{2}\left(3\sqrt{x^4+5x^2+3}-\frac{11}{2}\int \frac{1}{\sqrt{x^4+5x^2+3}}dx^2\right)$$

$$\Big\downarrow \text{ 1092}$$

$$\frac{1}{2}\left(3\sqrt{x^4+5x^2+3}-11\int \frac{1}{4-x^4}d\frac{2x^2+5}{\sqrt{x^4+5x^2+3}}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}\left(3\sqrt{x^4+5x^2+3}-\frac{11}{2}\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)\right)$$

input
```
Int[(x*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

output
```
(3*Sqrt[3 + 5*x^2 + x^4] - (11*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4
])])/2)/2
```

3.184.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

3.184.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

method	result	size
default	$-\frac{11 \ln \left(\frac{5}{2}+x^{2}+\sqrt{x^{4}+5 x^{2}+3}\right)}{4}+\frac{3 \sqrt{x^{4}+5 x^{2}+3}}{2}$	36
risch	$-\frac{11 \ln \left(\frac{5}{2}+x^{2}+\sqrt{x^{4}+5 x^{2}+3}\right)}{4}+\frac{3 \sqrt{x^{4}+5 x^{2}+3}}{2}$	36
elliptic	$-\frac{11 \ln \left(\frac{5}{2}+x^{2}+\sqrt{x^{4}+5 x^{2}+3}\right)}{4}+\frac{3 \sqrt{x^{4}+5 x^{2}+3}}{2}$	36
trager	$\frac{3 \sqrt{x^{4}+5 x^{2}+3}}{2}+\frac{11 \ln \left(-2 x^{2}+2 \sqrt{x^{4}+5 x^{2}+3}-5\right)}{4}$	40
pseudoelliptic	$-\frac{11 \ln \left(2 x^{2}+5+2 \sqrt{x^{4}+5 x^{2}+3}\right)}{4}+\frac{3 \sqrt{x^{4}+5 x^{2}+3}}{2}$	40

input
```
int(x*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-11/4*ln(5/2+x^2+(x^4+5*x^2+3)^(1/2))+3/2*(x^4+5*x^2+3)^(1/2)
```

3.184.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{x(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{3}{2}\sqrt{x^4+5\,x^2+3}+\frac{11}{4}\,\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)$$

input
```
integrate(x*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
3/2*sqrt(x^4 + 5*x^2 + 3) + 11/4*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

3.184.6 Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{x(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{3\sqrt{x^4+5x^2+3}}{2}-\frac{11\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)}{4}$$

input
```
integrate(x*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

output
```
3*sqrt(x**4 + 5*x**2 + 3)/2 - 11*log(2*x**2 + 2*sqrt(x**4 + 5*x**2 + 3) +
5)/4
```

3.184.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{x(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{3}{2}\sqrt{x^4+5\,x^2+3}-\frac{11}{4}\,\log\left(2\,x^2+2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

input
```
integrate(x*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
3/2*sqrt(x^4 + 5*x^2 + 3) - 11/4*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

3.184.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{x(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{3}{2}\,\sqrt{x^4+5\,x^2+3}+\frac{11}{4}\,\log\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

input
```
integrate(x*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
3/2*sqrt(x^4 + 5*x^2 + 3) + 11/4*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

3.184.9 Mupad [B] (verification not implemented)

Time = 8.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{x(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{3\,\sqrt{x^4+5\,x^2+3}}{2}-\frac{11\,\ln\left(\sqrt{x^4+5\,x^2+3}+x^2+\frac{5}{2}\right)}{4}$$

input
```
int((x*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2),x)
```

output
```
(3*(5*x^2 + x^4 + 3)^(1/2))/2 - (11*log((5*x^2 + x^4 + 3)^(1/2) + x^2 + 5/
2))/4
```

3.185 $\int \frac{2+3x^2}{x\sqrt{3+5x^2+x^4}}\,dx$

3.185.1 Optimal result

Integrand size = 25, antiderivative size = 69

$$\int \frac{2+3x^2}{x\sqrt{3+5x^2+x^4}}\,dx = \frac{3}{2}\text{arctanh}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right) - \frac{\text{arctanh}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{\sqrt{3}}$$

output
```
3/2*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))-1/3*arctanh(1/6*(5*x^2+6)*3
^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)
```

3.185.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \frac{2+3x^2}{x\sqrt{3+5x^2+x^4}}\,dx = \frac{2\text{arctanh}\left(\frac{x^2-\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{3}{2}\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)$$

input
```
Integrate[(2 + 3*x^2)/(x*Sqrt[3 + 5*x^2 + x^4]),x]
```

output
```
(2*ArcTanh[(x^2 - Sqrt[3 + 5*x^2 + x^4])/Sqrt[3]])/Sqrt[3] - (3*Log[-5 - 2
*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/2
```

3.185.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1578, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+2}{x\sqrt{x^4+5x^2+3}}\,dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{3x^2+2}{x^2\sqrt{x^4+5x^2+3}}dx^2$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{2}\left(3\int \frac{1}{\sqrt{x^4+5x^2+3}}dx^2+2\int \frac{1}{x^2\sqrt{x^4+5x^2+3}}dx^2\right)$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}\left(6\int \frac{1}{4-x^4}d\frac{2x^2+5}{\sqrt{x^4+5x^2+3}}+2\int \frac{1}{x^2\sqrt{x^4+5x^2+3}}dx^2\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(2\int \frac{1}{x^2\sqrt{x^4+5x^2+3}}dx^2+3\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)\right)$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{2}\left(3\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-4\int \frac{1}{12-x^4}d\frac{5x^2+6}{\sqrt{x^4+5x^2+3}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(3\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{2\text{arctanh}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{\sqrt{3}}\right)$$

input
```
Int[(2 + 3*x^2)/(x*Sqrt[3 + 5*x^2 + x^4]),x]
```

output
```
(3*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])] - (2*ArcTanh[(6 + 5*x^2)
/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/Sqrt[3])/2
```

3.185.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.185.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.75

method	result
default	$\frac{3\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{2}-\frac{\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{3}$
elliptic	$\frac{3\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{2}-\frac{\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{3}$
pseudoelliptic	$\frac{3\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)}{2}-\frac{\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{3}$
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{5\operatorname{RootOf}\left(\_Z^2-3\right)x^2+6\sqrt{x^4+5x^2+3}+6\operatorname{RootOf}\left(\_Z^2-3\right)}{x^2}\right)}{3}+\frac{3\ln\left(-2x^2-2\sqrt{x^4+5x^2+3}-5\right)}{2}$

input
```
int((3*x^2+2)/x/(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/2*ln(5/2+x^2+(x^4+5*x^2+3)^(1/2))-1/3*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4
+5*x^2+3)^(1/2))*3^(1/2)
```

3.185.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.09

$$\int \frac{2+3x^2}{x\sqrt{3+5x^2+x^4}}\,dx = \frac{1}{3}\sqrt{3}\log\left(\frac{25\,x^2-2\,\sqrt{3}(5\,x^2+6)-2\,\sqrt{x^4+5\,x^2+3}(5\,\sqrt{3}-6)+30}{x^2}\right) - \frac{3}{2}\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)$$

input
```
integrate((3*x^2+2)/x/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*x^2 + 3)*
(5*sqrt(3) - 6) + 30)/x^2) - 3/2*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

3.185.6 Sympy [F]

$$\int \frac{2+3x^2}{x\sqrt{3+5x^2+x^4}}\,dx = \int \frac{3x^2+2}{x\sqrt{x^4+5x^2+3}}\,dx$$

input
```
integrate((3*x**2+2)/x/(x**4+5*x**2+3)**(1/2),x)
```

output
```
Integral((3*x**2 + 2)/(x*sqrt(x**4 + 5*x**2 + 3)), x)
```

3.185.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

$$\int \frac{2+3x^2}{x\sqrt{3+5x^2+x^4}}\,dx = -\frac{1}{3}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right) + \frac{3}{2}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

input
```
integrate((3*x^2+2)/x/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 3/2*lo
g(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

3.185.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13

$$\int \frac{2+3x^2}{x\sqrt{3+5x^2+x^4}}\,dx = \frac{1}{3}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right) - \frac{3}{2}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

input
```
integrate((3*x^2+2)/x/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - s
qrt(x^4 + 5*x^2 + 3))) - 3/2*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

3.185.9 Mupad [B] (verification not implemented)

Time = 8.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.81

$$\int \frac{2+3x^2}{x\sqrt{3+5x^2+x^4}}\,dx = \frac{3\,\ln\left(\sqrt{x^4+5\,x^2+3}+x^2+\frac{5}{2}\right)}{2} - \frac{\sqrt{3}\,\left(\ln\left(\frac{1}{x^2}\right)+\ln\left(2\,\sqrt{3}\,\sqrt{x^4+5\,x^2+3}+5\,x^2+6\right)\right)}{3}$$

input
```
int((3*x^2 + 2)/(x*(5*x^2 + x^4 + 3)^(1/2)),x)
```

output
```
(3*log((5*x^2 + x^4 + 3)^(1/2) + x^2 + 5/2))/2 - (3^(1/2)*(log(1/x^2) + lo
g(2*3^(1/2)*(5*x^2 + x^4 + 3)^(1/2) + 5*x^2 + 6)))/3
```

3.186 $\int \frac{2+3x^2}{x^3\sqrt{3+5x^2+x^4}}\,dx$

3.186.1 Optimal result

Integrand size = 25, antiderivative size = 62

$$\int \frac{2+3x^2}{x^3\sqrt{3+5x^2+x^4}}\,dx = -\frac{\sqrt{3+5x^2+x^4}}{3x^2} - \frac{2\text{arctanh}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{3\sqrt{3}}$$

output
```
-2/9*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-1/3*(x^4+5
*x^2+3)^(1/2)/x^2
```

3.186.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int \frac{2+3x^2}{x^3\sqrt{3+5x^2+x^4}}\,dx = \frac{1}{9}\left(-\frac{3\sqrt{3+5x^2+x^4}}{x^2} + 4\sqrt{3}\text{arctanh}\left(\frac{x^2-\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right)\right)$$

input
```
Integrate[(2 + 3*x^2)/(x^3*Sqrt[3 + 5*x^2 + x^4]),x]
```

output
```
((-3*Sqrt[3 + 5*x^2 + x^4])/x^2 + 4*Sqrt[3]*ArcTanh[(x^2 - Sqrt[3 + 5*x^2
+ x^4])/Sqrt[3]])/9
```

3.186.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1578, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+2}{x^3\sqrt{x^4+5x^2+3}}dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{3x^2+2}{x^4\sqrt{x^4+5x^2+3}}dx^2$$

$$\Big\downarrow \text{1228}$$

$$\frac{1}{2}\left(\frac{4}{3}\int \frac{1}{x^2\sqrt{x^4+5x^2+3}}dx^2-\frac{2\sqrt{x^4+5x^2+3}}{3x^2}\right)$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{2}\left(-\frac{8}{3}\int \frac{1}{12-x^4}d\frac{5x^2+6}{\sqrt{x^4+5x^2+3}}-\frac{2\sqrt{x^4+5x^2+3}}{3x^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(-\frac{4\text{arctanh}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3\sqrt{3}}-\frac{2\sqrt{x^4+5x^2+3}}{3x^2}\right)$$

input
```
Int[(2 + 3*x^2)/(x^3*Sqrt[3 + 5*x^2 + x^4]),x]
```

output
```
((-2*Sqrt[3 + 5*x^2 + x^4])/(3*x^2) - (4*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sq
rt[3 + 5*x^2 + x^4])])/(3*Sqrt[3]))/2
```

3.186.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.186.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{9}-\frac{\sqrt{x^4+5x^2+3}}{3x^2}$	49
risch	$-\frac{2\,\mathrm{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{9}-\frac{\sqrt{x^4+5x^2+3}}{3x^2}$	49
elliptic	$-\frac{2\,\mathrm{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{9}-\frac{\sqrt{x^4+5x^2+3}}{3x^2}$	49
pseudoelliptic	$\frac{-2\,\mathrm{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}\,x^2-3\sqrt{x^4+5x^2+3}}{9x^2}$	54
trager	$-\frac{\sqrt{x^4+5x^2+3}}{3x^2}-\frac{2\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{5\,\mathrm{RootOf}\left(\_Z^2-3\right)x^2+6\sqrt{x^4+5x^2+3}+6\,\mathrm{RootOf}\left(\_Z^2-3\right)}{x^2}\right)}{9}$	67

input
```
int((3*x^2+2)/x^3/(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-1/3*(x^4+5
*x^2+3)^(1/2)/x^2
```

3.186.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.26

$$\int \frac{2+3x^2}{x^3\sqrt{3+5x^2+x^4}}\,dx = \frac{2\sqrt{3}x^2\log\left(\frac{25\,x^2-2\,\sqrt{3}(5\,x^2+6)-2\,\sqrt{x^4+5\,x^2+3}\left(5\,\sqrt{3}-6\right)+30}{x^2}\right)-3\,x^2-3\,\sqrt{x^4+5\,x^2+3}}{9\,x^2}$$

input
```
integrate((3*x^2+2)/x^3/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
1/9*(2*sqrt(3)*x^2*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*x^
2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 3*x^2 - 3*sqrt(x^4 + 5*x^2 + 3))/x^2
```

3.186.6 Sympy [F]

$$\int \frac{2+3x^2}{x^3\sqrt{3+5x^2+x^4}}\,dx = \int \frac{3x^2+2}{x^3\sqrt{x^4+5x^2+3}}\,dx$$

input
```
integrate((3*x**2+2)/x**3/(x**4+5*x**2+3)**(1/2),x)
```

output
```
Integral((3*x**2 + 2)/(x**3*sqrt(x**4 + 5*x**2 + 3)), x)
```

3.186.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.82

$$\int \frac{2+3x^2}{x^3\sqrt{3+5x^2+x^4}}\,dx = -\frac{2}{9}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)-\frac{\sqrt{x^4+5x^2+3}}{3x^2}$$

input
```
integrate((3*x^2+2)/x^3/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
-2/9*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) - 1/3*sq
rt(x^4 + 5*x^2 + 3)/x^2
```

3.186.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(48) = 96.

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.63

$$\int \frac{2+3x^2}{x^3\sqrt{3+5x^2+x^4}}\,dx = \frac{2}{9}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right) + \frac{5x^2-5\sqrt{x^4+5x^2+3}+6}{3\left(\left(x^2-\sqrt{x^4+5x^2+3}\right)^2-3\right)}$$

input
```
integrate((3*x^2+2)/x^3/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
2/9*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - s
qrt(x^4 + 5*x^2 + 3))) + 1/3*(5*x^2 - 5*sqrt(x^4 + 5*x^2 + 3) + 6)/((x^2 -
 sqrt(x^4 + 5*x^2 + 3))^2 - 3)
```

3.186.9 Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.34

$$\int \frac{2+3x^2}{x^3\sqrt{3+5x^2+x^4}}\,dx = \frac{5\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}\,(5\,x^2+6)}{6\,\sqrt{x^4+5\,x^2+3}}\right)}{18} - \frac{\sqrt{x^4+5\,x^2+3}}{3\,x^2} - \frac{\sqrt{3}\,\left(\ln\left(\frac{1}{x^2}\right)+\ln\left(2\,\sqrt{3}\,\sqrt{x^4+5\,x^2+3}+5\,x^2+6\right)\right)}{2}$$

input
```
int((3*x^2 + 2)/(x^3*(5*x^2 + x^4 + 3)^(1/2)),x)
```

output
```
(5*3^(1/2)*atanh((3^(1/2)*(5*x^2 + 6))/(6*(5*x^2 + x^4 + 3)^(1/2))))/18 -
(5*x^2 + x^4 + 3)^(1/2)/(3*x^2) - (3^(1/2)*(log(1/x^2) + log(2*3^(1/2)*(5*
x^2 + x^4 + 3)^(1/2) + 5*x^2 + 6)))/2
```

3.187 $\int \frac{2+3x^2}{x^5\sqrt{3+5x^2+x^4}}\,dx$

3.187.1 Optimal result

Integrand size = 25, antiderivative size = 83

$$\int \frac{2+3x^2}{x^5\sqrt{3+5x^2+x^4}}\,dx=-\frac{\sqrt{3+5x^2+x^4}}{6x^4}-\frac{\sqrt{3+5x^2+x^4}}{12x^2}+\frac{1}{8}\sqrt{3}\text{arctanh}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)$$

output
```
1/8*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-1/6*(x^4+5*
x^2+3)^(1/2)/x^4-1/12*(x^4+5*x^2+3)^(1/2)/x^2
```

3.187.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.76

$$\int \frac{2+3x^2}{x^5\sqrt{3+5x^2+x^4}}\,dx=-\frac{(2+x^2)\sqrt{3+5x^2+x^4}}{12x^4}-\frac{1}{4}\sqrt{3}\text{arctanh}\left(\frac{x^2-\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right)$$

input
```
Integrate[(2 + 3*x^2)/(x^5*Sqrt[3 + 5*x^2 + x^4]),x]
```

output
```
-1/12*((2 + x^2)*Sqrt[3 + 5*x^2 + x^4])/x^4 - (Sqrt[3]*ArcTanh[(x^2 - Sqrt
[3 + 5*x^2 + x^4])/Sqrt[3]])/4
```

3.187.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1578, 1237, 25, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+2}{x^5\sqrt{x^4+5x^2+3}}dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{3x^2+2}{x^6\sqrt{x^4+5x^2+3}}dx^2$$

$$\Big\downarrow \text{1237}$$

$$\frac{1}{2}\left(-\frac{1}{6}\int -\frac{3-2x^2}{x^4\sqrt{x^4+5x^2+3}}dx^2-\frac{\sqrt{x^4+5x^2+3}}{3x^4}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{6}\int \frac{3-2x^2}{x^4\sqrt{x^4+5x^2+3}}dx^2-\frac{\sqrt{x^4+5x^2+3}}{3x^4}\right)$$

$$\Big\downarrow \text{1228}$$

$$\frac{1}{2}\left(\frac{1}{6}\left(-\frac{9}{2}\int \frac{1}{x^2\sqrt{x^4+5x^2+3}}dx^2-\frac{\sqrt{x^4+5x^2+3}}{x^2}\right)-\frac{\sqrt{x^4+5x^2+3}}{3x^4}\right)$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{2}\left(\frac{1}{6}\left(9\int \frac{1}{12-x^4}d\frac{5x^2+6}{\sqrt{x^4+5x^2+3}}-\frac{\sqrt{x^4+5x^2+3}}{x^2}\right)-\frac{\sqrt{x^4+5x^2+3}}{3x^4}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{1}{6}\left(\frac{3}{2}\sqrt{3}\text{arctanh}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)-\frac{\sqrt{x^4+5x^2+3}}{x^2}\right)-\frac{\sqrt{x^4+5x^2+3}}{3x^4}\right)$$

input
```
Int[(2 + 3*x^2)/(x^5*Sqrt[3 + 5*x^2 + x^4]),x]
```

output
```
(-1/3*Sqrt[3 + 5*x^2 + x^4]/x^4 + (-(Sqrt[3 + 5*x^2 + x^4]/x^2) + (3*Sqrt[
3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/2)/6)/2
```

3.187.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.187. $\int \frac{2+3x^2}{x^5\sqrt{3+5x^2+x^4}} \, dx$

3.187.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.77

method	result	size
risch	$-\frac{x^{6}+7x^{4}+13x^{2}+6}{12x^{4}\sqrt{x^{4}+5x^{2}+3}}+\frac{\operatorname{arctanh}\left(\frac{\left(5x^{2}+6\right)\sqrt{3}}{6\sqrt{x^{4}+5x^{2}+3}}\right)\sqrt{3}}{8}$	64
default	$\frac{\operatorname{arctanh}\left(\frac{\left(5x^{2}+6\right)\sqrt{3}}{6\sqrt{x^{4}+5x^{2}+3}}\right)\sqrt{3}}{8}-\frac{\sqrt{x^{4}+5x^{2}+3}}{6x^{4}}-\frac{\sqrt{x^{4}+5x^{2}+3}}{12x^{2}}$	66
elliptic	$\frac{\operatorname{arctanh}\left(\frac{\left(5x^{2}+6\right)\sqrt{3}}{6\sqrt{x^{4}+5x^{2}+3}}\right)\sqrt{3}}{8}-\frac{\sqrt{x^{4}+5x^{2}+3}}{6x^{4}}-\frac{\sqrt{x^{4}+5x^{2}+3}}{12x^{2}}$	66
pseudoelliptic	$\frac{3\operatorname{arctanh}\left(\frac{\left(5x^{2}+6\right)\sqrt{3}}{6\sqrt{x^{4}+5x^{2}+3}}\right)\sqrt{3}\,x^{4}-2x^{2}\sqrt{x^{4}+5x^{2}+3}-4\sqrt{x^{4}+5x^{2}+3}}{24x^{4}}$	71
trager	$-\frac{\left(x^{2}+2\right)\sqrt{x^{4}+5x^{2}+3}}{12x^{4}}-\frac{\operatorname{RootOf}\left(\_Z^{2}-3\right)\ln\left(-\frac{-5\operatorname{RootOf}\left(\_Z^{2}-3\right)x^{2}+6\sqrt{x^{4}+5x^{2}+3}-6\operatorname{RootOf}\left(\_Z^{2}-3\right)}{x^{2}}\right)}{8}$	72

input
```
int((3*x^2+2)/x^5/(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(x^6+7*x^4+13*x^2+6)/x^4/(x^4+5*x^2+3)^(1/2)+1/8*arctanh(1/6*(5*x^2+
6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)
```

3.187.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \frac{2+3x^2}{x^5\sqrt{3+5x^2+x^4}}\,dx = \frac{3\sqrt{3}x^{4}\log\left(\frac{25\,x^{2}+2\,\sqrt{3}(5\,x^{2}+6)+2\,\sqrt{x^{4}+5\,x^{2}+3}\left(5\,\sqrt{3}+6\right)+30}{x^{2}}\right)-2\,x^{4}-2\,\sqrt{x^{4}+5\,x^{2}+3}(x^{2}+2)}{24\,x^{4}}$$

input
```
integrate((3*x^2+2)/x^5/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
1/24*(3*sqrt(3)*x^4*log((25*x^2 + 2*sqrt(3)*(5*x^2 + 6) + 2*sqrt(x^4 + 5*x
^2 + 3)*(5*sqrt(3) + 6) + 30)/x^2) - 2*x^4 - 2*sqrt(x^4 + 5*x^2 + 3)*(x^2
+ 2))/x^4
```

3.187.6 Sympy [F]

$$\int \frac{2+3x^2}{x^5\sqrt{3+5x^2+x^4}}\,dx = \int \frac{3x^2+2}{x^5\sqrt{x^4+5x^2+3}}\,dx$$

input
```
integrate((3*x**2+2)/x**5/(x**4+5*x**2+3)**(1/2),x)
```

output
```
Integral((3*x**2 + 2)/(x**5*sqrt(x**4 + 5*x**2 + 3)), x)
```

3.187.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.82

$$\int \frac{2+3x^2}{x^5\sqrt{3+5x^2+x^4}}\,dx = \frac{1}{8}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right) - \frac{\sqrt{x^4+5x^2+3}}{12x^2} - \frac{\sqrt{x^4+5x^2+3}}{6x^4}$$

input
```
integrate((3*x^2+2)/x^5/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
1/8*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) - 1/12*sq
rt(x^4 + 5*x^2 + 3)/x^2 - 1/6*sqrt(x^4 + 5*x^2 + 3)/x^4
```

3.187.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(65) = 130$.

Time = 0.29 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.75

$$\int \frac{2+3x^2}{x^5\sqrt{3+5x^2+x^4}}\,dx = -\frac{1}{8}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right) + \frac{9\left(x^2-\sqrt{x^4+5x^2+3}\right)^3+36\left(x^2-\sqrt{x^4+5x^2+3}\right)^2+47x^2-47\sqrt{x^4+5x^2+3}+12}{12\left(\left(x^2-\sqrt{x^4+5x^2+3}\right)^2-3\right)^2}$$

input
```
integrate((3*x^2+2)/x^5/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
-1/8*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) -
sqrt(x^4 + 5*x^2 + 3))) + 1/12*(9*(x^2 - sqrt(x^4 + 5*x^2 + 3))^3 + 36*(x^
2 - sqrt(x^4 + 5*x^2 + 3))^2 + 47*x^2 - 47*sqrt(x^4 + 5*x^2 + 3) + 12)/((x
^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 3)^2
```

3.187.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x^2}{x^5\sqrt{3+5x^2+x^4}}\,dx = \int \frac{3\,x^2+2}{x^5\,\sqrt{x^4+5\,x^2+3}}\,dx$$

input
```
int((3*x^2 + 2)/(x^5*(5*x^2 + x^4 + 3)^(1/2)),x)
```

output
```
int((3*x^2 + 2)/(x^5*(5*x^2 + x^4 + 3)^(1/2)), x)
```

3.188 $\int \frac{2+3x^2}{x^7\sqrt{3+5x^2+x^4}}\,dx$

3.188.1 Optimal result

Integrand size = 25, antiderivative size = 104

$$\int \frac{2+3x^2}{x^7\sqrt{3+5x^2+x^4}}\,dx=-\frac{\sqrt{3+5x^2+x^4}}{9x^6}-\frac{\sqrt{3+5x^2+x^4}}{54x^4}+\frac{13\sqrt{3+5x^2+x^4}}{108x^2}-\frac{61\text{arctanh}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{216\sqrt{3}}$$

output
```
-61/648*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-1/9*(x^
4+5*x^2+3)^(1/2)/x^6-1/54*(x^4+5*x^2+3)^(1/2)/x^4+13/108*(x^4+5*x^2+3)^(1/
2)/x^2
```

3.188.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.72

$$\int \frac{2+3x^2}{x^7\sqrt{3+5x^2+x^4}}\,dx=\frac{\sqrt{3+5x^2+x^4}(-12-2x^2+13x^4)}{108x^6}+\frac{61\text{arctanh}\left(\frac{x^2}{\sqrt{3}}-\frac{\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right)}{108\sqrt{3}}$$

input
```
Integrate[(2 + 3*x^2)/(x^7*Sqrt[3 + 5*x^2 + x^4]),x]
```

output
```
(Sqrt[3 + 5*x^2 + x^4]*(-12 - 2*x^2 + 13*x^4))/(108*x^6) + (61*ArcTanh[x^2
/Sqrt[3] - Sqrt[3 + 5*x^2 + x^4]/Sqrt[3]])/(108*Sqrt[3])
```

3.188.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {1578, 1237, 27, 1237, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+2}{x^7\sqrt{x^4+5x^2+3}}dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{3x^2+2}{x^8\sqrt{x^4+5x^2+3}}dx^2$$

$$\Big\downarrow \text{1237}$$

$$\frac{1}{2}\left(-\frac{1}{9}\int -\frac{2(1-2x^2)}{x^6\sqrt{x^4+5x^2+3}}dx^2-\frac{2\sqrt{x^4+5x^2+3}}{9x^6}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{2}{9}\int \frac{1-2x^2}{x^6\sqrt{x^4+5x^2+3}}dx^2-\frac{2\sqrt{x^4+5x^2+3}}{9x^6}\right)$$

$$\Big\downarrow \text{1237}$$

$$\frac{1}{2}\left(\frac{2}{9}\left(-\frac{1}{6}\int \frac{2x^2+39}{2x^4\sqrt{x^4+5x^2+3}}dx^2-\frac{\sqrt{x^4+5x^2+3}}{6x^4}\right)-\frac{2\sqrt{x^4+5x^2+3}}{9x^6}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{2}{9}\left(-\frac{1}{12}\int \frac{2x^2+39}{x^4\sqrt{x^4+5x^2+3}}dx^2-\frac{\sqrt{x^4+5x^2+3}}{6x^4}\right)-\frac{2\sqrt{x^4+5x^2+3}}{9x^6}\right)$$

$$\Big\downarrow \text{1228}$$

$$\frac{1}{2}\left(\frac{2}{9}\left(\frac{1}{12}\left(\frac{61}{2}\int \frac{1}{x^2\sqrt{x^4+5x^2+3}}dx^2+\frac{13\sqrt{x^4+5x^2+3}}{x^2}\right)-\frac{\sqrt{x^4+5x^2+3}}{6x^4}\right)-\frac{2\sqrt{x^4+5x^2+3}}{9x^6}\right)$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{2}\left(\frac{2}{9}\left(\frac{1}{12}\left(\frac{13\sqrt{x^4+5x^2+3}}{x^2}-61\int \frac{1}{12-x^4}d\frac{5x^2+6}{\sqrt{x^4+5x^2+3}}\right)-\frac{\sqrt{x^4+5x^2+3}}{6x^4}\right)-\frac{2\sqrt{x^4+5x^2+3}}{9x^6}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{2}{9}\left(\frac{1}{12}\left(\frac{13\sqrt{x^4+5x^2+3}}{x^2}-\frac{61\text{arctanh}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{2\sqrt{3}}\right)-\frac{\sqrt{x^4+5x^2+3}}{6x^4}\right)-\frac{2\sqrt{x^4+5x^2+3}}{9x^6}\right)$$

input
```
Int[(2 + 3*x^2)/(x^7*Sqrt[3 + 5*x^2 + x^4]),x]
```

output
```
((-2*Sqrt[3 + 5*x^2 + x^4])/(9*x^6) + (2*(-1/6*Sqrt[3 + 5*x^2 + x^4]/x^4 +
 ((13*Sqrt[3 + 5*x^2 + x^4])/x^2 - (61*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt
[3 + 5*x^2 + x^4])])/(2*Sqrt[3]))/12))/9)/2
```

3.188.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.188.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.63

method	result
pseudoelliptic	$\frac{-61\,\operatorname{arctanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}\,x^6+6\sqrt{x^4+5x^2+3}\,\left(13x^4-2x^2-12\right)}{648x^6}$
risch	$\frac{13x^8+63x^6+17x^4-66x^2-36}{108x^6\sqrt{x^4+5x^2+3}}-\frac{61\,\operatorname{arctanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{648}$
trager	$\frac{\left(13x^4-2x^2-12\right)\sqrt{x^4+5x^2+3}}{108x^6}+\frac{61\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{-5\,\mathrm{RootOf}\left(\_Z^2-3\right)x^2+6\sqrt{x^4+5x^2+3}-6\,\mathrm{RootOf}\left(\_Z^2-3\right)}{x^2}\right)}{648}$
default	$-\frac{61\,\operatorname{arctanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{648}-\frac{\sqrt{x^4+5x^2+3}}{9x^6}-\frac{\sqrt{x^4+5x^2+3}}{54x^4}+\frac{13\sqrt{x^4+5x^2+3}}{108x^2}$
elliptic	$-\frac{61\,\operatorname{arctanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{648}-\frac{\sqrt{x^4+5x^2+3}}{9x^6}-\frac{\sqrt{x^4+5x^2+3}}{54x^4}+\frac{13\sqrt{x^4+5x^2+3}}{108x^2}$

input
```
int((3*x^2+2)/x^7/(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/648*(-61*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)*x^6+
6*(x^4+5*x^2+3)^(1/2)*(13*x^4-2*x^2-12))/x^6
```

3.188.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

$$\int \frac{2+3x^2}{x^7\sqrt{3+5x^2+x^4}}\,dx$$
$$= \frac{61\sqrt{3}x^6\log\left(\frac{25\,x^2-2\,\sqrt{3}(5\,x^2+6)-2\,\sqrt{x^4+5\,x^2+3}\left(5\,\sqrt{3}-6\right)+30}{x^2}\right)+78\,x^6+6\,(13\,x^4-2\,x^2-12)\sqrt{x^4+5\,x^2+3}}{648\,x^6}$$

input
```
integrate((3*x^2+2)/x^7/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
1/648*(61*sqrt(3)*x^6*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5
*x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) + 78*x^6 + 6*(13*x^4 - 2*x^2 - 12)*sq
rt(x^4 + 5*x^2 + 3))/x^6
```

3.188.6 Sympy [F]

$$\int \frac{2+3x^2}{x^7\sqrt{3+5x^2+x^4}}\,dx = \int \frac{3x^2+2}{x^7\sqrt{x^4+5x^2+3}}\,dx$$

input
```
integrate((3*x**2+2)/x**7/(x**4+5*x**2+3)**(1/2),x)
```

output
```
Integral((3*x**2 + 2)/(x**7*sqrt(x**4 + 5*x**2 + 3)), x)
```

3.188.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.82

$$\int \frac{2+3x^2}{x^7\sqrt{3+5x^2+x^4}}\,dx = -\frac{61}{648}\,\sqrt{3}\log\left(\frac{2\,\sqrt{3}\sqrt{x^4+5\,x^2+3}}{x^2}+\frac{6}{x^2}+5\right) + \frac{13\,\sqrt{x^4+5\,x^2+3}}{108\,x^2} - \frac{\sqrt{x^4+5\,x^2+3}}{54\,x^4} - \frac{\sqrt{x^4+5\,x^2+3}}{9\,x^6}$$

input
```
integrate((3*x^2+2)/x^7/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
-61/648*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 13/
108*sqrt(x^4 + 5*x^2 + 3)/x^2 - 1/54*sqrt(x^4 + 5*x^2 + 3)/x^4 - 1/9*sqrt(
x^4 + 5*x^2 + 3)/x^6
```

3.188.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(82) = 164$.

Time = 0.30 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.61

$$\int \frac{2+3x^2}{x^7\sqrt{3+5x^2+x^4}}\,dx = \frac{61}{648}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5\,x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5\,x^2+3}}\right) - \frac{61\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^5 - 920\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^3 - 2052\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^2 - 1449\,x^2 + 1}{108\left(\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^2-3\right)^3}$$

input
```
integrate((3*x^2+2)/x^7/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
61/648*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3)
- sqrt(x^4 + 5*x^2 + 3))) - 1/108*(61*(x^2 - sqrt(x^4 + 5*x^2 + 3))^5 - 92
0*(x^2 - sqrt(x^4 + 5*x^2 + 3))^3 - 2052*(x^2 - sqrt(x^4 + 5*x^2 + 3))^2 -
 1449*x^2 + 1449*sqrt(x^4 + 5*x^2 + 3) - 108)/((x^2 - sqrt(x^4 + 5*x^2 + 3
))^2 - 3)^3
```

3.188.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x^2}{x^7\sqrt{3+5x^2+x^4}}\,dx = \int \frac{3\,x^2+2}{x^7\,\sqrt{x^4+5\,x^2+3}}\,dx$$

input
```
int((3*x^2 + 2)/(x^7*(5*x^2 + x^4 + 3)^(1/2)),x)
```

output
```
int((3*x^2 + 2)/(x^7*(5*x^2 + x^4 + 3)^(1/2)), x)
```

3.189 $\int \frac{x^4(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$

3.189.1 Optimal result

Integrand size = 25, antiderivative size = 298

$$\int \frac{x^4(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{419x(5+\sqrt{13}+2x^2)}{30\sqrt{3+5x^2+x^4}} - \frac{10}{3}x\sqrt{3+5x^2+x^4} + \frac{3}{5}x^3\sqrt{3+5x^2+x^4}$$
$$- \frac{419\sqrt{\frac{1}{6}(5+\sqrt{13})}\sqrt{\frac{6+(5-\sqrt{13})x^2}{6+(5+\sqrt{13})x^2}}\,(6+(5+\sqrt{13})x^2)\,E\left(\arctan\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right)|\frac{1}{6}(-13+5\sqrt{13})\right)}{30\sqrt{3+5x^2+x^4}}$$
$$+ \frac{5\sqrt{\frac{2}{3(5+\sqrt{13})}}\sqrt{\frac{6+(5-\sqrt{13})x^2}{6+(5+\sqrt{13})x^2}}\,(6+(5+\sqrt{13})x^2)\,\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right),\frac{1}{6}(-13+5\sqrt{13})\right)}{\sqrt{3+5x^2+x^4}}$$

output
```
419/30*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+3)^(1/2)-10/3*x*(x^4+5*x^2+3)^(1/2)
+3/5*x^3*(x^4+5*x^2+3)^(1/2)+5/3*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^
2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*1
3^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*6^(1/2)/
(5+13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+
5*x^2+3)^(1/2)-419/180*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13
^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^
(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(1/2))^(1
/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)
```

3.189.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.23 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.77

$$\int \frac{x^4(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{4x(-150-223x^2-5x^4+9x^6)+419i\sqrt{2}(-5+\sqrt{13})\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}E\left(i\text{arcsinh}\left(\sqrt{\frac{2}{5+\sqrt{13}}}\right.\right.}{60}$$

input
```
Integrate[(x^4*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

output
```
(4*x*(-150 - 223*x^2 - 5*x^4 + 9*x^6) + (419*I)*Sqrt[2]*(-5 + Sqrt[13])*Sq
rt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*Ell
ipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqr
t[2]*(-1795 + 419*Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*
Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x],
19/6 + (5*Sqrt[13])/6])/(60*Sqrt[3 + 5*x^2 + x^4])
```

3.189.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1602, 1602, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(3x^2+2)}{\sqrt{x^4+5x^2+3}}dx$$

$$\Big\downarrow \text{1602}$$

$$\frac{3}{5}x^3\sqrt{x^4+5x^2+3}-\frac{1}{5}\int \frac{x^2(50x^2+27)}{\sqrt{x^4+5x^2+3}}dx$$

$$\Big\downarrow \text{1602}$$

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{419x^2+150}{\sqrt{x^4+5x^2+3}}dx-\frac{50}{3}x\sqrt{x^4+5x^2+3}\right)+\frac{3}{5}\sqrt{x^4+5x^2+3}x^3$$

$$\Big\downarrow \text{1503}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(150\int \frac{1}{\sqrt{x^4+5x^2+3}}dx+419\int \frac{x^2}{\sqrt{x^4+5x^2+3}}dx\right)-\frac{50}{3}x\sqrt{x^4+5x^2+3}\right)+\frac{3}{5}\sqrt{x^4+5x^2+3}x^3$$

$\Big\downarrow$ 1412

$$\frac{1}{5}\left(\frac{1}{3}\left(419\int \frac{x^2}{\sqrt{x^4+5x^2+3}}dx+\frac{25\sqrt{\frac{6}{5+\sqrt{13}}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right.}\right.\right.}{\sqrt{x^4+5x^2+3}}\right.\right.$$

$$\frac{3}{5}\sqrt{x^4+5x^2+3}x^3$$

$\Big\downarrow$ 1455

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{25\sqrt{\frac{6}{5+\sqrt{13}}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}+\right.\right.$$

$$\frac{3}{5}\sqrt{x^4+5x^2+3}x^3$$

input
```
Int[(x^4*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

output
```
(3*x^3*Sqrt[3 + 5*x^2 + x^4])/5 + ((-50*x*Sqrt[3 + 5*x^2 + x^4])/3 + (419*
((x*(5 + Sqrt[13] + 2*x^2))/(2*Sqrt[3 + 5*x^2 + x^4]) - (Sqrt[(5 + Sqrt[13
])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sq
rt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13]
)/6])/(2*Sqrt[3 + 5*x^2 + x^4])) + (25*Sqrt[6/(5 + Sqrt[13])]*Sqrt[(6 + (5
 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*Ellip
ticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/Sqrt[3 + 5*x
^2 + x^4])/3)/5
```

3.189. $\int \frac{x^4(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$

3.189.3.1 Defintions of rubi rules used

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

3.189. $\int \frac{x^4(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$

3.189.4 Maple [A] (verified)

Time = 3.89 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.72

method	result
risch	$\frac{x(9x^2-50)\sqrt{x^4+5x^2+3}}{15}+\frac{60\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}-\frac{5028\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}}{}$
default	$\frac{3x^3\sqrt{x^4+5x^2+3}}{5}-\frac{10x\sqrt{x^4+5x^2+3}}{3}+\frac{60\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}-\frac{5028\sqrt{1-}}{}$
elliptic	$\frac{3x^3\sqrt{x^4+5x^2+3}}{5}-\frac{10x\sqrt{x^4+5x^2+3}}{3}+\frac{60\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}-\frac{5028\sqrt{1-}}{}$

input
```
int(x^4*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*x*(9*x^2-50)*(x^4+5*x^2+3)^(1/2)+60/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1
/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1
/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-5028/
5/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13
^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-30+
6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2
))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))
```

3.189.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.43

$$\int \frac{x^4(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{419\left(\sqrt{13}\sqrt{2}x-5\sqrt{2}x\right)\sqrt{\sqrt{13}-5}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{13}-5}}{2x}\right)\mid\frac{5}{6}\sqrt{13}+\frac{19}{6}\right)-\left(369\sqrt{13}\sqrt{2}x-2345\sqrt{2}x\right)\sqrt{}}{60\,x}$$

input
```
integrate(x^4*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
1/60*(419*(sqrt(13)*sqrt(2)*x - 5*sqrt(2)*x)*sqrt(sqrt(13) - 5)*elliptic_e
(arcsin(1/2*sqrt(2)*sqrt(sqrt(13) - 5)/x), 5/6*sqrt(13) + 19/6) - (369*sqr
t(13)*sqrt(2)*x - 2345*sqrt(2)*x)*sqrt(sqrt(13) - 5)*elliptic_f(arcsin(1/2
*sqrt(2)*sqrt(sqrt(13) - 5)/x), 5/6*sqrt(13) + 19/6) + 4*(9*x^4 - 50*x^2 +
 419)*sqrt(x^4 + 5*x^2 + 3))/x
```

3.189.6 Sympy [F]

$$\int \frac{x^4(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \int \frac{x^4 \cdot (3x^2+2)}{\sqrt{x^4+5x^2+3}}\,dx$$

input
```
integrate(x**4*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

output
```
Integral(x**4*(3*x**2 + 2)/sqrt(x**4 + 5*x**2 + 3), x)
```

3.189.7 Maxima [F]

$$\int \frac{x^4(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \int \frac{(3\,x^2+2)x^4}{\sqrt{x^4+5\,x^2+3}}\,dx$$

input
```
integrate(x^4*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 2)*x^4/sqrt(x^4 + 5*x^2 + 3), x)
```

3.189.8 Giac [F]

$$\int \frac{x^4(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \int \frac{(3\,x^2+2)x^4}{\sqrt{x^4+5\,x^2+3}}\,dx$$

input
```
integrate(x^4*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate((3*x^2 + 2)*x^4/sqrt(x^4 + 5*x^2 + 3), x)
```

3.189.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \int \frac{x^4\,(3\,x^2+2)}{\sqrt{x^4+5\,x^2+3}}\,dx$$

input
```
int((x^4*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2),x)
```

output
```
int((x^4*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2), x)
```

3.190 $\int \frac{x^2(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$

3.190.1 Optimal result

Integrand size = 25, antiderivative size = 270

$$\int \frac{x^2(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = -\frac{4x\left(5+\sqrt{13}+2x^2\right)}{\sqrt{3+5x^2+x^4}}+x\sqrt{3+5x^2+x^4}$$
$$+\frac{2\sqrt{\frac{2}{3}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{3+5x^2+x^4}}$$
$$-\frac{\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{3+5x^2+x^4}}$$

output
```
-4*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+3)^(1/2)+x*(x^4+5*x^2+3)^(1/2)-1/2*(1/(
36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*
(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))
^(1/2))*(6+x^2*(5+13^(1/2)))*6^(1/2)/(5+13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2
)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)+2/3*(1/(36+x^2*(30+6*1
3^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2)
)^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2
*(5+13^(1/2)))*(30+6*13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1
/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)
```

3.190.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.23 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.82

$$\int \frac{x^2(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$$
$$= \frac{2x(3+5x^2+x^4) - 4i\sqrt{2}(-5+\sqrt{13})\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)\cdots}{2\sqrt{3+5x^2\cdots}}$$

input
```
Integrate[(x^2*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

output
```
(2*x*(3 + 5*x^2 + x^4) - (4*I)*Sqrt[2]*(-5 + Sqrt[13])*Sqrt[(-5 + Sqrt[13]
 - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[
Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] + I*Sqrt[2]*(-17 + 4*Sqr
t[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2
*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6
])/(2*Sqrt[3 + 5*x^2 + x^4])
```

3.190.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1602, 27, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(3x^2+2)}{\sqrt{x^4+5x^2+3}}\,dx$$

$$\Big\downarrow \text{1602}$$

$$x\sqrt{x^4+5x^2+3} - \frac{1}{3}\int \frac{3(8x^2+3)}{\sqrt{x^4+5x^2+3}}dx$$

$$\Big\downarrow \text{27}$$

$$x\sqrt{x^4+5x^2+3} - \int \frac{8x^2+3}{\sqrt{x^4+5x^2+3}}dx$$

$$\Big\downarrow \text{1503}$$

$$-3\int \frac{1}{\sqrt{x^4+5x^2+3}}dx-8\int \frac{x^2}{\sqrt{x^4+5x^2+3}}dx+\sqrt{x^4+5x^2+3}x$$

$\big\downarrow$ 1412

$$-8\int \frac{x^2}{\sqrt{x^4+5x^2+3}}dx-\frac{\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}+\sqrt{x^4+5x^2+3}x$$

$\big\downarrow$ 1455

$$-\frac{\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}-8\left(\frac{x\left(2x^2+\sqrt{13}+5\right)}{2\sqrt{x^4+5x^2+3}}-\frac{\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}(-13\right.}{2\sqrt{x^4+5x^2+3}}\right.$$

$$\sqrt{x^4+5x^2+3}x$$

input
```
Int[(x^2*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

output
```
x*Sqrt[3 + 5*x^2 + x^4] - 8*((x*(5 + Sqrt[13] + 2*x^2))/(2*Sqrt[3 + 5*x^2
+ x^4]) - (Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 +
Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[1
3])/6]*x], (-13 + 5*Sqrt[13])/6])/(2*Sqrt[3 + 5*x^2 + x^4])) - (Sqrt[3/(2*
(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(
6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 +
 5*Sqrt[13])/6])/Sqrt[3 + 5*x^2 + x^4]
```

3.190. $\int \frac{x^2(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$

3.190.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

3.190. $\int \frac{x^2(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$

3.190.4 Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.77

method	result
default	$x\sqrt{x^4+5x^2+3}-\frac{18\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}+\frac{288\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{}}{}$
risch	$x\sqrt{x^4+5x^2+3}-\frac{18\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}+\frac{288\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{}}{}$
elliptic	$x\sqrt{x^4+5x^2+3}-\frac{18\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}+\frac{288\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{}}{}$

input
```
int(x^2*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*(x^4+5*x^2+3)^(1/2)-18/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2
)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/
6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))+288/(-30+6*13^(1/2))^
(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/
(x^4+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5
/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2
)+1/6*39^(1/2)))
```

3.190.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.45

$$\int \frac{x^2(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = -\frac{8\left(\sqrt{13}\sqrt{2}x-5\sqrt{2}x\right)\sqrt{\sqrt{13}-5}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{13}-5}}{2x}\right)\mid\frac{5}{6}\sqrt{13}+\frac{19}{6}\right)-\left(7\sqrt{13}\sqrt{2}x-45\sqrt{2}x\right)\sqrt{\sqrt{13}-}}{4x}$$

input
```
integrate(x^2*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*(8*(sqrt(13)*sqrt(2)*x - 5*sqrt(2)*x)*sqrt(sqrt(13) - 5)*elliptic_e(a
rcsin(1/2*sqrt(2)*sqrt(sqrt(13) - 5)/x), 5/6*sqrt(13) + 19/6) - (7*sqrt(13
)*sqrt(2)*x - 45*sqrt(2)*x)*sqrt(sqrt(13) - 5)*elliptic_f(arcsin(1/2*sqrt(
2)*sqrt(sqrt(13) - 5)/x), 5/6*sqrt(13) + 19/6) - 4*sqrt(x^4 + 5*x^2 + 3)*(
x^2 - 8))/x
```

3.190.6 Sympy [F]

$$\int \frac{x^2(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \int \frac{x^2\cdot(3x^2+2)}{\sqrt{x^4+5x^2+3}}\,dx$$

input
```
integrate(x**2*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

output
```
Integral(x**2*(3*x**2 + 2)/sqrt(x**4 + 5*x**2 + 3), x)
```

3.190.7 Maxima [F]

$$\int \frac{x^2(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \int \frac{(3\,x^2+2)x^2}{\sqrt{x^4+5\,x^2+3}}\,dx$$

input
```
integrate(x^2*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 2)*x^2/sqrt(x^4 + 5*x^2 + 3), x)
```

3.190.8 Giac [F]

$$\int \frac{x^2(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \int \frac{(3\,x^2+2)x^2}{\sqrt{x^4+5\,x^2+3}}\,dx$$

input
```
integrate(x^2*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate((3*x^2 + 2)*x^2/sqrt(x^4 + 5*x^2 + 3), x)
```

3.190.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \int \frac{x^2\,(3\,x^2+2)}{\sqrt{x^4+5\,x^2+3}}\,dx$$

input
```
int((x^2*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2),x)
```

output
```
int((x^2*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2), x)
```

3.191 $\int \frac{2+3x^2}{\sqrt{3+5x^2+x^4}}\,dx$

3.191.1 Optimal result

Integrand size = 22, antiderivative size = 257

$$\int \frac{2+3x^2}{\sqrt{3+5x^2+x^4}}\,dx = \frac{3x\left(5+\sqrt{13}+2x^2\right)}{2\sqrt{3+5x^2+x^4}} - \frac{\sqrt{\frac{3}{2}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{2\sqrt{3+5x^2+x^4}} + \frac{\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)\operatorname{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{3+5x^2+x^4}}$$

output
```
3/2*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+3)^(1/2)+1/3*(1/(36+x^2*(30+6*13^(1/2)
)))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13^(1/2))^(1/2)
/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^
(1/2))*6^(1/2)/(5+13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2
))))^(1/2)/(x^4+5*x^2+3)^(1/2)-1/4*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+
x^2*(30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6
*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*
13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x
^2+3)^(1/2)
```

3.191.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.12 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.62

$$\int \frac{2+3x^2}{\sqrt{3+5x^2+x^4}}\,dx = \frac{i\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\left(3\left(-5+\sqrt{13}\right)E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+\left(11-3\sqrt{13}\right)\text{Ellipt}\right.}{2\sqrt{2}\sqrt{3+5x^2+x^4}}$$

input
```
Integrate[(2 + 3*x^2)/Sqrt[3 + 5*x^2 + x^4],x]
```

output
```
((I/2)*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2
*x^2]*(3*(-5 + Sqrt[13])*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19
/6 + (5*Sqrt[13])/6] + (11 - 3*Sqrt[13])*EllipticF[I*ArcSinh[Sqrt[2/(5 + S
qrt[13])]*x], 19/6 + (5*Sqrt[13])/6]))/(Sqrt[2]*Sqrt[3 + 5*x^2 + x^4])
```

3.191.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+2}{\sqrt{x^4+5x^2+3}}\,dx$$

$$\Big\downarrow \text{1503}$$

$$2\int \frac{1}{\sqrt{x^4+5x^2+3}}dx+3\int \frac{x^2}{\sqrt{x^4+5x^2+3}}dx$$

$$\Big\downarrow \text{1412}$$

$$3\int \frac{x^2}{\sqrt{x^4+5x^2+3}}dx+ \frac{\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}$$

$$\Big\downarrow \text{1455}$$

$$\frac{\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}+$$
$$3\left(\frac{x\left(2x^2+\sqrt{13}+5\right)}{2\sqrt{x^4+5x^2+3}}-\frac{\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13\right.\right.}{2\sqrt{x^4+5x^2+3}}\right.$$

input
```
Int[(2 + 3*x^2)/Sqrt[3 + 5*x^2 + x^4],x]
```

output
```
3*((x*(5 + Sqrt[13] + 2*x^2))/(2*Sqrt[3 + 5*x^2 + x^4]) - (Sqrt[(5 + Sqrt[
13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 +
Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[1
3])/6])/(2*Sqrt[3 + 5*x^2 + x^4])) + (Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 +
 (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*El
lipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/Sqrt[3 +
5*x^2 + x^4]
```

3.191.3.1 Defintions of rubi rules used

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

3.191.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.75

method	result
default	$\frac{12\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}-\frac{108\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\left(F\left(\frac{x\sqrt{}}{}\right.\right.}{\sqrt{-30+6\sqrt{13}}\sqrt{x}}$
elliptic	$\frac{12\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}-\frac{108\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\left(F\left(\frac{x\sqrt{}}{}\right.\right.}{\sqrt{-30+6\sqrt{13}}\sqrt{x}}$

input
```
int((3*x^2+2)/(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
12/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*1
3^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(
1/2),5/6*3^(1/2)+1/6*39^(1/2))-108/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^
(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5
+13^(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2
))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))
```

3.191.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.46

$$\int \frac{2+3x^2}{\sqrt{3+5x^2+x^4}}\,dx = \frac{9\left(\sqrt{13}\sqrt{2}x-5\sqrt{2}x\right)\sqrt{\sqrt{13}-5}E(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{13}-5}}{2x}\right)\mid\frac{5}{6}\sqrt{13}+\frac{19}{6})-\left(7\sqrt{13}\sqrt{2}x-55\sqrt{2}x\right)\sqrt{\sqrt{13}-5}}{12x}$$

input
```
integrate((3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
1/12*(9*(sqrt(13)*sqrt(2)*x - 5*sqrt(2)*x)*sqrt(sqrt(13) - 5)*elliptic_e(a
rcsin(1/2*sqrt(2)*sqrt(sqrt(13) - 5)/x), 5/6*sqrt(13) + 19/6) - (7*sqrt(13
)*sqrt(2)*x - 55*sqrt(2)*x)*sqrt(sqrt(13) - 5)*elliptic_f(arcsin(1/2*sqrt(
2)*sqrt(sqrt(13) - 5)/x), 5/6*sqrt(13) + 19/6) + 36*sqrt(x^4 + 5*x^2 + 3))
/x
```

3.191.6 Sympy [F]

$$\int \frac{2+3x^2}{\sqrt{3+5x^2+x^4}}\,dx = \int \frac{3x^2+2}{\sqrt{x^4+5x^2+3}}\,dx$$

input
```
integrate((3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

output
```
Integral((3*x**2 + 2)/sqrt(x**4 + 5*x**2 + 3), x)
```

3.191.7 Maxima [F]

$$\int \frac{2+3x^2}{\sqrt{3+5x^2+x^4}}\,dx = \int \frac{3\,x^2+2}{\sqrt{x^4+5\,x^2+3}}\,dx$$

input
```
integrate((3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 2)/sqrt(x^4 + 5*x^2 + 3), x)
```

3.191.8 Giac [F]

$$\int \frac{2+3x^2}{\sqrt{3+5x^2+x^4}}\,dx = \int \frac{3\,x^2+2}{\sqrt{x^4+5\,x^2+3}}\,dx$$

input
```
integrate((3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate((3*x^2 + 2)/sqrt(x^4 + 5*x^2 + 3), x)
```

3.191.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x^2}{\sqrt{3+5x^2+x^4}}\,dx = \int \frac{3\,x^2+2}{\sqrt{x^4+5\,x^2+3}}\,dx$$

input
```
int((3*x^2 + 2)/(5*x^2 + x^4 + 3)^(1/2),x)
```

output
```
int((3*x^2 + 2)/(5*x^2 + x^4 + 3)^(1/2), x)
```

3.192 $\int \frac{2+3x^2}{x^2\sqrt{3+5x^2+x^4}}\,dx$

3.192.1 Optimal result

Integrand size = 25, antiderivative size = 278

$$\int \frac{2+3x^2}{x^2\sqrt{3+5x^2+x^4}}\,dx = \frac{x\left(5+\sqrt{13}+2x^2\right)}{3\sqrt{3+5x^2+x^4}} - \frac{2\sqrt{3+5x^2+x^4}}{3x} - \frac{\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{3\sqrt{3+5x^2+x^4}} + \frac{\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{3+5x^2+x^4}}$$

output
```
1/3*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+3)^(1/2)-2/3*(x^4+5*x^2+3)^(1/2)/x+1/2
*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*Ellipti
cF(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(
1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*6^(1/2)/(5+13^(1/2))^(1/2)*((6+x^2*(5-13
^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)-1/18*(1/(36+x^2*(
30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13
^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*
(6+x^2*(5+13^(1/2)))*(30+6*13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5
+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)
```

3.192.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.23 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.81

$$\int \frac{2+3x^2}{x^2\sqrt{3+5x^2+x^4}}\,dx$$
$$= \frac{-4(3+5x^2+x^4)+i\sqrt{2}(-5+\sqrt{13})\,x\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}E\left(i\text{arcsinh}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)}{6x\sqrt{3+5x^2+}}$$

input
```
Integrate[(2 + 3*x^2)/(x^2*Sqrt[3 + 5*x^2 + x^4]),x]
```

output
```
(-4*(3 + 5*x^2 + x^4) + I*Sqrt[2]*(-5 + Sqrt[13])*x*Sqrt[(-5 + Sqrt[13] -
2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqr
t[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqrt[2]*(4 + Sqrt[13])*
x*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]
*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(6
*x*Sqrt[3 + 5*x^2 + x^4])
```

3.192.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1604, 25, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+2}{x^2\sqrt{x^4+5x^2+3}}dx$$
$$\Big\downarrow \text{1604}$$
$$-\frac{1}{3}\int -\frac{2x^2+9}{\sqrt{x^4+5x^2+3}}dx-\frac{2\sqrt{x^4+5x^2+3}}{3x}$$
$$\Big\downarrow \text{25}$$
$$\frac{1}{3}\int \frac{2x^2+9}{\sqrt{x^4+5x^2+3}}dx-\frac{2\sqrt{x^4+5x^2+3}}{3x}$$
$$\Big\downarrow \text{1503}$$

$$\frac{1}{3}\left(9\int \frac{1}{\sqrt{x^4+5x^2+3}}dx+2\int \frac{x^2}{\sqrt{x^4+5x^2+3}}dx\right)-\frac{2\sqrt{x^4+5x^2+3}}{3x}$$

$\big\downarrow$ 1412

$$\frac{1}{3}\left(2\int \frac{x^2}{\sqrt{x^4+5x^2+3}}dx+\frac{3\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)\right.}{\sqrt{x^4+5x^2+3}}\right.$$

$$\frac{2\sqrt{x^4+5x^2+3}}{3x}$$

$\big\downarrow$ 1455

$$\frac{1}{3}\left(\frac{3\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}+2\right.$$

$$\frac{2\sqrt{x^4+5x^2+3}}{3x}$$

input
```
Int[(2 + 3*x^2)/(x^2*Sqrt[3 + 5*x^2 + x^4]),x]
```

output
```
(-2*Sqrt[3 + 5*x^2 + x^4])/(3*x) + (2*((x*(5 + Sqrt[13] + 2*x^2))/(2*Sqrt[
3 + 5*x^2 + x^4]) - (Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/
(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(
5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(2*Sqrt[3 + 5*x^2 + x^4])) + (
3*Sqrt[3/(2*(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[
13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6
]*x], (-13 + 5*Sqrt[13])/6])/Sqrt[3 + 5*x^2 + x^4])/3
```

3.192. $\int \frac{2+3x^2}{x^2\sqrt{3+5x^2+x^4}}\,dx$

3.192.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.192.4 Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.76

method	result
default	$\frac{18\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}-\frac{2\sqrt{x^4+5x^2+3}}{3x}-\frac{24\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\right.}}{}$
risch	$\frac{18\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}-\frac{2\sqrt{x^4+5x^2+3}}{3x}-\frac{24\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\right.}}{}$
elliptic	$\frac{18\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}-\frac{2\sqrt{x^4+5x^2+3}}{3x}-\frac{24\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\right.}}{}$

input
```
int((3*x^2+2)/x^2/(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
18/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*1
3^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(
1/2),5/6*3^(1/2)+1/6*39^(1/2))-2/3*(x^4+5*x^2+3)^(1/2)/x-24/(-30+6*13^(1/2
))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/
2)/(x^4+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2
),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(
1/2)+1/6*39^(1/2)))
```

3.192.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.45

$$\int \frac{2+3x^2}{x^2\sqrt{3+5x^2+x^4}}\,dx = -\frac{2\left(\sqrt{13}\sqrt{6}\sqrt{3}x-5\sqrt{6}\sqrt{3}x\right)\sqrt{\sqrt{13}-5}E\left(\arcsin\left(\frac{1}{6}\sqrt{6}x\sqrt{\sqrt{13}-5}\right)\mid\frac{5}{6}\sqrt{13}+\frac{19}{6}\right)+\left(7\sqrt{13}\sqrt{6}\sqrt{3}x+\right.}{108\,x}$$

input
```
integrate((3*x^2+2)/x^2/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
-1/108*(2*(sqrt(13)*sqrt(6)*sqrt(3)*x - 5*sqrt(6)*sqrt(3)*x)*sqrt(sqrt(13)
 - 5)*elliptic_e(arcsin(1/6*sqrt(6)*x*sqrt(sqrt(13) - 5)), 5/6*sqrt(13) +
19/6) + (7*sqrt(13)*sqrt(6)*sqrt(3)*x + 55*sqrt(6)*sqrt(3)*x)*sqrt(sqrt(13
) - 5)*elliptic_f(arcsin(1/6*sqrt(6)*x*sqrt(sqrt(13) - 5)), 5/6*sqrt(13) +
 19/6) + 72*sqrt(x^4 + 5*x^2 + 3))/x
```

3.192.6 Sympy [F]

$$\int \frac{2+3x^2}{x^2\sqrt{3+5x^2+x^4}}\,dx = \int \frac{3x^2+2}{x^2\sqrt{x^4+5x^2+3}}\,dx$$

input
```
integrate((3*x**2+2)/x**2/(x**4+5*x**2+3)**(1/2),x)
```

output
```
Integral((3*x**2 + 2)/(x**2*sqrt(x**4 + 5*x**2 + 3)), x)
```

3.192.7 Maxima [F]

$$\int \frac{2+3x^2}{x^2\sqrt{3+5x^2+x^4}}\,dx = \int \frac{3\,x^2+2}{\sqrt{x^4+5\,x^2+3}x^2}\,dx$$

input
```
integrate((3*x^2+2)/x^2/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^2), x)
```

3.192.8 Giac [F]

$$\int \frac{2+3x^2}{x^2\sqrt{3+5x^2+x^4}}\,dx = \int \frac{3\,x^2+2}{\sqrt{x^4+5\,x^2+3}x^2}\,dx$$

input
```
integrate((3*x^2+2)/x^2/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^2), x)
```

3.192.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x^2}{x^2\sqrt{3+5x^2+x^4}}\,dx=\int \frac{3\,x^2+2}{x^2\,\sqrt{x^4+5\,x^2+3}}\,dx$$

input
```
int((3*x^2 + 2)/(x^2*(5*x^2 + x^4 + 3)^(1/2)),x)
```

output
```
int((3*x^2 + 2)/(x^2*(5*x^2 + x^4 + 3)^(1/2)), x)
```

3.193 $\int \frac{2+3x^2}{x^4\sqrt{3+5x^2+x^4}}\,dx$

3.193.1 Optimal result

Integrand size = 25, antiderivative size = 302

$$\int \frac{2+3x^2}{x^4\sqrt{3+5x^2+x^4}}\,dx = \frac{7x\left(5+\sqrt{13}+2x^2\right)}{54\sqrt{3+5x^2+x^4}}-\frac{2\sqrt{3+5x^2+x^4}}{9x^3}-\frac{7\sqrt{3+5x^2+x^4}}{27x}$$
$$-\frac{7\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{54\sqrt{3+5x^2+x^4}}$$
$$-\frac{\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{9\sqrt{3+5x^2+x^4}}$$

output
```
7/54*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+3)^(1/2)-2/9*(x^4+5*x^2+3)^(1/2)/x^3-
7/27*(x^4+5*x^2+3)^(1/2)/x-1/27*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2
*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13
^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*6^(1/2)/(
5+13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5
*x^2+3)^(1/2)-7/324*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1
/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/
2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(1/2))^(1/2)
*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)
```

3.193.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.24 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.78

$$\int \frac{2+3x^2}{x^4\sqrt{3+5x^2+x^4}}\,dx$$
$$= \frac{-4(18+51x^2+41x^4+7x^6)+7i\sqrt{2}\left(-5+\sqrt{13}\right)x^3\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right.\right.}{108x^3}$$

input
```
Integrate[(2 + 3*x^2)/(x^4*Sqrt[3 + 5*x^2 + x^4]),x]
```

output
```
(-4*(18 + 51*x^2 + 41*x^4 + 7*x^6) + (7*I)*Sqrt[2]*(-5 + Sqrt[13])*x^3*Sqr
t[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*Elli
pticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqrt
[2]*(-47 + 7*Sqrt[13])*x^3*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*S
qrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 1
9/6 + (5*Sqrt[13])/6])/(108*x^3*Sqrt[3 + 5*x^2 + x^4])
```

3.193.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1604, 25, 1604, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+2}{x^4\sqrt{x^4+5x^2+3}}\,dx$$
$$\Big\downarrow \ 1604$$
$$-\frac{1}{9}\int -\frac{7-2x^2}{x^2\sqrt{x^4+5x^2+3}}dx-\frac{2\sqrt{x^4+5x^2+3}}{9x^3}$$
$$\Big\downarrow \ 25$$
$$\frac{1}{9}\int \frac{7-2x^2}{x^2\sqrt{x^4+5x^2+3}}dx-\frac{2\sqrt{x^4+5x^2+3}}{9x^3}$$
$$\Big\downarrow \ 1604$$

$$\frac{1}{9}\left(-\frac{1}{3}\int \frac{6-7x^2}{\sqrt{x^4+5x^2+3}}dx-\frac{7\sqrt{x^4+5x^2+3}}{3x}\right)-\frac{2\sqrt{x^4+5x^2+3}}{9x^3}$$

$$\Big\downarrow \text{1503}$$

$$\frac{1}{9}\left(\frac{1}{3}\left(7\int \frac{x^2}{\sqrt{x^4+5x^2+3}}dx-6\int \frac{1}{\sqrt{x^4+5x^2+3}}dx\right)-\frac{7\sqrt{x^4+5x^2+3}}{3x}\right)-\frac{2\sqrt{x^4+5x^2+3}}{9x^3}$$

$$\Big\downarrow \text{1412}$$

$$\frac{1}{9}\left(\frac{1}{3}\left(7\int \frac{x^2}{\sqrt{x^4+5x^2+3}}dx-\frac{\sqrt{\frac{6}{5+\sqrt{13}}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)\right.}{\sqrt{x^4+5x^2+3}}\right.\right.$$

$$\frac{2\sqrt{x^4+5x^2+3}}{9x^3}$$

$$\Big\downarrow \text{1455}$$

$$\frac{1}{9}\left(\frac{1}{3}\left(7\left(\frac{x\left(2x^2+\sqrt{13}+5\right)}{2\sqrt{x^4+5x^2+3}}-\frac{\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)\right|}{2\sqrt{x^4+5x^2+3}}\right.\right.\right.$$

$$\frac{2\sqrt{x^4+5x^2+3}}{9x^3}$$

input
```
Int[(2 + 3*x^2)/(x^4*Sqrt[3 + 5*x^2 + x^4]),x]
```

output
```
(-2*Sqrt[3 + 5*x^2 + x^4])/(9*x^3) + ((-7*Sqrt[3 + 5*x^2 + x^4])/(3*x) + (
7*((x*(5 + Sqrt[13] + 2*x^2))/(2*Sqrt[3 + 5*x^2 + x^4]) - (Sqrt[(5 + Sqrt[
13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 +
Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[1
3])/6])/(2*Sqrt[3 + 5*x^2 + x^4])) - (Sqrt[6/(5 + Sqrt[13])]*Sqrt[(6 + (5
- Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*Ellipt
icF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/Sqrt[3 + 5*x^
2 + x^4])/3)/9
```

3.193. $\int \frac{2+3x^2}{x^4\sqrt{3+5x^2+x^4}}\,dx$

3.193.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.193. $\int \frac{2+3x^2}{x^4\sqrt{3+5x^2+x^4}}\,dx$

3.193.4 Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.75

method	result
default	$-\frac{7\sqrt{x^4+5x^2+3}}{27x}-\frac{28\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\left(F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)-E\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)\right)}{3\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}\left(5+\sqrt{13}\right)}$
risch	$-\frac{7x^6+41x^4+51x^2+18}{27x^3\sqrt{x^4+5x^2+3}}-\frac{4\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{3\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}-\frac{28\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{}}{}$
elliptic	$-\frac{7\sqrt{x^4+5x^2+3}}{27x}-\frac{28\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\left(F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)-E\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)\right)}{3\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}\left(5+\sqrt{13}\right)}$

input
```
int((3*x^2+2)/x^4/(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-7/27*(x^4+5*x^2+3)^(1/2)/x-28/3/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1
/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+1
3^(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))
-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))-2/9*(x^
4+5*x^2+3)^(1/2)/x^3-4/3/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2
)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/
6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))
```

3.193.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.46

$$\int \frac{2+3x^2}{x^4\sqrt{3+5x^2+x^4}}\,dx =$$
$$-\frac{7\left(\sqrt{13}\sqrt{6}\sqrt{3}x^3-5\sqrt{6}\sqrt{3}x^3\right)\sqrt{\sqrt{13}-5}\,E\left(\arcsin\left(\frac{1}{6}\sqrt{6}x\sqrt{\sqrt{13}-5}\right)\mid\frac{5}{6}\sqrt{13}+\frac{19}{6}\right)-\left(13\sqrt{13}\sqrt{6}\sqrt{3}x\right.}{972}$$

input
```
integrate((3*x^2+2)/x^4/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
-1/972*(7*(sqrt(13)*sqrt(6)*sqrt(3)*x^3 - 5*sqrt(6)*sqrt(3)*x^3)*sqrt(sqrt
(13) - 5)*elliptic_e(arcsin(1/6*sqrt(6)*x*sqrt(sqrt(13) - 5)), 5/6*sqrt(13
) + 19/6) - (13*sqrt(13)*sqrt(6)*sqrt(3)*x^3 - 5*sqrt(6)*sqrt(3)*x^3)*sqrt
(sqrt(13) - 5)*elliptic_f(arcsin(1/6*sqrt(6)*x*sqrt(sqrt(13) - 5)), 5/6*sq
rt(13) + 19/6) + 36*sqrt(x^4 + 5*x^2 + 3)*(7*x^2 + 6))/x^3
```

3.193.6 Sympy [F]

$$\int \frac{2+3x^2}{x^4\sqrt{3+5x^2+x^4}}\,dx = \int \frac{3x^2+2}{x^4\sqrt{x^4+5x^2+3}}\,dx$$

input
```
integrate((3*x**2+2)/x**4/(x**4+5*x**2+3)**(1/2),x)
```

output
```
Integral((3*x**2 + 2)/(x**4*sqrt(x**4 + 5*x**2 + 3)), x)
```

3.193.7 Maxima [F]

$$\int \frac{2+3x^2}{x^4\sqrt{3+5x^2+x^4}}\,dx = \int \frac{3\,x^2+2}{\sqrt{x^4+5\,x^2+3}x^4}\,dx$$

input
```
integrate((3*x^2+2)/x^4/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^4), x)
```

3.193.8 Giac [F]

$$\int \frac{2+3x^2}{x^4\sqrt{3+5x^2+x^4}}\,dx = \int \frac{3\,x^2+2}{\sqrt{x^4+5\,x^2+3}x^4}\,dx$$

input
```
integrate((3*x^2+2)/x^4/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^4), x)
```

3.193.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x^2}{x^4\sqrt{3+5x^2+x^4}}\,dx = \int \frac{3\,x^2+2}{x^4\,\sqrt{x^4+5\,x^2+3}}\,dx$$

input
```
int((3*x^2 + 2)/(x^4*(5*x^2 + x^4 + 3)^(1/2)),x)
```

output
```
int((3*x^2 + 2)/(x^4*(5*x^2 + x^4 + 3)^(1/2)), x)
```

3.194 $\int \frac{x^5(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx$

3.194.1 Optimal result

Integrand size = 25, antiderivative size = 77

$$\int \frac{x^5(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = -\frac{x^2(33+47x^2)}{13\sqrt{3+5x^2+x^4}} + \frac{133}{26}\sqrt{3+5x^2+x^4} - \frac{41}{4}\text{arctanh}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)$$

output
```
-41/4*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))-1/13*x^2*(47*x^2+33)/(x^4
+5*x^2+3)^(1/2)+133/26*(x^4+5*x^2+3)^(1/2)
```

3.194.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.77

$$\int \frac{x^5(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \frac{399+599x^2+39x^4}{26\sqrt{3+5x^2+x^4}} + \frac{41}{4}\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)$$

input
```
Integrate[(x^5*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

output
```
(399 + 599*x^2 + 39*x^4)/(26*Sqrt[3 + 5*x^2 + x^4]) + (41*Log[-5 - 2*x^2 +
 2*Sqrt[3 + 5*x^2 + x^4]])/4
```

3.194.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1578, 1233, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(3x^2+2)}{(x^4+5x^2+3)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1578}$$

$$\frac{1}{2}\int \frac{x^4(3x^2+2)}{(x^4+5x^2+3)^{3/2}}dx^2$$

$$\Big\downarrow \text{ 1233}$$

$$\frac{1}{2}\left(\frac{2}{13}\int \frac{133x^2+66}{2\sqrt{x^4+5x^2+3}}dx^2-\frac{2x^2(47x^2+33)}{13\sqrt{x^4+5x^2+3}}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(\frac{1}{13}\int \frac{133x^2+66}{\sqrt{x^4+5x^2+3}}dx^2-\frac{2x^2(47x^2+33)}{13\sqrt{x^4+5x^2+3}}\right)$$

$$\Big\downarrow \text{ 1160}$$

$$\frac{1}{2}\left(\frac{1}{13}\left(133\sqrt{x^4+5x^2+3}-\frac{533}{2}\int \frac{1}{\sqrt{x^4+5x^2+3}}dx^2\right)-\frac{2x^2(47x^2+33)}{13\sqrt{x^4+5x^2+3}}\right)$$

$$\Big\downarrow \text{ 1092}$$

$$\frac{1}{2}\left(\frac{1}{13}\left(133\sqrt{x^4+5x^2+3}-533\int \frac{1}{4-x^4}d\frac{2x^2+5}{\sqrt{x^4+5x^2+3}}\right)-\frac{2x^2(47x^2+33)}{13\sqrt{x^4+5x^2+3}}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}\left(\frac{1}{13}\left(133\sqrt{x^4+5x^2+3}-\frac{533}{2}\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)\right)-\frac{2x^2(47x^2+33)}{13\sqrt{x^4+5x^2+3}}\right)$$

input
```
Int[(x^5*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

output
```
((-2*x^2*(33 + 47*x^2))/(13*Sqrt[3 + 5*x^2 + x^4]) + (133*Sqrt[3 + 5*x^2 +
 x^4] - (533*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/2)/13)/2
```

3.194.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.194. $\int \frac{x^5(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx$

3.194.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.62

method	result	size
risch	$\frac{39x^4+599x^2+399}{26\sqrt{x^4+5x^2+3}}-\frac{41\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{4}$	48
trager	$\frac{39x^4+599x^2+399}{26\sqrt{x^4+5x^2+3}}+\frac{41\ln\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)}{4}$	52
pseudoelliptic	$\frac{78x^4-533\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)\sqrt{x^4+5x^2+3}+1198x^2+798}{52\sqrt{x^4+5x^2+3}}$	63
default	$\frac{3x^4}{2\sqrt{x^4+5x^2+3}}+\frac{41x^2}{4\sqrt{x^4+5x^2+3}}-\frac{133}{8\sqrt{x^4+5x^2+3}}+\frac{\frac{665x^2}{52}+\frac{3325}{104}}{\sqrt{x^4+5x^2+3}}-\frac{41\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{4}$	91
elliptic	$\frac{3x^4}{2\sqrt{x^4+5x^2+3}}+\frac{41x^2}{4\sqrt{x^4+5x^2+3}}-\frac{133}{8\sqrt{x^4+5x^2+3}}+\frac{\frac{665x^2}{52}+\frac{3325}{104}}{\sqrt{x^4+5x^2+3}}-\frac{41\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{4}$	91

input
```
int(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/26*(39*x^4+599*x^2+399)/(x^4+5*x^2+3)^(1/2)-41/4*ln(5/2+x^2+(x^4+5*x^2+3
)^(1/2))
```

3.194.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.12

$$\int \frac{x^5(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \frac{1811\,x^4+9055\,x^2+1066\,(x^4+5\,x^2+3)\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)+4}{104\,(x^4+5\,x^2+3)}$$

input
```
integrate(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
1/104*(1811*x^4 + 9055*x^2 + 1066*(x^4 + 5*x^2 + 3)*log(-2*x^2 + 2*sqrt(x^
4 + 5*x^2 + 3) - 5) + 4*(39*x^4 + 599*x^2 + 399)*sqrt(x^4 + 5*x^2 + 3) + 5
433)/(x^4 + 5*x^2 + 3)
```

3.194.6 Sympy [F]

$$\int \frac{x^5(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \int \frac{x^5\cdot(3x^2+2)}{(x^4+5x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**5*(3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)
```

output
```
Integral(x**5*(3*x**2 + 2)/(x**4 + 5*x**2 + 3)**(3/2), x)
```

3.194.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.95

$$\int \frac{x^5(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \frac{3\,x^4}{2\,\sqrt{x^4+5\,x^2+3}} + \frac{599\,x^2}{26\,\sqrt{x^4+5\,x^2+3}} + \frac{399}{26\,\sqrt{x^4+5\,x^2+3}} - \frac{41}{4}\log\left(2\,x^2+2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

input
```
integrate(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
3/2*x^4/sqrt(x^4 + 5*x^2 + 3) + 599/26*x^2/sqrt(x^4 + 5*x^2 + 3) + 399/26/
sqrt(x^4 + 5*x^2 + 3) - 41/4*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

3.194.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.68

$$\int \frac{x^5(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \frac{(39\,x^2+599)x^2+399}{26\,\sqrt{x^4+5\,x^2+3}} + \frac{41}{4}\log\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

input
```
integrate(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

output
```
1/26*((39*x^2 + 599)*x^2 + 399)/sqrt(x^4 + 5*x^2 + 3) + 41/4*log(2*x^2 - 2
*sqrt(x^4 + 5*x^2 + 3) + 5)
```

3.194.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \int \frac{x^5\,(3\,x^2+2)}{(x^4+5\,x^2+3)^{3/2}}\,dx$$

input
```
int((x^5*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(3/2),x)
```

output
```
int((x^5*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(3/2), x)
```

3.195 $\int \frac{x^3(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx$

3.195.1 Optimal result

Integrand size = 25, antiderivative size = 56

$$\int \frac{x^3(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \frac{-33-47x^2}{13\sqrt{3+5x^2+x^4}}+\frac{3}{2}\text{arctanh}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)$$

output
```
3/2*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))+1/13*(-47*x^2-33)/(x^4+5*x^
2+3)^(1/2)
```

3.195.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \frac{x^3(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \frac{-33-47x^2}{13\sqrt{3+5x^2+x^4}}-\frac{3}{2}\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)$$

input
```
Integrate[(x^3*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

output
```
(-33 - 47*x^2)/(13*Sqrt[3 + 5*x^2 + x^4]) - (3*Log[-5 - 2*x^2 + 2*Sqrt[3 +
 5*x^2 + x^4]])/2
```

3.195.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1578, 1224, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(3x^2+2)}{(x^4+5x^2+3)^{3/2}}dx$$

$$\Big\downarrow \text{ 1578}$$

$$\frac{1}{2}\int \frac{x^2(3x^2+2)}{(x^4+5x^2+3)^{3/2}}dx^2$$

$$\Big\downarrow \text{ 1224}$$

$$\frac{1}{2}\left(3\int \frac{1}{\sqrt{x^4+5x^2+3}}dx^2-\frac{2(47x^2+33)}{13\sqrt{x^4+5x^2+3}}\right)$$

$$\Big\downarrow \text{ 1092}$$

$$\frac{1}{2}\left(6\int \frac{1}{4-x^4}d\frac{2x^2+5}{\sqrt{x^4+5x^2+3}}-\frac{2(47x^2+33)}{13\sqrt{x^4+5x^2+3}}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}\left(3\text{arctanh}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{2(47x^2+33)}{13\sqrt{x^4+5x^2+3}}\right)$$

input
```
Int[(x^3*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

output
```
((-2*(33 + 47*x^2))/(13*Sqrt[3 + 5*x^2 + x^4]) + 3*ArcTanh[(5 + 2*x^2)/(2*
Sqrt[3 + 5*x^2 + x^4])])/2
```

3.195.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1224
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x))*((a + b*x + c*x^2)^(p
+ 1)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c
*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c))   Int[(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -
1] &&  !(IntegerQ[p] && NeQ[a, 0] && NiceSqrtQ[b^2 - 4*a*c])
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.195.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

method	result	size
risch	$-\frac{47x^2+33}{13\sqrt{x^4+5x^2+3}}+\frac{3\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{2}$	43
trager	$-\frac{47x^2+33}{13\sqrt{x^4+5x^2+3}}+\frac{3\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)}{2}$	47
pseudoelliptic	$\frac{39\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)\sqrt{x^4+5x^2+3}-94x^2-66}{26\sqrt{x^4+5x^2+3}}$	58
elliptic	$\frac{11}{4\sqrt{x^4+5x^2+3}}-\frac{55(2x^2+5)}{52\sqrt{x^4+5x^2+3}}-\frac{3x^2}{2\sqrt{x^4+5x^2+3}}+\frac{3\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{2}$	74
default	$-\frac{3x^2}{2\sqrt{x^4+5x^2+3}}+\frac{15}{4\sqrt{x^4+5x^2+3}}-\frac{75(2x^2+5)}{52\sqrt{x^4+5x^2+3}}+\frac{3\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)}{2}+\frac{\frac{10x^2}{13}+\frac{12}{13}}{\sqrt{x^4+5x^2+3}}$	95

input
```
int(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/13*(47*x^2+33)/(x^4+5*x^2+3)^(1/2)+3/2*ln(5/2+x^2+(x^4+5*x^2+3)^(1/2))
```

3.195.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.45

$$\int \frac{x^3(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = -\frac{94\,x^4+470\,x^2+39\,(x^4+5\,x^2+3)\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)+2\,\sqrt{x^4+5\,x^2+3}(47\,x^2+33)+}{26\,(x^4+5\,x^2+3)}$$

input
```
integrate(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
-1/26*(94*x^4 + 470*x^2 + 39*(x^4 + 5*x^2 + 3)*log(-2*x^2 + 2*sqrt(x^4 + 5
*x^2 + 3) - 5) + 2*sqrt(x^4 + 5*x^2 + 3)*(47*x^2 + 33) + 282)/(x^4 + 5*x^2
 + 3)
```

3.195.6 Sympy [F]

$$\int \frac{x^3(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \int \frac{x^3\cdot(3x^2+2)}{(x^4+5x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3*(3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)
```

output
```
Integral(x**3*(3*x**2 + 2)/(x**4 + 5*x**2 + 3)**(3/2), x)
```

3.195.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \frac{x^3(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = -\frac{47\,x^2}{13\,\sqrt{x^4+5\,x^2+3}} - \frac{33}{13\,\sqrt{x^4+5\,x^2+3}} + \frac{3}{2}\,\log\left(2\,x^2+2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

input
```
integrate(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
-47/13*x^2/sqrt(x^4 + 5*x^2 + 3) - 33/13/sqrt(x^4 + 5*x^2 + 3) + 3/2*log(2
*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

3.195.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \frac{x^3(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = -\frac{47\,x^2+33}{13\,\sqrt{x^4+5\,x^2+3}} - \frac{3}{2}\,\log\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

input
```
integrate(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

output
```
-1/13*(47*x^2 + 33)/sqrt(x^4 + 5*x^2 + 3) - 3/2*log(2*x^2 - 2*sqrt(x^4 + 5
*x^2 + 3) + 5)
```

3.195.9 Mupad [B] (verification not implemented)

Time = 7.85 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\int \frac{x^3(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \frac{3\,\ln\left(\sqrt{x^4+5\,x^2+3}+x^2+\frac{5}{2}\right)}{2} - \frac{47\,x^2}{13\,\sqrt{x^4+5\,x^2+3}} - \frac{33}{13\,\sqrt{x^4+5\,x^2+3}}$$

input
```
int((x^3*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(3/2),x)
```

output
```
(3*log((5*x^2 + x^4 + 3)^(1/2) + x^2 + 5/2))/2 - (47*x^2)/(13*(5*x^2 + x^4
 + 3)^(1/2)) - 33/(13*(5*x^2 + x^4 + 3)^(1/2))
```

3.196 $\int \frac{x(2+3x^2)}{(3+5x^2+x^4)^{3/2}} dx$

3.196.1 Optimal result

Integrand size = 23, antiderivative size = 25

$$\int \frac{x(2+3x^2)}{(3+5x^2+x^4)^{3/2}} dx = \frac{8+11x^2}{13\sqrt{3+5x^2+x^4}}$$

output
```
1/13*(11*x^2+8)/(x^4+5*x^2+3)^(1/2)
```

3.196.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x(2+3x^2)}{(3+5x^2+x^4)^{3/2}} dx = \frac{8+11x^2}{13\sqrt{3+5x^2+x^4}}$$

input
```
Integrate[(x*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

output
```
(8 + 11*x^2)/(13*Sqrt[3 + 5*x^2 + x^4])
```

3.196.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1576, 1158}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(3x^2+2)}{(x^4+5x^2+3)^{3/2}}dx$$

$$\Big\downarrow \text{ 1576}$$

$$\frac{1}{2}\int \frac{3x^2+2}{(x^4+5x^2+3)^{3/2}}dx^2$$

$$\Big\downarrow \text{ 1158}$$

$$\frac{11x^2+8}{13\sqrt{x^4+5x^2+3}}$$

input
```
Int[(x*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

output
```
(8 + 11*x^2)/(13*Sqrt[3 + 5*x^2 + x^4])
```

3.196.3.1 Defintions of rubi rules used

rule 1158
```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

3.196.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
gosper	$\frac{11x^2+8}{13\sqrt{x^4+5x^2+3}}$	22
trager	$\frac{11x^2+8}{13\sqrt{x^4+5x^2+3}}$	22
risch	$\frac{11x^2+8}{13\sqrt{x^4+5x^2+3}}$	22
elliptic	$\frac{11x^2+8}{13\sqrt{x^4+5x^2+3}}$	22
pseudoelliptic	$\frac{11x^2+8}{13\sqrt{x^4+5x^2+3}}$	22
default	$-\frac{2(2x^2+5)}{13\sqrt{x^4+5x^2+3}}+\frac{\frac{15x^2}{13}+\frac{18}{13}}{\sqrt{x^4+5x^2+3}}$	44

input
```
int(x*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/13*(11*x^2+8)/(x^4+5*x^2+3)^(1/2)
```

3.196.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(21) = 42.

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \frac{x(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \frac{11\,x^4+55\,x^2+\sqrt{x^4+5\,x^2+3}(11\,x^2+8)+33}{13\,(x^4+5\,x^2+3)}$$

input
```
integrate(x*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
1/13*(11*x^4 + 55*x^2 + sqrt(x^4 + 5*x^2 + 3)*(11*x^2 + 8) + 33)/(x^4 + 5*
x^2 + 3)
```

3.196.6 Sympy [F]

$$\int \frac{x(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \int \frac{x(3x^2+2)}{(x^4+5x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)
```

output
```
Integral(x*(3*x**2 + 2)/(x**4 + 5*x**2 + 3)**(3/2), x)
```

3.196.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int \frac{x(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \frac{11\,x^2}{13\,\sqrt{x^4+5\,x^2+3}} + \frac{8}{13\,\sqrt{x^4+5\,x^2+3}}$$

input
```
integrate(x*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
11/13*x^2/sqrt(x^4 + 5*x^2 + 3) + 8/13/sqrt(x^4 + 5*x^2 + 3)
```

3.196.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{x(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \frac{11\,x^2+8}{13\,\sqrt{x^4+5\,x^2+3}}$$

input
```
integrate(x*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

output
```
1/13*(11*x^2 + 8)/sqrt(x^4 + 5*x^2 + 3)
```

3.196.9 Mupad [B] (verification not implemented)

Time = 7.58 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{x(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \frac{11\,x^2+8}{13\,\sqrt{x^4+5\,x^2+3}}$$

input
```
int((x*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(3/2),x)
```

output
```
(11*x^2 + 8)/(13*(5*x^2 + x^4 + 3)^(1/2))
```

3.197 $\int \frac{2+3x^2}{x(3+5x^2+x^4)^{3/2}}\,dx$

3.197.1 Optimal result

Integrand size = 25, antiderivative size = 66

$$\int \frac{2+3x^2}{x\left(3+5x^2+x^4\right)^{3/2}}\,dx=\frac{-7-8x^2}{39\sqrt{3+5x^2+x^4}}-\frac{\operatorname{arctanh}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{3\sqrt{3}}$$

output
```
-1/9*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)+1/39*(-8*x
^2-7)/(x^4+5*x^2+3)^(1/2)
```

3.197.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.94

$$\int \frac{2+3x^2}{x\left(3+5x^2+x^4\right)^{3/2}}\,dx=\frac{-7-8x^2}{39\sqrt{3+5x^2+x^4}}+\frac{2\operatorname{arctanh}\left(\frac{x^2-\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right)}{3\sqrt{3}}$$

input
```
Integrate[(2 + 3*x^2)/(x*(3 + 5*x^2 + x^4)^(3/2)),x]
```

output
```
(-7 - 8*x^2)/(39*Sqrt[3 + 5*x^2 + x^4]) + (2*ArcTanh[(x^2 - Sqrt[3 + 5*x^2
 + x^4])/Sqrt[3]])/(3*Sqrt[3])
```

3.197.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1578, 1235, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+2}{x\left(x^4+5x^2+3\right)^{3/2}}dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{3x^2+2}{x^2\left(x^4+5x^2+3\right)^{3/2}}dx^2$$

$$\Big\downarrow \text{1235}$$

$$\frac{1}{2}\left(-\frac{2}{39}\int -\frac{13}{x^2\sqrt{x^4+5x^2+3}}dx^2-\frac{2\left(8x^2+7\right)}{39\sqrt{x^4+5x^2+3}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{2}{3}\int \frac{1}{x^2\sqrt{x^4+5x^2+3}}dx^2-\frac{2\left(8x^2+7\right)}{39\sqrt{x^4+5x^2+3}}\right)$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{2}\left(-\frac{4}{3}\int \frac{1}{12-x^4}d\frac{5x^2+6}{\sqrt{x^4+5x^2+3}}-\frac{2\left(8x^2+7\right)}{39\sqrt{x^4+5x^2+3}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(-\frac{2\text{arctanh}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3\sqrt{3}}-\frac{2\left(8x^2+7\right)}{39\sqrt{x^4+5x^2+3}}\right)$$

input
```
Int[(2 + 3*x^2)/(x*(3 + 5*x^2 + x^4)^(3/2)),x]
```

output
```
((-2*(7 + 8*x^2))/(39*Sqrt[3 + 5*x^2 + x^4]) - (2*ArcTanh[(6 + 5*x^2)/(2*S
qrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(3*Sqrt[3]))/2
```

3.197.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.197. $\int \frac{2+3x^2}{x(3+5x^2+x^4)^{3/2}}\,dx$

3.197.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.80

method	result	size
risch	$-\frac{8x^2+7}{39\sqrt{x^4+5x^2+3}}-\frac{\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{9}$	53
pseudoelliptic	$\frac{-13\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}\sqrt{x^4+5x^2+3}-24x^2-21}{117\sqrt{x^4+5x^2+3}}$	64
default	$-\frac{4(2x^2+5)}{39\sqrt{x^4+5x^2+3}}+\frac{1}{3\sqrt{x^4+5x^2+3}}-\frac{\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{9}$	67
elliptic	$-\frac{4(2x^2+5)}{39\sqrt{x^4+5x^2+3}}+\frac{1}{3\sqrt{x^4+5x^2+3}}-\frac{\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{9}$	67
trager	$-\frac{8x^2+7}{39\sqrt{x^4+5x^2+3}}-\frac{\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{5\operatorname{RootOf}\left(\_Z^2-3\right)x^2+6\sqrt{x^4+5x^2+3}+6\operatorname{RootOf}\left(\_Z^2-3\right)}{x^2}\right)}{9}$	71

input
```
int((3*x^2+2)/x/(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/39*(8*x^2+7)/(x^4+5*x^2+3)^(1/2)-1/9*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4
+5*x^2+3)^(1/2))*3^(1/2)
```

3.197.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(52) = 104$.

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.62

$$\int \frac{2+3x^2}{x\left(3+5x^2+x^4\right)^{3/2}}\,dx =$$

$$-\frac{24\,x^4-13\,\sqrt{3}(x^4+5\,x^2+3)\log\left(\frac{25\,x^2-2\,\sqrt{3}(5\,x^2+6)-2\,\sqrt{x^4+5\,x^2+3}\left(5\,\sqrt{3}-6\right)+30}{x^2}\right)+120\,x^2+3\,\sqrt{x^4+5\,x^2+3}(8}{117\,(x^4+5\,x^2+3)}$$

input
```
integrate((3*x^2+2)/x/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
-1/117*(24*x^4 - 13*sqrt(3)*(x^4 + 5*x^2 + 3)*log((25*x^2 - 2*sqrt(3)*(5*x
^2 + 6) - 2*sqrt(x^4 + 5*x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) + 120*x^2 + 3
*sqrt(x^4 + 5*x^2 + 3)*(8*x^2 + 7) + 72)/(x^4 + 5*x^2 + 3)
```

3.197.6 Sympy [F]

$$\int \frac{2+3x^2}{x\left(3+5x^2+x^4\right)^{3/2}}\,dx = \int \frac{3x^2+2}{x\left(x^4+5x^2+3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((3*x**2+2)/x/(x**4+5*x**2+3)**(3/2),x)
```

output
```
Integral((3*x**2 + 2)/(x*(x**4 + 5*x**2 + 3)**(3/2)), x)
```

3.197.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98

$$\int \frac{2+3x^2}{x\left(3+5x^2+x^4\right)^{3/2}}\,dx = -\frac{8\,x^2}{39\,\sqrt{x^4+5\,x^2+3}} - \frac{1}{9}\,\sqrt{3}\log\left(\frac{2\,\sqrt{3}\sqrt{x^4+5\,x^2+3}}{x^2}+\frac{6}{x^2}+5\right) - \frac{7}{39\,\sqrt{x^4+5\,x^2+3}}$$

input
```
integrate((3*x^2+2)/x/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
-8/39*x^2/sqrt(x^4 + 5*x^2 + 3) - 1/9*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x
^2 + 3)/x^2 + 6/x^2 + 5) - 7/39/sqrt(x^4 + 5*x^2 + 3)
```

3.197.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.18

$$\int \frac{2+3x^2}{x(3+5x^2+x^4)^{3/2}}\,dx = -\frac{1}{9}\sqrt{3}\log\left(-x^2+\sqrt{3}+\sqrt{x^4+5x^2+3}\right) + \frac{1}{9}\sqrt{3}\log\left(-x^2-\sqrt{3}+\sqrt{x^4+5x^2+3}\right) - \frac{8x^2+7}{39\sqrt{x^4+5x^2+3}}$$

input
```
integrate((3*x^2+2)/x/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

output
```
-1/9*sqrt(3)*log(-x^2 + sqrt(3) + sqrt(x^4 + 5*x^2 + 3)) + 1/9*sqrt(3)*log
(-x^2 - sqrt(3) + sqrt(x^4 + 5*x^2 + 3)) - 1/39*(8*x^2 + 7)/sqrt(x^4 + 5*x
^2 + 3)
```

3.197.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x^2}{x(3+5x^2+x^4)^{3/2}}\,dx = \int \frac{3x^2+2}{x(x^4+5x^2+3)^{3/2}}\,dx$$

input
```
int((3*x^2 + 2)/(x*(5*x^2 + x^4 + 3)^(3/2)),x)
```

output
```
int((3*x^2 + 2)/(x*(5*x^2 + x^4 + 3)^(3/2)), x)
```

3.198 $\int \frac{2+3x^2}{x^3(3+5x^2+x^4)^{3/2}}\,dx$

3.198.1 Optimal result

Integrand size = 25, antiderivative size = 90

$$\int \frac{2+3x^2}{x^3\left(3+5x^2+x^4\right)^{3/2}}\,dx=\frac{-7-8x^2}{39x^2\sqrt{3+5x^2+x^4}}-\frac{2\sqrt{3+5x^2+x^4}}{39x^2}+\frac{\operatorname{arctanh}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{3\sqrt{3}}$$

output
```
1/9*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)+1/39*(-8*x^
2-7)/x^2/(x^4+5*x^2+3)^(1/2)-2/39*(x^4+5*x^2+3)^(1/2)/x^2
```

3.198.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.78

$$\int \frac{2+3x^2}{x^3\left(3+5x^2+x^4\right)^{3/2}}\,dx=\frac{-13-18x^2-2x^4}{39x^2\sqrt{3+5x^2+x^4}}-\frac{2\operatorname{arctanh}\left(\frac{x^2-\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right)}{3\sqrt{3}}$$

input
```
Integrate[(2 + 3*x^2)/(x^3*(3 + 5*x^2 + x^4)^(3/2)),x]
```

output
```
(-13 - 18*x^2 - 2*x^4)/(39*x^2*Sqrt[3 + 5*x^2 + x^4]) - (2*ArcTanh[(x^2 -
Sqrt[3 + 5*x^2 + x^4])/Sqrt[3]])/(3*Sqrt[3])
```

3.198.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1578, 1235, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+2}{x^3\left(x^4+5x^2+3\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{3x^2+2}{x^4\left(x^4+5x^2+3\right)^{3/2}}dx^2$$

$$\Big\downarrow \text{1235}$$

$$\frac{1}{2}\left(-\frac{2}{39}\int -\frac{2(3-4x^2)}{x^4\sqrt{x^4+5x^2+3}}dx^2-\frac{2(8x^2+7)}{39x^2\sqrt{x^4+5x^2+3}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{4}{39}\int \frac{3-4x^2}{x^4\sqrt{x^4+5x^2+3}}dx^2-\frac{2(8x^2+7)}{39x^2\sqrt{x^4+5x^2+3}}\right)$$

$$\Big\downarrow \text{1228}$$

$$\frac{1}{2}\left(\frac{4}{39}\left(-\frac{13}{2}\int \frac{1}{x^2\sqrt{x^4+5x^2+3}}dx^2-\frac{\sqrt{x^4+5x^2+3}}{x^2}\right)-\frac{2(8x^2+7)}{39x^2\sqrt{x^4+5x^2+3}}\right)$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{2}\left(\frac{4}{39}\left(13\int \frac{1}{12-x^4}d\frac{5x^2+6}{\sqrt{x^4+5x^2+3}}-\frac{\sqrt{x^4+5x^2+3}}{x^2}\right)-\frac{2(8x^2+7)}{39x^2\sqrt{x^4+5x^2+3}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{4}{39}\left(\frac{13\text{arctanh}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{2\sqrt{3}}-\frac{\sqrt{x^4+5x^2+3}}{x^2}\right)-\frac{2(8x^2+7)}{39x^2\sqrt{x^4+5x^2+3}}\right)$$

input
```
Int[(2 + 3*x^2)/(x^3*(3 + 5*x^2 + x^4)^(3/2)),x]
```

output
```
((-2*(7 + 8*x^2))/(39*x^2*Sqrt[3 + 5*x^2 + x^4]) + (4*(-(Sqrt[3 + 5*x^2 +
x^4]/x^2) + (13*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(2
*Sqrt[3])))/39)/2
```

3.198.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

3.198. $\int \frac{2+3x^2}{x^3(3+5x^2+x^4)^{3/2}}\,dx$

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.198.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.68

method	result	size
risch	$-\frac{2x^4+18x^2+13}{39x^2\sqrt{x^4+5x^2+3}}+\frac{\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{9}$	61
pseudoelliptic	$\frac{13\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}\,x^2\sqrt{x^4+5x^2+3}-6x^4-54x^2-39}{117x^2\sqrt{x^4+5x^2+3}}$	75
trager	$-\frac{2x^4+18x^2+13}{39x^2\sqrt{x^4+5x^2+3}}-\frac{\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{-5\operatorname{RootOf}\left(\_Z^2-3\right)x^2+6\sqrt{x^4+5x^2+3}-6\operatorname{RootOf}\left(\_Z^2-3\right)}{x^2}\right)}{9}$	79
default	$-\frac{1}{3\sqrt{x^4+5x^2+3}}-\frac{2x^2+5}{39\sqrt{x^4+5x^2+3}}+\frac{\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{9}-\frac{1}{3x^2\sqrt{x^4+5x^2+3}}$	84
elliptic	$-\frac{1}{3\sqrt{x^4+5x^2+3}}-\frac{2x^2+5}{39\sqrt{x^4+5x^2+3}}+\frac{\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{9}-\frac{1}{3x^2\sqrt{x^4+5x^2+3}}$	84

input
```
int((3*x^2+2)/x^3/(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/39*(2*x^4+18*x^2+13)/x^2/(x^4+5*x^2+3)^(1/2)+1/9*arctanh(1/6*(5*x^2+6)*
3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)
```

3.198.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.38

$$\int \frac{2+3x^2}{x^3\left(3+5x^2+x^4\right)^{3/2}}\,dx =$$

$$-\frac{6\,x^6+30\,x^4-13\,\sqrt{3}(x^6+5\,x^4+3\,x^2)\log\left(\frac{25\,x^2+2\,\sqrt{3}(5\,x^2+6)+2\,\sqrt{x^4+5\,x^2+3}\left(5\,\sqrt{3}+6\right)+30}{x^2}\right)+18\,x^2+3\,(2\,x^4+1}{117\,(x^6+5\,x^4+3\,x^2)}$$

input
```
integrate((3*x^2+2)/x^3/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
-1/117*(6*x^6 + 30*x^4 - 13*sqrt(3)*(x^6 + 5*x^4 + 3*x^2)*log((25*x^2 + 2*
sqrt(3)*(5*x^2 + 6) + 2*sqrt(x^4 + 5*x^2 + 3)*(5*sqrt(3) + 6) + 30)/x^2) +
 18*x^2 + 3*(2*x^4 + 18*x^2 + 13)*sqrt(x^4 + 5*x^2 + 3))/(x^6 + 5*x^4 + 3*
x^2)
```

3.198.6 Sympy [F]

$$\int \frac{2+3x^2}{x^3(3+5x^2+x^4)^{3/2}}\,dx = \int \frac{3x^2+2}{x^3\left(x^4+5x^2+3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((3*x**2+2)/x**3/(x**4+5*x**2+3)**(3/2),x)
```

output
```
Integral((3*x**2 + 2)/(x**3*(x**4 + 5*x**2 + 3)**(3/2)), x)
```

3.198.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.91

$$\int \frac{2+3x^2}{x^3(3+5x^2+x^4)^{3/2}}\,dx = -\frac{2\,x^2}{39\,\sqrt{x^4+5\,x^2+3}} + \frac{1}{9}\,\sqrt{3}\log\left(\frac{2\,\sqrt{3}\sqrt{x^4+5\,x^2+3}}{x^2}+\frac{6}{x^2}+5\right) - \frac{6}{13\,\sqrt{x^4+5\,x^2+3}} - \frac{1}{3\,\sqrt{x^4+5\,x^2+3}x^2}$$

input
```
integrate((3*x^2+2)/x^3/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
-2/39*x^2/sqrt(x^4 + 5*x^2 + 3) + 1/9*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x
^2 + 3)/x^2 + 6/x^2 + 5) - 6/13/sqrt(x^4 + 5*x^2 + 3) - 1/3/(sqrt(x^4 + 5*
x^2 + 3)*x^2)
```

3.198.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.36

$$\int \frac{2+3x^2}{x^3(3+5x^2+x^4)^{3/2}}\,dx = -\frac{1}{9}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5\,x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5\,x^2+3}}\right) + \frac{7\,x^2+11}{117\sqrt{x^4+5\,x^2+3}} + \frac{5\,x^2-5\sqrt{x^4+5\,x^2+3}+6}{9\left(\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^2-3\right)}$$

input
```
integrate((3*x^2+2)/x^3/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

output
```
-1/9*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) -
sqrt(x^4 + 5*x^2 + 3))) + 1/117*(7*x^2 + 11)/sqrt(x^4 + 5*x^2 + 3) + 1/9*(
5*x^2 - 5*sqrt(x^4 + 5*x^2 + 3) + 6)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 3)
```

3.198.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x^2}{x^3(3+5x^2+x^4)^{3/2}}\,dx = \int \frac{3\,x^2+2}{x^3\,(x^4+5\,x^2+3)^{3/2}}\,dx$$

input
```
int((3*x^2 + 2)/(x^3*(5*x^2 + x^4 + 3)^(3/2)),x)
```

output
```
int((3*x^2 + 2)/(x^3*(5*x^2 + x^4 + 3)^(3/2)), x)
```

3.199 $\int \frac{x^4(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx$

3.199.1 Optimal result

Integrand size = 25, antiderivative size = 307

$$\int \frac{x^4(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \frac{43x\left(5+\sqrt{13}+2x^2\right)}{13\sqrt{3+5x^2+x^4}}+\frac{x^3(8+11x^2)}{13\sqrt{3+5x^2+x^4}}-\frac{11}{13}x\sqrt{3+5x^2+x^4}$$
$$-\frac{43\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{13\sqrt{3+5x^2+x^4}}$$
$$+\frac{11\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{13\sqrt{3+5x^2+x^4}}$$

output
```
1/13*x^3*(11*x^2+8)/(x^4+5*x^2+3)^(1/2)+43/13*x*(5+2*x^2+13^(1/2))/(x^4+5*
x^2+3)^(1/2)-11/13*x*(x^4+5*x^2+3)^(1/2)+11/26*(1/(36+x^2*(30+6*13^(1/2))
)^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13^(1/2))^(1/2)/(
36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1
/2)))*6^(1/2)/(5+13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))
))^(1/2)/(x^4+5*x^2+3)^(1/2)-43/78*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+
x^2*(30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6
*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*
13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x
^2+3)^(1/2)
```

3.199.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.23 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.71

$$\int \frac{x^4(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \frac{-2x(33+47x^2)+43i\sqrt{2}(-5+\sqrt{13})\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}E\Big(i\text{arcsinh}}{}$$

input
```
Integrate[(x^4*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

output
```
(-2*x*(33 + 47*x^2) + (43*I)*Sqrt[2]*(-5 + Sqrt[13])*Sqrt[(-5 + Sqrt[13] -
 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sq
rt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqrt[2]*(-182 + 43*Sqr
t[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2
*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6
])/(26*Sqrt[3 + 5*x^2 + x^4])
```

3.199.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1598, 27, 1602, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(3x^2+2)}{(x^4+5x^2+3)^{3/2}}\,dx$$

$$\Big\downarrow \text{1598}$$

$$\frac{1}{13}\int -\frac{3x^2(11x^2+8)}{\sqrt{x^4+5x^2+3}}dx + \frac{(11x^2+8)\,x^3}{13\sqrt{x^4+5x^2+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^3(11x^2+8)}{13\sqrt{x^4+5x^2+3}} - \frac{3}{13}\int \frac{x^2(11x^2+8)}{\sqrt{x^4+5x^2+3}}dx$$

$$\Big\downarrow \text{1602}$$

$$\frac{x^3(11x^2+8)}{13\sqrt{x^4+5x^2+3}} - \frac{3}{13}\left(\frac{11}{3}x\sqrt{x^4+5x^2+3} - \frac{1}{3}\int \frac{86x^2+33}{\sqrt{x^4+5x^2+3}}dx\right)$$

$$\Big\downarrow \text{1503}$$

$$\frac{x^3(11x^2+8)}{13\sqrt{x^4+5x^2+3}}-\frac{3}{13}\left(\frac{1}{3}\left(-33\int\frac{1}{\sqrt{x^4+5x^2+3}}dx-86\int\frac{x^2}{\sqrt{x^4+5x^2+3}}dx\right)+\frac{11}{3}\sqrt{x^4+5x^2+3}x\right)$$

$$\Big\downarrow \text{1412}$$

$$\frac{x^3(11x^2+8)}{13\sqrt{x^4+5x^2+3}}-\frac{3}{13}\left(\frac{1}{3}\left(-86\int\frac{x^2}{\sqrt{x^4+5x^2+3}}dx-\frac{11\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\right.\right.\right.}{\sqrt{x^4+5x^2+3}}\right.\right.$$

$$\Big\downarrow \text{1455}$$

$$\frac{x^3(11x^2+8)}{13\sqrt{x^4+5x^2+3}}-\frac{3}{13}\left(\frac{1}{3}\left(-\frac{11\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}}\left(5+\sqrt{13}\right)x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right.\right.}{\sqrt{x^4+5x^2+3}}\right.\right.$$

input
```
Int[(x^4*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

output
```
(x^3*(8 + 11*x^2))/(13*Sqrt[3 + 5*x^2 + x^4]) - (3*((11*x*Sqrt[3 + 5*x^2 +
 x^4])/3 + (-86*((x*(5 + Sqrt[13] + 2*x^2))/(2*Sqrt[3 + 5*x^2 + x^4]) - (S
qrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^
2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (
-13 + 5*Sqrt[13])/6])/(2*Sqrt[3 + 5*x^2 + x^4])) - (11*Sqrt[3/(2*(5 + Sqrt
[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 +
Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[1
3])/6])/Sqrt[3 + 5*x^2 + x^4])/3))/13
```

3.199. $\int \frac{x^4(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx$

3.199.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1598
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Simp[f
^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1
)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
 GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.199. $\int \frac{x^4(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx$

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

3.199.4 Maple [A] (verified)

Time = 4.03 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.70

method	result
risch	$-\frac{x(47x^2+33)}{13\sqrt{x^4+5x^2+3}}+\frac{198\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{13\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}-\frac{3096\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\cdots}}{\cdots}$
elliptic	$-\frac{2\left(\frac{47}{26}x^3+\frac{33}{26}x\right)}{\sqrt{x^4+5x^2+3}}+\frac{198\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{13\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}-\frac{3096\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\cdots}}{\cdots}$
default	$-\frac{6\left(\frac{19}{26}x^3+\frac{15}{26}x\right)}{\sqrt{x^4+5x^2+3}}+\frac{198\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{13\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}-\frac{3096\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\cdots}}{\cdots}$

input
```
int(x^4*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/13*x*(47*x^2+33)/(x^4+5*x^2+3)^(1/2)+198/13/(-30+6*13^(1/2))^(1/2)*(1-(
-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2
+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))
-3096/13/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6
-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*
x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*
13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))
```

3.199.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.60

$$\int \frac{x^4(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \frac{86\left(\sqrt{13}\sqrt{2}(x^5+5\,x^3+3\,x)-5\,\sqrt{2}(x^5+5\,x^3+3\,x)\right)\sqrt{\sqrt{13}-5}E(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{}}}{2\,x}\right.}{}$$

input
```
integrate(x^4*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
1/52*(86*(sqrt(13)*sqrt(2)*(x^5 + 5*x^3 + 3*x) - 5*sqrt(2)*(x^5 + 5*x^3 +
3*x))*sqrt(sqrt(13) - 5)*elliptic_e(arcsin(1/2*sqrt(2)*sqrt(sqrt(13) - 5)/
x), 5/6*sqrt(13) + 19/6) - 5*(15*sqrt(13)*sqrt(2)*(x^5 + 5*x^3 + 3*x) - 97
*sqrt(2)*(x^5 + 5*x^3 + 3*x))*sqrt(sqrt(13) - 5)*elliptic_f(arcsin(1/2*sqr
t(2)*sqrt(sqrt(13) - 5)/x), 5/6*sqrt(13) + 19/6) + 4*(39*x^4 + 397*x^2 + 2
58)*sqrt(x^4 + 5*x^2 + 3))/(x^5 + 5*x^3 + 3*x)
```

3.199.6 Sympy [F]

$$\int \frac{x^4(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \int \frac{x^4\cdot(3x^2+2)}{(x^4+5x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4*(3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)
```

output
```
Integral(x**4*(3*x**2 + 2)/(x**4 + 5*x**2 + 3)**(3/2), x)
```

3.199.7 Maxima [F]

$$\int \frac{x^4(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \int \frac{(3\,x^2+2)x^4}{(x^4+5\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 2)*x^4/(x^4 + 5*x^2 + 3)^(3/2), x)
```

3.199.8 Giac [F]

$$\int \frac{x^4(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \int \frac{(3\,x^2+2)x^4}{(x^4+5\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

output
```
integrate((3*x^2 + 2)*x^4/(x^4 + 5*x^2 + 3)^(3/2), x)
```

3.199.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \int \frac{x^4\,(3\,x^2+2)}{(x^4+5\,x^2+3)^{3/2}}\,dx$$

input
```
int((x^4*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(3/2),x)
```

output
```
int((x^4*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(3/2), x)
```

3.200 $\int \frac{x^2(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx$

3.200.1 Optimal result

Integrand size = 25, antiderivative size = 286

$$\int \frac{x^2(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = -\frac{11x\left(5+\sqrt{13}+2x^2\right)}{26\sqrt{3+5x^2+x^4}}+\frac{x(8+11x^2)}{13\sqrt{3+5x^2+x^4}}$$
$$+\frac{11\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{26\sqrt{3+5x^2+x^4}}$$
$$-\frac{4\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{13\sqrt{3+5x^2+x^4}}$$

output
```
1/13*x*(11*x^2+8)/(x^4+5*x^2+3)^(1/2)-11/26*x*(5+2*x^2+13^(1/2))/(x^4+5*x^
2+3)^(1/2)-4/39*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2))
)^(1/2)*EllipticF(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1
/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*6^(1/2)/(5+13^(1/2))^(1/2
)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)+11
/156*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*Ell
ipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*
13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(1/2))^(1/2)*((6+x^2*(5-13^
(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)
```

3.200.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.25 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.77

$$\int \frac{x^2(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \frac{4x(8+11x^2)-11i\sqrt{2}(-5+\sqrt{13})\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}E\left(i\text{arcsinh}\left(\sqrt{\right.\right.}{}$$

input

```
Integrate[(x^2*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

output

```
(4*x*(8 + 11*x^2) - (11*I)*Sqrt[2]*(-5 + Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2
*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt
[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] + I*Sqrt[2]*(-39 + 11*Sqrt[1
3])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^
2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/
(52*Sqrt[3 + 5*x^2 + x^4])
```

3.200.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1598, 25, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(3x^2+2)}{(x^4+5x^2+3)^{3/2}}\,dx$$

$$\Big\downarrow \; 1598$$

$$\frac{1}{13}\int -\frac{11x^2+8}{\sqrt{x^4+5x^2+3}}dx+\frac{x(11x^2+8)}{13\sqrt{x^4+5x^2+3}}$$

$$\Big\downarrow \; 25$$

$$\frac{x(11x^2+8)}{13\sqrt{x^4+5x^2+3}}-\frac{1}{13}\int \frac{11x^2+8}{\sqrt{x^4+5x^2+3}}dx$$

$$\Big\downarrow \; 1503$$

$$\frac{1}{13}\left(-8\int \frac{1}{\sqrt{x^4+5x^2+3}}dx-11\int \frac{x^2}{\sqrt{x^4+5x^2+3}}dx\right)+\frac{x(11x^2+8)}{13\sqrt{x^4+5x^2+3}}$$

$$\Big\downarrow \text{1412}$$

$$\frac{1}{13}\left(-11\int \frac{x^2}{\sqrt{x^4+5x^2+3}}dx-\frac{4\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\right.\right.}{\sqrt{x^4+5x^2+3}}\right.$$

$$\frac{x\left(11x^2+8\right)}{13\sqrt{x^4+5x^2+3}}$$

$$\Big\downarrow \text{1455}$$

$$\frac{1}{13}\left(-\frac{4\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}-\right.$$

$$\frac{x\left(11x^2+8\right)}{13\sqrt{x^4+5x^2+3}}$$

input
```
Int[(x^2*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

output
```
(x*(8 + 11*x^2))/(13*Sqrt[3 + 5*x^2 + x^4]) + (-11*((x*(5 + Sqrt[13] + 2*x
^2))/(2*Sqrt[3 + 5*x^2 + x^4]) - (Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sq
rt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[
ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(2*Sqrt[3 + 5*x^2
 + x^4])) - (4*Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6
 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5
+ Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/Sqrt[3 + 5*x^2 + x^4])/13
```

3.200. $\int \frac{x^2(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx$

3.200.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1598
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Simp[f
^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1
)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
 GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.200. $\int \frac{x^2(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx$

3.200.4 Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.76

method	result
risch	$\frac{x(11x^2+8)}{13\sqrt{x^4+5x^2+3}} - \frac{48\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{13\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}} + \frac{396\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\right.}}{}$
elliptic	$-\frac{2\left(-\frac{11}{26}x^3-\frac{4}{13}x\right)}{\sqrt{x^4+5x^2+3}} - \frac{48\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{13\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}} + \frac{396\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-}}{}$
default	$-\frac{6\left(-\frac{5}{26}x^3-\frac{3}{13}x\right)}{\sqrt{x^4+5x^2+3}} - \frac{48\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{13\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}} + \frac{396\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-}}{}$

input
```
int(x^2*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/13*x*(11*x^2+8)/(x^4+5*x^2+3)^(1/2)-48/13/(-30+6*13^(1/2))^(1/2)*(1-(-5/
6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)
^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))+39
6/13/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6
*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-
30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(
1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))
```

3.200.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.63

$$\int \frac{x^2(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \frac{11\left(\sqrt{13}\sqrt{6}\sqrt{3}(x^4+5\,x^2+3)-5\,\sqrt{6}\sqrt{3}(x^4+5\,x^2+3)\right)\sqrt{\sqrt{13}-5}E(\arcsin\left(\frac{1}{6}\right.}{}$$

input
```
integrate(x^2*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
1/468*(11*(sqrt(13)*sqrt(6)*sqrt(3)*(x^4 + 5*x^2 + 3) - 5*sqrt(6)*sqrt(3)*
(x^4 + 5*x^2 + 3))*sqrt(sqrt(13) - 5)*elliptic_e(arcsin(1/6*sqrt(6)*x*sqrt
(sqrt(13) - 5)), 5/6*sqrt(13) + 19/6) - (3*sqrt(13)*sqrt(6)*sqrt(3)*(x^4 +
 5*x^2 + 3) - 95*sqrt(6)*sqrt(3)*(x^4 + 5*x^2 + 3))*sqrt(sqrt(13) - 5)*ell
iptic_f(arcsin(1/6*sqrt(6)*x*sqrt(sqrt(13) - 5)), 5/6*sqrt(13) + 19/6) + 3
6*sqrt(x^4 + 5*x^2 + 3)*(11*x^3 + 8*x))/(x^4 + 5*x^2 + 3)
```

3.200.6 Sympy [F]

$$\int \frac{x^2(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \int \frac{x^2\cdot(3x^2+2)}{(x^4+5x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*(3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)
```

output
```
Integral(x**2*(3*x**2 + 2)/(x**4 + 5*x**2 + 3)**(3/2), x)
```

3.200.7 Maxima [F]

$$\int \frac{x^2(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \int \frac{(3\,x^2+2)x^2}{(x^4+5\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 2)*x^2/(x^4 + 5*x^2 + 3)^(3/2), x)
```

3.200.8 Giac [F]

$$\int \frac{x^2(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \int \frac{(3\,x^2+2)x^2}{(x^4+5\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

output
```
integrate((3*x^2 + 2)*x^2/(x^4 + 5*x^2 + 3)^(3/2), x)
```

3.200.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \int \frac{x^2\,(3\,x^2+2)}{(x^4+5\,x^2+3)^{3/2}}\,dx$$

input
```
int((x^2*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(3/2),x)
```

output
```
int((x^2*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(3/2), x)
```

3.201 $\int \frac{2+3x^2}{(3+5x^2+x^4)^{3/2}} \, dx$

3.201.1 Optimal result

Integrand size = 22, antiderivative size = 282

$$\int \frac{2+3x^2}{(3+5x^2+x^4)^{3/2}} \, dx = \frac{4x\left(5+\sqrt{13}+2x^2\right)}{39\sqrt{3+5x^2+x^4}} - \frac{x(7+8x^2)}{39\sqrt{3+5x^2+x^4}} - \frac{2\sqrt{\frac{2}{3}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{39\sqrt{3+5x^2+x^4}} + \frac{11\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)\operatorname{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{13\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{3+5x^2+x^4}}$$

output

```
-1/39*x*(8*x^2+7)/(x^4+5*x^2+3)^(1/2)+4/39*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2
+3)^(1/2)-2/117*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2))
)^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1
/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(1/2))^(1/2)*((6
+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)+11/13*(
1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF
(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/
2))^(1/2))*(6+x^2*(5+13^(1/2)))*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2)))
)^(1/2)/(x^4+5*x^2+3)^(1/2)/(30+6*13^(1/2))^(1/2)
```

3.201.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.25 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.78

$$\int \frac{2+3x^2}{(3+5x^2+x^4)^{3/2}}\,dx = \frac{-2x(7+8x^2)+4i\sqrt{2}(-5+\sqrt{13})\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}E\left(i\operatorname{arcsinh}\left(\sqrt{\right.\right.}{}$$

input
```
Integrate[(2 + 3*x^2)/(3 + 5*x^2 + x^4)^(3/2),x]
```

output
```
(-2*x*(7 + 8*x^2) + (4*I)*Sqrt[2]*(-5 + Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*
x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[
2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqrt[2]*(13 + 4*Sqrt[13])
*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*
EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(78
*Sqrt[3 + 5*x^2 + x^4])
```

3.201.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1492, 25, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+2}{(x^4+5x^2+3)^{3/2}}dx$$

$$\Big\downarrow \text{ 1492}$$

$$-\frac{1}{39}\int -\frac{8x^2+33}{\sqrt{x^4+5x^2+3}}dx - \frac{x(8x^2+7)}{39\sqrt{x^4+5x^2+3}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{39}\int \frac{8x^2+33}{\sqrt{x^4+5x^2+3}}dx - \frac{x(8x^2+7)}{39\sqrt{x^4+5x^2+3}}$$

$$\Big\downarrow \text{ 1503}$$

$$\frac{1}{39}\left(33\int \frac{1}{\sqrt{x^4+5x^2+3}}dx + 8\int \frac{x^2}{\sqrt{x^4+5x^2+3}}dx\right) - \frac{x(8x^2+7)}{39\sqrt{x^4+5x^2+3}}$$

$$\Big\downarrow \text{1412}$$

$$\frac{1}{39}\left(8\int \frac{x^2}{\sqrt{x^4+5x^2+3}}dx+\frac{11\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\right.\right.}{\sqrt{x^4+5x^2+3}}\right.$$

$$\frac{x\left(8x^2+7\right)}{39\sqrt{x^4+5x^2+3}}$$

$$\Big\downarrow \text{1455}$$

$$\frac{1}{39}\left(\frac{11\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}+\right.$$

$$\frac{x\left(8x^2+7\right)}{39\sqrt{x^4+5x^2+3}}$$

input
```
Int[(2 + 3*x^2)/(3 + 5*x^2 + x^4)^(3/2),x]
```

output
```
-1/39*(x*(7 + 8*x^2))/Sqrt[3 + 5*x^2 + x^4] + (8*((x*(5 + Sqrt[13] + 2*x^2
))/(2*Sqrt[3 + 5*x^2 + x^4]) - (Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt
[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[Ar
cTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(2*Sqrt[3 + 5*x^2 +
 x^4])) + (11*Sqrt[3/(2*(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6
+ (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 +
 Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/Sqrt[3 + 5*x^2 + x^4])/39
```

3.201. $\int \frac{2+3x^2}{(3+5x^2+x^4)^{3/2}}\,dx$

3.201.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

3.201. $\int \frac{2+3x^2}{(3+5x^2+x^4)^{3/2}}\,dx$

3.201.4 Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.77

method	result
risch	$-\frac{x(8x^2+7)}{39\sqrt{x^4+5x^2+3}}+\frac{66\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{13\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}-\frac{96\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\right.}}{}$
elliptic	$-\frac{2\left(\frac{4}{39}x^3+\frac{7}{78}x\right)}{\sqrt{x^4+5x^2+3}}+\frac{66\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{13\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}-\frac{96\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\right.}}{}$
default	$-\frac{4\left(-\frac{19}{78}x-\frac{5}{78}x^3\right)}{\sqrt{x^4+5x^2+3}}+\frac{66\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{13\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}-\frac{96\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(\right.}}{}$

input
```
int((3*x^2+2)/(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/39*x*(8*x^2+7)/(x^4+5*x^2+3)^(1/2)+66/13/(-30+6*13^(1/2))^(1/2)*(1-(-5/
6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)
^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-96
/13/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*
13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-3
0+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1
/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))
```

3.201.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.63

$$\int \frac{2+3x^2}{(3+5x^2+x^4)^{3/2}}\,dx =$$
$$-\frac{8\left(\sqrt{13}\sqrt{6}\sqrt{3}(x^4+5\,x^2+3)-5\,\sqrt{6}\sqrt{3}(x^4+5\,x^2+3)\right)\sqrt{\sqrt{13}-5}\,E\left(\arcsin\left(\frac{1}{6}\,\sqrt{6}x\sqrt{\sqrt{13}-5}\right)\mid\frac{5}{6}\,\sqrt{13}+\right.}{}$$

input
```
integrate((3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
-1/1404*(8*(sqrt(13)*sqrt(6)*sqrt(3)*(x^4 + 5*x^2 + 3) - 5*sqrt(6)*sqrt(3)
*(x^4 + 5*x^2 + 3))*sqrt(sqrt(13) - 5)*elliptic_e(arcsin(1/6*sqrt(6)*x*sqr
t(sqrt(13) - 5)), 5/6*sqrt(13) + 19/6) + 5*(5*sqrt(13)*sqrt(6)*sqrt(3)*(x^
4 + 5*x^2 + 3) + 41*sqrt(6)*sqrt(3)*(x^4 + 5*x^2 + 3))*sqrt(sqrt(13) - 5)*
elliptic_f(arcsin(1/6*sqrt(6)*x*sqrt(sqrt(13) - 5)), 5/6*sqrt(13) + 19/6)
+ 36*sqrt(x^4 + 5*x^2 + 3)*(8*x^3 + 7*x))/(x^4 + 5*x^2 + 3)
```

3.201.6 Sympy [F]

$$\int \frac{2+3x^2}{(3+5x^2+x^4)^{3/2}}\,dx = \int \frac{3x^2+2}{(x^4+5x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate((3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)
```

output
```
Integral((3*x**2 + 2)/(x**4 + 5*x**2 + 3)**(3/2), x)
```

3.201.7 Maxima [F]

$$\int \frac{2+3x^2}{(3+5x^2+x^4)^{3/2}}\,dx = \int \frac{3\,x^2+2}{(x^4+5\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate((3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 2)/(x^4 + 5*x^2 + 3)^(3/2), x)
```

3.201.8 Giac [F]

$$\int \frac{2+3x^2}{(3+5x^2+x^4)^{3/2}}\,dx = \int \frac{3\,x^2+2}{(x^4+5\,x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate((3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

output
```
integrate((3*x^2 + 2)/(x^4 + 5*x^2 + 3)^(3/2), x)
```

3.201.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x^2}{(3+5x^2+x^4)^{3/2}}\,dx = \int \frac{3\,x^2+2}{\left(x^4+5\,x^2+3\right)^{3/2}}\,dx$$

input
```
int((3*x^2 + 2)/(5*x^2 + x^4 + 3)^(3/2),x)
```

output
```
int((3*x^2 + 2)/(5*x^2 + x^4 + 3)^(3/2), x)
```

3.202 $\int \frac{2+3x^2}{x^2(3+5x^2+x^4)^{3/2}}\,dx$

3.202.1 Optimal result

Integrand size = 25, antiderivative size = 309

$$\int \frac{2+3x^2}{x^2(3+5x^2+x^4)^{3/2}}\,dx = \frac{19x\left(5+\sqrt{13}+2x^2\right)}{234\sqrt{3+5x^2+x^4}} - \frac{7+8x^2}{39x\sqrt{3+5x^2+x^4}} - \frac{19\sqrt{3+5x^2+x^4}}{117x} - \frac{19\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{234\sqrt{3+5x^2+x^4}} - \frac{4\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{39\sqrt{3+5x^2+x^4}}$$

output
```
1/39*(-8*x^2-7)/x/(x^4+5*x^2+3)^(1/2)+19/234*x*(5+2*x^2+13^(1/2))/(x^4+5*x
^2+3)^(1/2)-19/117*(x^4+5*x^2+3)^(1/2)/x-4/117*(1/(36+x^2*(30+6*13^(1/2)))
)^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13^(1/2))^(1/2)/(
36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1
/2)))*6^(1/2)/(5+13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))
))^(1/2)/(x^4+5*x^2+3)^(1/2)-19/1404*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(3
6+x^2*(30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30
+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+
6*13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5
*x^2+3)^(1/2)
```

3.202.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.26 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.74

$$\int \frac{2+3x^2}{x^2\left(3+5x^2+x^4\right)^{3/2}}\,dx = \frac{-4(78+119x^2+19x^4)+19i\sqrt{2}\left(-5+\sqrt{13}\right)x\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}}{}$$

input
```
Integrate[(2 + 3*x^2)/(x^2*(3 + 5*x^2 + x^4)^(3/2)),x]
```

output
```
(-4*(78 + 119*x^2 + 19*x^4) + (19*I)*Sqrt[2]*(-5 + Sqrt[13])*x*Sqrt[(-5 +
Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*
ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqrt[2]*(-14
3 + 19*Sqrt[13])*x*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 +
Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5
*Sqrt[13])/6])/(468*x*Sqrt[3 + 5*x^2 + x^4])
```

3.202.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1600, 25, 1604, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+2}{x^2\left(x^4+5x^2+3\right)^{3/2}}dx$$

$$\Big\downarrow \text{1600}$$

$$-\frac{1}{39}\int -\frac{19-8x^2}{x^2\sqrt{x^4+5x^2+3}}dx-\frac{8x^2+7}{39x\sqrt{x^4+5x^2+3}}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{39}\int \frac{19-8x^2}{x^2\sqrt{x^4+5x^2+3}}dx-\frac{8x^2+7}{39x\sqrt{x^4+5x^2+3}}$$

$$\Big\downarrow \text{1604}$$

$$\frac{1}{39}\left(-\frac{1}{3}\int \frac{24-19x^2}{\sqrt{x^4+5x^2+3}}dx-\frac{19\sqrt{x^4+5x^2+3}}{3x}\right)-\frac{8x^2+7}{39x\sqrt{x^4+5x^2+3}}$$

$\downarrow$ 1503

$$\frac{1}{39}\left(\frac{1}{3}\left(19\int \frac{x^2}{\sqrt{x^4+5x^2+3}}dx-24\int \frac{1}{\sqrt{x^4+5x^2+3}}dx\right)-\frac{19\sqrt{x^4+5x^2+3}}{3x}\right)-\frac{8x^2+7}{39x\sqrt{x^4+5x^2+3}}$$

$\downarrow$ 1412

$$\frac{1}{39}\left(\frac{1}{3}\left(19\int \frac{x^2}{\sqrt{x^4+5x^2+3}}dx-\frac{4\sqrt{\frac{6}{5+\sqrt{13}}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\right.\right.}{\sqrt{x^4+5x^2+3}}\right.\right.$$

$$\frac{8x^2+7}{39x\sqrt{x^4+5x^2+3}}$$

$\downarrow$ 1455

$$\frac{1}{39}\left(\frac{1}{3}\left(19\left(\frac{x\left(2x^2+\sqrt{13}+5\right)}{2\sqrt{x^4+5x^2+3}}-\frac{\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right.\right.}{2\sqrt{x^4+5x^2+3}}\right.\right.\right.$$

$$\frac{8x^2+7}{39x\sqrt{x^4+5x^2+3}}$$

input
```
Int[(2 + 3*x^2)/(x^2*(3 + 5*x^2 + x^4)^(3/2)),x]
```

output
```
-1/39*(7 + 8*x^2)/(x*Sqrt[3 + 5*x^2 + x^4]) + ((-19*Sqrt[3 + 5*x^2 + x^4])
/(3*x) + (19*((x*(5 + Sqrt[13] + 2*x^2))/(2*Sqrt[3 + 5*x^2 + x^4]) - (Sqrt
[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]
*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13
 + 5*Sqrt[13])/6])/(2*Sqrt[3 + 5*x^2 + x^4])) - (4*Sqrt[6/(5 + Sqrt[13])]*
Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13]
)*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/
Sqrt[3 + 5*x^2 + x^4])/3)/39
```

3.202. $\int \frac{2+3x^2}{x^2\left(3+5x^2+x^4\right)^{3/2}}\,dx$

3.202.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1600
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^2 + c*x^4)^(p + 1)
*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2)/(2*a*f*(p + 1)*(b^2 - 4*a
*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^m*(a + b*x^2 + c
*x^4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1))
- a*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Int
egerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.202.4 Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{19x^4+119x^2+78}{117x\sqrt{x^4+5x^2+3}}-\frac{16\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{13\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}-\frac{76\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-}$
elliptic	$-\frac{2\left(-\frac{7}{234}x^3-\frac{11}{234}x\right)}{\sqrt{x^4+5x^2+3}}-\frac{2\sqrt{x^4+5x^2+3}}{9x}-\frac{16\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{13\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}-\frac{76\sqrt{1-(}$
default	$-\frac{6\left(-\frac{19}{78}x-\frac{5}{78}x^3\right)}{\sqrt{x^4+5x^2+3}}-\frac{16\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{13\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}-\frac{76\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-(}$

input
```
int((3*x^2+2)/x^2/(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/117*(19*x^4+119*x^2+78)/x/(x^4+5*x^2+3)^(1/2)-16/13/(-30+6*13^(1/2))^(1
/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x
^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*3
9^(1/2))-76/13/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1
-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^(1/2))*(Elliptic
F(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(
-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))
```

3.202.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.62

$$\int \frac{2+3x^2}{x^2 (3+5x^2+x^4)^{3/2}}\,dx =$$
$$-\frac{19 \left(\sqrt{13}\sqrt{6}\sqrt{3}(x^5+5\,x^3+3\,x) - 5\,\sqrt{6}\sqrt{3}(x^5+5\,x^3+3\,x)\right)\sqrt{\sqrt{13}-5}E(\arcsin\left(\frac{1}{6}\,\sqrt{6}x\sqrt{\sqrt{13}-5}\right)\mid \frac{5}{6}\,\sqrt{}}{}$$

input
```
integrate((3*x^2+2)/x^2/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
-1/4212*(19*(sqrt(13)*sqrt(6)*sqrt(3)*(x^5 + 5*x^3 + 3*x) - 5*sqrt(6)*sqrt
(3)*(x^5 + 5*x^3 + 3*x))*sqrt(sqrt(13) - 5)*elliptic_e(arcsin(1/6*sqrt(6)*
x*sqrt(sqrt(13) - 5)), 5/6*sqrt(13) + 19/6) - (43*sqrt(13)*sqrt(6)*sqrt(3)
*(x^5 + 5*x^3 + 3*x) + 25*sqrt(6)*sqrt(3)*(x^5 + 5*x^3 + 3*x))*sqrt(sqrt(1
3) - 5)*elliptic_f(arcsin(1/6*sqrt(6)*x*sqrt(sqrt(13) - 5)), 5/6*sqrt(13)
+ 19/6) + 36*(19*x^4 + 119*x^2 + 78)*sqrt(x^4 + 5*x^2 + 3))/(x^5 + 5*x^3 +
 3*x)
```

3.202.6 Sympy [F]

$$\int \frac{2+3x^2}{x^2 (3+5x^2+x^4)^{3/2}}\,dx = \int \frac{3x^2+2}{x^2 (x^4+5x^2+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate((3*x**2+2)/x**2/(x**4+5*x**2+3)**(3/2),x)
```

output
```
Integral((3*x**2 + 2)/(x**2*(x**4 + 5*x**2 + 3)**(3/2)), x)
```

3.202.7 Maxima [F]

$$\int \frac{2+3x^2}{x^2 (3+5x^2+x^4)^{3/2}}\,dx = \int \frac{3\,x^2+2}{(x^4+5\,x^2+3)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((3*x^2+2)/x^2/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 2)/((x^4 + 5*x^2 + 3)^(3/2)*x^2), x)
```

3.202.8 Giac [F]

$$\int \frac{2+3x^2}{x^2\left(3+5x^2+x^4\right)^{3/2}}\,dx=\int \frac{3\,x^2+2}{\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((3*x^2+2)/x^2/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

output
```
integrate((3*x^2 + 2)/((x^4 + 5*x^2 + 3)^(3/2)*x^2), x)
```

3.202.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x^2}{x^2\left(3+5x^2+x^4\right)^{3/2}}\,dx=\int \frac{3\,x^2+2}{x^2\,\left(x^4+5\,x^2+3\right)^{3/2}}\,dx$$

input
```
int((3*x^2 + 2)/(x^2*(5*x^2 + x^4 + 3)^(3/2)),x)
```

output
```
int((3*x^2 + 2)/(x^2*(5*x^2 + x^4 + 3)^(3/2)), x)
```

3.203 $\int \frac{2+3x^2}{x^4(3+5x^2+x^4)^{3/2}}\,dx$

3.203.1 Optimal result

Integrand size = 25, antiderivative size = 326

$$\int \frac{2+3x^2}{x^4\left(3+5x^2+x^4\right)^{3/2}}\,dx=-\frac{133x\left(5+\sqrt{13}+2x^2\right)}{1053\sqrt{3+5x^2+x^4}}-\frac{7+8x^2}{39x^3\sqrt{3+5x^2+x^4}}-\frac{5\sqrt{3+5x^2+x^4}}{351x^3}+\frac{266\sqrt{3+5x^2+x^4}}{1053x}+\frac{133\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)\middle|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{1053\sqrt{3+5x^2+x^4}}-\frac{5\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{351\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{3+5x^2+x^4}}$$

output
```
1/39*(-8*x^2-7)/x^3/(x^4+5*x^2+3)^(1/2)-133/1053*x*(5+2*x^2+13^(1/2))/(x^4
+5*x^2+3)^(1/2)-5/351*(x^4+5*x^2+3)^(1/2)/x^3+266/1053*(x^4+5*x^2+3)^(1/2)
/x+133/6318*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1
/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(
-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(1/2))^(1/2)*((6+x^2
*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)-5/351*(1/(3
6+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(
30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^
(1/2))*(6+x^2*(5+13^(1/2)))*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1
/2)/(x^4+5*x^2+3)^(1/2)/(30+6*13^(1/2))^(1/2)
```

3.203.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.26 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.72

$$\int \frac{2+3x^2}{x^4\left(3+5x^2+x^4\right)^{3/2}}\,dx = \frac{-468+1014x^2+2630x^4+532x^6-133i\sqrt{2}\left(-5+\sqrt{13}\right)x^3\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\cdots}}{\cdots}$$

input
```
Integrate[(2 + 3*x^2)/(x^4*(3 + 5*x^2 + x^4)^(3/2)),x]
```

output
```
(-468 + 1014*x^2 + 2630*x^4 + 532*x^6 - (133*I)*Sqrt[2]*(-5 + Sqrt[13])*x^
3*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]
*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] + I
*Sqrt[2]*(-650 + 133*Sqrt[13])*x^3*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt
[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13]
)]*x], 19/6 + (5*Sqrt[13])/6])/(2106*x^3*Sqrt[3 + 5*x^2 + x^4])
```

3.203.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {1600, 25, 1604, 1604, 25, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+2}{x^4\left(x^4+5x^2+3\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{1600}$$

$$-\frac{1}{39}\int -\frac{5-24x^2}{x^4\sqrt{x^4+5x^2+3}}dx-\frac{8x^2+7}{39x^3\sqrt{x^4+5x^2+3}}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{39}\int \frac{5-24x^2}{x^4\sqrt{x^4+5x^2+3}}dx-\frac{8x^2+7}{39x^3\sqrt{x^4+5x^2+3}}$$

$$\Big\downarrow \text{1604}$$

$$\frac{1}{39}\left(-\frac{1}{9}\int \frac{5x^2+266}{x^2\sqrt{x^4+5x^2+3}}dx-\frac{5\sqrt{x^4+5x^2+3}}{9x^3}\right)-\frac{8x^2+7}{39x^3\sqrt{x^4+5x^2+3}}$$

$$\Big\downarrow \text{ 1604}$$

$$\frac{1}{39}\left(\frac{1}{9}\left(\frac{1}{3}\int -\frac{266x^2+15}{\sqrt{x^4+5x^2+3}}dx+\frac{266\sqrt{x^4+5x^2+3}}{3x}\right)-\frac{5\sqrt{x^4+5x^2+3}}{9x^3}\right)- \frac{8x^2+7}{39x^3\sqrt{x^4+5x^2+3}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{39}\left(\frac{1}{9}\left(\frac{266\sqrt{x^4+5x^2+3}}{3x}-\frac{1}{3}\int \frac{266x^2+15}{\sqrt{x^4+5x^2+3}}dx\right)-\frac{5\sqrt{x^4+5x^2+3}}{9x^3}\right)-\frac{8x^2+7}{39x^3\sqrt{x^4+5x^2+3}}$$

$$\Big\downarrow \text{ 1503}$$

$$\frac{1}{39}\left(\frac{1}{9}\left(\frac{1}{3}\left(-15\int \frac{1}{\sqrt{x^4+5x^2+3}}dx-266\int \frac{x^2}{\sqrt{x^4+5x^2+3}}dx\right)+\frac{266\sqrt{x^4+5x^2+3}}{3x}\right)-\frac{5\sqrt{x^4+5x^2+3}}{9x^3}\right)- \frac{8x^2+7}{39x^3\sqrt{x^4+5x^2+3}}$$

$$\Big\downarrow \text{ 1412}$$

$$\frac{1}{39}\left(\frac{1}{9}\left(\frac{1}{3}\left(-266\int \frac{x^2}{\sqrt{x^4+5x^2+3}}dx-\frac{5\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}}\right.\right.}{\sqrt{x^4+5x^2+3}}\right.\right.\right.$$

$$\frac{8x^2+7}{39x^3\sqrt{x^4+5x^2+3}}$$

$$\Big\downarrow \text{ 1455}$$

$$\frac{1}{39}\left(\frac{1}{9}\left(\frac{1}{3}\left(-\frac{5\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\arctan\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(-13+5\sqrt{\right.\right.}{\sqrt{x^4+5x^2+3}}\right.\right.\right.$$

$$\frac{8x^2+7}{39x^3\sqrt{x^4+5x^2+3}}$$

input
```
Int[(2 + 3*x^2)/(x^4*(3 + 5*x^2 + x^4)^(3/2)),x]
```

3.203. $\int \frac{2+3x^2}{x^4(3+5x^2+x^4)^{3/2}}\,dx$

output
```
-1/39*(7 + 8*x^2)/(x^3*Sqrt[3 + 5*x^2 + x^4]) + ((-5*Sqrt[3 + 5*x^2 + x^4]
)/(9*x^3) + ((266*Sqrt[3 + 5*x^2 + x^4])/(3*x) + (-266*((x*(5 + Sqrt[13] +
 2*x^2))/(2*Sqrt[3 + 5*x^2 + x^4]) - (Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5
- Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*Ellipt
icE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(2*Sqrt[3 + 5
*x^2 + x^4])) - (5*Sqrt[3/(2*(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2
)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt
[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/Sqrt[3 + 5*x^2 + x^4])/3)/9)
/39
```

3.203.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

3.203. $\int \frac{2+3x^2}{x^4(3+5x^2+x^4)^{3/2}}\,dx$

rule 1600
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^2 + c*x^4)^(p + 1)
*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2)/(2*a*f*(p + 1)*(b^2 - 4*a
*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^m*(a + b*x^2 + c
*x^4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1))
- a*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Int
egerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.203.4 Maple [A] (verified)

Time = 2.98 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.70

method	result
risch	$\frac{266x^6+1315x^4+507x^2-234}{1053x^3\sqrt{x^4+5x^2+3}} - \frac{10\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6}, \frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{117\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}} + \frac{1064\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right.}}{}$
elliptic	$-\frac{2\left(\frac{11}{702}x^3+\frac{17}{351}x\right)}{\sqrt{x^4+5x^2+3}} - \frac{2\sqrt{x^4+5x^2+3}}{27x^3} + \frac{23\sqrt{x^4+5x^2+3}}{81x} - \frac{10\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6}, \frac{5\sqrt{3}}{6}+\right.}{117\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}$
default	$-\frac{6\left(\frac{19}{234}x^3+\frac{40}{117}x\right)}{\sqrt{x^4+5x^2+3}} + \frac{23\sqrt{x^4+5x^2+3}}{81x} - \frac{10\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6}, \frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{117\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}} + \frac{1064\sqrt{1-}}{}$

input
```
int((3*x^2+2)/x^4/(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/1053*(266*x^6+1315*x^4+507*x^2-234)/x^3/(x^4+5*x^2+3)^(1/2)-10/117/(-30+
6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))
*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6
*3^(1/2)+1/6*39^(1/2))+1064/117/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/
2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13
^(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-
EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))
```

3.203.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.63

$$\int \frac{2+3x^2}{x^4\left(3+5x^2+x^4\right)^{3/2}}\,dx = \frac{266\left(\sqrt{13}\sqrt{6}\sqrt{3}(x^7+5\,x^5+3\,x^3)-5\,\sqrt{6}\sqrt{3}(x^7+5\,x^5+3\,x^3)\right)\sqrt{\sqrt{13}-5}E(\mathrm{ar}}{}$$

input
```
integrate((3*x^2+2)/x^4/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

output
```
1/37908*(266*(sqrt(13)*sqrt(6)*sqrt(3)*(x^7 + 5*x^5 + 3*x^3) - 5*sqrt(6)*s
qrt(3)*(x^7 + 5*x^5 + 3*x^3))*sqrt(sqrt(13) - 5)*elliptic_e(arcsin(1/6*sqr
t(6)*x*sqrt(sqrt(13) - 5)), 5/6*sqrt(13) + 19/6) - (251*sqrt(13)*sqrt(6)*s
qrt(3)*(x^7 + 5*x^5 + 3*x^3) - 1405*sqrt(6)*sqrt(3)*(x^7 + 5*x^5 + 3*x^3))
*sqrt(sqrt(13) - 5)*elliptic_f(arcsin(1/6*sqrt(6)*x*sqrt(sqrt(13) - 5)), 5
/6*sqrt(13) + 19/6) + 36*(266*x^6 + 1315*x^4 + 507*x^2 - 234)*sqrt(x^4 + 5
*x^2 + 3))/(x^7 + 5*x^5 + 3*x^3)
```

3.203.6 Sympy [F]

$$\int \frac{2+3x^2}{x^4\left(3+5x^2+x^4\right)^{3/2}}\,dx = \int \frac{3x^2+2}{x^4\left(x^4+5x^2+3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((3*x**2+2)/x**4/(x**4+5*x**2+3)**(3/2),x)
```

output
```
Integral((3*x**2 + 2)/(x**4*(x**4 + 5*x**2 + 3)**(3/2)), x)
```

3.203.7 Maxima [F]

$$\int \frac{2+3x^2}{x^4\left(3+5x^2+x^4\right)^{3/2}}\,dx = \int \frac{3\,x^2+2}{\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate((3*x^2+2)/x^4/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 2)/((x^4 + 5*x^2 + 3)^(3/2)*x^4), x)
```

3.203.8 Giac [F]

$$\int \frac{2+3x^2}{x^4\left(3+5x^2+x^4\right)^{3/2}}\,dx = \int \frac{3\,x^2+2}{\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate((3*x^2+2)/x^4/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

output
```
integrate((3*x^2 + 2)/((x^4 + 5*x^2 + 3)^(3/2)*x^4), x)
```

3.203.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x^2}{x^4\left(3+5x^2+x^4\right)^{3/2}}\,dx = \int \frac{3\,x^2+2}{x^4\left(x^4+5\,x^2+3\right)^{3/2}}\,dx$$

input
```
int((3*x^2 + 2)/(x^4*(5*x^2 + x^4 + 3)^(3/2)),x)
```

output
```
int((3*x^2 + 2)/(x^4*(5*x^2 + x^4 + 3)^(3/2)), x)
```

3.204 $\int (fx)^{3/2} (d+ex^2) \sqrt{a+bx^2+cx^4}\,dx$

3.204.1 Optimal result

Integrand size = 31, antiderivative size = 297

$$\int (fx)^{3/2} (d+ex^2) \sqrt{a+bx^2+cx^4}\,dx = \frac{2d(fx)^{5/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{5}{4},-\frac{1}{2},-\frac{1}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{2e(fx)^{9/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{9}{4},-\frac{1}{2},-\frac{1}{2},\frac{13}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{9f^3\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

output
```
2/5*d*(f*x)^(5/2)*AppellF1(5/4,-1/2,-1/2,9/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2
)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f/(1+2*c*x^2/(b-
(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)+2/9*e*
(f*x)^(9/2)*AppellF1(9/4,-1/2,-1/2,13/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2
*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f^3/(1+2*c*x^2/(b-(-4
*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.204.2 Mathematica [A] (verified)

Time = 11.76 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.45

$$\int (fx)^{3/2} \left(d+ex^2\right) \sqrt{a+bx^2+cx^4}\,dx = \frac{2f\sqrt{fx}\Big(5(a+bx^2+cx^4)\left(-14b^2e+2bc(13d+5ex^2)+c(36ae+65cdx^2+45ce\right.}{}$$

input
```
Integrate[(f*x)^(3/2)*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(2*f*Sqrt[f*x]*(5*(a + b*x^2 + c*x^4)*(-14*b^2*e + 2*b*c*(13*d + 5*e*x^2)
+ c*(36*a*e + 65*c*d*x^2 + 45*c*e*x^4)) + 10*a*(-13*b*c*d + 7*b^2*e - 18*a
*c*e)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt
[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4,
1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^
2 - 4*a*c])] + 2*(-39*b^2*c*d + 130*a*c^2*d + 21*b^3*e - 79*a*b*c*e)*x^2*S
qrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + S
qrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/
2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a
*c])]))/(2925*c^2*Sqrt[a + b*x^2 + c*x^4])
```

3.204.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1674, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (fx)^{3/2} \left(d+ex^2\right) \sqrt{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \ 1674$$

$$\int \left(d(fx)^{3/2}\sqrt{a+bx^2+cx^4}+\frac{e(fx)^{7/2}\sqrt{a+bx^2+cx^4}}{f^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2d(fx)^{5/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{5}{4},-\frac{1}{2},-\frac{1}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2e(fx)^{9/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{9}{4},-\frac{1}{2},-\frac{1}{2},\frac{13}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{9f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[(f*x)^(3/2)*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(2*d*(f*x)^(5/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[5/4, -1/2, -1/2, 9/4, (-
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*
f*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt
[b^2 - 4*a*c])]) + (2*e*(f*x)^(9/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[9/4,
-1/2, -1/2, 13/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt
[b^2 - 4*a*c])])/(9*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1
 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

3.204.3.1 Defintions of rubi rules used

rule 1674
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.204.4 Maple [F]

$$\int (fx)^{\frac{3}{2}}\left(e\,x^2+d\right)\sqrt{c\,x^4+b\,x^2+a}dx$$

input
```
int((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x)
```

3.204.5 Fricas [F]

$$\int (fx)^{3/2} \left(d+ex^2\right) \sqrt{a+bx^2+cx^4}\,dx = \int \sqrt{cx^4+bx^2+a}(ex^2+d)(fx)^{\frac{3}{2}}\,dx$$

input
```
integrate((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas
")
```

output
```
integral((e*f*x^3 + d*f*x)*sqrt(c*x^4 + b*x^2 + a)*sqrt(f*x), x)
```

3.204.6 Sympy [F]

$$\int (fx)^{3/2} \left(d+ex^2\right) \sqrt{a+bx^2+cx^4}\,dx = \int (fx)^{\frac{3}{2}} \left(d+ex^2\right) \sqrt{a+bx^2+cx^4}\,dx$$

input
```
integrate((f*x)**(3/2)*(e*x**2+d)*(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((f*x)**(3/2)*(d + e*x**2)*sqrt(a + b*x**2 + c*x**4), x)
```

3.204.7 Maxima [F]

$$\int (fx)^{3/2} \left(d+ex^2\right) \sqrt{a+bx^2+cx^4}\,dx = \int \sqrt{cx^4+bx^2+a}(ex^2+d)(fx)^{\frac{3}{2}}\,dx$$

input
```
integrate((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*(f*x)^(3/2), x)
```

3.204.8 Giac [F]

$$\int (fx)^{3/2} \left(d+ex^2\right) \sqrt{a+bx^2+cx^4}\,dx = \int \sqrt{cx^4+bx^2+a}\left(ex^2+d\right)(fx)^{\frac{3}{2}}\,dx$$

input
```
integrate((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*(f*x)^(3/2), x)
```

3.204.9 Mupad [F(-1)]

Timed out.

$$\int (fx)^{3/2} \left(d+ex^2\right) \sqrt{a+bx^2+cx^4}\,dx = \int (f\,x)^{3/2}\,\left(e\,x^2+d\right)\,\sqrt{c\,x^4+b\,x^2+a}\,dx$$

input
```
int((f*x)^(3/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((f*x)^(3/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2), x)
```

3.205 $\int \sqrt{fx}(d+ex^2)\sqrt{a+bx^2+cx^4}\,dx$

3.205.1 Optimal result

Integrand size = 31, antiderivative size = 297

$$\int \sqrt{fx}(d+ex^2)\sqrt{a+bx^2+cx^4}\,dx = \frac{2d(fx)^{3/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{3}{4},-\frac{1}{2},-\frac{1}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{2e(fx)^{7/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{7}{4},-\frac{1}{2},-\frac{1}{2},\frac{11}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{7f^3\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

output
```
2/3*d*(f*x)^(3/2)*AppellF1(3/4,-1/2,-1/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2
)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f/(1+2*c*x^2/(b-
(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)+2/7*e*
(f*x)^(7/2)*AppellF1(7/4,-1/2,-1/2,11/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2
*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f^3/(1+2*c*x^2/(b-(-4
*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.205.2 Mathematica [A] (verified)

Time = 11.61 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.30

$$\int \sqrt{fx}(d+ex^2)\sqrt{a+bx^2+cx^4}\,dx$$
$$= \frac{2x\sqrt{fx}\left(21(11cd+2be+7cex^2)(a+bx^2+cx^4)+14a(22cd-3be)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{App}\right.}{}$$

input
```
Integrate[Sqrt[f*x]*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(2*x*Sqrt[f*x]*(21*(11*c*d + 2*b*e + 7*c*e*x^2)*(a + b*x^2 + c*x^4) + 14*a
*(22*c*d - 3*b*e)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4
*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Ap
pellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(
-b + Sqrt[b^2 - 4*a*c])] + 6*(11*b*c*d - 5*b^2*e + 14*a*c*e)*x^2*Sqrt[(b -
 Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2
- 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4,
 (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))
/(1617*c*Sqrt[a + b*x^2 + c*x^4])
```

3.205.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1674, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{fx}(d+ex^2)\sqrt{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{1674}$$

$$\int \left(d\sqrt{fx}\sqrt{a+bx^2+cx^4}+\frac{e(fx)^{5/2}\sqrt{a+bx^2+cx^4}}{f^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2d(fx)^{3/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{3}{4},-\frac{1}{2},-\frac{1}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2e(fx)^{7/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{7}{4},-\frac{1}{2},-\frac{1}{2},\frac{11}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{7f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[Sqrt[f*x]*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(2*d*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[3/4, -1/2, -1/2, 7/4, (-
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]])/(3*
f*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt
[b^2 - 4*a*c])]) + (2*e*(f*x)^(7/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[7/4,
-1/2, -1/2, 11/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt
[b^2 - 4*a*c])]])/(7*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1
 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

3.205.3.1 Defintions of rubi rules used

rule 1674
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.205.4 Maple [F]

$$\int \sqrt{fx}\left(e\,x^2+d\right)\sqrt{c\,x^4+b\,x^2+a}dx$$

input
```
int((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x)
```

3.205.5 Fricas [F]

$$\int \sqrt{fx}(d+ex^2)\sqrt{a+bx^2+cx^4}\,dx = \int \sqrt{cx^4+bx^2+a}(ex^2+d)\sqrt{fx}\,dx$$

input
```
integrate((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas
")
```

output
```
integral(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x), x)
```

3.205.6 Sympy [F]

$$\int \sqrt{fx}(d+ex^2)\sqrt{a+bx^2+cx^4}\,dx = \int \sqrt{fx}(d+ex^2)\sqrt{a+bx^2+cx^4}\,dx$$

input
```
integrate((f*x)**(1/2)*(e*x**2+d)*(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(f*x)*(d + e*x**2)*sqrt(a + b*x**2 + c*x**4), x)
```

3.205.7 Maxima [F]

$$\int \sqrt{fx}(d+ex^2)\sqrt{a+bx^2+cx^4}\,dx = \int \sqrt{cx^4+bx^2+a}(ex^2+d)\sqrt{fx}\,dx$$

input
```
integrate((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x), x)
```

3.205.8 Giac [F]

$$\int \sqrt{fx}(d+ex^2)\sqrt{a+bx^2+cx^4}\,dx = \int \sqrt{cx^4+bx^2+a}(ex^2+d)\sqrt{fx}\,dx$$

input
```
integrate((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x), x)
```

3.205.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{fx}(d+ex^2)\sqrt{a+bx^2+cx^4}\,dx = \int \sqrt{f\,x}\,\left(e\,x^2+d\right)\,\sqrt{c\,x^4+b\,x^2+a}\,dx$$

input
```
int((f*x)^(1/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((f*x)^(1/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2), x)
```

3.206 $\int \frac{(d+ex^2)\sqrt{a+bx^2+cx^4}}{\sqrt{fx}}\,dx$

3.206.1 Optimal result

Integrand size = 31, antiderivative size = 295

$$\begin{aligned}\int \frac{(d+ex^2)\sqrt{a+bx^2+cx^4}}{\sqrt{fx}}\,dx &= \frac{2d\sqrt{fx}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{1}{4},-\frac{1}{2},-\frac{1}{2},\frac{5}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} \\ &+ \frac{2e(fx)^{5/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{5}{4},-\frac{1}{2},-\frac{1}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f^3\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\end{aligned}$$

output
```
2/5*e*(f*x)^(5/2)*AppellF1(5/4,-1/2,-1/2,9/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2
)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f^3/(1+2*c*x^2/(
b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)+2*d*
AppellF1(1/4,-1/2,-1/2,5/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4
*a*c+b^2)^(1/2)))*(f*x)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/f/(1+2*c*x^2/(b-(-4*a*
c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.206.2 Mathematica [A] (verified)

Time = 11.56 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.31

$$\int \frac{(d+ex^2)\sqrt{a+bx^2+cx^4}}{\sqrt{fx}}\,dx$$
$$= \frac{2x\left(5(9cd+2be+5cex^2)(a+bx^2+cx^4)+10a(18cd-be)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1}{4}\right.\right.}{}$$

input
```
Integrate[((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4])/Sqrt[f*x],x]
```

output
```
(2*x*(5*(9*c*d + 2*b*e + 5*c*e*x^2)*(a + b*x^2 + c*x^4) + 10*a*(18*c*d - b
*e)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(
b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/
2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2
- 4*a*c])] + 2*(9*b*c*d - 3*b^2*e + 10*a*c*e)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a
*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(225*c*Sqrt[f*x
]*Sqrt[a + b*x^2 + c*x^4])
```

3.206.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1674, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)\sqrt{a+bx^2+cx^4}}{\sqrt{fx}}\,dx$$
$$\Big\downarrow \text{1674}$$
$$\int \left(\frac{d\sqrt{a+bx^2+cx^4}}{\sqrt{fx}}+\frac{e(fx)^{3/2}\sqrt{a+bx^2+cx^4}}{f^2}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{2d\sqrt{fx}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{1}{4},-\frac{1}{2},-\frac{1}{2},\frac{5}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2e(fx)^{5/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{5}{4},-\frac{1}{2},-\frac{1}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4])/Sqrt[f*x],x]
```

output
```
(2*d*Sqrt[f*x]*Sqrt[a + b*x^2 + c*x^4]*AppellF1[1/4, -1/2, -1/2, 5/4, (-2*
c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f*Sq
rt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c])]) + (2*e*(f*x)^(5/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[5/4, -1/2
, -1/2, 9/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])])/(5*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2
*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

3.206.3.1 Defintions of rubi rules used

rule 1674
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.206.4 Maple [F]

$$\int \frac{(e\,x^2+d)\sqrt{c\,x^4+b\,x^2+a}}{\sqrt{fx}}dx$$

input
```
int((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(1/2),x)
```

output
```
int((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(1/2),x)
```

3.206.5 Fricas [F]

$$\int \frac{(d+ex^2)\sqrt{a+bx^2+cx^4}}{\sqrt{fx}}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(ex^2+d)}{\sqrt{fx}}\,dx$$

input
```
integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(1/2),x, algorithm="fricas
")
```

output
```
integral(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x)/(f*x), x)
```

3.206.6 Sympy [F]

$$\int \frac{(d+ex^2)\sqrt{a+bx^2+cx^4}}{\sqrt{fx}}\,dx = \int \frac{(d+ex^2)\sqrt{a+bx^2+cx^4}}{\sqrt{fx}}\,dx$$

input
```
integrate((e*x**2+d)*(c*x**4+b*x**2+a)**(1/2)/(f*x)**(1/2),x)
```

output
```
Integral((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)/sqrt(f*x), x)
```

3.206.7 Maxima [F]

$$\int \frac{(d+ex^2)\sqrt{a+bx^2+cx^4}}{\sqrt{fx}}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(ex^2+d)}{\sqrt{fx}}\,dx$$

input
```
integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)/sqrt(f*x), x)
```

3.206.8 Giac [F]

$$\int \frac{(d+ex^2)\sqrt{a+bx^2+cx^4}}{\sqrt{fx}}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(ex^2+d)}{\sqrt{fx}}\,dx$$

input
```
integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)/sqrt(f*x), x)
```

3.206.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)\sqrt{a+bx^2+cx^4}}{\sqrt{fx}}\,dx = \int \frac{(e\,x^2+d)\,\sqrt{c\,x^4+b\,x^2+a}}{\sqrt{f\,x}}\,dx$$

input
```
int(((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2))/(f*x)^(1/2),x)
```

output
```
int(((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2))/(f*x)^(1/2), x)
```

3.207 $\int \frac{(d+ex^2)\sqrt{a+bx^2+cx^4}}{(fx)^{3/2}}\,dx$

3.207.1 Optimal result

Integrand size = 31, antiderivative size = 295

$$\int \frac{(d+ex^2)\sqrt{a+bx^2+cx^4}}{(fx)^{3/2}}\,dx = -\frac{2d\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(-\frac{1}{4},-\frac{1}{2},-\frac{1}{2},\frac{3}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{fx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{2e(fx)^{3/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{3}{4},-\frac{1}{2},-\frac{1}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f^3\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

output
```
2/3*e*(f*x)^(3/2)*AppellF1(3/4,-1/2,-1/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2
)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f^3/(1+2*c*x^2/(
b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)-2*d*
AppellF1(-1/4,-1/2,-1/2,3/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-
4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f/(f*x)^(1/2)/(1+2*c*x^2/(b-(-4*a
*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.207.2 Mathematica [A] (verified)

Time = 11.65 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.25

$$\int \frac{(d+ex^2)\sqrt{a+bx^2+cx^4}}{(fx)^{3/2}}\,dx = \frac{x\left(-42(7d-ex^2)(a+bx^2+cx^4)+28(7bd+2ae)x^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{b\cdots}\right.}{\cdots}$$

input
```
Integrate[((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4])/(f*x)^(3/2),x]
```

output
```
(x*(-42*(7*d - e*x^2)*(a + b*x^2 + c*x^4) + 28*(7*b*d + 2*a*e)*x^2*Sqrt[(b
 - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^
2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4
, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]
+ 12*(14*c*d + b*e)*x^4*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b
^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c
])]*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c
*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(147*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]
)
```

3.207.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1674, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)\sqrt{a+bx^2+cx^4}}{(fx)^{3/2}}\,dx$$

$$\Big\downarrow \text{1674}$$

$$\int \left(\frac{d\sqrt{a+bx^2+cx^4}}{(fx)^{3/2}}+\frac{e\sqrt{fx}\sqrt{a+bx^2+cx^4}}{f^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2e(fx)^{3/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{3}{4},-\frac{1}{2},-\frac{1}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}-\frac{2d\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(-\frac{1}{4},-\frac{1}{2},-\frac{1}{2},\frac{3}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{fx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4])/(f*x)^(3/2),x]
```

output
```
(-2*d*Sqrt[a + b*x^2 + c*x^4]*AppellF1[-1/4, -1/2, -1/2, 3/4, (-2*c*x^2)/(
b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f*Sqrt[f*x]*
Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c])]) + (2*e*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[3/4, -1
/2, -1/2, 7/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c])])/(3*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
(2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

3.207.3.1 Defintions of rubi rules used

rule 1674
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.207.4 Maple [F]

$$\int \frac{(e\,x^2+d)\,\sqrt{c\,x^4+b\,x^2+a}}{(fx)^{\frac{3}{2}}}dx$$

input
```
int((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(3/2),x)
```

output
```
int((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(3/2),x)
```

3.207.5 Fricas [F]

$$\int \frac{(d+ex^2)\sqrt{a+bx^2+cx^4}}{(fx)^{3/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(ex^2+d)}{(fx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(3/2),x, algorithm="fricas
")
```

output
```
integral(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x)/(f^2*x^2), x)
```

3.207.6 Sympy [F]

$$\int \frac{(d+ex^2)\sqrt{a+bx^2+cx^4}}{(fx)^{3/2}}\,dx = \int \frac{(d+ex^2)\sqrt{a+bx^2+cx^4}}{(fx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x**2+d)*(c*x**4+b*x**2+a)**(1/2)/(f*x)**(3/2),x)
```

output
```
Integral((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)/(f*x)**(3/2), x)
```

3.207.7 Maxima [F]

$$\int \frac{(d+ex^2)\sqrt{a+bx^2+cx^4}}{(fx)^{3/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(ex^2+d)}{(fx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(3/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)/(f*x)^(3/2), x)
```

3.207.8 Giac [F]

$$\int \frac{(d+ex^2)\sqrt{a+bx^2+cx^4}}{(fx)^{3/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(ex^2+d)}{(fx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)/(f*x)^(3/2), x)
```

3.207.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)\sqrt{a+bx^2+cx^4}}{(fx)^{3/2}}\,dx = \int \frac{(e\,x^2+d)\,\sqrt{c\,x^4+b\,x^2+a}}{(f\,x)^{3/2}}\,dx$$

input
```
int(((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2))/(f*x)^(3/2),x)
```

output
```
int(((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2))/(f*x)^(3/2), x)
```

3.208 $\int (fx)^{3/2} \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^{3/2} dx$

3.208.1 Optimal result

Integrand size = 31, antiderivative size = 299

$$\int (fx)^{3/2} \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^{3/2} dx = \frac{2ad(fx)^{5/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{5}{4},-\frac{3}{2},-\frac{3}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{2ae(fx)^{9/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{9}{4},-\frac{3}{2},-\frac{3}{2},\frac{13}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{9f^3\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

output
```
2/5*a*d*(f*x)^(5/2)*AppellF1(5/4,-3/2,-3/2,9/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1
/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f/(1+2*c*x^2/(
b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)+2/9*
a*e*(f*x)^(9/2)*AppellF1(9/4,-3/2,-3/2,13/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)
),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f^3/(1+2*c*x^2/(b
-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.208.2 Mathematica [A] (verified)

Time = 12.00 (sec) , antiderivative size = 567, normalized size of antiderivative = 1.90

$$\int (fx)^{3/2} \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^{3/2} dx = \frac{2f\sqrt{fx}\Big(5(a+bx^2+cx^4)\left(308b^4e-4b^3c(147d+55ex^2)+12b^2c(-167ae+5cx^2(7d+3ex^2))\right)}{}$$

input
```
Integrate[(f*x)^(3/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(2*f*Sqrt[f*x]*(5*(a + b*x^2 + c*x^4)*(308*b^4*e - 4*b^3*c*(147*d + 55*e*x
^2) + 12*b^2*c*(-167*a*e + 5*c*x^2*(7*d + 3*e*x^2)) + 3*b*c^2*(16*a*(77*d
+ 25*e*x^2) + 5*c*x^4*(399*d + 299*e*x^2)) + 3*c^2*(816*a^2*e + 65*c^2*x^6
*(21*d + 17*e*x^2) + 5*a*c*x^2*(637*d + 425*e*x^2))) - 20*a*(-147*b^3*c*d
+ 924*a*b*c^2*d + 77*b^4*e - 501*a*b^2*c*e + 612*a^2*c^2*e)*Sqrt[(b - Sqrt
[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a
*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] - 4*(-4
41*b^4*c*d + 3297*a*b^2*c^2*d - 5460*a^2*c^3*d + 231*b^5*e - 1778*a*b^3*c*
e + 3336*a^2*b*c^2*e)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt
[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a
*c])]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*
c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(348075*c^3*Sqrt[a + b*x^2 + c*x^4])
```

3.208.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1674, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (fx)^{3/2} \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^{3/2} dx$$

$$\Big\downarrow \text{1674}$$

$$\int \left(d(fx)^{3/2} \left(a+bx^2+cx^4\right)^{3/2}+\frac{e(fx)^{7/2} \left(a+bx^2+cx^4\right)^{3/2}}{f^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2ad(fx)^{5/2}\sqrt{a+bx^2+cx^4}\operatorname{AppellF1}\left(\frac{5}{4},-\frac{3}{2},-\frac{3}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2ae(fx)^{9/2}\sqrt{a+bx^2+cx^4}\operatorname{AppellF1}\left(\frac{9}{4},-\frac{3}{2},-\frac{3}{2},\frac{13}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{9f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[(f*x)^(3/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(2*a*d*(f*x)^(5/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[5/4, -3/2, -3/2, 9/4,
(-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(
5*f*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sq
rt[b^2 - 4*a*c])]) + (2*a*e*(f*x)^(9/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[9
/4, -3/2, -3/2, 13/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c])])/(9*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sq
rt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

3.208.3.1 Defintions of rubi rules used

rule 1674
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.208.4 Maple [F]

$$\int (fx)^{\frac{3}{2}}\left(e\,x^2+d\right)\left(c\,x^4+b\,x^2+a\right)^{\frac{3}{2}}dx$$

input
```
int((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
int((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x)
```

3.208.5 Fricas [F]

$$\int (fx)^{3/2} \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^{3/2} \, dx = \int \left(cx^4+bx^2+a\right)^{\frac{3}{2}} \left(ex^2+d\right) (fx)^{\frac{3}{2}} \, dx$$

input
```
integrate((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas
")
```

output
```
integral((c*e*f*x^7 + (c*d + b*e)*f*x^5 + (b*d + a*e)*f*x^3 + a*d*f*x)*sqr
t(c*x^4 + b*x^2 + a)*sqrt(f*x), x)
```

3.208.6 Sympy [F]

$$\int (fx)^{3/2} \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^{3/2} \, dx = \int (fx)^{\frac{3}{2}} \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^{\frac{3}{2}} \, dx$$

input
```
integrate((f*x)**(3/2)*(e*x**2+d)*(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral((f*x)**(3/2)*(d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2), x)
```

3.208.7 Maxima [F]

$$\int (fx)^{3/2} \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^{3/2} \, dx = \int \left(cx^4+bx^2+a\right)^{\frac{3}{2}} \left(ex^2+d\right) (fx)^{\frac{3}{2}} \, dx$$

input
```
integrate((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima
")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*(f*x)^(3/2), x)
```

3.208.8 Giac [F]

$$\int (fx)^{3/2} \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^{3/2} \, dx = \int \left(cx^4+bx^2+a\right)^{\frac{3}{2}} (ex^2+d)(fx)^{\frac{3}{2}} \, dx$$

input
```
integrate((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*(f*x)^(3/2), x)
```

3.208.9 Mupad [F(-1)]

Timed out.

$$\int (fx)^{3/2} \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^{3/2} \, dx = \int (f\,x)^{3/2}\,\left(e\,x^2+d\right)\,\left(c\,x^4+b\,x^2+a\right)^{3/2} \,dx$$

input
```
int((f*x)^(3/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int((f*x)^(3/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2), x)
```

3.209 $\int \sqrt{fx}(d+ex^2)\left(a+bx^2+cx^4\right)^{3/2} dx$

3.209.1 Optimal result

Integrand size = 31, antiderivative size = 299

$$\int \sqrt{fx}(d+ex^2)\left(a+bx^2+cx^4\right)^{3/2} dx = \frac{2ad(fx)^{3/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{3}{4},-\frac{3}{2},-\frac{3}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{2ae(fx)^{7/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{7}{4},-\frac{3}{2},-\frac{3}{2},\frac{11}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{7f^3\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

output
```
2/3*a*d*(f*x)^(3/2)*AppellF1(3/4,-3/2,-3/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1
/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f/(1+2*c*x^2/(
b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)+2/7*
a*e*(f*x)^(7/2)*AppellF1(7/4,-3/2,-3/2,11/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)
),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f^3/(1+2*c*x^2/(b
-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.209.2 Mathematica [A] (verified)

Time = 11.94 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.64

$$\int \sqrt{fx}(d+ex^2)\left(a+bx^2+cx^4\right)^{3/2}\,dx = \frac{2x\sqrt{fx}\left(7(a+bx^2+cx^4)\left(-108b^3e+12b^2c(19d+7ex^2)+bc(624ae+7cx^2(323d+231ex^2))\right.\right.}{}$$

input
```
Integrate[Sqrt[f*x]*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(2*x*Sqrt[f*x]*(7*(a + b*x^2 + c*x^4)*(-108*b^3*e + 12*b^2*c*(19*d + 7*e*x
^2) + b*c*(624*a*e + 7*c*x^2*(323*d + 231*e*x^2)) + c^2*(77*c*x^4*(19*d +
15*e*x^2) + a*(3971*d + 2415*e*x^2))) + 28*a*(-57*b^2*c*d + 836*a*c^2*d +
27*b^3*e - 156*a*b*c*e)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b
^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c
])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*
x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 12*(-95*b^3*c*d + 684*a*b*c^2*d + 45*b^4*
e - 309*a*b^2*c*e + 420*a^2*c^2*e)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x
^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(153615*c^2*Sqrt[a + b*x^
2 + c*x^4])
```

3.209.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1674, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{fx}(d+ex^2)\left(a+bx^2+cx^4\right)^{3/2}\,dx$$

$$\Big\downarrow \text{1674}$$

$$\int \left(d\sqrt{fx}\left(a+bx^2+cx^4\right)^{3/2}+\frac{e(fx)^{5/2}\left(a+bx^2+cx^4\right)^{3/2}}{f^2}\right)dx$$

$\downarrow$ 2009

$$\frac{2ad(fx)^{3/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{3}{4},-\frac{3}{2},-\frac{3}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2ae(fx)^{7/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{7}{4},-\frac{3}{2},-\frac{3}{2},\frac{11}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{7f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[Sqrt[f*x]*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(2*a*d*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[3/4, -3/2, -3/2, 7/4,
(-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(
3*f*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sq
rt[b^2 - 4*a*c])]) + (2*a*e*(f*x)^(7/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[7
/4, -3/2, -3/2, 11/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c])])/(7*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sq
rt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

3.209.3.1 Defintions of rubi rules used

rule 1674
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.209.4 Maple [F]

$$\int \sqrt{fx}\left(e\,x^2+d\right)\left(c\,x^4+b\,x^2+a\right)^{\frac{3}{2}}dx$$

input
```
int((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
int((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x)
```

3.209.5 Fricas [F]

$$\int \sqrt{fx}(d+ex^2)\left(a+bx^2+cx^4\right)^{3/2}\,dx = \int \left(cx^4+bx^2+a\right)^{\frac{3}{2}}\left(ex^2+d\right)\sqrt{fx}\,dx$$

input
```
integrate((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas
")
```

output
```
integral((c*e*x^6 + (c*d + b*e)*x^4 + (b*d + a*e)*x^2 + a*d)*sqrt(c*x^4 +
b*x^2 + a)*sqrt(f*x), x)
```

3.209.6 Sympy [F]

$$\int \sqrt{fx}(d+ex^2)\left(a+bx^2+cx^4\right)^{3/2}\,dx = \int \sqrt{fx}\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((f*x)**(1/2)*(e*x**2+d)*(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral(sqrt(f*x)*(d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2), x)
```

3.209.7 Maxima [F]

$$\int \sqrt{fx}(d+ex^2)\left(a+bx^2+cx^4\right)^{3/2}\,dx = \int \left(cx^4+bx^2+a\right)^{\frac{3}{2}}\left(ex^2+d\right)\sqrt{fx}\,dx$$

input
```
integrate((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima
")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*sqrt(f*x), x)
```

3.209.8 Giac [F]

$$\int \sqrt{fx}(d+ex^2)\left(a+bx^2+cx^4\right)^{3/2}\,dx=\int \left(cx^4+bx^2+a\right)^{\frac{3}{2}}(ex^2+d)\sqrt{fx}\,dx$$

input
```
integrate((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*sqrt(f*x), x)
```

3.209.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{fx}(d+ex^2)\left(a+bx^2+cx^4\right)^{3/2}\,dx=\int \sqrt{f\,x}\,\left(e\,x^2+d\right)\,\left(c\,x^4+b\,x^2+a\right)^{3/2}\,dx$$

input
```
int((f*x)^(1/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int((f*x)^(1/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2), x)
```

3.210 $\int \frac{(d+ex^2)\left(a+bx^2+cx^4\right)^{3/2}}{\sqrt{fx}}\,dx$

3.210.1 Optimal result

Integrand size = 31, antiderivative size = 297

$$\int \frac{(d+ex^2)\left(a+bx^2+cx^4\right)^{3/2}}{\sqrt{fx}}\,dx = \frac{2ad\sqrt{fx}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{1}{4},-\frac{3}{2},-\frac{3}{2},\frac{5}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$
$$+\frac{2ae(fx)^{5/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{5}{4},-\frac{3}{2},-\frac{3}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f^3\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

output
```
2/5*a*e*(f*x)^(5/2)*AppellF1(5/4,-3/2,-3/2,9/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1
/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f^3/(1+2*c*x^2
/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)+2*
a*d*AppellF1(1/4,-3/2,-3/2,5/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b
+(-4*a*c+b^2)^(1/2)))*(f*x)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/f/(1+2*c*x^2/(b-(-
4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.210.2 Mathematica [A] (verified)

Time = 11.91 (sec) , antiderivative size = 487, normalized size of antiderivative = 1.64

$$\int \frac{(d+ex^2)(a+bx^2+cx^4)^{3/2}}{\sqrt{fx}}\,dx = \frac{2x\Big(5(a+bx^2+cx^4)\left(-28b^3e+4b^2c(17d+5ex^2)+c^2(867ad+455aex\right.}{}$$

input
```
Integrate[((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2))/Sqrt[f*x],x]
```

output
```
(2*x*(5*(a + b*x^2 + c*x^4)*(-28*b^3*e + 4*b^2*c*(17*d + 5*e*x^2) + c^2*(8
67*a*d + 455*a*e*x^2 + 255*c*d*x^4 + 195*c*e*x^6) + b*c*(176*a*e + 5*c*x^2
*(85*d + 57*e*x^2))) + 20*a*(-17*b^2*c*d + 612*a*c^2*d + 7*b^3*e - 44*a*b*
c*e)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1
/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2
 - 4*a*c])] + 4*(-51*b^3*c*d + 476*a*b*c^2*d + 21*b^4*e - 157*a*b^2*c*e +
260*a^2*c^2*e)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 -
4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*A
ppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/
(-b + Sqrt[b^2 - 4*a*c])]))/(16575*c^2*Sqrt[f*x]*Sqrt[a + b*x^2 + c*x^4])
```

3.210.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1674, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(a+bx^2+cx^4)^{3/2}}{\sqrt{fx}}dx$$

$$\Big\downarrow \text{1674}$$

$$\int \left(\frac{d(a+bx^2+cx^4)^{3/2}}{\sqrt{fx}}+\frac{e(fx)^{3/2}(a+bx^2+cx^4)^{3/2}}{f^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2ad\sqrt{fx}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{1}{4},-\frac{3}{2},-\frac{3}{2},\frac{5}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2ae(fx)^{5/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{5}{4},-\frac{3}{2},-\frac{3}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2))/Sqrt[f*x],x]
```

output
```
(2*a*d*Sqrt[f*x]*Sqrt[a + b*x^2 + c*x^4]*AppellF1[1/4, -3/2, -3/2, 5/4, (-
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f*
Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c])]) + (2*a*e*(f*x)^(5/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[5/4,
-3/2, -3/2, 9/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c])])/(5*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1
+ (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

3.210.3.1 Defintions of rubi rules used

rule 1674
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.210.4 Maple [F]

$$\int \frac{(e\,x^2+d)\,(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{\sqrt{fx}}dx$$

input
```
int((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(1/2),x)
```

output
```
int((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(1/2),x)
```

3.210.5 Fricas [F]

$$\int \frac{(d+ex^2)\,(a+bx^2+cx^4)^{3/2}}{\sqrt{fx}}\,dx=\int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}(ex^2+d)}{\sqrt{fx}}\,dx$$

input
```
integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(1/2),x, algorithm="fricas
")
```

output
```
integral((c*e*x^6 + (c*d + b*e)*x^4 + (b*d + a*e)*x^2 + a*d)*sqrt(c*x^4 +
b*x^2 + a)*sqrt(f*x)/(f*x), x)
```

3.210.6 Sympy [F]

$$\int \frac{(d+ex^2)\,(a+bx^2+cx^4)^{3/2}}{\sqrt{fx}}\,dx=\int \frac{(d+ex^2)\,(a+bx^2+cx^4)^{\frac{3}{2}}}{\sqrt{fx}}\,dx$$

input
```
integrate((e*x**2+d)*(c*x**4+b*x**2+a)**(3/2)/(f*x)**(1/2),x)
```

output
```
Integral((d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)/sqrt(f*x), x)
```

3.210.7 Maxima [F]

$$\int \frac{(d+ex^2)\,(a+bx^2+cx^4)^{3/2}}{\sqrt{fx}}\,dx=\int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}(ex^2+d)}{\sqrt{fx}}\,dx$$

input
```
integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)/sqrt(f*x), x)
```

3.210.8 Giac [F]

$$\int \frac{(d+ex^2)(a+bx^2+cx^4)^{3/2}}{\sqrt{fx}}\,dx = \int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}(ex^2+d)}{\sqrt{fx}}\,dx$$

input
```
integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)/sqrt(f*x), x)
```

3.210.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)(a+bx^2+cx^4)^{3/2}}{\sqrt{fx}}\,dx = \int \frac{(e\,x^2+d)\,(c\,x^4+b\,x^2+a)^{3/2}}{\sqrt{f\,x}}\,dx$$

input
```
int(((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(f*x)^(1/2),x)
```

output
```
int(((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(f*x)^(1/2), x)
```

3.211 $\int \frac{(d+ex^2)\left(a+bx^2+cx^4\right)^{3/2}}{(fx)^{3/2}}\,dx$

3.211.1 Optimal result

Integrand size = 31, antiderivative size = 297

$$\int \frac{(d+ex^2)\left(a+bx^2+cx^4\right)^{3/2}}{(fx)^{3/2}}\,dx = -\frac{2ad\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(-\frac{1}{4},-\frac{3}{2},-\frac{3}{2},\frac{3}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{fx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{2ae(fx)^{3/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{3}{4},-\frac{3}{2},-\frac{3}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f^3\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

output
```
2/3*a*e*(f*x)^(3/2)*AppellF1(3/4,-3/2,-3/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1
/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f^3/(1+2*c*x^2
/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)-2*
a*d*AppellF1(-1/4,-3/2,-3/2,3/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(
b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f/(f*x)^(1/2)/(1+2*c*x^2/(b-(
-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.211.2 Mathematica [A] (verified)

Time = 11.75 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.51

$$\int \frac{(d+ex^2)(a+bx^2+cx^4)^{3/2}}{(fx)^{3/2}}\,dx = \frac{x\Big(14(a+bx^2+cx^4)\left(ac(-1155d+209ex^2)+x^2(12b^2e+7c^2x^2(15d+\right.}{}$$

input
```
Integrate[((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(f*x)^(3/2),x]
```

output
```
(x*(14*(a + b*x^2 + c*x^4)*(a*c*(-1155*d + 209*e*x^2) + x^2*(12*b^2*e + 7*
c^2*x^2*(15*d + 11*e*x^2) + b*c*(195*d + 119*e*x^2))) - 56*a*(-240*b*c*d +
 3*b^2*e - 44*a*c*e)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[
b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*
c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c
*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 24*(15*b^2*c*d + 420*a*c^2*d - 5*b^3*e +
 36*a*b*c*e)*x^4*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*
a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*App
ellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(
-b + Sqrt[b^2 - 4*a*c])]))/(8085*c*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4])
```

3.211.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1674, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(a+bx^2+cx^4)^{3/2}}{(fx)^{3/2}}\,dx$$

$$\Big\downarrow \ 1674$$

$$\int \left(\frac{d(a+bx^2+cx^4)^{3/2}}{(fx)^{3/2}}+\frac{e\sqrt{fx}(a+bx^2+cx^4)^{3/2}}{f^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2ae(fx)^{3/2}\sqrt{a+bx^2+cx^4}\operatorname{AppellF1}\left(\frac{3}{4},-\frac{3}{2},-\frac{3}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}-\frac{2ad\sqrt{a+bx^2+cx^4}\operatorname{AppellF1}\left(-\frac{1}{4},-\frac{3}{2},-\frac{3}{2},\frac{3}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{fx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(f*x)^(3/2),x]
```

output
```
(-2*a*d*Sqrt[a + b*x^2 + c*x^4]*AppellF1[-1/4, -3/2, -3/2, 3/4, (-2*c*x^2)
/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f*Sqrt[f*x
]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt
[b^2 - 4*a*c])]) + (2*a*e*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[3/4
, -3/2, -3/2, 7/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])])/(3*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

3.211.3.1 Defintions of rubi rules used

rule 1674
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.211.4 Maple [F]

$$\int \frac{(e\,x^2+d)\,(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{(fx)^{\frac{3}{2}}}dx$$

input
```
int((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(3/2),x)
```

output
```
int((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(3/2),x)
```

3.211.5 Fricas [F]

$$\int \frac{(d+ex^2)(a+bx^2+cx^4)^{3/2}}{(fx)^{3/2}}\,dx = \int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}(ex^2+d)}{(fx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(3/2),x, algorithm="fricas
")
```

output
```
integral((c*e*x^6 + (c*d + b*e)*x^4 + (b*d + a*e)*x^2 + a*d)*sqrt(c*x^4 +
b*x^2 + a)*sqrt(f*x)/(f^2*x^2), x)
```

3.211.6 Sympy [F]

$$\int \frac{(d+ex^2)(a+bx^2+cx^4)^{3/2}}{(fx)^{3/2}}\,dx = \int \frac{(d+ex^2)(a+bx^2+cx^4)^{\frac{3}{2}}}{(fx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x**2+d)*(c*x**4+b*x**2+a)**(3/2)/(f*x)**(3/2),x)
```

output
```
Integral((d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)/(f*x)**(3/2), x)
```

3.211.7 Maxima [F]

$$\int \frac{(d+ex^2)(a+bx^2+cx^4)^{3/2}}{(fx)^{3/2}}\,dx = \int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}(ex^2+d)}{(fx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(3/2),x, algorithm="maxima
")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)/(f*x)^(3/2), x)
```

3.211.8 Giac [F]

$$\int \frac{(d+ex^2)\,(a+bx^2+cx^4)^{3/2}}{(fx)^{3/2}}\,dx = \int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}(ex^2+d)}{(fx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)/(f*x)^(3/2), x)
```

3.211.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)\,(a+bx^2+cx^4)^{3/2}}{(fx)^{3/2}}\,dx = \int \frac{(e\,x^2+d)\,\left(c\,x^4+b\,x^2+a\right)^{3/2}}{(f\,x)^{3/2}}\,dx$$

input
```
int(((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(f*x)^(3/2),x)
```

output
```
int(((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(f*x)^(3/2), x)
```

3.212 $\int \frac{(fx)^{3/2}(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

3.212.1 Optimal result

Integrand size = 31, antiderivative size = 297

$$\int \frac{(fx)^{3/2}(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{2d(fx)^{5/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2c}{b+\sqrt{b}}\right.}{5f\sqrt{a+bx^2+cx^4}} + \frac{2e(fx)^{9/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{9}{4},\frac{1}{2},\frac{1}{2},\frac{13}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{9f^3\sqrt{a+bx^2+cx^4}}$$

output
```
2/5*d*(f*x)^(5/2)*AppellF1(5/4,1/2,1/2,9/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2))
,-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)
*(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/f/(c*x^4+b*x^2+a)^(1/2)+2/9*e*(f
*x)^(9/2)*AppellF1(9/4,1/2,1/2,13/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x
^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c
*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/f^3/(c*x^4+b*x^2+a)^(1/2)
```

3.212.2 Mathematica [A] (verified)

Time = 11.50 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.19

$$\int \frac{(fx)^{3/2}(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{2f\sqrt{fx}\left(5e(a+bx^2+cx^4)-5ae\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2}\right.\right.}{}$$

input
```
Integrate[((f*x)^(3/2)*(d + e*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(2*f*Sqrt[f*x]*(5*e*(a + b*x^2 + c*x^4) - 5*a*e*Sqrt[(b - Sqrt[b^2 - 4*a*c
] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + S
qrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (5*c*d - 3*b*e)*x
^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2
, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 -
 4*a*c])]))/(25*c*Sqrt[a + b*x^2 + c*x^4])
```

3.212.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1674, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^{3/2}\left(d+ex^2\right)}{\sqrt{a+bx^2+cx^4}}dx$$

$$\Big\downarrow \ 1674$$

$$\int \left(\frac{d(fx)^{3/2}}{\sqrt{a+bx^2+cx^4}}+\frac{e(fx)^{7/2}}{f^2\sqrt{a+bx^2+cx^4}}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2d(fx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{9/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{9}{4},\frac{1}{2},\frac{1}{2},\frac{13}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{9f^3\sqrt{a+bx^2+cx^4}}$$

input
```
Int[((f*x)^(3/2)*(d + e*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(2*d*(f*x)^(5/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b
- Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*f*Sqrt[a + b
*x^2 + c*x^4]) + (2*e*(f*x)^(9/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c
])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[9/4, 1/2, 1/2, 13
/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])
])/(9*f^3*Sqrt[a + b*x^2 + c*x^4])
```

3.212.3.1 Defintions of rubi rules used

rule 1674
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.212.4 Maple [F]

$$\int \frac{(fx)^{\frac{3}{2}} (e\,x^2+d)}{\sqrt{c\,x^4+b\,x^2+a}}dx$$

input
```
int((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

3.212.5 Fricas [F]

$$\int \frac{(fx)^{3/2} (d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(ex^2+d)(fx)^{\frac{3}{2}}}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas
")
```

output
```
integral((e*f*x^3 + d*f*x)*sqrt(f*x)/sqrt(c*x^4 + b*x^2 + a), x)
```

3.212.6 Sympy [F]

$$\int \frac{(fx)^{3/2}\left(d+ex^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{(fx)^{\frac{3}{2}}\left(d+ex^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((f*x)**(3/2)*(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((f*x)**(3/2)*(d + e*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

3.212.7 Maxima [F]

$$\int \frac{(fx)^{3/2}\left(d+ex^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{(ex^2+d)(fx)^{\frac{3}{2}}}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((e*x^2 + d)*(f*x)^(3/2)/sqrt(c*x^4 + b*x^2 + a), x)
```

3.212.8 Giac [F]

$$\int \frac{(fx)^{3/2}\left(d+ex^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{(ex^2+d)(fx)^{\frac{3}{2}}}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(f*x)^(3/2)/sqrt(c*x^4 + b*x^2 + a), x)
```

3.212.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^{3/2}\left(d+ex^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{{\left(f\,x\right)}^{3/2}\,\left(e\,x^2+d\right)}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int(((f*x)^(3/2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int(((f*x)^(3/2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.213 $\int \frac{\sqrt{fx}(d+ex^2)}{\sqrt{a+bx^2+cx^4}} dx$

3.213.1 Optimal result

Integrand size = 31, antiderivative size = 297

$$\int \frac{\sqrt{fx}(d+ex^2)}{\sqrt{a+bx^2+cx^4}} dx = \frac{2d(fx)^{3/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f\sqrt{a+bx^2+cx^4}} + \frac{2e(fx)^{7/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{7f^3\sqrt{a+bx^2+cx^4}}$$

output
```
2/3*d*(f*x)^(3/2)*AppellF1(3/4,1/2,1/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2))
,-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)
*(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/f/(c*x^4+b*x^2+a)^(1/2)+2/7*e*(f
*x)^(7/2)*AppellF1(7/4,1/2,1/2,11/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x
^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c
*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/f^3/(c*x^4+b*x^2+a)^(1/2)
```

3.213.2 Mathematica [A] (verified)

Time = 11.23 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{fx}(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{2\sqrt{fx}\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\left(7dx\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+3ex^3\,\text{AppellF1}\right.}{21\sqrt{a+bx^2+cx^4}}$$

input
```
Integrate[(Sqrt[f*x]*(d + e*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(2*Sqrt[f*x]*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]
)]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*(7*d*x*
AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)
/(-b + Sqrt[b^2 - 4*a*c])] + 3*e*x^3*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(21*Sqr
t[a + b*x^2 + c*x^4])
```

3.213.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1674, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{fx}(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$\Big\downarrow \; 1674$$

$$\int \left(\frac{d\sqrt{fx}}{\sqrt{a+bx^2+cx^4}}+\frac{e(fx)^{5/2}}{f^2\sqrt{a+bx^2+cx^4}}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{2d(fx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{7/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{7f^3\sqrt{a+bx^2+cx^4}}$$

input
```
Int[(Sqrt[f*x]*(d + e*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(2*d*(f*x)^(3/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b
- Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*f*Sqrt[a + b
*x^2 + c*x^4]) + (2*e*(f*x)^(7/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c
])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11
/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])
])/(7*f^3*Sqrt[a + b*x^2 + c*x^4])
```

3.213.3.1 Defintions of rubi rules used

rule 1674
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.213.4 Maple [F]

$$\int \frac{\sqrt{fx}\,(e\,x^2+d)}{\sqrt{c\,x^4+b\,x^2+a}}dx$$

input
```
int((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

3.213.5 Fricas [F]

$$\int \frac{\sqrt{fx}(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(ex^2+d)\sqrt{fx}}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas
")
```

output
```
integral((e*x^2 + d)*sqrt(f*x)/sqrt(c*x^4 + b*x^2 + a), x)
```

3.213.6 Sympy [F]

$$\int \frac{\sqrt{fx}(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{\sqrt{fx}(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((f*x)**(1/2)*(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(f*x)*(d + e*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

3.213.7 Maxima [F]

$$\int \frac{\sqrt{fx}(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(ex^2+d)\sqrt{fx}}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((e*x^2 + d)*sqrt(f*x)/sqrt(c*x^4 + b*x^2 + a), x)
```

3.213.8 Giac [F]

$$\int \frac{\sqrt{fx}(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(ex^2+d)\sqrt{fx}}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*sqrt(f*x)/sqrt(c*x^4 + b*x^2 + a), x)
```

3.213.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{fx}(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{\sqrt{f\,x}\,(e\,x^2+d)}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int(((f*x)^(1/2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int(((f*x)^(1/2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.214 $\int \frac{d+ex^2}{\sqrt{fx}\sqrt{a+bx^2+cx^4}}\,dx$

3.214.1 Optimal result

Integrand size = 31, antiderivative size = 295

$$\int \frac{d+ex^2}{\sqrt{fx}\sqrt{a+bx^2+cx^4}}\,dx = \frac{2d\sqrt{fx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{a+bx^2+cx^4}} + \frac{2e(fx)^{5/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f^3\sqrt{a+bx^2+cx^4}}$$

output
```
2/5*e*(f*x)^(5/2)*AppellF1(5/4,1/2,1/2,9/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2))
,-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)
*(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/f^3/(c*x^4+b*x^2+a)^(1/2)+2*d*Ap
pellF1(1/4,1/2,1/2,5/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c
+b^2)^(1/2)))*(f*x)^(1/2)*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*
x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/f/(c*x^4+b*x^2+a)^(1/2)
```

3.214.2 Mathematica [A] (verified)

Time = 11.22 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.82

$$\int \frac{d+ex^2}{\sqrt{fx}\sqrt{a+bx^2+cx^4}}\,dx = \frac{2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\left(5dx\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+ex^3\,\text{AppellF1}\left(\frac{5}{4},\right.\right.}{5\sqrt{fx}\sqrt{a+bx^2+cx^4}}$$

input
```
Integrate[(d + e*x^2)/(Sqrt[f*x]*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
(2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*(5*d*x*AppellF1[1
/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqr
t[b^2 - 4*a*c])] + e*x^3*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt
[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(5*Sqrt[f*x]*Sqrt[a
+ b*x^2 + c*x^4])
```

3.214.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1674, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{\sqrt{fx}\sqrt{a+bx^2+cx^4}}\,dx$$

$$\Big\downarrow \text{1674}$$

$$\int \left(\frac{d}{\sqrt{fx}\sqrt{a+bx^2+cx^4}}+\frac{e(fx)^{3/2}}{f^2\sqrt{a+bx^2+cx^4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2d\sqrt{fx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f^3\sqrt{a+bx^2+cx^4}}$$

input
```
Int[(d + e*x^2)/(Sqrt[f*x]*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
(2*d*Sqrt[f*x]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f*Sqrt[a + b*x^2
 + c*x^4]) + (2*e*(f*x)^(5/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4, (
-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5
*f^3*Sqrt[a + b*x^2 + c*x^4])
```

3.214.3.1 Defintions of rubi rules used

rule 1674
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.214.4 Maple [F]

$$\int \frac{e\,x^{2}+d}{\sqrt{f x}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}dx$$

input
```
int((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

3.214.5 Fricas [F]

$$\int \frac{d+ex^2}{\sqrt{fx}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{ex^2+d}{\sqrt{cx^4+bx^2+a}\sqrt{fx}}\,dx$$

input
```
integrate((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas
")
```

output
```
integral(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x)/(c*f*x^5 + b*f*x^3
+ a*f*x), x)
```

3.214.6 Sympy [F]

$$\int \frac{d+ex^2}{\sqrt{fx}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{d+ex^2}{\sqrt{fx}\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((e*x**2+d)/(f*x)**(1/2)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((d + e*x**2)/(sqrt(f*x)*sqrt(a + b*x**2 + c*x**4)), x)
```

3.214.7 Maxima [F]

$$\int \frac{d+ex^2}{\sqrt{fx}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{ex^2+d}{\sqrt{cx^4+bx^2+a}\sqrt{fx}}\,dx$$

input
```
integrate((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((e*x^2 + d)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(f*x)), x)
```

3.214.8 Giac [F]

$$\int \frac{d+ex^2}{\sqrt{fx}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{ex^2+d}{\sqrt{cx^4+bx^2+a}\sqrt{fx}}\,dx$$

input
```
integrate((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(f*x)), x)
```

3.214.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{\sqrt{fx}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{e\,x^2+d}{\sqrt{f\,x}\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((d + e*x^2)/((f*x)^(1/2)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int((d + e*x^2)/((f*x)^(1/2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.215 $\int \frac{d+ex^2}{(fx)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx$

3.215.1 Optimal result

Integrand size = 31, antiderivative size = 295

$$\int \frac{d+ex^2}{(fx)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx = -\frac{2d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(-\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{3}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{fx}\sqrt{a+bx^2+cx^4}} + \frac{2e(fx)^{3/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f^3\sqrt{a+bx^2+cx^4}}$$

output
```
2/3*e*(f*x)^(3/2)*AppellF1(3/4,1/2,1/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2))
,-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)
*(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/f^3/(c*x^4+b*x^2+a)^(1/2)-2*d*Ap
pellF1(-1/4,1/2,1/2,3/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*
c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^2/(b+(-4*
a*c+b^2)^(1/2)))^(1/2)/f/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

3.215.2 Mathematica [A] (verified)

Time = 11.54 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.21

$$\int \frac{d+ex^2}{(fx)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx = \frac{2x\left(-21d(a+bx^2+cx^4)+7(bd+ae)x^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,\mathrm{A}\right.}{}$$

input
```
Integrate[(d + e*x^2)/((f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
(2*x*(-21*d*(a + b*x^2 + c*x^4) + 7*(b*d + a*e)*x^2*Sqrt[(b - Sqrt[b^2 - 4
*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 9*c*d*x^4*Sqr
t[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqr
t[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2,
 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*
c])]))/(21*a*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4])
```

3.215.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1674, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{(fx)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx$$

$$\Big\downarrow \text{1674}$$

$$\int \left(\frac{d}{(fx)^{3/2}\sqrt{a+bx^2+cx^4}}+\frac{e\sqrt{fx}}{f^2\sqrt{a+bx^2+cx^4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2e(fx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\mathrm{AppellF1}\left(\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f^3\sqrt{a+bx^2+cx^4}}-\frac{2d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\mathrm{AppellF1}\left(-\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{3}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{fx}\sqrt{a+bx^2+cx^4}}$$

input
```
Int[(d + e*x^2)/((f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
(-2*d*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c])]*AppellF1[-1/4, 1/2, 1/2, 3/4, (-2*c*x^2)/(b - Sqrt[b^2
 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f*Sqrt[f*x]*Sqrt[a + b*x
^2 + c*x^4]) + (2*e*(f*x)^(3/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])
]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4,
 (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/
(3*f^3*Sqrt[a + b*x^2 + c*x^4])
```

3.215.3.1 Defintions of rubi rules used

rule 1674
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.215.4 Maple [F]

$$\int \frac{e\,x^{2}+d}{\left(f x \right)^{\frac{3}{2}} \sqrt{c\,x^{4}+b\,x^{2}+a}}dx$$

input
```
int((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

3.215.5 Fricas [F]

$$\int \frac{d+ex^2}{(fx)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{ex^2+d}{\sqrt{cx^4+bx^2+a}\,(fx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas
")
```

output
```
integral(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x)/(c*f^2*x^6 + b*f^2*
x^4 + a*f^2*x^2), x)
```

3.215.6 Sympy [F]

$$\int \frac{d+ex^2}{(fx)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{d+ex^2}{(fx)^{\frac{3}{2}}\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((e*x**2+d)/(f*x)**(3/2)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((d + e*x**2)/((f*x)**(3/2)*sqrt(a + b*x**2 + c*x**4)), x)
```

3.215.7 Maxima [F]

$$\int \frac{d+ex^2}{(fx)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{ex^2+d}{\sqrt{cx^4+bx^2+a}\,(fx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((e*x^2 + d)/(sqrt(c*x^4 + b*x^2 + a)*(f*x)^(3/2)), x)
```

3.215.8 Giac [F]

$$\int \frac{d+ex^2}{(fx)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{ex^2+d}{\sqrt{cx^4+bx^2+a}\,(fx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)/(sqrt(c*x^4 + b*x^2 + a)*(f*x)^(3/2)), x)
```

3.215.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{(fx)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{e\,x^2+d}{(f\,x)^{3/2}\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((d + e*x^2)/((f*x)^(3/2)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int((d + e*x^2)/((f*x)^(3/2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.216 $\int \frac{(fx)^{3/2}(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx$

3.216.1 Optimal result

Integrand size = 31, antiderivative size = 303

$$\int \frac{(fx)^{3/2}(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{2d(fx)^{5/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{5}{4},\frac{3}{2},\frac{3}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{}{b+\sqrt{}}\right.}{5af\sqrt{a+bx^2+cx^4}} + \frac{2e(fx)^{9/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{9}{4},\frac{3}{2},\frac{3}{2},\frac{13}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{9af^3\sqrt{a+bx^2+cx^4}}$$

output
```
2/5*d*(f*x)^(5/2)*AppellF1(5/4,3/2,3/2,9/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2))
,-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)
*(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/f/(c*x^4+b*x^2+a)^(1/2)+2/9*e*
(f*x)^(9/2)*AppellF1(9/4,3/2,3/2,13/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c
*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2
*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/f^3/(c*x^4+b*x^2+a)^(1/2)
```

3.216.2 Mathematica [A] (verified)

Time = 11.60 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.24

$$\int \frac{(fx)^{3/2}(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = -\frac{f\sqrt{fx}\left(5(bd-2ae+2cdx^2-bex^2)-5(bd-2ae)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\right.\right.}{5(b^2-}$$

input
```
Integrate[((f*x)^(3/2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
-1/5*(f*Sqrt[f*x]*(5*(b*d - 2*a*e + 2*c*d*x^2 - b*e*x^2) - 5*(b*d - 2*a*e)
*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b +
 Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2,
1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4
*a*c])] + (-2*c*d + b*e)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - S
qrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 -
4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]),
(2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^
4])
```

3.216.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1674, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^{3/2}\left(d+ex^2\right)}{\left(a+bx^2+cx^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{1674}$$

$$\int \left(\frac{d(fx)^{3/2}}{\left(a+bx^2+cx^4\right)^{3/2}}+\frac{e(fx)^{7/2}}{f^2\left(a+bx^2+cx^4\right)^{3/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2d(fx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{5}{4},\frac{3}{2},\frac{3}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5af\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{9/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{9}{4},\frac{3}{2},\frac{3}{2},\frac{13}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{9af^3\sqrt{a+bx^2+cx^4}}$$

input
```
Int[((f*x)^(3/2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(2*d*(f*x)^(5/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 3/2, 3/2, 9/4, (-2*c*x^2)/(b
- Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*a*f*Sqrt[a +
 b*x^2 + c*x^4]) + (2*e*(f*x)^(9/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a
*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[9/4, 3/2, 3/2,
13/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c
])])/(9*a*f^3*Sqrt[a + b*x^2 + c*x^4])
```

3.216.3.1 Defintions of rubi rules used

rule 1674
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.216.4 Maple [F]

$$\int \frac{(fx)^{\frac{3}{2}} (e\,x^2+d)}{(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
int((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

3.216.5 Fricas [F]

$$\int \frac{(fx)^{3/2} (d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(ex^2+d)(fx)^{\frac{3}{2}}}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas
")
```

output
```
integral((e*f*x^3 + d*f*x)*sqrt(c*x^4 + b*x^2 + a)*sqrt(f*x)/(c^2*x^8 + 2*
b*c*x^6 + (b^2 + 2*a*c)*x^4 + 2*a*b*x^2 + a^2), x)
```

3.216.6 Sympy [F]

$$\int \frac{(fx)^{3/2}(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(fx)^{\frac{3}{2}}(d+ex^2)}{(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x)**(3/2)*(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral((f*x)**(3/2)*(d + e*x**2)/(a + b*x**2 + c*x**4)**(3/2), x)
```

3.216.7 Maxima [F]

$$\int \frac{(fx)^{3/2}(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(ex^2+d)(fx)^{\frac{3}{2}}}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima
")
```

output
```
integrate((e*x^2 + d)*(f*x)^(3/2)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

3.216.8 Giac [F]

$$\int \frac{(fx)^{3/2}(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(ex^2+d)(fx)^{\frac{3}{2}}}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(f*x)^(3/2)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

3.216.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^{3/2}(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(f\,x)^{3/2}\,(e\,x^2+d)}{(c\,x^4+b\,x^2+a)^{3/2}}\,dx$$

input
```
int(((f*x)^(3/2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(((f*x)^(3/2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2), x)
```

3.217 $\int \frac{\sqrt{fx}(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx$

3.217.1 Optimal result

Integrand size = 31, antiderivative size = 303

$$\int \frac{\sqrt{fx}(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{2d(fx)^{3/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{3}{4},\frac{3}{2},\frac{3}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3af\sqrt{a+bx^2+cx^4}} + \frac{2e(fx)^{7/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{7}{4},\frac{3}{2},\frac{3}{2},\frac{11}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{7af^3\sqrt{a+bx^2+cx^4}}$$

output
```
2/3*d*(f*x)^(3/2)*AppellF1(3/4,3/2,3/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2))
,-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)
*(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/f/(c*x^4+b*x^2+a)^(1/2)+2/7*e*
(f*x)^(7/2)*AppellF1(7/4,3/2,3/2,11/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c
*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2
*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/f^3/(c*x^4+b*x^2+a)^(1/2)
```

3.217.2 Mathematica [A] (verified)

Time = 11.60 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{fx}(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{x\sqrt{fx}\Big(-21b^2d+21b(ae-cdx^2)+42ac(d+ex^2)+7(b^2d+2acd-3abe)\sqrt{\frac{b-\sqrt{\cdots}}{b\cdots}}\cdots}{\cdots}$$

input
```
Integrate[(Sqrt[f*x]*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(x*Sqrt[f*x]*(-21*b^2*d + 21*b*(a*e - c*d*x^2) + 42*a*c*(d + e*x^2) + 7*(b
^2*d + 2*a*c*d - 3*a*b*e)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt
[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a
*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*
c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 9*c*(b*d - 2*a*e)*x^2*Sqrt[(b - Sqrt[b^
2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(21*a*(
-b^2 + 4*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

3.217.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1674, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{fx}\left(d+ex^2\right)}{\left(a+bx^2+cx^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{1674}$$

$$\int \left(\frac{d\sqrt{fx}}{\left(a+bx^2+cx^4\right)^{3/2}}+\frac{e(fx)^{5/2}}{f^2\left(a+bx^2+cx^4\right)^{3/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2d(fx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{3}{4},\frac{3}{2},\frac{3}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3af\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{7/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{7}{4},\frac{3}{2},\frac{3}{2},\frac{11}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{7af^3\sqrt{a+bx^2+cx^4}}$$

input
```
Int[(Sqrt[f*x]*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(2*d*(f*x)^(3/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 3/2, 3/2, 7/4, (-2*c*x^2)/(b
- Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*a*f*Sqrt[a +
 b*x^2 + c*x^4]) + (2*e*(f*x)^(7/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a
*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 3/2, 3/2,
11/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c
])])/(7*a*f^3*Sqrt[a + b*x^2 + c*x^4])
```

3.217.3.1 Defintions of rubi rules used

rule 1674
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.217.4 Maple [F]

$$\int \frac{\sqrt{f x}\,(e\,x^{2}+d)}{(c\,x^{4}+b\,x^{2}+a)^{\frac{3}{2}}}dx$$

input
```
int((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
int((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

3.217.5 Fricas [F]

$$\int \frac{\sqrt{fx}(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(ex^2+d)\sqrt{fx}}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas
")
```

output
```
integral(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x)/(c^2*x^8 + 2*b*c*x^
6 + (b^2 + 2*a*c)*x^4 + 2*a*b*x^2 + a^2), x)
```

3.217.6 Sympy [F]

$$\int \frac{\sqrt{fx}(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{\sqrt{fx}\,(d+ex^2)}{(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x)**(1/2)*(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral(sqrt(f*x)*(d + e*x**2)/(a + b*x**2 + c*x**4)**(3/2), x)
```

3.217.7 Maxima [F]

$$\int \frac{\sqrt{fx}(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(ex^2+d)\sqrt{fx}}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima
")
```

output
```
integrate((e*x^2 + d)*sqrt(f*x)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

3.217.8 Giac [F]

$$\int \frac{\sqrt{fx}(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(ex^2+d)\sqrt{fx}}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*sqrt(f*x)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

3.217.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{fx}(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{\sqrt{f\,x}\,(e\,x^2+d)}{(c\,x^4+b\,x^2+a)^{3/2}}\,dx$$

input
```
int(((f*x)^(1/2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(((f*x)^(1/2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2), x)
```

3.218 $\int \frac{d+ex^2}{\sqrt{fx}\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

3.218.1 Optimal result

Integrand size = 31, antiderivative size = 301

$$\int \frac{d+ex^2}{\sqrt{fx}\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{2d\sqrt{fx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1}{4},\frac{3}{2},\frac{3}{2},\frac{5}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\right.}{af\sqrt{a+bx^2+cx^4}}$$
$$+\frac{2e(fx)^{5/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{5}{4},\frac{3}{2},\frac{3}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5af^3\sqrt{a+bx^2+cx^4}}$$

output
```
2/5*e*(f*x)^(5/2)*AppellF1(5/4,3/2,3/2,9/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2))
,-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)
*(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/f^3/(c*x^4+b*x^2+a)^(1/2)+2*d*
AppellF1(1/4,3/2,3/2,5/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a
*c+b^2)^(1/2)))*(f*x)^(1/2)*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*
c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/f/(c*x^4+b*x^2+a)^(1/2)
```

3.218.2 Mathematica [A] (verified)

Time = 11.57 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.31

$$\int \frac{d+ex^2}{\sqrt{fx}\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{x\left(-5b^2d+5b(ae-cdx^2)+10ac(d+ex^2)-5(b^2d-6acd+abe)\sqrt{\frac{b-\sqrt{b^2-\cdots}}{b-\sqrt{\cdots}}}\right.}{}$$

input
```
Integrate[(d + e*x^2)/(Sqrt[f*x]*(a + b*x^2 + c*x^4)^(3/2)),x]
```

output
```
(x*(-5*b^2*d + 5*b*(a*e - c*d*x^2) + 10*a*c*(d + e*x^2) - 5*(b^2*d - 6*a*c
*d + a*b*e)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])
]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1
[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + S
qrt[b^2 - 4*a*c])] + c*(b*d - 2*a*e)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c
*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(5*a*(-b^2 + 4*a*c)*Sqrt
[f*x]*Sqrt[a + b*x^2 + c*x^4])
```

3.218.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1674, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{\sqrt{fx}\left(a+bx^2+cx^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{1674}$$

$$\int \left(\frac{d}{\sqrt{fx}\left(a+bx^2+cx^4\right)^{3/2}}+\frac{e(fx)^{3/2}}{f^2\left(a+bx^2+cx^4\right)^{3/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2d\sqrt{fx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\operatorname{AppellF1}\left(\frac{1}{4},\frac{3}{2},\frac{3}{2},\frac{5}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{af\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\operatorname{AppellF1}\left(\frac{5}{4},\frac{3}{2},\frac{3}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5af^3\sqrt{a+bx^2+cx^4}}$$

input
```
Int[(d + e*x^2)/(Sqrt[f*x]*(a + b*x^2 + c*x^4)^(3/2)),x]
```

output
```
(2*d*Sqrt[f*x]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 3/2, 3/2, 5/4, (-2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(a*f*Sqrt[a + b*x
^2 + c*x^4]) + (2*e*(f*x)^(5/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])
]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 3/2, 3/2, 9/4,
 (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/
(5*a*f^3*Sqrt[a + b*x^2 + c*x^4])
```

3.218.3.1 Defintions of rubi rules used

rule 1674
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.218.4 Maple [F]

$$\int \frac{e\,x^{2}+d}{\sqrt{fx}\,\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}dx$$

input
```
int((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
int((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

3.218.5 Fricas [F]

$$\int \frac{d+ex^2}{\sqrt{fx}\,(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{ex^2+d}{(cx^4+bx^2+a)^{\frac{3}{2}}\sqrt{fx}}\,dx$$

input
```
integrate((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas
")
```

output
```
integral(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x)/(c^2*f*x^9 + 2*b*c*
f*x^7 + (b^2 + 2*a*c)*f*x^5 + 2*a*b*f*x^3 + a^2*f*x), x)
```

3.218.6 Sympy [F]

$$\int \frac{d+ex^2}{\sqrt{fx}\,(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{d+ex^2}{\sqrt{fx}\,(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x**2+d)/(f*x)**(1/2)/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral((d + e*x**2)/(sqrt(f*x)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

3.218.7 Maxima [F]

$$\int \frac{d+ex^2}{\sqrt{fx}\,(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{ex^2+d}{(cx^4+bx^2+a)^{\frac{3}{2}}\sqrt{fx}}\,dx$$

input
```
integrate((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima
")
```

output
```
integrate((e*x^2 + d)/((c*x^4 + b*x^2 + a)^(3/2)*sqrt(f*x)), x)
```

3.218.8 Giac [F]

$$\int \frac{d+ex^2}{\sqrt{fx}\,(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{ex^2+d}{(cx^4+bx^2+a)^{\frac{3}{2}}\sqrt{fx}}\,dx$$

input
```
integrate((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)/((c*x^4 + b*x^2 + a)^(3/2)*sqrt(f*x)), x)
```

3.218.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{\sqrt{fx}\,(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{e\,x^2+d}{\sqrt{f\,x}\,(c\,x^4+b\,x^2+a)^{3/2}}\,dx$$

input
```
int((d + e*x^2)/((f*x)^(1/2)*(a + b*x^2 + c*x^4)^(3/2)),x)
```

output
```
int((d + e*x^2)/((f*x)^(1/2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

3.219 $\int \frac{d+ex^2}{(fx)^{3/2}(a+bx^2+cx^4)^{3/2}}\,dx$

3.219.1 Optimal result

Integrand size = 31, antiderivative size = 301

$$\int \frac{d+ex^2}{(fx)^{3/2}(a+bx^2+cx^4)^{3/2}}\,dx = -\frac{2d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(-\frac{1}{4},\frac{3}{2},\frac{3}{2},\frac{3}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{af\sqrt{fx}\sqrt{a+bx^2+cx^4}} + \frac{2e(fx)^{3/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{3}{4},\frac{3}{2},\frac{3}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3af^3\sqrt{a+bx^2+cx^4}}$$

output
```
2/3*e*(f*x)^(3/2)*AppellF1(3/4,3/2,3/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2))
,-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)
*(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/f^3/(c*x^4+b*x^2+a)^(1/2)-2*d*
AppellF1(-1/4,3/2,3/2,3/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*
a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^2/(b+(-
4*a*c+b^2)^(1/2)))^(1/2)/a/f/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

3.219.2 Mathematica [A] (verified)

Time = 11.74 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.53

$$\int \frac{d+ex^2}{(fx)^{3/2} (a+bx^2+cx^4)^{3/2}} \, dx =$$
$$-\frac{x\Big(-21(-3b^2dx^2(b+cx^2)+a^2c(8d-2ex^2)+a(10c^2dx^4+b^2(-2d+ex^2)+bcx^2(11d+ex^2)))+7(-3b^3d}{}$$

input
```
Integrate[(d + e*x^2)/((f*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

output
```
-1/21*(x*(-21*(-3*b^2*d*x^2*(b + c*x^2) + a^2*c*(8*d - 2*e*x^2) + a*(10*c^
2*d*x^4 + b^2*(-2*d + e*x^2) + b*c*x^2*(11*d + e*x^2))) + 7*(-3*b^3*d + 9*
a*b*c*d + a*b^2*e + 2*a^2*c*e)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/
(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] - 9*c*(3*b^2*d - 10*a*c*d - a*b*
e)*x^4*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqr
t[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4,
 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[
b^2 - 4*a*c])]))/(a^2*(b^2 - 4*a*c)*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4])
```

3.219.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1674, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{(fx)^{3/2} (a+bx^2+cx^4)^{3/2}} \, dx$$
$$\Big\downarrow \text{1674}$$
$$\int \left(\frac{d}{(fx)^{3/2} (a+bx^2+cx^4)^{3/2}}+\frac{e\sqrt{fx}}{f^2 (a+bx^2+cx^4)^{3/2}}\right) dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{2e(fx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{3}{4},\frac{3}{2},\frac{3}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3af^3\sqrt{a+bx^2+cx^4}}-\frac{2d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(-\frac{1}{4},\frac{3}{2},\frac{3}{2},\frac{3}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{af\sqrt{fx}\sqrt{a+bx^2+cx^4}}$$

input
```
Int[(d + e*x^2)/((f*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

output
```
(-2*d*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c])]*AppellF1[-1/4, 3/2, 3/2, 3/4, (-2*c*x^2)/(b - Sqrt[b^2
 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(a*f*Sqrt[f*x]*Sqrt[a + b
*x^2 + c*x^4]) + (2*e*(f*x)^(3/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c
])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 3/2, 3/2, 7/
4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]
)/(3*a*f^3*Sqrt[a + b*x^2 + c*x^4])
```

3.219.3.1 Defintions of rubi rules used

rule 1674
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.219.4 Maple [F]

$$\int \frac{e\,x^2+d}{(fx)^{\frac{3}{2}}\,(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
int((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

3.219.5 Fricas [F]

$$\int \frac{d+ex^2}{(fx)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{ex^2+d}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}(fx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas
")
```

output
```
integral(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x)/(c^2*f^2*x^10 + 2*b
*c*f^2*x^8 + (b^2 + 2*a*c)*f^2*x^6 + 2*a*b*f^2*x^4 + a^2*f^2*x^2), x)
```

3.219.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{(fx)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((e*x**2+d)/(f*x)**(3/2)/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Timed out
```

3.219.7 Maxima [F]

$$\int \frac{d+ex^2}{(fx)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{ex^2+d}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}(fx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima
")
```

output
```
integrate((e*x^2 + d)/((c*x^4 + b*x^2 + a)^(3/2)*(f*x)^(3/2)), x)
```

3.219.8 Giac [F]

$$\int \frac{d+ex^2}{(fx)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{ex^2+d}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}(fx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)/((c*x^4 + b*x^2 + a)^(3/2)*(f*x)^(3/2)), x)
```

3.219.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{(fx)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{e\,x^2+d}{(f\,x)^{3/2}\left(c\,x^4+b\,x^2+a\right)^{3/2}}\,dx$$

input
```
int((d + e*x^2)/((f*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2)),x)
```

output
```
int((d + e*x^2)/((f*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

3.220 $\int (fx)^m (d+ex^2) (a+bx^2+cx^4)^3 \, dx$

3.220.1 Optimal result

Integrand size = 27, antiderivative size = 243

$$\begin{aligned}\int (fx)^m (d+ex^2) (a+bx^2+cx^4)^3 \, dx = {} & \frac{a^3d(fx)^{1+m}}{f(1+m)} + \frac{a^2(3bd+ae)(fx)^{3+m}}{f^3(3+m)} \\ & + \frac{3a(b^2d+acd+abe)(fx)^{5+m}}{f^5(5+m)} \\ & + \frac{(b^3d+6abcd+3ab^2e+3a^2ce)(fx)^{7+m}}{f^7(7+m)} \\ & + \frac{(3b^2cd+3ac^2d+b^3e+6abce)(fx)^{9+m}}{f^9(9+m)} \\ & + \frac{3c(bcd+b^2e+ace)(fx)^{11+m}}{f^{11}(11+m)} \\ & + \frac{c^2(cd+3be)(fx)^{13+m}}{f^{13}(13+m)} + \frac{c^3e(fx)^{15+m}}{f^{15}(15+m)}\end{aligned}$$

output
```
a^3*d*(f*x)^(1+m)/f/(1+m)+a^2*(a*e+3*b*d)*(f*x)^(3+m)/f^3/(3+m)+3*a*(a*b*e
+a*c*d+b^2*d)*(f*x)^(5+m)/f^5/(5+m)+(3*a^2*c*e+3*a*b^2*e+6*a*b*c*d+b^3*d)*
(f*x)^(7+m)/f^7/(7+m)+(6*a*b*c*e+3*a*c^2*d+b^3*e+3*b^2*c*d)*(f*x)^(9+m)/f^
9/(9+m)+3*c*(a*c*e+b^2*e+b*c*d)*(f*x)^(11+m)/f^11/(11+m)+c^2*(3*b*e+c*d)*(
f*x)^(13+m)/f^13/(13+m)+c^3*e*(f*x)^(15+m)/f^15/(15+m)
```

3.220.2 Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.79

$$\int (fx)^m (d+ex^2) (a+bx^2+cx^4)^3 \, dx = x(fx)^m \left(\frac{a^3d}{1+m}+\frac{a^2(3bd+ae)x^2}{3+m}+\frac{3a(b^2d+acd+abe)\,x^4}{5+m}+\frac{(b^3d+6abcd+3ab^2e+3a^2ce)\,x^6}{7+m}+\frac{(3b^2cd+3ac^2d+b^3e+6abce)\,x^8}{9+m}+\frac{3c(bcd+b^2e+ace)\,x^{10}}{11+m}+\frac{c^2(cd+3be)x^{12}}{13+m}+\frac{c^3ex^{14}}{15+m}\right)$$

input
```
Integrate[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

output
```
x*(f*x)^m*((a^3*d)/(1 + m) + (a^2*(3*b*d + a*e)*x^2)/(3 + m) + (3*a*(b^2*d
 + a*c*d + a*b*e)*x^4)/(5 + m) + ((b^3*d + 6*a*b*c*d + 3*a*b^2*e + 3*a^2*c
*e)*x^6)/(7 + m) + ((3*b^2*c*d + 3*a*c^2*d + b^3*e + 6*a*b*c*e)*x^8)/(9 +
m) + (3*c*(b*c*d + b^2*e + a*c*e)*x^10)/(11 + m) + (c^2*(c*d + 3*b*e)*x^12
)/(13 + m) + (c^3*e*x^14)/(15 + m))
```

3.220.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex^2)\,(fx)^m\,(a+bx^2+cx^4)^3\,dx$$

$$\Big\downarrow \text{1584}$$

$$\int \left(a^3d(fx)^m+\frac{(fx)^{m+6}\,(3a^2ce+3ab^2e+6abcd+b^3d)}{f^6}+\frac{a^2(fx)^{m+2}(ae+3bd)}{f^2}+\frac{3c(fx)^{m+10}\,(ace+b^2e+bc}{f^{10}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3d(fx)^{m+1}}{f(m+1)}+\frac{(fx)^{m+7}\left(3a^2ce+3ab^2e+6abcd+b^3d\right)}{f^7(m+7)}+\frac{a^2(fx)^{m+3}(ae+3bd)}{f^3(m+3)}+\frac{3c(fx)^{m+11}\left(ace+b^2e+bcd\right)}{f^{11}(m+11)}+\frac{3a(fx)^{m+5}\left(abe+acd+b^2d\right)}{f^5(m+5)}+\frac{(fx)^{m+9}\left(6abce+3ac^2d+b^3e+3b^2cd\right)}{f^9(m+9)}+\frac{c^2(fx)^{m+13}(3be+cd)}{f^{13}(m+13)}+\frac{c^3e(fx)^{m+15}}{f^{15}(m+15)}$$

input
```
Int[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

output
```
(a^3*d*(f*x)^(1 + m))/(f*(1 + m)) + (a^2*(3*b*d + a*e)*(f*x)^(3 + m))/(f^3
*(3 + m)) + (3*a*(b^2*d + a*c*d + a*b*e)*(f*x)^(5 + m))/(f^5*(5 + m)) + ((
b^3*d + 6*a*b*c*d + 3*a*b^2*e + 3*a^2*c*e)*(f*x)^(7 + m))/(f^7*(7 + m)) +
((3*b^2*c*d + 3*a*c^2*d + b^3*e + 6*a*b*c*e)*(f*x)^(9 + m))/(f^9*(9 + m))
+ (3*c*(b*c*d + b^2*e + a*c*e)*(f*x)^(11 + m))/(f^11*(11 + m)) + (c^2*(c*d
 + 3*b*e)*(f*x)^(13 + m))/(f^13*(13 + m)) + (c^3*e*(f*x)^(15 + m))/(f^15*(
15 + m))
```

3.220.3.1 Defintions of rubi rules used

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.220.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1934 vs. 2(243) = 486.

Time = 0.35 (sec) , antiderivative size = 1935, normalized size of antiderivative = 7.96

method	result	size
gosper	Expression too large to display	1935
risch	Expression too large to display	1935
parallelrisch	Expression too large to display	2737

input
```
int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
x*(c^3*e*m^7*x^14+49*c^3*e*m^6*x^14+3*b*c^2*e*m^7*x^12+c^3*d*m^7*x^12+973*
c^3*e*m^5*x^14+153*b*c^2*e*m^6*x^12+51*c^3*d*m^6*x^12+10045*c^3*e*m^4*x^14
+3*a*c^2*e*m^7*x^10+3*b^2*c*e*m^7*x^10+3*b*c^2*d*m^7*x^10+3135*b*c^2*e*m^5
*x^12+1045*c^3*d*m^5*x^12+57379*c^3*e*m^3*x^14+159*a*c^2*e*m^6*x^10+159*b^
2*c*e*m^6*x^10+159*b*c^2*d*m^6*x^10+33165*b*c^2*e*m^4*x^12+11055*c^3*d*m^4
*x^12+177331*c^3*e*m^2*x^14+6*a*b*c*e*m^7*x^8+3*a*c^2*d*m^7*x^8+3375*a*c^2
*e*m^5*x^10+b^3*e*m^7*x^8+3*b^2*c*d*m^7*x^8+3375*b^2*c*e*m^5*x^10+3375*b*c
^2*d*m^5*x^10+193017*b*c^2*e*m^3*x^12+64339*c^3*d*m^3*x^12+264207*c^3*e*m*
x^14+330*a*b*c*e*m^6*x^8+165*a*c^2*d*m^6*x^8+36795*a*c^2*e*m^4*x^10+55*b^3
*e*m^6*x^8+165*b^2*c*d*m^6*x^8+36795*b^2*c*e*m^4*x^10+36795*b*c^2*d*m^4*x^
10+604827*b*c^2*e*m^2*x^12+201609*c^3*d*m^2*x^12+135135*c^3*e*x^14+3*a^2*c
*e*m^7*x^6+3*a*b^2*e*m^7*x^6+6*a*b*c*d*m^7*x^6+7278*a*b*c*e*m^5*x^8+3639*a
*c^2*d*m^5*x^8+219417*a*c^2*e*m^3*x^10+b^3*d*m^7*x^6+1213*b^3*e*m^5*x^8+36
39*b^2*c*d*m^5*x^8+219417*b^2*c*e*m^3*x^10+219417*b*c^2*d*m^3*x^10+909765*
b*c^2*e*m*x^12+303255*c^3*d*m*x^12+171*a^2*c*e*m^6*x^6+171*a*b^2*e*m^6*x^6
+342*a*b*c*d*m^6*x^6+82338*a*b*c*e*m^4*x^8+41169*a*c^2*d*m^4*x^8+700461*a*
c^2*e*m^2*x^10+57*b^3*d*m^6*x^6+13723*b^3*e*m^4*x^8+41169*b^2*c*d*m^4*x^8+
700461*b^2*c*e*m^2*x^10+700461*b*c^2*d*m^2*x^10+467775*b*c^2*e*x^12+155925
*c^3*d*x^12+3*a^2*b*e*m^7*x^4+3*a^2*c*d*m^7*x^4+3927*a^2*c*e*m^5*x^6+3*a*b
^2*d*m^7*x^4+3927*a*b^2*e*m^5*x^6+7854*a*b*c*d*m^5*x^6+507282*a*b*c*e*m...
```

3.220.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1357 vs. $2(243) = 486$.

Time = 0.26 (sec) , antiderivative size = 1357, normalized size of antiderivative = 5.58

$$\int (fx)^m \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
((c^3*e*m^7 + 49*c^3*e*m^6 + 973*c^3*e*m^5 + 10045*c^3*e*m^4 + 57379*c^3*e
*m^3 + 177331*c^3*e*m^2 + 264207*c^3*e*m + 135135*c^3*e)*x^15 + ((c^3*d +
3*b*c^2*e)*m^7 + 51*(c^3*d + 3*b*c^2*e)*m^6 + 1045*(c^3*d + 3*b*c^2*e)*m^5
 + 11055*(c^3*d + 3*b*c^2*e)*m^4 + 155925*c^3*d + 467775*b*c^2*e + 64339*(
c^3*d + 3*b*c^2*e)*m^3 + 201609*(c^3*d + 3*b*c^2*e)*m^2 + 303255*(c^3*d +
3*b*c^2*e)*m)*x^13 + 3*((b*c^2*d + (b^2*c + a*c^2)*e)*m^7 + 53*(b*c^2*d +
(b^2*c + a*c^2)*e)*m^6 + 1125*(b*c^2*d + (b^2*c + a*c^2)*e)*m^5 + 12265*(b
*c^2*d + (b^2*c + a*c^2)*e)*m^4 + 184275*b*c^2*d + 73139*(b*c^2*d + (b^2*c
 + a*c^2)*e)*m^3 + 233487*(b*c^2*d + (b^2*c + a*c^2)*e)*m^2 + 184275*(b^2*
c + a*c^2)*e + 355815*(b*c^2*d + (b^2*c + a*c^2)*e)*m)*x^11 + ((3*(b^2*c +
 a*c^2)*d + (b^3 + 6*a*b*c)*e)*m^7 + 55*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*
b*c)*e)*m^6 + 1213*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)*m^5 + 13723*(
3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)*m^4 + 84547*(3*(b^2*c + a*c^2)*d
+ (b^3 + 6*a*b*c)*e)*m^3 + 277093*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e
)*m^2 + 675675*(b^2*c + a*c^2)*d + 225225*(b^3 + 6*a*b*c)*e + 430335*(3*(b
^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)*m)*x^9 + (((b^3 + 6*a*b*c)*d + 3*(a*b
^2 + a^2*c)*e)*m^7 + 57*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*m^6 + 13
09*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*m^5 + 15477*((b^3 + 6*a*b*c)*
d + 3*(a*b^2 + a^2*c)*e)*m^4 + 99715*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c
)*e)*m^3 + 340011*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*m^2 + 28957...
```

3.220.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11266 vs. $2(238) = 476$.

Time = 1.53 (sec) , antiderivative size = 11266, normalized size of antiderivative = 46.36

$$\int (fx)^m \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((f*x)**m*(e*x**2+d)*(c*x**4+b*x**2+a)**3,x)
```

output
```
Piecewise(((-a**3*d/(14*x**14) - a**3*e/(12*x**12) - a**2*b*d/(4*x**12) -
3*a**2*b*e/(10*x**10) - 3*a**2*c*d/(10*x**10) - 3*a**2*c*e/(8*x**8) - 3*a*
b**2*d/(10*x**10) - 3*a*b**2*e/(8*x**8) - 3*a*b*c*d/(4*x**8) - a*b*c*e/x**
6 - a*c**2*d/(2*x**6) - 3*a*c**2*e/(4*x**4) - b**3*d/(8*x**8) - b**3*e/(6*
x**6) - b**2*c*d/(2*x**6) - 3*b**2*c*e/(4*x**4) - 3*b*c**2*d/(4*x**4) - 3*
b*c**2*e/(2*x**2) - c**3*d/(2*x**2) + c**3*e*log(x))/f**15, Eq(m, -15)), (
(-a**3*d/(12*x**12) - a**3*e/(10*x**10) - 3*a**2*b*d/(10*x**10) - 3*a**2*b
*e/(8*x**8) - 3*a**2*c*d/(8*x**8) - a**2*c*e/(2*x**6) - 3*a*b**2*d/(8*x**8
) - a*b**2*e/(2*x**6) - a*b*c*d/x**6 - 3*a*b*c*e/(2*x**4) - 3*a*c**2*d/(4*
x**4) - 3*a*c**2*e/(2*x**2) - b**3*d/(6*x**6) - b**3*e/(4*x**4) - 3*b**2*c
*d/(4*x**4) - 3*b**2*c*e/(2*x**2) - 3*b*c**2*d/(2*x**2) + 3*b*c**2*e*log(x
) + c**3*d*log(x) + c**3*e*x**2/2)/f**13, Eq(m, -13)), ((-a**3*d/(10*x**10
) - a**3*e/(8*x**8) - 3*a**2*b*d/(8*x**8) - a**2*b*e/(2*x**6) - a**2*c*d/(
2*x**6) - 3*a**2*c*e/(4*x**4) - a*b**2*d/(2*x**6) - 3*a*b**2*e/(4*x**4) -
3*a*b*c*d/(2*x**4) - 3*a*b*c*e/x**2 - 3*a*c**2*d/(2*x**2) + 3*a*c**2*e*log
(x) - b**3*d/(4*x**4) - b**3*e/(2*x**2) - 3*b**2*c*d/(2*x**2) + 3*b**2*c*e
*log(x) + 3*b*c**2*d*log(x) + 3*b*c**2*e*x**2/2 + c**3*d*x**2/2 + c**3*e*x
**4/4)/f**11, Eq(m, -11)), ((-a**3*d/(8*x**8) - a**3*e/(6*x**6) - a**2*b*d
/(2*x**6) - 3*a**2*b*e/(4*x**4) - 3*a**2*c*d/(4*x**4) - 3*a**2*c*e/(2*x**2
) - 3*a*b**2*d/(4*x**4) - 3*a*b**2*e/(2*x**2) - 3*a*b*c*d/x**2 + 6*a*b*...
```

3.220.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.68

$$\int (fx)^m \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^3 \, dx = \frac{c^3 e f^m x^{15} x^m}{m+15} + \frac{c^3 d f^m x^{13} x^m}{m+13} + \frac{3\, b c^2 e f^m x^{13} x^m}{m+13} + \frac{3\, b c^2 d f^m x^{11} x^m}{m+11} + \frac{3\, b^2 c e f^m x^{11} x^m}{m+11} + \frac{3\, a c^2 e f^m x^{11} x^m}{m+11} + \frac{3\, b^2 c d f^m x^9 x^m}{m+9} + \frac{3\, a c^2 d f^m x^9 x^m}{m+9} + \frac{b^3 e f^m x^9 x^m}{m+9} + \frac{6\, abce f^m x^9 x^m}{m+9} + \frac{b^3 d f^m x^7 x^m}{m+7} + \frac{6\, abcd f^m x^7 x^m}{m+7} + \frac{3\, a b^2 e f^m x^7 x^m}{m+7} + \frac{3\, a^2 c e f^m x^7 x^m}{m+7} + \frac{3\, a b^2 d f^m x^5 x^m}{m+5} + \frac{3\, a^2 c d f^m x^5 x^m}{m+5} + \frac{3\, a^2 b e f^m x^5 x^m}{m+5} + \frac{3\, a^2 b d f^m x^3 x^m}{m+3} + \frac{a^3 e f^m x^3 x^m}{m+3} + \frac{(fx)^{m+1} a^3 d}{f(m+1)}$$

input
```
integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
c^3*e*f^m*x^15*x^m/(m + 15) + c^3*d*f^m*x^13*x^m/(m + 13) + 3*b*c^2*e*f^m*
x^13*x^m/(m + 13) + 3*b*c^2*d*f^m*x^11*x^m/(m + 11) + 3*b^2*c*e*f^m*x^11*x
^m/(m + 11) + 3*a*c^2*e*f^m*x^11*x^m/(m + 11) + 3*b^2*c*d*f^m*x^9*x^m/(m +
 9) + 3*a*c^2*d*f^m*x^9*x^m/(m + 9) + b^3*e*f^m*x^9*x^m/(m + 9) + 6*a*b*c*
e*f^m*x^9*x^m/(m + 9) + b^3*d*f^m*x^7*x^m/(m + 7) + 6*a*b*c*d*f^m*x^7*x^m/
(m + 7) + 3*a*b^2*e*f^m*x^7*x^m/(m + 7) + 3*a^2*c*e*f^m*x^7*x^m/(m + 7) +
3*a*b^2*d*f^m*x^5*x^m/(m + 5) + 3*a^2*c*d*f^m*x^5*x^m/(m + 5) + 3*a^2*b*e*
f^m*x^5*x^m/(m + 5) + 3*a^2*b*d*f^m*x^3*x^m/(m + 3) + a^3*e*f^m*x^3*x^m/(m
 + 3) + (f*x)^(m + 1)*a^3*d/(f*(m + 1))
```

3.220.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2736 vs. $2(243) = 486$.

Time = 0.36 (sec) , antiderivative size = 2736, normalized size of antiderivative = 11.26

$$\int (fx)^m \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
((f*x)^m*c^3*e*m^7*x^15 + 49*(f*x)^m*c^3*e*m^6*x^15 + (f*x)^m*c^3*d*m^7*x^
13 + 3*(f*x)^m*b*c^2*e*m^7*x^13 + 973*(f*x)^m*c^3*e*m^5*x^15 + 51*(f*x)^m*
c^3*d*m^6*x^13 + 153*(f*x)^m*b*c^2*e*m^6*x^13 + 10045*(f*x)^m*c^3*e*m^4*x^
15 + 3*(f*x)^m*b*c^2*d*m^7*x^11 + 3*(f*x)^m*b^2*c*e*m^7*x^11 + 3*(f*x)^m*a
*c^2*e*m^7*x^11 + 1045*(f*x)^m*c^3*d*m^5*x^13 + 3135*(f*x)^m*b*c^2*e*m^5*x
^13 + 57379*(f*x)^m*c^3*e*m^3*x^15 + 159*(f*x)^m*b*c^2*d*m^6*x^11 + 159*(f
*x)^m*b^2*c*e*m^6*x^11 + 159*(f*x)^m*a*c^2*e*m^6*x^11 + 11055*(f*x)^m*c^3*
d*m^4*x^13 + 33165*(f*x)^m*b*c^2*e*m^4*x^13 + 177331*(f*x)^m*c^3*e*m^2*x^1
5 + 3*(f*x)^m*b^2*c*d*m^7*x^9 + 3*(f*x)^m*a*c^2*d*m^7*x^9 + (f*x)^m*b^3*e*
m^7*x^9 + 6*(f*x)^m*a*b*c*e*m^7*x^9 + 3375*(f*x)^m*b*c^2*d*m^5*x^11 + 3375
*(f*x)^m*b^2*c*e*m^5*x^11 + 3375*(f*x)^m*a*c^2*e*m^5*x^11 + 64339*(f*x)^m*
c^3*d*m^3*x^13 + 193017*(f*x)^m*b*c^2*e*m^3*x^13 + 264207*(f*x)^m*c^3*e*m*
x^15 + 165*(f*x)^m*b^2*c*d*m^6*x^9 + 165*(f*x)^m*a*c^2*d*m^6*x^9 + 55*(f*x
)^m*b^3*e*m^6*x^9 + 330*(f*x)^m*a*b*c*e*m^6*x^9 + 36795*(f*x)^m*b*c^2*d*m^
4*x^11 + 36795*(f*x)^m*b^2*c*e*m^4*x^11 + 36795*(f*x)^m*a*c^2*e*m^4*x^11 +
 201609*(f*x)^m*c^3*d*m^2*x^13 + 604827*(f*x)^m*b*c^2*e*m^2*x^13 + 135135*
(f*x)^m*c^3*e*x^15 + (f*x)^m*b^3*d*m^7*x^7 + 6*(f*x)^m*a*b*c*d*m^7*x^7 + 3
*(f*x)^m*a*b^2*e*m^7*x^7 + 3*(f*x)^m*a^2*c*e*m^7*x^7 + 3639*(f*x)^m*b^2*c*
d*m^5*x^9 + 3639*(f*x)^m*a*c^2*d*m^5*x^9 + 1213*(f*x)^m*b^3*e*m^5*x^9 + 72
78*(f*x)^m*a*b*c*e*m^5*x^9 + 219417*(f*x)^m*b*c^2*d*m^3*x^11 + 219417*(...
```

3.220.9 Mupad [B] (verification not implemented)

Time = 8.07 (sec) , antiderivative size = 769, normalized size of antiderivative = 3.16

$$\int (fx)^m \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^3 \, dx$$
$$= \frac{x^7 (f x)^m (3 c e a^2 + 3 e a b^2 + 6 c d a b + d b^3) (m^7 + 57 m^6 + 1309 m^5 + 15477 m^4 + 99715 m^3 + 340011 m}{m^8 + 64 m^7 + 1708 m^6 + 24640 m^5 + 208054 m^4 + 1038016 m^3 + 2924172 m^2 + 4098240 m}$$
$$+ \frac{x^9 (f x)^m (e b^3 + 3 d b^2 c + 6 a e b c + 3 a d c^2) (m^7 + 55 m^6 + 1213 m^5 + 13723 m^4 + 84547 m^3 + 277093}{m^8 + 64 m^7 + 1708 m^6 + 24640 m^5 + 208054 m^4 + 1038016 m^3 + 2924172 m^2 + 4098240}$$
$$+ \frac{a^3 d x (f x)^m (m^7 + 63 m^6 + 1645 m^5 + 22995 m^4 + 185059 m^3 + 852957 m^2 + 2071215 m + 2027025)}{m^8 + 64 m^7 + 1708 m^6 + 24640 m^5 + 208054 m^4 + 1038016 m^3 + 2924172 m^2 + 4098240 m + 2027025}$$
$$+ \frac{c^3 e x^{15} (f x)^m (m^7 + 49 m^6 + 973 m^5 + 10045 m^4 + 57379 m^3 + 177331 m^2 + 264207 m + 135135)}{m^8 + 64 m^7 + 1708 m^6 + 24640 m^5 + 208054 m^4 + 1038016 m^3 + 2924172 m^2 + 4098240 m + 2027025}$$
$$+ \frac{3 a x^5 (f x)^m (d b^2 + a e b + a c d) (m^7 + 59 m^6 + 1413 m^5 + 17575 m^4 + 120179 m^3 + 437121 m^2 + 738}{m^8 + 64 m^7 + 1708 m^6 + 24640 m^5 + 208054 m^4 + 1038016 m^3 + 2924172 m^2 + 4098240 m +}$$
$$+ \frac{3 c x^{11} (f x)^m (e b^2 + c d b + a c e) (m^7 + 53 m^6 + 1125 m^5 + 12265 m^4 + 73139 m^3 + 233487 m^2 + 3558}{m^8 + 64 m^7 + 1708 m^6 + 24640 m^5 + 208054 m^4 + 1038016 m^3 + 2924172 m^2 + 4098240 m + 2}$$
$$+ \frac{a^2 x^3 (f x)^m (a e + 3 b d) (m^7 + 61 m^6 + 1525 m^5 + 20065 m^4 + 147859 m^3 + 594439 m^2 + 1140855 m +}{m^8 + 64 m^7 + 1708 m^6 + 24640 m^5 + 208054 m^4 + 1038016 m^3 + 2924172 m^2 + 4098240 m + 202}$$
$$+ \frac{c^2 x^{13} (f x)^m (3 b e + c d) (m^7 + 51 m^6 + 1045 m^5 + 11055 m^4 + 64339 m^3 + 201609 m^2 + 303255 m + 1}{m^8 + 64 m^7 + 1708 m^6 + 24640 m^5 + 208054 m^4 + 1038016 m^3 + 2924172 m^2 + 4098240 m + 2027}$$

input
```
int((f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^3,x)
```

output
```
(x^7*(f*x)^m*(b^3*d + 3*a*b^2*e + 3*a^2*c*e + 6*a*b*c*d)*(544095*m + 34001
1*m^2 + 99715*m^3 + 15477*m^4 + 1309*m^5 + 57*m^6 + m^7 + 289575))/(409824
0*m + 2924172*m^2 + 1038016*m^3 + 208054*m^4 + 24640*m^5 + 1708*m^6 + 64*m
^7 + m^8 + 2027025) + (x^9*(f*x)^m*(b^3*e + 3*a*c^2*d + 3*b^2*c*d + 6*a*b*
c*e)*(430335*m + 277093*m^2 + 84547*m^3 + 13723*m^4 + 1213*m^5 + 55*m^6 +
m^7 + 225225))/(4098240*m + 2924172*m^2 + 1038016*m^3 + 208054*m^4 + 24640
*m^5 + 1708*m^6 + 64*m^7 + m^8 + 2027025) + (a^3*d*x*(f*x)^m*(2071215*m +
852957*m^2 + 185059*m^3 + 22995*m^4 + 1645*m^5 + 63*m^6 + m^7 + 2027025))/
(4098240*m + 2924172*m^2 + 1038016*m^3 + 208054*m^4 + 24640*m^5 + 1708*m^6
 + 64*m^7 + m^8 + 2027025) + (c^3*e*x^15*(f*x)^m*(264207*m + 177331*m^2 +
57379*m^3 + 10045*m^4 + 973*m^5 + 49*m^6 + m^7 + 135135))/(4098240*m + 292
4172*m^2 + 1038016*m^3 + 208054*m^4 + 24640*m^5 + 1708*m^6 + 64*m^7 + m^8
+ 2027025) + (3*a*x^5*(f*x)^m*(b^2*d + a*b*e + a*c*d)*(738567*m + 437121*m
^2 + 120179*m^3 + 17575*m^4 + 1413*m^5 + 59*m^6 + m^7 + 405405))/(4098240*
m + 2924172*m^2 + 1038016*m^3 + 208054*m^4 + 24640*m^5 + 1708*m^6 + 64*m^7
 + m^8 + 2027025) + (3*c*x^11*(f*x)^m*(b^2*e + a*c*e + b*c*d)*(355815*m +
233487*m^2 + 73139*m^3 + 12265*m^4 + 1125*m^5 + 53*m^6 + m^7 + 184275))/(4
098240*m + 2924172*m^2 + 1038016*m^3 + 208054*m^4 + 24640*m^5 + 1708*m^6 +
 64*m^7 + m^8 + 2027025) + (a^2*x^3*(f*x)^m*(a*e + 3*b*d)*(1140855*m + 594
439*m^2 + 147859*m^3 + 20065*m^4 + 1525*m^5 + 61*m^6 + m^7 + 675675))/(...
```

3.220. $\int (fx)^m (d+ex^2) (a+bx^2+cx^4)^3 \, dx$

3.221 $\int (fx)^m (d+ex^2) (a+bx^2+cx^4)^2 \, dx$

3.221.1 Optimal result

Integrand size = 27, antiderivative size = 155

$$\int (fx)^m (d+ex^2) (a+bx^2+cx^4)^2 \, dx = \frac{a^2d(fx)^{1+m}}{f(1+m)} + \frac{a(2bd+ae)(fx)^{3+m}}{f^3(3+m)} + \frac{(b^2d+2acd+2abe)(fx)^{5+m}}{f^5(5+m)} + \frac{(2bcd+b^2e+2ace)(fx)^{7+m}}{f^7(7+m)} + \frac{c(cd+2be)(fx)^{9+m}}{f^9(9+m)} + \frac{c^2e(fx)^{11+m}}{f^{11}(11+m)}$$

output
```
a^2*d*(f*x)^(1+m)/f/(1+m)+a*(a*e+2*b*d)*(f*x)^(3+m)/f^3/(3+m)+(2*a*b*e+2*a
*c*d+b^2*d)*(f*x)^(5+m)/f^5/(5+m)+(2*a*c*e+b^2*e+2*b*c*d)*(f*x)^(7+m)/f^7/
(7+m)+c*(2*b*e+c*d)*(f*x)^(9+m)/f^9/(9+m)+c^2*e*(f*x)^(11+m)/f^11/(11+m)
```

3.221.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.75

$$\int (fx)^m \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^2 \, dx = x(fx)^m \left(\frac{a^2d}{1+m}+\frac{a(2bd+ae)x^2}{3+m} + \frac{\left(b^2d+2acd+2abe\right)x^4}{5+m} + \frac{\left(2bcd+b^2e+2ace\right)x^6}{7+m}+\frac{c(cd+2be)x^8}{9+m} + \frac{c^2ex^{10}}{11+m}\right)$$

input
```
Integrate[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

output
```
x*(f*x)^m*((a^2*d)/(1 + m) + (a*(2*b*d + a*e)*x^2)/(3 + m) + ((b^2*d + 2*a
*c*d + 2*a*b*e)*x^4)/(5 + m) + ((2*b*c*d + b^2*e + 2*a*c*e)*x^6)/(7 + m) +
 (c*(c*d + 2*b*e)*x^8)/(9 + m) + (c^2*e*x^10)/(11 + m))
```

3.221.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)(fx)^m \left(a+bx^2+cx^4\right)^2 \, dx$$

$$\Big\downarrow \; 1584$$

$$\int \left(a^2d(fx)^m+\frac{(fx)^{m+6}\left(2ace+b^2e+2bcd\right)}{f^6}+\frac{(fx)^{m+4}\left(2abe+2acd+b^2d\right)}{f^4}+\frac{a(fx)^{m+2}(ae+2bd)}{f^2}+\frac{c(fx)^m}{}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{a^2d(fx)^{m+1}}{f(m+1)}+\frac{(fx)^{m+7}\left(2ace+b^2e+2bcd\right)}{f^7(m+7)}+\frac{(fx)^{m+5}\left(2abe+2acd+b^2d\right)}{f^5(m+5)}+\frac{a(fx)^{m+3}(ae+2bd)}{f^3(m+3)}+\frac{c(fx)^{m+9}(2be+cd)}{f^9(m+9)}+\frac{c^2e(fx)^{m+11}}{f^{11}(m+11)}$$

input
```
Int[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

output
```
(a^2*d*(f*x)^(1 + m))/(f*(1 + m)) + (a*(2*b*d + a*e)*(f*x)^(3 + m))/(f^3*(
3 + m)) + ((b^2*d + 2*a*c*d + 2*a*b*e)*(f*x)^(5 + m))/(f^5*(5 + m)) + ((2*
b*c*d + b^2*e + 2*a*c*e)*(f*x)^(7 + m))/(f^7*(7 + m)) + (c*(c*d + 2*b*e)*(
f*x)^(9 + m))/(f^9*(9 + m)) + (c^2*e*(f*x)^(11 + m))/(f^11*(11 + m))
```

3.221.3.1 Defintions of rubi rules used

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.221.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 782 vs. $2(155) = 310$.

Time = 0.17 (sec) , antiderivative size = 783, normalized size of antiderivative = 5.05

method	result
gosper	$\frac{x\left(c^2 e\, m^5 x^{10}+25 c^2 e\, m^4 x^{10}+2 b c e\, m^5 x^8+c^2 d\, m^5 x^8+230 c^2 e\, m^3 x^{10}+54 b c e\, m^4 x^8+27 c^2 d\, m^4 x^8+950 c^2 e\, m^2 x^{10}+2 a c e\, m^5 x^6+b^2\right.}{}$
risch	$\frac{x\left(c^2 e\, m^5 x^{10}+25 c^2 e\, m^4 x^{10}+2 b c e\, m^5 x^8+c^2 d\, m^5 x^8+230 c^2 e\, m^3 x^{10}+54 b c e\, m^4 x^8+27 c^2 d\, m^4 x^8+950 c^2 e\, m^2 x^{10}+2 a c e\, m^5 x^6+b^2\right.}{}$
parallelrisch	Expression too large to display

input
```
int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
x*(c^2*e*m^5*x^10+25*c^2*e*m^4*x^10+2*b*c*e*m^5*x^8+c^2*d*m^5*x^8+230*c^2*
e*m^3*x^10+54*b*c*e*m^4*x^8+27*c^2*d*m^4*x^8+950*c^2*e*m^2*x^10+2*a*c*e*m^
5*x^6+b^2*e*m^5*x^6+2*b*c*d*m^5*x^6+524*b*c*e*m^3*x^8+262*c^2*d*m^3*x^8+16
89*c^2*e*m*x^10+58*a*c*e*m^4*x^6+29*b^2*e*m^4*x^6+58*b*c*d*m^4*x^6+2244*b*
c*e*m^2*x^8+1122*c^2*d*m^2*x^8+945*c^2*e*x^10+2*a*b*e*m^5*x^4+2*a*c*d*m^5*
x^4+604*a*c*e*m^3*x^6+b^2*d*m^5*x^4+302*b^2*e*m^3*x^6+604*b*c*d*m^3*x^6+40
82*b*c*e*m*x^8+2041*c^2*d*m*x^8+62*a*b*e*m^4*x^4+62*a*c*d*m^4*x^4+2732*a*c
*e*m^2*x^6+31*b^2*d*m^4*x^4+1366*b^2*e*m^2*x^6+2732*b*c*d*m^2*x^6+2310*b*c
*e*x^8+1155*c^2*d*x^8+a^2*e*m^5*x^2+2*a*b*d*m^5*x^2+700*a*b*e*m^3*x^4+700*
a*c*d*m^3*x^4+5154*a*c*e*m*x^6+350*b^2*d*m^3*x^4+2577*b^2*e*m*x^6+5154*b*c
*d*m*x^6+33*a^2*e*m^4*x^2+66*a*b*d*m^4*x^2+3460*a*b*e*m^2*x^4+3460*a*c*d*m
^2*x^4+2970*a*c*e*x^6+1730*b^2*d*m^2*x^4+1485*b^2*e*x^6+2970*b*c*d*x^6+a^2
*d*m^5+406*a^2*e*m^3*x^2+812*a*b*d*m^3*x^2+6978*a*b*e*m*x^4+6978*a*c*d*m*x
^4+3489*b^2*d*m*x^4+35*a^2*d*m^4+2262*a^2*e*m^2*x^2+4524*a*b*d*m^2*x^2+415
8*a*b*e*x^4+4158*a*c*d*x^4+2079*b^2*d*x^4+470*a^2*d*m^3+5353*a^2*e*m*x^2+1
0706*a*b*d*m*x^2+3010*a^2*d*m^2+3465*a^2*e*x^2+6930*a*b*d*x^2+9129*a^2*d*m
+10395*a^2*d)*(f*x)^m/(11+m)/(9+m)/(7+m)/(5+m)/(3+m)/(1+m)
```

3.221.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(155) = 310$.

Time = 0.25 (sec) , antiderivative size = 573, normalized size of antiderivative = 3.70

$$\int (fx)^m \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^2 dx$$
$$= \frac{((c^2em^5 + 25\,c^2em^4 + 230\,c^2em^3 + 950\,c^2em^2 + 1689\,c^2em + 945\,c^2e)x^{11} + ((c^2d + 2\,bce)m^5 + 27\,(c^2d +}{}$$

input
```
integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
((c^2*e*m^5 + 25*c^2*e*m^4 + 230*c^2*e*m^3 + 950*c^2*e*m^2 + 1689*c^2*e*m
+ 945*c^2*e)*x^11 + ((c^2*d + 2*b*c*e)*m^5 + 27*(c^2*d + 2*b*c*e)*m^4 + 26
2*(c^2*d + 2*b*c*e)*m^3 + 1155*c^2*d + 2310*b*c*e + 1122*(c^2*d + 2*b*c*e)
*m^2 + 2041*(c^2*d + 2*b*c*e)*m)*x^9 + ((2*b*c*d + (b^2 + 2*a*c)*e)*m^5 +
29*(2*b*c*d + (b^2 + 2*a*c)*e)*m^4 + 302*(2*b*c*d + (b^2 + 2*a*c)*e)*m^3 +
 2970*b*c*d + 1366*(2*b*c*d + (b^2 + 2*a*c)*e)*m^2 + 1485*(b^2 + 2*a*c)*e
+ 2577*(2*b*c*d + (b^2 + 2*a*c)*e)*m)*x^7 + ((2*a*b*e + (b^2 + 2*a*c)*d)*m
^5 + 31*(2*a*b*e + (b^2 + 2*a*c)*d)*m^4 + 350*(2*a*b*e + (b^2 + 2*a*c)*d)*
m^3 + 4158*a*b*e + 1730*(2*a*b*e + (b^2 + 2*a*c)*d)*m^2 + 2079*(b^2 + 2*a*
c)*d + 3489*(2*a*b*e + (b^2 + 2*a*c)*d)*m)*x^5 + ((2*a*b*d + a^2*e)*m^5 +
33*(2*a*b*d + a^2*e)*m^4 + 406*(2*a*b*d + a^2*e)*m^3 + 6930*a*b*d + 3465*a
^2*e + 2262*(2*a*b*d + a^2*e)*m^2 + 5353*(2*a*b*d + a^2*e)*m)*x^3 + (a^2*d
*m^5 + 35*a^2*d*m^4 + 470*a^2*d*m^3 + 3010*a^2*d*m^2 + 9129*a^2*d*m + 1039
5*a^2*d)*x)*(f*x)^m/(m^6 + 36*m^5 + 505*m^4 + 3480*m^3 + 12139*m^2 + 19524
*m + 10395)
```

3.221.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4068 vs. 2(146) = 292.

Time = 0.88 (sec) , antiderivative size = 4068, normalized size of antiderivative = 26.25

$$\int (fx)^m \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((f*x)**m*(e*x**2+d)*(c*x**4+b*x**2+a)**2,x)
```

output
```
Piecewise(((-a**2*d/(10*x**10) - a**2*e/(8*x**8) - a*b*d/(4*x**8) - a*b*e/
(3*x**6) - a*c*d/(3*x**6) - a*c*e/(2*x**4) - b**2*d/(6*x**6) - b**2*e/(4*x
**4) - b*c*d/(2*x**4) - b*c*e/x**2 - c**2*d/(2*x**2) + c**2*e*log(x))/f**1
1, Eq(m, -11)), ((-a**2*d/(8*x**8) - a**2*e/(6*x**6) - a*b*d/(3*x**6) - a*
b*e/(2*x**4) - a*c*d/(2*x**4) - a*c*e/x**2 - b**2*d/(4*x**4) - b**2*e/(2*x
**2) - b*c*d/x**2 + 2*b*c*e*log(x) + c**2*d*log(x) + c**2*e*x**2/2)/f**9,
Eq(m, -9)), ((-a**2*d/(6*x**6) - a**2*e/(4*x**4) - a*b*d/(2*x**4) - a*b*e/
x**2 - a*c*d/x**2 + 2*a*c*e*log(x) - b**2*d/(2*x**2) + b**2*e*log(x) + 2*b
*c*d*log(x) + b*c*e*x**2 + c**2*d*x**2/2 + c**2*e*x**4/4)/f**7, Eq(m, -7))
, ((-a**2*d/(4*x**4) - a**2*e/(2*x**2) - a*b*d/x**2 + 2*a*b*e*log(x) + 2*a
*c*d*log(x) + a*c*e*x**2 + b**2*d*log(x) + b**2*e*x**2/2 + b*c*d*x**2 + b*
c*e*x**4/2 + c**2*d*x**4/4 + c**2*e*x**6/6)/f**5, Eq(m, -5)), ((-a**2*d/(2
*x**2) + a**2*e*log(x) + 2*a*b*d*log(x) + a*b*e*x**2 + a*c*d*x**2 + a*c*e*
x**4/2 + b**2*d*x**2/2 + b**2*e*x**4/4 + b*c*d*x**4/2 + b*c*e*x**6/3 + c**
2*d*x**6/6 + c**2*e*x**8/8)/f**3, Eq(m, -3)), ((a**2*d*log(x) + a**2*e*x**
2/2 + a*b*d*x**2 + a*b*e*x**4/2 + a*c*d*x**4/2 + a*c*e*x**6/3 + b**2*d*x**
4/4 + b**2*e*x**6/6 + b*c*d*x**6/3 + b*c*e*x**8/4 + c**2*d*x**8/8 + c**2*e
*x**10/10)/f, Eq(m, -1)), (a**2*d*m**5*x*(f*x)**m/(m**6 + 36*m**5 + 505*m*
*4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 35*a**2*d*m**4*x*(f*x)**m
/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395)...
```

3.221.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.48

$$\int (fx)^m \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^2 \, dx = \frac{c^2 e f^m x^{11} x^m}{m+11} + \frac{c^2 d f^m x^9 x^m}{m+9} + \frac{2\, bcef^m x^9 x^m}{m+9} + \frac{2\, bcdf^m x^7 x^m}{m+7} + \frac{b^2 e f^m x^7 x^m}{m+7} + \frac{2\, acef^m x^7 x^m}{m+7} + \frac{b^2 d f^m x^5 x^m}{m+5} + \frac{2\, acdf^m x^5 x^m}{m+5} + \frac{2\, abef^m x^5 x^m}{m+5} + \frac{2\, abdf^m x^3 x^m}{m+3} + \frac{a^2 e f^m x^3 x^m}{m+3} + \frac{(fx)^{m+1} a^2 d}{f(m+1)}$$

input
```
integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
c^2*e*f^m*x^11*x^m/(m + 11) + c^2*d*f^m*x^9*x^m/(m + 9) + 2*b*c*e*f^m*x^9*
x^m/(m + 9) + 2*b*c*d*f^m*x^7*x^m/(m + 7) + b^2*e*f^m*x^7*x^m/(m + 7) + 2*
a*c*e*f^m*x^7*x^m/(m + 7) + b^2*d*f^m*x^5*x^m/(m + 5) + 2*a*c*d*f^m*x^5*x^
m/(m + 5) + 2*a*b*e*f^m*x^5*x^m/(m + 5) + 2*a*b*d*f^m*x^3*x^m/(m + 3) + a^
2*e*f^m*x^3*x^m/(m + 3) + (f*x)^(m + 1)*a^2*d/(f*(m + 1))
```

3.221.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1142 vs. 2(155) = 310.

Time = 0.32 (sec) , antiderivative size = 1142, normalized size of antiderivative = 7.37

$$\int (fx)^m\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^2\,dx=\text{Too large to display}$$

input
```
integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
((f*x)^m*c^2*e*m^5*x^11 + 25*(f*x)^m*c^2*e*m^4*x^11 + (f*x)^m*c^2*d*m^5*x^
9 + 2*(f*x)^m*b*c*e*m^5*x^9 + 230*(f*x)^m*c^2*e*m^3*x^11 + 27*(f*x)^m*c^2*
d*m^4*x^9 + 54*(f*x)^m*b*c*e*m^4*x^9 + 950*(f*x)^m*c^2*e*m^2*x^11 + 2*(f*x
)^m*b*c*d*m^5*x^7 + (f*x)^m*b^2*e*m^5*x^7 + 2*(f*x)^m*a*c*e*m^5*x^7 + 262*
(f*x)^m*c^2*d*m^3*x^9 + 524*(f*x)^m*b*c*e*m^3*x^9 + 1689*(f*x)^m*c^2*e*m*x
^11 + 58*(f*x)^m*b*c*d*m^4*x^7 + 29*(f*x)^m*b^2*e*m^4*x^7 + 58*(f*x)^m*a*c
*e*m^4*x^7 + 1122*(f*x)^m*c^2*d*m^2*x^9 + 2244*(f*x)^m*b*c*e*m^2*x^9 + 945
*(f*x)^m*c^2*e*x^11 + (f*x)^m*b^2*d*m^5*x^5 + 2*(f*x)^m*a*c*d*m^5*x^5 + 2*
(f*x)^m*a*b*e*m^5*x^5 + 604*(f*x)^m*b*c*d*m^3*x^7 + 302*(f*x)^m*b^2*e*m^3*
x^7 + 604*(f*x)^m*a*c*e*m^3*x^7 + 2041*(f*x)^m*c^2*d*m*x^9 + 4082*(f*x)^m*
b*c*e*m*x^9 + 31*(f*x)^m*b^2*d*m^4*x^5 + 62*(f*x)^m*a*c*d*m^4*x^5 + 62*(f*
x)^m*a*b*e*m^4*x^5 + 2732*(f*x)^m*b*c*d*m^2*x^7 + 1366*(f*x)^m*b^2*e*m^2*x
^7 + 2732*(f*x)^m*a*c*e*m^2*x^7 + 1155*(f*x)^m*c^2*d*x^9 + 2310*(f*x)^m*b*
c*e*x^9 + 2*(f*x)^m*a*b*d*m^5*x^3 + (f*x)^m*a^2*e*m^5*x^3 + 350*(f*x)^m*b^
2*d*m^3*x^5 + 700*(f*x)^m*a*c*d*m^3*x^5 + 700*(f*x)^m*a*b*e*m^3*x^5 + 5154
*(f*x)^m*b*c*d*m*x^7 + 2577*(f*x)^m*b^2*e*m*x^7 + 5154*(f*x)^m*a*c*e*m*x^7
 + 66*(f*x)^m*a*b*d*m^4*x^3 + 33*(f*x)^m*a^2*e*m^4*x^3 + 1730*(f*x)^m*b^2*
d*m^2*x^5 + 3460*(f*x)^m*a*c*d*m^2*x^5 + 3460*(f*x)^m*a*b*e*m^2*x^5 + 2970
*(f*x)^m*b*c*d*x^7 + 1485*(f*x)^m*b^2*e*x^7 + 2970*(f*x)^m*a*c*e*x^7 + (f*
x)^m*a^2*d*m^5*x + 812*(f*x)^m*a*b*d*m^3*x^3 + 406*(f*x)^m*a^2*e*m^3*x^...
```

3.221.9 Mupad [B] (verification not implemented)

Time = 7.87 (sec) , antiderivative size = 429, normalized size of antiderivative = 2.77

$$\int (fx)^m (d+ex^2) (a+bx^2+cx^4)^2 dx$$
$$= \frac{x^5 (f x)^m (d b^2 + 2 a e b + 2 a c d) (m^5 + 31 m^4 + 350 m^3 + 1730 m^2 + 3489 m + 2079)}{m^6 + 36 m^5 + 505 m^4 + 3480 m^3 + 12139 m^2 + 19524 m + 10395}$$
$$+ \frac{x^7 (f x)^m (e b^2 + 2 c d b + 2 a c e) (m^5 + 29 m^4 + 302 m^3 + 1366 m^2 + 2577 m + 1485)}{m^6 + 36 m^5 + 505 m^4 + 3480 m^3 + 12139 m^2 + 19524 m + 10395}$$
$$+ \frac{a^2 d x (f x)^m (m^5 + 35 m^4 + 470 m^3 + 3010 m^2 + 9129 m + 10395)}{m^6 + 36 m^5 + 505 m^4 + 3480 m^3 + 12139 m^2 + 19524 m + 10395}$$
$$+ \frac{a x^3 (f x)^m (a e + 2 b d) (m^5 + 33 m^4 + 406 m^3 + 2262 m^2 + 5353 m + 3465)}{m^6 + 36 m^5 + 505 m^4 + 3480 m^3 + 12139 m^2 + 19524 m + 10395}$$
$$+ \frac{c x^9 (f x)^m (2 b e + c d) (m^5 + 27 m^4 + 262 m^3 + 1122 m^2 + 2041 m + 1155)}{m^6 + 36 m^5 + 505 m^4 + 3480 m^3 + 12139 m^2 + 19524 m + 10395}$$
$$+ \frac{c^2 e x^{11} (f x)^m (m^5 + 25 m^4 + 230 m^3 + 950 m^2 + 1689 m + 945)}{m^6 + 36 m^5 + 505 m^4 + 3480 m^3 + 12139 m^2 + 19524 m + 10395}$$

input
```
int((f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^2,x)
```

output
```
(x^5*(f*x)^m*(b^2*d + 2*a*b*e + 2*a*c*d)*(3489*m + 1730*m^2 + 350*m^3 + 31
*m^4 + m^5 + 2079))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m
^6 + 10395) + (x^7*(f*x)^m*(b^2*e + 2*a*c*e + 2*b*c*d)*(2577*m + 1366*m^2
+ 302*m^3 + 29*m^4 + m^5 + 1485))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^
4 + 36*m^5 + m^6 + 10395) + (a^2*d*x*(f*x)^m*(9129*m + 3010*m^2 + 470*m^3
+ 35*m^4 + m^5 + 10395))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^
5 + m^6 + 10395) + (a*x^3*(f*x)^m*(a*e + 2*b*d)*(5353*m + 2262*m^2 + 406*m
^3 + 33*m^4 + m^5 + 3465))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*
m^5 + m^6 + 10395) + (c*x^9*(f*x)^m*(2*b*e + c*d)*(2041*m + 1122*m^2 + 262
*m^3 + 27*m^4 + m^5 + 1155))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 3
6*m^5 + m^6 + 10395) + (c^2*e*x^11*(f*x)^m*(1689*m + 950*m^2 + 230*m^3 + 2
5*m^4 + m^5 + 945))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m
^6 + 10395)
```

3.222 $\int (fx)^m (d+ex^2) (a+bx^2+cx^4) \, dx$

3.222.1 Optimal result

Integrand size = 25, antiderivative size = 83

$$\int (fx)^m (d+ex^2) (a+bx^2+cx^4) \, dx = \frac{ad(fx)^{1+m}}{f(1+m)} + \frac{(bd+ae)(fx)^{3+m}}{f^3(3+m)} + \frac{(cd+be)(fx)^{5+m}}{f^5(5+m)} + \frac{ce(fx)^{7+m}}{f^7(7+m)}$$

output
```
a*d*(f*x)^(1+m)/f/(1+m)+(a*e+b*d)*(f*x)^(3+m)/f^3/(3+m)+(b*e+c*d)*(f*x)^(5
+m)/f^5/(5+m)+c*e*(f*x)^(7+m)/f^7/(7+m)
```

3.222.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.71

$$\int (fx)^m (d+ex^2) (a+bx^2+cx^4) \, dx = x(fx)^m \left(\frac{ad}{1+m} + \frac{(bd+ae)x^2}{3+m} + \frac{(cd+be)x^4}{5+m} + \frac{cex^6}{7+m}\right)$$

input
```
Integrate[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4),x]
```

output
```
x*(f*x)^m*((a*d)/(1 + m) + ((b*d + a*e)*x^2)/(3 + m) + ((c*d + b*e)*x^4)/(
5 + m) + (c*e*x^6)/(7 + m))
```

3.222.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)(fx)^m \left(a+bx^2+cx^4\right) dx$$

$$\Big\downarrow \text{1584}$$

$$\int \left(\frac{(fx)^{m+2}(ae+bd)}{f^2}+ad(fx)^m+\frac{(fx)^{m+4}(be+cd)}{f^4}+\frac{ce(fx)^{m+6}}{f^6}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(fx)^{m+3}(ae+bd)}{f^3(m+3)}+\frac{ad(fx)^{m+1}}{f(m+1)}+\frac{(fx)^{m+5}(be+cd)}{f^5(m+5)}+\frac{ce(fx)^{m+7}}{f^7(m+7)}$$

input
```
Int[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4),x]
```

output
```
(a*d*(f*x)^(1 + m))/(f*(1 + m)) + ((b*d + a*e)*(f*x)^(3 + m))/(f^3*(3 + m)
) + ((c*d + b*e)*(f*x)^(5 + m))/(f^5*(5 + m)) + (c*e*(f*x)^(7 + m))/(f^7*(
7 + m))
```

3.222.3.1 Defintions of rubi rules used

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.222.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.99

method	result
norman	$\frac{(ae+bd)x^3 e^{m\ln(fx)}}{3+m}+\frac{(be+cd)x^5 e^{m\ln(fx)}}{5+m}+\frac{dax\,e^{m\ln(fx)}}{1+m}+\frac{ec\,x^7 e^{m\ln(fx)}}{7+m}$
gosper	$\frac{x(ce\,m^3x^6+9ce\,m^2x^6+be\,m^3x^4+cd\,m^3x^4+23cem\,x^6+11be\,m^2x^4+11cd\,m^2x^4+15ce\,x^6+ae\,m^3x^2+bd\,m^3x^2+31bem\,x^4+31cd}{(7+m)(5+}$
risch	$\frac{x(ce\,m^3x^6+9ce\,m^2x^6+be\,m^3x^4+cd\,m^3x^4+23cem\,x^6+11be\,m^2x^4+11cd\,m^2x^4+15ce\,x^6+ae\,m^3x^2+bd\,m^3x^2+31bem\,x^4+31cd}{(7+m)(5+}$
parallelrisch	$15x^7(fx)^m ce+21x^5(fx)^m be+21x^5(fx)^m cd+35x^3(fx)^m ae+35x^3(fx)^m bd+105x(fx)^m ad+71x(fx)^m adm+47x^3(fx)^m aem-$

input
```
int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
(a*e+b*d)/(3+m)*x^3*exp(m*ln(f*x))+(b*e+c*d)/(5+m)*x^5*exp(m*ln(f*x))+d*a/
(1+m)*x*exp(m*ln(f*x))+e*c/(7+m)*x^7*exp(m*ln(f*x))
```

3.222.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(83) = 166.

Time = 0.25 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.06

$$\int (fx)^m \left(d+ex^2\right)\left(a+bx^2+cx^4\right)\,dx$$
$$=\frac{((cem^3+9\,cem^2+23\,cem+15\,ce)x^7+((cd+be)m^3+11\,(cd+be)m^2+21\,cd+21\,be+31\,(cd+be)m)}{m^4+16\,}$$

input
```
integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
((c*e*m^3 + 9*c*e*m^2 + 23*c*e*m + 15*c*e)*x^7 + ((c*d + b*e)*m^3 + 11*(c*
d + b*e)*m^2 + 21*c*d + 21*b*e + 31*(c*d + b*e)*m)*x^5 + ((b*d + a*e)*m^3
+ 13*(b*d + a*e)*m^2 + 35*b*d + 35*a*e + 47*(b*d + a*e)*m)*x^3 + (a*d*m^3
+ 15*a*d*m^2 + 71*a*d*m + 105*a*d)*x)*(f*x)^m/(m^4 + 16*m^3 + 86*m^2 + 176
*m + 105)
```

3.222.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1015 vs. $2(71) = 142$.

Time = 0.47 (sec) , antiderivative size = 1015, normalized size of antiderivative = 12.23

$$\int (fx)^m (d+ex^2)(a+bx^2+cx^4)\,dx$$
$$= \begin{cases} \frac{-\frac{ad}{6x^6}-\frac{ae}{4x^4}-\frac{bd}{4x^4}-\frac{be}{2x^2}-\frac{cd}{2x^2}+ce\log(x)}{f^7} \\ \frac{-\frac{ad}{4x^4}-\frac{ae}{2x^2}-\frac{bd}{2x^2}+be\log(x)+cd\log(x)+\frac{cex^2}{2}}{f^5} \\ \frac{-\frac{ad}{2x^2}+ae\log(x)+bd\log(x)+\frac{bex^2}{2}+\frac{cdx^2}{2}+\frac{cex^4}{4}}{f^3} \\ \frac{ad\log(x)+\frac{aex^2}{2}+\frac{bdx^2}{2}+\frac{bex^4}{4}+\frac{cdx^4}{4}+\frac{cex^6}{6}}{f} \\ \frac{adm^3x(fx)^m}{m^4+16m^3+86m^2+176m+105}+\frac{15adm^2x(fx)^m}{m^4+16m^3+86m^2+176m+105}+\frac{71admx(fx)^m}{m^4+16m^3+86m^2+176m+105}+\frac{105adx(fx)^m}{m^4+16m^3+86m^2+176m+105}+ \end{cases}$$

input
```
integrate((f*x)**m*(e*x**2+d)*(c*x**4+b*x**2+a),x)
```

output
```
Piecewise(((-a*d/(6*x**6) - a*e/(4*x**4) - b*d/(4*x**4) - b*e/(2*x**2) - c
*d/(2*x**2) + c*e*log(x))/f**7, Eq(m, -7)), ((-a*d/(4*x**4) - a*e/(2*x**2)
 - b*d/(2*x**2) + b*e*log(x) + c*d*log(x) + c*e*x**2/2)/f**5, Eq(m, -5)),
((-a*d/(2*x**2) + a*e*log(x) + b*d*log(x) + b*e*x**2/2 + c*d*x**2/2 + c*e*
x**4/4)/f**3, Eq(m, -3)), ((a*d*log(x) + a*e*x**2/2 + b*d*x**2/2 + b*e*x**
4/4 + c*d*x**4/4 + c*e*x**6/6)/f, Eq(m, -1)), (a*d*m**3*x*(f*x)**m/(m**4 +
 16*m**3 + 86*m**2 + 176*m + 105) + 15*a*d*m**2*x*(f*x)**m/(m**4 + 16*m**3
 + 86*m**2 + 176*m + 105) + 71*a*d*m*x*(f*x)**m/(m**4 + 16*m**3 + 86*m**2
+ 176*m + 105) + 105*a*d*x*(f*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 10
5) + a*e*m**3*x**3*(f*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 13*
a*e*m**2*x**3*(f*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 47*a*e*m
*x**3*(f*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 35*a*e*x**3*(f*x
)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + b*d*m**3*x**3*(f*x)**m/(m*
*4 + 16*m**3 + 86*m**2 + 176*m + 105) + 13*b*d*m**2*x**3*(f*x)**m/(m**4 +
16*m**3 + 86*m**2 + 176*m + 105) + 47*b*d*m*x**3*(f*x)**m/(m**4 + 16*m**3
+ 86*m**2 + 176*m + 105) + 35*b*d*x**3*(f*x)**m/(m**4 + 16*m**3 + 86*m**2
+ 176*m + 105) + b*e*m**3*x**5*(f*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m
+ 105) + 11*b*e*m**2*x**5*(f*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105
) + 31*b*e*m*x**5*(f*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 21*b
*e*x**5*(f*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + c*d*m**3*x*...
```

3.222.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.25

$$\int (fx)^m \left(d+ex^2\right) \left(a+bx^2+cx^4\right) \, dx = \frac{cef^m x^7 x^m}{m+7} + \frac{cdf^m x^5 x^m}{m+5} + \frac{bef^m x^5 x^m}{m+5} + \frac{bdf^m x^3 x^m}{m+3} + \frac{aef^m x^3 x^m}{m+3} + \frac{(fx)^{m+1} ad}{f(m+1)}$$

input
```
integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
c*e*f^m*x^7*x^m/(m + 7) + c*d*f^m*x^5*x^m/(m + 5) + b*e*f^m*x^5*x^m/(m + 5
) + b*d*f^m*x^3*x^m/(m + 3) + a*e*f^m*x^3*x^m/(m + 3) + (f*x)^(m + 1)*a*d/
(f*(m + 1))
```

3.222.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. 2(83) = 166.

Time = 0.30 (sec) , antiderivative size = 338, normalized size of antiderivative = 4.07

$$\int (fx)^m \left(d+ex^2\right) \left(a+bx^2+cx^4\right) \, dx$$
$$= \frac{(fx)^m cem^3 x^7 + 9 (fx)^m cem^2 x^7 + (fx)^m cdm^3 x^5 + (fx)^m bem^3 x^5 + 23 (fx)^m cemx^7 + 11 (fx)^m cdm^2 x^5}{}$$

input
```
integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
((f*x)^m*c*e*m^3*x^7 + 9*(f*x)^m*c*e*m^2*x^7 + (f*x)^m*c*d*m^3*x^5 + (f*x)
^m*b*e*m^3*x^5 + 23*(f*x)^m*c*e*m*x^7 + 11*(f*x)^m*c*d*m^2*x^5 + 11*(f*x)^
m*b*e*m^2*x^5 + 15*(f*x)^m*c*e*x^7 + (f*x)^m*b*d*m^3*x^3 + (f*x)^m*a*e*m^3
*x^3 + 31*(f*x)^m*c*d*m*x^5 + 31*(f*x)^m*b*e*m*x^5 + 13*(f*x)^m*b*d*m^2*x^
3 + 13*(f*x)^m*a*e*m^2*x^3 + 21*(f*x)^m*c*d*x^5 + 21*(f*x)^m*b*e*x^5 + (f*
x)^m*a*d*m^3*x + 47*(f*x)^m*b*d*m*x^3 + 47*(f*x)^m*a*e*m*x^3 + 15*(f*x)^m*
a*d*m^2*x + 35*(f*x)^m*b*d*x^3 + 35*(f*x)^m*a*e*x^3 + 71*(f*x)^m*a*d*m*x +
 105*(f*x)^m*a*d*x)/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

3.222.9 Mupad [B] (verification not implemented)

Time = 7.68 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.06

$$\int (fx)^m (d+ex^2)(a+bx^2+cx^4)\,dx = (f\,x)^m \left(\frac{x^3\,(a\,e+b\,d)\,(m^3+13\,m^2+47\,m+35)}{m^4+16\,m^3+86\,m^2+176\,m+105} + \frac{x^5\,(b\,e+c\,d)\,(m^3+11\,m^2+31\,m+21)}{m^4+16\,m^3+86\,m^2+176\,m+105} + \frac{a\,d\,x\,(m^3+15\,m^2+71\,m+105)}{m^4+16\,m^3+86\,m^2+176\,m+105} + \frac{c\,e\,x^7\,(m^3+9\,m^2+23\,m+15)}{m^4+16\,m^3+86\,m^2+176\,m+105}\right)$$

input
```
int((f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4),x)
```

output
```
(f*x)^m*((x^3*(a*e + b*d)*(47*m + 13*m^2 + m^3 + 35))/(176*m + 86*m^2 + 16
*m^3 + m^4 + 105) + (x^5*(b*e + c*d)*(31*m + 11*m^2 + m^3 + 21))/(176*m +
86*m^2 + 16*m^3 + m^4 + 105) + (a*d*x*(71*m + 15*m^2 + m^3 + 105))/(176*m
+ 86*m^2 + 16*m^3 + m^4 + 105) + (c*e*x^7*(23*m + 9*m^2 + m^3 + 15))/(176*
m + 86*m^2 + 16*m^3 + m^4 + 105))
```

3.223 $\int \frac{(fx)^m(d+ex^2)}{a+bx^2+cx^4} dx$

3.223.1 Optimal result

Integrand size = 27, antiderivative size = 194

$$\int \frac{(fx)^m (d+ex^2)}{a+bx^2+cx^4} dx = \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(fx)^{1+m}\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)f(1+m)} + \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(fx)^{1+m}\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{\left(b+\sqrt{b^2-4ac}\right)f(1+m)}$$

output
```
(f*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-2*c*x^2/(b-(-4*a*c+b^2)^
(1/2)))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))/f/(1+m)/(b-(-4*a*c+b^2)^(1/2))
+(f*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-2*c*x^2/(b+(-4*a*c+b^2)
^(1/2)))*(e+(b*e-2*c*d)/(-4*a*c+b^2)^(1/2))/f/(1+m)/(b+(-4*a*c+b^2)^(1/2))
```

3.223.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.80

$$\int \frac{(fx)^m (d+ex^2)}{a+bx^2+cx^4} dx = \frac{x(fx)^m\left(\left(bd+\sqrt{b^2-4ac}d-2ae\right)\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+\left(-bd+\sqrt{b^2-4ac}d\right.\right.}{2a\sqrt{b^2-4ac}(1+m)}$$

input
```
Integrate[((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4),x]
```

output
```
(x*(f*x)^m*((b*d + Sqrt[b^2 - 4*a*c]*d - 2*a*e)*Hypergeometric2F1[1, (1 +
m)/2, (3 + m)/2, (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (-(b*d) + Sqrt[b^2
- 4*a*c]*d + 2*a*e)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/
(b + Sqrt[b^2 - 4*a*c])]))/(2*a*Sqrt[b^2 - 4*a*c]*(1 + m))
```

3.223.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1608, 27, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(fx)^m}{a+bx^2+cx^4}dx$$

$$\Big\downarrow \text{1608}$$

$$\frac{1}{2}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\int \frac{2(fx)^m}{2cx^2+b-\sqrt{b^2-4ac}}dx+\frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{2(fx)^m}{2cx^2+b+\sqrt{b^2-4ac}}dx$$

$$\Big\downarrow \text{27}$$

$$\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\int \frac{(fx)^m}{2cx^2+b-\sqrt{b^2-4ac}}dx+\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{(fx)^m}{2cx^2+b+\sqrt{b^2-4ac}}dx$$

$$\Big\downarrow \text{278}$$

$$\frac{(fx)^{m+1}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{f(m+1)\left(b-\sqrt{b^2-4ac}\right)}+\frac{(fx)^{m+1}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)\left(\sqrt{b^2-4ac}+b\right)}$$

input
```
Int[((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4),x]
```

output
```
((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*(f*x)^(1 + m)*Hypergeometric2F1[1,
(1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])/((b - Sqrt[b^2
- 4*a*c])*f*(1 + m)) + ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*(f*x)^(1 + m
)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c])])/((b + Sqrt[b^2 - 4*a*c])*f*(1 + m))
```

3.223.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 1608
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2))/((a_) + (b_.)*(x_)^2 + (c_.)
*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d -
b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d
- b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0]
```

3.223.4 Maple [F]

$$\int \frac{(fx)^m\,(e\,x^2+d)}{c\,x^4+b\,x^2+a}dx$$

input
```
int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

output
```
int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

3.223.5 Fricas [F]

$$\int \frac{(fx)^m (d+ex^2)}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)(fx)^m}{cx^4+bx^2+a}\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
integral((e*x^2 + d)*(f*x)^m/(c*x^4 + b*x^2 + a), x)
```

3.223.6 Sympy [F]

$$\int \frac{(fx)^m (d+ex^2)}{a+bx^2+cx^4}\,dx = \int \frac{(fx)^m (d+ex^2)}{a+bx^2+cx^4}\,dx$$

input
```
integrate((f*x)**m*(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

output
```
Integral((f*x)**m*(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

3.223.7 Maxima [F]

$$\int \frac{(fx)^m (d+ex^2)}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)(fx)^m}{cx^4+bx^2+a}\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)*(f*x)^m/(c*x^4 + b*x^2 + a), x)
```

3.223.8 Giac [F]

$$\int \frac{(fx)^m (d+ex^2)}{a+bx^2+cx^4} \, dx = \int \frac{(ex^2+d)(fx)^m}{cx^4+bx^2+a} \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(f*x)^m/(c*x^4 + b*x^2 + a), x)
```

3.223.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^m (d+ex^2)}{a+bx^2+cx^4} \, dx = \int \frac{(f\,x)^m\,(e\,x^2+d)}{c\,x^4+b\,x^2+a} \, dx$$

input
```
int(((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4),x)
```

output
```
int(((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4), x)
```

3.224 $\int \frac{(fx)^m(d+ex^2)}{(a+bx^2+cx^4)^2}\,dx$

3.224.1 Optimal result

Integrand size = 27, antiderivative size = 392

$$\int \frac{(fx)^m\left(d+ex^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^2\right)}{2a\left(b^2-4ac\right)f\left(a+bx^2+cx^4\right)}$$
$$+\frac{c\left(b\left(4ae+\sqrt{b^2-4ac}d(1-m)\right)-2a\left(\sqrt{b^2-4ac}e(1-m)+2cd(3-m)\right)+b^2(d-dm)\right)(fx)^{1+m}\operatorname{Hype}}{2a\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)f(1+m)}$$
$$-\frac{c\left(b\left(4ae-\sqrt{b^2-4ac}d(1-m)\right)+2a\left(\sqrt{b^2-4ac}e(1-m)-2cd(3-m)\right)+b^2d(1-m)\right)(fx)^{1+m}\operatorname{Hype}}{2a\left(b^2-4ac\right)^{3/2}\left(b+\sqrt{b^2-4ac}\right)f(1+m)}$$

output
```
1/2*(f*x)^(1+m)*(b^2*d-2*a*c*d-a*b*e+c*(-2*a*e+b*d)*x^2)/a/(-4*a*c+b^2)/f/
(c*x^4+b*x^2+a)-1/2*c*(f*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-2*
c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(b^2*d*(1-m)+b*(4*a*e-d*(1-m)*(-4*a*c+b^2)^(
1/2))+2*a*(-2*c*d*(3-m)+e*(1-m)*(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)^(3/2)/
f/(1+m)/(b+(-4*a*c+b^2)^(1/2))+1/2*c*(f*x)^(1+m)*hypergeom([1, 1/2+1/2*m],
[3/2+1/2*m],-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))*(b^2*(-d*m+d)+b*(4*a*e+d*(1-m
)*(-4*a*c+b^2)^(1/2))-2*a*(2*c*d*(3-m)+e*(1-m)*(-4*a*c+b^2)^(1/2)))/a/(-4*
a*c+b^2)^(3/2)/f/(1+m)/(b-(-4*a*c+b^2)^(1/2))
```

3.224.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 1.53 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.41

$$\int \frac{(fx)^m (d+ex^2)}{(a+bx^2+cx^4)^2}\,dx = \frac{x(fx)^m \left(d(3+m)\operatorname{AppellF1}\left(\frac{1+m}{2},2,2,\frac{3+m}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+e(1+m)x^2\operatorname{AppellF1}\left(\frac{3+m}{2},2,2,\right.\right.}{a^2(1+m)(3+m)}$$

input
```
Integrate[((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(x*(f*x)^m*(d*(3 + m)*AppellF1[(1 + m)/2, 2, 2, (3 + m)/2, (-2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + e*(1 + m)*x^2*A
ppellF1[(3 + m)/2, 2, 2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2
*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(a^2*(1 + m)*(3 + m))
```

3.224.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(fx)^m}{(a+bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \text{1600}$$

$$\frac{(fx)^{m+1}\left(cx^2(bd-2ae)-abe-2acd+b^2d\right)}{2af(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int -\frac{(fx)^m\left(d(1-m)b^2+ae(m+1)b+c(bd-2ae)(1-m)x^2-2acd(3-m)\right)}{cx^4+bx^2+a}dx}{2a(b^2-4ac)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int -\frac{(fx)^m\left(-((d-dm)b^2)-ae(m+1)b-c(bd-2ae)(1-m)x^2+2acd(3-m)\right)}{cx^4+bx^2+a}dx}{2a(b^2-4ac)}+\frac{(fx)^{m+1}\left(cx^2(bd-2ae)-abe-2acd+b^2d\right)}{2af(b^2-4ac)(a+bx^2+cx^4)}$$

$$\Big\downarrow \text{25}$$

$$\frac{(fx)^{m+1}\left(cx^2(bd-2ae)-abe-2acd+b^2d\right)}{2af\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{(fx)^m\left(d(1-m)b^2+ae(m+1)b+c(bd-2ae)(1-m)x^2-2acd(3-m)\right)}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{(fx)^m\left(-\left((d-dm)b^2\right)-ae(m+1)b-c(bd-2ae)(1-m)x^2+2acd(3-m)\right)}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}+\frac{(fx)^{m+1}\left(cx^2(bd-2ae)-abe-2acd+b^2d\right)}{2af\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{(fx)^{m+1}\left(cx^2(bd-2ae)-abe-2acd+b^2d\right)}{2af\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{(fx)^m\left(d(1-m)b^2+ae(m+1)b+c(bd-2ae)(1-m)x^2-2acd(3-m)\right)}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{(fx)^m\left(-\left((d-dm)b^2\right)-ae(m+1)b-c(bd-2ae)(1-m)x^2+2acd(3-m)\right)}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}+\frac{(fx)^{m+1}\left(cx^2(bd-2ae)-abe-2acd+b^2d\right)}{2af\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{(fx)^{m+1}\left(cx^2(bd-2ae)-abe-2acd+b^2d\right)}{2af\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{(fx)^m\left(d(1-m)b^2+ae(m+1)b+c(bd-2ae)(1-m)x^2-2acd(3-m)\right)}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{(fx)^m\left(-\left((d-dm)b^2\right)-ae(m+1)b-c(bd-2ae)(1-m)x^2+2acd(3-m)\right)}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}+\frac{(fx)^{m+1}\left(cx^2(bd-2ae)-abe-2acd+b^2d\right)}{2af\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{(fx)^{m+1}\left(cx^2(bd-2ae)-abe-2acd+b^2d\right)}{2af\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{(fx)^m\left(d(1-m)b^2+ae(m+1)b+c(bd-2ae)(1-m)x^2-2acd(3-m)\right)}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}$$

3.224. $\int \frac{(fx)^m\left(d+ex^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

$$\Big\downarrow 25$$

$$\frac{\int -\frac{(fx)^m\left(-\left((d-dm)b^2\right)-ae(m+1)b-c(bd-2ae)(1-m)x^2+2acd(3-m)\right)}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}+\frac{(fx)^{m+1}\left(cx^2(bd-2ae)-abe-2acd+b^2d\right)}{2af\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow 25$$

$$\frac{(fx)^{m+1}\left(cx^2(bd-2ae)-abe-2acd+b^2d\right)}{2af\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{(fx)^m\left(d(1-m)b^2+ae(m+1)b+c(bd-2ae)(1-m)x^2-2acd(3-m)\right)}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}$$

$$\Big\downarrow 25$$

$$\frac{\int -\frac{(fx)^m\left(-\left((d-dm)b^2\right)-ae(m+1)b-c(bd-2ae)(1-m)x^2+2acd(3-m)\right)}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}+\frac{(fx)^{m+1}\left(cx^2(bd-2ae)-abe-2acd+b^2d\right)}{2af\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow 25$$

$$\frac{(fx)^{m+1}\left(cx^2(bd-2ae)-abe-2acd+b^2d\right)}{2af\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{(fx)^m\left(d(1-m)b^2+ae(m+1)b+c(bd-2ae)(1-m)x^2-2acd(3-m)\right)}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}$$

$$\Big\downarrow 25$$

$$\frac{\int -\frac{(fx)^m\left(-\left((d-dm)b^2\right)-ae(m+1)b-c(bd-2ae)(1-m)x^2+2acd(3-m)\right)}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}+\frac{(fx)^{m+1}\left(cx^2(bd-2ae)-abe-2acd+b^2d\right)}{2af\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow 25$$

$$\frac{(fx)^{m+1}\left(cx^2(bd-2ae)-abe-2acd+b^2d\right)}{2af\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{(fx)^m\left(d(1-m)b^2+ae(m+1)b+c(bd-2ae)(1-m)x^2-2acd(3-m)\right)}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}$$

$$\Big\downarrow 25$$

3.224. $\int \frac{(fx)^m\left(d+ex^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

$$\frac{\int -\frac{(fx)^m\left(-\left((d-dm)b^2\right)-ae(m+1)b-c(bd-2ae)(1-m)x^2+2acd(3-m)\right)}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}+\frac{(fx)^{m+1}\left(cx^2(bd-2ae)-abe-2acd+b^2d\right)}{2af\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{(fx)^{m+1}\left(cx^2(bd-2ae)-abe-2acd+b^2d\right)}{2af\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{(fx)^m\left(d(1-m)b^2+ae(m+1)b+c(bd-2ae)(1-m)x^2-2acd(3-m)\right)}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{(fx)^m\left(-\left((d-dm)b^2\right)-ae(m+1)b-c(bd-2ae)(1-m)x^2+2acd(3-m)\right)}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}+\frac{(fx)^{m+1}\left(cx^2(bd-2ae)-abe-2acd+b^2d\right)}{2af\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{(fx)^{m+1}\left(cx^2(bd-2ae)-abe-2acd+b^2d\right)}{2af\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{(fx)^m\left(d(1-m)b^2+ae(m+1)b+c(bd-2ae)(1-m)x^2-2acd(3-m)\right)}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{(fx)^m\left(-\left((d-dm)b^2\right)-ae(m+1)b-c(bd-2ae)(1-m)x^2+2acd(3-m)\right)}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}+\frac{(fx)^{m+1}\left(cx^2(bd-2ae)-abe-2acd+b^2d\right)}{2af\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{(fx)^{m+1}\left(cx^2(bd-2ae)-abe-2acd+b^2d\right)}{2af\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{(fx)^m\left(d(1-m)b^2+ae(m+1)b+c(bd-2ae)(1-m)x^2-2acd(3-m)\right)}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{(fx)^m\left(-\left((d-dm)b^2\right)-ae(m+1)b-c(bd-2ae)(1-m)x^2+2acd(3-m)\right)}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}+\frac{(fx)^{m+1}\left(cx^2(bd-2ae)-abe-2acd+b^2d\right)}{2af\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

3.224. $\int \frac{(fx)^m\left(d+ex^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

$$\Big\downarrow \text{25}$$

$$\frac{(fx)^{m+1}\left(cx^2(bd-2ae)-abe-2acd+b^2d\right)}{2af\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{(fx)^m\left(d(1-m)b^2+ae(m+1)b+c(bd-2ae)(1-m)x^2-2acd(3-m)\right)}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int -\frac{(fx)^m\left(-\left((d-dm)b^2\right)-ae(m+1)b-c(bd-2ae)(1-m)x^2+2acd(3-m)\right)}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}+\frac{(fx)^{m+1}\left(cx^2(bd-2ae)-abe-2acd+b^2d\right)}{2af\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{(fx)^{m+1}\left(cx^2(bd-2ae)-abe-2acd+b^2d\right)}{2af\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{(fx)^m\left(d(1-m)b^2+ae(m+1)b+c(bd-2ae)(1-m)x^2-2acd(3-m)\right)}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int -\frac{(fx)^m\left(-\left((d-dm)b^2\right)-ae(m+1)b-c(bd-2ae)(1-m)x^2+2acd(3-m)\right)}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}+\frac{(fx)^{m+1}\left(cx^2(bd-2ae)-abe-2acd+b^2d\right)}{2af\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{(fx)^{m+1}\left(cx^2(bd-2ae)-abe-2acd+b^2d\right)}{2af\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{(fx)^m\left(d(1-m)b^2+ae(m+1)b+c(bd-2ae)(1-m)x^2-2acd(3-m)\right)}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int -\frac{(fx)^m\left(-\left((d-dm)b^2\right)-ae(m+1)b-c(bd-2ae)(1-m)x^2+2acd(3-m)\right)}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}+\frac{(fx)^{m+1}\left(cx^2(bd-2ae)-abe-2acd+b^2d\right)}{2af\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \text{25}$$

3.224. $\int \frac{(fx)^m\left(d+ex^2\right)}{\left(a+bx^2+cx^4\right)^2}dx$

$$\frac{(fx)^{m+1}\left(cx^2(bd-2ae)-abe-2acd+b^2d\right)}{2af\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{(fx)^m\left(d(1-m)b^2+ae(m+1)b+c(bd-2ae)(1-m)x^2-2acd(3-m)\right)}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{\int -\frac{(fx)^m\left(-\left((d-dm)b^2\right)-ae(m+1)b-c(bd-2ae)(1-m)x^2+2acd(3-m)\right)}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}+\frac{(fx)^{m+1}\left(cx^2(bd-2ae)-abe-2acd+b^2d\right)}{2af\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

input
```
Int[((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
$Aborted
```

3.224.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1600
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^2 + c*x^4)^(p + 1)
*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2)/(2*a*f*(p + 1)*(b^2 - 4*a
*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^m*(a + b*x^2 + c
*x^4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1))
- a*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Int
egerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.224.4 Maple [F]

$$\int \frac{(fx)^m\left(e\,x^2+d\right)}{\left(c\,x^4+b\,x^2+a\right)^2}dx$$

input
```
int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

output
```
int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

3.224.5 Fricas [F]

$$\int \frac{(fx)^m (d+ex^2)}{(a+bx^2+cx^4)^2}\, dx = \int \frac{(ex^2+d)(fx)^m}{(cx^4+bx^2+a)^2}\, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
integral((e*x^2 + d)*(f*x)^m/(c^2*x^8 + 2*b*c*x^6 + (b^2 + 2*a*c)*x^4 + 2*
a*b*x^2 + a^2), x)
```

3.224.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(fx)^m (d+ex^2)}{(a+bx^2+cx^4)^2}\, dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.224.7 Maxima [F]

$$\int \frac{(fx)^m (d+ex^2)}{(a+bx^2+cx^4)^2}\, dx = \int \frac{(ex^2+d)(fx)^m}{(cx^4+bx^2+a)^2}\, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)*(f*x)^m/(c*x^4 + b*x^2 + a)^2, x)
```

3.224.8 Giac [F]

$$\int \frac{(fx)^m (d+ex^2)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(ex^2+d)(fx)^m}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(f*x)^m/(c*x^4 + b*x^2 + a)^2, x)
```

3.224.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^m (d+ex^2)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(f\,x)^m\,(e\,x^2+d)}{(c\,x^4+b\,x^2+a)^2}\,dx$$

input
```
int(((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

output
```
int(((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^2, x)
```

3.225 $\int (fx)^m (d+ex^2) (a+bx^2+cx^4)^{3/2} dx$

3.225.1 Optimal result

Integrand size = 29, antiderivative size = 319

$$\int (fx)^m (d+ex^2) (a+bx^2+cx^4)^{3/2} dx = \frac{ad(fx)^{1+m}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{1+m}{2},-\frac{3}{2},-\frac{3}{2},\frac{3+m}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(1+m)\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{ae(fx)^{3+m}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{3+m}{2},-\frac{3}{2},-\frac{3}{2},\frac{5+m}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f^3(3+m)\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

output
```
a*d*(f*x)^(1+m)*AppellF1(1/2+1/2*m,-3/2,-3/2,3/2+1/2*m,-2*c*x^2/(b-(-4*a*c
+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f/(1+m
)/(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2
)))^(1/2)+a*e*(f*x)^(3+m)*AppellF1(3/2+1/2*m,-3/2,-3/2,5/2+1/2*m,-2*c*x^2/
(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1
/2)/f^3/(3+m)/(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a
*c+b^2)^(1/2)))^(1/2)
```

3.225.2 Mathematica [A] (warning: unable to verify)

Time = 2.90 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.46

$$\int (fx)^m \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^{3/2} dx = \frac{x(fx)^m\sqrt{a+bx^2+cx^4}\left(ad(105+71m+15m^2+m^3)\,\text{AppellF1}\left(\frac{1+m}{2},-\frac{1}{2},-\frac{1}{2},\frac{3+m}{2},-\frac{2cx^2}{b+\sqrt{b^2-}}\right.\right.}{}$$

input
```
Integrate[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(x*(f*x)^m*Sqrt[a + b*x^2 + c*x^4]*(a*d*(105 + 71*m + 15*m^2 + m^3)*Appell
F1[(1 + m)/2, -1/2, -1/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (
2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (1 + m)*x^2*((b*d + a*e)*(35 + 12*m +
 m^2)*AppellF1[(3 + m)/2, -1/2, -1/2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (3 + m)*x^2*((c*d + b*e)*
(7 + m)*AppellF1[(5 + m)/2, -1/2, -1/2, (7 + m)/2, (-2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + c*e*(5 + m)*x^2*AppellF
1[(7 + m)/2, -1/2, -1/2, (9 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2
*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))))/((1 + m)*(3 + m)*(5 + m)*(7 + m)*Sqr
t[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqr
t[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

3.225.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1674, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)(fx)^m \left(a+bx^2+cx^4\right)^{3/2} dx$$

$$\Big\downarrow \text{1674}$$

$$\int \left(d(fx)^m \left(a+bx^2+cx^4\right)^{3/2}+\frac{e(fx)^{m+2}\left(a+bx^2+cx^4\right)^{3/2}}{f^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ad(fx)^{m+1}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{m+1}{2},-\frac{3}{2},-\frac{3}{2},\frac{m+3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{ae(fx)^{m+3}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{m+3}{2},-\frac{3}{2},-\frac{3}{2},\frac{m+5}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f^3(m+3)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(a*d*(f*x)^(1 + m)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[(1 + m)/2, -3/2, -3/2,
 (3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 -
 4*a*c])])/(f*(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
 (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]) + (a*e*(f*x)^(3 + m)*Sqrt[a + b*x^2 +
 c*x^4]*AppellF1[(3 + m)/2, -3/2, -3/2, (5 + m)/2, (-2*c*x^2)/(b - Sqrt[b^
2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f^3*(3 + m)*Sqrt[1 + (2
*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
)])
```

3.225.3.1 Defintions of rubi rules used

rule 1674
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.225.4 Maple [F]

$$\int (fx)^m \left(e\,x^2+d\right) \left(c\,x^4+b\,x^2+a\right)^{\frac{3}{2}} dx$$

input
```
int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x)
```

3.225.5 Fricas [F]

$$\int (fx)^m (d+ex^2) (a+bx^2+cx^4)^{3/2} \, dx = \int (cx^4+bx^2+a)^{\frac{3}{2}} (ex^2+d)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
integral((c*e*x^6 + (c*d + b*e)*x^4 + (b*d + a*e)*x^2 + a*d)*sqrt(c*x^4 +
b*x^2 + a)*(f*x)^m, x)
```

3.225.6 Sympy [F]

$$\int (fx)^m (d+ex^2) (a+bx^2+cx^4)^{3/2} \, dx = \int (fx)^m (d+ex^2) (a+bx^2+cx^4)^{\frac{3}{2}} \, dx$$

input
```
integrate((f*x)**m*(e*x**2+d)*(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral((f*x)**m*(d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2), x)
```

3.225.7 Maxima [F]

$$\int (fx)^m (d+ex^2) (a+bx^2+cx^4)^{3/2} \, dx = \int (cx^4+bx^2+a)^{\frac{3}{2}} (ex^2+d)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*(f*x)^m, x)
```

3.225.8 Giac [F]

$$\int (fx)^m \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^{3/2} \, dx = \int \left(cx^4+bx^2+a\right)^{\frac{3}{2}} \left(ex^2+d\right) (fx)^m \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*(f*x)^m, x)
```

3.225.9 Mupad [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^{3/2} \, dx = \int (f\,x)^m \,\left(e\,x^2+d\right)\,\left(c\,x^4+b\,x^2+a\right)^{3/2} \,dx$$

input
```
int((f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int((f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2), x)
```

3.226 $\int (fx)^m (d+ex^2)\sqrt{a+bx^2+cx^4}\,dx$

3.226.1 Optimal result

Integrand size = 29, antiderivative size = 317

$$\int (fx)^m (d+ex^2)\sqrt{a+bx^2+cx^4}\,dx = \frac{d(fx)^{1+m}\sqrt{a+bx^2+cx^4}\operatorname{AppellF1}\left(\frac{1+m}{2},-\frac{1}{2},-\frac{1}{2},\frac{3+m}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(1+m)\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{e(fx)^{3+m}\sqrt{a+bx^2+cx^4}\operatorname{AppellF1}\left(\frac{3+m}{2},-\frac{1}{2},-\frac{1}{2},\frac{5+m}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f^3(3+m)\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

output
```
d*(f*x)^(1+m)*AppellF1(1/2+1/2*m,-1/2,-1/2,3/2+1/2*m,-2*c*x^2/(b-(-4*a*c+b
^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f/(1+m)/
(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2))
)^(1/2)+e*(f*x)^(3+m)*AppellF1(3/2+1/2*m,-1/2,-1/2,5/2+1/2*m,-2*c*x^2/(b-(
-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/
f^3/(3+m)/(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b
^2)^(1/2)))^(1/2)
```

3.226.2 Mathematica [A] (verified)

Time = 1.98 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.84

$$\int (fx)^m \left(d+ex^2\right) \sqrt{a+bx^2+cx^4}\,dx$$
$$= \frac{x(fx)^m\sqrt{a+bx^2+cx^4}\left(d(3+m)\,\text{AppellF1}\left(\frac{1+m}{2},-\frac{1}{2},-\frac{1}{2},\frac{3+m}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+e(1+m)x^2\,\text{A}\right.}{(1+m)(3+m)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}}$$

input
```
Integrate[(f*x)^m*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(x*(f*x)^m*Sqrt[a + b*x^2 + c*x^4]*(d*(3 + m)*AppellF1[(1 + m)/2, -1/2, -1
/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^
2 - 4*a*c])] + e*(1 + m)*x^2*AppellF1[(3 + m)/2, -1/2, -1/2, (5 + m)/2, (-
2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/((
1 + m)*(3 + m)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*
c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

3.226.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1674, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)(fx)^m\sqrt{a+bx^2+cx^4}\,dx$$
$$\Big\downarrow \; 1674$$
$$\int \left(d(fx)^m\sqrt{a+bx^2+cx^4}+\frac{e(fx)^{m+2}\sqrt{a+bx^2+cx^4}}{f^2}\right)dx$$
$$\Big\downarrow \; 2009$$

$$\frac{d(fx)^{m+1}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{m+1}{2},-\frac{1}{2},-\frac{1}{2},\frac{m+3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{e(fx)^{m+3}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{m+3}{2},-\frac{1}{2},-\frac{1}{2},\frac{m+5}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f^3(m+3)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[(f*x)^m*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(d*(f*x)^(1 + m)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[(1 + m)/2, -1/2, -1/2, (
3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4
*a*c])])/(f*(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (
2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]) + (e*(f*x)^(3 + m)*Sqrt[a + b*x^2 + c*x
^4]*AppellF1[(3 + m)/2, -1/2, -1/2, (5 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 -
4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f^3*(3 + m)*Sqrt[1 + (2*c*x
^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

3.226.3.1 Defintions of rubi rules used

rule 1674
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.226.4 Maple [F]

$$\int (fx)^m \left(e\,x^2+d\right)\sqrt{c\,x^4+b\,x^2+a}dx$$

input
```
int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x)
```

3.226.5 Fricas [F]

$$\int (fx)^m \left(d+ex^2\right) \sqrt{a+bx^2+cx^4}\,dx = \int \sqrt{cx^4+bx^2+a}\left(ex^2+d\right)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*(f*x)^m, x)
```

3.226.6 Sympy [F]

$$\int (fx)^m \left(d+ex^2\right) \sqrt{a+bx^2+cx^4}\,dx = \int (fx)^m \left(d+ex^2\right) \sqrt{a+bx^2+cx^4}\,dx$$

input
```
integrate((f*x)**m*(e*x**2+d)*(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((f*x)**m*(d + e*x**2)*sqrt(a + b*x**2 + c*x**4), x)
```

3.226.7 Maxima [F]

$$\int (fx)^m \left(d+ex^2\right) \sqrt{a+bx^2+cx^4}\,dx = \int \sqrt{cx^4+bx^2+a}\left(ex^2+d\right)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*(f*x)^m, x)
```

3.226.8 Giac [F]

$$\int (fx)^m \left(d+ex^2\right) \sqrt{a+bx^2+cx^4}\,dx = \int \sqrt{cx^4+bx^2+a}\left(ex^2+d\right)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*(f*x)^m, x)
```

3.226.9 Mupad [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right) \sqrt{a+bx^2+cx^4}\,dx = \int (f\,x)^m\,\left(e\,x^2+d\right)\,\sqrt{c\,x^4+b\,x^2+a}\,dx$$

input
```
int((f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2), x)
```

3.227 $\int \frac{(fx)^m(d+ex^2)}{\sqrt{a+bx^2+cx^4}} \, dx$

3.227.1 Optimal result

Integrand size = 29, antiderivative size = 317

$$\int \frac{(fx)^m (d+ex^2)}{\sqrt{a+bx^2+cx^4}} \, dx = \frac{d(fx)^{1+m}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1+m}{2},\frac{1}{2},\frac{1}{2},\frac{3+m}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(1+m)\sqrt{a+bx^2+cx^4}} + \frac{e(fx)^{3+m}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{3+m}{2},\frac{1}{2},\frac{1}{2},\frac{5+m}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f^3(3+m)\sqrt{a+bx^2+cx^4}}$$

output
```
d*(f*x)^(1+m)*AppellF1(1/2+1/2*m,1/2,1/2,3/2+1/2*m,-2*c*x^2/(b-(-4*a*c+b^2
)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)
))^(1/2)*(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/f/(1+m)/(c*x^4+b*x^2+a)^
(1/2)+e*(f*x)^(3+m)*AppellF1(3/2+1/2*m,1/2,1/2,5/2+1/2*m,-2*c*x^2/(b-(-4*a
*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)
^(1/2)))^(1/2)*(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/f^3/(3+m)/(c*x^4+b
*x^2+a)^(1/2)
```

3.227.2 Mathematica [A] (verified)

Time = 2.83 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.84

$$\int \frac{(fx)^m (d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \frac{x(fx)^m\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\left(d(3+m)\,\text{AppellF1}\left(\frac{1+m}{2},\frac{1}{2},\frac{1}{2},\frac{3+m}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+\right.}{(1+m)(3+m)\sqrt{a+bx^2+cx^4}}$$

input
```
Integrate[((f*x)^m*(d + e*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(x*(f*x)^m*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]
*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*(d*(3 + m
)*AppellF1[(1 + m)/2, 1/2, 1/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*
c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + e*(1 + m)*x^2*AppellF1[(3 + m)/
2, 1/2, 1/2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b
+ Sqrt[b^2 - 4*a*c])]))/((1 + m)*(3 + m)*Sqrt[a + b*x^2 + c*x^4])
```

3.227.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1674, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(fx)^m}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$\Big\downarrow \text{1674}$$
$$\int \left(\frac{d(fx)^m}{\sqrt{a+bx^2+cx^4}}+\frac{e(fx)^{m+2}}{f^2\sqrt{a+bx^2+cx^4}}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{d(fx)^{m+1}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{m+1}{2},\frac{1}{2},\frac{1}{2},\frac{m+3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)\sqrt{a+bx^2+cx^4}}+$$
$$\frac{e(fx)^{m+3}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{m+3}{2},\frac{1}{2},\frac{1}{2},\frac{m+5}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f^3(m+3)\sqrt{a+bx^2+cx^4}}$$

input
```
Int[((f*x)^m*(d + e*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(d*(f*x)^(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/2, 1/2, 1/2, (3 + m)/2, (-
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f*
(1 + m)*Sqrt[a + b*x^2 + c*x^4]) + (e*(f*x)^(3 + m)*Sqrt[1 + (2*c*x^2)/(b
- Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1
[(3 + m)/2, 1/2, 1/2, (5 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f^3*(3 + m)*Sqrt[a + b*x^2 + c*x^4])
```

3.227.3.1 Defintions of rubi rules used

rule 1674
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.227.4 Maple [F]

$$\int \frac{(fx)^m (e\,x^2 + d)}{\sqrt{c\,x^4 + b\,x^2 + a}}dx$$

input
```
int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

3.227.5 Fricas [F]

$$\int \frac{(fx)^m (d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(ex^2+d)(fx)^m}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral((e*x^2 + d)*(f*x)^m/sqrt(c*x^4 + b*x^2 + a), x)
```

3.227.6 Sympy [F]

$$\int \frac{(fx)^m (d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(fx)^m (d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((f*x)**m*(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((f*x)**m*(d + e*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

3.227.7 Maxima [F]

$$\int \frac{(fx)^m (d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(ex^2+d)(fx)^m}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)*(f*x)^m/sqrt(c*x^4 + b*x^2 + a), x)
```

3.227.8 Giac [F]

$$\int \frac{(fx)^m (d+ex^2)}{\sqrt{a+bx^2+cx^4}} dx = \int \frac{(ex^2+d)(fx)^m}{\sqrt{cx^4+bx^2+a}} dx$$

input
```
integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(f*x)^m/sqrt(c*x^4 + b*x^2 + a), x)
```

3.227.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^m (d+ex^2)}{\sqrt{a+bx^2+cx^4}} dx = \int \frac{(f\,x)^m\,(e\,x^2+d)}{\sqrt{c\,x^4+b\,x^2+a}} dx$$

input
```
int(((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int(((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.228 $\int \frac{(fx)^m(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx$

3.228.1 Optimal result

Integrand size = 29, antiderivative size = 323

$$\int \frac{(fx)^m\,(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{d(fx)^{1+m}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{1+m}{2},\frac{3}{2},\frac{3}{2},\frac{3+m}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},\right.}{af(1+m)\sqrt{a+bx^2+cx^4}}$$
$$+\frac{e(fx)^{3+m}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{3+m}{2},\frac{3}{2},\frac{3}{2},\frac{5+m}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{af^3(3+m)\sqrt{a+bx^2+cx^4}}$$

output
```
d*(f*x)^(1+m)*AppellF1(1/2+1/2*m,3/2,3/2,3/2+1/2*m,-2*c*x^2/(b-(-4*a*c+b^2
)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)
))^(1/2)*(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/f/(1+m)/(c*x^4+b*x^2+a
)^(1/2)+e*(f*x)^(3+m)*AppellF1(3/2+1/2*m,3/2,3/2,5/2+1/2*m,-2*c*x^2/(b-(-4
*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^
2)^(1/2)))^(1/2)*(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/f^3/(3+m)/(c*x
^4+b*x^2+a)^(1/2)
```

3.228.2 Mathematica [A] (verified)

Time = 11.34 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.95

$$\int \frac{(fx)^m (d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{x(fx)^m\left(-b+\sqrt{b^2-4ac}-2cx^2\right)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}\left(d(3+m\right.}{\left(-b+\right.}$$

input
```
Integrate[((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(x*(f*x)^m*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)*Sqrt[(b - Sqrt[b^2 - 4*a*c]
+ 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c]))^(3/2)*(d*(3 + m)*AppellF1[(1 + m)/2, 3/2, 3/2, (3 +
m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c
])] + e*(1 + m)*x^2*AppellF1[(3 + m)/2, 3/2, 3/2, (5 + m)/2, (-2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/((-b + Sqrt[b
^2 - 4*a*c])*(1 + m)*(3 + m)*(a + b*x^2 + c*x^4)^(3/2))
```

3.228.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1674, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(fx)^m}{(a+bx^2+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1674}$$

$$\int \left(\frac{d(fx)^m}{(a+bx^2+cx^4)^{3/2}}+\frac{e(fx)^{m+2}}{f^2(a+bx^2+cx^4)^{3/2}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{d(fx)^{m+1}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{m+1}{2},\frac{3}{2},\frac{3}{2},\frac{m+3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{af(m+1)\sqrt{a+bx^2+cx^4}}+\frac{e(fx)^{m+3}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{m+3}{2},\frac{3}{2},\frac{3}{2},\frac{m+5}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{af^3(m+3)\sqrt{a+bx^2+cx^4}}$$

input
```
Int[((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(d*(f*x)^(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/2, 3/2, 3/2, (3 + m)/2, (-
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(a*
f*(1 + m)*Sqrt[a + b*x^2 + c*x^4]) + (e*(f*x)^(3 + m)*Sqrt[1 + (2*c*x^2)/(
b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Appell
F1[(3 + m)/2, 3/2, 3/2, (5 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2
*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(a*f^3*(3 + m)*Sqrt[a + b*x^2 + c*x^4])
```

3.228.3.1 Defintions of rubi rules used

rule 1674
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.228.4 Maple [F]

$$\int \frac{(fx)^m (e\,x^2+d)}{(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

3.228.5 Fricas [F]

$$\int \frac{(fx)^m (d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(ex^2+d)(fx)^m}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*(f*x)^m/(c^2*x^8 + 2*b*c*x^6
+ (b^2 + 2*a*c)*x^4 + 2*a*b*x^2 + a^2), x)
```

3.228.6 Sympy [F]

$$\int \frac{(fx)^m (d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(fx)^m (d+ex^2)}{(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x)**m*(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral((f*x)**m*(d + e*x**2)/(a + b*x**2 + c*x**4)**(3/2), x)
```

3.228.7 Maxima [F]

$$\int \frac{(fx)^m (d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(ex^2+d)(fx)^m}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)*(f*x)^m/(c*x^4 + b*x^2 + a)^(3/2), x)
```

3.228.8 Giac [F]

$$\int \frac{(fx)^m (d+ex^2)}{(a+bx^2+cx^4)^{3/2}} \, dx = \int \frac{(ex^2+d)(fx)^m}{(cx^4+bx^2+a)^{\frac{3}{2}}} \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(f*x)^m/(c*x^4 + b*x^2 + a)^(3/2), x)
```

3.228.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^m (d+ex^2)}{(a+bx^2+cx^4)^{3/2}} \, dx = \int \frac{(f\,x)^m\,(e\,x^2+d)}{(c\,x^4+b\,x^2+a)^{3/2}} \, dx$$

input
```
int(((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2), x)
```

3.229 $\int \frac{x^9}{(d+ex^2)(a+cx^4)}\,dx$

3.229.1 Optimal result

Integrand size = 22, antiderivative size = 134

$$\int \frac{x^9}{(d+ex^2)(a+cx^4)}\,dx = -\frac{dx^2}{2ce^2}+\frac{x^4}{4ce}+\frac{a^{3/2}d\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2c^{3/2}(cd^2+ae^2)}+\frac{d^4\log(d+ex^2)}{2e^3(cd^2+ae^2)}-\frac{a^2e\log(a+cx^4)}{4c^2(cd^2+ae^2)}$$

output
```
-1/2*d*x^2/c/e^2+1/4*x^4/c/e+1/2*a^(3/2)*d*arctan(x^2*c^(1/2)/a^(1/2))/c^(
3/2)/(a*e^2+c*d^2)+1/2*d^4*ln(e*x^2+d)/e^3/(a*e^2+c*d^2)-1/4*a^2*e*ln(c*x^
4+a)/c^2/(a*e^2+c*d^2)
```

3.229.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00

$$\int \frac{x^9}{(d+ex^2)(a+cx^4)}\,dx = -\frac{dx^2}{2ce^2}+\frac{x^4}{4ce}+\frac{a^{3/2}d\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2c^{3/2}(cd^2+ae^2)}+\frac{d^4\log(d+ex^2)}{2e^3(cd^2+ae^2)}-\frac{a^2e\log(a+cx^4)}{4c^2(cd^2+ae^2)}$$

input
```
Integrate[x^9/((d + e*x^2)*(a + c*x^4)),x]
```

output
```
-1/2*(d*x^2)/(c*e^2) + x^4/(4*c*e) + (a^(3/2)*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[
a]])/(2*c^(3/2)*(c*d^2 + a*e^2)) + (d^4*Log[d + e*x^2])/(2*e^3*(c*d^2 + a*
e^2)) - (a^2*e*Log[a + c*x^4])/(4*c^2*(c*d^2 + a*e^2))
```

3.229.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1579, 604, 27, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{(a+cx^4)(d+ex^2)}dx$$

$$\Big\downarrow \text{1579}$$

$$\frac{1}{2}\int \frac{x^8}{(ex^2+d)(cx^4+a)}dx^2$$

$$\Big\downarrow \text{604}$$

$$\frac{1}{2}\left(\frac{\int -\frac{2(2cde^3x^6+e^2(cd^2+ae^2)x^4+2ade^3x^2+ad^2e^2)}{(ex^2+d)(cx^4+a)}dx^2}{2ce^4}+\frac{(d+ex^2)^2}{2ce^3}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{(d+ex^2)^2}{2ce^3}-\frac{\int \frac{2cde^3x^6+e^2(cd^2+ae^2)x^4+2ade^3x^2+ad^2e^2}{(ex^2+d)(cx^4+a)}dx^2}{ce^4}\right)$$

$$\Big\downarrow \text{2160}$$

$$\frac{1}{2}\left(\frac{(d+ex^2)^2}{2ce^3}-\frac{\int \left(-\frac{ce^2d^4}{(cd^2+ae^2)(ex^2+d)}+2e^2d+\frac{a^2e^4(ex^2-d)}{(cd^2+ae^2)(cx^4+a)}\right)dx^2}{ce^4}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{(d+ex^2)^2}{2ce^3}-\frac{-\frac{a^{3/2}de^4\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{c}(ae^2+cd^2)}+\frac{a^2e^5\log(a+cx^4)}{2c(ae^2+cd^2)}-\frac{cd^4e\log(d+ex^2)}{ae^2+cd^2}+2de^2x^2}{ce^4}\right)$$

input
```
Int[x^9/((d + e*x^2)*(a + c*x^4)),x]
```

output
```
((d + e*x^2)^2/(2*c*e^3) - (2*d*e^2*x^2 - (a^(3/2)*d*e^4*ArcTan[(Sqrt[c]*x
^2)/Sqrt[a]])/(Sqrt[c]*(c*d^2 + a*e^2)) - (c*d^4*e*Log[d + e*x^2])/(c*d^2
+ a*e^2) + (a^2*e^5*Log[a + c*x^4])/(2*c*(c*d^2 + a*e^2)))/(c*e^4))/2
```

3.229.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 604
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(b*d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*
x^2)^p*ExpandToSum[b*d^m*(m + n + 2*p + 1)*x^m - b*(m + n + 2*p + 1)*(c + d
*x)^m - (c + d*x)^(m - 2)*(a*d^2*(m + n - 1) - b*c^2*(m + n + 2*p + 1) - 2*
b*c*d*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && IGtQ[m,
 1] && NeQ[m + n + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.229.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.77

method	result
default	$\frac{(-e\,x^2+d)^2}{4c\,e^3}+\frac{a^2\left(-\frac{e\ln\left(c\,x^4+a\right)}{2c}+\frac{d\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{\sqrt{ac}}\right)}{2(a\,e^2+c\,d^2)c}+\frac{d^4\ln(e\,x^2+d)}{2e^3(a\,e^2+c\,d^2)}$
risch	$\frac{x^4}{4ce}-\frac{d\,x^2}{2c\,e^2}+\frac{d^2}{4c\,e^3}+\frac{d^4\ln(e\,x^2+d)}{2e^3(a\,e^2+c\,d^2)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\left(a\,c^2e^2+c^3d^2\right)\_Z^2+2a^2c\,e^3\_Z+e^4a^3\right)}\_R\ln\left(\left(\left(2a\,c^2e^4-2c^3d^2e^2\right)\_R^2\right.\right.}{}$

input
```
int(x^9/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-e*x^2+d)^2/c/e^3+1/2*a^2/(a*e^2+c*d^2)/c*(-1/2*e/c*ln(c*x^4+a)+d/(a*
c)^(1/2)*arctan(c*x^2/(a*c)^(1/2)))+1/2*d^4*ln(e*x^2+d)/e^3/(a*e^2+c*d^2)
```

3.229.5 Fricas [A] (verification not implemented)

Time = 2.89 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.07

$$\int\frac{x^9}{(d+ex^2)(a+cx^4)}\,dx$$
$$=\left[\frac{acde^3\sqrt{-\frac{a}{c}}\log\left(\frac{cx^4+2\,cx^2\sqrt{-\frac{a}{c}}-a}{cx^4+a}\right)-a^2e^4\log\left(cx^4+a\right)+2\,c^2d^4\log\left(ex^2+d\right)+(c^2d^2e^2+ace^4)x^4-2\,(c^2}{4\left(c^3d^2e^3+ac^2e^5\right)}\right.$$

input
```
integrate(x^9/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

output
```
[1/4*(a*c*d*e^3*sqrt(-a/c)*log((c*x^4 + 2*c*x^2*sqrt(-a/c) - a)/(c*x^4 + a
)) - a^2*e^4*log(c*x^4 + a) + 2*c^2*d^4*log(e*x^2 + d) + (c^2*d^2*e^2 + a*
c*e^4)*x^4 - 2*(c^2*d^3*e + a*c*d*e^3)*x^2)/(c^3*d^2*e^3 + a*c^2*e^5), 1/4
*(2*a*c*d*e^3*sqrt(a/c)*arctan(c*x^2*sqrt(a/c)/a) - a^2*e^4*log(c*x^4 + a)
 + 2*c^2*d^4*log(e*x^2 + d) + (c^2*d^2*e^2 + a*c*e^4)*x^4 - 2*(c^2*d^3*e +
 a*c*d*e^3)*x^2)/(c^3*d^2*e^3 + a*c^2*e^5)]
```

3.229.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^9}{(d+ex^2)(a+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x**9/(e*x**2+d)/(c*x**4+a),x)
```

output
```
Timed out
```

3.229.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.90

$$\int \frac{x^9}{(d+ex^2)(a+cx^4)}\,dx = \frac{d^4\log(ex^2+d)}{2(cd^2e^3+ae^5)} - \frac{a^2e\log(cx^4+a)}{4(c^3d^2+ac^2e^2)} + \frac{a^2d\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2(c^2d^2+ace^2)\sqrt{ac}} + \frac{ex^4-2\,dx^2}{4\,ce^2}$$

input
```
integrate(x^9/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

output
```
1/2*d^4*log(e*x^2 + d)/(c*d^2*e^3 + a*e^5) - 1/4*a^2*e*log(c*x^4 + a)/(c^3
*d^2 + a*c^2*e^2) + 1/2*a^2*d*arctan(c*x^2/sqrt(a*c))/((c^2*d^2 + a*c*e^2)
*sqrt(a*c)) + 1/4*(e*x^4 - 2*d*x^2)/(c*e^2)
```

3.229.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.92

$$\int \frac{x^9}{(d+ex^2)(a+cx^4)}\,dx = \frac{d^4\log(|ex^2+d|)}{2(cd^2e^3+ae^5)} - \frac{a^2e\log(cx^4+a)}{4(c^3d^2+ac^2e^2)} + \frac{a^2d\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2(c^2d^2+ace^2)\sqrt{ac}} + \frac{cex^4-2\,cdx^2}{4\,c^2e^2}$$

input
```
integrate(x^9/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

output
```
1/2*d^4*log(abs(e*x^2 + d))/(c*d^2*e^3 + a*e^5) - 1/4*a^2*e*log(c*x^4 + a)
/(c^3*d^2 + a*c^2*e^2) + 1/2*a^2*d*arctan(c*x^2/sqrt(a*c))/((c^2*d^2 + a*c
*e^2)*sqrt(a*c)) + 1/4*(c*e*x^4 - 2*c*d*x^2)/(c^2*e^2)
```

3.229.9 Mupad [B] (verification not implemented)

Time = 8.01 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.35

$$\int \frac{x^9}{(d+ex^2)(a+cx^4)}\,dx = \frac{\ln\left(\sqrt{-a^3\,c^5}+a\,c^3\,x^2\right)\,\left(d\,\sqrt{-a^3\,c^5}-a^2\,c^2\,e\right)}{4\,c^5\,d^2+4\,a\,c^4\,e^2} - \frac{\ln\left(\sqrt{-a^3\,c^5}-a\,c^3\,x^2\right)\,\left(d\,\sqrt{-a^3\,c^5}+a^2\,c^2\,e\right)}{4\,\left(c^5\,d^2+a\,c^4\,e^2\right)} + \frac{d^4\,\ln\left(e\,x^2+d\right)}{2\,c\,d^2\,e^3+2\,a\,e^5} + \frac{x^4}{4\,c\,e} - \frac{d\,x^2}{2\,c\,e^2}$$

input
```
int(x^9/((a + c*x^4)*(d + e*x^2)),x)
```

output
```
(log((-a^3*c^5)^(1/2) + a*c^3*x^2)*(d*(-a^3*c^5)^(1/2) - a^2*c^2*e))/(4*c^
5*d^2 + 4*a*c^4*e^2) - (log((-a^3*c^5)^(1/2) - a*c^3*x^2)*(d*(-a^3*c^5)^(1
/2) + a^2*c^2*e))/(4*(c^5*d^2 + a*c^4*e^2)) + (d^4*log(d + e*x^2))/(2*a*e^
5 + 2*c*d^2*e^3) + x^4/(4*c*e) - (d*x^2)/(2*c*e^2)
```

3.230 $\int \frac{x^7}{(d+ex^2)(a+cx^4)}\,dx$

3.230.1 Optimal result

Integrand size = 22, antiderivative size = 118

$$\int \frac{x^7}{(d+ex^2)(a+cx^4)}\,dx = \frac{x^2}{2ce} - \frac{a^{3/2}e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2c^{3/2}(cd^2+ae^2)} - \frac{d^3\log(d+ex^2)}{2e^2(cd^2+ae^2)} - \frac{ad\log(a+cx^4)}{4c(cd^2+ae^2)}$$

output
```
1/2*x^2/c/e-1/2*a^(3/2)*e*arctan(x^2*c^(1/2)/a^(1/2))/c^(3/2)/(a*e^2+c*d^2
)-1/2*d^3*ln(e*x^2+d)/e^2/(a*e^2+c*d^2)-1/4*a*d*ln(c*x^4+a)/c/(a*e^2+c*d^2
)
```

3.230.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.84

$$\int \frac{x^7}{(d+ex^2)(a+cx^4)}\,dx = \frac{-\frac{2a^{3/2}e^3\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{c^{3/2}} - 2d^3\log(d+ex^2) + \frac{e(2(cd^2+ae^2)x^2-ade\log(a+cx^4))}{c}}{4e^2(cd^2+ae^2)}$$

input
```
Integrate[x^7/((d + e*x^2)*(a + c*x^4)),x]
```

output
```
((-2*a^(3/2)*e^3*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/c^(3/2) - 2*d^3*Log[d + e*
x^2] + (e*(2*(c*d^2 + a*e^2)*x^2 - a*d*e*Log[a + c*x^4]))/c)/(4*e^2*(c*d^2
 + a*e^2))
```

3.230.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1579, 604, 25, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a+cx^4)(d+ex^2)}dx$$
$$\Big\downarrow \text{1579}$$
$$\frac{1}{2}\int \frac{x^6}{(ex^2+d)(cx^4+a)}dx^2$$
$$\Big\downarrow \text{604}$$
$$\frac{1}{2}\left(\frac{\int -\frac{cde^2x^4+ae^3x^2+ade^2}{(ex^2+d)(cx^4+a)}dx^2}{ce^3}+\frac{d+ex^2}{ce^2}\right)$$
$$\Big\downarrow \text{25}$$
$$\frac{1}{2}\left(\frac{d+ex^2}{ce^2}-\frac{\int \frac{cde^2x^4+ae^3x^2+ade^2}{(ex^2+d)(cx^4+a)}dx^2}{ce^3}\right)$$
$$\Big\downarrow \text{2160}$$
$$\frac{1}{2}\left(\frac{d+ex^2}{ce^2}-\frac{\int \left(\frac{ce^2d^3}{(cd^2+ae^2)(ex^2+d)}+\frac{ae^3(cdx^2+ae)}{(cd^2+ae^2)(cx^4+a)}\right)dx^2}{ce^3}\right)$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{2}\left(\frac{d+ex^2}{ce^2}-\frac{\frac{a^{3/2}e^4\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{c}(ae^2+cd^2)}+\frac{ade^3\log(a+cx^4)}{2(ae^2+cd^2)}+\frac{cd^3e\log(d+ex^2)}{ae^2+cd^2}}{ce^3}\right)$$

input
```
Int[x^7/((d + e*x^2)*(a + c*x^4)),x]
```

output
```
((d + e*x^2)/(c*e^2) - ((a^(3/2)*e^4*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[
c]*(c*d^2 + a*e^2)) + (c*d^3*e*Log[d + e*x^2])/(c*d^2 + a*e^2) + (a*d*e^3*
Log[a + c*x^4])/(2*(c*d^2 + a*e^2)))/(c*e^3))/2
```

3.230.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 604
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(b*d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*
x^2)^p*ExpandToSum[b*d^m*(m + n + 2*p + 1)*x^m - b*(m + n + 2*p + 1)*(c + d
*x)^m - (c + d*x)^(m - 2)*(a*d^2*(m + n - 1) - b*c^2*(m + n + 2*p + 1) - 2*
b*c*d*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && IGtQ[m,
 1] && NeQ[m + n + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.230.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.78

method	result
default	$\frac{x^2}{2ce}-\frac{a\left(\frac{d\ln\left(c\,x^4+a\right)}{2}+\frac{ae\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{\sqrt{ac}}\right)}{2(a\,e^2+c\,d^2)c}-\frac{d^3\ln\left(e\,x^2+d\right)}{2e^2(a\,e^2+c\,d^2)}$
risch	$\frac{x^2}{2ce}+\frac{a\ln\left(\left(-\sqrt{-ac}\,a^2e^5+3\sqrt{-ac}\,ac\,d^2e^3-4\sqrt{-ac}\,c^2d^4e+3a^2cd\,e^4-3a\,c^2d^3e^2+2c^3d^5\right)x^2-3\sqrt{-ac}\,a^2d\,e^4+3\sqrt{-ac}\,ac\,d^3e^2-2\sqrt{-a}}{4c^2(a\,e^2+c\,d^2)}$

input
```
int(x^7/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/c/e-1/2*a/(a*e^2+c*d^2)/c*(1/2*d*ln(c*x^4+a)+a*e/(a*c)^(1/2)*arcta
n(c*x^2/(a*c)^(1/2)))-1/2*d^3*ln(e*x^2+d)/e^2/(a*e^2+c*d^2)
```

3.230.5 Fricas [A] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.80

$$\int \frac{x^7}{(d+ex^2)(a+cx^4)}\,dx = \left[\frac{ae^3\sqrt{-\frac{a}{c}}\log\left(\frac{cx^4-2\,cx^2\sqrt{-\frac{a}{c}}-a}{cx^4+a}\right)-ade^2\log\left(cx^4+a\right)-2\,cd^3\log\left(ex^2+d\right)+2\,(cd^2e+ae^3)x^2}{4\,(c^2d^2e^2+ace^4)},\ -\frac{2\,ae^3\sqrt{\frac{a}{c}}\arctan\left(\frac{cx^2\sqrt{\frac{a}{c}}}{a}\right)+ade^2\log\left(cx^4+a\right)+2\,cd^3\log\left(ex^2+d\right)-2\,(cd^2e+ae^3)x^2}{4\,(c^2d^2e^2+ace^4)}\right]$$

input
```
integrate(x^7/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

output
```
[1/4*(a*e^3*sqrt(-a/c)*log((c*x^4 - 2*c*x^2*sqrt(-a/c) - a)/(c*x^4 + a)) -
 a*d*e^2*log(c*x^4 + a) - 2*c*d^3*log(e*x^2 + d) + 2*(c*d^2*e + a*e^3)*x^2
)/(c^2*d^2*e^2 + a*c*e^4), -1/4*(2*a*e^3*sqrt(a/c)*arctan(c*x^2*sqrt(a/c)/
a) + a*d*e^2*log(c*x^4 + a) + 2*c*d^3*log(e*x^2 + d) - 2*(c*d^2*e + a*e^3)
*x^2)/(c^2*d^2*e^2 + a*c*e^4)]
```

3.230.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^7}{(d+ex^2)(a+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x**7/(e*x**2+d)/(c*x**4+a),x)
```

output
```
Timed out
```

3.230.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91

$$\int \frac{x^7}{(d+ex^2)(a+cx^4)}\,dx = -\frac{d^3\log(ex^2+d)}{2(cd^2e^2+ae^4)} - \frac{a^2e\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2(c^2d^2+ace^2)\sqrt{ac}} - \frac{ad\log(cx^4+a)}{4(c^2d^2+ace^2)} + \frac{x^2}{2\,ce}$$

input
```
integrate(x^7/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

output
```
-1/2*d^3*log(e*x^2 + d)/(c*d^2*e^2 + a*e^4) - 1/2*a^2*e*arctan(c*x^2/sqrt(
a*c))/((c^2*d^2 + a*c*e^2)*sqrt(a*c)) - 1/4*a*d*log(c*x^4 + a)/(c^2*d^2 +
a*c*e^2) + 1/2*x^2/(c*e)
```

3.230.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.92

$$\int \frac{x^7}{(d+ex^2)(a+cx^4)}\,dx = -\frac{d^3\log(|ex^2+d|)}{2(cd^2e^2+ae^4)} - \frac{a^2e\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2(c^2d^2+ace^2)\sqrt{ac}} - \frac{ad\log(cx^4+a)}{4(c^2d^2+ace^2)} + \frac{x^2}{2\,ce}$$

input
```
integrate(x^7/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

output
```
-1/2*d^3*log(abs(e*x^2 + d))/(c*d^2*e^2 + a*e^4) - 1/2*a^2*e*arctan(c*x^2/
sqrt(a*c))/((c^2*d^2 + a*c*e^2)*sqrt(a*c)) - 1/4*a*d*log(c*x^4 + a)/(c^2*d
^2 + a*c*e^2) + 1/2*x^2/(c*e)
```

3.230.9 Mupad [B] (verification not implemented)

Time = 7.88 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.41

$$\begin{aligned}\int \frac{x^7}{(d+ex^2)(a+cx^4)}\,dx &= \frac{x^2}{2\,c\,e} - \frac{d^3\,\ln(e\,x^2+d)}{2\,c\,d^2\,e^2+2\,a\,e^4} \\ &\quad - \frac{\ln\left(\sqrt{-a^3\,c^3}+a\,c^2\,x^2\right)\,\left(e\,\sqrt{-a^3\,c^3}+a\,c^2\,d\right)}{4\,(c^4\,d^2+a\,c^3\,e^2)} \\ &\quad + \frac{\ln\left(\sqrt{-a^3\,c^3}-a\,c^2\,x^2\right)\,\left(e\,\sqrt{-a^3\,c^3}-a\,c^2\,d\right)}{4\,c^4\,d^2+4\,a\,c^3\,e^2}\end{aligned}$$

input
```
int(x^7/((a + c*x^4)*(d + e*x^2)),x)
```

output
```
x^2/(2*c*e) - (d^3*log(d + e*x^2))/(2*a*e^4 + 2*c*d^2*e^2) - (log((-a^3*c^
3)^(1/2) + a*c^2*x^2)*(e*(-a^3*c^3)^(1/2) + a*c^2*d))/(4*(c^4*d^2 + a*c^3*
e^2)) + (log((-a^3*c^3)^(1/2) - a*c^2*x^2)*(e*(-a^3*c^3)^(1/2) - a*c^2*d))
/(4*c^4*d^2 + 4*a*c^3*e^2)
```

3.230. $\int \frac{x^7}{(d+ex^2)(a+cx^4)}\,dx$

3.231 $\int \frac{x^5}{(d+ex^2)(a+cx^4)}\,dx$

3.231.1 Optimal result

Integrand size = 22, antiderivative size = 105

$$\int \frac{x^5}{(d+ex^2)(a+cx^4)}\,dx = -\frac{\sqrt{a}d\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{c}\,(cd^2+ae^2)} + \frac{d^2\log(d+ex^2)}{2e\,(cd^2+ae^2)} + \frac{ae\log(a+cx^4)}{4c\,(cd^2+ae^2)}$$

output
```
1/2*d^2*ln(e*x^2+d)/e/(a*e^2+c*d^2)+1/4*a*e*ln(c*x^4+a)/c/(a*e^2+c*d^2)-1/
2*d*arctan(x^2*c^(1/2)/a^(1/2))*a^(1/2)/(a*e^2+c*d^2)/c^(1/2)
```

3.231.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.73

$$\int \frac{x^5}{(d+ex^2)(a+cx^4)}\,dx = \frac{-2\sqrt{a}\sqrt{c}de\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right) + 2cd^2\log(d+ex^2) + ae^2\log(a+cx^4)}{4c^2d^2e+4ace^3}$$

input
```
Integrate[x^5/((d + e*x^2)*(a + c*x^4)),x]
```

output
```
(-2*Sqrt[a]*Sqrt[c]*d*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]] + 2*c*d^2*Log[d + e*
x^2] + a*e^2*Log[a + c*x^4])/(4*c^2*d^2*e + 4*a*c*e^3)
```

3.231.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1579, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+cx^4)(d+ex^2)}dx$$

$$\Big\downarrow \text{1579}$$

$$\frac{1}{2}\int \frac{x^4}{(ex^2+d)(cx^4+a)}dx^2$$

$$\Big\downarrow \text{615}$$

$$\frac{1}{2}\int \left(\frac{d^2}{(cd^2+ae^2)(ex^2+d)}-\frac{a(d-ex^2)}{(cd^2+ae^2)(cx^4+a)}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{\sqrt{a}d\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{c}(ae^2+cd^2)}+\frac{ae\log(a+cx^4)}{2c(ae^2+cd^2)}+\frac{d^2\log(d+ex^2)}{e(ae^2+cd^2)}\right)$$

input
```
Int[x^5/((d + e*x^2)*(a + c*x^4)),x]
```

output
```
(-((Sqrt[a]*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[c]*(c*d^2 + a*e^2))) +
(d^2*Log[d + e*x^2])/(e*(c*d^2 + a*e^2)) + (a*e*Log[a + c*x^4])/(2*c*(c*d^
2 + a*e^2)))/2
```

3.231.3.1 Defintions of rubi rules used

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.231.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.76

method	result
default	$-\frac{a\left(-\frac{e\ln\left(c\,x^{4}+a\right)}{2c}+\frac{d\arctan\left(\frac{c\,x^{2}}{\sqrt{ac}}\right)}{\sqrt{ac}}\right)}{2(a\,e^{2}+c\,d^{2})}+\frac{d^{2}\ln\left(e\,x^{2}+d\right)}{2e(a\,e^{2}+c\,d^{2})}$
risch	$\frac{\ln\left(\left(-a^{2}c\,e^{4}+5a\,c^{2}d^{2}e^{2}-3\sqrt{-ac}\,acd\,e^{3}+5\sqrt{-ac}\,c^{2}d^{3}e-2c^{3}d^{4}\right)x^{2}-3a^{2}cd\,e^{3}+5a\,c^{2}d^{3}e+\sqrt{-ac}\,a^{2}e^{4}-5\sqrt{-ac}\,ac\,d^{2}e^{2}+2\sqrt{-ac}\,c^{2}d^{4}\right)d}{4(a\,e^{2}+c\,d^{2})c}$

input
```
int(x^5/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/(a*e^2+c*d^2)*(-1/2*e/c*ln(c*x^4+a)+d/(a*c)^(1/2)*arctan(c*x^2/(a*c
)^(1/2)))+1/2*d^2*ln(e*x^2+d)/e/(a*e^2+c*d^2)
```

3.231.5 Fricas [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.62

$$\int \frac{x^5}{(d+ex^2)(a+cx^4)}\,dx = \left[\frac{cde\sqrt{-\frac{a}{c}}\log\left(\frac{cx^4-2\,cx^2\sqrt{-\frac{a}{c}}-a}{cx^4+a}\right)+ae^2\log\left(cx^4+a\right)+2\,cd^2\log\left(ex^2+d\right)}{4\left(c^2d^2e+ace^3\right)}, -\frac{2\,cde\sqrt{\frac{a}{c}}\arctan\left(\frac{cx^2\sqrt{\frac{a}{c}}}{a}\right)-ae^2\log\left(cx^4+a\right)-2\,cd^2\log\left(ex^2+d\right)}{4\left(c^2d^2e+ace^3\right)}\right]$$

input
```
integrate(x^5/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

output
```
[1/4*(c*d*e*sqrt(-a/c)*log((c*x^4 - 2*c*x^2*sqrt(-a/c) - a)/(c*x^4 + a)) +
 a*e^2*log(c*x^4 + a) + 2*c*d^2*log(e*x^2 + d))/(c^2*d^2*e + a*c*e^3), -1/
4*(2*c*d*e*sqrt(a/c)*arctan(c*x^2*sqrt(a/c)/a) - a*e^2*log(c*x^4 + a) - 2*
c*d^2*log(e*x^2 + d))/(c^2*d^2*e + a*c*e^3)]
```

3.231.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^5}{(d+ex^2)(a+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x**5/(e*x**2+d)/(c*x**4+a),x)
```

output
```
Timed out
```

3.231.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.85

$$\int \frac{x^5}{(d+ex^2)(a+cx^4)}\,dx = \frac{ae\log(cx^4+a)}{4(c^2d^2+ace^2)} + \frac{d^2\log(ex^2+d)}{2(cd^2e+ae^3)} - \frac{ad\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2(cd^2+ae^2)\sqrt{ac}}$$

input
```
integrate(x^5/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

output
```
1/4*a*e*log(c*x^4 + a)/(c^2*d^2 + a*c*e^2) + 1/2*d^2*log(e*x^2 + d)/(c*d^2
*e + a*e^3) - 1/2*a*d*arctan(c*x^2/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a*c))
```

3.231.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.86

$$\int \frac{x^5}{(d+ex^2)(a+cx^4)}\,dx = \frac{ae\log(cx^4+a)}{4(c^2d^2+ace^2)} + \frac{d^2\log(|ex^2+d|)}{2(cd^2e+ae^3)} - \frac{ad\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2(cd^2+ae^2)\sqrt{ac}}$$

input
```
integrate(x^5/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

output
```
1/4*a*e*log(c*x^4 + a)/(c^2*d^2 + a*c*e^2) + 1/2*d^2*log(abs(e*x^2 + d))/(
c*d^2*e + a*e^3) - 1/2*a*d*arctan(c*x^2/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a
*c))
```

3.231.9 Mupad [B] (verification not implemented)

Time = 8.06 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.31

$$\int \frac{x^5}{(d+ex^2)(a+cx^4)}\,dx = \frac{d^2\,\ln\left(e\,x^2+d\right)}{2\,c\,d^2\,e+2\,a\,e^3} - \frac{\ln\left(\sqrt{-a\,c^3}+c^2\,x^2\right)\,\left(d\,\sqrt{-a\,c^3}-a\,c\,e\right)}{4\,\left(c^3\,d^2+a\,c^2\,e^2\right)} + \frac{\ln\left(\sqrt{-a\,c^3}-c^2\,x^2\right)\,\left(d\,\sqrt{-a\,c^3}+a\,c\,e\right)}{4\,c^3\,d^2+4\,a\,c^2\,e^2}$$

input
```
int(x^5/((a + c*x^4)*(d + e*x^2)),x)
```

output
```
(d^2*log(d + e*x^2))/(2*a*e^3 + 2*c*d^2*e) - (log((-a*c^3)^(1/2) + c^2*x^2
)*(d*(-a*c^3)^(1/2) - a*c*e))/(4*(c^3*d^2 + a*c^2*e^2)) + (log((-a*c^3)^(1
/2) - c^2*x^2)*(d*(-a*c^3)^(1/2) + a*c*e))/(4*c^3*d^2 + 4*a*c^2*e^2)
```

3.232 $\int \frac{x^3}{(d+ex^2)(a+cx^4)}\,dx$

3.232.1 Optimal result

Integrand size = 22, antiderivative size = 96

$$\int \frac{x^3}{(d+ex^2)(a+cx^4)}\,dx = \frac{\sqrt{a}e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{c}(cd^2+ae^2)} - \frac{d\log(d+ex^2)}{2(cd^2+ae^2)} + \frac{d\log(a+cx^4)}{4(cd^2+ae^2)}$$

output
```
-1/2*d*ln(e*x^2+d)/(a*e^2+c*d^2)+1/4*d*ln(c*x^4+a)/(a*e^2+c*d^2)+1/2*e*arc
tan(x^2*c^(1/2)/a^(1/2))*a^(1/2)/(a*e^2+c*d^2)/c^(1/2)
```

3.232.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{(d+ex^2)(a+cx^4)}\,dx = \frac{\frac{2\sqrt{a}e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{c}} - 2d\log(d+ex^2) + d\log(a+cx^4)}{4cd^2+4ae^2}$$

input
```
Integrate[x^3/((d + e*x^2)*(a + c*x^4)),x]
```

output
```
((2*Sqrt[a]*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/Sqrt[c] - 2*d*Log[d + e*x^2]
+ d*Log[a + c*x^4])/(4*c*d^2 + 4*a*e^2)
```

3.232.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1579, 587, 16, 452, 218, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+cx^4)(d+ex^2)}dx$$

$$\Big\downarrow \text{1579}$$

$$\frac{1}{2}\int \frac{x^2}{(ex^2+d)(cx^4+a)}dx^2$$

$$\Big\downarrow \text{587}$$

$$\frac{1}{2}\left(\frac{\int \frac{cdx^2+ae}{cx^4+a}dx^2}{ae^2+cd^2}-\frac{de\int \frac{1}{ex^2+d}dx^2}{ae^2+cd^2}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\frac{\int \frac{cdx^2+ae}{cx^4+a}dx^2}{ae^2+cd^2}-\frac{d\log(d+ex^2)}{ae^2+cd^2}\right)$$

$$\Big\downarrow \text{452}$$

$$\frac{1}{2}\left(\frac{cd\int \frac{x^2}{cx^4+a}dx^2+ae\int \frac{1}{cx^4+a}dx^2}{ae^2+cd^2}-\frac{d\log(d+ex^2)}{ae^2+cd^2}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(\frac{cd\int \frac{x^2}{cx^4+a}dx^2+\frac{\sqrt{a}e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{c}}}{ae^2+cd^2}-\frac{d\log(d+ex^2)}{ae^2+cd^2}\right)$$

$$\Big\downarrow \text{240}$$

$$\frac{1}{2}\left(\frac{\frac{\sqrt{a}e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{c}}+\frac{1}{2}d\log(a+cx^4)}{ae^2+cd^2}-\frac{d\log(d+ex^2)}{ae^2+cd^2}\right)$$

input
```
Int[x^3/((d + e*x^2)*(a + c*x^4)),x]
```

output
```
(-((d*Log[d + e*x^2])/(c*d^2 + a*e^2)) + ((Sqrt[a]*e*ArcTan[(Sqrt[c]*x^2)/
Sqrt[a]])/Sqrt[c] + (d*Log[a + c*x^4])/2)/(c*d^2 + a*e^2))/2
```

3.232.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 587
```
Int[(x_.)/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[(-
c)*(d/(b*c^2 + a*d^2))   Int[1/(c + d*x), x], x] + Simp[1/(b*c^2 + a*d^2)
 Int[(a*d + b*c*x)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
^2 + a*d^2, 0]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

3.232.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.75

method	result
default	$\frac{\frac{d\ln\left(c\,x^4+a\right)}{2}+\frac{ae\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{\sqrt{ac}}}{2a\,e^2+2c\,d^2}-\frac{d\ln\left(e\,x^2+d\right)}{2\left(a\,e^2+c\,d^2\right)}$
risch	$\frac{\ln\left(\left(\left(\sqrt{-ac}\,a\,e^3-7\sqrt{-ac}\,c\,d^2e+5acd\,e^2-3c^2d^3\right)x^2+5\sqrt{-ac}\,ad\,e^2-3\sqrt{-ac}\,c\,d^3-e^3a^2+7ac\,d^2e\right)e\sqrt{-ac}}{4c\left(a\,e^2+c\,d^2\right)}+\frac{\ln\left(\left(\sqrt{-ac}\,a\,e^3-7\sqrt{-ac}\,c\,d\right.\right.}{}$

input
```
int(x^3/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/(a*e^2+c*d^2)*(1/2*d*ln(c*x^4+a)+a*e/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1
/2)))-1/2*d*ln(e*x^2+d)/(a*e^2+c*d^2)
```

3.232.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.51

$$\int \frac{x^3}{(d+ex^2)(a+cx^4)}\,dx = \left[\frac{e\sqrt{-\frac{a}{c}}\log\left(\frac{cx^4+2\,cx^2\sqrt{-\frac{a}{c}}-a}{cx^4+a}\right)+d\log\left(cx^4+a\right)-2\,d\log\left(ex^2+d\right)}{4\left(cd^2+ae^2\right)}, \frac{2\,e\sqrt{\frac{a}{c}}\arctan\left(\frac{cx^2\sqrt{\frac{a}{c}}}{a}\right)+d\log\left(cx^4\right.}{4\left(cd^2+ae^2\right)}\right.$$

input
```
integrate(x^3/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

output
```
[1/4*(e*sqrt(-a/c)*log((c*x^4 + 2*c*x^2*sqrt(-a/c) - a)/(c*x^4 + a)) + d*l
og(c*x^4 + a) - 2*d*log(e*x^2 + d))/(c*d^2 + a*e^2), 1/4*(2*e*sqrt(a/c)*ar
ctan(c*x^2*sqrt(a/c)/a) + d*log(c*x^4 + a) - 2*d*log(e*x^2 + d))/(c*d^2 +
a*e^2)]
```

3.232.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3}{(d+ex^2)(a+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x**3/(e*x**2+d)/(c*x**4+a),x)
```

output
```
Timed out
```

3.232.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int \frac{x^3}{(d+ex^2)(a+cx^4)}\,dx = \frac{ae\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\,(cd^2+ae^2)\sqrt{ac}} + \frac{d\log\left(cx^4+a\right)}{4\,(cd^2+ae^2)} - \frac{d\log\left(ex^2+d\right)}{2\,(cd^2+ae^2)}$$

input
```
integrate(x^3/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

output
```
1/2*a*e*arctan(c*x^2/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a*c)) + 1/4*d*log(c*
x^4 + a)/(c*d^2 + a*e^2) - 1/2*d*log(e*x^2 + d)/(c*d^2 + a*e^2)
```

3.232.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int \frac{x^3}{(d+ex^2)(a+cx^4)}\,dx = -\frac{de\log\left(|ex^2+d|\right)}{2\,(cd^2e+ae^3)} + \frac{ae\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\,(cd^2+ae^2)\sqrt{ac}} + \frac{d\log\left(cx^4+a\right)}{4\,(cd^2+ae^2)}$$

input
```
integrate(x^3/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

output
```
-1/2*d*e*log(abs(e*x^2 + d))/(c*d^2*e + a*e^3) + 1/2*a*e*arctan(c*x^2/sqrt
(a*c))/((c*d^2 + a*e^2)*sqrt(a*c)) + 1/4*d*log(c*x^4 + a)/(c*d^2 + a*e^2)
```

3.232.9 Mupad [B] (verification not implemented)

Time = 8.74 (sec) , antiderivative size = 944, normalized size of antiderivative = 9.83

$$\int \frac{x^3}{(d+ex^2)(a+cx^4)}\,dx$$
$$= \frac{c\,d\,\ln\Big(a^4\,e^6 - 9\,a\,c^3\,d^6 - 39\,a^3\,c\,d^2\,e^4 + a^3\,e^6\,x^2\,\sqrt{-a\,c} - 9\,c^3\,d^6\,x^2\,\sqrt{-a\,c} + 79\,a^2\,c^2\,d^4\,e^2 - 42\,c\,d^5\,e\,(-a\,c}{}$$
$$- \frac{d\,\ln(e\,x^2+d)}{2\,(c\,d^2+a\,e^2)}$$
$$+ \frac{c\,d\,\ln\big(9\,a\,c^3\,d^6 - a^4\,e^6 + 39\,a^3\,c\,d^2\,e^4 + a^3\,e^6\,x^2\,\sqrt{-a\,c} - 9\,c^3\,d^6\,x^2\,\sqrt{-a\,c} - 79\,a^2\,c^2\,d^4\,e^2 + 10\,a^3\,d\,e^5\,\sqrt{-}}{}$$
$$- \frac{e\,\ln\Big(a^4\,e^6 - 9\,a\,c^3\,d^6 - 39\,a^3\,c\,d^2\,e^4 + a^3\,e^6\,x^2\,\sqrt{-a\,c} - 9\,c^3\,d^6\,x^2\,\sqrt{-a\,c} + 79\,a^2\,c^2\,d^4\,e^2 - 42\,c\,d^5\,e\,(-a\,c}{}$$
$$+ \frac{e\,\ln\big(9\,a\,c^3\,d^6 - a^4\,e^6 + 39\,a^3\,c\,d^2\,e^4 + a^3\,e^6\,x^2\,\sqrt{-a\,c} - 9\,c^3\,d^6\,x^2\,\sqrt{-a\,c} - 79\,a^2\,c^2\,d^4\,e^2 + 10\,a^3\,d\,e^5\,\sqrt{-}}{}$$

input
```
int(x^3/((a + c*x^4)*(d + e*x^2)),x)
```

output
```
(c*d*log(a^4*e^6 - 9*a*c^3*d^6 - 39*a^3*c*d^2*e^4 + a^3*e^6*x^2*(-a*c)^(1/
2) - 9*c^3*d^6*x^2*(-a*c)^(1/2) + 79*a^2*c^2*d^4*e^2 - 42*c*d^5*e*(-a*c)^(
3/2) + 76*a*d^3*e^3*(-a*c)^(3/2) + 10*a^3*d*e^5*(-a*c)^(1/2) + 76*a^2*c^2*
d^3*e^3*x^2 - 42*a*c^3*d^5*e*x^2 - 10*a^3*c*d*e^5*x^2 + 39*a*d^2*e^4*x^2*(
-a*c)^(3/2) - 79*c*d^4*e^2*x^2*(-a*c)^(3/2)))/(4*c^2*d^2 + 4*a*c*e^2) - (d
*log(d + e*x^2))/(2*(a*e^2 + c*d^2)) + (c*d*log(9*a*c^3*d^6 - a^4*e^6 + 39
*a^3*c*d^2*e^4 + a^3*e^6*x^2*(-a*c)^(1/2) - 9*c^3*d^6*x^2*(-a*c)^(1/2) - 7
9*a^2*c^2*d^4*e^2 + 10*a^3*d*e^5*(-a*c)^(1/2) + 42*a*c^2*d^5*e*(-a*c)^(1/2
) - 76*a^2*c^2*d^3*e^3*x^2 + 42*a*c^3*d^5*e*x^2 + 10*a^3*c*d*e^5*x^2 - 76*
a^2*c*d^3*e^3*(-a*c)^(1/2) + 79*a*c^2*d^4*e^2*x^2*(-a*c)^(1/2) - 39*a^2*c*
d^2*e^4*x^2*(-a*c)^(1/2)))/(4*c^2*d^2 + 4*a*c*e^2) - (e*log(a^4*e^6 - 9*a*
c^3*d^6 - 39*a^3*c*d^2*e^4 + a^3*e^6*x^2*(-a*c)^(1/2) - 9*c^3*d^6*x^2*(-a*
c)^(1/2) + 79*a^2*c^2*d^4*e^2 - 42*c*d^5*e*(-a*c)^(3/2) + 76*a*d^3*e^3*(-a
*c)^(3/2) + 10*a^3*d*e^5*(-a*c)^(1/2) + 76*a^2*c^2*d^3*e^3*x^2 - 42*a*c^3*
d^5*e*x^2 - 10*a^3*c*d*e^5*x^2 + 39*a*d^2*e^4*x^2*(-a*c)^(3/2) - 79*c*d^4*
e^2*x^2*(-a*c)^(3/2))*(-a*c)^(1/2))/(4*c^2*d^2 + 4*a*c*e^2) + (e*log(9*a*c
^3*d^6 - a^4*e^6 + 39*a^3*c*d^2*e^4 + a^3*e^6*x^2*(-a*c)^(1/2) - 9*c^3*d^6
*x^2*(-a*c)^(1/2) - 79*a^2*c^2*d^4*e^2 + 10*a^3*d*e^5*(-a*c)^(1/2) + 42*a*
c^2*d^5*e*(-a*c)^(1/2) - 76*a^2*c^2*d^3*e^3*x^2 + 42*a*c^3*d^5*e*x^2 + 10*
a^3*c*d*e^5*x^2 - 76*a^2*c*d^3*e^3*(-a*c)^(1/2) + 79*a*c^2*d^4*e^2*x^2*...
```

3.233 $\int \frac{x}{(d+ex^2)(a+cx^4)}\,dx$

3.233.1 Optimal result

Integrand size = 20, antiderivative size = 96

$$\int \frac{x}{(d+ex^2)(a+cx^4)}\,dx=\frac{\sqrt{c}d\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}(cd^2+ae^2)}+\frac{e\log(d+ex^2)}{2(cd^2+ae^2)}-\frac{e\log(a+cx^4)}{4(cd^2+ae^2)}$$

output
```
1/2*e*ln(e*x^2+d)/(a*e^2+c*d^2)-1/4*e*ln(c*x^4+a)/(a*e^2+c*d^2)+1/2*d*arct
an(x^2*c^(1/2)/a^(1/2))*c^(1/2)/(a*e^2+c*d^2)/a^(1/2)
```

3.233.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.70

$$\int \frac{x}{(d+ex^2)(a+cx^4)}\,dx=\frac{\frac{2\sqrt{c}d\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}}+2e\log(d+ex^2)-e\log(a+cx^4)}{4cd^2+4ae^2}$$

input
```
Integrate[x/((d + e*x^2)*(a + c*x^4)),x]
```

output
```
((2*Sqrt[c]*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/Sqrt[a] + 2*e*Log[d + e*x^2]
- e*Log[a + c*x^4])/(4*c*d^2 + 4*a*e^2)
```

3.233.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1577, 479, 452, 218, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+cx^4)(d+ex^2)}dx$$

$$\Big\downarrow \text{1577}$$

$$\frac{1}{2}\int \frac{1}{(ex^2+d)(cx^4+a)}dx^2$$

$$\Big\downarrow \text{479}$$

$$\frac{1}{2}\left(\frac{c\int \frac{d-ex^2}{cx^4+a}dx^2}{ae^2+cd^2}+\frac{e\log(d+ex^2)}{ae^2+cd^2}\right)$$

$$\Big\downarrow \text{452}$$

$$\frac{1}{2}\left(\frac{c\left(d\int \frac{1}{cx^4+a}dx^2-e\int \frac{x^2}{cx^4+a}dx^2\right)}{ae^2+cd^2}+\frac{e\log(d+ex^2)}{ae^2+cd^2}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(\frac{c\left(\frac{d\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}-e\int \frac{x^2}{cx^4+a}dx^2\right)}{ae^2+cd^2}+\frac{e\log(d+ex^2)}{ae^2+cd^2}\right)$$

$$\Big\downarrow \text{240}$$

$$\frac{1}{2}\left(\frac{c\left(\frac{d\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}-\frac{e\log(a+cx^4)}{2c}\right)}{ae^2+cd^2}+\frac{e\log(d+ex^2)}{ae^2+cd^2}\right)$$

input
```
Int[x/((d + e*x^2)*(a + c*x^4)),x]
```

output
```
((e*Log[d + e*x^2])/(c*d^2 + a*e^2) + (c*((d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]]
)/(Sqrt[a]*Sqrt[c]) - (e*Log[a + c*x^4])/(2*c)))/(c*d^2 + a*e^2))/2
```

3.233.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 479
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[d*(Log
[RemoveContent[c + d*x, x]]/(b*c^2 + a*d^2)), x] + Simp[b/(b*c^2 + a*d^2)
 Int[(c - d*x)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

3.233.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.78

method	result
default	$\frac{c\left(-\frac{e\ln\left(c\,x^{4}+a\right)}{2c}+\frac{d\arctan\left(\frac{c\,x^{2}}{\sqrt{ac}}\right)}{\sqrt{ac}}\right)}{2a\,e^{2}+2c\,d^{2}}+\frac{e\ln\left(e\,x^{2}+d\right)}{2a\,e^{2}+2c\,d^{2}}$
risch	$\frac{\ln\left(\left(-3a^{2}c\,e^{3}+5a\,c^{2}d^{2}e+7\sqrt{-ac}\,acd\,e^{2}-\sqrt{-ac}\,c^{2}d^{3}\right)x^{2}-7a^{2}cd\,e^{2}+a\,c^{2}d^{3}-3\sqrt{-ac}\,a^{2}e^{3}+5\sqrt{-ac}\,ac\,d^{2}e\right)d\sqrt{-ac}}{4a\left(a\,e^{2}+c\,d^{2}\right)}-\ln((-3a^{2}c\,e^{3}+$

input
```
int(x/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*c/(a*e^2+c*d^2)*(-1/2*e/c*ln(c*x^4+a)+d/(a*c)^(1/2)*arctan(c*x^2/(a*c)
^(1/2)))+1/2*e*ln(e*x^2+d)/(a*e^2+c*d^2)
```

3.233.5 Fricas [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.52

$$\int \frac{x}{(d+ex^2)(a+cx^4)}\,dx = \left[\frac{d\sqrt{-\frac{c}{a}}\log\left(\frac{cx^4+2\,ax^2\sqrt{-\frac{c}{a}}-a}{cx^4+a}\right)-e\log\left(cx^4+a\right)+2\,e\log\left(ex^2+d\right)}{4\left(cd^2+ae^2\right)}, -\frac{2\,d\sqrt{\frac{c}{a}}\arctan\left(\frac{a\sqrt{\frac{c}{a}}}{cx^2}\right)+e\log\left(cx^4+a\right)-2\,e\log\left(ex^2+d\right)}{4\left(cd^2+ae^2\right)}\right]$$

input
```
integrate(x/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

output
```
[1/4*(d*sqrt(-c/a)*log((c*x^4 + 2*a*x^2*sqrt(-c/a) - a)/(c*x^4 + a)) - e*l
og(c*x^4 + a) + 2*e*log(e*x^2 + d))/(c*d^2 + a*e^2), -1/4*(2*d*sqrt(c/a)*a
rctan(a*sqrt(c/a)/(c*x^2)) + e*log(c*x^4 + a) - 2*e*log(e*x^2 + d))/(c*d^2
 + a*e^2)]
```

3.233.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x}{(d+ex^2)(a+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x/(e*x**2+d)/(c*x**4+a),x)
```

output
```
Timed out
```

3.233.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int \frac{x}{(d+ex^2)(a+cx^4)}\,dx = \frac{cd\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2(cd^2+ae^2)\sqrt{ac}} - \frac{e\log(cx^4+a)}{4(cd^2+ae^2)} + \frac{e\log(ex^2+d)}{2(cd^2+ae^2)}$$

input
```
integrate(x/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

output
```
1/2*c*d*arctan(c*x^2/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a*c)) - 1/4*e*log(c*
x^4 + a)/(c*d^2 + a*e^2) + 1/2*e*log(e*x^2 + d)/(c*d^2 + a*e^2)
```

3.233.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.90

$$\int \frac{x}{(d+ex^2)(a+cx^4)}\,dx = \frac{e^2\log(|ex^2+d|)}{2(cd^2e+ae^3)} + \frac{cd\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2(cd^2+ae^2)\sqrt{ac}} - \frac{e\log(cx^4+a)}{4(cd^2+ae^2)}$$

input
```
integrate(x/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

output
```
1/2*e^2*log(abs(e*x^2 + d))/(c*d^2*e + a*e^3) + 1/2*c*d*arctan(c*x^2/sqrt(
a*c))/((c*d^2 + a*e^2)*sqrt(a*c)) - 1/4*e*log(c*x^4 + a)/(c*d^2 + a*e^2)
```

3.233.9 Mupad [B] (verification not implemented)

Time = 8.31 (sec) , antiderivative size = 328, normalized size of antiderivative = 3.42

$$\int \frac{x}{(d+ex^2)(a+cx^4)}\,dx = \frac{e\,\ln(e\,x^2+d)}{2\,c\,d^2+2\,a\,e^2}$$

$$-\frac{\ln\left(a\,c^5\,d^6\,x^2-c^3\,d^6\,(-a\,c)^{3/2}-9\,a^3\,e^6\,(-a\,c)^{3/2}+9\,a^4\,c^2\,e^6\,x^2+19\,a\,d^2\,e^4\,(-a\,c)^{5/2}+11\,c\,d^4\,e^2\,(-a\,c)^{5}\right.}{4\,(a^2\,e^2+c\,a\,d^2)}$$

$$-\frac{\ln\left(9\,a^3\,e^6\,(-a\,c)^{3/2}+c^3\,d^6\,(-a\,c)^{3/2}+a\,c^5\,d^6\,x^2+9\,a^4\,c^2\,e^6\,x^2-19\,a\,d^2\,e^4\,(-a\,c)^{5/2}-11\,c\,d^4\,e^2\,(-a\,c)^{5}\right.}{4\,(a^2\,e^2+c\,a\,d^2)}$$

input
```
int(x/((a + c*x^4)*(d + e*x^2)),x)
```

output
```
(e*log(d + e*x^2))/(2*a*e^2 + 2*c*d^2) - (log(a*c^5*d^6*x^2 - c^3*d^6*(-a*
c)^(3/2) - 9*a^3*e^6*(-a*c)^(3/2) + 9*a^4*c^2*e^6*x^2 + 19*a*d^2*e^4*(-a*c
)^(5/2) + 11*c*d^4*e^2*(-a*c)^(5/2) + 11*a^2*c^4*d^4*e^2*x^2 + 19*a^3*c^3*
d^2*e^4*x^2)*(a*e - d*(-a*c)^(1/2)))/(4*(a^2*e^2 + a*c*d^2)) - (log(9*a^3*
e^6*(-a*c)^(3/2) + c^3*d^6*(-a*c)^(3/2) + a*c^5*d^6*x^2 + 9*a^4*c^2*e^6*x^
2 - 19*a*d^2*e^4*(-a*c)^(5/2) - 11*c*d^4*e^2*(-a*c)^(5/2) + 11*a^2*c^4*d^4
*e^2*x^2 + 19*a^3*c^3*d^2*e^4*x^2)*(a*e + d*(-a*c)^(1/2)))/(4*(a^2*e^2 + a
*c*d^2))
```

3.233. $\int \frac{x}{(d+ex^2)(a+cx^4)}\,dx$

3.234 $\int \frac{1}{x(d+ex^2)(a+cx^4)}\,dx$

3.234.1 Optimal result

Integrand size = 22, antiderivative size = 114

$$\int \frac{1}{x(d+ex^2)(a+cx^4)}\,dx = -\frac{\sqrt{c}e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}(cd^2+ae^2)}+\frac{\log(x)}{ad}-\frac{e^2\log(d+ex^2)}{2d(cd^2+ae^2)}-\frac{cd\log(a+cx^4)}{4a(cd^2+ae^2)}$$

output
```
ln(x)/a/d-1/2*e^2*ln(e*x^2+d)/d/(a*e^2+c*d^2)-1/4*c*d*ln(c*x^4+a)/a/(a*e^2
+c*d^2)-1/2*e*arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/(a*e^2+c*d^2)/a^(1/2)
```

3.234.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.18

$$\int \frac{1}{x(d+ex^2)(a+cx^4)}\,dx = \frac{2\sqrt{a}\sqrt{c}de\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\sqrt{a}\sqrt{c}de\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+4cd^2\log(x)+4ae^2\log(x)-2ae^2\log}{4acd^3+4a^2de^2}$$

input
```
Integrate[1/(x*(d + e*x^2)*(a + c*x^4)),x]
```

output
```
(2*Sqrt[a]*Sqrt[c]*d*e*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*Sqrt[a]
*Sqrt[c]*d*e*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 4*c*d^2*Log[x] + 4*
a*e^2*Log[x] - 2*a*e^2*Log[d + e*x^2] - c*d^2*Log[a + c*x^4])/(4*a*c*d^3 +
 4*a^2*d*e^2)
```

3.234.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1579, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+cx^4\right)\left(d+ex^2\right)}dx$$

$$\Big\downarrow \text{1579}$$

$$\frac{1}{2}\int \frac{1}{x^2\left(ex^2+d\right)\left(cx^4+a\right)}dx^2$$

$$\Big\downarrow \text{615}$$

$$\frac{1}{2}\int \left(-\frac{e^3}{d\left(cd^2+ae^2\right)\left(ex^2+d\right)}-\frac{c\left(cdx^2+ae\right)}{a\left(cd^2+ae^2\right)\left(cx^4+a\right)}+\frac{1}{adx^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{\sqrt{c}e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}\left(ae^2+cd^2\right)}-\frac{cd\log\left(a+cx^4\right)}{2a\left(ae^2+cd^2\right)}-\frac{e^2\log\left(d+ex^2\right)}{d\left(ae^2+cd^2\right)}+\frac{\log\left(x^2\right)}{ad}\right)$$

input
```
Int[1/(x*(d + e*x^2)*(a + c*x^4)),x]
```

output
```
(-((Sqrt[c]*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*(c*d^2 + a*e^2))) +
Log[x^2]/(a*d) - (e^2*Log[d + e*x^2])/(d*(c*d^2 + a*e^2)) - (c*d*Log[a + c
*x^4])/(2*a*(c*d^2 + a*e^2)))/2
```

3.234.3.1 Defintions of rubi rules used

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.234.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.79

method	result
default	$\frac{\ln(x)}{ad}-\frac{c\left(\frac{d\ln\left(c\,x^4+a\right)}{2}+\frac{ae\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{\sqrt{ac}}\right)}{2(a\,e^2+c\,d^2)a}-\frac{e^2\ln(e\,x^2+d)}{2d(a\,e^2+c\,d^2)}$
risch	$\frac{\ln(x)}{ad}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(e^2a^3+c\,a^2d^2\right)\_Z^2+2ad\_Zc+c\right)}\_R\ln\left(\left(\left(-6e^4a^3-7a^2c\,d^2e^2-5a\,c^2d^4\right)\_R^2+\left(-22acd\,e^2-5c^2d^3\right)\_R-1\right.\right.\right.}{4}$

input
```
int(1/x/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
ln(x)/a/d-1/2*c/(a*e^2+c*d^2)/a*(1/2*d*ln(c*x^4+a)+a*e/(a*c)^(1/2)*arctan(
c*x^2/(a*c)^(1/2)))-1/2*e^2*ln(e*x^2+d)/d/(a*e^2+c*d^2)
```

3.234.5 Fricas [A] (verification not implemented)

Time = 4.43 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.76

$$\int \frac{1}{x(d+ex^2)(a+cx^4)}\,dx = \left[\frac{ade\sqrt{-\frac{c}{a}}\log\left(\frac{cx^4-2\,ax^2\sqrt{-\frac{c}{a}}-a}{cx^4+a}\right)-cd^2\log(cx^4+a)-2\,ae^2\log(ex^2+d)+4\,(cd^2+ae^2)\log(x)}{4\,(acd^3+a^2de^2)}, \frac{2\,ade\sqrt{}}{}\right.$$

input
```
integrate(1/x/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

output
```
[1/4*(a*d*e*sqrt(-c/a)*log((c*x^4 - 2*a*x^2*sqrt(-c/a) - a)/(c*x^4 + a)) -
 c*d^2*log(c*x^4 + a) - 2*a*e^2*log(e*x^2 + d) + 4*(c*d^2 + a*e^2)*log(x))
/(a*c*d^3 + a^2*d*e^2), 1/4*(2*a*d*e*sqrt(c/a)*arctan(a*sqrt(c/a)/(c*x^2))
 - c*d^2*log(c*x^4 + a) - 2*a*e^2*log(e*x^2 + d) + 4*(c*d^2 + a*e^2)*log(x
))/(a*c*d^3 + a^2*d*e^2)]
```

3.234.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x(d+ex^2)(a+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(e*x**2+d)/(c*x**4+a),x)
```

output
```
Timed out
```

3.234.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.89

$$\int \frac{1}{x(d+ex^2)(a+cx^4)}\,dx = -\frac{cd\log(cx^4+a)}{4\,(acd^2+a^2e^2)}-\frac{e^2\log(ex^2+d)}{2\,(cd^3+ade^2)} - \frac{ce\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\,(cd^2+ae^2)\sqrt{ac}}+\frac{\log(x^2)}{2\,ad}$$

input
```
integrate(1/x/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

output
```
-1/4*c*d*log(c*x^4 + a)/(a*c*d^2 + a^2*e^2) - 1/2*e^2*log(e*x^2 + d)/(c*d^
3 + a*d*e^2) - 1/2*c*e*arctan(c*x^2/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a*c))
 + 1/2*log(x^2)/(a*d)
```

3.234.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90

$$\int \frac{1}{x(d+ex^2)(a+cx^4)}\,dx = -\frac{e^3\log\left(|ex^2+d|\right)}{2\left(cd^3e+ade^3\right)} - \frac{cd\log\left(cx^4+a\right)}{4\left(acd^2+a^2e^2\right)} - \frac{ce\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\left(cd^2+ae^2\right)\sqrt{ac}} + \frac{\log\left(x^2\right)}{2\,ad}$$

input
```
integrate(1/x/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

output
```
-1/2*e^3*log(abs(e*x^2 + d))/(c*d^3*e + a*d*e^3) - 1/4*c*d*log(c*x^4 + a)/
(a*c*d^2 + a^2*e^2) - 1/2*c*e*arctan(c*x^2/sqrt(a*c))/((c*d^2 + a*e^2)*sqr
t(a*c)) + 1/2*log(x^2)/(a*d)
```

3.234.9 Mupad [B] (verification not implemented)

Time = 8.04 (sec) , antiderivative size = 527, normalized size of antiderivative = 4.62

$$\int \frac{1}{x(d+ex^2)(a+cx^4)}\,dx = \frac{\ln\Big(64\,a^7\,c\,e^{10}\,x^2 - 64\,a^6\,e^{10}\,\sqrt{-a^3\,c} - 25\,a\,c^5\,d^{10}\,\sqrt{-a^3\,c} + 25\,a^2\,c^6\,d^{10}\,x^2 + 180\,a^2\,d^2\,e^8\,{\left(-a^3\,c\right)}^{3/2} - 41\,c^2 \cdots}{\cdots} - \frac{\ln\Big(64\,a^6\,e^{10}\,\sqrt{-a^3\,c} + 64\,a^7\,c\,e^{10}\,x^2 + 25\,a\,c^5\,d^{10}\,\sqrt{-a^3\,c} + 25\,a^2\,c^6\,d^{10}\,x^2 - 180\,a^2\,d^2\,e^8\,{\left(-a^3\,c\right)}^{3/2} + 41 \cdots}{\cdots} - \frac{e^2\,\ln\left(e\,x^2+d\right)}{2\,c\,d^3+2\,a\,d\,e^2} + \frac{\ln\left(x\right)}{a\,d}$$

input
```
int(1/(x*(a + c*x^4)*(d + e*x^2)),x)
```

output
```
(log(64*a^7*c*e^10*x^2 - 64*a^6*e^10*(-a^3*c)^(1/2) - 25*a*c^5*d^10*(-a^3*
c)^(1/2) + 25*a^2*c^6*d^10*x^2 + 180*a^2*d^2*e^8*(-a^3*c)^(3/2) - 41*c^2*d
^6*e^4*(-a^3*c)^(3/2) - 9*a^3*c^5*d^8*e^2*x^2 - 41*a^4*c^4*d^6*e^4*x^2 + 1
09*a^5*c^3*d^4*e^6*x^2 + 180*a^6*c^2*d^2*e^8*x^2 + 9*a^2*c^4*d^8*e^2*(-a^3
*c)^(1/2) + 109*a*c*d^4*e^6*(-a^3*c)^(3/2))*(e*(-a^3*c)^(1/2) - a*c*d))/(4
*a^3*e^2 + 4*a^2*c*d^2) - (log(64*a^6*e^10*(-a^3*c)^(1/2) + 64*a^7*c*e^10*
x^2 + 25*a*c^5*d^10*(-a^3*c)^(1/2) + 25*a^2*c^6*d^10*x^2 - 180*a^2*d^2*e^8
*(-a^3*c)^(3/2) + 41*c^2*d^6*e^4*(-a^3*c)^(3/2) - 9*a^3*c^5*d^8*e^2*x^2 -
41*a^4*c^4*d^6*e^4*x^2 + 109*a^5*c^3*d^4*e^6*x^2 + 180*a^6*c^2*d^2*e^8*x^2
 - 9*a^2*c^4*d^8*e^2*(-a^3*c)^(1/2) - 109*a*c*d^4*e^6*(-a^3*c)^(3/2))*(e*(
-a^3*c)^(1/2) + a*c*d))/(4*(a^3*e^2 + a^2*c*d^2)) - (e^2*log(d + e*x^2))/(
2*c*d^3 + 2*a*d*e^2) + log(x)/(a*d)
```

3.234. $\int \frac{1}{x(d+ex^2)(a+cx^4)}\,dx$

3.235 $\int \frac{1}{x^3(d+ex^2)(a+cx^4)}\,dx$

3.235.1 Optimal result

Integrand size = 22, antiderivative size = 129

$$\int \frac{1}{x^3\left(d+ex^2\right)\left(a+cx^4\right)}\,dx = -\frac{1}{2adx^2}-\frac{c^{3/2}d\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}\left(cd^2+ae^2\right)}-\frac{e\log(x)}{ad^2}+\frac{e^3\log\left(d+ex^2\right)}{2d^2\left(cd^2+ae^2\right)}+\frac{ce\log\left(a+cx^4\right)}{4a\left(cd^2+ae^2\right)}$$

output
```
-1/2/a/d/x^2-1/2*c^(3/2)*d*arctan(x^2*c^(1/2)/a^(1/2))/a^(3/2)/(a*e^2+c*d^
2)-e*ln(x)/a/d^2+1/2*e^3*ln(e*x^2+d)/d^2/(a*e^2+c*d^2)+1/4*c*e*ln(c*x^4+a)
/a/(a*e^2+c*d^2)
```

3.235.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.31

$$\int \frac{1}{x^3\left(d+ex^2\right)\left(a+cx^4\right)}\,dx = \frac{2c^{3/2}d^3x^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2c^{3/2}d^3x^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+\sqrt{a}(-2cd^3-2ade^2-4e(cd^2+ae^2)\,x}{4a^{3/2}d^2\left(cd^2+ae^2\right)x^2}$$

input
```
Integrate[1/(x^3*(d + e*x^2)*(a + c*x^4)),x]
```

output
```
(2*c^(3/2)*d^3*x^2*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*c^(3/2)*d^3
*x^2*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + Sqrt[a]*(-2*c*d^3 - 2*a*d*e
^2 - 4*e*(c*d^2 + a*e^2)*x^2*Log[x] + 2*a*e^3*x^2*Log[d + e*x^2] + c*d^2*e
*x^2*Log[a + c*x^4]))/(4*a^(3/2)*d^2*(c*d^2 + a*e^2)*x^2)
```

3.235.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1579, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+cx^4)(d+ex^2)} \, dx$$

$$\Big\downarrow \text{1579}$$

$$\frac{1}{2}\int \frac{1}{x^4 (ex^2+d)(cx^4+a)}dx^2$$

$$\Big\downarrow \text{615}$$

$$\frac{1}{2}\int \left(\frac{e^4}{d^2 (cd^2+ae^2)(ex^2+d)}-\frac{e}{ad^2x^2}-\frac{c^2(d-ex^2)}{a(cd^2+ae^2)(cx^4+a)}+\frac{1}{adx^4}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{c^{3/2}d\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{a^{3/2}(ae^2+cd^2)}+\frac{ce\log(a+cx^4)}{2a(ae^2+cd^2)}+\frac{e^3\log(d+ex^2)}{d^2(ae^2+cd^2)}-\frac{e\log(x^2)}{ad^2}-\frac{1}{adx^2}\right)$$

input
```
Int[1/(x^3*(d + e*x^2)*(a + c*x^4)),x]
```

output
```
(-(1/(a*d*x^2)) - (c^(3/2)*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(a^(3/2)*(c*d^
2 + a*e^2)) - (e*Log[x^2])/(a*d^2) + (e^3*Log[d + e*x^2])/(d^2*(c*d^2 + a*
e^2)) + (c*e*Log[a + c*x^4])/(2*a*(c*d^2 + a*e^2)))/2
```

3.235.3.1 Defintions of rubi rules used

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.235.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.83

method	result
default	$-\frac{1}{2ad\,x^2}-\frac{e\ln(x)}{a\,d^2}-\frac{c^2\left(-\frac{e\ln\left(c\,x^4+a\right)}{2c}+\frac{d\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{\sqrt{ac}}\right)}{2(a\,e^2+c\,d^2)a}+\frac{e^3\ln\left(e\,x^2+d\right)}{2d^2(a\,e^2+c\,d^2)}$
risch	$-\frac{1}{2ad\,x^2}-\frac{e\ln(x)}{a\,d^2}+\frac{e^3\ln\left(-e\,x^2-d\right)}{2d^2(a\,e^2+c\,d^2)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(e^2a^4+a^3c\,d^2\right)\_Z^2-2a^2ce\_Z+c^2\right)}\_R\ln\left(\left(\left(-6a^5d^2e^4-7a^4c\,d^4e^2-5a^3\right.\right.\right.\right.}{}$

input
```
int(1/x^3/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/d/x^2-e*ln(x)/a/d^2-1/2*c^2/(a*e^2+c*d^2)/a*(-1/2*e/c*ln(c*x^4+a)+d
/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2)))+1/2*e^3*ln(e*x^2+d)/d^2/(a*e^2+c*d
^2)
```

3.235.5 Fricas [A] (verification not implemented)

Time = 22.70 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.05

$$\int \frac{1}{x^3(d+ex^2)(a+cx^4)}\,dx$$
$$= \left[\frac{cd^3x^2\sqrt{-\frac{c}{a}}\log\left(\frac{cx^4-2\,ax^2\sqrt{-\frac{c}{a}}-a}{cx^4+a}\right)+cd^2ex^2\log\left(cx^4+a\right)+2\,ae^3x^2\log\left(ex^2+d\right)-2\,cd^3-2\,ade^2-4\,(cd}{4\,(acd^4+a^2d^2e^2)x^2} \right.$$

input
```
integrate(1/x^3/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

output
```
[1/4*(c*d^3*x^2*sqrt(-c/a)*log((c*x^4 - 2*a*x^2*sqrt(-c/a) - a)/(c*x^4 + a
)) + c*d^2*e*x^2*log(c*x^4 + a) + 2*a*e^3*x^2*log(e*x^2 + d) - 2*c*d^3 - 2
*a*d*e^2 - 4*(c*d^2*e + a*e^3)*x^2*log(x))/((a*c*d^4 + a^2*d^2*e^2)*x^2),
1/4*(2*c*d^3*x^2*sqrt(c/a)*arctan(a*sqrt(c/a)/(c*x^2)) + c*d^2*e*x^2*log(c
*x^4 + a) + 2*a*e^3*x^2*log(e*x^2 + d) - 2*c*d^3 - 2*a*d*e^2 - 4*(c*d^2*e
+ a*e^3)*x^2*log(x))/((a*c*d^4 + a^2*d^2*e^2)*x^2)]
```

3.235.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3(d+ex^2)(a+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/x**3/(e*x**2+d)/(c*x**4+a),x)
```

output
```
Timed out
```

3.235.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^3(d+ex^2)(a+cx^4)}\,dx = \frac{e^3\log(ex^2+d)}{2(cd^4+ad^2e^2)} - \frac{c^2d\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2(acd^2+a^2e^2)\sqrt{ac}} + \frac{ce\log(cx^4+a)}{4(acd^2+a^2e^2)} - \frac{e\log(x^2)}{2\,ad^2} - \frac{1}{2\,adx^2}$$

input
```
integrate(1/x^3/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

output
```
1/2*e^3*log(e*x^2 + d)/(c*d^4 + a*d^2*e^2) - 1/2*c^2*d*arctan(c*x^2/sqrt(a
*c))/((a*c*d^2 + a^2*e^2)*sqrt(a*c)) + 1/4*c*e*log(c*x^4 + a)/(a*c*d^2 + a
^2*e^2) - 1/2*e*log(x^2)/(a*d^2) - 1/2/(a*d*x^2)
```

3.235.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^3(d+ex^2)(a+cx^4)}\,dx = \frac{e^4\log(|ex^2+d|)}{2(cd^4e+ad^2e^3)} - \frac{c^2d\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2(acd^2+a^2e^2)\sqrt{ac}} + \frac{ce\log(cx^4+a)}{4(acd^2+a^2e^2)} - \frac{e\log(x^2)}{2\,ad^2} + \frac{ex^2-d}{2\,ad^2x^2}$$

input
```
integrate(1/x^3/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

output
```
1/2*e^4*log(abs(e*x^2 + d))/(c*d^4*e + a*d^2*e^3) - 1/2*c^2*d*arctan(c*x^2
/sqrt(a*c))/((a*c*d^2 + a^2*e^2)*sqrt(a*c)) + 1/4*c*e*log(c*x^4 + a)/(a*c*
d^2 + a^2*e^2) - 1/2*e*log(x^2)/(a*d^2) + 1/2*(e*x^2 - d)/(a*d^2*x^2)
```

3.235.9 Mupad [B] (verification not implemented)

Time = 8.29 (sec) , antiderivative size = 820, normalized size of antiderivative = 6.36

$$\int \frac{1}{x^3(d+ex^2)(a+cx^4)}\,dx$$
$$= \frac{\ln\left(a^6\,c^{12}\,d^{16}\,x^2+64\,a^{14}\,c^4\,e^{16}\,x^2+a^2\,c^7\,d^{16}\,(-a^3\,c^3)^{3/2}-64\,a^{13}\,c^2\,e^{16}\,\sqrt{-a^3\,c^3}+63\,a^3\,d^8\,e^8\,(-a^3\,c^3)^{5/2}+\right.}{}$$
$$-\frac{\ln\left(a^6\,c^{12}\,d^{16}\,x^2+64\,a^{14}\,c^4\,e^{16}\,x^2-a^2\,c^7\,d^{16}\,(-a^3\,c^3)^{3/2}+64\,a^{13}\,c^2\,e^{16}\,\sqrt{-a^3\,c^3}-63\,a^3\,d^8\,e^8\,(-a^3\,c^3)^{5/2}\right.}{}$$
$$+\frac{e^3\,\ln(e\,x^2+d)}{2\,c\,d^4+2\,a\,d^2\,e^2}-\frac{1}{2\,a\,d\,x^2}-\frac{e\,\ln(x)}{a\,d^2}$$

input
```
int(1/(x^3*(a + c*x^4)*(d + e*x^2)),x)
```

output
```
(log(a^6*c^12*d^16*x^2 + 64*a^14*c^4*e^16*x^2 + a^2*c^7*d^16*(-a^3*c^3)^(3
/2) - 64*a^13*c^2*e^16*(-a^3*c^3)^(1/2) + 63*a^3*d^8*e^8*(-a^3*c^3)^(5/2)
+ 224*a^9*d^2*e^14*(-a^3*c^3)^(3/2) - 28*c^3*d^14*e^2*(-a^3*c^3)^(5/2) + 2
8*a^7*c^11*d^14*e^2*x^2 + 114*a^8*c^10*d^12*e^4*x^2 + 108*a^9*c^9*d^10*e^6
*x^2 - 63*a^10*c^8*d^8*e^8*x^2 - 32*a^11*c^7*d^6*e^10*x^2 + 212*a^12*c^6*d
^4*e^12*x^2 + 224*a^13*c^5*d^2*e^14*x^2 - 114*a*c^2*d^12*e^4*(-a^3*c^3)^(5
/2) - 108*a^2*c*d^10*e^6*(-a^3*c^3)^(5/2) + 212*a^8*c*d^4*e^12*(-a^3*c^3)^
(3/2) - 32*a^7*c^2*d^6*e^10*(-a^3*c^3)^(3/2))*(d*(-a^3*c^3)^(1/2) + a^2*c*
e))/(4*a^4*e^2 + 4*a^3*c*d^2) - (log(a^6*c^12*d^16*x^2 + 64*a^14*c^4*e^16*
x^2 - a^2*c^7*d^16*(-a^3*c^3)^(3/2) + 64*a^13*c^2*e^16*(-a^3*c^3)^(1/2) -
63*a^3*d^8*e^8*(-a^3*c^3)^(5/2) - 224*a^9*d^2*e^14*(-a^3*c^3)^(3/2) + 28*c
^3*d^14*e^2*(-a^3*c^3)^(5/2) + 28*a^7*c^11*d^14*e^2*x^2 + 114*a^8*c^10*d^1
2*e^4*x^2 + 108*a^9*c^9*d^10*e^6*x^2 - 63*a^10*c^8*d^8*e^8*x^2 - 32*a^11*c
^7*d^6*e^10*x^2 + 212*a^12*c^6*d^4*e^12*x^2 + 224*a^13*c^5*d^2*e^14*x^2 +
114*a*c^2*d^12*e^4*(-a^3*c^3)^(5/2) + 108*a^2*c*d^10*e^6*(-a^3*c^3)^(5/2)
- 212*a^8*c*d^4*e^12*(-a^3*c^3)^(3/2) + 32*a^7*c^2*d^6*e^10*(-a^3*c^3)^(3/
2))*(d*(-a^3*c^3)^(1/2) - a^2*c*e))/(4*(a^4*e^2 + a^3*c*d^2)) + (e^3*log(d
 + e*x^2))/(2*c*d^4 + 2*a*d^2*e^2) - 1/(2*a*d*x^2) - (e*log(x))/(a*d^2)
```

3.236 $\int \frac{1}{x^5(d+ex^2)(a+cx^4)}\,dx$

3.236.1 Optimal result

Integrand size = 22, antiderivative size = 156

$$\int \frac{1}{x^5(d+ex^2)(a+cx^4)}\,dx = -\frac{1}{4adx^4}+\frac{e}{2ad^2x^2}+\frac{c^{3/2}e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}(cd^2+ae^2)} - \frac{(cd^2-ae^2)\log(x)}{a^2d^3}-\frac{e^4\log(d+ex^2)}{2d^3(cd^2+ae^2)}+\frac{c^2d\log(a+cx^4)}{4a^2(cd^2+ae^2)}$$

output
```
-1/4/a/d/x^4+1/2*e/a/d^2/x^2+1/2*c^(3/2)*e*arctan(x^2*c^(1/2)/a^(1/2))/a^(
3/2)/(a*e^2+c*d^2)-(-a*e^2+c*d^2)*ln(x)/a^2/d^3-1/2*e^4*ln(e*x^2+d)/d^3/(a
*e^2+c*d^2)+1/4*c^2*d*ln(c*x^4+a)/a^2/(a*e^2+c*d^2)
```

3.236.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.34

$$\int \frac{1}{x^5(d+ex^2)(a+cx^4)}\,dx = -\frac{acd^4+a^2d^2e^2-2acd^3ex^2-2a^2de^3x^2+2\sqrt{a}c^{3/2}d^3ex^4\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\sqrt{a}c^{3/2}d^3ex^4\arctan\left(1\right.}{4a^2d^3(cd^2+ae}$$

input
```
Integrate[1/(x^5*(d + e*x^2)*(a + c*x^4)),x]
```

output
```
-1/4*(a*c*d^4 + a^2*d^2*e^2 - 2*a*c*d^3*e*x^2 - 2*a^2*d*e^3*x^2 + 2*Sqrt[a
]*c^(3/2)*d^3*e*x^4*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*Sqrt[a]*c^
(3/2)*d^3*e*x^4*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 4*c^2*d^4*x^4*Lo
g[x] - 4*a^2*e^4*x^4*Log[x] + 2*a^2*e^4*x^4*Log[d + e*x^2] - c^2*d^4*x^4*L
og[a + c*x^4])/(a^2*d^3*(c*d^2 + a*e^2)*x^4)
```

3.236.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1579, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (a+cx^4) (d+ex^2)} \, dx$$

$$\Big\downarrow \ 1579$$

$$\frac{1}{2} \int \frac{1}{x^6 (ex^2+d) (cx^4+a)} dx^2$$

$$\Big\downarrow \ 615$$

$$\frac{1}{2} \int \left(-\frac{e^5}{d^3 (cd^2+ae^2) (ex^2+d)} - \frac{e}{ad^2x^4} + \frac{c^2 (cdx^2+ae)}{a^2 (cd^2+ae^2) (cx^4+a)} + \frac{ae^2-cd^2}{a^2d^3x^2} + \frac{1}{adx^6} \right) dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2} \left(\frac{c^{3/2} e \arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{a^{3/2} (ae^2+cd^2)} + \frac{c^2 d \log(a+cx^4)}{2a^2 (ae^2+cd^2)} - \frac{\log(x^2) (cd^2-ae^2)}{a^2d^3} - \frac{e^4 \log(d+ex^2)}{d^3 (ae^2+cd^2)} + \frac{e}{ad^2x^2} - \frac{1}{2adx^4} \right)$$

input
```
Int[1/(x^5*(d + e*x^2)*(a + c*x^4)),x]
```

output
```
(-1/2*1/(a*d*x^4) + e/(a*d^2*x^2) + (c^(3/2)*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a
]])/(a^(3/2)*(c*d^2 + a*e^2)) - ((c*d^2 - a*e^2)*Log[x^2])/(a^2*d^3) - (e^
4*Log[d + e*x^2])/(d^3*(c*d^2 + a*e^2)) + (c^2*d*Log[a + c*x^4])/(2*a^2*(c
*d^2 + a*e^2)))/2
```

3.236.3.1 Defintions of rubi rules used

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.236.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.81

method	result
default	$-\frac{1}{4 a d \,x^{4}}+\frac{\left(a \,e^{2}-c \,d^{2}\right) \ln \left(x \right)}{d^{3} a^{2}}+\frac{e}{2 a \,d^{2} x^{2}}+\frac{c^{2} \left(\frac{d \ln \left(c \,x^{4}+a \right)}{2}+\frac{a e \arctan \left(\frac{c \,x^{2}}{\sqrt{a c}}\right)}{\sqrt{a c}}\right)}{2 \left(a \,e^{2}+c \,d^{2}\right) a^{2}}-\frac{e^{4} \ln \left(e \,x^{2}+d \right)}{2 d^{3} \left(a \,e^{2}+c \,d^{2}\right)}$
risch	$\frac{\frac{e \,x^{2}}{2 d^{2} a}-\frac{1}{4 d a}}{x^{4}}+\frac{\ln \left(x \right) e^{2}}{d^{3} a}-\frac{\ln \left(x \right) c}{d \,a^{2}}-\frac{e^{4} \ln \left(e \,x^{2}+d \right)}{2 d^{3} \left(a \,e^{2}+c \,d^{2}\right)}+\frac{\left(\sum_{\textit{\_R} =\operatorname{RootOf}\left(\left(a^{5} e^{2}+d^{2} a^{4} c \right) \textit{\_Z}^{2}-2 a^{2} c^{2} d \textit{\_Z} +c^{3}\right)}\textit{\_R} \ln \left(\left(\left(-6 a^{6} d^{4} e^{4}-7 \right.\right.\right.$

input
```
int(1/x^5/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4/a/d/x^4+(a*e^2-c*d^2)/d^3/a^2*ln(x)+1/2*e/a/d^2/x^2+1/2*c^2/(a*e^2+c*
d^2)/a^2*(1/2*d*ln(c*x^4+a)+a*e/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2)))-1/2
*e^4*ln(e*x^2+d)/d^3/(a*e^2+c*d^2)
```

3.236.5 Fricas [A] (verification not implemented)

Time = 49.46 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.17

$$\int \frac{1}{x^5 (d + ex^2) (a + cx^4)} dx$$
$$= \left[\frac{acd^3ex^4\sqrt{-\frac{c}{a}}\log\left(\frac{cx^4+2\,ax^2\sqrt{-\frac{c}{a}}-a}{cx^4+a}\right) + c^2d^4x^4\log(cx^4+a) - 2\,a^2e^4x^4\log(ex^2+d) - acd^4 - a^2d^2e^2 - 4}{4\,(a^2cd^5 + a^3d^3e^2)x^4} \right.$$
$$- \frac{2\,acd^3ex^4\sqrt{\frac{c}{a}}\arctan\left(\frac{a\sqrt{\frac{c}{a}}}{cx^2}\right) - c^2d^4x^4\log(cx^4+a) + 2\,a^2e^4x^4\log(ex^2+d) + acd^4 + a^2d^2e^2 + 4\,(c^2d^4}{4\,(a^2cd^5 + a^3d^3e^2)x^4}$$

input
```
integrate(1/x^5/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

output
```
[1/4*(a*c*d^3*e*x^4*sqrt(-c/a)*log((c*x^4 + 2*a*x^2*sqrt(-c/a) - a)/(c*x^4
 + a)) + c^2*d^4*x^4*log(c*x^4 + a) - 2*a^2*e^4*x^4*log(e*x^2 + d) - a*c*d
^4 - a^2*d^2*e^2 - 4*(c^2*d^4 - a^2*e^4)*x^4*log(x) + 2*(a*c*d^3*e + a^2*d
*e^3)*x^2)/((a^2*c*d^5 + a^3*d^3*e^2)*x^4), -1/4*(2*a*c*d^3*e*x^4*sqrt(c/a
)*arctan(a*sqrt(c/a)/(c*x^2)) - c^2*d^4*x^4*log(c*x^4 + a) + 2*a^2*e^4*x^4
*log(e*x^2 + d) + a*c*d^4 + a^2*d^2*e^2 + 4*(c^2*d^4 - a^2*e^4)*x^4*log(x)
 - 2*(a*c*d^3*e + a^2*d*e^3)*x^2)/((a^2*c*d^5 + a^3*d^3*e^2)*x^4)]
```

3.236.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (d + ex^2) (a + cx^4)} dx = \text{Timed out}$$

input
```
integrate(1/x**5/(e*x**2+d)/(c*x**4+a),x)
```

output
```
Timed out
```

3.236.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^5 (d + ex^2) (a + cx^4)} \, dx = -\frac{e^4 \log (ex^2 + d)}{2 (cd^5 + ad^3e^2)} + \frac{c^2 d \log (cx^4 + a)}{4 (a^2 cd^2 + a^3 e^2)} + \frac{c^2 e \arctan \left(\frac{cx^2}{\sqrt{ac}}\right)}{2 (acd^2 + a^2 e^2)\sqrt{ac}} - \frac{(cd^2 - ae^2) \log (x^2)}{2 a^2 d^3} + \frac{2 ex^2 - d}{4 ad^2 x^4}$$

input
```
integrate(1/x^5/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

output
```
-1/2*e^4*log(e*x^2 + d)/(c*d^5 + a*d^3*e^2) + 1/4*c^2*d*log(c*x^4 + a)/(a^
2*c*d^2 + a^3*e^2) + 1/2*c^2*e*arctan(c*x^2/sqrt(a*c))/((a*c*d^2 + a^2*e^2
)*sqrt(a*c)) - 1/2*(c*d^2 - a*e^2)*log(x^2)/(a^2*d^3) + 1/4*(2*e*x^2 - d)/
(a*d^2*x^4)
```

3.236.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^5 (d + ex^2) (a + cx^4)} \, dx = -\frac{e^5 \log (|ex^2 + d|)}{2 (cd^5 e + ad^3 e^3)} + \frac{c^2 d \log (cx^4 + a)}{4 (a^2 cd^2 + a^3 e^2)} + \frac{c^2 e \arctan \left(\frac{cx^2}{\sqrt{ac}}\right)}{2 (acd^2 + a^2 e^2)\sqrt{ac}} - \frac{(cd^2 - ae^2) \log (x^2)}{2 a^2 d^3} + \frac{3 cd^2 x^4 - 3 ae^2 x^4 + 2 adex^2 - ad^2}{4 a^2 d^3 x^4}$$

input
```
integrate(1/x^5/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

output
```
-1/2*e^5*log(abs(e*x^2 + d))/(c*d^5*e + a*d^3*e^3) + 1/4*c^2*d*log(c*x^4 +
 a)/(a^2*c*d^2 + a^3*e^2) + 1/2*c^2*e*arctan(c*x^2/sqrt(a*c))/((a*c*d^2 +
a^2*e^2)*sqrt(a*c)) - 1/2*(c*d^2 - a*e^2)*log(x^2)/(a^2*d^3) + 1/4*(3*c*d^
2*x^4 - 3*a*e^2*x^4 + 2*a*d*e*x^2 - a*d^2)/(a^2*d^3*x^4)
```

3.236.9 Mupad [B] (verification not implemented)

Time = 8.60 (sec) , antiderivative size = 1017, normalized size of antiderivative = 6.52

$$\int \frac{1}{x^5 (d+ex^2)(a+cx^4)}\,dx$$
$$= \frac{\ln\Big(25\,a^2\,c^9\,d^{20}\,(-a^5\,c^3)^{3/2} - 64\,a^{19}\,c^4\,e^{20}\,x^2 - 25\,a^9\,c^{14}\,d^{20}\,x^2 - 64\,a^{17}\,c^2\,e^{20}\,\sqrt{-a^5\,c^3} + 100\,a^3\,d^8\,e^{12}\,(-a^5\,c}{}$$
$$- \frac{e^4\,\ln(e\,x^2+d)}{2\,(c\,d^5+a\,d^3\,e^2)}$$
$$- \frac{\ln\Big(25\,a^9\,c^{14}\,d^{20}\,x^2 + 64\,a^{19}\,c^4\,e^{20}\,x^2 + 25\,a^2\,c^9\,d^{20}\,(-a^5\,c^3)^{3/2} - 64\,a^{17}\,c^2\,e^{20}\,\sqrt{-a^5\,c^3} + 100\,a^3\,d^8\,e^{12}\,(-a}{}$$
$$- \frac{\frac{1}{4\,a\,d} - \frac{e\,x^2}{2\,a\,d^2}}{x^4} + \frac{\ln(x)\,(a\,e^2 - c\,d^2)}{a^2\,d^3}$$

input
```
int(1/(x^5*(a + c*x^4)*(d + e*x^2)),x)
```

output
```
(log(25*a^2*c^9*d^20*(-a^5*c^3)^(3/2) - 64*a^19*c^4*e^20*x^2 - 25*a^9*c^14
*d^20*x^2 - 64*a^17*c^2*e^20*(-a^5*c^3)^(1/2) + 100*a^3*d^8*e^12*(-a^5*c^3
)^(5/2) + 128*a^11*d^2*e^18*(-a^5*c^3)^(3/2) - 112*c^3*d^14*e^6*(-a^5*c^3)
^(5/2) - 76*a^10*c^13*d^18*e^2*x^2 - 138*a^11*c^12*d^16*e^4*x^2 - 112*a^12
*c^11*d^14*e^6*x^2 + 55*a^13*c^10*d^12*e^8*x^2 + 104*a^14*c^9*d^10*e^10*x^
2 + 100*a^15*c^8*d^8*e^12*x^2 + 172*a^16*c^7*d^6*e^14*x^2 + 32*a^17*c^6*d^
4*e^16*x^2 - 128*a^18*c^5*d^2*e^18*x^2 + 55*a*c^2*d^12*e^8*(-a^5*c^3)^(5/2
) + 104*a^2*c*d^10*e^10*(-a^5*c^3)^(5/2) - 32*a^10*c*d^4*e^16*(-a^5*c^3)^(
3/2) + 76*a^3*c^8*d^18*e^2*(-a^5*c^3)^(3/2) + 138*a^4*c^7*d^16*e^4*(-a^5*c
^3)^(3/2) - 172*a^9*c^2*d^6*e^14*(-a^5*c^3)^(3/2))*(e*(-a^5*c^3)^(1/2) + a
^2*c^2*d))/(4*a^5*e^2 + 4*a^4*c*d^2) - (e^4*log(d + e*x^2))/(2*(c*d^5 + a*
d^3*e^2)) - (log(25*a^9*c^14*d^20*x^2 + 64*a^19*c^4*e^20*x^2 + 25*a^2*c^9*
d^20*(-a^5*c^3)^(3/2) - 64*a^17*c^2*e^20*(-a^5*c^3)^(1/2) + 100*a^3*d^8*e^
12*(-a^5*c^3)^(5/2) + 128*a^11*d^2*e^18*(-a^5*c^3)^(3/2) - 112*c^3*d^14*e^
6*(-a^5*c^3)^(5/2) + 76*a^10*c^13*d^18*e^2*x^2 + 138*a^11*c^12*d^16*e^4*x^
2 + 112*a^12*c^11*d^14*e^6*x^2 - 55*a^13*c^10*d^12*e^8*x^2 - 104*a^14*c^9*
d^10*e^10*x^2 - 100*a^15*c^8*d^8*e^12*x^2 - 172*a^16*c^7*d^6*e^14*x^2 - 32
*a^17*c^6*d^4*e^16*x^2 + 128*a^18*c^5*d^2*e^18*x^2 + 55*a*c^2*d^12*e^8*(-a
^5*c^3)^(5/2) + 104*a^2*c*d^10*e^10*(-a^5*c^3)^(5/2) - 32*a^10*c*d^4*e^16*
(-a^5*c^3)^(3/2) + 76*a^3*c^8*d^18*e^2*(-a^5*c^3)^(3/2) + 138*a^4*c^7*d...
```

3.237 $\int \frac{x^8}{(d+ex^2)(a+cx^4)} dx$

3.237.1 Optimal result

Integrand size = 22, antiderivative size = 359

$$\begin{aligned}\int \frac{x^8}{(d+ex^2)(a+cx^4)} dx = &-\frac{dx}{ce^2}+\frac{x^3}{3ce}+\frac{d^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{5/2}(cd^2+ae^2)} \\ &-\frac{a^{5/4}\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{7/4}(cd^2+ae^2)} \\ &+\frac{a^{5/4}\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{7/4}(cd^2+ae^2)} \\ &-\frac{a^{5/4}\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}c^{7/4}(cd^2+ae^2)} \\ &+\frac{a^{5/4}\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}c^{7/4}(cd^2+ae^2)}\end{aligned}$$

output
```
-d*x/c/e^2+1/3*x^3/c/e+d^(7/2)*arctan(x*e^(1/2)/d^(1/2))/e^(5/2)/(a*e^2+c*
d^2)+1/4*a^(5/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2
))/c^(7/4)/(a*e^2+c*d^2)*2^(1/2)+1/4*a^(5/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^
(1/4))*(-e*a^(1/2)+d*c^(1/2))/c^(7/4)/(a*e^2+c*d^2)*2^(1/2)-1/8*a^(5/4)*ln
(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/c^(
7/4)/(a*e^2+c*d^2)*2^(1/2)+1/8*a^(5/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2
)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/c^(7/4)/(a*e^2+c*d^2)*2^(1/2)
```

3.237.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.96

$$\int \frac{x^8}{(d+ex^2)(a+cx^4)}\,dx$$
$$= \frac{-24c^{3/4}d\sqrt{e}(cd^2+ae^2)\,x+8c^{3/4}e^{3/2}(cd^2+ae^2)\,x^3+24c^{7/4}d^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)+6\sqrt{2}a^{5/4}e^{5/2}(-\sqrt{c}d+\sqrt{c}}{}$$

input
```
Integrate[x^8/((d + e*x^2)*(a + c*x^4)),x]
```

output
```
(-24*c^(3/4)*d*Sqrt[e]*(c*d^2 + a*e^2)*x + 8*c^(3/4)*e^(3/2)*(c*d^2 + a*e^
2)*x^3 + 24*c^(7/4)*d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + 6*Sqrt[2]*a^(5/4
)*e^(5/2)*(-(Sqrt[c]*d) + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4
)] - 6*Sqrt[2]*a^(5/4)*e^(5/2)*(-(Sqrt[c]*d) + Sqrt[a]*e)*ArcTan[1 + (Sqrt
[2]*c^(1/4)*x)/a^(1/4)] - 3*Sqrt[2]*a*e^(5/2)*(a^(1/4)*Sqrt[c]*d + a^(3/4)
*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + 3*Sqrt[2]*a*e
^(5/2)*(a^(1/4)*Sqrt[c]*d + a^(3/4)*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/
4)*x + Sqrt[c]*x^2])/(24*c^(7/4)*e^(5/2)*(c*d^2 + a*e^2))
```

3.237.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1611, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+cx^4)(d+ex^2)}\,dx$$
$$\Big\downarrow \; 1611$$
$$\int \left(\frac{a^2(d-ex^2)}{c(a+cx^4)(ae^2+cd^2)}+\frac{d^4}{e^2(d+ex^2)(ae^2+cd^2)}-\frac{d}{ce^2}+\frac{x^2}{ce}\right)dx$$
$$\Big\downarrow \; 2009$$

$$-\frac{a^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(\sqrt{c}d-\sqrt{a}e\right)}{2\sqrt{2}c^{7/4}\left(ae^2+cd^2\right)}+\frac{a^{5/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{c}d-\sqrt{a}e\right)}{2\sqrt{2}c^{7/4}\left(ae^2+cd^2\right)}-\frac{a^{5/4}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{7/4}\left(ae^2+cd^2\right)}+\frac{a^{5/4}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{7/4}\left(ae^2+cd^2\right)}+\frac{d^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{5/2}\left(ae^2+cd^2\right)}-\frac{dx}{ce^2}+\frac{x^3}{3ce}$$

input
```
Int[x^8/((d + e*x^2)*(a + c*x^4)),x]
```

output
```
-((d*x)/(c*e^2)) + x^3/(3*c*e) + (d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^
(5/2)*(c*d^2 + a*e^2)) - (a^(5/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt
[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) + (a^(5/4)*(S
qrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*
c^(7/4)*(c*d^2 + a*e^2)) - (a^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^
2)) + (a^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/
4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2))
```

3.237.3.1 Defintions of rubi rules used

rule 1611
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a + c*x^4)), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.237.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.78

method	result
default	$-\frac{-\frac{1}{3}e\,x^3+dx}{c\,e^2}+\frac{a^2\left(\frac{d\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}-\frac{e\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\right.\right.}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+}\right.\right.}{(a\,e^2+c\,d^2)c}$
risch	$\frac{x^3}{3ce}-\frac{dx}{c\,e^2}+\frac{\sqrt{-ed}\,d^3\ln\left(\left(-16(-ed)^{\frac{5}{2}}a\,c^7d^{12}e^2+16(-ed)^{\frac{5}{2}}c^8d^{14}+14a^4c^4d^7e^9(-ed)^{\frac{3}{2}}-4a^3c^5d^9e^7(-ed)^{\frac{3}{2}}-2a^2c^6d^{11}e^5(-ed)^{\frac{3}{2}}\right.\right.}{}$

input
```
int(x^8/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/c/e^2*(-1/3*e*x^3+d*x)+a^2/(a*e^2+c*d^2)/c*(1/8*d*(a/c)^(1/4)/a*2^(1/2)
*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a
/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)
*x-1))-1/8*e/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1
/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)
*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1)))+1/e^2*d^4/(a*e^2+c*d^2)/(e*d)^(1
/2)*arctan(e*x/(e*d)^(1/2))
```

3.237.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2197 vs. 2(268) = 536.

Time = 5.53 (sec) , antiderivative size = 4414, normalized size of antiderivative = 12.30

$$\int \frac{x^8}{(d+ex^2)\,(a+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(x^8/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

output
```
[1/12*(6*c*d^3*sqrt(-d/e)*log((e*x^2 + 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d))
+ 4*(c*d^2*e + a*e^3)*x^3 - 3*(c^2*d^2*e^2 + a*c*e^4)*sqrt((2*a^3*d*e + (c
^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e
^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8
*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))*log(-
(a^3*c*d^2 - a^4*e^2)*x + (a^2*c^3*d^3 - a^3*c^2*d*e^2 + (c^7*d^4*e + 2*a*
c^6*d^2*e^3 + a^2*c^5*e^5)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)
/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^
4*c^7*e^8)))*sqrt((2*a^3*d*e + (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*s
qrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^
2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*
c^4*d^2*e^2 + a^2*c^3*e^4))) + 3*(c^2*d^2*e^2 + a*c*e^4)*sqrt((2*a^3*d*e +
 (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^
2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*
c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))*lo
g(-(a^3*c*d^2 - a^4*e^2)*x - (a^2*c^3*d^3 - a^3*c^2*d*e^2 + (c^7*d^4*e + 2
*a*c^6*d^2*e^3 + a^2*c^5*e^5)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e
^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 +
 a^4*c^7*e^8)))*sqrt((2*a^3*d*e + (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4
)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*...
```

3.237.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^8}{(d+ex^2)(a+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x**8/(e*x**2+d)/(c*x**4+a),x)
```

output
```
Timed out
```

3.237.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8}{(d+ex^2)(a+cx^4)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^8/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.237.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.03

$$\begin{aligned}\int \frac{x^8}{(d+ex^2)(a+cx^4)}\,dx &= \frac{d^4\arctan\left(\frac{ex}{\sqrt{de}}\right)}{(cd^2e^2+ae^4)\sqrt{de}} \\ &+ \frac{\left((ac^3)^{\frac14}ac^2d-(ac^3)^{\frac34}ae\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac14}\right)}{2\left(\frac{a}{c}\right)^{\frac14}}\right)}{2\left(\sqrt{2}c^5d^2+\sqrt{2}ac^4e^2\right)} \\ &+ \frac{\left((ac^3)^{\frac14}ac^2d-(ac^3)^{\frac34}ae\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac14}\right)}{2\left(\frac{a}{c}\right)^{\frac14}}\right)}{2\left(\sqrt{2}c^5d^2+\sqrt{2}ac^4e^2\right)} \\ &+ \frac{\left((ac^3)^{\frac14}ac^2d+(ac^3)^{\frac34}ae\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac14}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}c^5d^2+\sqrt{2}ac^4e^2\right)} \\ &- \frac{\left((ac^3)^{\frac14}ac^2d+(ac^3)^{\frac34}ae\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac14}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}c^5d^2+\sqrt{2}ac^4e^2\right)} \\ &+ \frac{c^2e^2x^3-3\,c^2dex}{3\,c^3e^3}\end{aligned}$$

input
```
integrate(x^8/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

output
```
d^4*arctan(e*x/sqrt(d*e))/((c*d^2*e^2 + a*e^4)*sqrt(d*e)) + 1/2*((a*c^3)^(
1/4)*a*c^2*d - (a*c^3)^(3/4)*a*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^
(1/4))/(a/c)^(1/4))/(sqrt(2)*c^5*d^2 + sqrt(2)*a*c^4*e^2) + 1/2*((a*c^3)^(
1/4)*a*c^2*d - (a*c^3)^(3/4)*a*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^
(1/4))/(a/c)^(1/4))/(sqrt(2)*c^5*d^2 + sqrt(2)*a*c^4*e^2) + 1/4*((a*c^3)^(
1/4)*a*c^2*d + (a*c^3)^(3/4)*a*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a
/c))/(sqrt(2)*c^5*d^2 + sqrt(2)*a*c^4*e^2) - 1/4*((a*c^3)^(1/4)*a*c^2*d +
(a*c^3)^(3/4)*a*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*c
^5*d^2 + sqrt(2)*a*c^4*e^2) + 1/3*(c^2*e^2*x^3 - 3*c^2*d*e*x)/(c^3*e^3)
```

3.237.9 Mupad [B] (verification not implemented)

Time = 8.48 (sec) , antiderivative size = 6097, normalized size of antiderivative = 16.98

$$\int \frac{x^8}{(d+ex^2)(a+cx^4)}\,dx = \text{Too large to display}$$

input
```
int(x^8/((a + c*x^4)*(d + e*x^2)),x)
```

output
```
(log(a^7*d^4*e^26 + 16*c^7*d^18*e^12 - 16*c^7*x*(-d^7*e^5)^(5/2) + 2*a^6*c
*d^6*e^24 + 16*a^3*c^4*d^12*e^18 + a^5*c^2*d^8*e^22 - a^7*e^24*x*(-d^7*e^5
)^(1/2) - a^5*c^2*d^4*e^20*x*(-d^7*e^5)^(1/2) + 16*a^3*c^4*d*e^11*x*(-d^7*
e^5)^(3/2) - 2*a^6*c*d^2*e^22*x*(-d^7*e^5)^(1/2))*(-d^7*e^5)^(1/2))/(2*a*e
^7 + 2*c*d^2*e^5) - atan((((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 +
 192*a^5*c^6*d^2*e^9)/(c^3*e^3) - (2*x*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-
a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e
^2)))^(1/2)*(256*a^5*c^7*e^12 - 256*a^2*c^10*d^6*e^6 - 256*a^3*c^9*d^4*e^8
 + 256*a^4*c^8*d^2*e^10))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^
5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2
)))^(1/2) + (2*x*(64*a^2*c^8*d^9*e + 56*a^6*c^4*d*e^9 - 8*a^4*c^6*d^5*e^5
- 16*a^5*c^5*d^3*e^7))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c
^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))
^(1/2) - (16*a^3*c^6*d^9 + 4*a^7*c^2*d*e^8 - 64*a^4*c^5*d^7*e^2 + 64*a^5*c
^4*d^5*e^4 + 4*a^6*c^3*d^3*e^6)/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^
2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d
^2*e^2)))^(1/2) - (2*x*(a^8*e^8 + 2*a^4*c^4*d^8))/(c^3*e^3))*((c*d^2*(-a^5
*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c
^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2)*1i - (((((192*a^3*c^8*d^6*e^5 + 384*a^4*
c^7*d^4*e^7 + 192*a^5*c^6*d^2*e^9)/(c^3*e^3) + (2*x*((c*d^2*(-a^5*c^7)^...
```

3.238 $\int \frac{x^6}{(d+ex^2)(a+cx^4)}\,dx$

3.238.1 Optimal result

Integrand size = 22, antiderivative size = 345

$$\begin{aligned}\int \frac{x^6}{(d+ex^2)(a+cx^4)}\,dx &= \frac{x}{ce}-\frac{d^{5/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}(cd^2+ae^2)}\\
&+\frac{a^{3/4}(\sqrt{c}d+\sqrt{a}e)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{5/4}(cd^2+ae^2)}\\
&-\frac{a^{3/4}(\sqrt{c}d+\sqrt{a}e)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{5/4}(cd^2+ae^2)}\\
&-\frac{a^{3/4}(\sqrt{c}d-\sqrt{a}e)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}c^{5/4}(cd^2+ae^2)}\\
&+\frac{a^{3/4}(\sqrt{c}d-\sqrt{a}e)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}c^{5/4}(cd^2+ae^2)}\end{aligned}$$

output
```
x/c/e-d^(5/2)*arctan(x*e^(1/2)/d^(1/2))/e^(3/2)/(a*e^2+c*d^2)-1/8*a^(3/4)*
ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e*a^(1/2)+d*c^(1/2))/
c^(5/4)/(a*e^2+c*d^2)*2^(1/2)+1/8*a^(3/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(
1/2)+x^2*c^(1/2))*(-e*a^(1/2)+d*c^(1/2))/c^(5/4)/(a*e^2+c*d^2)*2^(1/2)-1/4
*a^(3/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+d*c^(1/2))/c^(5/4
)/(a*e^2+c*d^2)*2^(1/2)-1/4*a^(3/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e
*a^(1/2)+d*c^(1/2))/c^(5/4)/(a*e^2+c*d^2)*2^(1/2)
```

3.238.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.08

$$\int \frac{x^6}{(d+ex^2)(a+cx^4)}\,dx = \frac{x}{ce} - \frac{d^{5/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}(cd^2+ae^2)} - \frac{\left(a^{3/4}cd+a^{5/4}\sqrt{c}e\right)\arctan\left(\frac{-\sqrt{2}\sqrt[4]{a}+2\sqrt[4]{c}x}{\sqrt{2}\sqrt[4]{a}}\right)}{2\sqrt{2}c^{7/4}(cd^2+ae^2)} - \frac{\left(a^{3/4}cd+a^{5/4}\sqrt{c}e\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}+2\sqrt[4]{c}x}{\sqrt{2}\sqrt[4]{a}}\right)}{2\sqrt{2}c^{7/4}(cd^2+ae^2)} - \frac{\left(a^{3/4}cd-a^{5/4}\sqrt{c}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}c^{7/4}(cd^2+ae^2)} + \frac{\left(a^{3/4}cd-a^{5/4}\sqrt{c}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}c^{7/4}(cd^2+ae^2)}$$

input
```
Integrate[x^6/((d + e*x^2)*(a + c*x^4)),x]
```

output
```
x/(c*e) - (d^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(3/2)*(c*d^2 + a*e^2))
- ((a^(3/4)*c*d + a^(5/4)*Sqrt[c]*e)*ArcTan[(-(Sqrt[2]*a^(1/4)) + 2*c^(1/4
)*x)/(Sqrt[2]*a^(1/4))])/(2*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) - ((a^(3/4)*c
*d + a^(5/4)*Sqrt[c]*e)*ArcTan[(Sqrt[2]*a^(1/4) + 2*c^(1/4)*x)/(Sqrt[2]*a^
(1/4))])/(2*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) - ((a^(3/4)*c*d - a^(5/4)*Sqr
t[c]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]
*c^(7/4)*(c*d^2 + a*e^2)) + ((a^(3/4)*c*d - a^(5/4)*Sqrt[c]*e)*Log[Sqrt[a]
 + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(7/4)*(c*d^2 + a
*e^2))
```

3.238.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1611, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+cx^4)(d+ex^2)}\,dx$$

$$\Big\downarrow \text{ 1611}$$

$$\int \left(-\frac{a(ae+cdx^2)}{c(a+cx^4)(ae^2+cd^2)}-\frac{d^3}{e(d+ex^2)(ae^2+cd^2)}+\frac{1}{ce}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e+\sqrt{c}d\right)}{2\sqrt{2}c^{5/4}(ae^2+cd^2)}-\frac{a^{3/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}e+\sqrt{c}d\right)}{2\sqrt{2}c^{5/4}(ae^2+cd^2)}-\frac{a^{3/4}\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{5/4}(ae^2+cd^2)}+\frac{a^{3/4}\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{5/4}(ae^2+cd^2)}-\frac{d^{5/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}(ae^2+cd^2)}+\frac{x}{ce}$$

input
```
Int[x^6/((d + e*x^2)*(a + c*x^4)),x]
```

output
```
x/(c*e) - (d^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(3/2)*(c*d^2 + a*e^2))
+ (a^(3/4)*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]
)/(2*Sqrt[2]*c^(5/4)*(c*d^2 + a*e^2)) - (a^(3/4)*(Sqrt[c]*d + Sqrt[a]*e)*A
rcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(5/4)*(c*d^2 + a*e^2)
) - (a^(3/4)*(Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(5/4)*(c*d^2 + a*e^2)) + (a^(3/4)*(Sqrt[c]
*d - Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4
*Sqrt[2]*c^(5/4)*(c*d^2 + a*e^2))
```

3.238.3.1 Defintions of rubi rules used

rule 1611
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a + c*x^4)), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.238.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.76

method	result
default	$\frac{x}{ce}-\frac{a\left(\frac{e\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8}+\frac{d\sqrt{2}\left(\ln\left(\frac{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan \cdots}{8\left(\frac{a}{c}\right) \cdots}\right)}{\left(a\,e^{2}+c\,d^{2}\right)c \cdots}$
risch	$\frac{x}{ce}+\frac{\sqrt{-ed}\,d^{2}\ln\left(\left(-16(-ed)^{\frac{5}{2}}a\,c^{5}d^{8}e^{2}+16(-ed)^{\frac{5}{2}}c^{6}d^{10}-14a^{3}c^{3}d^{5}e^{7}(-ed)^{\frac{3}{2}}+4a^{2}c^{4}d^{7}e^{5}(-ed)^{\frac{3}{2}}+2a\,c^{5}d^{9}(-ed)^{\frac{3}{2}}e^{3}+16c^{6}d^{11}(- \cdots}{\cdots}$

input
```
int(x^6/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
x/c/e-a/(a*e^2+c*d^2)/c*(1/8*e*(a/c)^(1/4)*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*
2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(
1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/8*d/(a/c)^(1/4)
*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^
(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/
c)^(1/4)*x-1)))-1/e*d^3/(a*e^2+c*d^2)/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2))
```

3.238.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2167 vs. 2(256) = 512.

Time = 1.08 (sec) , antiderivative size = 4354, normalized size of antiderivative = 12.62

$$\int \frac{x^6}{(d+ex^2)(a+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(x^6/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

output
```
[1/4*(2*c*d^2*sqrt(-d/e)*log((e*x^2 - 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d)) +
 (c^2*d^2*e + a*c*e^3)*sqrt(-(2*a^2*d*e + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2
*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c
^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d
^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))*log(-(a^2*c*d^2 - a^3*e^2)*x + (a^2*c
^2*d^2*e - a^3*c*e^3 - (c^6*d^5 + 2*a*c^5*d^3*e^2 + a^2*c^4*d*e^4)*sqrt(-(
a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^
2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))*sqrt(-(2*a^2*d*e + (c^4
*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2
 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2
*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))) - (c^2*d
^2*e + a*c*e^3)*sqrt(-(2*a^2*d*e + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^
4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*
e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*
a*c^3*d^2*e^2 + a^2*c^2*e^4))*log(-(a^2*c*d^2 - a^3*e^2)*x - (a^2*c^2*d^2*
e - a^3*c*e^3 - (c^6*d^5 + 2*a*c^5*d^3*e^2 + a^2*c^4*d*e^4)*sqrt(-(a^3*c^2
*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d
^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))*sqrt(-(2*a^2*d*e + (c^4*d^4 +
2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*
e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6...
```

3.238.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^6}{(d+ex^2)(a+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x**6/(e*x**2+d)/(c*x**4+a),x)
```

output
```
Timed out
```

3.238.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^6}{(d+ex^2)(a+cx^4)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^6/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.238.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.98

$$\begin{aligned}\int \frac{x^6}{(d+ex^2)(a+cx^4)}\,dx &= -\frac{d^3\arctan\left(\frac{ex}{\sqrt{de}}\right)}{(cd^2e+ae^3)\sqrt{de}} \\ &\quad - \frac{\left((ac^3)^{\frac{1}{4}}ace+(ac^3)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}c^4d^2+\sqrt{2}ac^3e^2\right)} \\ &\quad - \frac{\left((ac^3)^{\frac{1}{4}}ace+(ac^3)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}c^4d^2+\sqrt{2}ac^3e^2\right)} \\ &\quad - \frac{\left((ac^3)^{\frac{1}{4}}ace-(ac^3)^{\frac{3}{4}}d\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}c^4d^2+\sqrt{2}ac^3e^2\right)} \\ &\quad + \frac{\left((ac^3)^{\frac{1}{4}}ace-(ac^3)^{\frac{3}{4}}d\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}c^4d^2+\sqrt{2}ac^3e^2\right)} + \frac{x}{ce}\end{aligned}$$

input
```
integrate(x^6/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

output
```
-d^3*arctan(e*x/sqrt(d*e))/((c*d^2*e + a*e^3)*sqrt(d*e)) - 1/2*((a*c^3)^(1
/4)*a*c*e + (a*c^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4)
)/(a/c)^(1/4))/(sqrt(2)*c^4*d^2 + sqrt(2)*a*c^3*e^2) - 1/2*((a*c^3)^(1/4)*
a*c*e + (a*c^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a
/c)^(1/4))/(sqrt(2)*c^4*d^2 + sqrt(2)*a*c^3*e^2) - 1/4*((a*c^3)^(1/4)*a*c*
e - (a*c^3)^(3/4)*d)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)
*c^4*d^2 + sqrt(2)*a*c^3*e^2) + 1/4*((a*c^3)^(1/4)*a*c*e - (a*c^3)^(3/4)*d
)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*c^4*d^2 + sqrt(2)*
a*c^3*e^2) + x/(c*e)
```

3.238.9 Mupad [B] (verification not implemented)

Time = 8.44 (sec) , antiderivative size = 5908, normalized size of antiderivative = 17.12

$$\int \frac{x^6}{(d+ex^2)(a+cx^4)}\,dx = \text{Too large to display}$$

input
```
int(x^6/((a + c*x^4)*(d + e*x^2)),x)
```

output
```
atan((((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^3*e^6)/(c
*e) - (2*x*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*
d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2)*(256*a^5*c^5*e^
10 - 256*a^2*c^8*d^6*e^4 - 256*a^3*c^7*d^4*e^6 + 256*a^4*c^6*d^2*e^8))/(c*
e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(1
6*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) + (2*x*(64*a^2*c^6*d^7
*e - 56*a^5*c^3*d*e^7 + 8*a^3*c^5*d^5*e^3 + 16*a^4*c^4*d^3*e^5))/(c*e))*(-
(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7
*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (48*a^3*c^4*d^6*e - 60*a^4
*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5)/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2
*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^
2*e^2)))^(1/2) - (2*x*(a^6*e^6 - 2*a^3*c^3*d^6))/(c*e))*(-(a*e^2*(-a^3*c^5
)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e
^4 + 2*a*c^6*d^2*e^2)))^(1/2)*1i - (((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*
e^4 + 128*a^4*c^5*d^3*e^6)/(c*e) + (2*x*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*
(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2
*e^2)))^(1/2)*(256*a^5*c^5*e^10 - 256*a^2*c^8*d^6*e^4 - 256*a^3*c^7*d^4*e^
6 + 256*a^4*c^6*d^2*e^8))/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c
^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))
^(1/2) - (2*x*(64*a^2*c^6*d^7*e - 56*a^5*c^3*d*e^7 + 8*a^3*c^5*d^5*e^3 ...
```

3.239 $\int \frac{x^4}{(d+ex^2)(a+cx^4)}\,dx$

3.239.1 Optimal result

Integrand size = 22, antiderivative size = 336

$$\int \frac{x^4}{(d+ex^2)(a+cx^4)}\,dx = \frac{d^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}\,(cd^2+ae^2)} + \frac{\sqrt[4]{a}\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{3/4}\,(cd^2+ae^2)} - \frac{\sqrt[4]{a}\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{3/4}\,(cd^2+ae^2)} + \frac{\sqrt[4]{a}\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}\,(cd^2+ae^2)} - \frac{\sqrt[4]{a}\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}\,(cd^2+ae^2)}$$

output
```
-1/4*a^(1/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/c
^(3/4)/(a*e^2+c*d^2)*2^(1/2)-1/4*a^(1/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4
))*(-e*a^(1/2)+d*c^(1/2))/c^(3/4)/(a*e^2+c*d^2)*2^(1/2)+1/8*a^(1/4)*ln(-a^
(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/c^(3/4)
/(a*e^2+c*d^2)*2^(1/2)-1/8*a^(1/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^
2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/c^(3/4)/(a*e^2+c*d^2)*2^(1/2)+d^(3/2)*arc
tan(x*e^(1/2)/d^(1/2))/(a*e^2+c*d^2)/e^(1/2)
```

3.239.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.69

$$\int \frac{x^4}{(d+ex^2)(a+cx^4)}\,dx$$
$$= \frac{8c^{3/4}d^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)+\sqrt{2}\sqrt[4]{a}\sqrt{e}\left(2(\sqrt{c}d-\sqrt{a}e)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+(-2\sqrt{c}d+2\sqrt{a}e)\arctan\left(1\right.\right.}{8c^{3/4}\sqrt{e}\,(cd^2}$$

input
```
Integrate[x^4/((d + e*x^2)*(a + c*x^4)),x]
```

output
```
(8*c^(3/4)*d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + Sqrt[2]*a^(1/4)*Sqrt[e]*(
2*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + (-2*Sq
rt[c]*d + 2*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + (Sqrt[c]*
d + Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])))/(8*c^(3/4)*Sqrt[e
]*(c*d^2 + a*e^2))
```

3.239.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1611, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+cx^4)(d+ex^2)}\,dx$$
$$\Big\downarrow \; 1611$$
$$\int \left(\frac{d^2}{(d+ex^2)(ae^2+cd^2)}-\frac{a(d-ex^2)}{(a+cx^4)(ae^2+cd^2)}\right)dx$$
$$\Big\downarrow \; 2009$$

$$\frac{\sqrt[4]{a}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(\sqrt{c}d-\sqrt{a}e\right)}{2\sqrt{2}c^{3/4}\left(ae^2+cd^2\right)}-\frac{\sqrt[4]{a}\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{c}d-\sqrt{a}e\right)}{2\sqrt{2}c^{3/4}\left(ae^2+cd^2\right)}+\frac{d^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{a}(\sqrt{a}e+\sqrt{c}d)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}\left(ae^2+cd^2\right)}-\frac{\sqrt[4]{a}(\sqrt{a}e+\sqrt{c}d)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}\left(ae^2+cd^2\right)}$$

input
```
Int[x^4/((d + e*x^2)*(a + c*x^4)),x]
```

output
```
(d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*(c*d^2 + a*e^2)) + (a^(1/4)
*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[
2]*c^(3/4)*(c*d^2 + a*e^2)) - (a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 +
(Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)) + (a^(1/
4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[
c]*x^2])/(4*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)) - (a^(1/4)*(Sqrt[c]*d + Sqrt[
a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c
^(3/4)*(c*d^2 + a*e^2))
```

3.239.3.1 Defintions of rubi rules used

rule 1611
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a + c*x^4)), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.239.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.76

method	result
default	$-\frac{a\left(\frac{d\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}-\frac{e\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\cdots\right.\right.}{8c\left(\frac{a}{c}\right)^{\frac{1}{4}}}\cdots\right.}{a\,e^2+c\,d^2}$
risch	$\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^2c^3e^4+2a\,c^4d^2e^2+c^5d^4\right)\_Z^4-4a\,c^2de\_Z^2+a\right)}\_R\ln\left(\left(\left(-2a^3c^3e^8-2a^2c^4d^2e^6+2a\,c^5d^4e^4+2c^6d^6e^2\right)\_R^5+(7a^2c^2\cdots\right.\right.\right.$

input
```
int(x^4/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-a/(a*e^2+c*d^2)*(1/8*d*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/
2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/
(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))-1/8*e/c/(a/c)^(1/4)*2^
(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/
2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^
(1/4)*x-1)))+d^2/(a*e^2+c*d^2)/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2))
```

3.239.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2010 vs. 2(247) = 494.

Time = 0.47 (sec) , antiderivative size = 4040, normalized size of antiderivative = 12.02

$$\int \frac{x^4}{(d+ex^2)(a+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

output
```
[1/4*((c*d^2 + a*e^2)*sqrt((2*a*d*e + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e
^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e
^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a
*c^2*d^2*e^2 + a^2*c*e^4))*log(-(c*d^2 - a*e^2)*x + (c^2*d^3 - a*c*d*e^2 +
 (c^4*d^4*e + 2*a*c^3*d^2*e^3 + a^2*c^2*e^5)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^
2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^
4*d^2*e^6 + a^4*c^3*e^8)))*sqrt((2*a*d*e + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^
2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*
d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4
+ 2*a*c^2*d^2*e^2 + a^2*c*e^4))) - (c*d^2 + a*e^2)*sqrt((2*a*d*e + (c^3*d^
4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*
e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 +
a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))*log(-(c*d^2 - a*e^
2)*x - (c^2*d^3 - a*c*d*e^2 + (c^4*d^4*e + 2*a*c^3*d^2*e^3 + a^2*c^2*e^5)*
sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 +
 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))*sqrt((2*a*d*e + (c
^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 +
 a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e
^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))) + (c*d^2 + a
*e^2)*sqrt((2*a*d*e - (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*...
```

3.239.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4}{(d+ex^2)(a+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x**4/(e*x**2+d)/(c*x**4+a),x)
```

output
```
Timed out
```

3.239.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{(d+ex^2)(a+cx^4)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.239.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.99

$$\int \frac{x^4}{(d+ex^2)(a+cx^4)}\,dx = \frac{d^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{(cd^2+ae^2)\sqrt{de}} - \frac{\left((ac^3)^{\frac{1}{4}}c^2d-(ac^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}c^4d^2+\sqrt{2}ac^3e^2\right)} - \frac{\left((ac^3)^{\frac{1}{4}}c^2d-(ac^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}c^4d^2+\sqrt{2}ac^3e^2\right)} - \frac{\left((ac^3)^{\frac{1}{4}}c^2d+(ac^3)^{\frac{3}{4}}e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}c^4d^2+\sqrt{2}ac^3e^2\right)} + \frac{\left((ac^3)^{\frac{1}{4}}c^2d+(ac^3)^{\frac{3}{4}}e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}c^4d^2+\sqrt{2}ac^3e^2\right)}$$

input
```
integrate(x^4/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

output
```
d^2*arctan(e*x/sqrt(d*e))/((c*d^2 + a*e^2)*sqrt(d*e)) - 1/2*((a*c^3)^(1/4)
*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(
a/c)^(1/4))/(sqrt(2)*c^4*d^2 + sqrt(2)*a*c^3*e^2) - 1/2*((a*c^3)^(1/4)*c^2
*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)
^(1/4))/(sqrt(2)*c^4*d^2 + sqrt(2)*a*c^3*e^2) - 1/4*((a*c^3)^(1/4)*c^2*d +
 (a*c^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*c^
4*d^2 + sqrt(2)*a*c^3*e^2) + 1/4*((a*c^3)^(1/4)*c^2*d + (a*c^3)^(3/4)*e)*l
og(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*c^4*d^2 + sqrt(2)*a*c
^3*e^2)
```

3.239.9 Mupad [B] (verification not implemented)

Time = 8.65 (sec) , antiderivative size = 5111, normalized size of antiderivative = 15.21

$$\int \frac{x^4}{(d+ex^2)(a+cx^4)} dx = \text{Too large to display}$$

input
```
int(x^4/((a + c*x^4)*(d + e*x^2)),x)
```

output
```
atan(-((((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(
c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*((x*(112*a^4*c^3*d*e^6 +
112*a^2*c^5*d^5*e^2 - 32*a^3*c^4*d^3*e^4) + ((a*e^2*(-a*c^3)^(1/2) - c*d^2
*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^
2)))^(1/2)*(64*a^2*c^6*d^6*e^2 - x*((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)
^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2
)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c
^5*d^2*e^7) + 128*a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6))*((a*e^2*(-a*c^3)^
(1/2) - c*d^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2
*a*c^4*d^2*e^2)))^(1/2) + 16*a^2*c^4*d^5*e + 4*a^4*c^2*d*e^5 - 60*a^3*c^3*
d^3*e^3) - x*(2*a^4*c*e^5 + 4*a^2*c^3*d^4*e))*((a*e^2*(-a*c^3)^(1/2) - c*d
^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*
e^2)))^(1/2)*1i + (((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(1/2) + 2*a*c^2
*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*((x*(112*a^4*c
^3*d*e^6 + 112*a^2*c^5*d^5*e^2 - 32*a^3*c^4*d^3*e^4) - ((a*e^2*(-a*c^3)^(1
/2) - c*d^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a
*c^4*d^2*e^2)))^(1/2)*(x*((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(1/2) + 2
*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*(512*a^5
*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7
) + 64*a^2*c^6*d^6*e^2 + 128*a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6))*((a...
```

3.240 $\int \frac{x^2}{(d+ex^2)(a+cx^4)}\,dx$

3.240.1 Optimal result

Integrand size = 22, antiderivative size = 337

$$\int \frac{x^2}{(d+ex^2)(a+cx^4)}\,dx = -\frac{\sqrt{d}\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{cd^2+ae^2} - \frac{(\sqrt{c}d+\sqrt{a}e)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}(cd^2+ae^2)} + \frac{(\sqrt{c}d+\sqrt{a}e)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}(cd^2+ae^2)} + \frac{(\sqrt{c}d-\sqrt{a}e)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}(cd^2+ae^2)} - \frac{(\sqrt{c}d-\sqrt{a}e)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}(cd^2+ae^2)}$$

output
```
1/8*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e*a^(1/2)+d*c^(1/
2))/a^(1/4)/c^(1/4)/(a*e^2+c*d^2)*2^(1/2)-1/8*ln(a^(1/4)*c^(1/4)*x*2^(1/2)
+a^(1/2)+x^2*c^(1/2))*(-e*a^(1/2)+d*c^(1/2))/a^(1/4)/c^(1/4)/(a*e^2+c*d^2)
*2^(1/2)+1/4*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+d*c^(1/2))/a^
(1/4)/c^(1/4)/(a*e^2+c*d^2)*2^(1/2)+1/4*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4)
)*(e*a^(1/2)+d*c^(1/2))/a^(1/4)/c^(1/4)/(a*e^2+c*d^2)*2^(1/2)-arctan(x*e^(
1/2)/d^(1/2))*d^(1/2)*e^(1/2)/(a*e^2+c*d^2)
```

3.240.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.69

$$\int \frac{x^2}{(d+ex^2)(a+cx^4)}\,dx$$
$$= \frac{-8\sqrt[4]{a}\sqrt[4]{c}\sqrt{d}\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)+\sqrt{2}\left(-2(\sqrt{c}d+\sqrt{a}e)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2(\sqrt{c}d+\sqrt{a}e)\arctan\left(1\right.\right.}{8\sqrt[4]{a}\sqrt[4]{c}\,(cd^2\,-}$$

input
```
Integrate[x^2/((d + e*x^2)*(a + c*x^4)),x]
```

output
```
(-8*a^(1/4)*c^(1/4)*Sqrt[d]*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + Sqrt[2]*
(-2*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*(S
qrt[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + (Sqrt[c]*d
 - Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Lo
g[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])))/(8*a^(1/4)*c^(1/4)
*(c*d^2 + a*e^2))
```

3.240.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1611, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+cx^4)(d+ex^2)}\,dx$$
$$\Big\downarrow \text{1611}$$
$$\int \left(\frac{ae+cdx^2}{(a+cx^4)(ae^2+cd^2)}-\frac{de}{(d+ex^2)(ae^2+cd^2)}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{d}\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{ae^2+cd^2}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e+\sqrt{c}d\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\left(ae^2+cd^2\right)}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}e+\sqrt{c}d\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\left(ae^2+cd^2\right)}+\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\left(ae^2+cd^2\right)}-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\left(ae^2+cd^2\right)}$$

input
```
Int[x^2/((d + e*x^2)*(a + c*x^4)),x]
```

output
```
-((Sqrt[d]*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)) - ((Sqrt[
c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1
/4)*c^(1/4)*(c*d^2 + a*e^2)) + ((Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2
]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*c^(1/4)*(c*d^2 + a*e^2)) + ((Sqr
t[c]*d - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]
)/(4*Sqrt[2]*a^(1/4)*c^(1/4)*(c*d^2 + a*e^2)) - ((Sqrt[c]*d - Sqrt[a]*e)*L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(1/4)*
c^(1/4)*(c*d^2 + a*e^2))
```

3.240.3.1 Defintions of rubi rules used

rule 1611
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a + c*x^4)), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.240.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.73

method	result
default	$\frac{\frac{e\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8}+\frac{d\sqrt{2}\left(\ln\left(\frac{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+\cdots\right)\right)}{8\left(\frac{a}{c}\right)^{\frac{1}{4}}}}{a\,e^{2}+c\,d^{2}}$
risch	$\frac{\sqrt{-ed}\,\ln\left(\left(-16(-ed)^{\frac{5}{2}}a^{2}c\,e^{3}+16(-ed)^{\frac{5}{2}}a\,c^{2}d^{2}e-14a^{2}cd\,e^{4}(-ed)^{\frac{3}{2}}+20a\,c^{2}d^{3}e^{2}(-ed)^{\frac{3}{2}}+2d^{5}(-ed)^{\frac{3}{2}}c^{3}-\sqrt{-ed}\,a^{3}e^{7}+3\sqrt{-ed}\,a\,c^{2}d^{\cdots}\right.\right.}{2a\,e^{2}+2c\,d^{2}}$

input
```
int(x^2/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/(a*e^2+c*d^2)*(1/8*e*(a/c)^(1/4)*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+
(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/
c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/8*d/(a/c)^(1/4)*2^(1/2)
*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a
/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)
*x-1)))-d*e/(a*e^2+c*d^2)/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2))
```

3.240.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1936 vs. $2(248) = 496$.

Time = 0.36 (sec) , antiderivative size = 3892, normalized size of antiderivative = 11.55

$$\int \frac{x^2}{(d+ex^2)(a+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

output
```
[-1/4*((c*d^2 + a*e^2)*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*
sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 +
 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2
*e^2 + a^2*e^4))*log(-(c*d^2 - a*e^2)*x + (a*c*d^2*e - a^2*e^3 - (a*c^3*d^
5 + 2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*
e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^
6 + a^5*c*e^8)))*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-
(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3
*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 +
 a^2*e^4))) - (c*d^2 + a*e^2)*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^
2*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^
6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*
a*c*d^2*e^2 + a^2*e^4))*log(-(c*d^2 - a*e^2)*x - (a*c*d^2*e - a^2*e^3 - (a
*c^3*d^5 + 2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2
 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2
*d^2*e^6 + a^5*c*e^8)))*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)
*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2
+ 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^
2*e^2 + a^2*e^4))) + (c*d^2 + a*e^2)*sqrt(-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e
^2 + a^2*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*...
```

3.240.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(d+ex^2)(a+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x**2/(e*x**2+d)/(c*x**4+a),x)
```

output
```
Timed out
```

3.240.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(d+ex^2)\,(a+cx^4)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.240.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.01

$$\begin{aligned}\int \frac{x^2}{(d+ex^2)\,(a+cx^4)}\,dx = & -\frac{de\arctan\left(\frac{ex}{\sqrt{de}}\right)}{(cd^2+ae^2)\sqrt{de}} \\ & + \frac{\left((ac^3)^{\frac{1}{4}}ace+(ac^3)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)} \\ & + \frac{\left((ac^3)^{\frac{1}{4}}ace+(ac^3)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)} \\ & + \frac{\left((ac^3)^{\frac{1}{4}}ace-(ac^3)^{\frac{3}{4}}d\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)} \\ & - \frac{\left((ac^3)^{\frac{1}{4}}ace-(ac^3)^{\frac{3}{4}}d\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)}\end{aligned}$$

input
```
integrate(x^2/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

output
```
-d*e*arctan(e*x/sqrt(d*e))/((c*d^2 + a*e^2)*sqrt(d*e)) + 1/2*((a*c^3)^(1/4
)*a*c*e + (a*c^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/
(a/c)^(1/4))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + 1/2*((a*c^3)^(1/4
)*a*c*e + (a*c^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/
(a/c)^(1/4))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + 1/4*((a*c^3)^(1/4
)*a*c*e - (a*c^3)^(3/4)*d)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(s
qrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) - 1/4*((a*c^3)^(1/4)*a*c*e - (a*c^
3)^(3/4)*d)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^3*d^
2 + sqrt(2)*a^2*c^2*e^2)
```

3.240.9 Mupad [B] (verification not implemented)

Time = 8.46 (sec) , antiderivative size = 4720, normalized size of antiderivative = 14.01

$$\int \frac{x^2}{(d+ex^2)(a+cx^4)}\,dx = \text{Too large to display}$$

input
```
int(x^2/((a + c*x^4)*(d + e*x^2)),x)
```

output
```
(log(a^2*d*e^7 + c^2*d^5*e^3 - c^2*d*x*(-d*e)^(7/2) + 2*a*c*d^3*e^5 + a^2*
e^7*x*(-d*e)^(1/2) + 2*a*c*e^3*x*(-d*e)^(5/2))*(-d*e)^(1/2))/(2*a*e^2 + 2*
c*d^2) - atan((((-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(1
6*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(((-(c*d^2*(-a*c)^(1
/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c
^2*d^2*e^2)))^(1/2)*(x*(-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*
d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(512*a^5*c^4*
e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 1
92*a^4*c^4*d*e^7 + 192*a^2*c^6*d^5*e^3 + 384*a^3*c^5*d^3*e^5) - x*(16*a*c^
6*d^5*e^2 - 112*a^3*c^4*d*e^6 + 160*a^2*c^5*d^3*e^4))*(-(c*d^2*(-a*c)^(1/2
) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2
*d^2*e^2)))^(1/2) + 4*a*c^5*d^4*e^2 + 52*a^2*c^4*d^2*e^4) + x*(2*a^2*c^3*e
^5 - 4*a*c^4*d^2*e^3))*(-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*
d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*1i - ((-(c*d^
2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^
4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(((-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2
) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(19
2*a^4*c^4*d*e^7 - x*(-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e
)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9
 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 1...
```

3.241 $\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx$

3.241.1 Optimal result

Integrand size = 19, antiderivative size = 336

$$\begin{aligned}\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx =& \frac{e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2+ae^2)} - \frac{\sqrt[4]{c}(\sqrt{c}d-\sqrt{a}e)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(cd^2+ae^2)} \\ &+ \frac{\sqrt[4]{c}(\sqrt{c}d-\sqrt{a}e)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(cd^2+ae^2)} \\ &- \frac{\sqrt[4]{c}(\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\,(cd^2+ae^2)} \\ &+ \frac{\sqrt[4]{c}(\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\,(cd^2+ae^2)}\end{aligned}$$

output
```
1/4*c^(1/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/a^
(3/4)/(a*e^2+c*d^2)*2^(1/2)+1/4*c^(1/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4)
)*(-e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)*2^(1/2)-1/8*c^(1/4)*ln(-a^(
1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/a^(3/4)/
(a*e^2+c*d^2)*2^(1/2)+1/8*c^(1/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2
*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)*2^(1/2)+e^(3/2)*arct
an(x*e^(1/2)/d^(1/2))/(a*e^2+c*d^2)/d^(1/2)
```

3.241.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.70

$$\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx$$
$$= \frac{8a^{3/4}e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)+\sqrt{2}\sqrt[4]{c}\sqrt{d}\left(\left(-2\sqrt{c}d+2\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(1\right.\right.}{8a^{3/4}\sqrt{d}\left(cd^2\right.}$$

input
```
Integrate[1/((d + e*x^2)*(a + c*x^4)),x]
```

output
```
(8*a^(3/4)*e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + Sqrt[2]*c^(1/4)*Sqrt[d]*(
(-2*Sqrt[c]*d + 2*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*(
Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - (Sqrt[c]*
d + Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])))/(8*a^(3/4)*Sqrt[d
]*(c*d^2 + a*e^2))
```

3.241.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1485, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^4)(d+ex^2)}\,dx$$
$$\Big\downarrow \text{1485}$$
$$\int \left(\frac{e^2}{(d+ex^2)(ae^2+cd^2)}+\frac{c(d-ex^2)}{(a+cx^4)(ae^2+cd^2)}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt[4]{c}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(\sqrt{c}d-\sqrt{a}e\right)}{2\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{c}\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{c}d-\sqrt{a}e\right)}{2\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)}-\frac{\sqrt[4]{c}(\sqrt{a}e+\sqrt{c}d)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{c}(\sqrt{a}e+\sqrt{c}d)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)}+\frac{e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(ae^2+cd^2\right)}$$

input
```
Int[1/((d + e*x^2)*(a + c*x^4)),x]
```

output
```
(e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 + a*e^2)) - (c^(1/4)
*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[
2]*a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 +
(Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)) - (c^(1/
4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[
c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]*d + Sqrt[
a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a
^(3/4)*(c*d^2 + a*e^2))
```

3.241.3.1 Defintions of rubi rules used

rule 1485
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.241.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.75

method	result
default	$\frac{c\left(\frac{d\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}-\frac{e\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right.\right.}{8c\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right.}{a\,e^2+c\,d^2}$
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(a^5e^4+2a^4c\,d^2e^2+a^3c^2d^4\right)\_Z^4-4a^2cde\_Z^2+c\right)}\_R\ln\left(\left(\left(-2a^5e^7-2a^4c\,d^2e^5+2a^3c^2d^4e^3+2a^2c^3d^6e\right)\_R^4+\left(15a^2cd\right.\right.\right.\right.}{4}$

input
```
int(1/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
c/(a*e^2+c*d^2)*(1/8*d*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2
)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(
a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))-1/8*e/c/(a/c)^(1/4)*2^(
1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2
)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(
1/4)*x-1)))+e^2/(a*e^2+c*d^2)/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2))
```

3.241.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2030 vs. 2(247) = 494.

Time = 0.68 (sec) , antiderivative size = 4084, normalized size of antiderivative = 12.15

$$\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

output
```
[-1/4*((c*d^2 + a*e^2)*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*
e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^
3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 +
2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^2)*x + (a*c^2*d^3 - a^2*
c*d*e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a
*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4
*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d
^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d
^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/
(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) - (c*d^2 + a*e^2)*sqrt((2*c*d*e
+ (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2
 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6
*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2
*d^2 - a*c*e^2)*x - (a*c^2*d^3 - a^2*c*d*e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^
2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^
8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*s
qrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*
a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^
4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^
4))) + (c*d^2 + a*e^2)*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a...
```

3.241.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x**2+d)/(c*x**4+a),x)
```

output
```
Timed out
```

3.241.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.241.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.02

$$\begin{aligned}\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx = & \frac{e^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{(cd^2+ae^2)\sqrt{de}} \\ & + \frac{\left((ac^3)^{\frac{1}{4}}c^2d-(ac^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)} \\ & + \frac{\left((ac^3)^{\frac{1}{4}}c^2d-(ac^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)} \\ & + \frac{\left((ac^3)^{\frac{1}{4}}c^2d+(ac^3)^{\frac{3}{4}}e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)} \\ & - \frac{\left((ac^3)^{\frac{1}{4}}c^2d+(ac^3)^{\frac{3}{4}}e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)}\end{aligned}$$

input
```
integrate(1/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

output
```
e^2*arctan(e*x/sqrt(d*e))/((c*d^2 + a*e^2)*sqrt(d*e)) + 1/2*((a*c^3)^(1/4)
*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(
a/c)^(1/4))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + 1/2*((a*c^3)^(1/4)
*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(
a/c)^(1/4))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + 1/4*((a*c^3)^(1/4)
*c^2*d + (a*c^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sq
rt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) - 1/4*((a*c^3)^(1/4)*c^2*d + (a*c^3
)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^3*d^2
 + sqrt(2)*a^2*c^2*e^2)
```

3.241.9 Mupad [B] (verification not implemented)

Time = 8.52 (sec) , antiderivative size = 4802, normalized size of antiderivative = 14.29

$$\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx = \text{Too large to display}$$

input
```
int(1/((a + c*x^4)*(d + e*x^2)),x)
```

output
```
atan(((((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a
^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(4*c^6*d^3*e^3 - (((a*e^2*
(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^
2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 + x*((a*e^2*(-a^3*c)^(1/
2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^
4*c*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*
d^4*e^5 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 +
448*a^3*c^5*d^2*e^6) + x*(16*c^7*d^5*e^2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*
d*e^6))*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(
a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2) + 20*a*c^5*d*e^5) - 6*c^5
*e^5*x)*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(
a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*1i - (((a*e^2*(-a^3*c)^(1
/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a
^4*c*d^2*e^2)))^(1/2)*(4*c^6*d^3*e^3 - (((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a
^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))
^(1/2)*(256*a^4*c^4*e^8 - x*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2)
+ 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(512*
a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*
e^7) - 64*a*c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6) - x*(
16*c^7*d^5*e^2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*((a*e^2*(-a^3*c...
```

3.242 $\int \frac{1}{x^2(d+ex^2)(a+cx^4)}\,dx$

3.242.1 Optimal result

Integrand size = 22, antiderivative size = 348

$$\int \frac{1}{x^2(d+ex^2)(a+cx^4)}\,dx = -\frac{1}{adx} - \frac{e^{5/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}(cd^2+ae^2)} + \frac{c^{3/4}\left(\sqrt{c}d+\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{5/4}(cd^2+ae^2)} - \frac{c^{3/4}\left(\sqrt{c}d+\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{5/4}(cd^2+ae^2)} - \frac{c^{3/4}\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{5/4}(cd^2+ae^2)} + \frac{c^{3/4}\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{5/4}(cd^2+ae^2)}$$

output
```
-1/a/d/x-e^(5/2)*arctan(x*e^(1/2)/d^(1/2))/d^(3/2)/(a*e^2+c*d^2)-1/8*c^(3/
4)*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e*a^(1/2)+d*c^(1/2
))/a^(5/4)/(a*e^2+c*d^2)*2^(1/2)+1/8*c^(3/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+
a^(1/2)+x^2*c^(1/2))*(-e*a^(1/2)+d*c^(1/2))/a^(5/4)/(a*e^2+c*d^2)*2^(1/2)-
1/4*c^(3/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+d*c^(1/2))/a^(
5/4)/(a*e^2+c*d^2)*2^(1/2)-1/4*c^(3/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))
*(e*a^(1/2)+d*c^(1/2))/a^(5/4)/(a*e^2+c*d^2)*2^(1/2)
```

3.242.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^2 (d+ex^2) (a+cx^4)} \, dx$$
$$= \frac{-8a^{5/4}e^{5/2}x \arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) - \sqrt{d}\left(8\sqrt[4]{a}cd^2 + 8a^{5/4}e^2 - 2\sqrt{2}c^{3/4}d(\sqrt{c}d+\sqrt{a}e)\,x\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right) + 2\cdots\right.}{}$$

input
```
Integrate[1/(x^2*(d + e*x^2)*(a + c*x^4)),x]
```

output
```
(-8*a^(5/4)*e^(5/2)*x*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - Sqrt[d]*(8*a^(1/4)*c*d
^2 + 8*a^(5/4)*e^2 - 2*Sqrt[2]*c^(3/4)*d*(Sqrt[c]*d + Sqrt[a]*e)*x*ArcTan[
1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*Sqrt[2]*c^(3/4)*d*(Sqrt[c]*d + Sqrt[a
]*e)*x*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + Sqrt[2]*c^(5/4)*d^2*x*Log
[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Sqrt[2]*Sqrt[a]*c^(3
/4)*d*e*x*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Sqrt[2]
*c^(5/4)*d^2*x*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Sq
rt[2]*Sqrt[a]*c^(3/4)*d*e*x*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2]))/(8*a^(5/4)*d^(3/2)*(c*d^2 + a*e^2)*x)
```

3.242.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1611, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+cx^4) (d+ex^2)} \, dx$$

$$\Big\downarrow \text{ 1611}$$

$$\int \left(-\frac{c(ae+cdx^2)}{a(a+cx^4)(ae^2+cd^2)} - \frac{e^3}{d(d+ex^2)(ae^2+cd^2)} + \frac{1}{adx^2}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e+\sqrt{c}d\right)}{2\sqrt{2}a^{5/4}\left(ae^2+cd^2\right)}-\frac{c^{3/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}e+\sqrt{c}d\right)}{2\sqrt{2}a^{5/4}\left(ae^2+cd^2\right)}-\frac{c^{3/4}\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{5/4}\left(ae^2+cd^2\right)}+\frac{c^{3/4}\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{5/4}\left(ae^2+cd^2\right)}-\frac{e^{5/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\left(ae^2+cd^2\right)}-\frac{1}{adx}$$

input
```
Int[1/(x^2*(d + e*x^2)*(a + c*x^4)),x]
```

output
```
-(1/(a*d*x)) - (e^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*(c*d^2 + a*e
^2)) + (c^(3/4)*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(
1/4)])/(2*Sqrt[2]*a^(5/4)*(c*d^2 + a*e^2)) - (c^(3/4)*(Sqrt[c]*d + Sqrt[a]
*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(5/4)*(c*d^2 + a
*e^2)) - (c^(3/4)*(Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^
(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(5/4)*(c*d^2 + a*e^2)) + (c^(3/4)*(Sq
rt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2
])/(4*Sqrt[2]*a^(5/4)*(c*d^2 + a*e^2))
```

3.242.3.1 Defintions of rubi rules used

rule 1611
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a + c*x^4)), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.242.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.76

method	result
default	$-\frac{c\left(\frac{e\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8}+\frac{d\sqrt{2}\left(\ln\left(\frac{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\cdots\right.}{8\left(\frac{a}{c}\right)^{\frac{1}{4}}}\cdots\right)}{\left(a\,e^{2}+c\,d^{2}\right)a}\cdots$
risch	$-\frac{1}{adx}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^{7}e^{4}+2a^{6}c\,d^{2}e^{2}+a^{5}c^{2}d^{4}\right)\_Z^{4}+4a^{3}c^{2}de\_Z^{2}+c^{3}\right)}\_R\ln\left(\left(\left(6e^{8}d^{3}a^{9}+19e^{6}d^{5}c\,a^{8}+25e^{4}d^{7}c^{2}a^{7}+17e^{2}d^{9}c^{3}a\cdots\right.\right.\right.}{\cdots}$

input
```
int(1/x^2/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-c/(a*e^2+c*d^2)/a*(1/8*e*(a/c)^(1/4)*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/
2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/
(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/8*d/(a/c)^(1/4)*2^(1
/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)
+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1
/4)*x-1)))-1/d*e^3/(a*e^2+c*d^2)/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2))-1/a/d
/x
```

3.242.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2169 vs. 2(259) = 518.

Time = 1.81 (sec) , antiderivative size = 4362, normalized size of antiderivative = 12.53

$$\int \frac{1}{x^2\left(d+ex^2\right)\left(a+cx^4\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

output
```
[1/4*(2*a*e^2*x*sqrt(-e/d)*log((e*x^2 - 2*d*x*sqrt(-e/d) - d)/(e*x^2 + d))
 + (a*c*d^3 + a^2*d*e^2)*x*sqrt(-(2*c^2*d*e + (a^2*c^2*d^4 + 2*a^3*c*d^2*e
^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8
 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a
^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))*log(-(c^3*d^2 - a*c^2*e^2)*x + (a
^2*c^2*d^2*e - a^3*c*e^3 - (a^4*c^2*d^5 + 2*a^5*c*d^3*e^2 + a^6*d*e^4)*sqr
t(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*
e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))*sqrt(-(2*c^2*d*e +
(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2
 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a
^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))) - (a
*c*d^3 + a^2*d*e^2)*x*sqrt(-(2*c^2*d*e + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 +
a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*
a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^
2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))*log(-(c^3*d^2 - a*c^2*e^2)*x - (a^2*c^
2*d^2*e - a^3*c*e^3 - (a^4*c^2*d^5 + 2*a^5*c*d^3*e^2 + a^6*d*e^4)*sqrt(-(c
^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 +
 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))*sqrt(-(2*c^2*d*e + (a^2*
c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^
2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8...
```

3.242.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2(d+ex^2)(a+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/x**2/(e*x**2+d)/(c*x**4+a),x)
```

output
```
Timed out
```

3.242.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^2(d+ex^2)(a+cx^4)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^2/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.242.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.01

$$\begin{aligned}\int \frac{1}{x^2(d+ex^2)(a+cx^4)}\,dx = &-\frac{e^3\arctan\left(\frac{ex}{\sqrt{de}}\right)}{(cd^3+ade^2)\sqrt{de}} \\ &-\frac{\left((ac^3)^{\frac{1}{4}}ace+(ac^3)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^2c^2d^2+\sqrt{2}a^3ce^2\right)} \\ &-\frac{\left((ac^3)^{\frac{1}{4}}ace+(ac^3)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^2c^2d^2+\sqrt{2}a^3ce^2\right)} \\ &-\frac{\left((ac^3)^{\frac{1}{4}}ace-(ac^3)^{\frac{3}{4}}d\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}a^2c^2d^2+\sqrt{2}a^3ce^2\right)} \\ &+\frac{\left((ac^3)^{\frac{1}{4}}ace-(ac^3)^{\frac{3}{4}}d\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}a^2c^2d^2+\sqrt{2}a^3ce^2\right)}-\frac{1}{adx}\end{aligned}$$

input
```
integrate(1/x^2/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

output
```
-e^3*arctan(e*x/sqrt(d*e))/((c*d^3 + a*d*e^2)*sqrt(d*e)) - 1/2*((a*c^3)^(1
/4)*a*c*e + (a*c^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4)
)/(a/c)^(1/4))/(sqrt(2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) - 1/2*((a*c^3)^(1
/4)*a*c*e + (a*c^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4)
)/(a/c)^(1/4))/(sqrt(2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) - 1/4*((a*c^3)^(1
/4)*a*c*e - (a*c^3)^(3/4)*d)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/
(sqrt(2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) + 1/4*((a*c^3)^(1/4)*a*c*e - (a*
c^3)^(3/4)*d)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^
2*d^2 + sqrt(2)*a^3*c*e^2) - 1/(a*d*x)
```

3.242.9 Mupad [B] (verification not implemented)

Time = 8.68 (sec) , antiderivative size = 5761, normalized size of antiderivative = 16.55

$$\int \frac{1}{x^2(d+ex^2)(a+cx^4)}\,dx = \text{Too large to display}$$

input
```
int(1/(x^2*(a + c*x^4)*(d + e*x^2)),x)
```

output
```
atan(((x*(2*a^7*c^7*d^9*e^5 - 4*a^8*c^6*d^7*e^7) - (-(a*e^2*(-a^5*c^3)^(1/
2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 +
2*a^6*c*d^2*e^2)))^(1/2)*(((-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1
/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)
*(x*(-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(1
6*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^15*e^3
 + 512*a^12*c^6*d^13*e^5 - 512*a^13*c^5*d^11*e^7 - 512*a^14*c^4*d^9*e^9) -
 192*a^10*c^7*d^14*e^3 - 128*a^11*c^6*d^12*e^5 + 320*a^12*c^5*d^10*e^7 + 2
56*a^13*c^4*d^8*e^9) + x*(16*a^8*c^8*d^14*e^2 + 32*a^9*c^7*d^12*e^4 - 112*
a^10*c^6*d^10*e^6 + 128*a^11*c^5*d^8*e^8))*(-(a*e^2*(-a^5*c^3)^(1/2) - c*d
^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*
d^2*e^2)))^(1/2) - 4*a^7*c^8*d^13*e^2 - 4*a^8*c^7*d^11*e^4 + 16*a^10*c^5*d
^7*e^8))*(-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*
e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*1i + (x*(2*a^7*c^
7*d^9*e^5 - 4*a^8*c^6*d^7*e^7) - (-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c
^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))
^(1/2)*(((-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*
e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(x*(-(a*e^2*(-a^5
*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c
^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^15*e^3 + 512*a^12*c^6...
```

3.243 $\int \frac{1}{x^4(d+ex^2)(a+cx^4)}\,dx$

3.243.1 Optimal result

Integrand size = 22, antiderivative size = 360

$$\begin{aligned}\int \frac{1}{x^4\left(d+ex^2\right)\left(a+cx^4\right)}\,dx = &-\frac{1}{3adx^3}+\frac{e}{ad^2x}+\frac{e^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\left(cd^2+ae^2\right)}\\ &+\frac{c^{5/4}\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{7/4}\left(cd^2+ae^2\right)}\\ &-\frac{c^{5/4}\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{7/4}\left(cd^2+ae^2\right)}\\ &+\frac{c^{5/4}\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{7/4}\left(cd^2+ae^2\right)}\\ &-\frac{c^{5/4}\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{7/4}\left(cd^2+ae^2\right)}\end{aligned}$$

output
```
-1/3/a/d/x^3+e/a/d^2/x+e^(7/2)*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)/(a*e^2+c*
d^2)-1/4*c^(5/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2
))/a^(7/4)/(a*e^2+c*d^2)*2^(1/2)-1/4*c^(5/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^
(1/4))*(-e*a^(1/2)+d*c^(1/2))/a^(7/4)/(a*e^2+c*d^2)*2^(1/2)+1/8*c^(5/4)*ln
(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/a^(
7/4)/(a*e^2+c*d^2)*2^(1/2)-1/8*c^(5/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2
)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/a^(7/4)/(a*e^2+c*d^2)*2^(1/2)
```

3.243.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^4(d+ex^2)(a+cx^4)}\,dx = \frac{-8ad^{3/2}(cd^2+ae^2)+24a\sqrt{d}e(cd^2+ae^2)\,x^2+24a^2e^{7/2}x^3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)+6\sqrt{2}\sqrt[4]{a}c^{5/4}d^{5/2}\left(\sqrt{c}d-\sqrt{a}e\right)x}{}$$

input
```
Integrate[1/(x^4*(d + e*x^2)*(a + c*x^4)),x]
```

output
```
(-8*a*d^(3/2)*(c*d^2 + a*e^2) + 24*a*Sqrt[d]*e*(c*d^2 + a*e^2)*x^2 + 24*a^
2*e^(7/2)*x^3*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + 6*Sqrt[2]*a^(1/4)*c^(5/4)*d^(5
/2)*(Sqrt[c]*d - Sqrt[a]*e)*x^3*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] +
6*Sqrt[2]*a^(1/4)*c^(5/4)*d^(5/2)*(-(Sqrt[c]*d) + Sqrt[a]*e)*x^3*ArcTan[1
+ (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 3*Sqrt[2]*c^(5/4)*d^(5/2)*(a^(1/4)*Sqrt[c
]*d + a^(3/4)*e)*x^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2
] - 3*Sqrt[2]*c^(5/4)*d^(5/2)*(a^(1/4)*Sqrt[c]*d + a^(3/4)*e)*x^3*Log[Sqrt
[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(24*a^2*d^(5/2)*(c*d^2 + a
*e^2)*x^3)
```

3.243.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1611, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4(a+cx^4)(d+ex^2)}\,dx$$

$$\Big\downarrow \; 1611$$

$$\int \left(-\frac{c^2(d-ex^2)}{a(a+cx^4)(ae^2+cd^2)}+\frac{e^4}{d^2(d+ex^2)(ae^2+cd^2)}-\frac{e}{ad^2x^2}+\frac{1}{adx^4}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{c^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(\sqrt{c}d-\sqrt{a}e\right)}{2\sqrt{2}a^{7/4}\left(ae^2+cd^2\right)}-\frac{c^{5/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{c}d-\sqrt{a}e\right)}{2\sqrt{2}a^{7/4}\left(ae^2+cd^2\right)}+\frac{c^{5/4}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{7/4}\left(ae^2+cd^2\right)}-\frac{c^{5/4}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{7/4}\left(ae^2+cd^2\right)}+\frac{e^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\left(ae^2+cd^2\right)}+\frac{e}{ad^2x}-\frac{1}{3adx^3}$$

input
```
Int[1/(x^4*(d + e*x^2)*(a + c*x^4)),x]
```

output
```
-1/3*1/(a*d*x^3) + e/(a*d^2*x) + (e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^
(5/2)*(c*d^2 + a*e^2)) + (c^(5/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt
[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) - (c^(5/4)*(S
qrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*
a^(7/4)*(c*d^2 + a*e^2)) + (c^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^
2)) - (c^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/
4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2))
```

3.243.3.1 Defintions of rubi rules used

rule 1611
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a + c*x^4)), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.243.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.79

method	result
default	$-\frac{1}{3adx^3}+\frac{e}{a\,d^2x}-\frac{c^2\left(\frac{d\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}-\frac{e\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\cdots}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\cdots}\cdots\right.\right.}{\cdots}\cdots\right)}{(a\,e^2+c\,d^2)a}\cdots$
risch	$\frac{\frac{e\,x^2}{d^2a}-\frac{1}{3da}}{x^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^2d^5e^4+2c\,e^2a\,d^7+c^2d^9\right)\_Z^2+e^7\right)}\_R\ln\left(\left(\left(48a^{11}d^5e^8+152a^{10}c\,d^7e^6+200a^9c^2d^9e^4+136a^8c^3d^{11}e^2+\cdots\right.\right.\right.\right.}{\cdots}$

input
```
int(1/x^4/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3/a/d/x^3+e/a/d^2/x-c^2/(a*e^2+c*d^2)/a*(1/8*d*(a/c)^(1/4)/a*2^(1/2)*(l
n((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)
^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-
1))-1/8*e/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)
)/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+
1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1)))+1/d^2*e^4/(a*e^2+c*d^2)/(e*d)^(1/2)
*arctan(e*x/(e*d)^(1/2))
```

3.243.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2209 vs. 2(269) = 538.

Time = 5.98 (sec) , antiderivative size = 4442, normalized size of antiderivative = 12.34

$$\int \frac{1}{x^4\left(d+ex^2\right)\left(a+cx^4\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^4/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

output
```
[1/12*(6*a*e^3*x^3*sqrt(-e/d)*log((e*x^2 + 2*d*x*sqrt(-e/d) - d)/(e*x^2 +
d)) + 3*(a*c*d^4 + a^2*d^2*e^2)*x^3*sqrt((2*c^3*d*e + (a^3*c^2*d^4 + 2*a^4
*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7
*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11
*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))*log(-(c^5*d^2 - a*c^4*e
^2)*x + (a^2*c^4*d^3 - a^3*c^3*d*e^2 + (a^6*c^2*d^4*e + 2*a^7*c*d^2*e^3 +
a^8*e^5)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*
a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))*sqrt(
(2*c^3*d*e + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*
a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*
d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 +
a^5*e^4))) - 3*(a*c*d^4 + a^2*d^2*e^2)*x^3*sqrt((2*c^3*d*e + (a^3*c^2*d^4
+ 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^
4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6
 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))*log(-(c^5*d^2 -
a*c^4*e^2)*x - (a^2*c^4*d^3 - a^3*c^3*d*e^2 + (a^6*c^2*d^4*e + 2*a^7*c*d^2
*e^3 + a^8*e^5)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d
^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8))
)*sqrt((2*c^3*d*e + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d
^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + ...
```

3.243.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (d + ex^2) (a + cx^4)} \, dx = \text{Timed out}$$

input
```
integrate(1/x**4/(e*x**2+d)/(c*x**4+a),x)
```

output
```
Timed out
```

3.243.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^4 (d + ex^2) (a + cx^4)} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^4/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.243.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.02

$$\begin{aligned}\int \frac{1}{x^4 (d + ex^2) (a + cx^4)} \, dx = &\frac{e^4 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{(cd^4 + ad^2e^2)\sqrt{de}} \\ &- \frac{\left((ac^3)^{\frac{1}{4}} c^2 d - (ac^3)^{\frac{3}{4}} e\right) \arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^2c^2d^2 + \sqrt{2}a^3ce^2\right)} \\ &- \frac{\left((ac^3)^{\frac{1}{4}} c^2 d - (ac^3)^{\frac{3}{4}} e\right) \arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^2c^2d^2 + \sqrt{2}a^3ce^2\right)} \\ &- \frac{\left((ac^3)^{\frac{1}{4}} c^2 d + (ac^3)^{\frac{3}{4}} e\right) \log\left(x^2 + \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}a^2c^2d^2 + \sqrt{2}a^3ce^2\right)} \\ &+ \frac{\left((ac^3)^{\frac{1}{4}} c^2 d + (ac^3)^{\frac{3}{4}} e\right) \log\left(x^2 - \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}a^2c^2d^2 + \sqrt{2}a^3ce^2\right)} \\ &+ \frac{3\,ex^2 - d}{3\,ad^2x^3}\end{aligned}$$

input
```
integrate(1/x^4/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

output
```
e^4*arctan(e*x/sqrt(d*e))/((c*d^4 + a*d^2*e^2)*sqrt(d*e)) - 1/2*((a*c^3)^(
1/4)*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4
))/(a/c)^(1/4))/(sqrt(2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) - 1/2*((a*c^3)^(
1/4)*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4
))/(a/c)^(1/4))/(sqrt(2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) - 1/4*((a*c^3)^(
1/4)*c^2*d + (a*c^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))
/(sqrt(2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) + 1/4*((a*c^3)^(1/4)*c^2*d + (a
*c^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c
^2*d^2 + sqrt(2)*a^3*c*e^2) + 1/3*(3*e*x^2 - d)/(a*d^2*x^3)
```

3.243.9 Mupad [B] (verification not implemented)

Time = 8.71 (sec) , antiderivative size = 5972, normalized size of antiderivative = 16.59

$$\int \frac{1}{x^4 (d + ex^2) (a + cx^4)} \, dx = \text{Too large to display}$$

input
```
int(1/(x^4*(a + c*x^4)*(d + e*x^2)),x)
```

output
```
atan(((x*(2*a^5*c^9*d^18*e^5 + 4*a^7*c^7*d^14*e^9) - ((a*e^2*(-a^7*c^5)^(1
/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 +
 2*a^8*c*d^2*e^2)))^(1/2)*((((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1
/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)
*(x*((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16
*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^24*e^3
+ 512*a^12*c^6*d^22*e^5 - 512*a^13*c^5*d^20*e^7 - 512*a^14*c^4*d^18*e^9) -
 64*a^9*c^8*d^24*e^2 + 128*a^10*c^7*d^22*e^4 + 192*a^11*c^6*d^20*e^6 - 256
*a^12*c^5*d^18*e^8 - 256*a^13*c^4*d^16*e^10) - x*(16*a^7*c^9*d^23*e^2 + 32
*a^8*c^8*d^21*e^4 - 112*a^9*c^7*d^19*e^6 - 128*a^11*c^5*d^15*e^10))*((a*e^
2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4
+ a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2) - 4*a^6*c^9*d^21*e^3 - 4*a^7*c^8*
d^19*e^5 + 48*a^9*c^6*d^15*e^9))*((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^
5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^
(1/2)*1i + (x*(2*a^5*c^9*d^18*e^5 + 4*a^7*c^7*d^14*e^9) - ((a*e^2*(-a^7*c^
5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*
d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*((((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^
5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^
(1/2)*(x*((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e
)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d...
```

3.244 $\int \frac{x^9}{(d+ex^2)(a+cx^4)^2}\,dx$

3.244.1 Optimal result

Integrand size = 22, antiderivative size = 169

$$\int \frac{x^9}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{a(ae+cdx^2)}{4c^2(cd^2+ae^2)(a+cx^4)} - \frac{\sqrt{a}d(3cd^2+ae^2)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4c^{3/2}(cd^2+ae^2)^2} + \frac{d^4\log(d+ex^2)}{2e(cd^2+ae^2)^2} + \frac{ae(2cd^2+ae^2)\log(a+cx^4)}{4c^2(cd^2+ae^2)^2}$$

output
```
1/4*a*(c*d*x^2+a*e)/c^2/(a*e^2+c*d^2)/(c*x^4+a)+1/2*d^4*ln(e*x^2+d)/e/(a*e
^2+c*d^2)^2+1/4*a*e*(a*e^2+2*c*d^2)*ln(c*x^4+a)/c^2/(a*e^2+c*d^2)^2-1/4*d*
(a*e^2+3*c*d^2)*arctan(x^2*c^(1/2)/a^(1/2))*a^(1/2)/c^(3/2)/(a*e^2+c*d^2)^
2
```

3.244.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.94

$$\int \frac{x^9}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{-\sqrt{a}\sqrt{c}de(3cd^2+ae^2)(a+cx^4)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right) + 2c^2d^4(a+cx^4)\log(d+ex^2) + ae((cd^2+ae^2)(ae+cdx

input
```
Integrate[x^9/((d + e*x^2)*(a + c*x^4)^2),x]
```

output
```
(-(Sqrt[a]*Sqrt[c]*d*e*(3*c*d^2 + a*e^2)*(a + c*x^4)*ArcTan[(Sqrt[c]*x^2)/
Sqrt[a]]) + 2*c^2*d^4*(a + c*x^4)*Log[d + e*x^2] + a*e*((c*d^2 + a*e^2)*(a
*e + c*d*x^2) + e*(2*c*d^2 + a*e^2)*(a + c*x^4)*Log[a + c*x^4]))/(4*c^2*e*
(c*d^2 + a*e^2)^2*(a + c*x^4))
```

3.244.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1579, 601, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{(a+cx^4)^2(d+ex^2)}dx$$

$$\Big\downarrow \text{1579}$$

$$\frac{1}{2}\int \frac{x^8}{(ex^2+d)(cx^4+a)^2}dx^2$$

$$\Big\downarrow \text{601}$$

$$\frac{1}{2}\left(\frac{a(ae+cdx^2)}{2c^2(a+cx^4)(ae^2+cd^2)}-\frac{\int \frac{-\frac{2ax^4}{c}-\frac{a^2dex^2}{c(cd^2+ae^2)}+\frac{a^2d^2}{c(cd^2+ae^2)}}{(ex^2+d)(cx^4+a)}dx^2}{2a}\right)$$

$$\Big\downarrow \text{2160}$$

$$\frac{1}{2}\left(\frac{a(ae+cdx^2)}{2c^2(a+cx^4)(ae^2+cd^2)}-\frac{\int \left(\frac{a^2(d(3cd^2+ae^2)-2e(2cd^2+ae^2)x^2)}{c(cd^2+ae^2)^2(cx^4+a)}-\frac{2ad^4}{(cd^2+ae^2)^2(ex^2+d)}\right)dx^2}{2a}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{a(ae+cdx^2)}{2c^2(a+cx^4)(ae^2+cd^2)}-\frac{\frac{a^{3/2}d\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)(ae^2+3cd^2)}{c^{3/2}(ae^2+cd^2)^2}-\frac{a^2e(ae^2+2cd^2)\log(a+cx^4)}{c^2(ae^2+cd^2)^2}-\frac{2ad^4\log(d+ex^2)}{e(ae^2+cd^2)^2}}{2a}\right)$$

input
```
Int[x^9/((d + e*x^2)*(a + c*x^4)^2),x]
```

output
```
((a*(a*e + c*d*x^2))/(2*c^2*(c*d^2 + a*e^2)*(a + c*x^4)) - ((a^(3/2)*d*(3*
c*d^2 + a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(c^(3/2)*(c*d^2 + a*e^2)^2)
- (2*a*d^4*Log[d + e*x^2])/(e*(c*d^2 + a*e^2)^2) - (a^2*e*(2*c*d^2 + a*e^2
)*Log[a + c*x^4])/(c^2*(c*d^2 + a*e^2)^2))/(2*a))/2
```

3.244.3.1 Defintions of rubi rules used

rule 601
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c
+ d*x)^n*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Qx)/(c + d*x)^n + (e*
(2*p + 3))/(c + d*x)^n, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 1]
 && LtQ[p, -1] && ILtQ[n, 0] && NeQ[b*c^2 + a*d^2, 0]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.244.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.95

method	result
default	$-\frac{a\left(\frac{-\frac{d\left(a e^2+c d^2\right)x^2}{2c}-\frac{ae\left(a e^2+c d^2\right)}{2c^2}}{c x^4+a}+\frac{\frac{\left(-2a e^3-4c d^2e\right)\ln\left(c x^4+a\right)}{2c}+\frac{\left(d e^2a+3d^3c\right)\arctan\left(\frac{c x^2}{\sqrt{ac}}\right)}{\sqrt{ac}}}{2c}\right)}{2(a e^2+c d^2)^2}+\frac{d^4\ln\left(e x^2+d\right)}{2e(a e^2+c d^2)^2}$
risch	$\frac{\frac{da x^2}{4c\left(a e^2+c d^2\right)}+\frac{a^2e}{4c^2\left(a e^2+c d^2\right)}}{c x^4+a}+\frac{d^4\ln\left(e x^2+d\right)}{2e(a^2e^4+2ac d^2e^2+c^2d^4)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(c^4e^4a^2+2a c^5d^2e^2+c^6d^4\right)\_Z^2+\left(-4e^3c^2a^2-8a c^3d^2e\right)\right.}\right.}{}$

input
```
int(x^9/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/(a*e^2+c*d^2)^2*((-1/2*d*(a*e^2+c*d^2)/c*x^2-1/2*a*e*(a*e^2+c*d^2)/
c^2)/(c*x^4+a)+1/2/c*(1/2*(-2*a*e^3-4*c*d^2*e)/c*ln(c*x^4+a)+(a*d*e^2+3*c*
d^3)/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2))))+1/2*d^4*ln(e*x^2+d)/e/(a*e^2+
c*d^2)^2
```

3.244.5 Fricas [A] (verification not implemented)

Time = 8.77 (sec) , antiderivative size = 555, normalized size of antiderivative = 3.28

$$\int \frac{x^9}{(d+ex^2)(a+cx^4)^2}\,dx$$
$$=\left[\frac{2a^2cd^2e^2+2a^3e^4+2(ac^2d^3e+a^2cde^3)x^2+(3ac^2d^3e+a^2cde^3+(3c^3d^3e+ac^2de^3)x^4)\sqrt{-\frac{a}{c}}\log\left(\frac{cx^4-}{}\right.}{8(ac^4d^4e+2a^2c^3d^2e^3+a^3c^2e^5}\right.$$

input
```
integrate(x^9/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
[1/8*(2*a^2*c*d^2*e^2 + 2*a^3*e^4 + 2*(a*c^2*d^3*e + a^2*c*d*e^3)*x^2 + (3
*a*c^2*d^3*e + a^2*c*d*e^3 + (3*c^3*d^3*e + a*c^2*d*e^3)*x^4)*sqrt(-a/c)*l
og((c*x^4 - 2*c*x^2*sqrt(-a/c) - a)/(c*x^4 + a)) + 2*(2*a^2*c*d^2*e^2 + a^
3*e^4 + (2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^4)*log(c*x^4 + a) + 4*(c^3*d^4*x^4
 + a*c^2*d^4)*log(e*x^2 + d))/(a*c^4*d^4*e + 2*a^2*c^3*d^2*e^3 + a^3*c^2*e
^5 + (c^5*d^4*e + 2*a*c^4*d^2*e^3 + a^2*c^3*e^5)*x^4), 1/4*(a^2*c*d^2*e^2
+ a^3*e^4 + (a*c^2*d^3*e + a^2*c*d*e^3)*x^2 - (3*a*c^2*d^3*e + a^2*c*d*e^3
 + (3*c^3*d^3*e + a*c^2*d*e^3)*x^4)*sqrt(a/c)*arctan(c*x^2*sqrt(a/c)/a) +
(2*a^2*c*d^2*e^2 + a^3*e^4 + (2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^4)*log(c*x^4
+ a) + 2*(c^3*d^4*x^4 + a*c^2*d^4)*log(e*x^2 + d))/(a*c^4*d^4*e + 2*a^2*c^
3*d^2*e^3 + a^3*c^2*e^5 + (c^5*d^4*e + 2*a*c^4*d^2*e^3 + a^2*c^3*e^5)*x^4)
]
```

3.244.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^9}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**9/(e*x**2+d)/(c*x**4+a)**2,x)
```

output
```
Timed out
```

3.244.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.30

$$\begin{aligned}\int \frac{x^9}{(d+ex^2)(a+cx^4)^2}\,dx &= \frac{d^4\log(ex^2+d)}{2(c^2d^4e+2\,acd^2e^3+a^2e^5)} + \frac{(2\,acd^2e+a^2e^3)\log(cx^4+a)}{4(c^4d^4+2\,ac^3d^2e^2+a^2c^2e^4)} \\ &\quad - \frac{(3\,acd^3+a^2de^2)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4(c^3d^4+2\,ac^2d^2e^2+a^2ce^4)\sqrt{ac}} \\ &\quad + \frac{acdx^2+a^2e}{4(ac^3d^2+a^2c^2e^2+(c^4d^2+ac^3e^2)x^4)}\end{aligned}$$

input
```
integrate(x^9/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/2*d^4*log(e*x^2 + d)/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5) + 1/4*(2*a*c*
d^2*e + a^2*e^3)*log(c*x^4 + a)/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)
- 1/4*(3*a*c*d^3 + a^2*d*e^2)*arctan(c*x^2/sqrt(a*c))/((c^3*d^4 + 2*a*c^2*
d^2*e^2 + a^2*c*e^4)*sqrt(a*c)) + 1/4*(a*c*d*x^2 + a^2*e)/(a*c^3*d^2 + a^2
*c^2*e^2 + (c^4*d^2 + a*c^3*e^2)*x^4)
```

3.244.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.53

$$\int \frac{x^9}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{d^4\log(|ex^2+d|)}{2(c^2d^4e+2acd^2e^3+a^2e^5)} + \frac{(2acd^2e+a^2e^3)\log(cx^4+a)}{4(c^4d^4+2ac^3d^2e^2+a^2c^2e^4)} - \frac{(3acd^3+a^2de^2)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4(c^3d^4+2ac^2d^2e^2+a^2ce^4)\sqrt{ac}} - \frac{2acd^2ex^4+a^2e^3x^4-acd^3x^2-a^2de^2x^2+a^2d^2e}{4(c^3d^4+2ac^2d^2e^2+a^2ce^4)(cx^4+a)}$$

input
```
integrate(x^9/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/2*d^4*log(abs(e*x^2 + d))/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5) + 1/4*(2
*a*c*d^2*e + a^2*e^3)*log(c*x^4 + a)/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*
e^4) - 1/4*(3*a*c*d^3 + a^2*d*e^2)*arctan(c*x^2/sqrt(a*c))/((c^3*d^4 + 2*a
*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(a*c)) - 1/4*(2*a*c*d^2*e*x^4 + a^2*e^3*x^4
- a*c*d^3*x^2 - a^2*d*e^2*x^2 + a^2*d^2*e)/((c^3*d^4 + 2*a*c^2*d^2*e^2 + a
^2*c*e^4)*(c*x^4 + a))
```

3.244.9 Mupad [B] (verification not implemented)

Time = 8.25 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.80

$$\int \frac{x^9}{(d+ex^2)(a+cx^4)^2}\,dx$$
$$= \frac{\frac{a^2 e}{4 c^2 (c d^2+a e^2)}+\frac{a d x^2}{4 c (c d^2+a e^2)}}{c x^4+a} - \frac{\ln\left(\sqrt{-a c^5}+c^3 x^2\right)\left(3 c d^3 \sqrt{-a c^5}-2 a^2 c^2 e^3-4 a c^3 d^2 e+a d e^2 \sqrt{-a c^5}\right)}{8\left(a^2 c^4 e^4+2 a c^5 d^2 e^2+c^6 d^4\right)} + \frac{\ln\left(\sqrt{-a c^5}-c^3 x^2\right)\left(3 c d^3 \sqrt{-a c^5}+2 a^2 c^2 e^3+4 a c^3 d^2 e+a d e^2 \sqrt{-a c^5}\right)}{8\left(a^2 c^4 e^4+2 a c^5 d^2 e^2+c^6 d^4\right)} + \frac{d^4 \ln\left(e x^2+d\right)}{2 a^2 e^5+4 a c d^2 e^3+2 c^2 d^4 e}$$

input
```
int(x^9/((a + c*x^4)^2*(d + e*x^2)),x)
```

output
```
((a^2*e)/(4*c^2*(a*e^2 + c*d^2)) + (a*d*x^2)/(4*c*(a*e^2 + c*d^2)))/(a + c
*x^4) - (log((-a*c^5)^(1/2) + c^3*x^2)*(3*c*d^3*(-a*c^5)^(1/2) - 2*a^2*c^2
*e^3 - 4*a*c^3*d^2*e + a*d*e^2*(-a*c^5)^(1/2)))/(8*(c^6*d^4 + a^2*c^4*e^4
+ 2*a*c^5*d^2*e^2)) + (log((-a*c^5)^(1/2) - c^3*x^2)*(3*c*d^3*(-a*c^5)^(1/
2) + 2*a^2*c^2*e^3 + 4*a*c^3*d^2*e + a*d*e^2*(-a*c^5)^(1/2)))/(8*(c^6*d^4
+ a^2*c^4*e^4 + 2*a*c^5*d^2*e^2)) + (d^4*log(d + e*x^2))/(2*a^2*e^5 + 2*c^
2*d^4*e + 4*a*c*d^2*e^3)
```

3.245 $\int \frac{x^7}{(d+ex^2)(a+cx^4)^2}\,dx$

3.245.1 Optimal result

Integrand size = 22, antiderivative size = 150

$$\int \frac{x^7}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{a(d-ex^2)}{4c(cd^2+ae^2)(a+cx^4)} + \frac{\sqrt{a}e(3cd^2+ae^2)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4c^{3/2}(cd^2+ae^2)^2} - \frac{d^3\log(d+ex^2)}{2(cd^2+ae^2)^2} + \frac{d^3\log(a+cx^4)}{4(cd^2+ae^2)^2}$$

output
```
1/4*a*(-e*x^2+d)/c/(a*e^2+c*d^2)/(c*x^4+a)-1/2*d^3*ln(e*x^2+d)/(a*e^2+c*d^
2)^2+1/4*d^3*ln(c*x^4+a)/(a*e^2+c*d^2)^2+1/4*e*(a*e^2+3*c*d^2)*arctan(x^2*
c^(1/2)/a^(1/2))*a^(1/2)/c^(3/2)/(a*e^2+c*d^2)^2
```

3.245.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.95

$$\int \frac{x^7}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{\sqrt{a}e(3cd^2+ae^2)(a+cx^4)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right) + \sqrt{c}(a(cd^2+ae^2)(d-ex^2) - 2cd^3(a+cx^4)\log(d+ex^2) + cd}{4c^{3/2}(cd^2+ae^2)^2(a+cx^4)}$$

input
```
Integrate[x^7/((d + e*x^2)*(a + c*x^4)^2),x]
```

output
```
(Sqrt[a]*e*(3*c*d^2 + a*e^2)*(a + c*x^4)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]] + S
qrt[c]*(a*(c*d^2 + a*e^2)*(d - e*x^2) - 2*c*d^3*(a + c*x^4)*Log[d + e*x^2]
 + c*d^3*(a + c*x^4)*Log[a + c*x^4]))/(4*c^(3/2)*(c*d^2 + a*e^2)^2*(a + c*
x^4))
```

3.245.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1579, 601, 25, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{\left(a+cx^4\right)^2\left(d+ex^2\right)}dx$$

$$\Big\downarrow \text{1579}$$

$$\frac{1}{2}\int \frac{x^6}{\left(ex^2+d\right)\left(cx^4+a\right)^2}dx^2$$

$$\Big\downarrow \text{601}$$

$$\frac{1}{2}\left(\frac{a\left(d-ex^2\right)}{2c\left(a+cx^4\right)\left(ae^2+cd^2\right)}-\frac{\int -\frac{a\left(c\left(2d^2+\frac{ae^2}{c}\right)x^2+ade\right)}{c(cd^2+ae^2)(ex^2+d)(cx^4+a)}dx^2}{2a}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{\int \frac{a\left(\left(2cd^2+ae^2\right)x^2+ade\right)}{c(cd^2+ae^2)(ex^2+d)(cx^4+a)}dx^2}{2a}+\frac{a\left(d-ex^2\right)}{2c\left(a+cx^4\right)\left(ae^2+cd^2\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\int \frac{\left(2cd^2+ae^2\right)x^2+ade}{(ex^2+d)(cx^4+a)}dx^2}{2c\left(ae^2+cd^2\right)}+\frac{a\left(d-ex^2\right)}{2c\left(a+cx^4\right)\left(ae^2+cd^2\right)}\right)$$

$$\Big\downarrow \text{657}$$

$$\frac{1}{2}\left(\frac{\int \left(\frac{2c^2x^2d^3+3aced^2+a^2e^3}{(cd^2+ae^2)(cx^4+a)}-\frac{2cd^3e}{(cd^2+ae^2)(ex^2+d)}\right)dx^2}{2c\left(ae^2+cd^2\right)}+\frac{a\left(d-ex^2\right)}{2c\left(a+cx^4\right)\left(ae^2+cd^2\right)}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{\frac{\sqrt{a}e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)(ae^2+3cd^2)}{\sqrt{c}(ae^2+cd^2)}+\frac{cd^3\log(a+cx^4)}{ae^2+cd^2}-\frac{2cd^3\log(d+ex^2)}{ae^2+cd^2}}{2c\,(ae^2+cd^2)}+\frac{a(d-ex^2)}{2c\,(a+cx^4)\,(ae^2+cd^2)}\right)$$

input
```
Int[x^7/((d + e*x^2)*(a + c*x^4)^2),x]
```

output
```
((a*(d - e*x^2))/(2*c*(c*d^2 + a*e^2)*(a + c*x^4)) + ((Sqrt[a]*e*(3*c*d^2
+ a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[c]*(c*d^2 + a*e^2)) - (2*c*d
^3*Log[d + e*x^2])/(c*d^2 + a*e^2) + (c*d^3*Log[a + c*x^4])/(c*d^2 + a*e^2
))/(2*c*(c*d^2 + a*e^2)))/2
```

3.245.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 601
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c
+ d*x)^n*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Qx)/(c + d*x)^n + (e*
(2*p + 3))/(c + d*x)^n, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 1]
 && LtQ[p, -1] && ILtQ[n, 0] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.245.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{\frac{-\frac{ae(ae^2+cd^2)x^2}{2c}+\frac{da(ae^2+cd^2)}{2c}}{cx^4+a}+\frac{cd^3\ln(cx^4+a)+\frac{(e^3a^2+3acd^2e)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{\sqrt{ac}}}{2c}}{2(ae^2+cd^2)^2}-\frac{d^3\ln(ex^2+d)}{2(ae^2+cd^2)^2}$	146
risch	Expression too large to display	1428

input
```
int(x^7/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/(a*e^2+c*d^2)^2*((-1/2*a*e*(a*e^2+c*d^2)/c*x^2+1/2*d*a*(a*e^2+c*d^2)/c
)/(c*x^4+a)+1/2/c*(c*d^3*ln(c*x^4+a)+(a^2*e^3+3*a*c*d^2*e)/(a*c)^(1/2)*arc
tan(c*x^2/(a*c)^(1/2))))-1/2*d^3*ln(e*x^2+d)/(a*e^2+c*d^2)^2
```

3.245.5 Fricas [A] (verification not implemented)

Time = 4.23 (sec) , antiderivative size = 457, normalized size of antiderivative = 3.05

$$\int \frac{x^7}{(d+ex^2)(a+cx^4)^2}\,dx$$
$$= \left[\frac{2\,acd^3+2\,a^2de^2-2\,(acd^2e+a^2e^3)x^2+(3\,acd^2e+a^2e^3+(3\,c^2d^2e+ace^3)x^4)\sqrt{-\frac{a}{c}}\log\left(\frac{cx^4+2\,cx^2\sqrt{-\frac{a}{c}}-a}{cx^4+a}\right.}{8\,(ac^3d^4+2\,a^2c^2d^2e^2+a^3ce^4+(c^4d^4+2\,ac^3d^2e^2}\right.$$

input
```
integrate(x^7/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
[1/8*(2*a*c*d^3 + 2*a^2*d*e^2 - 2*(a*c*d^2*e + a^2*e^3)*x^2 + (3*a*c*d^2*e
 + a^2*e^3 + (3*c^2*d^2*e + a*c*e^3)*x^4)*sqrt(-a/c)*log((c*x^4 + 2*c*x^2*
sqrt(-a/c) - a)/(c*x^4 + a)) + 2*(c^2*d^3*x^4 + a*c*d^3)*log(c*x^4 + a) -
4*(c^2*d^3*x^4 + a*c*d^3)*log(e*x^2 + d))/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 +
 a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4), 1/4*(a*c*d^3
+ a^2*d*e^2 - (a*c*d^2*e + a^2*e^3)*x^2 + (3*a*c*d^2*e + a^2*e^3 + (3*c^2*
d^2*e + a*c*e^3)*x^4)*sqrt(a/c)*arctan(c*x^2*sqrt(a/c)/a) + (c^2*d^3*x^4 +
 a*c*d^3)*log(c*x^4 + a) - 2*(c^2*d^3*x^4 + a*c*d^3)*log(e*x^2 + d))/(a*c^
3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c
^2*e^4)*x^4)]
```

3.245.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^7}{(d+ex^2)\,(a+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**7/(e*x**2+d)/(c*x**4+a)**2,x)
```

output
```
Timed out
```

3.245.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.31

$$\int \frac{x^7}{(d+ex^2)\,(a+cx^4)^2}\,dx = \frac{d^3\log(cx^4+a)}{4\,(c^2d^4+2\,acd^2e^2+a^2e^4)} - \frac{d^3\log(ex^2+d)}{2\,(c^2d^4+2\,acd^2e^2+a^2e^4)} + \frac{(3\,acd^2e+a^2e^3)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\,(c^3d^4+2\,ac^2d^2e^2+a^2ce^4)\sqrt{ac}} - \frac{aex^2-ad}{4\,(ac^2d^2+a^2ce^2+(c^3d^2+ac^2e^2)x^4)}$$

input
```
integrate(x^7/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*d^3*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) - 1/2*d^3*log(e
*x^2 + d)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/4*(3*a*c*d^2*e + a^2*e^3
)*arctan(c*x^2/sqrt(a*c))/((c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(a*
c)) - 1/4*(a*e*x^2 - a*d)/(a*c^2*d^2 + a^2*c*e^2 + (c^3*d^2 + a*c^2*e^2)*x
^4)
```

3.245.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.53

$$\int \frac{x^7}{(d+ex^2)(a+cx^4)^2}\,dx = -\frac{d^3e\log(|ex^2+d|)}{2(c^2d^4e+2acd^2e^3+a^2e^5)} + \frac{d^3\log(cx^4+a)}{4(c^2d^4+2acd^2e^2+a^2e^4)} + \frac{(3acd^2e+a^2e^3)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4(c^3d^4+2ac^2d^2e^2+a^2ce^4)\sqrt{ac}} - \frac{c^2d^3x^4+acd^2ex^2+a^2e^3x^2-a^2de^2}{4(c^3d^4+2ac^2d^2e^2+a^2ce^4)(cx^4+a)}$$

input
```
integrate(x^7/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
-1/2*d^3*e*log(abs(e*x^2 + d))/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5) + 1/4
*d^3*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/4*(3*a*c*d^2*e
 + a^2*e^3)*arctan(c*x^2/sqrt(a*c))/((c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^
4)*sqrt(a*c)) - 1/4*(c^2*d^3*x^4 + a*c*d^2*e*x^2 + a^2*e^3*x^2 - a^2*d*e^2
)/((c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*(c*x^4 + a))
```

3.245.9 Mupad [B] (verification not implemented)

Time = 8.26 (sec) , antiderivative size = 647, normalized size of antiderivative = 4.31

$$\int \frac{x^7}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{\frac{a\,d}{4\,c\,(c\,d^2+a\,e^2)} - \frac{a\,e\,x^2}{4\,c\,(c\,d^2+a\,e^2)}}{c\,x^4+a} - \frac{d^3\,\ln(e\,x^2+d)}{2\,(a^2\,e^4+2\,a\,c\,d^2\,e^2+c^2\,d^4)} + \frac{\ln\left(36\,c^8\,d^{10}\,x^2+36\,c^6\,d^{10}\,\sqrt{-a\,c^3}+a^5\,c\,e^{10}\,\sqrt{-a\,c^3}+a^5\,c^3\,e^{10}\,x^2-22\,a^2\,d^4\,e^6\,(-a\,c^3)^{3/2}-81\,c^2\,d^8\,e^2\,(\cdots\right.}{} - \frac{\ln\left(36\,c^8\,d^{10}\,x^2-36\,c^6\,d^{10}\,\sqrt{-a\,c^3}-a^5\,c\,e^{10}\,\sqrt{-a\,c^3}+a^5\,c^3\,e^{10}\,x^2+22\,a^2\,d^4\,e^6\,(-a\,c^3)^{3/2}+81\,c^2\,d^8\,e^2\,(\cdots\right.}{}$$

input
```
int(x^7/((a + c*x^4)^2*(d + e*x^2)),x)
```

output
```
((a*d)/(4*c*(a*e^2 + c*d^2)) - (a*e*x^2)/(4*c*(a*e^2 + c*d^2)))/(a + c*x^4
) - (d^3*log(d + e*x^2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (log(36
*c^8*d^10*x^2 + 36*c^6*d^10*(-a*c^3)^(1/2) + a^5*c*e^10*(-a*c^3)^(1/2) + a
^5*c^3*e^10*x^2 - 22*a^2*d^4*e^6*(-a*c^3)^(3/2) - 81*c^2*d^8*e^2*(-a*c^3)^
(3/2) + 60*a^2*c^6*d^6*e^4*x^2 + 22*a^3*c^5*d^4*e^6*x^2 + 8*a^4*c^4*d^2*e^
8*x^2 + 8*a^4*c^2*d^2*e^8*(-a*c^3)^(1/2) - 60*a*c*d^6*e^4*(-a*c^3)^(3/2) +
 81*a*c^7*d^8*e^2*x^2)*(2*c^3*d^3 + a*e^3*(-a*c^3)^(1/2) + 3*c*d^2*e*(-a*c
^3)^(1/2)))/(8*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)) - (log(36*c^8*d^
10*x^2 - 36*c^6*d^10*(-a*c^3)^(1/2) - a^5*c*e^10*(-a*c^3)^(1/2) + a^5*c^3*
e^10*x^2 + 22*a^2*d^4*e^6*(-a*c^3)^(3/2) + 81*c^2*d^8*e^2*(-a*c^3)^(3/2) +
 60*a^2*c^6*d^6*e^4*x^2 + 22*a^3*c^5*d^4*e^6*x^2 + 8*a^4*c^4*d^2*e^8*x^2 -
 8*a^4*c^2*d^2*e^8*(-a*c^3)^(1/2) + 60*a*c*d^6*e^4*(-a*c^3)^(3/2) + 81*a*c
^7*d^8*e^2*x^2)*(a*e^3*(-a*c^3)^(1/2) - 2*c^3*d^3 + 3*c*d^2*e*(-a*c^3)^(1/
2)))/(8*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2))
```

3.245. $\int \frac{x^7}{(d+ex^2)(a+cx^4)^2}\,dx$

3.246 $\int \frac{x^5}{(d+ex^2)(a+cx^4)^2}\,dx$

3.246.1 Optimal result

Integrand size = 22, antiderivative size = 155

$$\int \frac{x^5}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{-ae-cdx^2}{4c(cd^2+ae^2)(a+cx^4)} + \frac{d(cd^2-ae^2)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4\sqrt{a}\sqrt{c}(cd^2+ae^2)^2} + \frac{d^2e\log(d+ex^2)}{2(cd^2+ae^2)^2} - \frac{d^2e\log(a+cx^4)}{4(cd^2+ae^2)^2}$$

output
```
1/4*(-c*d*x^2-a*e)/c/(a*e^2+c*d^2)/(c*x^4+a)+1/2*d^2*e*ln(e*x^2+d)/(a*e^2+
c*d^2)^2-1/4*d^2*e*ln(c*x^4+a)/(a*e^2+c*d^2)^2+1/4*d*(-a*e^2+c*d^2)*arctan
(x^2*c^(1/2)/a^(1/2))/(a*e^2+c*d^2)^2/a^(1/2)/c^(1/2)
```

3.246.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.96

$$\int \frac{x^5}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{\sqrt{c}d(cd^2-ae^2)(a+cx^4)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right) - \sqrt{a}((cd^2+ae^2)(ae+cdx^2) - 2cd^2e(a+cx^4)\log(d+ex^2) + c}{4\sqrt{ac}(cd^2+ae^2)^2(a+cx^4)}$$

input
```
Integrate[x^5/((d + e*x^2)*(a + c*x^4)^2),x]
```

output
```
(Sqrt[c]*d*(c*d^2 - a*e^2)*(a + c*x^4)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]] - Sqr
t[a]*((c*d^2 + a*e^2)*(a*e + c*d*x^2) - 2*c*d^2*e*(a + c*x^4)*Log[d + e*x^
2] + c*d^2*e*(a + c*x^4)*Log[a + c*x^4]))/(4*Sqrt[a]*c*(c*d^2 + a*e^2)^2*(
a + c*x^4))
```

3.246.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1579, 601, 25, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\left(a+cx^4\right)^2\left(d+ex^2\right)}dx$$

$$\Big\downarrow \text{1579}$$

$$\frac{1}{2}\int \frac{x^4}{\left(ex^2+d\right)\left(cx^4+a\right)^2}dx^2$$

$$\Big\downarrow \text{601}$$

$$\frac{1}{2}\left(-\frac{\int -\frac{ad(d-ex^2)}{(cd^2+ae^2)(ex^2+d)(cx^4+a)}dx^2}{2a}-\frac{ae+cdx^2}{2c\left(a+cx^4\right)\left(ae^2+cd^2\right)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{\int \frac{ad(d-ex^2)}{(cd^2+ae^2)(ex^2+d)(cx^4+a)}dx^2}{2a}-\frac{ae+cdx^2}{2c\left(a+cx^4\right)\left(ae^2+cd^2\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{d\int \frac{d-ex^2}{(ex^2+d)(cx^4+a)}dx^2}{2\left(ae^2+cd^2\right)}-\frac{ae+cdx^2}{2c\left(a+cx^4\right)\left(ae^2+cd^2\right)}\right)$$

$$\Big\downarrow \text{657}$$

$$\frac{1}{2}\left(\frac{d\int \left(\frac{2de^2}{(cd^2+ae^2)(ex^2+d)}+\frac{cd^2-2cex^2d-ae^2}{(cd^2+ae^2)(cx^4+a)}\right)dx^2}{2\left(ae^2+cd^2\right)}-\frac{ae+cdx^2}{2c\left(a+cx^4\right)\left(ae^2+cd^2\right)}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{d\left(\frac{\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)(cd^2-ae^2)}{\sqrt{a}\sqrt{c}(ae^2+cd^2)}-\frac{de\log(a+cx^4)}{ae^2+cd^2}+\frac{2de\log(d+ex^2)}{ae^2+cd^2}\right)}{2\left(ae^2+cd^2\right)}-\frac{ae+cdx^2}{2c\left(a+cx^4\right)\left(ae^2+cd^2\right)}\right)$$

input
```
Int[x^5/((d + e*x^2)*(a + c*x^4)^2),x]
```

output
```
(-1/2*(a*e + c*d*x^2)/(c*(c*d^2 + a*e^2)*(a + c*x^4)) + (d*(((c*d^2 - a*e^
2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]*(c*d^2 + a*e^2)) + (2*d
*e*Log[d + e*x^2])/(c*d^2 + a*e^2) - (d*e*Log[a + c*x^4])/(c*d^2 + a*e^2))
)/(2*(c*d^2 + a*e^2)))/2
```

3.246.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 601
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c
+ d*x)^n*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Qx)/(c + d*x)^n + (e*
(2*p + 3))/(c + d*x)^n, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 1]
 && LtQ[p, -1] && ILtQ[n, 0] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.246.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.88

method	result
default	$-\frac{\frac{\left(\frac{1}{2} d\,e^{2} a+\frac{1}{2} d^{3} c\right) x^{2}+\frac{a e \left(a\,e^{2}+c\,d^{2}\right)}{2 c}}{c\,x^{4}+a}+\frac{d \left(d e \ln \left(c\,x^{4}+a\right)+\frac{\left(a\,e^{2}-c\,d^{2}\right) \arctan \left(\frac{c\,x^{2}}{\sqrt{a c}}\right)}{\sqrt{a c}}\right)}{2}}{2 \left(a\,e^{2}+c\,d^{2}\right)^{2}}+\frac{d^{2} e \ln \left(e\,x^{2}+d\right)}{2 \left(a\,e^{2}+c\,d^{2}\right)^{2}}$
risch	$\frac{-\frac{d\,x^{2}}{4 \left(a\,e^{2}+c\,d^{2}\right)}-\frac{a e}{4 c \left(a\,e^{2}+c\,d^{2}\right)}}{c\,x^{4}+a}+\frac{d^{2} e \ln \left(e\,x^{2}+d\right)}{2 a^{2} e^{4}+4 a c\,d^{2} e^{2}+2 c^{2} d^{4}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^{3} c\,e^{4}+2 a^{2} c^{2} d^{2} e^{2}+a\,c^{3} d^{4}\right) \_Z^{2}+4 a c\,d^{2} e \_Z+d^{2}\right)}\_R \ln$

input
```
int(x^5/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/(a*e^2+c*d^2)^2*(((1/2*d*e^2*a+1/2*d^3*c)*x^2+1/2*a*e*(a*e^2+c*d^2)/c
)/(c*x^4+a)+1/2*d*(d*e*ln(c*x^4+a)+(a*e^2-c*d^2)/(a*c)^(1/2)*arctan(c*x^2/
(a*c)^(1/2)))+1/2*d^2*e*ln(e*x^2+d)/(a*e^2+c*d^2)^2
```

3.246.5 Fricas [A] (verification not implemented)

Time = 1.79 (sec) , antiderivative size = 487, normalized size of antiderivative = 3.14

$$\int \frac{x^5}{(d+ex^2)(a+cx^4)^2}\,dx$$
$$= \left[-\frac{2\,a^2cd^2e+2\,a^3e^3+2\,(ac^2d^3+a^2cde^2)x^2-(acd^3-a^2de^2+(c^2d^3-acde^2)x^4)\sqrt{-ac}\log\left(\frac{cx^4+2\sqrt{-ac}x^2-}{cx^4+a}\right.}{8\,(a^2c^3d^4+2\,a^3c^2d^2e^2+a^4ce^4+(ac^4d^4+2\,a}\right.$$
$$-\frac{a^2cd^2e+a^3e^3+(ac^2d^3+a^2cde^2)x^2+(acd^3-a^2de^2+(c^2d^3-acde^2)x^4)\sqrt{ac}\arctan\left(\frac{\sqrt{ac}}{cx^2}\right)+(ac^2d^2ex}{4\,(a^2c^3d^4+2\,a^3c^2d^2e^2+a^4ce^4+(ac^4d^4+2\,a^2c^3d^2e^2}$$

input
```
integrate(x^5/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
[-1/8*(2*a^2*c*d^2*e + 2*a^3*e^3 + 2*(a*c^2*d^3 + a^2*c*d*e^2)*x^2 - (a*c*
d^3 - a^2*d*e^2 + (c^2*d^3 - a*c*d*e^2)*x^4)*sqrt(-a*c)*log((c*x^4 + 2*sqr
t(-a*c)*x^2 - a)/(c*x^4 + a)) + 2*(a*c^2*d^2*e*x^4 + a^2*c*d^2*e)*log(c*x^
4 + a) - 4*(a*c^2*d^2*e*x^4 + a^2*c*d^2*e)*log(e*x^2 + d))/(a^2*c^3*d^4 +
2*a^3*c^2*d^2*e^2 + a^4*c*e^4 + (a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e
^4)*x^4), -1/4*(a^2*c*d^2*e + a^3*e^3 + (a*c^2*d^3 + a^2*c*d*e^2)*x^2 + (a
*c*d^3 - a^2*d*e^2 + (c^2*d^3 - a*c*d*e^2)*x^4)*sqrt(a*c)*arctan(sqrt(a*c)
/(c*x^2)) + (a*c^2*d^2*e*x^4 + a^2*c*d^2*e)*log(c*x^4 + a) - 2*(a*c^2*d^2*
e*x^4 + a^2*c*d^2*e)*log(e*x^2 + d))/(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^
4*c*e^4 + (a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*x^4)]
```

3.246.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^5}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**5/(e*x**2+d)/(c*x**4+a)**2,x)
```

output
```
Timed out
```

3.246.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.24

$$\begin{aligned}\int \frac{x^5}{(d+ex^2)(a+cx^4)^2}\,dx = & -\frac{d^2e\log(cx^4+a)}{4(c^2d^4+2acd^2e^2+a^2e^4)} + \frac{d^2e\log(ex^2+d)}{2(c^2d^4+2acd^2e^2+a^2e^4)} \\ & + \frac{(cd^3-ade^2)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4(c^2d^4+2acd^2e^2+a^2e^4)\sqrt{ac}} \\ & - \frac{cdx^2+ae}{4(ac^2d^2+a^2ce^2+(c^3d^2+ac^2e^2)x^4)}\end{aligned}$$

input
```
integrate(x^5/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
-1/4*d^2*e*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/2*d^2*e*
log(e*x^2 + d)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/4*(c*d^3 - a*d*e^2)
*arctan(c*x^2/sqrt(a*c))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(a*c)) -
 1/4*(c*d*x^2 + a*e)/(a*c^2*d^2 + a^2*c*e^2 + (c^3*d^2 + a*c^2*e^2)*x^4)
```

3.246.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.47

$$\int \frac{x^5}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{d^2e^2\log(|ex^2+d|)}{2(c^2d^4e+2acd^2e^3+a^2e^5)} - \frac{d^2e\log(cx^4+a)}{4(c^2d^4+2acd^2e^2+a^2e^4)} + \frac{(cd^3-ade^2)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4(c^2d^4+2acd^2e^2+a^2e^4)\sqrt{ac}} + \frac{c^2d^2ex^4-c^2d^3x^2-acde^2x^2-a^2e^3}{4(c^3d^4+2ac^2d^2e^2+a^2ce^4)(cx^4+a)}$$

input
```
integrate(x^5/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/2*d^2*e^2*log(abs(e*x^2 + d))/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5) - 1/
4*d^2*e*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/4*(c*d^3 -
a*d*e^2)*arctan(c*x^2/sqrt(a*c))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt
(a*c)) + 1/4*(c^2*d^2*e*x^4 - c^2*d^3*x^2 - a*c*d*e^2*x^2 - a^2*e^3)/((c^3
*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*(c*x^4 + a))
```

3.246.9 Mupad [B] (verification not implemented)

Time = 8.28 (sec) , antiderivative size = 528, normalized size of antiderivative = 3.41

$$\int \frac{x^5}{(d+ex^2)(a+cx^4)^2}\,dx$$
$$= \frac{\ln\left(a^4\,e^8\,\sqrt{-a\,c}+c^4\,d^8\,\sqrt{-a\,c}+70\,d^4\,e^4\,(-a\,c)^{5/2}+c^5\,d^8\,x^2+a^4\,c\,e^8\,x^2-36\,a^2\,d^2\,e^6\,(-a\,c)^{3/2}-36\,c^2\,d^6\,e\right.}{a^3\,c\,e^4+2\,a^2\,c^2\,d}$$
$$- \frac{\frac{d\,x^2}{4\,(c\,d^2+a\,e^2)}+\frac{a\,e}{4\,c\,(c\,d^2+a\,e^2)}}{c\,x^4+a}$$
$$- \frac{\ln\left(c^5\,d^8\,x^2-c^4\,d^8\,\sqrt{-a\,c}-70\,d^4\,e^4\,(-a\,c)^{5/2}-a^4\,e^8\,\sqrt{-a\,c}+a^4\,c\,e^8\,x^2+36\,a^2\,d^2\,e^6\,(-a\,c)^{3/2}+36\,c^2\,d\right.}{a^3\,c\,e^4+2\,a^2\,c^2}$$
$$+ \frac{d^2\,e\,\ln(e\,x^2+d)}{2\,(a^2\,e^4+2\,a\,c\,d^2\,e^2+c^2\,d^4)}$$

input
```
int(x^5/((a + c*x^4)^2*(d + e*x^2)),x)
```

output
```
(log(a^4*e^8*(-a*c)^(1/2) + c^4*d^8*(-a*c)^(1/2) + 70*d^4*e^4*(-a*c)^(5/2)
 + c^5*d^8*x^2 + a^4*c*e^8*x^2 - 36*a^2*d^2*e^6*(-a*c)^(3/2) - 36*c^2*d^6*
e^2*(-a*c)^(3/2) + 70*a^2*c^3*d^4*e^4*x^2 + 36*a^3*c^2*d^2*e^6*x^2 + 36*a*
c^4*d^6*e^2*x^2)*(c*((d^3*(-a*c)^(1/2))/8 - (a*d^2*e)/4) - (a*d*e^2*(-a*c)
^(1/2))/8))/(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2) - ((d*x^2)/(4*(a*e
^2 + c*d^2)) + (a*e)/(4*c*(a*e^2 + c*d^2)))/(a + c*x^4) - (log(c^5*d^8*x^2
 - c^4*d^8*(-a*c)^(1/2) - 70*d^4*e^4*(-a*c)^(5/2) - a^4*e^8*(-a*c)^(1/2) +
 a^4*c*e^8*x^2 + 36*a^2*d^2*e^6*(-a*c)^(3/2) + 36*c^2*d^6*e^2*(-a*c)^(3/2)
 + 70*a^2*c^3*d^4*e^4*x^2 + 36*a^3*c^2*d^2*e^6*x^2 + 36*a*c^4*d^6*e^2*x^2)
*(c*((d^3*(-a*c)^(1/2))/8 + (a*d^2*e)/4) - (a*d*e^2*(-a*c)^(1/2))/8))/(a*c
^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2) + (d^2*e*log(d + e*x^2))/(2*(a^2*e
^4 + c^2*d^4 + 2*a*c*d^2*e^2))
```

3.246. $\int \frac{x^5}{(d+ex^2)(a+cx^4)^2}\,dx$

3.247 $\int \frac{x^3}{(d+ex^2)(a+cx^4)^2}\,dx$

3.247.1 Optimal result

Integrand size = 22, antiderivative size = 149

$$\int \frac{x^3}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{-d+ex^2}{4(cd^2+ae^2)(a+cx^4)} - \frac{e(cd^2-ae^2)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4\sqrt{a}\sqrt{c}(cd^2+ae^2)^2} - \frac{de^2\log(d+ex^2)}{2(cd^2+ae^2)^2} + \frac{de^2\log(a+cx^4)}{4(cd^2+ae^2)^2}$$

output
```
1/4*(e*x^2-d)/(a*e^2+c*d^2)/(c*x^4+a)-1/2*d*e^2*ln(e*x^2+d)/(a*e^2+c*d^2)^
2+1/4*d*e^2*ln(c*x^4+a)/(a*e^2+c*d^2)^2-1/4*e*(-a*e^2+c*d^2)*arctan(x^2*c^
(1/2)/a^(1/2))/(a*e^2+c*d^2)^2/a^(1/2)/c^(1/2)
```

3.247.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.77

$$\int \frac{x^3}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{\frac{(cd^2+ae^2)(-d+ex^2)}{a+cx^4} + \frac{e(-cd^2+ae^2)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}} - 2de^2\log(d+ex^2) + de^2\log(a+cx^4)}{4(cd^2+ae^2)^2}$$

input
```
Integrate[x^3/((d + e*x^2)*(a + c*x^4)^2),x]
```

output
```
(((c*d^2 + a*e^2)*(-d + e*x^2))/(a + c*x^4) + (e*(-(c*d^2) + a*e^2)*ArcTan
[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) - 2*d*e^2*Log[d + e*x^2] + d*e^
2*Log[a + c*x^4])/(4*(c*d^2 + a*e^2)^2)
```

3.247.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1579, 593, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+cx^4)^2(d+ex^2)}dx$$

$$\Big\downarrow \text{1579}$$

$$\frac{1}{2}\int \frac{x^2}{(ex^2+d)(cx^4+a)^2}dx^2$$

$$\Big\downarrow \text{593}$$

$$\frac{1}{2}\left(\frac{e\int -\frac{d-ex^2}{(ex^2+d)(cx^4+a)}dx^2}{2(ae^2+cd^2)}-\frac{d-ex^2}{2(a+cx^4)(ae^2+cd^2)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(-\frac{e\int \frac{d-ex^2}{(ex^2+d)(cx^4+a)}dx^2}{2(ae^2+cd^2)}-\frac{d-ex^2}{2(a+cx^4)(ae^2+cd^2)}\right)$$

$$\Big\downarrow \text{657}$$

$$\frac{1}{2}\left(-\frac{e\int \left(\frac{2de^2}{(cd^2+ae^2)(ex^2+d)}+\frac{cd^2-2cex^2d-ae^2}{(cd^2+ae^2)(cx^4+a)}\right)dx^2}{2(ae^2+cd^2)}-\frac{d-ex^2}{2(a+cx^4)(ae^2+cd^2)}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{e\left(\frac{\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)(cd^2-ae^2)}{\sqrt{a}\sqrt{c}(ae^2+cd^2)}-\frac{de\log(a+cx^4)}{ae^2+cd^2}+\frac{2de\log(d+ex^2)}{ae^2+cd^2}\right)}{2(ae^2+cd^2)}-\frac{d-ex^2}{2(a+cx^4)(ae^2+cd^2)}\right)$$

input
```
Int[x^3/((d + e*x^2)*(a + c*x^4)^2),x]
```

output
```
(-1/2*(d - e*x^2)/((c*d^2 + a*e^2)*(a + c*x^4)) - (e*(((c*d^2 - a*e^2)*Arc
Tan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]*(c*d^2 + a*e^2)) + (2*d*e*Log
[d + e*x^2])/(c*d^2 + a*e^2) - (d*e*Log[a + c*x^4])/(c*d^2 + a*e^2)))/(2*(
c*d^2 + a*e^2)))/2
```

3.247.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 593
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[(c + d*x)^(n + 1)*(c - d*x)*((a + b*x^2)^(p + 1)/(2*(p + 1)*(b*c^2 +
a*d^2))), x] - Simp[d/(2*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a + b*
x^2)^(p + 1)*(c*n - d*(n + 2*p + 4)*x), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && LtQ[p, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.247.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.89

method	result
default	$\frac{\frac{\left(\frac{1}{2}a e^{3}+\frac{1}{2}c d^{2} e\right)x^{2}-\frac{d\left(a e^{2}+c d^{2}\right)}{2}}{c x^{4}+a}+\frac{e\left(d e \ln\left(c x^{4}+a\right)+\frac{\left(a e^{2}-c d^{2}\right)\arctan\left(\frac{c x^{2}}{\sqrt{a c}}\right)}{\sqrt{a c}}\right)}{2}}{2\left(a e^{2}+c d^{2}\right)^{2}}-\frac{d e^{2}\ln\left(e x^{2}+d\right)}{2\left(a e^{2}+c d^{2}\right)^{2}}$
risch	$\frac{\frac{e x^{2}}{4a e^{2}+4c d^{2}}-\frac{d}{4\left(a e^{2}+c d^{2}\right)}}{c x^{4}+a}-\frac{d e^{2}\ln\left(e x^{2}+d\right)}{2\left(a^{2}e^{4}+2ac\,d^{2}e^{2}+c^{2}d^{4}\right)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^{3}c\,e^{4}+2a^{2}c^{2}d^{2}e^{2}+a\,c^{3}d^{4}\right)\_Z^{2}-4acd\,e^{2}\_Z+e^{2}\right)}\_R\ln\left(\left(\right.\right.\right.}{}$

input
```
int(x^3/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/(a*e^2+c*d^2)^2*(((1/2*a*e^3+1/2*c*d^2*e)*x^2-1/2*d*(a*e^2+c*d^2))/(c*
x^4+a)+1/2*e*(d*e*ln(c*x^4+a)+(a*e^2-c*d^2)/(a*c)^(1/2)*arctan(c*x^2/(a*c)
^(1/2))))-1/2*d*e^2*ln(e*x^2+d)/(a*e^2+c*d^2)^2
```

3.247.5 Fricas [A] (verification not implemented)

Time = 1.75 (sec) , antiderivative size = 492, normalized size of antiderivative = 3.30

$$\int \frac{x^3}{(d+ex^2)(a+cx^4)^2}\,dx$$
$$= \left[-\frac{2\,ac^2d^3+2\,a^2cde^2-2\,(ac^2d^2e+a^2ce^3)x^2-(acd^2e-a^2e^3+(c^2d^2e-ace^3)x^4)\sqrt{-ac}\log\left(\frac{cx^4-2\sqrt{-ac}x^2}{cx^4+a}\right.}{8\,(a^2c^3d^4+2\,a^3c^2d^2e^2+a^4ce^4+(ac^4d^4+2\,}\right.$$
$$-\frac{ac^2d^3+a^2cde^2-(ac^2d^2e+a^2ce^3)x^2-(acd^2e-a^2e^3+(c^2d^2e-ace^3)x^4)\sqrt{ac}\arctan\left(\frac{\sqrt{ac}}{cx^2}\right)-(ac^2de^2}{4\,(a^2c^3d^4+2\,a^3c^2d^2e^2+a^4ce^4+(ac^4d^4+2\,a^2c^3d^2e^2}$$

input
```
integrate(x^3/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
[-1/8*(2*a*c^2*d^3 + 2*a^2*c*d*e^2 - 2*(a*c^2*d^2*e + a^2*c*e^3)*x^2 - (a*
c*d^2*e - a^2*e^3 + (c^2*d^2*e - a*c*e^3)*x^4)*sqrt(-a*c)*log((c*x^4 - 2*s
qrt(-a*c)*x^2 - a)/(c*x^4 + a)) - 2*(a*c^2*d*e^2*x^4 + a^2*c*d*e^2)*log(c*
x^4 + a) + 4*(a*c^2*d*e^2*x^4 + a^2*c*d*e^2)*log(e*x^2 + d))/(a^2*c^3*d^4
+ 2*a^3*c^2*d^2*e^2 + a^4*c*e^4 + (a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2
*e^4)*x^4), -1/4*(a*c^2*d^3 + a^2*c*d*e^2 - (a*c^2*d^2*e + a^2*c*e^3)*x^2
- (a*c*d^2*e - a^2*e^3 + (c^2*d^2*e - a*c*e^3)*x^4)*sqrt(a*c)*arctan(sqrt(
a*c)/(c*x^2)) - (a*c^2*d*e^2*x^4 + a^2*c*d*e^2)*log(c*x^4 + a) + 2*(a*c^2*
d*e^2*x^4 + a^2*c*d*e^2)*log(e*x^2 + d))/(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2
+ a^4*c*e^4 + (a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*x^4)]
```

3.247.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**3/(e*x**2+d)/(c*x**4+a)**2,x)
```

output
```
Timed out
```

3.247.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.25

$$\begin{aligned}\int \frac{x^3}{(d+ex^2)(a+cx^4)^2}\,dx &= \frac{de^2\log(cx^4+a)}{4(c^2d^4+2\,acd^2e^2+a^2e^4)} - \frac{de^2\log(ex^2+d)}{2(c^2d^4+2\,acd^2e^2+a^2e^4)} \\ &\quad - \frac{(cd^2e-ae^3)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4(c^2d^4+2\,acd^2e^2+a^2e^4)\sqrt{ac}} \\ &\quad + \frac{ex^2-d}{4((c^2d^2+ace^2)x^4+acd^2+a^2e^2)}\end{aligned}$$

input
```
integrate(x^3/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*d*e^2*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) - 1/2*d*e^2*l
og(e*x^2 + d)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) - 1/4*(c*d^2*e - a*e^3)*
arctan(c*x^2/sqrt(a*c))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(a*c)) +
1/4*(e*x^2 - d)/((c^2*d^2 + a*c*e^2)*x^4 + a*c*d^2 + a^2*e^2)
```

3.247.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.32

$$\int \frac{x^3}{(d+ex^2)(a+cx^4)^2}\,dx = -\frac{de^3\log(|ex^2+d|)}{2(c^2d^4e+2acd^2e^3+a^2e^5)}+\frac{de^2\log(cx^4+a)}{4(c^2d^4+2acd^2e^2+a^2e^4)} - \frac{(cd^2e-ae^3)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4(c^2d^4+2acd^2e^2+a^2e^4)\sqrt{ac}}-\frac{cd^3+ade^2-(cd^2e+ae^3)x^2}{4(cx^4+a)(cd^2+ae^2)^2}$$

input
```
integrate(x^3/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
-1/2*d*e^3*log(abs(e*x^2 + d))/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5) + 1/4
*d*e^2*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) - 1/4*(c*d^2*e -
 a*e^3)*arctan(c*x^2/sqrt(a*c))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(
a*c)) - 1/4*(c*d^3 + a*d*e^2 - (c*d^2*e + a*e^3)*x^2)/((c*x^4 + a)*(c*d^2
+ a*e^2)^2)
```

3.247.9 Mupad [B] (verification not implemented)

Time = 8.24 (sec) , antiderivative size = 527, normalized size of antiderivative = 3.54

$$\int \frac{x^3}{(d+ex^2)(a+cx^4)^2}\,dx$$

$$= \frac{\ln\left(a^4\,e^8\,\sqrt{-a\,c}+c^4\,d^8\,\sqrt{-a\,c}+70\,d^4\,e^4\,(-a\,c)^{5/2}+c^5\,d^8\,x^2+a^4\,c\,e^8\,x^2-36\,a^2\,d^2\,e^6\,(-a\,c)^{3/2}-36\,c^2\,d^6\,e\right.}{a^3\,c\,e^4+2\,a^2\,c^2\,d^2}$$

$$-\frac{\frac{d}{4\,(c\,d^2+a\,e^2)}-\frac{e\,x^2}{4\,(c\,d^2+a\,e^2)}}{c\,x^4+a}$$

$$-\frac{\ln\left(c^5\,d^8\,x^2-c^4\,d^8\,\sqrt{-a\,c}-70\,d^4\,e^4\,(-a\,c)^{5/2}-a^4\,e^8\,\sqrt{-a\,c}+a^4\,c\,e^8\,x^2+36\,a^2\,d^2\,e^6\,(-a\,c)^{3/2}+36\,c^2\,d\right.}{a^3\,c\,e^4+2\,a^2\,c^2}$$

$$-\frac{d\,e^2\,\ln(e\,x^2+d)}{2\,(a^2\,e^4+2\,a\,c\,d^2\,e^2+c^2\,d^4)}$$

input
```
int(x^3/((a + c*x^4)^2*(d + e*x^2)),x)
```

output
```
(log(a^4*e^8*(-a*c)^(1/2) + c^4*d^8*(-a*c)^(1/2) + 70*d^4*e^4*(-a*c)^(5/2)
 + c^5*d^8*x^2 + a^4*c*e^8*x^2 - 36*a^2*d^2*e^6*(-a*c)^(3/2) - 36*c^2*d^6*
e^2*(-a*c)^(3/2) + 70*a^2*c^3*d^4*e^4*x^2 + 36*a^3*c^2*d^2*e^6*x^2 + 36*a*
c^4*d^6*e^2*x^2)*(a*((e^3*(-a*c)^(1/2))/8 + (c*d*e^2)/4) - (c*d^2*e*(-a*c)
^(1/2))/8))/(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2) - (d/(4*(a*e^2 + c
*d^2)) - (e*x^2)/(4*(a*e^2 + c*d^2)))/(a + c*x^4) - (log(c^5*d^8*x^2 - c^4
*d^8*(-a*c)^(1/2) - 70*d^4*e^4*(-a*c)^(5/2) - a^4*e^8*(-a*c)^(1/2) + a^4*c
*e^8*x^2 + 36*a^2*d^2*e^6*(-a*c)^(3/2) + 36*c^2*d^6*e^2*(-a*c)^(3/2) + 70*
a^2*c^3*d^4*e^4*x^2 + 36*a^3*c^2*d^2*e^6*x^2 + 36*a*c^4*d^6*e^2*x^2)*(a*((
e^3*(-a*c)^(1/2))/8 - (c*d*e^2)/4) - (c*d^2*e*(-a*c)^(1/2))/8))/(a*c^3*d^4
 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2) - (d*e^2*log(d + e*x^2))/(2*(a^2*e^4 + c
^2*d^4 + 2*a*c*d^2*e^2))
```

3.247. $\int \frac{x^3}{(d+ex^2)(a+cx^4)^2}\,dx$

3.248 $\int \frac{x}{(d+ex^2)(a+cx^4)^2} dx$

3.248.1 Optimal result

Integrand size = 20, antiderivative size = 151

$$\int \frac{x}{(d+ex^2)(a+cx^4)^2} dx = \frac{ae+cdx^2}{4a(cd^2+ae^2)(a+cx^4)} + \frac{\sqrt{c}d(cd^2+3ae^2)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}(cd^2+ae^2)^2} + \frac{e^3\log(d+ex^2)}{2(cd^2+ae^2)^2} - \frac{e^3\log(a+cx^4)}{4(cd^2+ae^2)^2}$$

output
```
1/4*(c*d*x^2+a*e)/a/(a*e^2+c*d^2)/(c*x^4+a)+1/2*e^3*ln(e*x^2+d)/(a*e^2+c*d
^2)^2-1/4*e^3*ln(c*x^4+a)/(a*e^2+c*d^2)^2+1/4*d*(3*a*e^2+c*d^2)*arctan(x^2
*c^(1/2)/a^(1/2))*c^(1/2)/a^(3/2)/(a*e^2+c*d^2)^2
```

3.248.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.95

$$\int \frac{x}{(d+ex^2)(a+cx^4)^2} dx = \frac{\sqrt{c}d(cd^2+3ae^2)(a+cx^4)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right) + \sqrt{a}((cd^2+ae^2)(ae+cdx^2) + 2ae^3(a+cx^4)\log(d+ex^2) - a}{4a^{3/2}(cd^2+ae^2)^2(a+cx^4)}$$

input
```
Integrate[x/((d + e*x^2)*(a + c*x^4)^2),x]
```

output
```
(Sqrt[c]*d*(c*d^2 + 3*a*e^2)*(a + c*x^4)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]] + S
qrt[a]*((c*d^2 + a*e^2)*(a*e + c*d*x^2) + 2*a*e^3*(a + c*x^4)*Log[d + e*x^
2] - a*e^3*(a + c*x^4)*Log[a + c*x^4]))/(4*a^(3/2)*(c*d^2 + a*e^2)^2*(a +
c*x^4))
```

3.248.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1577, 496, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+cx^4)^2(d+ex^2)}dx$$

$$\Big\downarrow \text{1577}$$

$$\frac{1}{2}\int \frac{1}{(ex^2+d)(cx^4+a)^2}dx^2$$

$$\Big\downarrow \text{496}$$

$$\frac{1}{2}\left(\frac{ae+cdx^2}{2a(a+cx^4)(ae^2+cd^2)}-\frac{\int -\frac{cd^2+cex^2d+2ae^2}{(ex^2+d)(cx^4+a)}dx^2}{2a(ae^2+cd^2)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{\int \frac{cd^2+cex^2d+2ae^2}{(ex^2+d)(cx^4+a)}dx^2}{2a(ae^2+cd^2)}+\frac{ae+cdx^2}{2a(a+cx^4)(ae^2+cd^2)}\right)$$

$$\Big\downarrow \text{657}$$

$$\frac{1}{2}\left(\frac{\int \left(\frac{2ae^4}{(cd^2+ae^2)(ex^2+d)}+\frac{c(cd^3+3ae^2d-2ae^3x^2)}{(cd^2+ae^2)(cx^4+a)}\right)dx^2}{2a(ae^2+cd^2)}+\frac{ae+cdx^2}{2a(a+cx^4)(ae^2+cd^2)}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{\frac{\sqrt{c}d\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)(3ae^2+cd^2)}{\sqrt{a}(ae^2+cd^2)}-\frac{ae^3\log(a+cx^4)}{ae^2+cd^2}+\frac{2ae^3\log(d+ex^2)}{ae^2+cd^2}}{2a(ae^2+cd^2)}+\frac{ae+cdx^2}{2a(a+cx^4)(ae^2+cd^2)}\right)$$

input
```
Int[x/((d + e*x^2)*(a + c*x^4)^2),x]
```

output
```
((a*e + c*d*x^2)/(2*a*(c*d^2 + a*e^2)*(a + c*x^4)) + ((Sqrt[c]*d*(c*d^2 +
3*a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*(c*d^2 + a*e^2)) + (2*a*e
^3*Log[d + e*x^2])/(c*d^2 + a*e^2) - (a*e^3*Log[a + c*x^4])/(c*d^2 + a*e^2
))/(2*a*(c*d^2 + a*e^2)))/2
```

3.248.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.248.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.97

method	result
default	$\frac{c\left(\frac{\frac{d\left(a e^{2}+c d^{2}\right)x^{2}}{2a}+\frac{e\left(a e^{2}+c d^{2}\right)}{2c}}{c x^{4}+a}+\frac{-\frac{a e^{3}\ln\left(c x^{4}+a\right)}{c}+\frac{\left(3d e^{2}a+d^{3}c\right)\arctan\left(\frac{c x^{2}}{\sqrt{ac}}\right)}{\sqrt{ac}}}{2a}\right)}{2\left(a e^{2}+c d^{2}\right)^{2}}+\frac{e^{3}\ln\left(e x^{2}+d\right)}{2\left(a e^{2}+c d^{2}\right)^{2}}$
risch	$\frac{\frac{c d x^{2}}{4a\left(a e^{2}+c d^{2}\right)}+\frac{e}{4a e^{2}+4c d^{2}}}{c x^{4}+a}+\frac{e^{3}\ln\left(e x^{2}+d\right)}{2a^{2}e^{4}+4ac d^{2}e^{2}+2c^{2}d^{4}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^{5}e^{4}+2a^{4}c d^{2}e^{2}+a^{3}c^{2}d^{4}\right)\_Z^{2}+4a^{3}e^{3}\_Z+4a e^{2}+c d^{2}\right)}\_R\right.}{}$

input
```
int(x/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*c/(a*e^2+c*d^2)^2*((1/2*d*(a*e^2+c*d^2)/a*x^2+1/2*e*(a*e^2+c*d^2)/c)/(
c*x^4+a)+1/2/a*(-a*e^3/c*ln(c*x^4+a)+(3*a*d*e^2+c*d^3)/(a*c)^(1/2)*arctan(
c*x^2/(a*c)^(1/2))))+1/2*e^3*ln(e*x^2+d)/(a*e^2+c*d^2)^2
```

3.248.5 Fricas [A] (verification not implemented)

Time = 4.22 (sec) , antiderivative size = 458, normalized size of antiderivative = 3.03

$$\int \frac{x}{(d+ex^2)(a+cx^4)^2}\,dx$$
$$= \left[\frac{2\,acd^2e+2\,a^2e^3+2\,(c^2d^3+acde^2)x^2+(acd^3+3\,a^2de^2+(c^2d^3+3\,acde^2)x^4)\sqrt{-\frac{c}{a}}\log\left(\frac{cx^4+2\,ax^2\sqrt{-\frac{c}{a}}-a}{cx^4+a}\right.}{8\,(a^2c^2d^4+2\,a^3cd^2e^2+a^4e^4+(ac^3d^4+2\,a^2c^2d^2e}\right.$$

input
```
integrate(x/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
[1/8*(2*a*c*d^2*e + 2*a^2*e^3 + 2*(c^2*d^3 + a*c*d*e^2)*x^2 + (a*c*d^3 + 3
*a^2*d*e^2 + (c^2*d^3 + 3*a*c*d*e^2)*x^4)*sqrt(-c/a)*log((c*x^4 + 2*a*x^2*
sqrt(-c/a) - a)/(c*x^4 + a)) - 2*(a*c*e^3*x^4 + a^2*e^3)*log(c*x^4 + a) +
4*(a*c*e^3*x^4 + a^2*e^3)*log(e*x^2 + d))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 +
 a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4), 1/4*(a*c*d^2*
e + a^2*e^3 + (c^2*d^3 + a*c*d*e^2)*x^2 - (a*c*d^3 + 3*a^2*d*e^2 + (c^2*d^
3 + 3*a*c*d*e^2)*x^4)*sqrt(c/a)*arctan(a*sqrt(c/a)/(c*x^2)) - (a*c*e^3*x^4
 + a^2*e^3)*log(c*x^4 + a) + 2*(a*c*e^3*x^4 + a^2*e^3)*log(e*x^2 + d))/(a^
2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a
^3*c*e^4)*x^4)]
```

3.248.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x/(e*x**2+d)/(c*x**4+a)**2,x)
```

output
```
Timed out
```

3.248.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.30

$$\int \frac{x}{(d+ex^2)(a+cx^4)^2}\,dx = -\frac{e^3\log(cx^4+a)}{4(c^2d^4+2\,acd^2e^2+a^2e^4)} + \frac{e^3\log(ex^2+d)}{2(c^2d^4+2\,acd^2e^2+a^2e^4)} + \frac{(c^2d^3+3\,acde^2)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4(ac^2d^4+2\,a^2cd^2e^2+a^3e^4)\sqrt{ac}} + \frac{cdx^2+ae}{4(a^2cd^2+a^3e^2+(ac^2d^2+a^2ce^2)x^4)}$$

input
```
integrate(x/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
-1/4*e^3*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/2*e^3*log(
e*x^2 + d)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/4*(c^2*d^3 + 3*a*c*d*e^
2)*arctan(c*x^2/sqrt(a*c))/((a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(a
*c)) + 1/4*(c*d*x^2 + a*e)/(a^2*c*d^2 + a^3*e^2 + (a*c^2*d^2 + a^2*c*e^2)*
x^4)
```

3.248.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.38

$$\int \frac{x}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{e^4\log(|ex^2+d|)}{2(c^2d^4e+2acd^2e^3+a^2e^5)} - \frac{e^3\log(cx^4+a)}{4(c^2d^4+2acd^2e^2+a^2e^4)} + \frac{(c^2d^3+3acde^2)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4(ac^2d^4+2a^2cd^2e^2+a^3e^4)\sqrt{ac}} + \frac{acd^2e+a^2e^3+(c^2d^3+acde^2)x^2}{4(cx^4+a)(cd^2+ae^2)^2a}$$

input
```
integrate(x/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/2*e^4*log(abs(e*x^2 + d))/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5) - 1/4*e^
3*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/4*(c^2*d^3 + 3*a*
c*d*e^2)*arctan(c*x^2/sqrt(a*c))/((a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*
sqrt(a*c)) + 1/4*(a*c*d^2*e + a^2*e^3 + (c^2*d^3 + a*c*d*e^2)*x^2)/((c*x^4
 + a)*(c*d^2 + a*e^2)^2*a)
```

3.248.9 Mupad [B] (verification not implemented)

Time = 8.41 (sec) , antiderivative size = 649, normalized size of antiderivative = 4.30

$$\int \frac{x}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{\frac{e}{4\,(c\,d^2+a\,e^2)}+\frac{c\,d\,x^2}{4\,a\,(c\,d^2+a\,e^2)}}{c\,x^4+a} + \frac{e^3\,\ln\left(e\,x^2+d\right)}{2\,\left(a^2\,e^4+2\,a\,c\,d^2\,e^2+c^2\,d^4\right)} + \frac{\ln\left(36\,a^6\,e^{10}\,\sqrt{-a^3\,c}+36\,a^7\,c\,e^{10}\,x^2+a\,c^5\,d^{10}\,\sqrt{-a^3\,c}+a^2\,c^6\,d^{10}\,x^2-81\,a^2\,d^2\,e^8\,\left(-a^3\,c\right)^{3/2}-22\,c^2\,d^6\,e^4\right.}{} - \frac{\ln\left(36\,a^7\,c\,e^{10}\,x^2-36\,a^6\,e^{10}\,\sqrt{-a^3\,c}-a\,c^5\,d^{10}\,\sqrt{-a^3\,c}+a^2\,c^6\,d^{10}\,x^2+81\,a^2\,d^2\,e^8\,\left(-a^3\,c\right)^{3/2}+22\,c^2\,d^6\,e^4\right.}{}$$

input
```
int(x/((a + c*x^4)^2*(d + e*x^2)),x)
```

output
```
(e/(4*(a*e^2 + c*d^2)) + (c*d*x^2)/(4*a*(a*e^2 + c*d^2)))/(a + c*x^4) + (e
^3*log(d + e*x^2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (log(36*a^6*e
^10*(-a^3*c)^(1/2) + 36*a^7*c*e^10*x^2 + a*c^5*d^10*(-a^3*c)^(1/2) + a^2*c
^6*d^10*x^2 - 81*a^2*d^2*e^8*(-a^3*c)^(3/2) - 22*c^2*d^6*e^4*(-a^3*c)^(3/2
) + 8*a^3*c^5*d^8*e^2*x^2 + 22*a^4*c^4*d^6*e^4*x^2 + 60*a^5*c^3*d^4*e^6*x^
2 + 81*a^6*c^2*d^2*e^8*x^2 + 8*a^2*c^4*d^8*e^2*(-a^3*c)^(1/2) - 60*a*c*d^4
*e^6*(-a^3*c)^(3/2))*(c*d^3*(-a^3*c)^(1/2) - 2*a^3*e^3 + 3*a*d*e^2*(-a^3*c
)^(1/2)))/(8*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)) - (log(36*a^7*c*e^
10*x^2 - 36*a^6*e^10*(-a^3*c)^(1/2) - a*c^5*d^10*(-a^3*c)^(1/2) + a^2*c^6*
d^10*x^2 + 81*a^2*d^2*e^8*(-a^3*c)^(3/2) + 22*c^2*d^6*e^4*(-a^3*c)^(3/2) +
 8*a^3*c^5*d^8*e^2*x^2 + 22*a^4*c^4*d^6*e^4*x^2 + 60*a^5*c^3*d^4*e^6*x^2 +
 81*a^6*c^2*d^2*e^8*x^2 - 8*a^2*c^4*d^8*e^2*(-a^3*c)^(1/2) + 60*a*c*d^4*e^
6*(-a^3*c)^(3/2))*(2*a^3*e^3 + c*d^3*(-a^3*c)^(1/2) + 3*a*d*e^2*(-a^3*c)^(
1/2)))/(8*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2))
```

3.248. $\int \frac{x}{(d+ex^2)(a+cx^4)^2}\,dx$

3.249 $\int \frac{1}{x(d+ex^2)(a+cx^4)^2} dx$

3.249.1 Optimal result

Integrand size = 22, antiderivative size = 209

$$\int \frac{1}{x(d+ex^2)(a+cx^4)^2} dx = \frac{c(d-ex^2)}{4a(cd^2+ae^2)(a+cx^4)} - \frac{\sqrt{c}e^3\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}(cd^2+ae^2)^2} - \frac{\sqrt{c}e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}(cd^2+ae^2)} + \frac{\log(x)}{a^2d} - \frac{e^4\log(d+ex^2)}{2d(cd^2+ae^2)^2} - \frac{cd(cd^2+2ae^2)\log(a+cx^4)}{4a^2(cd^2+ae^2)^2}$$

output
```
1/4*c*(-e*x^2+d)/a/(a*e^2+c*d^2)/(c*x^4+a)+ln(x)/a^2/d-1/2*e^4*ln(e*x^2+d)
/d/(a*e^2+c*d^2)^2-1/4*c*d*(2*a*e^2+c*d^2)*ln(c*x^4+a)/a^2/(a*e^2+c*d^2)^2
-1/4*e*arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/a^(3/2)/(a*e^2+c*d^2)-1/2*e^3*a
rctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/(a*e^2+c*d^2)^2/a^(1/2)
```

3.249.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.15

$$\int \frac{1}{x(d+ex^2)(a+cx^4)^2} dx = \frac{acd(cd^2+ae^2)(d-ex^2)+\sqrt{a}\sqrt{c}de(cd^2+3ae^2)(a+cx^4)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+\sqrt{a}\sqrt{c}de(cd^2+3ae^2)(}{}$$

input
```
Integrate[1/(x*(d + e*x^2)*(a + c*x^4)^2),x]
```

output
```
(a*c*d*(c*d^2 + a*e^2)*(d - e*x^2) + Sqrt[a]*Sqrt[c]*d*e*(c*d^2 + 3*a*e^2)
*(a + c*x^4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + Sqrt[a]*Sqrt[c]*d*e
*(c*d^2 + 3*a*e^2)*(a + c*x^4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 4
*(c*d^2 + a*e^2)^2*(a + c*x^4)*Log[x] - 2*a^2*e^4*(a + c*x^4)*Log[d + e*x^
2] - c*d^2*(c*d^2 + 2*a*e^2)*(a + c*x^4)*Log[a + c*x^4])/(4*a^2*d*(c*d^2 +
 a*e^2)^2*(a + c*x^4))
```

3.249.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1579, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+cx^4\right)^2\left(d+ex^2\right)}dx$$

$$\Big\downarrow \text{1579}$$

$$\frac{1}{2}\int \frac{1}{x^2\left(ex^2+d\right)\left(cx^4+a\right)^2}dx^2$$

$$\Big\downarrow \text{615}$$

$$\frac{1}{2}\int \left(-\frac{e^5}{d\left(cd^2+ae^2\right)^2\left(ex^2+d\right)}+\frac{c\left(-a^2e^3-cd\left(cd^2+2ae^2\right)x^2\right)}{a^2\left(cd^2+ae^2\right)^2\left(cx^4+a\right)}+\frac{1}{a^2dx^2}-\frac{c\left(cdx^2+ae\right)}{a\left(cd^2+ae^2\right)\left(cx^4+a\right)^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{\sqrt{c}e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}\left(ae^2+cd^2\right)}-\frac{cd\left(2ae^2+cd^2\right)\log\left(a+cx^4\right)}{2a^2\left(ae^2+cd^2\right)^2}+\frac{\log\left(x^2\right)}{a^2d}-\frac{\sqrt{c}e^3\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}\left(ae^2+cd^2\right)^2}+\frac{c\left(d-ex^2\right)}{2a\left(a+cx^4\right)\left(ae^2+\right.}\right.$$

input
```
Int[1/(x*(d + e*x^2)*(a + c*x^4)^2),x]
```

output
```
((c*(d - e*x^2))/(2*a*(c*d^2 + a*e^2)*(a + c*x^4)) - (Sqrt[c]*e^3*ArcTan[(
Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*(c*d^2 + a*e^2)^2) - (Sqrt[c]*e*ArcTan[(Sq
rt[c]*x^2)/Sqrt[a]])/(2*a^(3/2)*(c*d^2 + a*e^2)) + Log[x^2]/(a^2*d) - (e^4
*Log[d + e*x^2])/(d*(c*d^2 + a*e^2)^2) - (c*d*(c*d^2 + 2*a*e^2)*Log[a + c*
x^4])/(2*a^2*(c*d^2 + a*e^2)^2))/2
```

3.249.3.1 Defintions of rubi rules used

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.249.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.82

method	result
default	$\frac{\ln(x)}{a^2 d}-\frac{c\left(\frac{\left(\frac{1}{2}e^3a^2+\frac{1}{2}acd^2e\right)x^2-\frac{ad\left(ae^2+cd^2\right)}{2}}{cx^4+a}+\frac{\left(4acde^2+2c^2d^3\right)\ln\left(cx^4+a\right)}{4c}+\frac{\left(3e^3a^2+acd^2e\right)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\sqrt{ac}}\right)}{2(ae^2+cd^2)^2a^2}-\frac{e^4\ln(ex^2+d)}{2d(ae^2+cd^2)^2}$
risch	$\frac{-\frac{ecx^2}{4a\left(ae^2+cd^2\right)}+\frac{cd}{4a\left(ae^2+cd^2\right)}}{cx^4+a}+\frac{\ln(x)}{a^2d}-\frac{e^4\ln(ex^2+d)}{2d(a^2e^4+2acd^2e^2+c^2d^4)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^6e^4+2d^2a^5ce^2+a^4d^4c^2\right)\_Z^2+\left(8a^3cde^2+\right.\right.}\right.}{}$

input
```
int(1/x/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
ln(x)/a^2/d-1/2*c/(a*e^2+c*d^2)^2/a^2*(((1/2*e^3*a^2+1/2*a*c*d^2*e)*x^2-1/
2*a*d*(a*e^2+c*d^2))/(c*x^4+a)+1/4*(4*a*c*d*e^2+2*c^2*d^3)/c*ln(c*x^4+a)+1
/2*(3*a^2*e^3+a*c*d^2*e)/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2)))-1/2*e^4*ln
(e*x^2+d)/d/(a*e^2+c*d^2)^2
```

3.249.5 Fricas [A] (verification not implemented)

Time = 74.98 (sec) , antiderivative size = 686, normalized size of antiderivative = 3.28

$$\int \frac{1}{x\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx$$
$$= \left[\frac{2\,ac^2d^4 + 2\,a^2cd^2e^2 - 2\,(ac^2d^3e + a^2cde^3)x^2 + (a^2cd^3e + 3\,a^3de^3 + (ac^2d^3e + 3\,a^2cde^3)x^4)\sqrt{-\frac{c}{a}}\log\left(\frac{cx^4 \cdots}{\cdots}\right.}{\cdots}\right.$$

input
```
integrate(1/x/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
[1/8*(2*a*c^2*d^4 + 2*a^2*c*d^2*e^2 - 2*(a*c^2*d^3*e + a^2*c*d*e^3)*x^2 +
(a^2*c*d^3*e + 3*a^3*d*e^3 + (a*c^2*d^3*e + 3*a^2*c*d*e^3)*x^4)*sqrt(-c/a)
*log((c*x^4 - 2*a*x^2*sqrt(-c/a) - a)/(c*x^4 + a)) - 2*(a*c^2*d^4 + 2*a^2*
c*d^2*e^2 + (c^3*d^4 + 2*a*c^2*d^2*e^2)*x^4)*log(c*x^4 + a) - 4*(a^2*c*e^4
*x^4 + a^3*e^4)*log(e*x^2 + d) + 8*(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4
+ (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^4)*log(x))/(a^3*c^2*d^5 + 2*a^
4*c*d^3*e^2 + a^5*d*e^4 + (a^2*c^3*d^5 + 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*
x^4), 1/4*(a*c^2*d^4 + a^2*c*d^2*e^2 - (a*c^2*d^3*e + a^2*c*d*e^3)*x^2 + (
a^2*c*d^3*e + 3*a^3*d*e^3 + (a*c^2*d^3*e + 3*a^2*c*d*e^3)*x^4)*sqrt(c/a)*a
rctan(a*sqrt(c/a)/(c*x^2)) - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + (c^3*d^4 + 2*a
*c^2*d^2*e^2)*x^4)*log(c*x^4 + a) - 2*(a^2*c*e^4*x^4 + a^3*e^4)*log(e*x^2
+ d) + 4*(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e
^2 + a^2*c*e^4)*x^4)*log(x))/(a^3*c^2*d^5 + 2*a^4*c*d^3*e^2 + a^5*d*e^4 +
(a^2*c^3*d^5 + 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4)*x^4)]
```

3.249.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(e*x**2+d)/(c*x**4+a)**2,x)
```

output
```
Timed out
```

3.249.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.09

$$\int \frac{1}{x(d+ex^2)(a+cx^4)^2}\,dx = -\frac{e^4\log(ex^2+d)}{2(c^2d^5+2\,acd^3e^2+a^2de^4)} - \frac{(c^2d^3+2\,acde^2)\log(cx^4+a)}{4(a^2c^2d^4+2\,a^3cd^2e^2+a^4e^4)} - \frac{(c^2d^2e+3\,ace^3)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4(ac^2d^4+2\,a^2cd^2e^2+a^3e^4)\sqrt{ac}} - \frac{cex^2-cd}{4(a^2cd^2+a^3e^2+(ac^2d^2+a^2ce^2)x^4)} + \frac{\log(x^2)}{2\,a^2d}$$

input
```
integrate(1/x/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
-1/2*e^4*log(e*x^2 + d)/(c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4) - 1/4*(c^2*d
^3 + 2*a*c*d*e^2)*log(c*x^4 + a)/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)
 - 1/4*(c^2*d^2*e + 3*a*c*e^3)*arctan(c*x^2/sqrt(a*c))/((a*c^2*d^4 + 2*a^2
*c*d^2*e^2 + a^3*e^4)*sqrt(a*c)) - 1/4*(c*e*x^2 - c*d)/(a^2*c*d^2 + a^3*e^
2 + (a*c^2*d^2 + a^2*c*e^2)*x^4) + 1/2*log(x^2)/(a^2*d)
```

3.249.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.38

$$\int \frac{1}{x(d+ex^2)(a+cx^4)^2}\,dx = -\frac{e^5\log(|ex^2+d|)}{2(c^2d^5e+2\,acd^3e^3+a^2de^5)} - \frac{(c^2d^3+2\,acde^2)\log(cx^4+a)}{4(a^2c^2d^4+2\,a^3cd^2e^2+a^4e^4)} - \frac{(c^2d^2e+3\,ace^3)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4(ac^2d^4+2\,a^2cd^2e^2+a^3e^4)\sqrt{ac}} + \frac{c^3d^3x^4+2\,ac^2de^2x^4-ac^2d^2ex^2-a^2ce^3x^2+2\,ac^2d^3+3\,a^2cde^2}{4(a^2c^2d^4+2\,a^3cd^2e^2+a^4e^4)(cx^4+a)} + \frac{\log(x^2)}{2\,a^2d}$$

input
```
integrate(1/x/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
-1/2*e^5*log(abs(e*x^2 + d))/(c^2*d^5*e + 2*a*c*d^3*e^3 + a^2*d*e^5) - 1/4
*(c^2*d^3 + 2*a*c*d*e^2)*log(c*x^4 + a)/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a
^4*e^4) - 1/4*(c^2*d^2*e + 3*a*c*e^3)*arctan(c*x^2/sqrt(a*c))/((a*c^2*d^4
+ 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(a*c)) + 1/4*(c^3*d^3*x^4 + 2*a*c^2*d*e^2
*x^4 - a*c^2*d^2*e*x^2 - a^2*c*e^3*x^2 + 2*a*c^2*d^3 + 3*a^2*c*d*e^2)/((a^
2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*(c*x^4 + a)) + 1/2*log(x^2)/(a^2*d)
```

3.249.9 Mupad [B] (verification not implemented)

Time = 9.05 (sec) , antiderivative size = 1082, normalized size of antiderivative = 5.18

$$\int \frac{1}{x\,(d+ex^2)\,(a+cx^4)^2}\,dx = \frac{\frac{c\,d}{4\,a\,(c\,d^2+a\,e^2)}-\frac{c\,e\,x^2}{4\,a\,(c\,d^2+a\,e^2)}}{c\,x^4+a}$$

$$-\frac{\ln\left(400\,a^9\,c^{12}\,d^{20}\,x^2-10481\,d^4\,e^{16}\,(-a^5\,c)^{7/2}-1024\,a^{12}\,e^{20}\,(-a^5\,c)^{3/2}+1024\,a^{19}\,c^2\,e^{20}\,x^2-400\,a^2\,c^{10}\,d}{}$$

$$+\frac{\ln\left(1024\,a^{12}\,e^{20}\,(-a^5\,c)^{3/2}+10481\,d^4\,e^{16}\,(-a^5\,c)^{7/2}+400\,a^9\,c^{12}\,d^{20}\,x^2+1024\,a^{19}\,c^2\,e^{20}\,x^2+400\,a^2\,c^{10}\,d}{}$$

$$-\frac{e^4\,\ln\left(e\,x^2+d\right)}{2\,a^2\,d\,e^4+4\,a\,c\,d^3\,e^2+2\,c^2\,d^5}+\frac{\ln\left(x\right)}{a^2\,d}$$

input
```
int(1/(x*(a + c*x^4)^2*(d + e*x^2)),x)
```

output
```
((c*d)/(4*a*(a*e^2 + c*d^2)) - (c*e*x^2)/(4*a*(a*e^2 + c*d^2)))/(a + c*x^4
) - (log(400*a^9*c^12*d^20*x^2 - 10481*d^4*e^16*(-a^5*c)^(7/2) - 1024*a^12
*e^20*(-a^5*c)^(3/2) + 1024*a^19*c^2*e^20*x^2 - 400*a^2*c^10*d^20*(-a^5*c)
^(3/2) + 5840*a^6*d^2*e^18*(-a^5*c)^(5/2) + 33710*c^6*d^14*e^6*(-a^5*c)^(5
/2) + 4104*a^10*c^11*d^18*e^2*x^2 + 16689*a^11*c^10*d^16*e^4*x^2 + 33710*a
^12*c^9*d^14*e^6*x^2 + 33391*a^13*c^8*d^12*e^8*x^2 + 10748*a^14*c^7*d^10*e
^10*x^2 - 3585*a^15*c^6*d^8*e^12*x^2 + 3998*a^16*c^5*d^6*e^14*x^2 + 10481*
a^17*c^4*d^4*e^16*x^2 + 5840*a^18*c^3*d^2*e^18*x^2 + 10748*a^2*c^4*d^10*e^
10*(-a^5*c)^(5/2) - 3585*a^3*c^3*d^8*e^12*(-a^5*c)^(5/2) + 3998*a^4*c^2*d^
6*e^14*(-a^5*c)^(5/2) - 4104*a^3*c^9*d^18*e^2*(-a^5*c)^(3/2) - 16689*a^4*c
^8*d^16*e^4*(-a^5*c)^(3/2) + 33391*a*c^5*d^12*e^8*(-a^5*c)^(5/2))*(3*a*e^3
*(-a^5*c)^(1/2) + 2*a^2*c^2*d^3 + 4*a^3*c*d*e^2 + c*d^2*e*(-a^5*c)^(1/2)))
/(8*(a^6*e^4 + a^4*c^2*d^4 + 2*a^5*c*d^2*e^2)) + (log(1024*a^12*e^20*(-a^5
*c)^(3/2) + 10481*d^4*e^16*(-a^5*c)^(7/2) + 400*a^9*c^12*d^20*x^2 + 1024*a
^19*c^2*e^20*x^2 + 400*a^2*c^10*d^20*(-a^5*c)^(3/2) - 5840*a^6*d^2*e^18*(-
a^5*c)^(5/2) - 33710*c^6*d^14*e^6*(-a^5*c)^(5/2) + 4104*a^10*c^11*d^18*e^2
*x^2 + 16689*a^11*c^10*d^16*e^4*x^2 + 33710*a^12*c^9*d^14*e^6*x^2 + 33391*
a^13*c^8*d^12*e^8*x^2 + 10748*a^14*c^7*d^10*e^10*x^2 - 3585*a^15*c^6*d^8*e
^12*x^2 + 3998*a^16*c^5*d^6*e^14*x^2 + 10481*a^17*c^4*d^4*e^16*x^2 + 5840*
a^18*c^3*d^2*e^18*x^2 - 10748*a^2*c^4*d^10*e^10*(-a^5*c)^(5/2) + 3585*a...
```

3.249. $\int \frac{1}{x(d+ex^2)(a+cx^4)^2}\,dx$

3.250 $\int \frac{1}{x^3(d+ex^2)(a+cx^4)^2} dx$

3.250.1 Optimal result

Integrand size = 22, antiderivative size = 236

$$\int \frac{1}{x^3(d+ex^2)(a+cx^4)^2} dx = -\frac{1}{2a^2dx^2} - \frac{c(ae+cdx^2)}{4a^2(cd^2+ae^2)(a+cx^4)} - \frac{c^{3/2}d\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{5/2}(cd^2+ae^2)} - \frac{c^{3/2}d(cd^2+2ae^2)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{5/2}(cd^2+ae^2)^2} - \frac{e\log(x)}{a^2d^2} + \frac{e^5\log(d+ex^2)}{2d^2(cd^2+ae^2)^2} + \frac{ce(cd^2+2ae^2)\log(a+cx^4)}{4a^2(cd^2+ae^2)^2}$$

output
```
-1/2/a^2/d/x^2-1/4*c*(c*d*x^2+a*e)/a^2/(a*e^2+c*d^2)/(c*x^4+a)-1/4*c^(3/2)
*d*arctan(x^2*c^(1/2)/a^(1/2))/a^(5/2)/(a*e^2+c*d^2)-1/2*c^(3/2)*d*(2*a*e^
2+c*d^2)*arctan(x^2*c^(1/2)/a^(1/2))/a^(5/2)/(a*e^2+c*d^2)^2-e*ln(x)/a^2/d
^2+1/2*e^5*ln(e*x^2+d)/d^2/(a*e^2+c*d^2)^2+1/4*c*e*(2*a*e^2+c*d^2)*ln(c*x^
4+a)/a^2/(a*e^2+c*d^2)^2
```

3.250.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^3 (d+ex^2) (a+cx^4)^2} dx = \frac{1}{4}\left(-\frac{2}{a^2dx^2} - \frac{c(ae+cdx^2)}{a^2 (cd^2+ae^2) (a+cx^4)} + \frac{c^{3/2}d(3cd^2+5ae^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{5/2} (cd^2+ae^2)^2} + \frac{c^{3/2}d(3cd^2+5ae^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{5/2} (cd^2+ae^2)^2} - \frac{4e\log(x)}{a^2d^2} + \frac{2e^5\log(d+ex^2)}{(cd^3+ade^2)^2} + \frac{c(cd^2e+2ae^3)\log(a+cx^4)}{a^2 (cd^2+ae^2)^2}\right)$$

input
```
Integrate[1/(x^3*(d + e*x^2)*(a + c*x^4)^2),x]
```

output
```
(-2/(a^2*d*x^2) - (c*(a*e + c*d*x^2))/(a^2*(c*d^2 + a*e^2)*(a + c*x^4)) +
(c^(3/2)*d*(3*c*d^2 + 5*a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a
^(5/2)*(c*d^2 + a*e^2)^2) + (c^(3/2)*d*(3*c*d^2 + 5*a*e^2)*ArcTan[1 + (Sqr
t[2]*c^(1/4)*x)/a^(1/4)])/(a^(5/2)*(c*d^2 + a*e^2)^2) - (4*e*Log[x])/(a^2*
d^2) + (2*e^5*Log[d + e*x^2])/(c*d^3 + a*d*e^2)^2 + (c*(c*d^2*e + 2*a*e^3)
*Log[a + c*x^4])/(a^2*(c*d^2 + a*e^2)^2))/4
```

3.250.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1579, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+cx^4)^2 (d+ex^2)} dx$$

$$\Big\downarrow \text{ 1579}$$

$$\frac{1}{2}\int \frac{1}{x^4\left(ex^2+d\right)\left(cx^4+a\right)^2}dx^2$$

$$\Big\downarrow \text{615}$$

$$\frac{1}{2}\int \left(\frac{e^6}{d^2\left(cd^2+ae^2\right)^2\left(ex^2+d\right)}-\frac{e}{a^2d^2x^2}-\frac{c^2\left(cd^2+2ae^2\right)\left(d-ex^2\right)}{a^2\left(cd^2+ae^2\right)^2\left(cx^4+a\right)}-\frac{c^2\left(d-ex^2\right)}{a\left(cd^2+ae^2\right)\left(cx^4+a\right)^2}+\frac{1}{a^2dx^4}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{c^{3/2}d\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)\left(2ae^2+cd^2\right)}{a^{5/2}\left(ae^2+cd^2\right)^2}-\frac{c^{3/2}d\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{5/2}\left(ae^2+cd^2\right)}+\frac{ce\left(2ae^2+cd^2\right)\log\left(a+cx^4\right)}{2a^2\left(ae^2+cd^2\right)^2}-\frac{c\left(ae+cd\right.}{2a^2\left(a+cx^4\right)\left(a\right.}\right.$$

input
```
Int[1/(x^3*(d + e*x^2)*(a + c*x^4)^2),x]
```

output
```
(-(1/(a^2*d*x^2)) - (c*(a*e + c*d*x^2))/(2*a^2*(c*d^2 + a*e^2)*(a + c*x^4)
) - (c^(3/2)*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*a^(5/2)*(c*d^2 + a*e^2))
- (c^(3/2)*d*(c*d^2 + 2*a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(a^(5/2)*(c*
d^2 + a*e^2)^2) - (e*Log[x^2])/(a^2*d^2) + (e^5*Log[d + e*x^2])/(d^2*(c*d^
2 + a*e^2)^2) + (c*e*(c*d^2 + 2*a*e^2)*Log[a + c*x^4])/(2*a^2*(c*d^2 + a*e
^2)^2))/2
```

3.250.3.1 Defintions of rubi rules used

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.250.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.77

method	result
default	$-\frac{1}{2a^{2}d\,x^{2}}-\frac{e\ln\left(x\right)}{a^{2}d^{2}}-\frac{c^{2}\left(\frac{\left(\frac{1}{2}d\,e^{2}a+\frac{1}{2}d^{3}c\right)x^{2}+\frac{ae\left(a\,e^{2}+c\,d^{2}\right)}{2c}}{c\,x^{4}+a}+\frac{\left(-4a\,e^{3}-2c\,d^{2}e\right)\ln\left(c\,x^{4}+a\right)}{4c}+\frac{\left(5d\,e^{2}a+3d^{3}c\right)\arctan\left(\frac{c\,x^{2}}{\sqrt{ac}}\right)}{2\sqrt{ac}}\right)}{2\left(a\,e^{2}+c\,d^{2}\right)^{2}a^{2}}+\frac{e^{5}}{2d^{2}}$
risch	$\frac{-\frac{c\left(2a\,e^{2}+3c\,d^{2}\right)x^{4}}{4d\,a^{2}\left(a\,e^{2}+c\,d^{2}\right)}-\frac{ec\,x^{2}}{4a\left(a\,e^{2}+c\,d^{2}\right)}-\frac{1}{2da}}{x^{2}\left(c\,x^{4}+a\right)}-\frac{e\ln\left(x\right)}{a^{2}d^{2}}+\frac{e^{5}\ln\left(-e\,x^{2}-d\right)}{2d^{2}\left(a^{2}e^{4}+2ac\,d^{2}e^{2}+c^{2}d^{4}\right)}+\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^{7}e^{4}+2a^{6}c\,d^{2}e^{2}+a^{5}c^{2}d^{4}\right)\_Z^{2}\right.}\right.$

input
```
int(1/x^3/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^2/d/x^2-e*ln(x)/a^2/d^2-1/2*c^2/(a*e^2+c*d^2)^2/a^2*(((1/2*d*e^2*a+
1/2*d^3*c)*x^2+1/2*a*e*(a*e^2+c*d^2)/c)/(c*x^4+a)+1/4*(-4*a*e^3-2*c*d^2*e)
/c*ln(c*x^4+a)+1/2*(5*a*d*e^2+3*c*d^3)/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2
)))+1/2*e^5*ln(e*x^2+d)/d^2/(a*e^2+c*d^2)^2
```

3.250.5 Fricas [A] (verification not implemented)

Time = 224.20 (sec) , antiderivative size = 852, normalized size of antiderivative = 3.61

$$\int \frac{1}{x^{3}\left(d+ex^{2}\right)\left(a+cx^{4}\right)^{2}}\,dx$$

$$= \left[-\frac{4\,ac^{2}d^{5}+8\,a^{2}cd^{3}e^{2}+4\,a^{3}de^{4}+2\left(3\,c^{3}d^{5}+5\,ac^{2}d^{3}e^{2}+2\,a^{2}cde^{4}\right)x^{4}+2\left(ac^{2}d^{4}e+a^{2}cd^{2}e^{3}\right)x^{2}-\left(\left(3\,c^{3}d^{5}\right.\right.}{}\right.$$

$$-\frac{2\,ac^{2}d^{5}+4\,a^{2}cd^{3}e^{2}+2\,a^{3}de^{4}+\left(3\,c^{3}d^{5}+5\,ac^{2}d^{3}e^{2}+2\,a^{2}cde^{4}\right)x^{4}+\left(ac^{2}d^{4}e+a^{2}cd^{2}e^{3}\right)x^{2}-\left(\left(3\,c^{3}d^{5}+5\right.\right.}{}$$

input
```
integrate(1/x^3/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
[-1/8*(4*a*c^2*d^5 + 8*a^2*c*d^3*e^2 + 4*a^3*d*e^4 + 2*(3*c^3*d^5 + 5*a*c^
2*d^3*e^2 + 2*a^2*c*d*e^4)*x^4 + 2*(a*c^2*d^4*e + a^2*c*d^2*e^3)*x^2 - ((3
*c^3*d^5 + 5*a*c^2*d^3*e^2)*x^6 + (3*a*c^2*d^5 + 5*a^2*c*d^3*e^2)*x^2)*sqr
t(-c/a)*log((c*x^4 - 2*a*x^2*sqrt(-c/a) - a)/(c*x^4 + a)) - 2*((c^3*d^4*e
+ 2*a*c^2*d^2*e^3)*x^6 + (a*c^2*d^4*e + 2*a^2*c*d^2*e^3)*x^2)*log(c*x^4 +
a) - 4*(a^2*c*e^5*x^6 + a^3*e^5*x^2)*log(e*x^2 + d) + 8*((c^3*d^4*e + 2*a*
c^2*d^2*e^3 + a^2*c*e^5)*x^6 + (a*c^2*d^4*e + 2*a^2*c*d^2*e^3 + a^3*e^5)*x
^2)*log(x))/((a^2*c^3*d^6 + 2*a^3*c^2*d^4*e^2 + a^4*c*d^2*e^4)*x^6 + (a^3*
c^2*d^6 + 2*a^4*c*d^4*e^2 + a^5*d^2*e^4)*x^2), -1/4*(2*a*c^2*d^5 + 4*a^2*c
*d^3*e^2 + 2*a^3*d*e^4 + (3*c^3*d^5 + 5*a*c^2*d^3*e^2 + 2*a^2*c*d*e^4)*x^4
 + (a*c^2*d^4*e + a^2*c*d^2*e^3)*x^2 - ((3*c^3*d^5 + 5*a*c^2*d^3*e^2)*x^6
+ (3*a*c^2*d^5 + 5*a^2*c*d^3*e^2)*x^2)*sqrt(c/a)*arctan(a*sqrt(c/a)/(c*x^2
)) - ((c^3*d^4*e + 2*a*c^2*d^2*e^3)*x^6 + (a*c^2*d^4*e + 2*a^2*c*d^2*e^3)*
x^2)*log(c*x^4 + a) - 2*(a^2*c*e^5*x^6 + a^3*e^5*x^2)*log(e*x^2 + d) + 4*(
(c^3*d^4*e + 2*a*c^2*d^2*e^3 + a^2*c*e^5)*x^6 + (a*c^2*d^4*e + 2*a^2*c*d^2
*e^3 + a^3*e^5)*x^2)*log(x))/((a^2*c^3*d^6 + 2*a^3*c^2*d^4*e^2 + a^4*c*d^2
*e^4)*x^6 + (a^3*c^2*d^6 + 2*a^4*c*d^4*e^2 + a^5*d^2*e^4)*x^2)]
```

3.250.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/x**3/(e*x**2+d)/(c*x**4+a)**2,x)
```

output
```
Timed out
```

3.250.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.18

$$\int \frac{1}{x^3\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx = \frac{e^5\log\left(ex^2+d\right)}{2\left(c^2d^6+2\,acd^4e^2+a^2d^2e^4\right)} + \frac{\left(c^2d^2e+2\,ace^3\right)\log\left(cx^4+a\right)}{4\left(a^2c^2d^4+2\,a^3cd^2e^2+a^4e^4\right)} - \frac{\left(3\,c^3d^3+5\,ac^2de^2\right)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\left(a^2c^2d^4+2\,a^3cd^2e^2+a^4e^4\right)\sqrt{ac}} - \frac{acdex^2+\left(3\,c^2d^2+2\,ace^2\right)x^4+2\,acd^2+2\,a^2e^2}{4\left(\left(a^2c^2d^3+a^3cde^2\right)x^6+\left(a^3cd^3+a^4de^2\right)x^2\right)} - \frac{e\log\left(x^2\right)}{2\,a^2d^2}$$

input
```
integrate(1/x^3/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/2*e^5*log(e*x^2 + d)/(c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4) + 1/4*(c^2*
d^2*e + 2*a*c*e^3)*log(c*x^4 + a)/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4
) - 1/4*(3*c^3*d^3 + 5*a*c^2*d*e^2)*arctan(c*x^2/sqrt(a*c))/((a^2*c^2*d^4
+ 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(a*c)) - 1/4*(a*c*d*e*x^2 + (3*c^2*d^2 +
2*a*c*e^2)*x^4 + 2*a*c*d^2 + 2*a^2*e^2)/((a^2*c^2*d^3 + a^3*c*d*e^2)*x^6 +
 (a^3*c*d^3 + a^4*d*e^2)*x^2) - 1/2*e*log(x^2)/(a^2*d^2)
```

3.250.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.51

$$\int \frac{1}{x^3(d+ex^2)(a+cx^4)^2}\,dx = \frac{e^6\log(|ex^2+d|)}{2(c^2d^6e+2acd^4e^3+a^2d^2e^5)} + \frac{(c^2d^2e+2ace^3)\log(cx^4+a)}{4(a^2c^2d^4+2a^3cd^2e^2+a^4e^4)} - \frac{(3c^3d^3+5ac^2de^2)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4(a^2c^2d^4+2a^3cd^2e^2+a^4e^4)\sqrt{ac}} + \frac{2a^2ce^5x^6-9c^3d^5x^4-15ac^2d^3e^2x^4-6a^2cde^4x^4-3ac^2d^4ex^2-3a^2cd^2e^3x^2+2a^3e^5x^2-6ac^2d^5-12}{12(a^2c^2d^6+2a^3cd^4e^2+a^4d^2e^4)(cx^6+ax^2)} - \frac{e\log(x^2)}{2a^2d^2}$$

input
```
integrate(1/x^3/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/2*e^6*log(abs(e*x^2 + d))/(c^2*d^6*e + 2*a*c*d^4*e^3 + a^2*d^2*e^5) + 1/
4*(c^2*d^2*e + 2*a*c*e^3)*log(c*x^4 + a)/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 +
a^4*e^4) - 1/4*(3*c^3*d^3 + 5*a*c^2*d*e^2)*arctan(c*x^2/sqrt(a*c))/((a^2*c
^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(a*c)) + 1/12*(2*a^2*c*e^5*x^6 - 9
*c^3*d^5*x^4 - 15*a*c^2*d^3*e^2*x^4 - 6*a^2*c*d*e^4*x^4 - 3*a*c^2*d^4*e*x^
2 - 3*a^2*c*d^2*e^3*x^2 + 2*a^3*e^5*x^2 - 6*a*c^2*d^5 - 12*a^2*c*d^3*e^2 -
 6*a^3*d*e^4)/((a^2*c^2*d^6 + 2*a^3*c*d^4*e^2 + a^4*d^2*e^4)*(c*x^6 + a*x^
2)) - 1/2*e*log(x^2)/(a^2*d^2)
```

3.250.9 Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 1337, normalized size of antiderivative = 5.67

$$\int \frac{1}{x^3 (d + ex^2) (a + cx^4)^2} \, dx = \text{Too large to display}$$

input
```
int(1/(x^3*(a + c*x^4)^2*(d + e*x^2)),x)
```

output
```
(log(81*a^10*c^16*d^24*x^2 + 1024*a^22*c^4*e^24*x^2 - 81*a^3*c^11*d^24*(-a
^5*c^3)^(3/2) + 1024*a^20*c^2*e^24*(-a^5*c^3)^(1/2) - 14496*a^6*d^8*e^16*(
-a^5*c^3)^(5/2) - 5120*a^14*d^2*e^22*(-a^5*c^3)^(3/2) + 11647*c^6*d^20*e^4
*(-a^5*c^3)^(5/2) + 1638*a^11*c^15*d^22*e^2*x^2 + 11647*a^12*c^14*d^20*e^4
*x^2 + 43524*a^13*c^13*d^18*e^6*x^2 + 97311*a^14*c^12*d^16*e^8*x^2 + 13333
4*a^15*c^11*d^14*e^10*x^2 + 103633*a^16*c^10*d^12*e^12*x^2 + 29456*a^17*c^
9*d^10*e^14*x^2 - 14496*a^18*c^8*d^8*e^16*x^2 - 7984*a^19*c^7*d^6*e^18*x^2
 + 5888*a^20*c^6*d^4*e^20*x^2 + 5120*a^21*c^5*d^2*e^22*x^2 + 43524*a*c^5*d
^18*e^6*(-a^5*c^3)^(5/2) + 29456*a^5*c*d^10*e^14*(-a^5*c^3)^(5/2) - 5888*a
^13*c*d^4*e^20*(-a^5*c^3)^(3/2) + 97311*a^2*c^4*d^16*e^8*(-a^5*c^3)^(5/2)
+ 133334*a^3*c^3*d^14*e^10*(-a^5*c^3)^(5/2) + 103633*a^4*c^2*d^12*e^12*(-a
^5*c^3)^(5/2) - 1638*a^4*c^10*d^22*e^2*(-a^5*c^3)^(3/2) + 7984*a^12*c^2*d^
6*e^18*(-a^5*c^3)^(3/2))*(4*a^4*c*e^3 - 3*c*d^3*(-a^5*c^3)^(1/2) + 2*a^3*c
^2*d^2*e - 5*a*d*e^2*(-a^5*c^3)^(1/2)))/(8*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*
c*d^2*e^2)) - (1/(2*a*d) + (c*e*x^2)/(4*a*(a*e^2 + c*d^2)) + (c*x^4*(2*a*e
^2 + 3*c*d^2))/(4*a^2*d*(a*e^2 + c*d^2)))/(a*x^2 + c*x^6) + (log(81*a^10*c
^16*d^24*x^2 + 1024*a^22*c^4*e^24*x^2 + 81*a^3*c^11*d^24*(-a^5*c^3)^(3/2)
- 1024*a^20*c^2*e^24*(-a^5*c^3)^(1/2) + 14496*a^6*d^8*e^16*(-a^5*c^3)^(5/2
) + 5120*a^14*d^2*e^22*(-a^5*c^3)^(3/2) - 11647*c^6*d^20*e^4*(-a^5*c^3)^(5
/2) + 1638*a^11*c^15*d^22*e^2*x^2 + 11647*a^12*c^14*d^20*e^4*x^2 + 4352...
```

3.251 $\int \frac{1}{x^5(d+ex^2)(a+cx^4)^2}\,dx$

3.251.1 Optimal result

Integrand size = 22, antiderivative size = 265

$$\int \frac{1}{x^5(d+ex^2)(a+cx^4)^2}\,dx = -\frac{1}{4a^2dx^4}+\frac{e}{2a^2d^2x^2}-\frac{c^2(d-ex^2)}{4a^2(cd^2+ae^2)(a+cx^4)} + \frac{c^{3/2}e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{5/2}(cd^2+ae^2)}+\frac{c^{3/2}e(cd^2+2ae^2)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{5/2}(cd^2+ae^2)^2} - \frac{(2cd^2-ae^2)\log(x)}{a^3d^3}-\frac{e^6\log(d+ex^2)}{2d^3(cd^2+ae^2)^2} + \frac{c^2d(2cd^2+3ae^2)\log(a+cx^4)}{4a^3(cd^2+ae^2)^2}$$

output
```
-1/4/a^2/d/x^4+1/2*e/a^2/d^2/x^2-1/4*c^2*(-e*x^2+d)/a^2/(a*e^2+c*d^2)/(c*x
^4+a)+1/4*c^(3/2)*e*arctan(x^2*c^(1/2)/a^(1/2))/a^(5/2)/(a*e^2+c*d^2)+1/2*
c^(3/2)*e*(2*a*e^2+c*d^2)*arctan(x^2*c^(1/2)/a^(1/2))/a^(5/2)/(a*e^2+c*d^2
)^2-(-a*e^2+2*c*d^2)*ln(x)/a^3/d^3-1/2*e^6*ln(e*x^2+d)/d^3/(a*e^2+c*d^2)^2
+1/4*c^2*d*(3*a*e^2+2*c*d^2)*ln(c*x^4+a)/a^3/(a*e^2+c*d^2)^2
```

3.251.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^5 (d+ex^2) (a+cx^4)^2} \, dx = \frac{1}{4} \left(-\frac{1}{a^2 d x^4} + \frac{2e}{a^2 d^2 x^2} + \frac{c^2(-d+ex^2)}{a^2 (cd^2+ae^2) (a+cx^4)} - \frac{c^{3/2} e (3cd^2+5ae^2) \arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{5/2} (cd^2+ae^2)^2} - \frac{c^{3/2} e (3cd^2+5ae^2) \arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{5/2} (cd^2+ae^2)^2} + \frac{4(-2cd^2+ae^2)\log(x)}{a^3 d^3} - \frac{2e^6 \log(d+ex^2)}{d^3 (cd^2+ae^2)^2} + \frac{c^2(2cd^3+3ade^2)\log(a+cx^4)}{a^3 (cd^2+ae^2)^2} \right)$$

input
```
Integrate[1/(x^5*(d + e*x^2)*(a + c*x^4)^2),x]
```

output
```
(-(1/(a^2*d*x^4)) + (2*e)/(a^2*d^2*x^2) + (c^2*(-d + e*x^2))/(a^2*(c*d^2 +
 a*e^2)*(a + c*x^4)) - (c^(3/2)*e*(3*c*d^2 + 5*a*e^2)*ArcTan[1 - (Sqrt[2]*
c^(1/4)*x)/a^(1/4)])/(a^(5/2)*(c*d^2 + a*e^2)^2) - (c^(3/2)*e*(3*c*d^2 + 5
*a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(5/2)*(c*d^2 + a*e^2)^
2) + (4*(-2*c*d^2 + a*e^2)*Log[x])/(a^3*d^3) - (2*e^6*Log[d + e*x^2])/(d^3
*(c*d^2 + a*e^2)^2) + (c^2*(2*c*d^3 + 3*a*d*e^2)*Log[a + c*x^4])/(a^3*(c*d
^2 + a*e^2)^2))/4
```

3.251.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1579, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (a+cx^4)^2 (d+ex^2)} \, dx$$
$$\Big\downarrow \text{1579}$$
$$\frac{1}{2}\int \frac{1}{x^6 (ex^2+d) (cx^4+a)^2} dx^2$$
$$\Big\downarrow \text{615}$$
$$\frac{1}{2}\int \left(-\frac{e^7}{d^3 (cd^2+ae^2)^2 (ex^2+d)} - \frac{e}{a^2d^2x^4} + \frac{c^2 (cd(2cd^2+3ae^2) x^2 + ae(cd^2+2ae^2))}{a^3 (cd^2+ae^2)^2 (cx^4+a)} + \frac{ae^2-2cd^2}{a^3d^3x^2} + \frac{c^2}{a^2 (cd^2 \cdot$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{2}\left(\frac{c^{3/2}e \arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right) (2ae^2+cd^2)}{a^{5/2} (ae^2+cd^2)^2} + \frac{c^{3/2}e \arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{5/2} (ae^2+cd^2)} + \frac{c^2d(3ae^2+2cd^2) \log(a+cx^4)}{2a^3 (ae^2+cd^2)^2} - \frac{\log(x^2) (2cd^2 -}{a^3d^3}\right.$$

input
```
Int[1/(x^5*(d + e*x^2)*(a + c*x^4)^2),x]
```

output
```
(-1/2*1/(a^2*d*x^4) + e/(a^2*d^2*x^2) - (c^2*(d - e*x^2))/(2*a^2*(c*d^2 +
a*e^2)*(a + c*x^4)) + (c^(3/2)*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*a^(5/2)
*(c*d^2 + a*e^2)) + (c^(3/2)*e*(c*d^2 + 2*a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt
[a]])/(a^(5/2)*(c*d^2 + a*e^2)^2) - ((2*c*d^2 - a*e^2)*Log[x^2])/(a^3*d^3)
 - (e^6*Log[d + e*x^2])/(d^3*(c*d^2 + a*e^2)^2) + (c^2*d*(2*c*d^2 + 3*a*e^
2)*Log[a + c*x^4])/(2*a^3*(c*d^2 + a*e^2)^2))/2
```

3.251.3.1 Defintions of rubi rules used

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.251.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.79

method	result
default	$-\frac{1}{4a^2 d x^4}+\frac{(a e^2-2c d^2)\ln(x)}{a^3 d^3}+\frac{e}{2a^2 d^2 x^2}+\frac{c^2\left(\frac{\left(\frac{1}{2}e^3 a^2+\frac{1}{2}ac d^2 e\right)x^2-\frac{ad\left(a e^2+c d^2\right)}{2}}{c x^4+a}+\frac{\left(6acd e^2+4c^2 d^3\right)\ln\left(c x^4+a\right)}{4c}+\frac{\left(5e^3 a^2+3a\right.}{}\right.}{2(a e^2+c d^2)^2 a^3}$
risch	$\frac{\frac{ec\left(2a e^2+3c d^2\right)x^6}{4\left(a e^2+c d^2\right)a^2 d^2}-\frac{c\left(a e^2+2c d^2\right)x^4}{4d a^2\left(a e^2+c d^2\right)}+\frac{e x^2}{2d^2 a}-\frac{1}{4da}}{x^4(c x^4+a)}+\frac{\ln(x)e^2}{a^2 d^3}-\frac{2\ln(x)c}{a^3 d}-\frac{e^6\ln(e x^2+d)}{2d^3(a^2 e^4+2ac d^2 e^2+c^2 d^4)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^8 e^4+2\right.\right.}\right.}{}$

input
```
int(1/x^5/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4/a^2/d/x^4+(a*e^2-2*c*d^2)/a^3/d^3*ln(x)+1/2*e/a^2/d^2/x^2+1/2*c^2/(a*
e^2+c*d^2)^2/a^3*(((1/2*e^3*a^2+1/2*a*c*d^2*e)*x^2-1/2*a*d*(a*e^2+c*d^2))/
(c*x^4+a)+1/4*(6*a*c*d*e^2+4*c^2*d^3)/c*ln(c*x^4+a)+1/2*(5*a^2*e^3+3*a*c*d
^2*e)/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2)))-1/2*e^6*ln(e*x^2+d)/d^3/(a*e^
2+c*d^2)^2
```

3.251.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (d+ex^2) (a+cx^4)^2} dx = \text{Timed out}$$

input
```
integrate(1/x^5/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.251.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (d+ex^2) (a+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/x**5/(e*x**2+d)/(c*x**4+a)**2,x)
```

output
```
Timed out
```

3.251.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.25

$$\begin{aligned}\int \frac{1}{x^5 (d+ex^2) (a+cx^4)^2}\,dx &= -\frac{e^6 \log (ex^2+d)}{2 (c^2d^7+2\,acd^5e^2+a^2d^3e^4)} + \frac{(2\,c^3d^3+3\,ac^2de^2) \log (cx^4+a)}{4 (a^3c^2d^4+2\,a^4cd^2e^2+a^5e^4)} \\ &\quad + \frac{(3\,c^3d^2e+5\,ac^2e^3) \arctan \left(\frac{cx^2}{\sqrt{ac}}\right)}{4 (a^2c^2d^4+2\,a^3cd^2e^2+a^4e^4)\sqrt{ac}} \\ &\quad + \frac{(3\,c^2d^2e+2\,ace^3)x^6-acd^3-a^2de^2-(2\,c^2d^3+acde^2)x^4+2\,(acd^2e+a^2e^3)x^2}{4\,((a^2c^2d^4+a^3cd^2e^2)x^8+(a^3cd^4+a^4d^2e^2)x^4)} \\ &\quad - \frac{(2\,cd^2-ae^2) \log (x^2)}{2\,a^3d^3}\end{aligned}$$

input
```
integrate(1/x^5/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
-1/2*e^6*log(e*x^2 + d)/(c^2*d^7 + 2*a*c*d^5*e^2 + a^2*d^3*e^4) + 1/4*(2*c
^3*d^3 + 3*a*c^2*d*e^2)*log(c*x^4 + a)/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^
5*e^4) + 1/4*(3*c^3*d^2*e + 5*a*c^2*e^3)*arctan(c*x^2/sqrt(a*c))/((a^2*c^2
*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(a*c)) + 1/4*((3*c^2*d^2*e + 2*a*c*e
^3)*x^6 - a*c*d^3 - a^2*d*e^2 - (2*c^2*d^3 + a*c*d*e^2)*x^4 + 2*(a*c*d^2*e
 + a^2*e^3)*x^2)/((a^2*c^2*d^4 + a^3*c*d^2*e^2)*x^8 + (a^3*c*d^4 + a^4*d^2
*e^2)*x^4) - 1/2*(2*c*d^2 - a*e^2)*log(x^2)/(a^3*d^3)
```

3.251.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.36

$$\int \frac{1}{x^5 (d+ex^2) (a+cx^4)^2} \, dx = -\frac{e^7 \log (|ex^2+d|)}{2 (c^2d^7e+2 acd^5e^3+a^2d^3e^5)} + \frac{(2 c^3d^3+3 ac^2de^2) \log (cx^4+a)}{4 (a^3c^2d^4+2 a^4cd^2e^2+a^5e^4)} + \frac{(3 c^3d^2e+5 ac^2e^3) \arctan \left(\frac{cx^2}{\sqrt{ac}}\right)}{4 (a^2c^2d^4+2 a^3cd^2e^2+a^4e^4)\sqrt{ac}} - \frac{2 c^4d^3x^4+3 ac^3de^2x^4-ac^3d^2ex^2-a^2c^2e^3x^2+3 ac^3d^3+4 a^2c^2de^2}{4 (a^3c^2d^4+2 a^4cd^2e^2+a^5e^4)(cx^4+a)} - \frac{(2 cd^2-ae^2) \log (x^2)}{2 a^3d^3} + \frac{6 cd^2x^4-3 ae^2x^4+2 adex^2-ad^2}{4 a^3d^3x^4}$$

input
```
integrate(1/x^5/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
-1/2*e^7*log(abs(e*x^2 + d))/(c^2*d^7*e + 2*a*c*d^5*e^3 + a^2*d^3*e^5) + 1
/4*(2*c^3*d^3 + 3*a*c^2*d*e^2)*log(c*x^4 + a)/(a^3*c^2*d^4 + 2*a^4*c*d^2*e
^2 + a^5*e^4) + 1/4*(3*c^3*d^2*e + 5*a*c^2*e^3)*arctan(c*x^2/sqrt(a*c))/((
a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(a*c)) - 1/4*(2*c^4*d^3*x^4 +
 3*a*c^3*d*e^2*x^4 - a*c^3*d^2*e*x^2 - a^2*c^2*e^3*x^2 + 3*a*c^3*d^3 + 4*a
^2*c^2*d*e^2)/((a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*(c*x^4 + a)) - 1/
2*(2*c*d^2 - a*e^2)*log(x^2)/(a^3*d^3) + 1/4*(6*c*d^2*x^4 - 3*a*e^2*x^4 +
2*a*d*e*x^2 - a*d^2)/(a^3*d^3*x^4)
```

3.251.9 Mupad [B] (verification not implemented)

Time = 9.49 (sec) , antiderivative size = 1545, normalized size of antiderivative = 5.83

$$\int \frac{1}{x^5 (d+ex^2) (a+cx^4)^2} \, dx = \text{Too large to display}$$

input
```
int(1/(x^5*(a + c*x^4)^2*(d + e*x^2)),x)
```

output
```
(log(6400*a^13*c^18*d^28*x^2 + 1024*a^27*c^4*e^28*x^2 - 6400*a^3*c^13*d^28
*(-a^7*c^3)^(3/2) + 1024*a^24*c^2*e^28*(-a^7*c^3)^(1/2) - 10688*a^6*d^8*e^
20*(-a^7*c^3)^(5/2) - 2048*a^16*d^2*e^26*(-a^7*c^3)^(3/2) + 536959*c^6*d^2
0*e^8*(-a^7*c^3)^(5/2) + 54944*a^14*c^17*d^26*e^2*x^2 + 200881*a^15*c^16*d
^24*e^4*x^2 + 413414*a^16*c^15*d^22*e^6*x^2 + 536959*a^17*c^14*d^20*e^8*x^
2 + 465092*a^18*c^13*d^18*e^10*x^2 + 256991*a^19*c^12*d^16*e^12*x^2 + 5282
2*a^20*c^11*d^14*e^14*x^2 - 37423*a^21*c^10*d^12*e^16*x^2 - 27472*a^22*c^9
*d^10*e^18*x^2 - 10688*a^23*c^8*d^8*e^20*x^2 - 10288*a^24*c^7*d^6*e^22*x^2
 - 3584*a^25*c^6*d^4*e^24*x^2 + 2048*a^26*c^5*d^2*e^26*x^2 + 465092*a*c^5*
d^18*e^10*(-a^7*c^3)^(5/2) - 27472*a^5*c*d^10*e^18*(-a^7*c^3)^(5/2) + 3584
*a^15*c*d^4*e^24*(-a^7*c^3)^(3/2) + 256991*a^2*c^4*d^16*e^12*(-a^7*c^3)^(5
/2) + 52822*a^3*c^3*d^14*e^14*(-a^7*c^3)^(5/2) - 37423*a^4*c^2*d^12*e^16*(
-a^7*c^3)^(5/2) - 54944*a^4*c^12*d^26*e^2*(-a^7*c^3)^(3/2) - 200881*a^5*c^
11*d^24*e^4*(-a^7*c^3)^(3/2) - 413414*a^6*c^10*d^22*e^6*(-a^7*c^3)^(3/2) +
 10288*a^14*c^2*d^6*e^22*(-a^7*c^3)^(3/2))*(4*a^3*c^3*d^3 + 5*a*e^3*(-a^7*
c^3)^(1/2) + 6*a^4*c^2*d*e^2 + 3*c*d^2*e*(-a^7*c^3)^(1/2)))/(8*(a^8*e^4 +
a^6*c^2*d^4 + 2*a^7*c*d^2*e^2)) - (e^6*log(d + e*x^2))/(2*(c^2*d^7 + a^2*d
^3*e^4 + 2*a*c*d^5*e^2)) - (1/(4*a*d) - (e*x^2)/(2*a*d^2) + (x^4*(2*c^2*d^
2 + a*c*e^2))/(4*a^2*d*(a*e^2 + c*d^2)) - (c*e*x^6*(2*a*e^2 + 3*c*d^2))/(4
*a^2*d^2*(a*e^2 + c*d^2)))/(a*x^4 + c*x^8) + (log(6400*a^13*c^18*d^28*x...
```

3.251. $\int \frac{1}{x^5(d+ex^2)(a+cx^4)^2}\,dx$

3.252 $\int \frac{x^8}{(d+ex^2)(a+cx^4)^2} dx$

3.252.1 Optimal result

Integrand size = 22, antiderivative size = 712

$$\int \frac{x^8}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{dx}{4c(cd^2+ae^2)} - \frac{x^3(ae+cdx^2)}{4c(cd^2+ae^2)(a+cx^4)} + \frac{d^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}(cd^2+ae^2)^2}$$
$$+ \frac{\sqrt[4]{a}d^2(\sqrt{c}d-\sqrt{a}e)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{3/4}(cd^2+ae^2)^2}$$
$$+ \frac{\sqrt[4]{a}(\sqrt{c}d-3\sqrt{a}e)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}c^{7/4}(cd^2+ae^2)}$$
$$- \frac{\sqrt[4]{a}d^2(\sqrt{c}d-\sqrt{a}e)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{3/4}(cd^2+ae^2)^2}$$
$$- \frac{\sqrt[4]{a}(\sqrt{c}d-3\sqrt{a}e)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}c^{7/4}(cd^2+ae^2)}$$
$$+ \frac{\sqrt[4]{a}d^2(\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}(cd^2+ae^2)^2}$$
$$+ \frac{\sqrt[4]{a}(\sqrt{c}d+3\sqrt{a}e)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}c^{7/4}(cd^2+ae^2)}$$
$$- \frac{\sqrt[4]{a}d^2(\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}(cd^2+ae^2)^2}$$
$$- \frac{\sqrt[4]{a}(\sqrt{c}d+3\sqrt{a}e)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}c^{7/4}(cd^2+ae^2)}$$

output
```
1/4*d*x/c/(a*e^2+c*d^2)-1/4*x^3*(c*d*x^2+a*e)/c/(a*e^2+c*d^2)/(c*x^4+a)-1/
16*a^(1/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-3*e*a^(1/2)+d*c^(1/2))/c
^(7/4)/(a*e^2+c*d^2)*2^(1/2)-1/16*a^(1/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/
4))*(-3*e*a^(1/2)+d*c^(1/2))/c^(7/4)/(a*e^2+c*d^2)*2^(1/2)-1/4*a^(1/4)*d^2
*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/c^(3/4)/(a*e^
2+c*d^2)^2*2^(1/2)-1/4*a^(1/4)*d^2*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e
*a^(1/2)+d*c^(1/2))/c^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)+1/8*a^(1/4)*d^2*ln(-a^
(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/c^(3/4)
/(a*e^2+c*d^2)^2*2^(1/2)-1/8*a^(1/4)*d^2*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1
/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/c^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)+1/3
2*a^(1/4)*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(3*e*a^(1/2)+
d*c^(1/2))/c^(7/4)/(a*e^2+c*d^2)*2^(1/2)-1/32*a^(1/4)*ln(a^(1/4)*c^(1/4)*x
*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(3*e*a^(1/2)+d*c^(1/2))/c^(7/4)/(a*e^2+c*d^2
)*2^(1/2)+d^(7/2)*arctan(x*e^(1/2)/d^(1/2))/(a*e^2+c*d^2)^2/e^(1/2)
```

3.252.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 431, normalized size of antiderivative = 0.61

$$\int \frac{x^8}{(d+ex^2)(a+cx^4)^2}\,dx$$

$$= \frac{\frac{8a(cd^2+ae^2)x(d-ex^2)}{c(a+cx^4)} + \frac{32d^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} - \frac{2\sqrt{2}\sqrt[4]{a}(-5c^{3/2}d^3+7\sqrt{a}cd^2e-a\sqrt{c}de^2+3a^{3/2}e^3)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{c^{7/4}} + \frac{2\sqrt{2}\sqrt[4]{a}(-\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[x^8/((d + e*x^2)*(a + c*x^4)^2),x]
```

output
```
((8*a*(c*d^2 + a*e^2)*x*(d - e*x^2))/(c*(a + c*x^4)) + (32*d^(7/2)*ArcTan[
(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (2*Sqrt[2]*a^(1/4)*(-5*c^(3/2)*d^3 + 7*Sqr
t[a]*c*d^2*e - a*Sqrt[c]*d*e^2 + 3*a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4
)*x)/a^(1/4)])/c^(7/4) + (2*Sqrt[2]*a^(1/4)*(-5*c^(3/2)*d^3 + 7*Sqrt[a]*c*
d^2*e - a*Sqrt[c]*d*e^2 + 3*a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^
(1/4)])/c^(7/4) + (Sqrt[2]*a^(1/4)*(5*c^(3/2)*d^3 + 7*Sqrt[a]*c*d^2*e + a*
Sqrt[c]*d*e^2 + 3*a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + S
qrt[c]*x^2])/c^(7/4) - (Sqrt[2]*a^(1/4)*(5*c^(3/2)*d^3 + 7*Sqrt[a]*c*d^2*e
 + a*Sqrt[c]*d*e^2 + 3*a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*
x + Sqrt[c]*x^2])/c^(7/4))/(32*(c*d^2 + a*e^2)^2)
```

3.252.3 Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 652, normalized size of antiderivative = 0.92, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {1651, 1599, 25, 1603, 27, 1482, 27, 1476, 1082, 217, 1479, 25, 27, 1103, 1611, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+cx^4)^2(d+ex^2)}dx$$

$$\Big\downarrow \text{ 1651}$$

$$\frac{d^2\int \frac{x^4}{(ex^2+d)(cx^4+a)}dx}{ae^2+cd^2}-\frac{a\int \frac{x^4(d-ex^2)}{(cx^4+a)^2}dx}{ae^2+cd^2}$$

$$\Big\downarrow \text{ 1599}$$

$$\frac{d^2\int \frac{x^4}{(ex^2+d)(cx^4+a)}dx}{ae^2+cd^2}-\frac{a\left(\frac{\int -\frac{x^2(cdx^2+3ae)}{cx^4+a}dx}{4ac}+\frac{x^3(ae+cdx^2)}{4ac(a+cx^4)}\right)}{ae^2+cd^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{d^2\int \frac{x^4}{(ex^2+d)(cx^4+a)}dx}{ae^2+cd^2}-\frac{a\left(\frac{x^3(ae+cdx^2)}{4ac(a+cx^4)}-\frac{\int \frac{x^2(cdx^2+3ae)}{cx^4+a}dx}{4ac}\right)}{ae^2+cd^2}$$

$$\Big\downarrow \text{ 1603}$$

$$\frac{d^2\int \frac{x^4}{(ex^2+d)(cx^4+a)}dx}{ae^2+cd^2}-\frac{a\left(\frac{x^3(ae+cdx^2)}{4ac(a+cx^4)}-\frac{dx-\frac{\int \frac{ac(d-3ex^2)}{cx^4+a}dx}{c}}{4ac}\right)}{ae^2+cd^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{d^2\int \frac{x^4}{(ex^2+d)(cx^4+a)}dx}{ae^2+cd^2}-\frac{a\left(\frac{x^3(ae+cdx^2)}{4ac(a+cx^4)}-\frac{dx-a\int \frac{d-3ex^2}{cx^4+a}dx}{4ac}\right)}{ae^2+cd^2}$$

$$\Big\downarrow \text{ 1482}$$

$$\frac{d^2\int\frac{x^4}{(ex^2+d)(cx^4+a)}dx}{ae^2+cd^2}-\frac{a\left(\frac{x^3(ae+cdx^2)}{4ac(a+cx^4)}-\frac{dx-a\left(\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+3e\right)\int\frac{\sqrt{c}\left(\sqrt{a}-\sqrt{c}x^2\right)}{cx^4+a}dx}{2c}+\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-3e\right)\int\frac{\sqrt{c}\left(\sqrt{c}x^2+\sqrt{a}\right)}{cx^4+a}dx}{2c}\right)}{4ac}\right)}{ae^2+cd^2}$$

$$\Big\downarrow \ 27$$

$$\frac{d^2\int\frac{x^4}{(ex^2+d)(cx^4+a)}dx}{ae^2+cd^2}-\frac{a\left(\frac{x^3(ae+cdx^2)}{4ac(a+cx^4)}-\frac{dx-a\left(\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+3e\right)\int\frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}+\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-3e\right)\int\frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{c}}\right)}{4ac}\right)}{ae^2+cd^2}$$

$$\Big\downarrow \ 1476$$

$$\frac{d^2\int\frac{x^4}{(ex^2+d)(cx^4+a)}dx}{ae^2+cd^2}-\frac{a\left(\frac{x^3(ae+cdx^2)}{4ac(a+cx^4)}-\frac{dx-a\left(\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-3e\right)\left(\frac{\int\frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int\frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+3e\right)\int\frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}\right)}{4ac}\right)}{ae^2+cd^2}$$

$$\Big\downarrow \ 1082$$

3.252. $\int\frac{x^8}{(d+ex^2)(a+cx^4)^2}\,dx$

$$\frac{d^2\int \frac{x^4}{(ex^2+d)(cx^4+a)}dx}{ae^2+cd^2}-$$

$$\frac{a\left(\frac{x^3(ae+cdx^2)}{4ac(a+cx^4)}-\frac{dx-a\left(\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+3e\right)\int\frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}+\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-3e\right)\left(\frac{\int\frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int\frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+\right.}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right.}{2\sqrt{c}}\right.}{4ac}\right.}{ae^2+cd^2}$$

$\downarrow$ 217

$$\frac{d^2\int \frac{x^4}{(ex^2+d)(cx^4+a)}dx}{ae^2+cd^2}-$$

$$\frac{a\left(\frac{x^3(ae+cdx^2)}{4ac(a+cx^4)}-\frac{dx-a\left(\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+3e\right)\int\frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{\sqrt{c}d}{\sqrt{a}}-3e\right)}{2\sqrt{c}}\right)}{4ac}\right)}{ae^2+cd^2}$$

$\downarrow$ 1479

3.252. $\int \frac{x^8}{(d+ex^2)(a+cx^4)^2}\,dx$

$$\frac{d^2\int \frac{x^4}{(ex^2+d)(cx^4+a)}dx}{ae^2+cd^2} - \frac{a\left(\frac{x^3(ae+cdx^2)}{4ac(a+cx^4)} - \frac{dx - a\left(\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+3e\right)\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}} - \frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)}{2\sqrt{c}} + \frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}} - \frac{\arctan\left(\cdots\right.}{\sqrt{2}\cdots}\right.}{2\sqrt{c}\cdots}\right.}{4ac\cdots}\right.}{ae^2+cd^2\cdots}$$

$\big\downarrow$ 25

3.252. $\int \frac{x^8}{(d+ex^2)(a+cx^4)^2}\,dx$

$$\frac{d^2\int \frac{x^4}{(ex^2+d)(cx^4+a)}dx}{ae^2+cd^2}-$$

$$\frac{a\left(\frac{x^3(ae+cdx^2)}{4ac(a+cx^4)}-\frac{dx-a\left(\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+3e\right)\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{}}{}\right.}{\sqrt{2}\sqrt[4]{a}}\right.}{2\sqrt{c}}}{4ac}}{ae^2+cd^2}$$

$\downarrow$ 27

3.252. $\int \frac{x^8}{(d+ex^2)(a+cx^4)^2}\,dx$

$$\frac{d^2\int \frac{x^4}{(ex^2+d)(cx^4+a)}dx}{ae^2+cd^2} - \frac{a\left(\frac{x^3(ae+cdx^2)}{4ac(a+cx^4)} - \frac{dx - a\left(\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+3e\right)\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}} + \frac{\int \frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}\right)}{2\sqrt{c}} + \frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{\sqrt{c}\cdots}{\sqrt{c}\cdots}\right.\cdots}{2\sqrt{c}}\cdots\right)}{4ac}\cdots\right)}{ae^2+cd^2}$$

$$\Big\downarrow \; 1103$$

$$\frac{d^2\int \frac{x^4}{(ex^2+d)(cx^4+a)}dx}{ae^2+cd^2} - \frac{a\left(\frac{x^3(ae+cdx^2)}{4ac(a+cx^4)} - \frac{dx - a\left(\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{\sqrt{c}d}{\sqrt{a}}-3e\right)}{2\sqrt{c}} + \frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+3e\right)\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}} - \frac{\log\left(-\sqrt{2}\sqrt[4]{\cdots}\right)}{2\sqrt{\cdots}}\cdots\right.}{2\sqrt{c}}\cdots\right)}{4ac}\cdots\right)}{ae^2+cd^2}$$

$$\Big\downarrow \; 1611$$

3.252. $\int \frac{x^8}{(d+ex^2)(a+cx^4)^2}\,dx$

$$\frac{d^2\int\left(\frac{d^2}{(cd^2+ae^2)(ex^2+d)}-\frac{a(d-ex^2)}{(cd^2+ae^2)(cx^4+a)}\right)dx}{ae^2+cd^2}-$$

$$\frac{a\left(\frac{x^3(ae+cdx^2)}{4ac(a+cx^4)}-\frac{dx-a\left(\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{\sqrt{c}d}{\sqrt{a}}-3e\right)}{2\sqrt{c}}+\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+3e\right)\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{\ldots}\right.}{2\sqrt{\ldots}}\right.}{2\sqrt{c}}\right.}{4ac}\right.}{ae^2+cd^2}$$

$\downarrow$ 2009

$$\frac{d^2\left(\frac{\sqrt[4]{a}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)(\sqrt{c}d-\sqrt{a}e)}{2\sqrt{2}c^{3/4}(ae^2+cd^2)}-\frac{\sqrt[4]{a}\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)(\sqrt{c}d-\sqrt{a}e)}{2\sqrt{2}c^{3/4}(ae^2+cd^2)}+\frac{d^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}(ae^2+cd^2)}+\frac{\sqrt[4]{a}(\sqrt{a}e+\sqrt{c}d)\log\left(-\sqrt{2}\sqrt[4]{\ldots}\right.}{4\sqrt{2}c^{3/4}(ae^2+\ldots}\right.}{ae^2+cd^2}$$

$$\frac{a\left(\frac{x^3(ae+cdx^2)}{4ac(a+cx^4)}-\frac{dx-a\left(\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{\sqrt{c}d}{\sqrt{a}}-3e\right)}{2\sqrt{c}}+\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+3e\right)\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{\ldots}\right.}{2\sqrt{\ldots}}\right.}{2\sqrt{c}}\right.}{4ac}\right.}{ae^2+cd^2}$$

input
```
Int[x^8/((d + e*x^2)*(a + c*x^4)^2),x]
```

3.252. $\int \frac{x^8}{(d+ex^2)(a+cx^4)^2}\,dx$

output
```
(d^2*((d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*(c*d^2 + a*e^2)) + (a
^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2
*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)) - (a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTa
n[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)) +
(a^(1/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x +
 Sqrt[c]*x^2])/(4*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)) - (a^(1/4)*(Sqrt[c]*d +
 Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqr
t[2]*c^(3/4)*(c*d^2 + a*e^2))))/(c*d^2 + a*e^2) - (a*((x^3*(a*e + c*d*x^2)
)/(4*a*c*(a + c*x^4)) - (d*x - a*((((Sqrt[c]*d)/Sqrt[a] - 3*e)*(-(ArcTan[1
 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4))) + ArcTan[1 + (S
qrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4))))/(2*Sqrt[c]) + (((Sq
rt[c]*d)/Sqrt[a] + 3*e)*(-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sq
rt[c]*x^2]/(Sqrt[2]*a^(1/4)*c^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/
4)*x + Sqrt[c]*x^2]/(2*Sqrt[2]*a^(1/4)*c^(1/4))))/(2*Sqrt[c]))/(4*a*c)))/
(c*d^2 + a*e^2)
```

3.252.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.252. $\int \frac{x^8}{(d+ex^2)(a+cx^4)^2}\,dx$

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 1599
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1)*((a*e - c*d*x^2)/(4*a*
c*(p + 1))), x] - Simp[f^2/(4*a*c*(p + 1))   Int[(f*x)^(m - 2)*(a + c*x^4)^
(p + 1)*(a*e*(m - 1) - c*d*(4*p + 4 + m + 1)*x^2), x], x] /; FreeQ[{a, c, d
, e, f}, x] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

rule 1603
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
 1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ
[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[
m])
```

rule 1611
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a + c*x^4)), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

3.252. $\int \frac{x^8}{(d+ex^2)(a+cx^4)^2}\,dx$

rule 1651
```
Int[(((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^4)^(p_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-a)*(f^4/(c*d^2 + a*e^2))   Int[(f*x)^(m - 4)*(d - e*x^2
)*(a + c*x^4)^p, x], x] + Simp[d^2*(f^4/(c*d^2 + a*e^2))   Int[(f*x)^(m - 4
)*((a + c*x^4)^(p + 1)/(d + e*x^2)), x], x] /; FreeQ[{a, c, d, e, f}, x] &&
 LtQ[p, -1] && GtQ[m, 2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.252.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.47

method	result
default	$-\frac{a\left(\frac{\frac{e(ae^2+cd^2)x^3}{4c}-\frac{d(ae^2+cd^2)x}{4c}}{cx^4+a}+\frac{\frac{(de^2a+5d^3c)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}}{4c}\right)}{(ae^2+cd^2)^2}$
risch	Expression too large to display

input
```
int(x^8/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-a/(a*e^2+c*d^2)^2*((1/4*e*(a*e^2+c*d^2)/c*x^3-1/4*d*(a*e^2+c*d^2)/c*x)/(c
*x^4+a)+1/4/c*(1/8*(a*d*e^2+5*c*d^3)*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^
(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*ar
ctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/8*(-3*a
*e^3-7*c*d^2*e)/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)
^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1
/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))))+1/(a*e^2+c*d^2)^2*d^4/(e*d)^
(1/2)*arctan(e*x/(e*d)^(1/2))
```

3.252.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4918 vs. 2(538) = 1076.

Time = 9.62 (sec) , antiderivative size = 9856, normalized size of antiderivative = 13.84

$$\int \frac{x^8}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^8/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.252.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^8}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**8/(e*x**2+d)/(c*x**4+a)**2,x)
```

output
```
Timed out
```

3.252.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^8/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.252.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 599, normalized size of antiderivative = 0.84

$$\int \frac{x^8}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{d^4\arctan\left(\frac{ex}{\sqrt{de}}\right)}{(c^2d^4+2\,acd^2e^2+a^2e^4)\sqrt{de}} - \frac{\left(5\,(ac^3)^{\frac14}c^3d^3+(ac^3)^{\frac14}ac^2de^2-7\,(ac^3)^{\frac34}cd^2e-3\,(ac^3)^{\frac34}ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac14}\right)}{2\left(\frac{a}{c}\right)^{\frac14}}\right)}{8\left(\sqrt{2}c^6d^4+2\,\sqrt{2}ac^5d^2e^2+\sqrt{2}a^2c^4e^4\right)} - \frac{\left(5\,(ac^3)^{\frac14}c^3d^3+(ac^3)^{\frac14}ac^2de^2-7\,(ac^3)^{\frac34}cd^2e-3\,(ac^3)^{\frac34}ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac14}\right)}{2\left(\frac{a}{c}\right)^{\frac14}}\right)}{8\left(\sqrt{2}c^6d^4+2\,\sqrt{2}ac^5d^2e^2+\sqrt{2}a^2c^4e^4\right)} - \frac{\left(5\,(ac^3)^{\frac14}c^3d^3+(ac^3)^{\frac14}ac^2de^2+7\,(ac^3)^{\frac34}cd^2e+3\,(ac^3)^{\frac34}ae^3\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac14}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}c^6d^4+2\,\sqrt{2}ac^5d^2e^2+\sqrt{2}a^2c^4e^4\right)} + \frac{\left(5\,(ac^3)^{\frac14}c^3d^3+(ac^3)^{\frac14}ac^2de^2+7\,(ac^3)^{\frac34}cd^2e+3\,(ac^3)^{\frac34}ae^3\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac14}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}c^6d^4+2\,\sqrt{2}ac^5d^2e^2+\sqrt{2}a^2c^4e^4\right)} - \frac{aex^3-adx}{4\,(cx^4+a)(c^2d^2+ace^2)}$$

input
```
integrate(x^8/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
d^4*arctan(e*x/sqrt(d*e))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(d*e))
- 1/8*(5*(a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a*c^2*d*e^2 - 7*(a*c^3)^(3/
4)*c*d^2*e - 3*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c
)^(1/4))/(a/c)^(1/4))/(sqrt(2)*c^6*d^4 + 2*sqrt(2)*a*c^5*d^2*e^2 + sqrt(2)
*a^2*c^4*e^4) - 1/8*(5*(a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a*c^2*d*e^2 -
 7*(a*c^3)^(3/4)*c*d^2*e - 3*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x
- sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*c^6*d^4 + 2*sqrt(2)*a*c^5*d^2
*e^2 + sqrt(2)*a^2*c^4*e^4) - 1/16*(5*(a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4
)*a*c^2*d*e^2 + 7*(a*c^3)^(3/4)*c*d^2*e + 3*(a*c^3)^(3/4)*a*e^3)*log(x^2 +
 sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*c^6*d^4 + 2*sqrt(2)*a*c^5*d^2
*e^2 + sqrt(2)*a^2*c^4*e^4) + 1/16*(5*(a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4
)*a*c^2*d*e^2 + 7*(a*c^3)^(3/4)*c*d^2*e + 3*(a*c^3)^(3/4)*a*e^3)*log(x^2 -
 sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*c^6*d^4 + 2*sqrt(2)*a*c^5*d^2
*e^2 + sqrt(2)*a^2*c^4*e^4) - 1/4*(a*e*x^3 - a*d*x)/((c*x^4 + a)*(c^2*d^2
+ a*c*e^2))
```

3.252.9 Mupad [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 18343, normalized size of antiderivative = 25.76

$$\int \frac{x^8}{(d+ex^2)\,(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(x^8/((a + c*x^4)^2*(d + e*x^2)),x)
```

output
```
((a*d*x)/(4*c*(a*e^2 + c*d^2)) - (a*e*x^3)/(4*c*(a*e^2 + c*d^2)))/(a + c*x
^4) + atan((((5120*a^2*c^8*d^13*e + 432*a^8*c^2*d*e^13 - 17232*a^3*c^7*d^
11*e^3 - 37776*a^4*c^6*d^9*e^5 - 13600*a^5*c^5*d^7*e^7 + 4320*a^6*c^4*d^5*
e^9 + 2928*a^7*c^3*d^3*e^11)/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2
 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - (((81920*a^5*c^9*d^8*e^8 - 73
728*a^3*c^11*d^12*e^4 - 61440*a^4*c^10*d^10*e^6 - 20480*a^2*c^12*d^14*e^2
+ 184320*a^6*c^8*d^6*e^10 + 122880*a^7*c^7*d^4*e^12 + 28672*a^8*c^6*d^2*e^
14)/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*
a^3*c^4*d^2*e^6)) - (x*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1
/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4
*e^2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^
4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1
/2)*(65536*a^9*c^7*e^17 - 65536*a^2*c^14*d^14*e^3 - 327680*a^3*c^13*d^12*e
^5 - 589824*a^4*c^12*d^10*e^7 - 327680*a^5*c^11*d^8*e^9 + 327680*a^6*c^10*
d^6*e^11 + 589824*a^7*c^9*d^4*e^13 + 327680*a^8*c^8*d^2*e^15))/(128*(c^7*d
^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6
)))*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e
^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2)
 - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^1
0*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2) + (x*(1920*a...
```

3.253 $\int \frac{x^6}{(d+ex^2)(a+cx^4)^2}\,dx$

3.253.1 Optimal result

Integrand size = 22, antiderivative size = 687

$$\int \frac{x^6}{(d+ex^2)(a+cx^4)^2}\,dx = -\frac{x(ae+cdx^2)}{4c(cd^2+ae^2)(a+cx^4)} - \frac{d^{5/2}\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2}$$
$$- \frac{d^2(\sqrt{c}d+\sqrt{a}e)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}(cd^2+ae^2)^2}$$
$$+ \frac{(\sqrt{c}d-\sqrt{a}e)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\sqrt[4]{a}c^{5/4}(cd^2+ae^2)}$$
$$+ \frac{d^2(\sqrt{c}d+\sqrt{a}e)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}(cd^2+ae^2)^2}$$
$$- \frac{(\sqrt{c}d-\sqrt{a}e)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\sqrt[4]{a}c^{5/4}(cd^2+ae^2)}$$
$$+ \frac{d^2(\sqrt{c}d-\sqrt{a}e)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}(cd^2+ae^2)^2}$$
$$- \frac{(\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}\sqrt[4]{a}c^{5/4}(cd^2+ae^2)}$$
$$- \frac{d^2(\sqrt{c}d-\sqrt{a}e)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}(cd^2+ae^2)^2}$$
$$+ \frac{(\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}\sqrt[4]{a}c^{5/4}(cd^2+ae^2)}$$

output
```
-1/4*x*(c*d*x^2+a*e)/c/(a*e^2+c*d^2)/(c*x^4+a)-1/16*arctan(-1+c^(1/4)*x*2^
(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/a^(1/4)/c^(5/4)/(a*e^2+c*d^2)*2^(1/2
)-1/16*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/a^(1/4)/
c^(5/4)/(a*e^2+c*d^2)*2^(1/2)+1/8*d^2*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2
)+x^2*c^(1/2))*(-e*a^(1/2)+d*c^(1/2))/a^(1/4)/c^(1/4)/(a*e^2+c*d^2)^2*2^(1
/2)-1/8*d^2*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e*a^(1/2)+
d*c^(1/2))/a^(1/4)/c^(1/4)/(a*e^2+c*d^2)^2*2^(1/2)+1/4*d^2*arctan(-1+c^(1/
4)*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+d*c^(1/2))/a^(1/4)/c^(1/4)/(a*e^2+c*d^2)^
2*2^(1/2)+1/4*d^2*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+d*c^(1/2)
)/a^(1/4)/c^(1/4)/(a*e^2+c*d^2)^2*2^(1/2)-1/32*ln(-a^(1/4)*c^(1/4)*x*2^(1/
2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/a^(1/4)/c^(5/4)/(a*e^2+c*d^2
)*2^(1/2)+1/32*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2
)+d*c^(1/2))/a^(1/4)/c^(5/4)/(a*e^2+c*d^2)*2^(1/2)-d^(5/2)*arctan(x*e^(1/2
)/d^(1/2))*e^(1/2)/(a*e^2+c*d^2)^2
```

3.253.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 428, normalized size of antiderivative = 0.62

$$\int \frac{x^6}{(d+ex^2)(a+cx^4)^2}\,dx =$$

$$-\frac{\frac{8(cd^2+ae^2)(aex+cdx^3)}{c(a+cx^4)} + 32d^{5/2}\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) + \frac{2\sqrt{2}(3c^{3/2}d^3+5\sqrt{a}cd^2e-a\sqrt{c}de^2+a^{3/2}e^3)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}c^{5/4}} - \frac{2\sqrt{2}(}{}}{}$$

input
```
Integrate[x^6/((d + e*x^2)*(a + c*x^4)^2),x]
```

output
```
-1/32*((8*(c*d^2 + a*e^2)*(a*e*x + c*d*x^3))/(c*(a + c*x^4)) + 32*d^(5/2)*
Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + (2*Sqrt[2]*(3*c^(3/2)*d^3 + 5*Sqrt[a
]*c*d^2*e - a*Sqrt[c]*d*e^2 + a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/
a^(1/4)])/(a^(1/4)*c^(5/4)) - (2*Sqrt[2]*(3*c^(3/2)*d^3 + 5*Sqrt[a]*c*d^2*
e - a*Sqrt[c]*d*e^2 + a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]
)/(a^(1/4)*c^(5/4)) + (Sqrt[2]*(-3*c^(3/2)*d^3 + 5*Sqrt[a]*c*d^2*e + a*Sqr
t[c]*d*e^2 + a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c
]*x^2])/(a^(1/4)*c^(5/4)) - (Sqrt[2]*(-3*c^(3/2)*d^3 + 5*Sqrt[a]*c*d^2*e +
 a*Sqrt[c]*d*e^2 + a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x +
Sqrt[c]*x^2])/(a^(1/4)*c^(5/4)))/(c*d^2 + a*e^2)^2
```

3.253.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 641, normalized size of antiderivative = 0.93, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {1651, 1599, 25, 1482, 27, 1476, 1082, 217, 1479, 25, 27, 1103, 1611, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+cx^4)^2(d+ex^2)}dx$$

$$\Big\downarrow \text{1651}$$

$$\frac{d^2\int \frac{x^2}{(ex^2+d)(cx^4+a)}dx}{ae^2+cd^2}-\frac{a\int \frac{x^2(d-ex^2)}{(cx^4+a)^2}dx}{ae^2+cd^2}$$

$$\Big\downarrow \text{1599}$$

$$\frac{d^2\int \frac{x^2}{(ex^2+d)(cx^4+a)}dx}{ae^2+cd^2}-\frac{a\left(\frac{\int -\frac{ae-cdx^2}{cx^4+a}dx}{4ac}+\frac{x(ae+cdx^2)}{4ac(a+cx^4)}\right)}{ae^2+cd^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{d^2\int \frac{x^2}{(ex^2+d)(cx^4+a)}dx}{ae^2+cd^2}-\frac{a\left(\frac{x(ae+cdx^2)}{4ac(a+cx^4)}-\frac{\int \frac{ae-cdx^2}{cx^4+a}dx}{4ac}\right)}{ae^2+cd^2}$$

$$\Big\downarrow \text{1482}$$

$$\frac{d^2\int \frac{x^2}{(ex^2+d)(cx^4+a)}dx}{ae^2+cd^2}-\frac{a\left(\frac{x(ae+cdx^2)}{4ac(a+cx^4)}-\frac{\frac{1}{2}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\int \frac{\sqrt{c}\left(\sqrt{a}-\sqrt{c}x^2\right)}{cx^4+a}dx-\frac{1}{2}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{\sqrt{c}\left(\sqrt{c}x^2+\sqrt{a}\right)}{cx^4+a}dx}{4ac}\right)}{ae^2+cd^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^2\int \frac{x^2}{(ex^2+d)(cx^4+a)}dx}{ae^2+cd^2}-\frac{a\left(\frac{x(ae+cdx^2)}{4ac(a+cx^4)}-\frac{\frac{1}{2}\sqrt{c}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx-\frac{1}{2}\sqrt{c}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{4ac}\right)}{ae^2+cd^2}$$

$$\Big\downarrow \text{1476}$$

$$\frac{d^2\int \frac{x^2}{(ex^2+d)(cx^4+a)}dx}{ae^2+cd^2} - \frac{a\left(\frac{x(ae+cdx^2)}{4ac(a+cx^4)} - \frac{\frac{1}{2}\sqrt{c}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\int\frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx - \frac{1}{2}\sqrt{c}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\left(\frac{\int\frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int\frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}\right)}{4ac}\right)}{ae^2+cd^2}$$

$$\Big\downarrow \; 1082$$

$$\frac{d^2\int \frac{x^2}{(ex^2+d)(cx^4+a)}dx}{ae^2+cd^2} - \frac{a\left(\frac{x(ae+cdx^2)}{4ac(a+cx^4)} - \frac{\frac{1}{2}\sqrt{c}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\int\frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx - \frac{1}{2}\sqrt{c}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\left(\frac{\int\frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int\frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+\right.}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right.}{4ac}\right.}{ae^2+cd^2}$$

$$\Big\downarrow \; 217$$

$$\frac{d^2\int \frac{x^2}{(ex^2+d)(cx^4+a)}dx}{ae^2+cd^2} - \frac{a\left(\frac{x(ae+cdx^2)}{4ac(a+cx^4)} - \frac{\frac{1}{2}\sqrt{c}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\int\frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx - \frac{1}{2}\sqrt{c}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)}{4ac}\right)}{ae^2+cd^2}$$

$$\Big\downarrow \; 1479$$

3.253. $\int \frac{x^6}{(d+ex^2)(a+cx^4)^2}\,dx$

$$\frac{d^2\int \frac{x^2}{(ex^2+d)(cx^4+a)}dx}{ae^2+cd^2} - \frac{a\left(\frac{x(ae+cdx^2)}{4ac(a+cx^4)} - \frac{\frac{1}{2}\sqrt{c}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}} - \frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right) - \frac{1}{2}\sqrt{c}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}} - \frac{\arctan \cdots}{\cdots}\right.\cdots}{4ac}\right.\cdots}{ae^2+cd^2}$$

$\Big\downarrow$ 25

$$\frac{d^2\int \frac{x^2}{(ex^2+d)(cx^4+a)}dx}{ae^2+cd^2} - \frac{a\left(\frac{x(ae+cdx^2)}{4ac(a+cx^4)} - \frac{\frac{1}{2}\sqrt{c}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}} + \frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right) - \frac{1}{2}\sqrt{c}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}} - \frac{\arctan\left(1-\cdots\right)}{\sqrt{2}\sqrt[4]{\cdots}}\right.\cdots}{4ac}\right.\cdots}{ae^2+cd^2}$$

$\Big\downarrow$ 27

3.253. $\int \frac{x^6}{(d+ex^2)(a+cx^4)^2}\,dx$

$$\frac{d^2\int \frac{x^2}{(ex^2+d)(cx^4+a)}dx}{ae^2+cd^2}-\frac{a\left(\frac{x(ae+cdx^2)}{4ac(a+cx^4)}-\frac{\frac{1}{2}\sqrt{c}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}\right)-\frac{1}{2}\sqrt{c}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(d\cdots\right)}{4ac}\right)}{ae^2+cd^2}$$

$\downarrow$ 1103

$$\frac{d^2\int \frac{x^2}{(ex^2+d)(cx^4+a)}dx}{ae^2+cd^2}-\frac{a\left(\frac{x(ae+cdx^2)}{4ac(a+cx^4)}-\frac{\frac{1}{2}\sqrt{c}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)-\frac{1}{2}\sqrt{c}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\cdots\right)}{\sqrt{2}\sqrt[4]{a}\cdots}\right)}{4ac}\right)}{ae^2+cd^2}$$

$\downarrow$ 1611

$$\frac{d^2\int \left(\frac{cdx^2+ae}{(cd^2+ae^2)(cx^4+a)}-\frac{de}{(cd^2+ae^2)(ex^2+d)}\right)dx}{ae^2+cd^2}-\frac{a\left(\frac{x(ae+cdx^2)}{4ac(a+cx^4)}-\frac{\frac{1}{2}\sqrt{c}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)-\frac{1}{2}\sqrt{c}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\cdots\right)}{\sqrt{2}\sqrt[4]{a}\cdots}\right)}{4ac}\right)}{ae^2+cd^2}$$

$\downarrow$ 2009

3.253. $\int \frac{x^6}{(d+ex^2)(a+cx^4)^2}\,dx$

$$\frac{d^2\left(-\frac{\sqrt{d}\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{ae^2+cd^2}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e+\sqrt{c}d\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}(ae^2+cd^2)}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}e+\sqrt{c}d\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}(ae^2+cd^2)}+\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\cdots\right.}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}(ae^2+cd^2)}\cdots}{ae^2+cd^2}$$

$$\frac{a\left(\frac{x(ae+cdx^2)}{4ac(a+cx^4)}-\frac{\frac{1}{2}\sqrt{c}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)-\frac{1}{2}\sqrt{c}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\cdots\right.}{\sqrt{2}\sqrt[4]{a}\cdots}\cdots\right.}{4ac}\cdots\right.}{ae^2+cd^2}$$

input
```
Int[x^6/((d + e*x^2)*(a + c*x^4)^2),x]
```

output
```
(d^2*(-((Sqrt[d]*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)) - (
(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2
]*a^(1/4)*c^(1/4)*(c*d^2 + a*e^2)) + ((Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 + (
Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*c^(1/4)*(c*d^2 + a*e^2)) +
 ((Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c
]*x^2])/(4*Sqrt[2]*a^(1/4)*c^(1/4)*(c*d^2 + a*e^2)) - ((Sqrt[c]*d - Sqrt[a
]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^
(1/4)*c^(1/4)*(c*d^2 + a*e^2))))/(c*d^2 + a*e^2) - (a*((x*(a*e + c*d*x^2))
/(4*a*c*(a + c*x^4)) - (-1/2*(Sqrt[c]*(d - (Sqrt[a]*e)/Sqrt[c])*(-(ArcTan[
1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4))) + ArcTan[1 + (
Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4)))) + (Sqrt[c]*(d + (S
qrt[a]*e)/Sqrt[c])*(-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]
*x^2]/(Sqrt[2]*a^(1/4)*c^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x
+ Sqrt[c]*x^2]/(2*Sqrt[2]*a^(1/4)*c^(1/4))))/2)/(4*a*c))/(c*d^2 + a*e^2)
```

3.253.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 1599
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1)*((a*e - c*d*x^2)/(4*a*
c*(p + 1))), x] - Simp[f^2/(4*a*c*(p + 1))   Int[(f*x)^(m - 2)*(a + c*x^4)^
(p + 1)*(a*e*(m - 1) - c*d*(4*p + 4 + m + 1)*x^2), x], x] /; FreeQ[{a, c, d
, e, f}, x] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

3.253. $\int \frac{x^6}{(d+ex^2)(a+cx^4)^2} \, dx$

rule 1611
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a + c*x^4)), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

rule 1651
```
Int[(((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^4)^(p_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-a)*(f^4/(c*d^2 + a*e^2))   Int[(f*x)^(m - 4)*(d - e*x^2
)*(a + c*x^4)^p, x], x] + Simp[d^2*(f^4/(c*d^2 + a*e^2))   Int[(f*x)^(m - 4
)*((a + c*x^4)^(p + 1)/(d + e*x^2)), x], x] /; FreeQ[{a, c, d, e, f}, x] &&
 LtQ[p, -1] && GtQ[m, 2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.253.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 339, normalized size of antiderivative = 0.49

method	result
default	$\frac{\frac{\left(-\frac{1}{4} d\, e^{2} a-\frac{1}{4} d^{3} c\right) x^{3}-\frac{a e \left(a\, e^{2}+c\, d^{2}\right) x}{4 c}}{c\, x^{4}+a}+\frac{\frac{\left(e^{3} a^{2}+5 a c\, d^{2} e\right) \left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2}\, \left(\ln \left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2 \arctan \left(\frac{\sqrt{2}\, x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2 \arctan \left(\frac{\sqrt{2}\, x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8 a}}{4c}}{\left(a\, e^{2}+c\, d^{2}\right)^{2}}$
risch	Expression too large to display

input
```
int(x^6/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/(a*e^2+c*d^2)^2*(((-1/4*d*e^2*a-1/4*d^3*c)*x^3-1/4*a*e*(a*e^2+c*d^2)/c*x
)/(c*x^4+a)+1/4/c*(1/8*(a^2*e^3+5*a*c*d^2*e)*(a/c)^(1/4)/a*2^(1/2)*(ln((x^
2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2
)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1
/8*(-a*c*d*e^2+3*c^2*d^3)/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(
1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2
)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))))-e/(a*e^2+c*d^2)^2*
d^3/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2))
```

3.253.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4901 vs. 2(515) = 1030.

Time = 6.61 (sec) , antiderivative size = 9822, normalized size of antiderivative = 14.30

$$\int \frac{x^6}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^6/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.253.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^6}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**6/(e*x**2+d)/(c*x**4+a)**2,x)
```

output
```
Timed out
```

3.253.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^6}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^6/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.253.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 614, normalized size of antiderivative = 0.89

$$\int \frac{x^6}{(d+ex^2)(a+cx^4)^2}\,dx = -\frac{d^3e\arctan\left(\frac{ex}{\sqrt{de}}\right)}{(c^2d^4+2\,acd^2e^2+a^2e^4)\sqrt{de}} + \frac{\left(5\,(ac^3)^{\frac14}\,ac^2d^2e+(ac^3)^{\frac14}\,a^2ce^3+3\,(ac^3)^{\frac34}\,cd^3-(ac^3)^{\frac34}\,ade^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac14}\right)}{2\left(\frac{a}{c}\right)^{\frac14}}\right)}{8\left(\sqrt{2}ac^5d^4+2\,\sqrt{2}a^2c^4d^2e^2+\sqrt{2}a^3c^3e^4\right)} + \frac{\left(5\,(ac^3)^{\frac14}\,ac^2d^2e+(ac^3)^{\frac14}\,a^2ce^3+3\,(ac^3)^{\frac34}\,cd^3-(ac^3)^{\frac34}\,ade^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac14}\right)}{2\left(\frac{a}{c}\right)^{\frac14}}\right)}{8\left(\sqrt{2}ac^5d^4+2\,\sqrt{2}a^2c^4d^2e^2+\sqrt{2}a^3c^3e^4\right)} + \frac{\left(5\,(ac^3)^{\frac14}\,ac^2d^2e+(ac^3)^{\frac14}\,a^2ce^3-3\,(ac^3)^{\frac34}\,cd^3+(ac^3)^{\frac34}\,ade^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac14}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}ac^5d^4+2\,\sqrt{2}a^2c^4d^2e^2+\sqrt{2}a^3c^3e^4\right)} - \frac{\left(5\,(ac^3)^{\frac14}\,ac^2d^2e+(ac^3)^{\frac14}\,a^2ce^3-3\,(ac^3)^{\frac34}\,cd^3+(ac^3)^{\frac34}\,ade^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac14}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}ac^5d^4+2\,\sqrt{2}a^2c^4d^2e^2+\sqrt{2}a^3c^3e^4\right)} - \frac{cdx^3+aex}{4\,(cx^4+a)(c^2d^2+ace^2)}$$

input
```
integrate(x^6/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
-d^3*e*arctan(e*x/sqrt(d*e))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(d*e
)) + 1/8*(5*(a*c^3)^(1/4)*a*c^2*d^2*e + (a*c^3)^(1/4)*a^2*c*e^3 + 3*(a*c^3
)^(3/4)*c*d^3 - (a*c^3)^(3/4)*a*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(
a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2*c^4*d^2*e^2 +
sqrt(2)*a^3*c^3*e^4) + 1/8*(5*(a*c^3)^(1/4)*a*c^2*d^2*e + (a*c^3)^(1/4)*a^
2*c*e^3 + 3*(a*c^3)^(3/4)*c*d^3 - (a*c^3)^(3/4)*a*d*e^2)*arctan(1/2*sqrt(2
)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*
a^2*c^4*d^2*e^2 + sqrt(2)*a^3*c^3*e^4) + 1/16*(5*(a*c^3)^(1/4)*a*c^2*d^2*e
 + (a*c^3)^(1/4)*a^2*c*e^3 - 3*(a*c^3)^(3/4)*c*d^3 + (a*c^3)^(3/4)*a*d*e^2
)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^5*d^4 + 2*sqrt
(2)*a^2*c^4*d^2*e^2 + sqrt(2)*a^3*c^3*e^4) - 1/16*(5*(a*c^3)^(1/4)*a*c^2*d
^2*e + (a*c^3)^(1/4)*a^2*c*e^3 - 3*(a*c^3)^(3/4)*c*d^3 + (a*c^3)^(3/4)*a*d
*e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^5*d^4 + 2*
sqrt(2)*a^2*c^4*d^2*e^2 + sqrt(2)*a^3*c^3*e^4) - 1/4*(c*d*x^3 + a*e*x)/((c
*x^4 + a)*(c^2*d^2 + a*c*e^2))
```

3.253.9 Mupad [B] (verification not implemented)

Time = 9.02 (sec) , antiderivative size = 17909, normalized size of antiderivative = 26.07

$$\int \frac{x^6}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(x^6/((a + c*x^4)^2*(d + e*x^2)),x)
```

output
```
atan(((((432*a*c^7*d^12*e^2 + 13040*a^2*c^6*d^10*e^4 + 12000*a^3*c^5*d^8*e
^6 - 1056*a^4*c^4*d^6*e^8 - 400*a^5*c^3*d^4*e^10 + 48*a^6*c^2*d^2*e^12)/(2
56*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*
d^2*e^6)) + (((45056*a^2*c^10*d^13*e^3 - 4096*a^8*c^4*d*e^15 + 221184*a^3*
c^9*d^11*e^5 + 430080*a^4*c^8*d^9*e^7 + 409600*a^5*c^7*d^7*e^9 + 184320*a^
6*c^6*d^5*e^11 + 24576*a^7*c^5*d^3*e^13)/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c
^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) - (x*(-(a^3*e^6*(-a*c
^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3
 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*
c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*
d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2)*(65536*a^9*c^5*e^17 - 65536*a^2*c^12*
d^14*e^3 - 327680*a^3*c^11*d^12*e^5 - 589824*a^4*c^10*d^10*e^7 - 327680*a^
5*c^9*d^8*e^9 + 327680*a^6*c^8*d^6*e^11 + 589824*a^7*c^7*d^4*e^13 + 327680
*a^8*c^6*d^2*e^15))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^
3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a
*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*
c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d
^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e
^6)))^(1/2) + (x*(1152*a*c^9*d^13*e^2 + 1152*a^7*c^3*d*e^14 + 21248*a^2*c^
8*d^11*e^4 + 25472*a^3*c^7*d^9*e^6 - 5632*a^4*c^6*d^7*e^8 - 7296*a^5*c^...
```

3.254 $\int \frac{x^4}{(d+ex^2)(a+cx^4)^2} \, dx$

3.254.1 Optimal result

Integrand size = 22, antiderivative size = 685

$$\int \frac{x^4}{(d+ex^2)(a+cx^4)^2}\,dx = -\frac{x(d-ex^2)}{4(cd^2+ae^2)(a+cx^4)} + \frac{d^{3/2}e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} - \frac{\sqrt[4]{c}d^2(\sqrt{c}d-\sqrt{a}e)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(cd^2+ae^2)^2} + \frac{(3\sqrt{c}d-\sqrt{a}e)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{3/4}c^{3/4}(cd^2+ae^2)} + \frac{\sqrt[4]{c}d^2(\sqrt{c}d-\sqrt{a}e)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(cd^2+ae^2)^2} - \frac{(3\sqrt{c}d-\sqrt{a}e)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{3/4}c^{3/4}(cd^2+ae^2)} - \frac{\sqrt[4]{c}d^2(\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(cd^2+ae^2)^2} + \frac{(3\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{3/4}c^{3/4}(cd^2+ae^2)} + \frac{\sqrt[4]{c}d^2(\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(cd^2+ae^2)^2} - \frac{(3\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{3/4}c^{3/4}(cd^2+ae^2)}$$

output
```
-1/4*x*(-e*x^2+d)/(a*e^2+c*d^2)/(c*x^4+a)+d^(3/2)*e^(3/2)*arctan(x*e^(1/2)
/d^(1/2))/(a*e^2+c*d^2)^2+1/4*c^(1/4)*d^2*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1
/4))*(-e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)+1/4*c^(1/4)*d^
2*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^
2+c*d^2)^2*2^(1/2)-1/8*c^(1/4)*d^2*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x
^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)+1/8*c^(1
/4)*d^2*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(
1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)-1/16*arctan(-1+c^(1/4)*x*2^(1/2)/a^(
1/4))*(-e*a^(1/2)+3*d*c^(1/2))/a^(3/4)/c^(3/4)/(a*e^2+c*d^2)*2^(1/2)-1/16*
arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+3*d*c^(1/2))/a^(3/4)/c^(3/
4)/(a*e^2+c*d^2)*2^(1/2)+1/32*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^
(1/2))*(e*a^(1/2)+3*d*c^(1/2))/a^(3/4)/c^(3/4)/(a*e^2+c*d^2)*2^(1/2)-1/32*
ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+3*d*c^(1/2))/
a^(3/4)/c^(3/4)/(a*e^2+c*d^2)*2^(1/2)
```

3.254.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 423, normalized size of antiderivative = 0.62

$$\int \frac{x^4}{(d+ex^2)(a+cx^4)^2}\,dx$$

$$= \frac{\frac{8(cd^2+ae^2)(-dx+ex^3)}{a+cx^4} + 32d^{3/2}e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) - \frac{2\sqrt{2}\left(c^{3/2}d^3-3\sqrt{a}cd^2e-3a\sqrt{c}de^2+a^{3/2}e^3\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{3/4}c^{3/4}} + \frac{2\sqrt{2}(c^{3/}\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[x^4/((d + e*x^2)*(a + c*x^4)^2),x]
```

output
```
((8*(c*d^2 + a*e^2)*(-(d*x) + e*x^3))/(a + c*x^4) + 32*d^(3/2)*e^(3/2)*Arc
Tan[(Sqrt[e]*x)/Sqrt[d]] - (2*Sqrt[2]*(c^(3/2)*d^3 - 3*Sqrt[a]*c*d^2*e - 3
*a*Sqrt[c]*d*e^2 + a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(
a^(3/4)*c^(3/4)) + (2*Sqrt[2]*(c^(3/2)*d^3 - 3*Sqrt[a]*c*d^2*e - 3*a*Sqrt[
c]*d*e^2 + a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(3/4)*
c^(3/4)) + (Sqrt[2]*(-(c^(3/2)*d^3) - 3*Sqrt[a]*c*d^2*e + 3*a*Sqrt[c]*d*e^
2 + a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(
a^(3/4)*c^(3/4)) + (Sqrt[2]*(c^(3/2)*d^3 + 3*Sqrt[a]*c*d^2*e - 3*a*Sqrt[c]
*d*e^2 - a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(a^(3/4)*c^(3/4)))/(32*(c*d^2 + a*e^2)^2)
```

3.254.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 635, normalized size of antiderivative = 0.93, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {1651, 1485, 1493, 25, 1482, 27, 1476, 1082, 217, 1479, 25, 27, 1103, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+cx^4)^2(d+ex^2)}dx$$

$$\Big\downarrow \text{ 1651}$$

$$\frac{d^2\int \frac{1}{(ex^2+d)(cx^4+a)}dx}{ae^2+cd^2}-\frac{a\int \frac{d-ex^2}{(cx^4+a)^2}dx}{ae^2+cd^2}$$

$$\Big\downarrow \text{ 1485}$$

$$\frac{d^2\int \left(\frac{e^2}{(cd^2+ae^2)(ex^2+d)}+\frac{c(d-ex^2)}{(cd^2+ae^2)(cx^4+a)}\right)dx}{ae^2+cd^2}-\frac{a\int \frac{d-ex^2}{(cx^4+a)^2}dx}{ae^2+cd^2}$$

$$\Big\downarrow \text{ 1493}$$

$$\frac{d^2\int \left(\frac{e^2}{(cd^2+ae^2)(ex^2+d)}+\frac{c(d-ex^2)}{(cd^2+ae^2)(cx^4+a)}\right)dx}{ae^2+cd^2}-\frac{a\left(\frac{x(d-ex^2)}{4a(a+cx^4)}-\frac{\int -\frac{3d-ex^2}{cx^4+a}dx}{4a}\right)}{ae^2+cd^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{d^2\int \left(\frac{e^2}{(cd^2+ae^2)(ex^2+d)}+\frac{c(d-ex^2)}{(cd^2+ae^2)(cx^4+a)}\right)dx}{ae^2+cd^2}-\frac{a\left(\frac{\int \frac{3d-ex^2}{cx^4+a}dx}{4a}+\frac{x(d-ex^2)}{4a(a+cx^4)}\right)}{ae^2+cd^2}$$

$$\Big\downarrow \text{ 1482}$$

$$\frac{d^2\int \left(\frac{e^2}{(cd^2+ae^2)(ex^2+d)}+\frac{c(d-ex^2)}{(cd^2+ae^2)(cx^4+a)}\right)dx}{ae^2+cd^2}-\frac{a\left(\frac{\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}+e\right)\int \frac{\sqrt{c}\left(\sqrt{a}-\sqrt{c}x^2\right)}{cx^4+a}dx}{2c}+\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{\sqrt{c}\left(\sqrt{c}x^2+\sqrt{a}\right)}{cx^4+a}dx}{2c}}{4a}+\frac{x(d-ex^2)}{4a(a+cx^4)}\right)}{ae^2+cd^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{d^2\int\left(\frac{e^2}{(cd^2+ae^2)(ex^2+d)}+\frac{c(d-ex^2)}{(cd^2+ae^2)(cx^4+a)}\right)dx}{ae^2+cd^2}-\frac{a\left(\frac{\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}+e\right)\int\frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}+\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)\int\frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{c}}}{4a}+\frac{x(d-ex^2)}{4a(a+cx^4)}\right)}{ae^2+cd^2}$$

$\Big\downarrow$ 1476

$$\frac{d^2\int\left(\frac{e^2}{(cd^2+ae^2)(ex^2+d)}+\frac{c(d-ex^2)}{(cd^2+ae^2)(cx^4+a)}\right)dx}{ae^2+cd^2}-\frac{a\left(\frac{\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)\left(\frac{\int\frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int\frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}+e\right)\int\frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}}{4a}+\frac{x(d-ex^2)}{4a(a+cx^4)}\right)}{ae^2+cd^2}$$

$\Big\downarrow$ 1082

$$\frac{d^2\int\left(\frac{e^2}{(cd^2+ae^2)(ex^2+d)}+\frac{c(d-ex^2)}{(cd^2+ae^2)(cx^4+a)}\right)dx}{ae^2+cd^2}-\frac{a\left(\frac{\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}+e\right)\int\frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}+\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)\left(\frac{\int\frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int\frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)}{2\sqrt{c}}}{4a}+\frac{x(d-ex^2)}{4a(a+cx^4)}\right)}{ae^2+cd^2}$$

$\Big\downarrow$ 217

3.254. $\int\frac{x^4}{(d+ex^2)(a+cx^4)^2}\,dx$

$$\frac{d^2\int\left(\frac{e^2}{(cd^2+ae^2)(ex^2+d)}+\frac{c(d-ex^2)}{(cd^2+ae^2)(cx^4+a)}\right)dx}{ae^2+cd^2}-\frac{a\left(\frac{\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}+e\right)\int\frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)}{2\sqrt{c}}}{4a}+\frac{x(d-ex^2)}{4a(a+cx^4)}\right)}{ae^2+cd^2}$$

$\downarrow$ 1479

$$\frac{d^2\int\left(\frac{e^2}{(cd^2+ae^2)(ex^2+d)}+\frac{c(d-ex^2)}{(cd^2+ae^2)(cx^4+a)}\right)dx}{ae^2+cd^2}-\frac{a\left(\frac{\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}+e\right)\left(-\frac{\int-\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)\cdots}{2\sqrt{c}}}{4a}\cdots\right)}{ae^2+cd^2}$$

$\downarrow$ 25

3.254. $\int\frac{x^4}{(d+ex^2)(a+cx^4)^2}\,dx$

$$\frac{d^2\int\left(\frac{e^2}{(cd^2+ae^2)(ex^2+d)}+\frac{c(d-ex^2)}{(cd^2+ae^2)(cx^4+a)}\right)dx}{ae^2+cd^2}-\frac{a\left(\frac{\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}+e\right)\left(\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)}{2\sqrt{c}}}{4a}+\right.}{ae^2+cd^2}$$

$\Big\downarrow$ 27

$$\frac{d^2\int\left(\frac{e^2}{(cd^2+ae^2)(ex^2+d)}+\frac{c(d-ex^2)}{(cd^2+ae^2)(cx^4+a)}\right)dx}{ae^2+cd^2}-\frac{a\left(\frac{\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}+e\right)\left(\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)}{2\sqrt{c}}}{4a}+\frac{x(d-ex^2)}{4a(a+cx^4)}\right)}{ae^2+cd^2}$$

$\Big\downarrow$ 1103

3.254. $\int \frac{x^4}{(d+ex^2)(a+cx^4)^2}\,dx$

$$\frac{d^2\int\left(\frac{e^2}{(cd^2+ae^2)(ex^2+d)}+\frac{c(d-ex^2)}{(cd^2+ae^2)(cx^4+a)}\right)dx}{ae^2+cd^2}-$$

$$\frac{a\left(\frac{\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)}{2\sqrt{c}}+\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}+e\right)\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)}{2\sqrt{c}}}{4a}+\right.}{ae^2+cd^2}$$

$\downarrow$ 2009

$$\frac{d^2\left(-\frac{\sqrt[4]{c}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)(\sqrt{c}d-\sqrt{a}e)}{2\sqrt{2}a^{3/4}(ae^2+cd^2)}+\frac{\sqrt[4]{c}\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)(\sqrt{c}d-\sqrt{a}e)}{2\sqrt{2}a^{3/4}(ae^2+cd^2)}-\frac{\sqrt[4]{c}(\sqrt{a}e+\sqrt{c}d)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(ae^2+cd^2)}\right.}{ae^2+cd^2}$$

$$\frac{a\left(\frac{\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)}{2\sqrt{c}}+\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}+e\right)\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)}{2\sqrt{c}}}{4a}+\right.}{ae^2+cd^2}$$

input
```
Int[x^4/((d + e*x^2)*(a + c*x^4)^2),x]
```

3.254. $\int \frac{x^4}{(d+ex^2)(a+cx^4)^2}\,dx$

output
```
(d^2*((e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 + a*e^2)) - (c
^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2
*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTa
n[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)) -
(c^(1/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x +
 Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]*d +
 Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqr
t[2]*a^(3/4)*(c*d^2 + a*e^2))))/(c*d^2 + a*e^2) - (a*((x*(d - e*x^2))/(4*a
*(a + c*x^4)) + ((((3*Sqrt[c]*d)/Sqrt[a] - e)*(-(ArcTan[1 - (Sqrt[2]*c^(1/
4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*x)
/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4))))/(2*Sqrt[c]) + (((3*Sqrt[c]*d)/Sqrt[a
] + e)*(-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(Sqrt[
2]*a^(1/4)*c^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2]/(2*Sqrt[2]*a^(1/4)*c^(1/4))))/(2*Sqrt[c]))/(4*a)))/(c*d^2 + a*e^2)
```

3.254.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.254. $\int \frac{x^4}{(d+ex^2)(a+cx^4)^2}\,dx$

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 1485
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

rule 1493
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Simp[1/(4*a*(p + 1)
)   Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && Integer
Q[2*p]
```

rule 1651
```
Int[(((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^4)^(p_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-a)*(f^4/(c*d^2 + a*e^2))   Int[(f*x)^(m - 4)*(d - e*x^2
)*(a + c*x^4)^p, x], x] + Simp[d^2*(f^4/(c*d^2 + a*e^2))   Int[(f*x)^(m - 4
)*((a + c*x^4)^(p + 1)/(d + e*x^2)), x], x] /; FreeQ[{a, c, d, e, f}, x] &&
 LtQ[p, -1] && GtQ[m, 2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.254. $\int \frac{x^4}{(d+ex^2)(a+cx^4)^2}\,dx$

3.254.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.48

method	result
default	$-\frac{\frac{\left(-\frac{1}{4}a e^{3}-\frac{1}{4}c d^{2}e\right)x^{3}+\left(\frac{1}{4}d e^{2}a+\frac{1}{4}d^{3}c\right)x}{c x^{4}+a}+\frac{\left(3d e^{2}a-d^{3}c\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right.}{32a}}{\left(a e^{2}+c d^{2}\right)^{2}}$
risch	Expression too large to display

input
```
int(x^4/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/(a*e^2+c*d^2)^2*(((-1/4*a*e^3-1/4*c*d^2*e)*x^3+(1/4*d*e^2*a+1/4*d^3*c)*
x)/(c*x^4+a)+1/32*(3*a*d*e^2-c*d^3)*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(
1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arc
tan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/32*(-a*e
^3+3*c*d^2*e)/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(
1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4
)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1)))+d^2*e^2/(a*e^2+c*d^2)^2/(e*d)^(
1/2)*arctan(e*x/(e*d)^(1/2))
```

3.254.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4829 vs. 2(514) = 1028.

Time = 5.78 (sec) , antiderivative size = 9678, normalized size of antiderivative = 14.13

$$\int \frac{x^4}{(d+ex^2)\,(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.254.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**4/(e*x**2+d)/(c*x**4+a)**2,x)
```

output
```
Timed out
```

3.254.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.254.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 607, normalized size of antiderivative = 0.89

$$\int \frac{x^4}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{d^2e^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{(c^2d^4+2\,acd^2e^2+a^2e^4)\sqrt{de}}$$
$$+\frac{\left((ac^3)^{\frac14}c^3d^3-3\,(ac^3)^{\frac14}ac^2de^2-3\,(ac^3)^{\frac34}cd^2e+(ac^3)^{\frac34}ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac14}\right)}{2\left(\frac{a}{c}\right)^{\frac14}}\right)}{8\left(\sqrt{2}ac^5d^4+2\,\sqrt{2}a^2c^4d^2e^2+\sqrt{2}a^3c^3e^4\right)}$$
$$+\frac{\left((ac^3)^{\frac14}c^3d^3-3\,(ac^3)^{\frac14}ac^2de^2-3\,(ac^3)^{\frac34}cd^2e+(ac^3)^{\frac34}ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac14}\right)}{2\left(\frac{a}{c}\right)^{\frac14}}\right)}{8\left(\sqrt{2}ac^5d^4+2\,\sqrt{2}a^2c^4d^2e^2+\sqrt{2}a^3c^3e^4\right)}$$
$$+\frac{\left((ac^3)^{\frac14}c^3d^3-3\,(ac^3)^{\frac14}ac^2de^2+3\,(ac^3)^{\frac34}cd^2e-(ac^3)^{\frac34}ae^3\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac14}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}ac^5d^4+2\,\sqrt{2}a^2c^4d^2e^2+\sqrt{2}a^3c^3e^4\right)}$$
$$-\frac{\left((ac^3)^{\frac14}c^3d^3-3\,(ac^3)^{\frac14}ac^2de^2+3\,(ac^3)^{\frac34}cd^2e-(ac^3)^{\frac34}ae^3\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac14}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}ac^5d^4+2\,\sqrt{2}a^2c^4d^2e^2+\sqrt{2}a^3c^3e^4\right)}$$
$$+\frac{ex^3-dx}{4\,(cx^4+a)(cd^2+ae^2)}$$

input
```
integrate(x^4/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
d^2*e^2*arctan(e*x/sqrt(d*e))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(d*
e)) + 1/8*((a*c^3)^(1/4)*c^3*d^3 - 3*(a*c^3)^(1/4)*a*c^2*d*e^2 - 3*(a*c^3)
^(3/4)*c*d^2*e + (a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a
/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2*c^4*d^2*e^2 + s
qrt(2)*a^3*c^3*e^4) + 1/8*((a*c^3)^(1/4)*c^3*d^3 - 3*(a*c^3)^(1/4)*a*c^2*d
*e^2 - 3*(a*c^3)^(3/4)*c*d^2*e + (a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(
2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2
*c^4*d^2*e^2 + sqrt(2)*a^3*c^3*e^4) + 1/16*((a*c^3)^(1/4)*c^3*d^3 - 3*(a*c
^3)^(1/4)*a*c^2*d*e^2 + 3*(a*c^3)^(3/4)*c*d^2*e - (a*c^3)^(3/4)*a*e^3)*log
(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a
^2*c^4*d^2*e^2 + sqrt(2)*a^3*c^3*e^4) - 1/16*((a*c^3)^(1/4)*c^3*d^3 - 3*(a
*c^3)^(1/4)*a*c^2*d*e^2 + 3*(a*c^3)^(3/4)*c*d^2*e - (a*c^3)^(3/4)*a*e^3)*l
og(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)
*a^2*c^4*d^2*e^2 + sqrt(2)*a^3*c^3*e^4) + 1/4*(e*x^3 - d*x)/((c*x^4 + a)*(
c*d^2 + a*e^2))
```

3.254.9 Mupad [B] (verification not implemented)

Time = 10.46 (sec) , antiderivative size = 17180, normalized size of antiderivative = 25.08

$$\int \frac{x^4}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(x^4/((a + c*x^4)^2*(d + e*x^2)),x)
```

output
```
- atan((((((28672*a^2*c^8*d^10*e^4 - 4096*a*c^9*d^12*e^2 + 155648*a^3*c^7
*d^8*e^6 + 253952*a^4*c^6*d^6*e^8 + 176128*a^5*c^5*d^4*e^10 + 45056*a^6*c^
4*d^2*e^12)/(256*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4))
- (x*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5
*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1
/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 +
 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2)*(65536
*a^9*c^4*e^17 - 65536*a^2*c^11*d^14*e^3 - 327680*a^3*c^10*d^12*e^5 - 58982
4*a^4*c^9*d^10*e^7 - 327680*a^5*c^8*d^8*e^9 + 327680*a^6*c^7*d^6*e^11 + 58
9824*a^7*c^6*d^4*e^13 + 327680*a^8*c^5*d^2*e^15))/(128*(a^4*e^8 + c^4*d^8
+ 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((a^3*e^6*(-a^3
*c^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5
 - 20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e
^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 +
 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) + (x*(256*a*c^8*d^11*e^4 -
 128*c^9*d^13*e^2 + 2944*a^6*c^3*d*e^14 + 21632*a^2*c^7*d^9*e^6 + 32256*a^
3*c^6*d^7*e^8 + 4224*a^4*c^5*d^5*e^10 - 3840*a^5*c^4*d^3*e^12))/(128*(a^4*
e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*(
(a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6
*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2)...
```

3.255 $\int \frac{x^2}{(d+ex^2)(a+cx^4)^2}\,dx$

3.255.1 Optimal result

Integrand size = 22, antiderivative size = 685

$$\int \frac{x^2}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{x(ae+cdx^2)}{4a(cd^2+ae^2)(a+cx^4)} - \frac{\sqrt{d}e^{5/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} + \frac{\sqrt[4]{c}de(\sqrt{c}d-\sqrt{a}e)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(cd^2+ae^2)^2} - \frac{(\sqrt{c}d+3\sqrt{a}e)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{5/4}\sqrt[4]{c}(cd^2+ae^2)} - \frac{\sqrt[4]{c}de(\sqrt{c}d-\sqrt{a}e)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(cd^2+ae^2)^2} + \frac{(\sqrt{c}d+3\sqrt{a}e)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{5/4}\sqrt[4]{c}(cd^2+ae^2)} + \frac{\sqrt[4]{c}de(\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(cd^2+ae^2)^2} + \frac{(\sqrt{c}d-3\sqrt{a}e)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{5/4}\sqrt[4]{c}(cd^2+ae^2)} - \frac{\sqrt[4]{c}de(\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(cd^2+ae^2)^2} - \frac{(\sqrt{c}d-3\sqrt{a}e)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{5/4}\sqrt[4]{c}(cd^2+ae^2)}$$

output
```
1/4*x*(c*d*x^2+a*e)/a/(a*e^2+c*d^2)/(c*x^4+a)+1/32*ln(-a^(1/4)*c^(1/4)*x*2
^(1/2)+a^(1/2)+x^2*c^(1/2))*(-3*e*a^(1/2)+d*c^(1/2))/a^(5/4)/c^(1/4)/(a*e^
2+c*d^2)*2^(1/2)-1/32*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-
3*e*a^(1/2)+d*c^(1/2))/a^(5/4)/c^(1/4)/(a*e^2+c*d^2)*2^(1/2)-1/4*c^(1/4)*d
*e*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*
e^2+c*d^2)^2*2^(1/2)-1/4*c^(1/4)*d*e*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(
-e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)+1/8*c^(1/4)*d*e*ln(-
a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/a^(3/
4)/(a*e^2+c*d^2)^2*2^(1/2)-1/8*c^(1/4)*d*e*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^
(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)+1
/16*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*e*a^(1/2)+d*c^(1/2))/a^(5/4)/c
^(1/4)/(a*e^2+c*d^2)*2^(1/2)+1/16*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*e
*a^(1/2)+d*c^(1/2))/a^(5/4)/c^(1/4)/(a*e^2+c*d^2)*2^(1/2)-e^(5/2)*arctan(x
*e^(1/2)/d^(1/2))*d^(1/2)/(a*e^2+c*d^2)^2
```

3.255.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 428, normalized size of antiderivative = 0.62

$$\int \frac{x^2}{(d+ex^2)\,(a+cx^4)^2}\,dx$$

$$= \frac{\frac{8(cd^2+ae^2)(aex+cdx^3)}{a(a+cx^4)} - 32\sqrt{d}e^{5/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) - \frac{2\sqrt{2}(c^{3/2}d^3-\sqrt{a}cd^2e+5a\sqrt{c}de^2+3a^{3/2}e^3)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{5/4}\sqrt[4]{c}} + \frac{2\sqrt{2}(c^{3/}}{}}{}$$

input
```
Integrate[x^2/((d + e*x^2)*(a + c*x^4)^2),x]
```

output
```
((8*(c*d^2 + a*e^2)*(a*e*x + c*d*x^3))/(a*(a + c*x^4)) - 32*Sqrt[d]*e^(5/2
)*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - (2*Sqrt[2]*(c^(3/2)*d^3 - Sqrt[a]*c*d^2*e
+ 5*a*Sqrt[c]*d*e^2 + 3*a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4
)])/(a^(5/4)*c^(1/4)) + (2*Sqrt[2]*(c^(3/2)*d^3 - Sqrt[a]*c*d^2*e + 5*a*Sq
rt[c]*d*e^2 + 3*a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(
5/4)*c^(1/4)) + (Sqrt[2]*(c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + 5*a*Sqrt[c]*d*e^
2 - 3*a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])
/(a^(5/4)*c^(1/4)) - (Sqrt[2]*(c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + 5*a*Sqrt[c]
*d*e^2 - 3*a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*
x^2])/(a^(5/4)*c^(1/4)))/(32*(c*d^2 + a*e^2)^2)
```

3.255.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 633, normalized size of antiderivative = 0.92, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {1653, 1485, 1493, 25, 1482, 27, 1476, 1082, 217, 1479, 25, 27, 1103, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+cx^4)^2(d+ex^2)}dx$$

$$\Big\downarrow \ 1653$$

$$\frac{\int \frac{cdx^2+ae}{(cx^4+a)^2}dx}{ae^2+cd^2}-\frac{de\int \frac{1}{(ex^2+d)(cx^4+a)}dx}{ae^2+cd^2}$$

$$\Big\downarrow \ 1485$$

$$\frac{\int \frac{cdx^2+ae}{(cx^4+a)^2}dx}{ae^2+cd^2}-\frac{de\int \left(\frac{e^2}{(cd^2+ae^2)(ex^2+d)}+\frac{c(d-ex^2)}{(cd^2+ae^2)(cx^4+a)}\right)dx}{ae^2+cd^2}$$

$$\Big\downarrow \ 1493$$

$$\frac{\frac{x(ae+cdx^2)}{4a(a+cx^4)}-\frac{\int -\frac{cdx^2+3ae}{cx^4+a}dx}{4a}}{ae^2+cd^2}-\frac{de\int \left(\frac{e^2}{(cd^2+ae^2)(ex^2+d)}+\frac{c(d-ex^2)}{(cd^2+ae^2)(cx^4+a)}\right)dx}{ae^2+cd^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{cdx^2+3ae}{cx^4+a}dx}{4a}+\frac{x(ae+cdx^2)}{4a(a+cx^4)}}{ae^2+cd^2}-\frac{de\int \left(\frac{e^2}{(cd^2+ae^2)(ex^2+d)}+\frac{c(d-ex^2)}{(cd^2+ae^2)(cx^4+a)}\right)dx}{ae^2+cd^2}$$

$$\Big\downarrow \ 1482$$

$$\frac{\frac{\frac{1}{2}\left(\frac{3\sqrt{a}e}{\sqrt{c}}+d\right)\int \frac{\sqrt{c}\left(\sqrt{c}x^2+\sqrt{a}\right)}{cx^4+a}dx-\frac{1}{2}\left(d-\frac{3\sqrt{a}e}{\sqrt{c}}\right)\int \frac{\sqrt{c}\left(\sqrt{a}-\sqrt{c}x^2\right)}{cx^4+a}dx}{4a}+\frac{x(ae+cdx^2)}{4a(a+cx^4)}}{ae^2+cd^2}-\frac{de\int \left(\frac{e^2}{(cd^2+ae^2)(ex^2+d)}+\frac{c(d-ex^2)}{(cd^2+ae^2)(cx^4+a)}\right)dx}{ae^2+cd^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{1}{2}\sqrt{c}\left(\frac{3\sqrt{a}e}{\sqrt{c}}+d\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx-\frac{1}{2}\sqrt{c}\left(d-\frac{3\sqrt{a}e}{\sqrt{c}}\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{4a}+\frac{x(ae+cdx^2)}{4a(a+cx^4)}}{ae^2+cd^2}-\frac{de\int \left(\frac{e^2}{(cd^2+ae^2)(ex^2+d)}+\frac{c(d-ex^2)}{(cd^2+ae^2)(cx^4+a)}\right)dx}{ae^2+cd^2}$$

$$\Big\downarrow \ 1476$$

$$\frac{\frac{\frac{1}{2}\sqrt{c}\left(\frac{3\sqrt{a}e}{\sqrt{c}}+d\right)\left(\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}\right)-\frac{1}{2}\sqrt{c}\left(d-\frac{3\sqrt{a}e}{\sqrt{c}}\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{4a}+\frac{x(ae+cdx^2)}{4a(a+cx^4)}}{ae^2+cd^2}-\frac{de\int \left(\frac{e^2}{(cd^2+ae^2)(ex^2+d)}+\frac{c(d-ex^2)}{(cd^2+ae^2)(cx^4+a)}\right)dx}{ae^2+cd^2}$$

$$\Big\downarrow \ 1082$$

$$\frac{\frac{\frac{1}{2}\sqrt{c}\left(\frac{3\sqrt{a}e}{\sqrt{c}}+d\right)\left(\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)-\frac{1}{2}\sqrt{c}\left(d-\frac{3\sqrt{a}e}{\sqrt{c}}\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{4a}+\frac{x(ae+cdx^2)}{4a(a+cx^4)}}{ae^2+cd^2}-\frac{de\int \left(\frac{e^2}{(cd^2+ae^2)(ex^2+d)}+\frac{c(d-ex^2)}{(cd^2+ae^2)(cx^4+a)}\right)dx}{ae^2+cd^2}$$

$$\Big\downarrow \ 217$$

$$\frac{\frac{\frac{1}{2}\sqrt{c}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{3\sqrt{a}e}{\sqrt{c}}+d\right)-\frac{1}{2}\sqrt{c}\left(d-\frac{3\sqrt{a}e}{\sqrt{c}}\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{4a}+\frac{x(ae+cdx^2)}{4a(a+cx^4)}}{ae^2+cd^2}-\frac{de\int \left(\frac{e^2}{(cd^2+ae^2)(ex^2+d)}+\frac{c(d-ex^2)}{(cd^2+ae^2)(cx^4+a)}\right)dx}{ae^2+cd^2}$$

$$\Big\downarrow \ 1479$$

$$\frac{\frac{\frac{1}{2}\sqrt{c}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{3\sqrt{a}e}{\sqrt{c}}+d\right)-\frac{1}{2}\sqrt{c}\left(d-\frac{3\sqrt{a}e}{\sqrt{c}}\right)\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\cdots\right)}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\cdots\right.}{4a}\cdots}{ae^2+cd^2}-\frac{de\int \left(\frac{e^2}{(cd^2+ae^2)(ex^2+d)}+\frac{c(d-ex^2)}{(cd^2+ae^2)(cx^4+a)}\right)dx}{ae^2+cd^2}$$

3.255. $\int \frac{x^2}{(d+ex^2)(a+cx^4)^2}\,dx$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{1}{2}\sqrt{c}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{3\sqrt{a}e}{\sqrt{c}}+d\right)-\frac{1}{2}\sqrt{c}\left(d-\frac{3\sqrt{a}e}{\sqrt{c}}\right)\left(\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}d}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right.}{4a}}{ae^2+cd^2}$$

$$\frac{de\int\left(\frac{e^2}{(cd^2+ae^2)(ex^2+d)}+\frac{c(d-ex^2)}{(cd^2+ae^2)(cx^4+a)}\right)dx}{ae^2+cd^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{1}{2}\sqrt{c}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{3\sqrt{a}e}{\sqrt{c}}+d\right)-\frac{1}{2}\sqrt{c}\left(d-\frac{3\sqrt{a}e}{\sqrt{c}}\right)\left(\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}\right)}{4a}+\frac{x(ae+c}{4a(a+}}{ae^2+cd^2}$$

$$\frac{de\int\left(\frac{e^2}{(cd^2+ae^2)(ex^2+d)}+\frac{c(d-ex^2)}{(cd^2+ae^2)(cx^4+a)}\right)dx}{ae^2+cd^2}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{\frac{1}{2}\sqrt{c}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{3\sqrt{a}e}{\sqrt{c}}+d\right)-\frac{1}{2}\sqrt{c}\left(d-\frac{3\sqrt{a}e}{\sqrt{c}}\right)\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right.}{4a}}{ae^2+cd^2}$$

$$\frac{de\int\left(\frac{e^2}{(cd^2+ae^2)(ex^2+d)}+\frac{c(d-ex^2)}{(cd^2+ae^2)(cx^4+a)}\right)dx}{ae^2+cd^2}$$

$$\Big\downarrow \; 2009$$

3.255. $\int \frac{x^2}{(d+ex^2)(a+cx^4)^2}\,dx$

$$\frac{\frac{\frac{1}{2}\sqrt{c}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{3\sqrt{a}e}{\sqrt{c}}+d\right)-\frac{1}{2}\sqrt{c}\left(d-\frac{3\sqrt{a}e}{\sqrt{c}}\right)\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right.}{4a}}{ae^2+cd^2}$$

$$\frac{de\left(-\frac{\sqrt[4]{c}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)(\sqrt{c}d-\sqrt{a}e)}{2\sqrt{2}a^{3/4}(ae^2+cd^2)}+\frac{\sqrt[4]{c}\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)(\sqrt{c}d-\sqrt{a}e)}{2\sqrt{2}a^{3/4}(ae^2+cd^2)}-\frac{\sqrt[4]{c}(\sqrt{a}e+\sqrt{c}d)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(ae^2+cd^2)}\right.}{ae^2+cd^2}$$

input
```
Int[x^2/((d + e*x^2)*(a + c*x^4)^2),x]
```

output
```
-((d*e*((e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 + a*e^2)) -
(c^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/
(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*Arc
Tan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2))
- (c^(1/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x
 + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]*d
 + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*S
qrt[2]*a^(3/4)*(c*d^2 + a*e^2))))/(c*d^2 + a*e^2)) + ((x*(a*e + c*d*x^2))/
(4*a*(a + c*x^4)) + ((Sqrt[c]*(d + (3*Sqrt[a]*e)/Sqrt[c])*(-(ArcTan[1 - (S
qrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2
]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4))))/2 - (Sqrt[c]*(d - (3*Sqr
t[a]*e)/Sqrt[c])*(-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x
^2]/(Sqrt[2]*a^(1/4)*c^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x +
Sqrt[c]*x^2]/(2*Sqrt[2]*a^(1/4)*c^(1/4))))/2)/(4*a))/(c*d^2 + a*e^2)
```

3.255.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 1485
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

rule 1493
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Simp[1/(4*a*(p + 1)
)   Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && Integer
Q[2*p]
```

3.255. $\int \frac{x^2}{(d+ex^2)(a+cx^4)^2}\,dx$

rule 1653
```
Int[(((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^4)^(p_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[f^2/(c*d^2 + a*e^2)   Int[(f*x)^(m - 2)*(a*e + c*d*x^2)*(
a + c*x^4)^p, x], x] - Simp[d*e*(f^2/(c*d^2 + a*e^2))   Int[(f*x)^(m - 2)*(
(a + c*x^4)^(p + 1)/(d + e*x^2)), x], x] /; FreeQ[{a, c, d, e, f}, x] && Lt
Q[p, -1] && GtQ[m, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.255.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 339, normalized size of antiderivative = 0.49

method	result
default	$\frac{\frac{\frac{cd\left(a\,e^{2}+c\,d^{2}\right)x^{3}}{4a}+\left(\frac{1}{4}a\,e^{3}+\frac{1}{4}c\,d^{2}e\right)x}{c\,x^{4}+a}+\frac{\frac{\left(3e^{3}a^{2}-ac\,d^{2}e\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}}{4a}}{\left(a\,e^{2}+c\,d^{2}\right)^{2}}$
risch	Expression too large to display

input
```
int(x^2/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/(a*e^2+c*d^2)^2*((1/4*c*d*(a*e^2+c*d^2)/a*x^3+(1/4*a*e^3+1/4*c*d^2*e)*x)
/(c*x^4+a)+1/4/a*(1/8*(3*a^2*e^3-a*c*d^2*e)*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2
+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)
))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/
8*(5*a*c*d*e^2+c^2*d^3)/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/
2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/
(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))))-e^3*d/(a*e^2+c*d^2)^
2/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2))
```

3.255.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4877 vs. 2(513) = 1026.

Time = 6.45 (sec) , antiderivative size = 9774, normalized size of antiderivative = 14.27

$$\int \frac{x^2}{(d+ex^2)(a+cx^4)^2} dx = \text{Too large to display}$$

input
```
integrate(x^2/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.255.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(d+ex^2)(a+cx^4)^2} dx = \text{Timed out}$$

input
```
integrate(x**2/(e*x**2+d)/(c*x**4+a)**2,x)
```

output
```
Timed out
```

3.255.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(d+ex^2)(a+cx^4)^2} dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.255.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 622, normalized size of antiderivative = 0.91

$$
\begin{aligned}
\int \frac{x^2}{(d+ex^2)(a+cx^4)^2}\,dx = & -\frac{de^3\arctan\left(\frac{ex}{\sqrt{de}}\right)}{(c^2d^4+2\,acd^2e^2+a^2e^4)\sqrt{de}} \\
& - \frac{\left((ac^3)^{\frac{1}{4}}\,ac^2d^2e-3\,(ac^3)^{\frac{1}{4}}\,a^2ce^3-(ac^3)^{\frac{3}{4}}\,cd^3-5\,(ac^3)^{\frac{3}{4}}\,ade^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}a^2c^4d^4+2\sqrt{2}a^3c^3d^2e^2+\sqrt{2}a^4c^2e^4\right)} \\
& - \frac{\left((ac^3)^{\frac{1}{4}}\,ac^2d^2e-3\,(ac^3)^{\frac{1}{4}}\,a^2ce^3-(ac^3)^{\frac{3}{4}}\,cd^3-5\,(ac^3)^{\frac{3}{4}}\,ade^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}a^2c^4d^4+2\sqrt{2}a^3c^3d^2e^2+\sqrt{2}a^4c^2e^4\right)} \\
& - \frac{\left((ac^3)^{\frac{1}{4}}\,ac^2d^2e-3\,(ac^3)^{\frac{1}{4}}\,a^2ce^3+(ac^3)^{\frac{3}{4}}\,cd^3+5\,(ac^3)^{\frac{3}{4}}\,ade^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}a^2c^4d^4+2\sqrt{2}a^3c^3d^2e^2+\sqrt{2}a^4c^2e^4\right)} \\
& + \frac{\left((ac^3)^{\frac{1}{4}}\,ac^2d^2e-3\,(ac^3)^{\frac{1}{4}}\,a^2ce^3+(ac^3)^{\frac{3}{4}}\,cd^3+5\,(ac^3)^{\frac{3}{4}}\,ade^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}a^2c^4d^4+2\sqrt{2}a^3c^3d^2e^2+\sqrt{2}a^4c^2e^4\right)} \\
& + \frac{cdx^3+aex}{4\,(cx^4+a)(acd^2+a^2e^2)}
\end{aligned}
$$

input

```
integrate(x^2/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

output

```
-d*e^3*arctan(e*x/sqrt(d*e))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(d*e
)) - 1/8*((a*c^3)^(1/4)*a*c^2*d^2*e - 3*(a*c^3)^(1/4)*a^2*c*e^3 - (a*c^3)^
(3/4)*c*d^3 - 5*(a*c^3)^(3/4)*a*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(
a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a^3*c^3*d^2*e^2
+ sqrt(2)*a^4*c^2*e^4) - 1/8*((a*c^3)^(1/4)*a*c^2*d^2*e - 3*(a*c^3)^(1/4)*
a^2*c*e^3 - (a*c^3)^(3/4)*c*d^3 - 5*(a*c^3)^(3/4)*a*d*e^2)*arctan(1/2*sqrt
(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt
(2)*a^3*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) - 1/16*((a*c^3)^(1/4)*a*c^2*d^2
*e - 3*(a*c^3)^(1/4)*a^2*c*e^3 + (a*c^3)^(3/4)*c*d^3 + 5*(a*c^3)^(3/4)*a*d
*e^2)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*d^4 +
2*sqrt(2)*a^3*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) + 1/16*((a*c^3)^(1/4)*a*c
^2*d^2*e - 3*(a*c^3)^(1/4)*a^2*c*e^3 + (a*c^3)^(3/4)*c*d^3 + 5*(a*c^3)^(3/
4)*a*d*e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*
d^4 + 2*sqrt(2)*a^3*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) + 1/4*(c*d*x^3 + a*
e*x)/((c*x^4 + a)*(a*c*d^2 + a^2*e^2))
```

3.255.9 Mupad [B] (verification not implemented)

Time = 9.66 (sec) , antiderivative size = 17812, normalized size of antiderivative = 26.00

$$\int \frac{x^2}{(d+ex^2)(a+cx^4)^2} \, dx = \text{Too large to display}$$

input
```
int(x^2/((a + c*x^4)^2*(d + e*x^2)),x)
```

output
```
((e*x)/(4*(a*e^2 + c*d^2)) + (c*d*x^3)/(4*a*(a*e^2 + c*d^2)))/(a + c*x^4)
+ atan((((((53248*a^9*c^4*d*e^15 + 4096*a^3*c^10*d^13*e^3 + 73728*a^4*c^9
*d^11*e^5 + 307200*a^5*c^8*d^9*e^7 + 573440*a^6*c^7*d^7*e^9 + 552960*a^7*c
^6*d^5*e^11 + 270336*a^8*c^5*d^3*e^13)/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5
*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) - (x*(-(c^3*d^6*(-a^5
*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3
 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^
5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*
d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2)*(65536*a^11*c^4*e^17 - 65536*a^4*c^11
*d^14*e^3 - 327680*a^5*c^10*d^12*e^5 - 589824*a^6*c^9*d^10*e^7 - 327680*a^
7*c^8*d^8*e^9 + 327680*a^8*c^7*d^6*e^11 + 589824*a^9*c^6*d^4*e^13 + 327680
*a^10*c^5*d^2*e^15))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3
*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*
(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*
a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*
c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^
2*e^6)))^(1/2) + (x*(128*a*c^10*d^13*e^2 - 14208*a^7*c^4*d*e^14 + 768*a^2*
c^9*d^11*e^4 + 3968*a^3*c^8*d^9*e^6 + 27136*a^4*c^7*d^7*e^8 + 30592*a^5*c^
6*d^5*e^10 - 7424*a^6*c^5*d^3*e^12))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c
*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)...
```

3.256 $\int \frac{1}{(d+ex^2)(a+cx^4)^2}\,dx$

3.256.1 Optimal result

Integrand size = 19, antiderivative size = 689

$$\begin{aligned}\int \frac{1}{(d+ex^2)(a+cx^4)^2}\,dx &= \frac{cx(d-ex^2)}{4a(cd^2+ae^2)(a+cx^4)}+\frac{e^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}(cd^2+ae^2)^2}\\
&-\frac{\sqrt[4]{c}e^2(\sqrt{c}d-\sqrt{a}e)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(cd^2+ae^2)^2}\\
&-\frac{\sqrt[4]{c}(3\sqrt{c}d-\sqrt{a}e)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(cd^2+ae^2)}\\
&+\frac{\sqrt[4]{c}e^2(\sqrt{c}d-\sqrt{a}e)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(cd^2+ae^2)^2}\\
&+\frac{\sqrt[4]{c}(3\sqrt{c}d-\sqrt{a}e)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(cd^2+ae^2)}\\
&-\frac{\sqrt[4]{c}e^2(\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(cd^2+ae^2)^2}\\
&-\frac{\sqrt[4]{c}(3\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}(cd^2+ae^2)}\\
&+\frac{\sqrt[4]{c}e^2(\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(cd^2+ae^2)^2}\\
&+\frac{\sqrt[4]{c}(3\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}(cd^2+ae^2)}\end{aligned}$$

output
```
1/4*c*x*(-e*x^2+d)/a/(a*e^2+c*d^2)/(c*x^4+a)+1/4*c^(1/4)*e^2*arctan(-1+c^(
1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^(
1/2)+1/4*c^(1/4)*e^2*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(
1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)-1/8*c^(1/4)*e^2*ln(-a^(1/4)*c^(1/4)*
x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)
^2*2^(1/2)+1/8*c^(1/4)*e^2*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2
))*(e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)+1/16*c^(1/4)*arct
an(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+3*d*c^(1/2))/a^(7/4)/(a*e^2+c
*d^2)*2^(1/2)+1/16*c^(1/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)
+3*d*c^(1/2))/a^(7/4)/(a*e^2+c*d^2)*2^(1/2)-1/32*c^(1/4)*ln(-a^(1/4)*c^(1/
4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+3*d*c^(1/2))/a^(7/4)/(a*e^2+c
*d^2)*2^(1/2)+1/32*c^(1/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2
))*(e*a^(1/2)+3*d*c^(1/2))/a^(7/4)/(a*e^2+c*d^2)*2^(1/2)+e^(7/2)*arctan(x*
e^(1/2)/d^(1/2))/(a*e^2+c*d^2)^2/d^(1/2)
```

3.256.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 429, normalized size of antiderivative = 0.62

$$\int \frac{1}{(d+ex^2)(a+cx^4)^2}\,dx$$

$$= \frac{\frac{8c(cd^2+ae^2)x(d-ex^2)}{a(a+cx^4)} + \frac{32e^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}} + \frac{2\sqrt{2}\sqrt[4]{c}(-3c^{3/2}d^3+\sqrt{a}cd^2e-7a\sqrt{c}de^2+5a^{3/2}e^3)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{7/4}} - \frac{2\sqrt{2}\sqrt[4]{c}(-3}{}$$

input
```
Integrate[1/((d + e*x^2)*(a + c*x^4)^2),x]
```

output
```
((8*c*(c*d^2 + a*e^2)*x*(d - e*x^2))/(a*(a + c*x^4)) + (32*e^(7/2)*ArcTan[
(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] + (2*Sqrt[2]*c^(1/4)*(-3*c^(3/2)*d^3 + Sqrt[
a]*c*d^2*e - 7*a*Sqrt[c]*d*e^2 + 5*a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4
)*x)/a^(1/4)])/a^(7/4) - (2*Sqrt[2]*c^(1/4)*(-3*c^(3/2)*d^3 + Sqrt[a]*c*d^
2*e - 7*a*Sqrt[c]*d*e^2 + 5*a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^
(1/4)])/a^(7/4) - (Sqrt[2]*c^(1/4)*(3*c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + 7*a*
Sqrt[c]*d*e^2 + 5*a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + S
qrt[c]*x^2])/a^(7/4) + (Sqrt[2]*c^(1/4)*(3*c^(3/2)*d^3 + Sqrt[a]*c*d^2*e +
 7*a*Sqrt[c]*d*e^2 + 5*a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*
x + Sqrt[c]*x^2])/a^(7/4))/(32*(c*d^2 + a*e^2)^2)
```

3.256.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 689, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1568, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^4)^2(d+ex^2)}dx$$

$$\Big\downarrow \text{1568}$$

$$\int \left(-\frac{ce^2(ex^2-d)}{(a+cx^4)(ae^2+cd^2)^2}+\frac{c(d-ex^2)}{(a+cx^4)^2(ae^2+cd^2)}+\frac{e^4}{(d+ex^2)(ae^2+cd^2)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt[4]{c}e^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)(\sqrt{c}d-\sqrt{a}e)}{2\sqrt{2}a^{3/4}(ae^2+cd^2)^2}+\frac{\sqrt[4]{c}e^2\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)(\sqrt{c}d-\sqrt{a}e)}{2\sqrt{2}a^{3/4}(ae^2+cd^2)^2}-\frac{\sqrt[4]{c}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)(3\sqrt{c}d-\sqrt{a}e)}{8\sqrt{2}a^{7/4}(ae^2+cd^2)}+\frac{\sqrt[4]{c}\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)(3\sqrt{c}d-\sqrt{a}e)}{8\sqrt{2}a^{7/4}(ae^2+cd^2)}-\frac{\sqrt[4]{c}e^2(\sqrt{a}e+\sqrt{c}d)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(ae^2+cd^2)^2}+\frac{\sqrt[4]{c}e^2(\sqrt{a}e+\sqrt{c}d)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(ae^2+cd^2)^2}-\frac{\sqrt[4]{c}(\sqrt{a}e+3\sqrt{c}d)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}(ae^2+cd^2)}+\frac{\sqrt[4]{c}(\sqrt{a}e+3\sqrt{c}d)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}(ae^2+cd^2)}+\frac{e^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}(ae^2+cd^2)^2}+\frac{cx(d-ex^2)}{4a(a+cx^4)(ae^2+cd^2)}$$

input
```
Int[1/((d + e*x^2)*(a + c*x^4)^2),x]
```

output
```
(c*x*(d - e*x^2))/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) + (e^(7/2)*ArcTan[(Sqr
t[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 + a*e^2)^2) - (c^(1/4)*e^2*(Sqrt[c]*d -
Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*
d^2 + a*e^2)^2) - (c^(1/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c
^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) + (c^(1/4)*e^2*(Sq
rt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a
^(3/4)*(c*d^2 + a*e^2)^2) + (c^(1/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 +
(Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) - (c^(1/
4)*e^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + S
qrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) - (c^(1/4)*(3*Sqrt[c]*d
 + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*
Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) + (c^(1/4)*e^2*(Sqrt[c]*d + Sqrt[a]*e)*Lo
g[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(
c*d^2 + a*e^2)^2) + (c^(1/4)*(3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2))
```

3.256.3.1 Defintions of rubi rules used

rule 1568
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e,
p, q}, x] && ((IntegerQ[p] && IntegerQ[q]) || IGtQ[p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.256.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.48

method	result
default	$\frac{c\left(\frac{-\frac{e\left(a e^2+c d^2\right)x^3}{4a}+\frac{d\left(a e^2+c d^2\right)x}{4a}}{c x^4+a}+\frac{\frac{\left(7d e^2 a+3d^3 c\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}-}{4a}\right.}{\left(a e^2+c d^2\right)^2}$
risch	Expression too large to display

input
```
int(1/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
c/(a*e^2+c*d^2)^2*((-1/4*e*(a*e^2+c*d^2)/a*x^3+1/4*d*(a*e^2+c*d^2)/a*x)/(c
*x^4+a)+1/4/a*(1/8*(7*a*d*e^2+3*c*d^3)*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c
)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*
arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/8*(-5
*a*e^3-c*d^2*e)/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)
^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1
/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))))+e^4/(a*e^2+c*d^2)^2/(e*d)^(1
/2)*arctan(e*x/(e*d)^(1/2))
```

3.256.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4934 vs. $2(518) = 1036$.

Time = 11.96 (sec) , antiderivative size = 9892, normalized size of antiderivative = 14.36

$$\int \frac{1}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.256.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x**2+d)/(c*x**4+a)**2,x)
```

output
```
Timed out
```

3.256.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.256.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 621, normalized size of antiderivative = 0.90

$$\begin{aligned}\int \frac{1}{(d+ex^2)(a+cx^4)^2}\,dx &= \frac{e^4\arctan\left(\frac{ex}{\sqrt{de}}\right)}{(c^2d^4+2\,acd^2e^2+a^2e^4)\sqrt{de}} \\ &+ \frac{\left(3\,(ac^3)^{\frac14}c^3d^3+7\,(ac^3)^{\frac14}ac^2de^2-(ac^3)^{\frac34}cd^2e-5\,(ac^3)^{\frac34}ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac14}\right)}{2\left(\frac{a}{c}\right)^{\frac14}}\right)}{8\left(\sqrt{2}a^2c^4d^4+2\,\sqrt{2}a^3c^3d^2e^2+\sqrt{2}a^4c^2e^4\right)} \\ &+ \frac{\left(3\,(ac^3)^{\frac14}c^3d^3+7\,(ac^3)^{\frac14}ac^2de^2-(ac^3)^{\frac34}cd^2e-5\,(ac^3)^{\frac34}ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac14}\right)}{2\left(\frac{a}{c}\right)^{\frac14}}\right)}{8\left(\sqrt{2}a^2c^4d^4+2\,\sqrt{2}a^3c^3d^2e^2+\sqrt{2}a^4c^2e^4\right)} \\ &+ \frac{\left(3\,(ac^3)^{\frac14}c^3d^3+7\,(ac^3)^{\frac14}ac^2de^2+(ac^3)^{\frac34}cd^2e+5\,(ac^3)^{\frac34}ae^3\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac14}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}a^2c^4d^4+2\,\sqrt{2}a^3c^3d^2e^2+\sqrt{2}a^4c^2e^4\right)} \\ &- \frac{\left(3\,(ac^3)^{\frac14}c^3d^3+7\,(ac^3)^{\frac14}ac^2de^2+(ac^3)^{\frac34}cd^2e+5\,(ac^3)^{\frac34}ae^3\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac14}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}a^2c^4d^4+2\,\sqrt{2}a^3c^3d^2e^2+\sqrt{2}a^4c^2e^4\right)} \\ &- \frac{cex^3-cdx}{4\,(cx^4+a)(acd^2+a^2e^2)}\end{aligned}$$

input
```
integrate(1/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
e^4*arctan(e*x/sqrt(d*e))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(d*e))
+ 1/8*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*c^3)^(1/4)*a*c^2*d*e^2 - (a*c^3)^(3/
4)*c*d^2*e - 5*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c
)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a^3*c^3*d^2*e^2 + s
qrt(2)*a^4*c^2*e^4) + 1/8*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*c^3)^(1/4)*a*c^2
*d*e^2 - (a*c^3)^(3/4)*c*d^2*e - 5*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)
*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)
*a^3*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) + 1/16*(3*(a*c^3)^(1/4)*c^3*d^3 +
7*(a*c^3)^(1/4)*a*c^2*d*e^2 + (a*c^3)^(3/4)*c*d^2*e + 5*(a*c^3)^(3/4)*a*e^
3)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*d^4 + 2*s
qrt(2)*a^3*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) - 1/16*(3*(a*c^3)^(1/4)*c^3*
d^3 + 7*(a*c^3)^(1/4)*a*c^2*d*e^2 + (a*c^3)^(3/4)*c*d^2*e + 5*(a*c^3)^(3/4
)*a*e^3)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*d^4
 + 2*sqrt(2)*a^3*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) - 1/4*(c*e*x^3 - c*d*x
)/((c*x^4 + a)*(a*c*d^2 + a^2*e^2))
```

3.256.9 Mupad [B] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 17945, normalized size of antiderivative = 26.04

$$\int \frac{1}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((a + c*x^4)^2*(d + e*x^2)),x)
```

output
```
((c*d*x)/(4*a*(a*e^2 + c*d^2)) - (c*e*x^3)/(4*a*(a*e^2 + c*d^2)))/(a + c*x
^4) - atan((((((65536*a^11*c^4*e^16 - 12288*a^4*c^11*d^14*e^2 - 57344*a^5
*c^10*d^12*e^4 - 36864*a^6*c^9*d^10*e^6 + 245760*a^7*c^8*d^8*e^8 + 634880*
a^8*c^7*d^6*e^10 + 663552*a^9*c^6*d^4*e^12 + 331776*a^10*c^5*d^2*e^14)/(25
6*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2
*d^4*e^4)) - (x*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6
*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-
a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d
^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*(65
536*a^13*c^4*e^17 - 65536*a^6*c^11*d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 5
89824*a^8*c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11
 + 589824*a^11*c^6*d^4*e^13 + 327680*a^12*c^5*d^2*e^15))/(128*(a^8*e^8 + a
^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((
9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 4
4*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*
a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*
e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (x*(1152*a^2*c^11*d
^13*e^2 - 49024*a^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^
9*e^6 + 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3
*e^12))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e...
```

3.256. $\int \frac{1}{(d+ex^2)(a+cx^4)^2}\,dx$

3.257 $\int \frac{1}{x^2(d+ex^2)(a+cx^4)^2}\,dx$

3.257.1 Optimal result

Integrand size = 22, antiderivative size = 745

$$\int \frac{1}{x^2 (d+ex^2) (a+cx^4)^2} \, dx$$

$$= -\frac{1}{a^2dx} - \frac{cx(ae+cdx^2)}{4a^2 (cd^2+ae^2) (a+cx^4)} - \frac{e^{9/2} \arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2} (cd^2+ae^2)^2}$$
$$+ \frac{c^{3/4}(\sqrt{c}d+3\sqrt{a}e) \arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{9/4} (cd^2+ae^2)}$$
$$+ \frac{c^{3/4}(a^{3/2}e^3+\sqrt{c}d(cd^2+2ae^2)) \arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{9/4} (cd^2+ae^2)^2}$$
$$- \frac{c^{3/4}(\sqrt{c}d+3\sqrt{a}e) \arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{9/4} (cd^2+ae^2)}$$
$$- \frac{c^{3/4}(a^{3/2}e^3+\sqrt{c}d(cd^2+2ae^2)) \arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{9/4} (cd^2+ae^2)^2}$$
$$- \frac{c^{3/4}(\sqrt{c}d-3\sqrt{a}e) \log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{9/4} (cd^2+ae^2)}$$
$$+ \frac{c^{3/4}(a^{3/2}e^3-\sqrt{c}d(cd^2+2ae^2)) \log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{9/4} (cd^2+ae^2)^2}$$
$$+ \frac{c^{3/4}(\sqrt{c}d-3\sqrt{a}e) \log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{9/4} (cd^2+ae^2)}$$
$$- \frac{c^{3/4}(a^{3/2}e^3-\sqrt{c}d(cd^2+2ae^2)) \log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{9/4} (cd^2+ae^2)^2}$$

output
```
-1/a^2/d/x-1/4*c*x*(c*d*x^2+a*e)/a^2/(a*e^2+c*d^2)/(c*x^4+a)-e^(9/2)*arcta
n(x*e^(1/2)/d^(1/2))/d^(3/2)/(a*e^2+c*d^2)^2-1/32*c^(3/4)*ln(-a^(1/4)*c^(1
/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-3*e*a^(1/2)+d*c^(1/2))/a^(9/4)/(a*e^2
+c*d^2)*2^(1/2)+1/32*c^(3/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1
/2))*(-3*e*a^(1/2)+d*c^(1/2))/a^(9/4)/(a*e^2+c*d^2)*2^(1/2)-1/16*c^(3/4)*a
rctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*e*a^(1/2)+d*c^(1/2))/a^(9/4)/(a*e^2
+c*d^2)*2^(1/2)-1/16*c^(3/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*e*a^(1
/2)+d*c^(1/2))/a^(9/4)/(a*e^2+c*d^2)*2^(1/2)+1/8*c^(3/4)*ln(-a^(1/4)*c^(1/
4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(a^(3/2)*e^3-d*(2*a*e^2+c*d^2)*c^(1/2))/
a^(9/4)/(a*e^2+c*d^2)^2*2^(1/2)-1/8*c^(3/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a
^(1/2)+x^2*c^(1/2))*(a^(3/2)*e^3-d*(2*a*e^2+c*d^2)*c^(1/2))/a^(9/4)/(a*e^2
+c*d^2)^2*2^(1/2)-1/4*c^(3/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(a^(3/2
)*e^3+d*(2*a*e^2+c*d^2)*c^(1/2))/a^(9/4)/(a*e^2+c*d^2)^2*2^(1/2)-1/4*c^(3/
4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(a^(3/2)*e^3+d*(2*a*e^2+c*d^2)*c^(1
/2))/a^(9/4)/(a*e^2+c*d^2)^2*2^(1/2)
```

3.257.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 499, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^2 (d+ex^2) (a+cx^4)^2} \, dx$$
$$= \frac{1}{32}\left(-\frac{32}{a^2dx} - \frac{8cx(ae+cdx^2)}{a^2(cd^2+ae^2)(a+cx^4)} - \frac{32e^{9/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}(cd^2+ae^2)^2} + \frac{2\sqrt{2}c^{3/4}\left(5c^{3/2}d^3+3\sqrt{a}cd^2e+9a\sqrt{c}de^2+7a^{3/2}e^3\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{9/4}(cd^2+ae^2)^2} - \frac{2\sqrt{2}c^{3/4}\left(5c^{3/2}d^3+3\sqrt{a}cd^2e+9a\sqrt{c}de^2+7a^{3/2}e^3\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{9/4}(cd^2+ae^2)^2} + \frac{\sqrt{2}c^{3/4}\left(-5c^{3/2}d^3+3\sqrt{a}cd^2e-9a\sqrt{c}de^2+7a^{3/2}e^3\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{a^{9/4}(cd^2+ae^2)^2} + \frac{\sqrt{2}c^{3/4}\left(5c^{3/2}d^3-3\sqrt{a}cd^2e+9a\sqrt{c}de^2-7a^{3/2}e^3\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{a^{9/4}(cd^2+ae^2)^2}\right)$$

input
```
Integrate[1/(x^2*(d + e*x^2)*(a + c*x^4)^2),x]
```

output
```
(-32/(a^2*d*x) - (8*c*x*(a*e + c*d*x^2))/(a^2*(c*d^2 + a*e^2)*(a + c*x^4))
 - (32*e^(9/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*(c*d^2 + a*e^2)^2) +
(2*Sqrt[2]*c^(3/4)*(5*c^(3/2)*d^3 + 3*Sqrt[a]*c*d^2*e + 9*a*Sqrt[c]*d*e^2
+ 7*a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(9/4)*(c*d^2
+ a*e^2)^2) - (2*Sqrt[2]*c^(3/4)*(5*c^(3/2)*d^3 + 3*Sqrt[a]*c*d^2*e + 9*a*
Sqrt[c]*d*e^2 + 7*a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a
^(9/4)*(c*d^2 + a*e^2)^2) + (Sqrt[2]*c^(3/4)*(-5*c^(3/2)*d^3 + 3*Sqrt[a]*c
*d^2*e - 9*a*Sqrt[c]*d*e^2 + 7*a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*
c^(1/4)*x + Sqrt[c]*x^2])/(a^(9/4)*(c*d^2 + a*e^2)^2) + (Sqrt[2]*c^(3/4)*(
5*c^(3/2)*d^3 - 3*Sqrt[a]*c*d^2*e + 9*a*Sqrt[c]*d*e^2 - 7*a^(3/2)*e^3)*Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(a^(9/4)*(c*d^2 + a*e
^2)^2))/32
```

3.257.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 745, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1675, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+cx^4)^2 (d+ex^2)}dx$$

$$\Big\downarrow \text{ 1675}$$

$$\int \left(\frac{c\left(-a^2e^3-cdx^2\left(2ae^2+cd^2\right)\right)}{a^2 (a+cx^4) (ae^2+cd^2)^2}+\frac{1}{a^2dx^2}-\frac{c\left(ae+cdx^2\right)}{a (a+cx^4)^2 (ae^2+cd^2)}-\frac{e^5}{d (d+ex^2) (ae^2+cd^2)^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)(3\sqrt{a}e+\sqrt{c}d)}{8\sqrt{2}a^{9/4}(ae^2+cd^2)}-\frac{c^{3/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)(3\sqrt{a}e+\sqrt{c}d)}{8\sqrt{2}a^{9/4}(ae^2+cd^2)}+\frac{c^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)(a^{3/2}e^3+\sqrt{c}d(2ae^2+cd^2))}{2\sqrt{2}a^{9/4}(ae^2+cd^2)^2}-\frac{c^{3/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)(a^{3/2}e^3+\sqrt{c}d(2ae^2+cd^2))}{2\sqrt{2}a^{9/4}(ae^2+cd^2)^2}-\frac{c^{3/4}(\sqrt{c}d-3\sqrt{a}e)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{9/4}(ae^2+cd^2)}+\frac{c^{3/4}(\sqrt{c}d-3\sqrt{a}e)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{9/4}(ae^2+cd^2)}+\frac{c^{3/4}(a^{3/2}e^3-\sqrt{c}d(2ae^2+cd^2))\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{9/4}(ae^2+cd^2)^2}-\frac{c^{3/4}(a^{3/2}e^3-\sqrt{c}d(2ae^2+cd^2))\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{9/4}(ae^2+cd^2)^2}-\frac{cx(ae+cdx^2)}{4a^2(a+cx^4)(ae^2+cd^2)}-\frac{1}{a^2dx}-\frac{e^{9/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}(ae^2+cd^2)^2}$$

input
```
Int[1/(x^2*(d + e*x^2)*(a + c*x^4)^2),x]
```

output
```
-(1/(a^2*d*x)) - (c*x*(a*e + c*d*x^2))/(4*a^2*(c*d^2 + a*e^2)*(a + c*x^4))
 - (e^(9/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*(c*d^2 + a*e^2)^2) + (c^
(3/4)*(Sqrt[c]*d + 3*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(
8*Sqrt[2]*a^(9/4)*(c*d^2 + a*e^2)) + (c^(3/4)*(a^(3/2)*e^3 + Sqrt[c]*d*(c*
d^2 + 2*a*e^2))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(9/4
)*(c*d^2 + a*e^2)^2) - (c^(3/4)*(Sqrt[c]*d + 3*Sqrt[a]*e)*ArcTan[1 + (Sqrt
[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(9/4)*(c*d^2 + a*e^2)) - (c^(3/4)*(a
^(3/2)*e^3 + Sqrt[c]*d*(c*d^2 + 2*a*e^2))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a
^(1/4)])/(2*Sqrt[2]*a^(9/4)*(c*d^2 + a*e^2)^2) - (c^(3/4)*(Sqrt[c]*d - 3*S
qrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt
[2]*a^(9/4)*(c*d^2 + a*e^2)) + (c^(3/4)*(a^(3/2)*e^3 - Sqrt[c]*d*(c*d^2 +
2*a*e^2))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[
2]*a^(9/4)*(c*d^2 + a*e^2)^2) + (c^(3/4)*(Sqrt[c]*d - 3*Sqrt[a]*e)*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(9/4)*(c*d^
2 + a*e^2)) - (c^(3/4)*(a^(3/2)*e^3 - Sqrt[c]*d*(c*d^2 + 2*a*e^2))*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(9/4)*(c*d^2
 + a*e^2)^2)
```

3.257. $\int \frac{1}{x^2(d+ex^2)(a+cx^4)^2}\,dx$

3.257.3.1 Defintions of rubi rules used

rule 1675
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && (IGtQ[p, 0] || IGtQ[q, 0] |
| IntegersQ[m, q])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.257.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 355, normalized size of antiderivative = 0.48

method	result
default	$-\frac{1}{a^2 d x}-\frac{c\left(\frac{\left(\frac{1}{4}acd\,e^2+\frac{1}{4}c^2d^3\right)x^3+\left(\frac{1}{4}e^3a^2+\frac{1}{4}ac\,d^2e\right)x}{c\,x^4+a}+\frac{\left(7e^3a^2+3ac\,d^2e\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\right.}{32a}}{(a\,e^2+c\,d^2}$
risch	Expression too large to display

input
```
int(1/x^2/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/a^2/d/x-c/(a*e^2+c*d^2)^2/a^2*(((1/4*a*c*d*e^2+1/4*c^2*d^3)*x^3+(1/4*e^
3*a^2+1/4*a*c*d^2*e)*x)/(c*x^4+a)+1/32*(7*a^2*e^3+3*a*c*d^2*e)*(a/c)^(1/4)
/a*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*
2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(
a/c)^(1/4)*x-1))+1/32*(9*a*c*d*e^2+5*c^2*d^3)/c/(a/c)^(1/4)*2^(1/2)*(ln((x
^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/
2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1)))
-1/d*e^5/(a*e^2+c*d^2)^2/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2))
```

3.257.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5082 vs. $2(573) = 1146$.

Time = 29.01 (sec) , antiderivative size = 10188, normalized size of antiderivative = 13.68

$$\int \frac{1}{x^2 (d + ex^2) (a + cx^4)^2} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.257.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (d + ex^2) (a + cx^4)^2} \, dx = \text{Timed out}$$

input
```
integrate(1/x**2/(e*x**2+d)/(c*x**4+a)**2,x)
```

output
```
Timed out
```

3.257.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (d + ex^2) (a + cx^4)^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^2/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.257.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 659, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2(d+ex^2)(a+cx^4)^2}\,dx = -\frac{e^5\arctan\left(\frac{ex}{\sqrt{de}}\right)}{(c^2d^5+2\,acd^3e^2+a^2de^4)\sqrt{de}} - \frac{\left(3\,(ac^3)^{\frac{1}{4}}\,ac^2d^2e+7\,(ac^3)^{\frac{1}{4}}\,a^2ce^3+5\,(ac^3)^{\frac{3}{4}}\,cd^3+9\,(ac^3)^{\frac{3}{4}}\,ade^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}a^3c^3d^4+2\,\sqrt{2}a^4c^2d^2e^2+\sqrt{2}a^5ce^4\right)} - \frac{\left(3\,(ac^3)^{\frac{1}{4}}\,ac^2d^2e+7\,(ac^3)^{\frac{1}{4}}\,a^2ce^3+5\,(ac^3)^{\frac{3}{4}}\,cd^3+9\,(ac^3)^{\frac{3}{4}}\,ade^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}a^3c^3d^4+2\,\sqrt{2}a^4c^2d^2e^2+\sqrt{2}a^5ce^4\right)} - \frac{\left(3\,(ac^3)^{\frac{1}{4}}\,ac^2d^2e+7\,(ac^3)^{\frac{1}{4}}\,a^2ce^3-5\,(ac^3)^{\frac{3}{4}}\,cd^3-9\,(ac^3)^{\frac{3}{4}}\,ade^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}a^3c^3d^4+2\,\sqrt{2}a^4c^2d^2e^2+\sqrt{2}a^5ce^4\right)} + \frac{\left(3\,(ac^3)^{\frac{1}{4}}\,ac^2d^2e+7\,(ac^3)^{\frac{1}{4}}\,a^2ce^3-5\,(ac^3)^{\frac{3}{4}}\,cd^3-9\,(ac^3)^{\frac{3}{4}}\,ade^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}a^3c^3d^4+2\,\sqrt{2}a^4c^2d^2e^2+\sqrt{2}a^5ce^4\right)} - \frac{5\,c^2d^2x^4+4\,ace^2x^4+acdex^2+4\,acd^2+4\,a^2e^2}{4\,(a^2cd^3+a^3de^2)(cx^5+ax)}$$

input
```
integrate(1/x^2/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
-e^5*arctan(e*x/sqrt(d*e))/((c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4)*sqrt(d*e
)) - 1/8*(3*(a*c^3)^(1/4)*a*c^2*d^2*e + 7*(a*c^3)^(1/4)*a^2*c*e^3 + 5*(a*c
^3)^(3/4)*c*d^3 + 9*(a*c^3)^(3/4)*a*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(
2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^3*c^3*d^4 + 2*sqrt(2)*a^4*c^2*d^2*
e^2 + sqrt(2)*a^5*c*e^4) - 1/8*(3*(a*c^3)^(1/4)*a*c^2*d^2*e + 7*(a*c^3)^(1
/4)*a^2*c*e^3 + 5*(a*c^3)^(3/4)*c*d^3 + 9*(a*c^3)^(3/4)*a*d*e^2)*arctan(1/
2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^3*c^3*d^4 +
2*sqrt(2)*a^4*c^2*d^2*e^2 + sqrt(2)*a^5*c*e^4) - 1/16*(3*(a*c^3)^(1/4)*a*c
^2*d^2*e + 7*(a*c^3)^(1/4)*a^2*c*e^3 - 5*(a*c^3)^(3/4)*c*d^3 - 9*(a*c^3)^(
3/4)*a*d*e^2)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^3*c^
3*d^4 + 2*sqrt(2)*a^4*c^2*d^2*e^2 + sqrt(2)*a^5*c*e^4) + 1/16*(3*(a*c^3)^(
1/4)*a*c^2*d^2*e + 7*(a*c^3)^(1/4)*a^2*c*e^3 - 5*(a*c^3)^(3/4)*c*d^3 - 9*(
a*c^3)^(3/4)*a*d*e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2
)*a^3*c^3*d^4 + 2*sqrt(2)*a^4*c^2*d^2*e^2 + sqrt(2)*a^5*c*e^4) - 1/4*(5*c^
2*d^2*x^4 + 4*a*c*e^2*x^4 + a*c*d*e*x^2 + 4*a*c*d^2 + 4*a^2*e^2)/((a^2*c*d
^3 + a^3*d*e^2)*(c*x^5 + a*x))
```

3.257.9 Mupad [B] (verification not implemented)

Time = 10.73 (sec) , antiderivative size = 24015, normalized size of antiderivative = 32.23

$$\int \frac{1}{x^2 (d+ex^2) (a+cx^4)^2} \, dx = \text{Too large to display}$$

input
```
int(1/(x^2*(a + c*x^4)^2*(d + e*x^2)),x)
```

output
```
- (1/(a*d) + (c*e*x^2)/(4*a*(a*e^2 + c*d^2)) + (c*x^4*(4*a*e^2 + 5*c*d^2))
/(4*a^2*d*(a*e^2 + c*d^2)))/(a*x + c*x^5) - atan(((11875*a^5*c^10*d^15*e -
 a^9*c^3*(72128*a^3*d*e^15 + 265655*c^3*d^7*e^9 - 76440*a*c^2*d^5*e^11 - 1
78585*a^2*c*d^3*e^13) + 68800*a^6*c^9*d^13*e^3 + 89403*a^7*c^8*d^11*e^5 -
126488*a^8*c^7*d^9*e^7)*(a^25*d^2*e^19*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) -
25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a
^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^
9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*
e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*2i - a^15*c^2*e^17*x*(-(49*a^3*e^6*(-a^9*
c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*
d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c
*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*
a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*3136i - a^11*c^10*d^19*x*(-(
49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5
*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)
^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^
12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*25i - a^16*
c^9*d^20*e*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2)
+ 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^
4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 +...
```

3.258 $\int \frac{1}{x^4(d+ex^2)(a+cx^4)^2}\,dx$

3.258.1 Optimal result

Integrand size = 22, antiderivative size = 751

$$\begin{aligned}
&\int \frac{1}{x^4(d+ex^2)(a+cx^4)^2}\,dx \\
&= -\frac{1}{3a^2dx^3}+\frac{e}{a^2d^2x}-\frac{c^2x(d-ex^2)}{4a^2(cd^2+ae^2)(a+cx^4)} \\
&+\frac{e^{11/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}(cd^2+ae^2)^2}+\frac{c^{5/4}(3\sqrt{c}d-\sqrt{a}e)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{11/4}(cd^2+ae^2)} \\
&+\frac{c^{5/4}(\sqrt{c}d-\sqrt{a}e)(cd^2+2ae^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{11/4}(cd^2+ae^2)^2} \\
&-\frac{c^{5/4}(3\sqrt{c}d-\sqrt{a}e)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{11/4}(cd^2+ae^2)} \\
&-\frac{c^{5/4}(\sqrt{c}d-\sqrt{a}e)(cd^2+2ae^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{11/4}(cd^2+ae^2)^2} \\
&+\frac{c^{5/4}(3\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{11/4}(cd^2+ae^2)} \\
&+\frac{c^{5/4}(\sqrt{c}d+\sqrt{a}e)(cd^2+2ae^2)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{11/4}(cd^2+ae^2)^2} \\
&-\frac{c^{5/4}(3\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{11/4}(cd^2+ae^2)} \\
&-\frac{c^{5/4}(\sqrt{c}d+\sqrt{a}e)(cd^2+2ae^2)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{11/4}(cd^2+ae^2)^2}
\end{aligned}$$

output
```
-1/3/a^2/d/x^3+e/a^2/d^2/x-1/4*c^2*x*(-e*x^2+d)/a^2/(a*e^2+c*d^2)/(c*x^4+a
)+e^(11/2)*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)/(a*e^2+c*d^2)^2-1/4*c^(5/4)*(
2*a*e^2+c*d^2)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))
/a^(11/4)/(a*e^2+c*d^2)^2*2^(1/2)-1/4*c^(5/4)*(2*a*e^2+c*d^2)*arctan(1+c^(
1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/a^(11/4)/(a*e^2+c*d^2)^2*2^
(1/2)+1/8*c^(5/4)*(2*a*e^2+c*d^2)*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^
2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/a^(11/4)/(a*e^2+c*d^2)^2*2^(1/2)-1/8*c^(5
/4)*(2*a*e^2+c*d^2)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a
^(1/2)+d*c^(1/2))/a^(11/4)/(a*e^2+c*d^2)^2*2^(1/2)-1/16*c^(5/4)*arctan(-1+
c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+3*d*c^(1/2))/a^(11/4)/(a*e^2+c*d^2)
*2^(1/2)-1/16*c^(5/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+3*d*
c^(1/2))/a^(11/4)/(a*e^2+c*d^2)*2^(1/2)+1/32*c^(5/4)*ln(-a^(1/4)*c^(1/4)*x
*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+3*d*c^(1/2))/a^(11/4)/(a*e^2+c*d^
2)*2^(1/2)-1/32*c^(5/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*
(e*a^(1/2)+3*d*c^(1/2))/a^(11/4)/(a*e^2+c*d^2)*2^(1/2)
```

3.258.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 513, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^4\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx$$
$$= \frac{1}{96}\left(-\frac{32}{a^2dx^3}+\frac{96e}{a^2d^2x}-\frac{24c^2x(d-ex^2)}{a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)}+\frac{96e^{11/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\left(cd^2+ae^2\right)^2}\right.$$
$$+\frac{6\sqrt{2}c^{5/4}\left(7c^{3/2}d^3-5\sqrt{a}cd^2e+11a\sqrt{c}de^2-9a^{3/2}e^3\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{11/4}\left(cd^2+ae^2\right)^2}$$
$$+\frac{6\sqrt{2}c^{5/4}\left(-7c^{3/2}d^3+5\sqrt{a}cd^2e-11a\sqrt{c}de^2+9a^{3/2}e^3\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{11/4}\left(cd^2+ae^2\right)^2}$$
$$+\frac{3\sqrt{2}c^{5/4}\left(7c^{3/2}d^3+5\sqrt{a}cd^2e+11a\sqrt{c}de^2+9a^{3/2}e^3\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{a^{11/4}\left(cd^2+ae^2\right)^2}$$
$$\left.-\frac{3\sqrt{2}c^{5/4}\left(7c^{3/2}d^3+5\sqrt{a}cd^2e+11a\sqrt{c}de^2+9a^{3/2}e^3\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{a^{11/4}\left(cd^2+ae^2\right)^2}\right)$$

input
```
Integrate[1/(x^4*(d + e*x^2)*(a + c*x^4)^2),x]
```

output
```
(-32/(a^2*d*x^3) + (96*e)/(a^2*d^2*x) - (24*c^2*x*(d - e*x^2))/(a^2*(c*d^2
 + a*e^2)*(a + c*x^4)) + (96*e^(11/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(5/2
)*(c*d^2 + a*e^2)^2) + (6*Sqrt[2]*c^(5/4)*(7*c^(3/2)*d^3 - 5*Sqrt[a]*c*d^2
*e + 11*a*Sqrt[c]*d*e^2 - 9*a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^
(1/4)])/(a^(11/4)*(c*d^2 + a*e^2)^2) + (6*Sqrt[2]*c^(5/4)*(-7*c^(3/2)*d^3
+ 5*Sqrt[a]*c*d^2*e - 11*a*Sqrt[c]*d*e^2 + 9*a^(3/2)*e^3)*ArcTan[1 + (Sqrt
[2]*c^(1/4)*x)/a^(1/4)])/(a^(11/4)*(c*d^2 + a*e^2)^2) + (3*Sqrt[2]*c^(5/4)
*(7*c^(3/2)*d^3 + 5*Sqrt[a]*c*d^2*e + 11*a*Sqrt[c]*d*e^2 + 9*a^(3/2)*e^3)*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(a^(11/4)*(c*d^2 +
 a*e^2)^2) - (3*Sqrt[2]*c^(5/4)*(7*c^(3/2)*d^3 + 5*Sqrt[a]*c*d^2*e + 11*a*
Sqrt[c]*d*e^2 + 9*a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + S
qrt[c]*x^2])/(a^(11/4)*(c*d^2 + a*e^2)^2))/96
```

3.258.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 751, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1675, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a+cx^4\right)^2 \left(d+ex^2\right)} \, dx$$

$$\Big\downarrow \text{1675}$$

$$\int \left(-\frac{c^2 \left(d-ex^2\right) \left(2ae^2+cd^2\right)}{a^2 \left(a+cx^4\right) \left(ae^2+cd^2\right)^2}-\frac{e}{a^2 d^2 x^2}+\frac{1}{a^2 d x^4}-\frac{c^2 \left(d-ex^2\right)}{a \left(a+cx^4\right)^2 \left(ae^2+cd^2\right)}+\frac{e^6}{d^2 \left(d+ex^2\right) \left(ae^2+cd^2\right)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(\sqrt{c}d-\sqrt{a}e\right)\left(2ae^2+cd^2\right)}{2\sqrt{2}a^{11/4}\left(ae^2+cd^2\right)^2}+\frac{c^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(3\sqrt{c}d-\sqrt{a}e\right)}{8\sqrt{2}a^{11/4}\left(ae^2+cd^2\right)}-\frac{c^{5/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{c}d-\sqrt{a}e\right)\left(2ae^2+cd^2\right)}{2\sqrt{2}a^{11/4}\left(ae^2+cd^2\right)^2}-\frac{c^{5/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(3\sqrt{c}d-\sqrt{a}e\right)}{8\sqrt{2}a^{11/4}\left(ae^2+cd^2\right)}+\frac{c^{5/4}\left(\sqrt{a}e+\sqrt{c}d\right)\left(2ae^2+cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{11/4}\left(ae^2+cd^2\right)^2}+\frac{c^{5/4}\left(\sqrt{a}e+3\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{11/4}\left(ae^2+cd^2\right)}-\frac{c^{5/4}\left(\sqrt{a}e+\sqrt{c}d\right)\left(2ae^2+cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{11/4}\left(ae^2+cd^2\right)^2}-\frac{c^{5/4}\left(\sqrt{a}e+3\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{11/4}\left(ae^2+cd^2\right)}-\frac{c^2x\left(d-ex^2\right)}{4a^2\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{e}{a^2d^2x}-\frac{1}{3a^2dx^3}+\frac{e^{11/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\left(ae^2+cd^2\right)^2}$$

input
```
Int[1/(x^4*(d + e*x^2)*(a + c*x^4)^2),x]
```

output
```
-1/3*1/(a^2*d*x^3) + e/(a^2*d^2*x) - (c^2*x*(d - e*x^2))/(4*a^2*(c*d^2 + a
*e^2)*(a + c*x^4)) + (e^(11/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(5/2)*(c*d^
2 + a*e^2)^2) + (c^(5/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(
1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(11/4)*(c*d^2 + a*e^2)) + (c^(5/4)*(Sqrt[c]
*d - Sqrt[a]*e)*(c*d^2 + 2*a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])
/(2*Sqrt[2]*a^(11/4)*(c*d^2 + a*e^2)^2) - (c^(5/4)*(3*Sqrt[c]*d - Sqrt[a]*
e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(11/4)*(c*d^2 + a
*e^2)) - (c^(5/4)*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + 2*a*e^2)*ArcTan[1 + (Sq
rt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(11/4)*(c*d^2 + a*e^2)^2) + (c^(5/
4)*(3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqr
t[c]*x^2])/(16*Sqrt[2]*a^(11/4)*(c*d^2 + a*e^2)) + (c^(5/4)*(Sqrt[c]*d + S
qrt[a]*e)*(c*d^2 + 2*a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2])/(4*Sqrt[2]*a^(11/4)*(c*d^2 + a*e^2)^2) - (c^(5/4)*(3*Sqrt[c]*d +
 Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sq
rt[2]*a^(11/4)*(c*d^2 + a*e^2)) - (c^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*(c*d^2
+ 2*a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt
[2]*a^(11/4)*(c*d^2 + a*e^2)^2)
```

3.258. $\int \frac{1}{x^4(d+ex^2)(a+cx^4)^2}\,dx$

3.258.3.1 Defintions of rubi rules used

rule 1675
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && (IGtQ[p, 0] || IGtQ[q, 0] |
| IntegersQ[m, q])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.258.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 355, normalized size of antiderivative = 0.47

method	result
default	$-\frac{1}{3a^2 d x^3}+\frac{e}{a^2 d^2 x}-\frac{c^2\left(\frac{\left(-\frac{1}{4}a e^3-\frac{1}{4}c d^2 e\right)x^3+\left(\frac{1}{4}d e^2 a+\frac{1}{4}d^3 c\right)x}{c x^4+a}+\frac{\left(11 d e^2 a+7 d^3 c\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\cdots\right)\cdots\right)}{32a}\cdots\right)}{\cdots}$
risch	Expression too large to display

input
```
int(1/x^4/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3/a^2/d/x^3+e/a^2/d^2/x-c^2/(a*e^2+c*d^2)^2/a^2*(((-1/4*a*e^3-1/4*c*d^2
*e)*x^3+(1/4*d*e^2*a+1/4*d^3*c)*x)/(c*x^4+a)+1/32*(11*a*d*e^2+7*c*d^3)*(a/
c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^
(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2
^(1/2)/(a/c)^(1/4)*x-1))+1/32*(-9*a*e^3-5*c*d^2*e)/c/(a/c)^(1/4)*2^(1/2)*(
ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c
)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x
-1)))+1/d^2*e^6/(a*e^2+c*d^2)^2/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2))
```

3.258.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5164 vs. $2(578) = 1156$.

Time = 65.77 (sec) , antiderivative size = 10352, normalized size of antiderivative = 13.78

$$\int \frac{1}{x^4 (d+ex^2) (a+cx^4)^2} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^4/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.258.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (d+ex^2) (a+cx^4)^2} \, dx = \text{Timed out}$$

input
```
integrate(1/x**4/(e*x**2+d)/(c*x**4+a)**2,x)
```

output
```
Timed out
```

3.258.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^4 (d+ex^2) (a+cx^4)^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^4/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.258.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 645, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^4(d+ex^2)(a+cx^4)^2}\,dx = \frac{e^6\arctan\left(\frac{ex}{\sqrt{de}}\right)}{(c^2d^6+2\,acd^4e^2+a^2d^2e^4)\sqrt{de}} - \frac{\left(7\,(ac^3)^{\frac{1}{4}}c^3d^3+11\,(ac^3)^{\frac{1}{4}}ac^2de^2-5\,(ac^3)^{\frac{3}{4}}cd^2e-9\,(ac^3)^{\frac{3}{4}}ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}a^3c^3d^4+2\,\sqrt{2}a^4c^2d^2e^2+\sqrt{2}a^5ce^4\right)} - \frac{\left(7\,(ac^3)^{\frac{1}{4}}c^3d^3+11\,(ac^3)^{\frac{1}{4}}ac^2de^2-5\,(ac^3)^{\frac{3}{4}}cd^2e-9\,(ac^3)^{\frac{3}{4}}ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}a^3c^3d^4+2\,\sqrt{2}a^4c^2d^2e^2+\sqrt{2}a^5ce^4\right)} - \frac{\left(7\,(ac^3)^{\frac{1}{4}}c^3d^3+11\,(ac^3)^{\frac{1}{4}}ac^2de^2+5\,(ac^3)^{\frac{3}{4}}cd^2e+9\,(ac^3)^{\frac{3}{4}}ae^3\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}a^3c^3d^4+2\,\sqrt{2}a^4c^2d^2e^2+\sqrt{2}a^5ce^4\right)} + \frac{\left(7\,(ac^3)^{\frac{1}{4}}c^3d^3+11\,(ac^3)^{\frac{1}{4}}ac^2de^2+5\,(ac^3)^{\frac{3}{4}}cd^2e+9\,(ac^3)^{\frac{3}{4}}ae^3\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}a^3c^3d^4+2\,\sqrt{2}a^4c^2d^2e^2+\sqrt{2}a^5ce^4\right)} + \frac{c^2ex^3-c^2dx}{4\,(a^2cd^2+a^3e^2)(cx^4+a)} + \frac{3\,ex^2-d}{3\,a^2d^2x^3}$$

input
```
integrate(1/x^4/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
e^6*arctan(e*x/sqrt(d*e))/((c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4)*sqrt(d*
e)) - 1/8*(7*(a*c^3)^(1/4)*c^3*d^3 + 11*(a*c^3)^(1/4)*a*c^2*d*e^2 - 5*(a*c
^3)^(3/4)*c*d^2*e - 9*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(
2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^3*c^3*d^4 + 2*sqrt(2)*a^4*c^2*d^2*
e^2 + sqrt(2)*a^5*c*e^4) - 1/8*(7*(a*c^3)^(1/4)*c^3*d^3 + 11*(a*c^3)^(1/4)
*a*c^2*d*e^2 - 5*(a*c^3)^(3/4)*c*d^2*e - 9*(a*c^3)^(3/4)*a*e^3)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^3*c^3*d^4 + 2
*sqrt(2)*a^4*c^2*d^2*e^2 + sqrt(2)*a^5*c*e^4) - 1/16*(7*(a*c^3)^(1/4)*c^3*
d^3 + 11*(a*c^3)^(1/4)*a*c^2*d*e^2 + 5*(a*c^3)^(3/4)*c*d^2*e + 9*(a*c^3)^(
3/4)*a*e^3)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^3*c^3*
d^4 + 2*sqrt(2)*a^4*c^2*d^2*e^2 + sqrt(2)*a^5*c*e^4) + 1/16*(7*(a*c^3)^(1/
4)*c^3*d^3 + 11*(a*c^3)^(1/4)*a*c^2*d*e^2 + 5*(a*c^3)^(3/4)*c*d^2*e + 9*(a
*c^3)^(3/4)*a*e^3)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a
^3*c^3*d^4 + 2*sqrt(2)*a^4*c^2*d^2*e^2 + sqrt(2)*a^5*c*e^4) + 1/4*(c^2*e*x
^3 - c^2*d*x)/((a^2*c*d^2 + a^3*e^2)*(c*x^4 + a)) + 1/3*(3*e*x^2 - d)/(a^2
*d^2*x^3)
```

3.258.9 Mupad [B] (verification not implemented)

Time = 11.04 (sec) , antiderivative size = 20828, normalized size of antiderivative = 27.73

$$\int \frac{1}{x^4 (d+ex^2) (a+cx^4)^2} dx = \text{Too large to display}$$

input
```
int(1/(x^4*(a + c*x^4)^2*(d + e*x^2)),x)
```

output
```
atan(((x*(4917248*a^10*c^18*d^36*e^5 + 50677760*a^11*c^17*d^34*e^7 + 23049
8304*a^12*c^16*d^32*e^9 + 607559680*a^13*c^15*d^30*e^11 + 1026486272*a^14*
c^14*d^28*e^13 + 1166602240*a^15*c^13*d^26*e^15 + 923508736*a^16*c^12*d^24
*e^17 + 539500544*a^17*c^11*d^22*e^19 + 259409920*a^18*c^10*d^20*e^21 + 10
9709312*a^19*c^9*d^18*e^23 + 34537472*a^20*c^8*d^16*e^25 + 5308416*a^21*c^
7*d^14*e^27) - ((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/
2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^
2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^
15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2
*d^4*e^4)))^(1/2)*((x*(1787297792*a^19*c^13*d^31*e^12 - 147587072*a^15*c^1
7*d^39*e^4 - 698089472*a^16*c^16*d^37*e^6 - 1660157952*a^17*c^15*d^35*e^8
- 1588068352*a^18*c^14*d^33*e^10 - 12845056*a^14*c^18*d^41*e^2 + 783967846
4*a^20*c^12*d^29*e^14 + 11879841792*a^21*c^11*d^27*e^16 + 10631249920*a^22
*c^10*d^25*e^18 + 6274940928*a^23*c^9*d^23*e^20 + 2652110848*a^24*c^8*d^21
*e^22 + 891027456*a^25*c^7*d^19*e^24 + 234881024*a^26*c^6*d^17*e^26 + 3355
4432*a^27*c^5*d^15*e^28) + ((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a
^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^
3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/
2))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2
+ 6*a^13*c^2*d^4*e^4)))^(1/2)*(x*((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^...
```

3.259 $\int \frac{x^2}{(1+x^2)\sqrt{1+x^4}}\,dx$

3.259.1 Optimal result

Integrand size = 20, antiderivative size = 70

$$\int \frac{x^2}{(1+x^2)\sqrt{1+x^4}}\,dx = -\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{2\sqrt{2}} + \frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{4\sqrt{1+x^4}}$$

output
```
-1/4*arctan(x*2^(1/2)/(x^4+1)^(1/2))*2^(1/2)+1/4*(x^2+1)*(cos(2*arctan(x))
^2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2*2^(1/2))*((x^4+1
)/(x^2+1)^2)^(1/2)/(x^4+1)^(1/2)
```

3.259.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.57

$$\int \frac{x^2}{(1+x^2)\sqrt{1+x^4}}\,dx = \sqrt[4]{-1}\left(-\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt[4]{-1}x\right),-1\right) + \text{EllipticPi}\left(-i,i\,\text{arcsinh}\left(\sqrt[4]{-1}x\right),-1\right)\right)$$

input
```
Integrate[x^2/((1 + x^2)*Sqrt[1 + x^4]),x]
```

output
```
(-1)^(1/4)*(-EllipticF[I*ArcSinh[(-1)^(1/4)*x], -1] + EllipticPi[-I, I*Arc
Sinh[(-1)^(1/4)*x], -1])
```

3.259.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1655, 761, 2213, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(x^2+1)\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{1655}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{x^4+1}}dx-\frac{1}{2}\int \frac{1-x^2}{(x^2+1)\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{761}$$

$$\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{4\sqrt{x^4+1}}-\frac{1}{2}\int \frac{1-x^2}{(x^2+1)\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{2213}$$

$$\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{4\sqrt{x^4+1}}-\frac{1}{2}\int \frac{1}{\frac{2x^2}{x^4+1}+1}d\frac{x}{\sqrt{x^4+1}}$$

$$\Big\downarrow \text{216}$$

$$\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{4\sqrt{x^4+1}}-\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}$$

input
```
Int[x^2/((1 + x^2)*Sqrt[1 + x^4]),x]
```

output
```
-1/2*ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2] + ((1 + x^2)*Sqrt[(1 + x^4)
/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(4*Sqrt[1 + x^4])
```

3.259.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1655
```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
Simp[1/(2*e)   Int[1/Sqrt[a + c*x^4], x], x] - Simp[1/(2*e)   Int[(d - e*x^
2)/((d + e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && PosQ[
c/a] && EqQ[c*d^2 - a*e^2, 0]
```

rule 2213
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Simp[A   Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^
4]], x] /; FreeQ[{a, c, d, e, A, B}, x] && EqQ[c*d^2 - a*e^2, 0] && EqQ[B*d
 + A*e, 0]
```

3.259.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.06 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.57

method	result	size
default	$\frac{\sqrt{-i x^2+1}\,\sqrt{i x^2+1}\,F\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}+\frac{(-1)^{\frac{3}{4}}\sqrt{-i x^2+1}\,\sqrt{i x^2+1}\,\Pi\left((-1)^{\frac{1}{4}}x,i,-\sqrt{-i}\,(-1)^{\frac{3}{4}}\right)}{\sqrt{x^4+1}}$	110
elliptic	$\frac{\sqrt{-i x^2+1}\,\sqrt{i x^2+1}\,F\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}+\frac{(-1)^{\frac{3}{4}}\sqrt{-i x^2+1}\,\sqrt{i x^2+1}\,\Pi\left((-1)^{\frac{1}{4}}x,i,-\sqrt{-i}\,(-1)^{\frac{3}{4}}\right)}{\sqrt{x^4+1}}$	110

input
```
int(x^2/(x^2+1)/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2
)*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)+(-1)^(3/4)*(1-I*x^2)^(1/2)*(1
+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,I,(-I)^(1/2)/(-1)^(1/4
))
```

3.259.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.46

$$\int \frac{x^2}{(1+x^2)\sqrt{1+x^4}}\,dx = -\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right) - \frac{1}{2}i\sqrt{i}F\left(\arcsin\left(\sqrt{i}x\right)|-1\right)$$

input
```
integrate(x^2/(x^2+1)/(x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*sqrt(2)*arctan(sqrt(2)*x/sqrt(x^4 + 1)) - 1/2*I*sqrt(I)*elliptic_f(ar
csin(sqrt(I)*x), -1)
```

3.259.6 Sympy [F]

$$\int \frac{x^2}{(1+x^2)\sqrt{1+x^4}}\,dx = \int \frac{x^2}{(x^2+1)\sqrt{x^4+1}}\,dx$$

input
```
integrate(x**2/(x**2+1)/(x**4+1)**(1/2),x)
```

output
```
Integral(x**2/((x**2 + 1)*sqrt(x**4 + 1)), x)
```

3.259.7 Maxima [F]

$$\int \frac{x^2}{(1+x^2)\sqrt{1+x^4}}\,dx = \int \frac{x^2}{\sqrt{x^4+1}(x^2+1)}\,dx$$

input
```
integrate(x^2/(x^2+1)/(x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(x^4 + 1)*(x^2 + 1)), x)
```

3.259.8 Giac [F]

$$\int \frac{x^2}{(1+x^2)\sqrt{1+x^4}}\,dx = \int \frac{x^2}{\sqrt{x^4+1}(x^2+1)}\,dx$$

input
```
integrate(x^2/(x^2+1)/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/(sqrt(x^4 + 1)*(x^2 + 1)), x)
```

3.259.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(1+x^2)\sqrt{1+x^4}}\,dx = \int \frac{x^2}{(x^2+1)\sqrt{x^4+1}}\,dx$$

input
```
int(x^2/((x^2 + 1)*(x^4 + 1)^(1/2)),x)
```

output
```
int(x^2/((x^2 + 1)*(x^4 + 1)^(1/2)), x)
```

3.260 $\int \frac{x^2}{(1-x^2)\sqrt{1+x^4}}\,dx$

3.260.1 Optimal result

Integrand size = 22, antiderivative size = 70

$$\int \frac{x^2}{(1-x^2)\sqrt{1+x^4}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{2\sqrt{2}} - \frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\operatorname{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{4\sqrt{1+x^4}}$$

output
```
1/4*arctanh(x*2^(1/2)/(x^4+1)^(1/2))*2^(1/2)-1/4*(x^2+1)*(cos(2*arctan(x))
^2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2*2^(1/2))*((x^4+1
)/(x^2+1)^2)^(1/2)/(x^4+1)^(1/2)
```

3.260.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.51

$$\int \frac{x^2}{(1-x^2)\sqrt{1+x^4}}\,dx = \sqrt[4]{-1}\left(\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt[4]{-1}x\right),-1\right) - \operatorname{EllipticPi}\left(i,\arcsin\left((-1)^{3/4}x\right),-1\right)\right)$$

input
```
Integrate[x^2/((1 - x^2)*Sqrt[1 + x^4]),x]
```

output
```
(-1)^(1/4)*(EllipticF[I*ArcSinh[(-1)^(1/4)*x], -1] - EllipticPi[I, ArcSin[
(-1)^(3/4)*x], -1])
```

3.260.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1655, 761, 2213, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(1-x^2)\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{ 1655}$$

$$\frac{1}{2}\int \frac{x^2+1}{(1-x^2)\sqrt{x^4+1}}dx-\frac{1}{2}\int \frac{1}{\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{ 761}$$

$$\frac{1}{2}\int \frac{x^2+1}{(1-x^2)\sqrt{x^4+1}}dx-\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{4\sqrt{x^4+1}}$$

$$\Big\downarrow \text{ 2213}$$

$$\frac{1}{2}\int \frac{1}{1-\frac{2x^2}{x^4+1}}d\frac{x}{\sqrt{x^4+1}}-\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{4\sqrt{x^4+1}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}-\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{4\sqrt{x^4+1}}$$

input
```
Int[x^2/((1 - x^2)*Sqrt[1 + x^4]),x]
```

output
```
ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]]/(2*Sqrt[2]) - ((1 + x^2)*Sqrt[(1 + x^4)
/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(4*Sqrt[1 + x^4])
```

3.260.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1655
```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
Simp[1/(2*e)   Int[1/Sqrt[a + c*x^4], x], x] - Simp[1/(2*e)   Int[(d - e*x^
2)/((d + e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && PosQ[
c/a] && EqQ[c*d^2 - a*e^2, 0]
```

rule 2213
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Simp[A   Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^
4]], x] /; FreeQ[{a, c, d, e, A, B}, x] && EqQ[c*d^2 - a*e^2, 0] && EqQ[B*d
 + A*e, 0]
```

3.260.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.60

method	result	size
default	$-\frac{\sqrt{-i x^{2}+1}\,\sqrt{i x^{2}+1}\,F\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^{4}+1}}-\frac{(-1)^{\frac{3}{4}}\sqrt{-i x^{2}+1}\,\sqrt{i x^{2}+1}\,\Pi\left((-1)^{\frac{1}{4}}x,-i,-\sqrt{-i}\,(-1)^{\frac{3}{4}}\right)}{\sqrt{x^{4}+1}}$	112
elliptic	$-\frac{\sqrt{-i x^{2}+1}\,\sqrt{i x^{2}+1}\,F\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^{4}+1}}-\frac{(-1)^{\frac{3}{4}}\sqrt{-i x^{2}+1}\,\sqrt{i x^{2}+1}\,\Pi\left((-1)^{\frac{1}{4}}x,-i,-\sqrt{-i}\,(-1)^{\frac{3}{4}}\right)}{\sqrt{x^{4}+1}}$	112

input
```
int(x^2/(-x^2+1)/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/
2)*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)-(-1)^(3/4)*(1-I*x^2)^(1/2)*(
1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,-I,(-I)^(1/2)/(-1)^(1
/4))
```

3.260.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.80

$$\int \frac{x^2}{(1-x^2)\sqrt{1+x^4}}\,dx = \frac{1}{2}i\sqrt{i}F\left(\arcsin\left(\sqrt{i}x\right)\mid -1\right) + \frac{1}{8}\sqrt{2}\log\left(\frac{x^4+2\sqrt{2}\sqrt{x^4+1}x+2x^2+1}{x^4-2x^2+1}\right)$$

input
```
integrate(x^2/(-x^2+1)/(x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*I*sqrt(I)*elliptic_f(arcsin(sqrt(I)*x), -1) + 1/8*sqrt(2)*log((x^4 + 2
*sqrt(2)*sqrt(x^4 + 1)*x + 2*x^2 + 1)/(x^4 - 2*x^2 + 1))
```

3.260.6 Sympy [F]

$$\int \frac{x^2}{(1-x^2)\sqrt{1+x^4}}\,dx = -\int \frac{x^2}{x^2\sqrt{x^4+1}-\sqrt{x^4+1}}\,dx$$

input
```
integrate(x**2/(-x**2+1)/(x**4+1)**(1/2),x)
```

output
```
-Integral(x**2/(x**2*sqrt(x**4 + 1) - sqrt(x**4 + 1)), x)
```

3.260.7 Maxima [F]

$$\int \frac{x^2}{(1-x^2)\sqrt{1+x^4}}\,dx = \int -\frac{x^2}{\sqrt{x^4+1}(x^2-1)}\,dx$$

input
```
integrate(x^2/(-x^2+1)/(x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(x^2/(sqrt(x^4 + 1)*(x^2 - 1)), x)
```

3.260.8 Giac [F]

$$\int \frac{x^2}{(1-x^2)\sqrt{1+x^4}}\,dx = \int -\frac{x^2}{\sqrt{x^4+1}(x^2-1)}\,dx$$

input
```
integrate(x^2/(-x^2+1)/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-x^2/(sqrt(x^4 + 1)*(x^2 - 1)), x)
```

3.260.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(1-x^2)\sqrt{1+x^4}}\,dx = -\int \frac{x^2}{(x^2-1)\sqrt{x^4+1}}\,dx$$

input
```
int(-x^2/((x^2 - 1)*(x^4 + 1)^(1/2)),x)
```

output
```
-int(x^2/((x^2 - 1)*(x^4 + 1)^(1/2)), x)
```

3.261 $\int \frac{x^2}{(1+x^2)\sqrt{1-x^4}}\,dx$

3.261.1 Optimal result

Integrand size = 22, antiderivative size = 99

$$\int \frac{x^2}{(1+x^2)\sqrt{1-x^4}}\,dx = -\frac{x(1-x^2)}{2\sqrt{1-x^4}} - \frac{\sqrt{1-x^2}\sqrt{1+x^2}E(\arcsin(x)|-1)}{2\sqrt{1-x^4}} + \frac{\sqrt{1-x^2}\sqrt{1+x^2}\,\text{EllipticF}(\arcsin(x),-1)}{\sqrt{1-x^4}}$$

output
```
-1/2*x*(-x^2+1)/(-x^4+1)^(1/2)-1/2*EllipticE(x,I)*(-x^2+1)^(1/2)*(x^2+1)^(
1/2)/(-x^4+1)^(1/2)+EllipticF(x,I)*(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(
1/2)
```

3.261.2 Mathematica [A] (verified)

Time = 10.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.46

$$\int \frac{x^2}{(1+x^2)\sqrt{1-x^4}}\,dx = \frac{1}{2}\left(-\frac{x}{\sqrt{1-x^4}} + \frac{x^3}{\sqrt{1-x^4}} - E(\arcsin(x)|-1) + 2\,\text{EllipticF}(\arcsin(x),-1)\right)$$

input
```
Integrate[x^2/((1 + x^2)*Sqrt[1 - x^4]),x]
```

output
```
(-(x/Sqrt[1 - x^4]) + x^3/Sqrt[1 - x^4] - EllipticE[ArcSin[x], -1] + 2*Ell
ipticF[ArcSin[x], -1])/2
```

3.261.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.43, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1388, 373, 326, 284, 327, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(x^2+1)\sqrt{1-x^4}}dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{x^2}{\sqrt{1-x^2}(x^2+1)^{3/2}}dx$$

$$\Big\downarrow \text{373}$$

$$\frac{1}{2}\int \frac{\sqrt{1-x^2}}{\sqrt{x^2+1}}dx-\frac{x\sqrt{1-x^2}}{2\sqrt{x^2+1}}$$

$$\Big\downarrow \text{326}$$

$$\frac{1}{2}\left(2\int \frac{1}{\sqrt{1-x^2}\sqrt{x^2+1}}dx-\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^2}}dx\right)-\frac{x\sqrt{1-x^2}}{2\sqrt{x^2+1}}$$

$$\Big\downarrow \text{284}$$

$$\frac{1}{2}\left(2\int \frac{1}{\sqrt{1-x^4}}dx-\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^2}}dx\right)-\frac{x\sqrt{1-x^2}}{2\sqrt{x^2+1}}$$

$$\Big\downarrow \text{327}$$

$$\frac{1}{2}\left(2\int \frac{1}{\sqrt{1-x^4}}dx-E(\arcsin(x)|-1)\right)-\frac{x\sqrt{1-x^2}}{2\sqrt{x^2+1}}$$

$$\Big\downarrow \text{762}$$

$$\frac{1}{2}(2\,\text{EllipticF}(\arcsin(x),-1)-E(\arcsin(x)|-1))-\frac{x\sqrt{1-x^2}}{2\sqrt{x^2+1}}$$

input
```
Int[x^2/((1 + x^2)*Sqrt[1 - x^4]),x]
```

output
```
-1/2*(x*Sqrt[1 - x^2])/Sqrt[1 + x^2] + (-EllipticE[ArcSin[x], -1] + 2*Elli
pticF[ArcSin[x], -1])/2
```

3.261.3.1 Defintions of rubi rules used

rule 284
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

3.261. $\int \frac{x^2}{(1+x^2)\sqrt{1-x^4}}\,dx$

3.261.4 Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.89

method	result	size
risch	$\frac{x(x^2-1)}{2\sqrt{-x^4+1}}+\frac{F(x,i)\sqrt{-x^2+1}\,\sqrt{x^2+1}}{2\sqrt{-x^4+1}}+\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(F(x,i)-E(x,i))}{2\sqrt{-x^4+1}}$	88
default	$\frac{F(x,i)\sqrt{-x^2+1}\,\sqrt{x^2+1}}{2\sqrt{-x^4+1}}-\frac{(-x^2+1)x}{2\sqrt{(-x^2+1)(x^2+1)}}+\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(F(x,i)-E(x,i))}{2\sqrt{-x^4+1}}$	96
elliptic	$\frac{F(x,i)\sqrt{-x^2+1}\,\sqrt{x^2+1}}{2\sqrt{-x^4+1}}-\frac{(-x^2+1)x}{2\sqrt{(-x^2+1)(x^2+1)}}+\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(F(x,i)-E(x,i))}{2\sqrt{-x^4+1}}$	96

input
```
int(x^2/(x^2+1)/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(x^2-1)/(-x^4+1)^(1/2)+1/2*EllipticF(x,I)*(-x^2+1)^(1/2)*(x^2+1)^(1/
2)/(-x^4+1)^(1/2)+1/2*(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/2)*(Ellipti
cF(x,I)-EllipticE(x,I))
```

3.261.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.42

$$\int \frac{x^2}{(1+x^2)\sqrt{1-x^4}}\,dx = -\frac{(x^2+1)E(\arcsin(x)\,|-1)-2\,(x^2+1)F(\arcsin(x)\,|-1)+\sqrt{-x^4+1}x}{2\,(x^2+1)}$$

input
```
integrate(x^2/(x^2+1)/(-x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*((x^2 + 1)*elliptic_e(arcsin(x), -1) - 2*(x^2 + 1)*elliptic_f(arcsin(
x), -1) + sqrt(-x^4 + 1)*x)/(x^2 + 1)
```

3.261.6 Sympy [F]

$$\int \frac{x^2}{(1+x^2)\sqrt{1-x^4}}\,dx = \int \frac{x^2}{\sqrt{-(x-1)(x+1)(x^2+1)}\,(x^2+1)}\,dx$$

input
```
integrate(x**2/(x**2+1)/(-x**4+1)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(-(x - 1)*(x + 1)*(x**2 + 1))*(x**2 + 1)), x)
```

3.261.7 Maxima [F]

$$\int \frac{x^2}{(1+x^2)\sqrt{1-x^4}}\,dx = \int \frac{x^2}{\sqrt{-x^4+1}(x^2+1)}\,dx$$

input
```
integrate(x^2/(x^2+1)/(-x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(-x^4 + 1)*(x^2 + 1)), x)
```

3.261.8 Giac [F]

$$\int \frac{x^2}{(1+x^2)\sqrt{1-x^4}}\,dx = \int \frac{x^2}{\sqrt{-x^4+1}(x^2+1)}\,dx$$

input
```
integrate(x^2/(x^2+1)/(-x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/(sqrt(-x^4 + 1)*(x^2 + 1)), x)
```

3.261.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(1+x^2)\sqrt{1-x^4}}\,dx = \int \frac{x^2}{(x^2+1)\sqrt{1-x^4}}\,dx$$

input
```
int(x^2/((x^2 + 1)*(1 - x^4)^(1/2)),x)
```

output
```
int(x^2/((x^2 + 1)*(1 - x^4)^(1/2)), x)
```

3.262 $\int \frac{x^2}{(1-x^2)\sqrt{1-x^4}}\,dx$

3.262.1 Optimal result

Integrand size = 24, antiderivative size = 61

$$\int \frac{x^2}{(1-x^2)\sqrt{1-x^4}}\,dx = \frac{x(1+x^2)}{2\sqrt{1-x^4}} - \frac{\sqrt{1-x^2}\sqrt{1+x^2}E(\arcsin(x)|-1)}{2\sqrt{1-x^4}}$$

output
```
1/2*x*(x^2+1)/(-x^4+1)^(1/2)-1/2*EllipticE(x,I)*(-x^2+1)^(1/2)*(x^2+1)^(1/
2)/(-x^4+1)^(1/2)
```

3.262.2 Mathematica [A] (verified)

Time = 10.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.61

$$\int \frac{x^2}{(1-x^2)\sqrt{1-x^4}}\,dx = \frac{x+x^3-\sqrt{1-x^4}E(\arcsin(x)|-1)}{2\sqrt{1-x^4}}$$

input
```
Integrate[x^2/((1 - x^2)*Sqrt[1 - x^4]),x]
```

output
```
(x + x^3 - Sqrt[1 - x^4]*EllipticE[ArcSin[x], -1])/(2*Sqrt[1 - x^4])
```

3.262.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.56, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1388, 373, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(1-x^2)\sqrt{1-x^4}}\,dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{x^2}{(1-x^2)^{3/2}\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{373}$$

$$\frac{x\sqrt{x^2+1}}{2\sqrt{1-x^2}}-\frac{1}{2}\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{327}$$

$$\frac{x\sqrt{x^2+1}}{2\sqrt{1-x^2}}-\frac{1}{2}E(\arcsin(x)|-1)$$

input
```
Int[x^2/((1 - x^2)*Sqrt[1 - x^4]),x]
```

output
```
(x*Sqrt[1 + x^2])/(2*Sqrt[1 - x^2]) - EllipticE[ArcSin[x], -1]/2
```

3.262.3.1 Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

3.262.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.44

method	result	s
risch	$\frac{x(x^2+1)}{2\sqrt{-x^4+1}}-\frac{F(x,i)\sqrt{-x^2+1}\,\sqrt{x^2+1}}{2\sqrt{-x^4+1}}+\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(F(x,i)-E(x,i))}{2\sqrt{-x^4+1}}$	8
elliptic	$-\frac{(-x^2-1)x}{2\sqrt{(x^2-1)(-x^2-1)}}-\frac{F(x,i)\sqrt{-x^2+1}\,\sqrt{x^2+1}}{2\sqrt{-x^4+1}}+\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(F(x,i)-E(x,i))}{2\sqrt{-x^4+1}}$	9
default	$-\frac{F(x,i)\sqrt{-x^2+1}\,\sqrt{x^2+1}}{2\sqrt{-x^4+1}}-\frac{-x^3+x^2-x+1}{4\sqrt{(x+1)(-x^3+x^2-x+1)}}+\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(F(x,i)-E(x,i))}{2\sqrt{-x^4+1}}-\frac{-x^3-x^2-x-1}{4\sqrt{(x-1)(-x^3-x^2-x-1)}}$	1

input
```
int(x^2/(-x^2+1)/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(x^2+1)/(-x^4+1)^(1/2)-1/2*EllipticF(x,I)*(-x^2+1)^(1/2)*(x^2+1)^(1/
2)/(-x^4+1)^(1/2)+1/2*(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/2)*(Ellipti
cF(x,I)-EllipticE(x,I))
```

3.262.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.51

$$\int \frac{x^2}{(1-x^2)\sqrt{1-x^4}}\,dx = -\frac{(x^2-1)E(\arcsin(x)\,|-1)+\sqrt{-x^4+1}x}{2(x^2-1)}$$

input
```
integrate(x^2/(-x^2+1)/(-x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*((x^2 - 1)*elliptic_e(arcsin(x), -1) + sqrt(-x^4 + 1)*x)/(x^2 - 1)
```

3.262.6 Sympy [F]

$$\int \frac{x^2}{(1-x^2)\sqrt{1-x^4}}\,dx = -\int \frac{x^2}{x^2\sqrt{1-x^4}-\sqrt{1-x^4}}\,dx$$

input
```
integrate(x**2/(-x**2+1)/(-x**4+1)**(1/2),x)
```

output
```
-Integral(x**2/(x**2*sqrt(1 - x**4) - sqrt(1 - x**4)), x)
```

3.262.7 Maxima [F]

$$\int \frac{x^2}{(1-x^2)\sqrt{1-x^4}}\,dx = \int -\frac{x^2}{\sqrt{-x^4+1}(x^2-1)}\,dx$$

input
```
integrate(x^2/(-x^2+1)/(-x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(x^2/(sqrt(-x^4 + 1)*(x^2 - 1)), x)
```

3.262.8 Giac [F]

$$\int \frac{x^2}{(1-x^2)\sqrt{1-x^4}}\,dx = \int -\frac{x^2}{\sqrt{-x^4+1}(x^2-1)}\,dx$$

input
```
integrate(x^2/(-x^2+1)/(-x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-x^2/(sqrt(-x^4 + 1)*(x^2 - 1)), x)
```

3.262.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(1-x^2)\sqrt{1-x^4}}\,dx = -\int \frac{x^2}{(x^2-1)\sqrt{1-x^4}}\,dx$$

input
```
int(-x^2/((x^2 - 1)*(1 - x^4)^(1/2)),x)
```

output
```
-int(x^2/((x^2 - 1)*(1 - x^4)^(1/2)), x)
```

3.263 $\int \frac{x^2}{(1+x^2)\sqrt{-1+x^4}}\,dx$

3.263.1 Optimal result

Integrand size = 20, antiderivative size = 113

$$\int \frac{x^2}{(1+x^2)\sqrt{-1+x^4}}\,dx = -\frac{x(1-x^2)}{2\sqrt{-1+x^4}} - \frac{\sqrt{1-x^2}\sqrt{1+x^2}E(\arcsin(x)|-1)}{2\sqrt{-1+x^4}} + \frac{\sqrt{-1+x^2}\sqrt{1+x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{-1+x^2}}\right),\frac{1}{2}\right)}{\sqrt{2}\sqrt{-1+x^4}}$$

output
```
-1/2*x*(-x^2+1)/(x^4-1)^(1/2)-1/2*EllipticE(x,I)*(-x^2+1)^(1/2)*(x^2+1)^(1
/2)/(x^4-1)^(1/2)+1/2*EllipticF(x*2^(1/2)/(x^2-1)^(1/2),1/2*2^(1/2))*(x^2-
1)^(1/2)*(x^2+1)^(1/2)*2^(1/2)/(x^4-1)^(1/2)
```

3.263.2 Mathematica [A] (verified)

Time = 10.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.48

$$\int \frac{x^2}{(1+x^2)\sqrt{-1+x^4}}\,dx = \frac{-x+x^3-\sqrt{1-x^4}E(\arcsin(x)|-1)+2\sqrt{1-x^4}\,\text{EllipticF}(\arcsin(x),-1)}{2\sqrt{-1+x^4}}$$

input
```
Integrate[x^2/((1 + x^2)*Sqrt[-1 + x^4]),x]
```

output
```
(-x + x^3 - Sqrt[1 - x^4]*EllipticE[ArcSin[x], -1] + 2*Sqrt[1 - x^4]*Ellip
ticF[ArcSin[x], -1])/(2*Sqrt[-1 + x^4])
```

3.263.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.59, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1393}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(x^2+1)\sqrt{x^4-1}}\,dx$$

$$\Big\downarrow \; 1393$$

$$\frac{\sqrt{x^2-1}\sqrt{x^2+1}E\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4-1}}-\frac{x}{\sqrt{x^4-1}}$$

input
```
Int[x^2/((1 + x^2)*Sqrt[-1 + x^4]),x]
```

output
```
-(x/Sqrt[-1 + x^4]) + (Sqrt[-1 + x^2]*Sqrt[1 + x^2]*EllipticE[ArcSin[(Sqrt
[2]*x)/Sqrt[-1 + x^2]], 1/2])/(Sqrt[2]*Sqrt[-1 + x^4])
```

3.263.3.1 Defintions of rubi rules used

rule 1393
```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
Simp[-x/(e*Sqrt[a + c*x^4]), x] + Simp[(Sqrt[-1 + (e/d)*x^2]*Sqrt[1 + (e/d)
*x^2]*EllipticE[ArcSin[(Sqrt[2]*Rt[e/d, 2]*x)/Sqrt[-1 + (e/d)*x^2]], 1/2])/
(Sqrt[2]*e*Rt[e/d, 2]*Sqrt[a + c*x^4]), x] /; FreeQ[{a, c, d, e}, x] && EqQ
[c*d^2 + a*e^2, 0] && LtQ[a, 0] && GtQ[c, 0] && PosQ[e/d]
```

3.263.4 Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.82

method	result	size
risch	$\frac{x(x^2-1)}{2\sqrt{x^4-1}}-\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,F(ix,i)}{2\sqrt{x^4-1}}+\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,(F(ix,i)-E(ix,i))}{2\sqrt{x^4-1}}$	93
default	$-\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,F(ix,i)}{2\sqrt{x^4-1}}+\frac{(x^2-1)x}{2\sqrt{(x^2+1)(x^2-1)}}+\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,(F(ix,i)-E(ix,i))}{2\sqrt{x^4-1}}$	99
elliptic	$-\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,F(ix,i)}{2\sqrt{x^4-1}}+\frac{(x^2-1)x}{2\sqrt{(x^2+1)(x^2-1)}}+\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,(F(ix,i)-E(ix,i))}{2\sqrt{x^4-1}}$	99

input
```
int(x^2/(x^2+1)/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(x^2-1)/(x^4-1)^(1/2)-1/2*I*(x^2+1)^(1/2)*(-x^2+1)^(1/2)/(x^4-1)^(1/
2)*EllipticF(I*x,I)+1/2*I*(x^2+1)^(1/2)*(-x^2+1)^(1/2)/(x^4-1)^(1/2)*(Elli
pticF(I*x,I)-EllipticE(I*x,I))
```

3.263.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.39

$$\int \frac{x^2}{(1+x^2)\sqrt{-1+x^4}}\,dx = \frac{(i\,x^2+i)E(\arcsin{(x)}\mid -1)-2\,(i\,x^2+i)F(\arcsin{(x)}\mid -1)+\sqrt{x^4-1}x}{2\,(x^2+1)}$$

input
```
integrate(x^2/(x^2+1)/(x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*((I*x^2 + I)*elliptic_e(arcsin(x), -1) - 2*(I*x^2 + I)*elliptic_f(arcs
in(x), -1) + sqrt(x^4 - 1)*x)/(x^2 + 1)
```

3.263.6 Sympy [F]

$$\int \frac{x^2}{(1+x^2)\sqrt{-1+x^4}}\,dx = \int \frac{x^2}{\sqrt{(x-1)(x+1)(x^2+1)}\,(x^2+1)}\,dx$$

input
```
integrate(x**2/(x**2+1)/(x**4-1)**(1/2),x)
```

output
```
Integral(x**2/(sqrt((x - 1)*(x + 1)*(x**2 + 1))*(x**2 + 1)), x)
```

3.263.7 Maxima [F]

$$\int \frac{x^2}{(1+x^2)\sqrt{-1+x^4}}\,dx = \int \frac{x^2}{\sqrt{x^4-1}(x^2+1)}\,dx$$

input
```
integrate(x^2/(x^2+1)/(x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(x^4 - 1)*(x^2 + 1)), x)
```

3.263.8 Giac [F]

$$\int \frac{x^2}{(1+x^2)\sqrt{-1+x^4}}\,dx = \int \frac{x^2}{\sqrt{x^4-1}(x^2+1)}\,dx$$

input
```
integrate(x^2/(x^2+1)/(x^4-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/(sqrt(x^4 - 1)*(x^2 + 1)), x)
```

3.263.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(1+x^2)\sqrt{-1+x^4}}\,dx = \int \frac{x^2}{(x^2+1)\,\sqrt{x^4-1}}\,dx$$

input
```
int(x^2/((x^2 + 1)*(x^4 - 1)^(1/2)),x)
```

output
```
int(x^2/((x^2 + 1)*(x^4 - 1)^(1/2)), x)
```

3.264 $\int \frac{x^2}{(1-x^2)\sqrt{-1+x^4}}\,dx$

3.264.1 Optimal result

Integrand size = 22, antiderivative size = 57

$$\int \frac{x^2}{(1-x^2)\sqrt{-1+x^4}}\,dx = \frac{x(1+x^2)}{2\sqrt{-1+x^4}} - \frac{\sqrt{1-x^2}\sqrt{1+x^2}E(\arcsin(x)|-1)}{2\sqrt{-1+x^4}}$$

output
```
1/2*x*(x^2+1)/(x^4-1)^(1/2)-1/2*EllipticE(x,I)*(-x^2+1)^(1/2)*(x^2+1)^(1/2
)/(x^4-1)^(1/2)
```

3.264.2 Mathematica [A] (verified)

Time = 10.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.61

$$\int \frac{x^2}{(1-x^2)\sqrt{-1+x^4}}\,dx = \frac{x+x^3-\sqrt{1-x^4}E(\arcsin(x)|-1)}{2\sqrt{-1+x^4}}$$

input
```
Integrate[x^2/((1 - x^2)*Sqrt[-1 + x^4]),x]
```

output
```
(x + x^3 - Sqrt[1 - x^4]*EllipticE[ArcSin[x], -1])/(2*Sqrt[-1 + x^4])
```

3.264.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.74, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(1-x^2)\sqrt{x^4-1}}\,dx$$

$$\Big\downarrow \ 1394$$

$$\frac{x\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{x^2-1}}{\sqrt{x^4-1}}\right),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2}}-\frac{xE\left(\arcsin\left(\frac{\sqrt{2}\sqrt{x^2-1}}{\sqrt{x^4-1}}\right)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2}}+\frac{x}{\sqrt{x^4-1}}$$

input
```
Int[x^2/((1 - x^2)*Sqrt[-1 + x^4]),x]
```

output
```
x/Sqrt[-1 + x^4] - (x*EllipticE[ArcSin[(Sqrt[2]*Sqrt[-1 + x^2])/Sqrt[-1 +
x^4]], 1/2])/(Sqrt[2]*Sqrt[x^2]) + (x*EllipticF[ArcSin[(Sqrt[2]*Sqrt[-1 +
x^2])/Sqrt[-1 + x^4]], 1/2])/(Sqrt[2]*Sqrt[x^2])
```

3.264.3.1 Defintions of rubi rules used

rule 1394
```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
Simp[-x/(e*Sqrt[a + c*x^4]), x] + (Simp[(x/(e*Sqrt[-2*a]*Sqrt[(-e/d)*x^2]))
*EllipticE[ArcSin[(Sqrt[-2*a]*Sqrt[-1 - (e/d)*x^2])/Sqrt[a + c*x^4]], 1/2],
 x] - Simp[(x/(e*Sqrt[-2*a]*Sqrt[(-e/d)*x^2]))*EllipticF[ArcSin[(Sqrt[-2*a]
*Sqrt[-1 - (e/d)*x^2])/Sqrt[a + c*x^4]], 1/2], x]) /; FreeQ[{a, c, d, e}, x
] && EqQ[c*d^2 + a*e^2, 0] && LtQ[a, 0] && GtQ[c, 0] && NegQ[e/d]
```

3.264.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(45) = 90.

Time = 1.15 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.63

method	result	siz
risch	$\frac{x(x^2+1)}{2\sqrt{x^4-1}} + \frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,F(ix,i)}{2\sqrt{x^4-1}} + \frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,(F(ix,i)-E(ix,i))}{2\sqrt{x^4-1}}$	93
elliptic	$\frac{(x^2+1)x}{2\sqrt{(x^2+1)(x^2-1)}} + \frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,F(ix,i)}{2\sqrt{x^4-1}} + \frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,(F(ix,i)-E(ix,i))}{2\sqrt{x^4-1}}$	99
default	$\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,F(ix,i)}{2\sqrt{x^4-1}} + \frac{x^3-x^2+x-1}{4\sqrt{(x+1)(x^3-x^2+x-1)}} + \frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,(F(ix,i)-E(ix,i))}{2\sqrt{x^4-1}} + \frac{x^3+x^2+x+1}{4\sqrt{(x-1)(x^3+x^2+x+1)}}$	13

input
```
int(x^2/(-x^2+1)/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(x^2+1)/(x^4-1)^(1/2)+1/2*I*(x^2+1)^(1/2)*(-x^2+1)^(1/2)/(x^4-1)^(1/
2)*EllipticF(I*x,I)+1/2*I*(x^2+1)^(1/2)*(-x^2+1)^(1/2)/(x^4-1)^(1/2)*(Elli
pticF(I*x,I)-EllipticE(I*x,I))
```

3.264.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.54

$$\int \frac{x^2}{(1-x^2)\sqrt{-1+x^4}}\,dx = \frac{(i\,x^2-i)E(\arcsin(x)\mid -1)+\sqrt{x^4-1}x}{2\,(x^2-1)}$$

input
```
integrate(x^2/(-x^2+1)/(x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*((I*x^2 - I)*elliptic_e(arcsin(x), -1) + sqrt(x^4 - 1)*x)/(x^2 - 1)
```

3.264.6 Sympy [F]

$$\int \frac{x^2}{(1-x^2)\sqrt{-1+x^4}}\,dx = -\int \frac{x^2}{x^2\sqrt{x^4-1}-\sqrt{x^4-1}}\,dx$$

input
```
integrate(x**2/(-x**2+1)/(x**4-1)**(1/2),x)
```

output
```
-Integral(x**2/(x**2*sqrt(x**4 - 1) - sqrt(x**4 - 1)), x)
```

3.264.7 Maxima [F]

$$\int \frac{x^2}{(1-x^2)\sqrt{-1+x^4}}\,dx = \int -\frac{x^2}{\sqrt{x^4-1}(x^2-1)}\,dx$$

input
```
integrate(x^2/(-x^2+1)/(x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(x^2/(sqrt(x^4 - 1)*(x^2 - 1)), x)
```

3.264.8 Giac [F]

$$\int \frac{x^2}{(1-x^2)\sqrt{-1+x^4}}\,dx = \int -\frac{x^2}{\sqrt{x^4-1}(x^2-1)}\,dx$$

input
```
integrate(x^2/(-x^2+1)/(x^4-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-x^2/(sqrt(x^4 - 1)*(x^2 - 1)), x)
```

3.264.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(1-x^2)\sqrt{-1+x^4}}\,dx = -\int \frac{x^2}{(x^2-1)\sqrt{x^4-1}}\,dx$$

input
```
int(-x^2/((x^2 - 1)*(x^4 - 1)^(1/2)),x)
```

output
```
-int(x^2/((x^2 - 1)*(x^4 - 1)^(1/2)), x)
```

3.265 $\int \frac{x^2}{(1+x^2)\sqrt{-1-x^4}}\,dx$

3.265.1 Optimal result

Integrand size = 22, antiderivative size = 74

$$\int \frac{x^2}{(1+x^2)\sqrt{-1-x^4}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{-1-x^4}}\right)}{2\sqrt{2}} + \frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\operatorname{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{4\sqrt{-1-x^4}}$$

output
```
-1/4*arctanh(x*2^(1/2)/(-x^4-1)^(1/2))*2^(1/2)+1/4*(x^2+1)*(cos(2*arctan(x
))^2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2*2^(1/2))*((x^4
+1)/(x^2+1)^2)^(1/2)/(-x^4-1)^(1/2)
```

3.265.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{(1+x^2)\sqrt{-1-x^4}}\,dx = \frac{\sqrt[4]{-1}\sqrt{1+x^4}\left(-\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt[4]{-1}x\right),-1\right)+\operatorname{EllipticPi}\left(-i,i\operatorname{arcsinh}\left(\sqrt[4]{-1}x\right),-1\right)\right)}{\sqrt{-1-x^4}}$$

input
```
Integrate[x^2/((1 + x^2)*Sqrt[-1 - x^4]),x]
```

output
```
((-1)^(1/4)*Sqrt[1 + x^4]*(-EllipticF[I*ArcSinh[(-1)^(1/4)*x], -1] + Ellip
ticPi[-I, I*ArcSinh[(-1)^(1/4)*x], -1]))/Sqrt[-1 - x^4]
```

3.265.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1655, 761, 2213, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(x^2+1)\sqrt{-x^4-1}}dx$$

$$\Big\downarrow \text{1655}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{-x^4-1}}dx-\frac{1}{2}\int \frac{1-x^2}{(x^2+1)\sqrt{-x^4-1}}dx$$

$$\Big\downarrow \text{761}$$

$$\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{4\sqrt{-x^4-1}}-\frac{1}{2}\int \frac{1-x^2}{(x^2+1)\sqrt{-x^4-1}}dx$$

$$\Big\downarrow \text{2213}$$

$$\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{4\sqrt{-x^4-1}}-\frac{1}{2}\int \frac{1}{1-\frac{2x^2}{-x^4-1}}d\frac{x}{\sqrt{-x^4-1}}$$

$$\Big\downarrow \text{219}$$

$$\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{4\sqrt{-x^4-1}}-\frac{\text{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{-x^4-1}}\right)}{2\sqrt{2}}$$

input
```
Int[x^2/((1 + x^2)*Sqrt[-1 - x^4]),x]
```

output
```
-1/2*ArcTanh[(Sqrt[2]*x)/Sqrt[-1 - x^4]]/Sqrt[2] + ((1 + x^2)*Sqrt[(1 + x^
4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(4*Sqrt[-1 - x^4])
```

3.265.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1655
```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
Simp[1/(2*e)   Int[1/Sqrt[a + c*x^4], x], x] - Simp[1/(2*e)   Int[(d - e*x^
2)/((d + e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && PosQ[
c/a] && EqQ[c*d^2 - a*e^2, 0]
```

rule 2213
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Simp[A   Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^
4]], x] /; FreeQ[{a, c, d, e, A, B}, x] && EqQ[c*d^2 - a*e^2, 0] && EqQ[B*d
 + A*e, 0]
```

3.265.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.40 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.27

method	result
default	$\frac{\sqrt{i x^2+1}\,\sqrt{-i x^2+1}\,F\left(\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)x,i\right)}{\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)\sqrt{-x^4-1}}-\frac{i\sqrt{-i}\,\sqrt{i x^2+1}\,\sqrt{-i x^2+1}\,\Pi\left(\sqrt{-i}\,x,-i,\frac{(-1)^{\frac{1}{4}}}{\sqrt{-i}}\right)}{2\sqrt{-x^4-1}}-\frac{\sqrt{i x^2+1}\,\sqrt{-i x^2+1}\,\Pi\left(\sqrt{-i}\,x,-i,\frac{(-1}{\sqrt{-}}\right.}{2\sqrt{-i}\,\sqrt{-x^4-1}}$
elliptic	$\frac{\sqrt{i x^2+1}\,\sqrt{-i x^2+1}\,F\left(\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)x,i\right)}{\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)\sqrt{-x^4-1}}-\frac{i\sqrt{-i}\,\sqrt{i x^2+1}\,\sqrt{-i x^2+1}\,\Pi\left(\sqrt{-i}\,x,-i,\frac{(-1)^{\frac{1}{4}}}{\sqrt{-i}}\right)}{2\sqrt{-x^4-1}}-\frac{\sqrt{i x^2+1}\,\sqrt{-i x^2+1}\,\Pi\left(\sqrt{-i}\,x,-i,\frac{(-1}{\sqrt{-}}\right.}{2\sqrt{-i}\,\sqrt{-x^4-1}}$

input
```
int(x^2/(x^2+1)/(-x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(1/2*2^(1/2)-1/2*I*2^(1/2))*(1+I*x^2)^(1/2)*(1-I*x^2)^(1/2)/(-x^4-1)^(1/
2)*EllipticF((1/2*2^(1/2)-1/2*I*2^(1/2))*x,I)-1/2*I*(-I)^(1/2)*(1+I*x^2)^(
1/2)*(1-I*x^2)^(1/2)/(-x^4-1)^(1/2)*EllipticPi((-I)^(1/2)*x,-I,(-1)^(1/4)/
(-I)^(1/2))-1/2/(-I)^(1/2)*(1+I*x^2)^(1/2)*(1-I*x^2)^(1/2)/(-x^4-1)^(1/2)*
EllipticPi((-I)^(1/2)*x,-I,(-1)^(1/4)/(-I)^(1/2))
```

3.265.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01

$$\int \frac{x^2}{(1+x^2)\sqrt{-1-x^4}}\,dx = -\frac{1}{2}\sqrt{i}F\left(\arcsin\left(\sqrt{i}x\right)\middle| -1\right) - \frac{1}{8}\sqrt{2}\log\left(\frac{\sqrt{2}x+\sqrt{-x^4-1}}{x^2+1}\right) + \frac{1}{8}\sqrt{2}\log\left(-\frac{\sqrt{2}x-\sqrt{-x^4-1}}{x^2+1}\right)$$

input
```
integrate(x^2/(x^2+1)/(-x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(I)*elliptic_f(arcsin(sqrt(I)*x), -1) - 1/8*sqrt(2)*log((sqrt(2)*
x + sqrt(-x^4 - 1))/(x^2 + 1)) + 1/8*sqrt(2)*log(-(sqrt(2)*x - sqrt(-x^4 -
 1))/(x^2 + 1))
```

3.265.6 Sympy [F]

$$\int \frac{x^2}{(1+x^2)\sqrt{-1-x^4}}\,dx = \int \frac{x^2}{(x^2+1)\sqrt{-x^4-1}}\,dx$$

input
```
integrate(x**2/(x**2+1)/(-x**4-1)**(1/2),x)
```

output
```
Integral(x**2/((x**2 + 1)*sqrt(-x**4 - 1)), x)
```

3.265.7 Maxima [F]

$$\int \frac{x^2}{(1+x^2)\sqrt{-1-x^4}}\,dx = \int \frac{x^2}{\sqrt{-x^4-1}(x^2+1)}\,dx$$

input
```
integrate(x^2/(x^2+1)/(-x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(-x^4 - 1)*(x^2 + 1)), x)
```

3.265.8 Giac [F]

$$\int \frac{x^2}{(1+x^2)\sqrt{-1-x^4}}\,dx = \int \frac{x^2}{\sqrt{-x^4-1}(x^2+1)}\,dx$$

input
```
integrate(x^2/(x^2+1)/(-x^4-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/(sqrt(-x^4 - 1)*(x^2 + 1)), x)
```

3.265.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(1+x^2)\sqrt{-1-x^4}}\,dx = \int \frac{x^2}{(x^2+1)\,\sqrt{-x^4-1}}\,dx$$

input
```
int(x^2/((x^2 + 1)*(- x^4 - 1)^(1/2)),x)
```

output
```
int(x^2/((x^2 + 1)*(- x^4 - 1)^(1/2)), x)
```

3.266 $\int \frac{x^2}{(1-x^2)\sqrt{-1-x^4}}\,dx$

3.266.1 Optimal result

Integrand size = 24, antiderivative size = 74

$$\int \frac{x^2}{(1-x^2)\sqrt{-1-x^4}}\,dx = \frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt{-1-x^4}}\right)}{2\sqrt{2}} - \frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{4\sqrt{-1-x^4}}$$

output
```
1/4*arctan(x*2^(1/2)/(-x^4-1)^(1/2))*2^(1/2)-1/4*(x^2+1)*(cos(2*arctan(x))
^2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2*2^(1/2))*((x^4+1
)/(x^2+1)^2)^(1/2)/(-x^4-1)^(1/2)
```

3.266.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.17 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.76

$$\int \frac{x^2}{(1-x^2)\sqrt{-1-x^4}}\,dx = \frac{\sqrt[4]{-1}\sqrt{1+x^4}\left(\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt[4]{-1}x\right),-1\right)-\text{EllipticPi}\left(i,\arcsin\left((-1)^{3/4}x\right),-1\right)\right)}{\sqrt{-1-x^4}}$$

input
```
Integrate[x^2/((1 - x^2)*Sqrt[-1 - x^4]),x]
```

output
```
((-1)^(1/4)*Sqrt[1 + x^4]*(EllipticF[I*ArcSinh[(-1)^(1/4)*x], -1] - Ellipt
icPi[I, ArcSin[(-1)^(3/4)*x], -1]))/Sqrt[-1 - x^4]
```

3.266.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1655, 761, 2213, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(1-x^2)\sqrt{-x^4-1}}dx$$

$$\Big\downarrow \text{ 1655}$$

$$\frac{1}{2}\int \frac{x^2+1}{(1-x^2)\sqrt{-x^4-1}}dx-\frac{1}{2}\int \frac{1}{\sqrt{-x^4-1}}dx$$

$$\Big\downarrow \text{ 761}$$

$$\frac{1}{2}\int \frac{x^2+1}{(1-x^2)\sqrt{-x^4-1}}dx-\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{4\sqrt{-x^4-1}}$$

$$\Big\downarrow \text{ 2213}$$

$$\frac{1}{2}\int \frac{1}{\frac{2x^2}{-x^4-1}+1}d\frac{x}{\sqrt{-x^4-1}}-\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{4\sqrt{-x^4-1}}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt{-x^4-1}}\right)}{2\sqrt{2}}-\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{4\sqrt{-x^4-1}}$$

input
```
Int[x^2/((1 - x^2)*Sqrt[-1 - x^4]),x]
```

output
```
ArcTan[(Sqrt[2]*x)/Sqrt[-1 - x^4]]/(2*Sqrt[2]) - ((1 + x^2)*Sqrt[(1 + x^4)
/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(4*Sqrt[-1 - x^4])
```

3.266.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1655
```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
Simp[1/(2*e)   Int[1/Sqrt[a + c*x^4], x], x] - Simp[1/(2*e)   Int[(d - e*x^
2)/((d + e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && PosQ[
c/a] && EqQ[c*d^2 - a*e^2, 0]
```

rule 2213
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Simp[A   Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^
4]], x] /; FreeQ[{a, c, d, e, A, B}, x] && EqQ[c*d^2 - a*e^2, 0] && EqQ[B*d
 + A*e, 0]
```

3.266.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.47 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.55

method	result	size
default	$-\frac{\sqrt{i x^2+1}\,\sqrt{-i x^2+1}\,F\left(\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)x,i\right)}{\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)\sqrt{-x^4-1}}+\frac{\sqrt{i x^2+1}\,\sqrt{-i x^2+1}\,\Pi\left(\sqrt{-i}\,x,i,\frac{(-1)^{\frac{1}{4}}}{\sqrt{-i}}\right)}{\sqrt{-i}\,\sqrt{-x^4-1}}$	115
elliptic	$-\frac{\sqrt{i x^2+1}\,\sqrt{-i x^2+1}\,F\left(\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)x,i\right)}{\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)\sqrt{-x^4-1}}+\frac{\sqrt{i x^2+1}\,\sqrt{-i x^2+1}\,\Pi\left(\sqrt{-i}\,x,i,\frac{(-1)^{\frac{1}{4}}}{\sqrt{-i}}\right)}{\sqrt{-i}\,\sqrt{-x^4-1}}$	115

input
```
int(x^2/(-x^2+1)/(-x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(1/2*2^(1/2)-1/2*I*2^(1/2))*(1+I*x^2)^(1/2)*(1-I*x^2)^(1/2)/(-x^4-1)^(1
/2)*EllipticF((1/2*2^(1/2)-1/2*I*2^(1/2))*x,I)+1/(-I)^(1/2)*(1+I*x^2)^(1/2
)*(1-I*x^2)^(1/2)/(-x^4-1)^(1/2)*EllipticPi((-I)^(1/2)*x,I,(-1)^(1/4)/(-I)
^(1/2))
```

3.266.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1-x^2)\sqrt{-1-x^4}}\,dx = \frac{1}{2}\sqrt{i}F\left(\arcsin\left(\sqrt{i}x\right)\,|-1\right) - \frac{1}{8}i\sqrt{2}\log\left(\frac{i\sqrt{2}x+\sqrt{-x^4-1}}{x^2-1}\right) + \frac{1}{8}i\sqrt{2}\log\left(\frac{-i\sqrt{2}x+\sqrt{-x^4-1}}{x^2-1}\right)$$

input
```
integrate(x^2/(-x^2+1)/(-x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(I)*elliptic_f(arcsin(sqrt(I)*x), -1) - 1/8*I*sqrt(2)*log((I*sqrt(
2)*x + sqrt(-x^4 - 1))/(x^2 - 1)) + 1/8*I*sqrt(2)*log((-I*sqrt(2)*x + sqrt
(-x^4 - 1))/(x^2 - 1))
```

3.266.6 Sympy [F]

$$\int \frac{x^2}{(1-x^2)\sqrt{-1-x^4}}\,dx = -\int \frac{x^2}{x^2\sqrt{-x^4-1}-\sqrt{-x^4-1}}\,dx$$

input
```
integrate(x**2/(-x**2+1)/(-x**4-1)**(1/2),x)
```

output
```
-Integral(x**2/(x**2*sqrt(-x**4 - 1) - sqrt(-x**4 - 1)), x)
```

3.266.7 Maxima [F]

$$\int \frac{x^2}{(1-x^2)\sqrt{-1-x^4}}\,dx = \int -\frac{x^2}{\sqrt{-x^4-1}(x^2-1)}\,dx$$

input
```
integrate(x^2/(-x^2+1)/(-x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(x^2/(sqrt(-x^4 - 1)*(x^2 - 1)), x)
```

3.266.8 Giac [F]

$$\int \frac{x^2}{(1-x^2)\sqrt{-1-x^4}}\,dx = \int -\frac{x^2}{\sqrt{-x^4-1}(x^2-1)}\,dx$$

input
```
integrate(x^2/(-x^2+1)/(-x^4-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-x^2/(sqrt(-x^4 - 1)*(x^2 - 1)), x)
```

3.266.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(1-x^2)\sqrt{-1-x^4}}\,dx = -\int \frac{x^2}{(x^2-1)\sqrt{-x^4-1}}\,dx$$

input
```
int(-x^2/((x^2 - 1)*(- x^4 - 1)^(1/2)),x)
```

output
```
-int(x^2/((x^2 - 1)*(- x^4 - 1)^(1/2)), x)
```

3.267 $\int x^2\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$

3.267.1 Optimal result

Integrand size = 37, antiderivative size = 243

$$\int x^2\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = -\frac{c(bc-2ad)x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{16d^2(a+bx^2)} - \frac{(bc-2ad)x^3\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{8d(a+bx^2)} + \frac{bx^3(c+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{6d(a+bx^2)} + \frac{c^2(bc-2ad)\sqrt{a^2+2abx^2+b^2x^4}\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16d^{5/2}(a+bx^2)}$$

output
```
1/6*b*x^3*(d*x^2+c)^(3/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)+1/16*c^2*(-2*a*d
+b*c)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))*((b*x^2+a)^2)^(1/2)/d^(5/2)/(b*x^
2+a)-1/16*c*(-2*a*d+b*c)*x*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/d^2/(b*x^2+
a)-1/8*(-2*a*d+b*c)*x^3*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)
```

3.267.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.53

$$\int x^2\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$$
$$=\frac{\sqrt{(a+bx^2)^2}\left(\sqrt{d}x\sqrt{c+dx^2}(6ad(c+2dx^2)+b(-3c^2+2cdx^2+8d^2x^4))+6c^2(bc-2ad)\text{arctanh}\left(\frac{\sqrt{d}x}{-\sqrt{c}+\sqrt{c}}\right.\right.}{48d^{5/2}\,(a+bx^2)}$$

input
```
Integrate[x^2*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
(Sqrt[(a + b*x^2)^2]*(Sqrt[d]*x*Sqrt[c + d*x^2]*(6*a*d*(c + 2*d*x^2) + b*(
-3*c^2 + 2*c*d*x^2 + 8*d^2*x^4)) + 6*c^2*(b*c - 2*a*d)*ArcTanh[(Sqrt[d]*x)
/(-Sqrt[c] + Sqrt[c + d*x^2])]))/(48*d^(5/2)*(a + b*x^2))
```

3.267.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.59, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {1384, 27, 363, 248, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int bx^2\left(bx^2+a\right)\sqrt{dx^2+c}dx}{b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int x^2\left(bx^2+a\right)\sqrt{dx^2+c}dx}{a+bx^2}$$

$$\Big\downarrow \text{363}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{bx^3\left(c+dx^2\right)^{3/2}}{6d}-\frac{(bc-2ad)\int x^2\sqrt{dx^2+c}dx}{2d}\right)}{a+bx^2}$$

$$\Big\downarrow \text{248}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{bx^3(c+dx^2)^{3/2}}{6d}-\frac{(bc-2ad)\left(\frac{1}{4}c\int\frac{x^2}{\sqrt{dx^2+c}}dx+\frac{1}{4}x^3\sqrt{c+dx^2}\right)}{2d}\right)}{a+bx^2}$$

$$\Big\downarrow \text{262}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{bx^3(c+dx^2)^{3/2}}{6d}-\frac{(bc-2ad)\left(\frac{1}{4}c\left(\frac{x\sqrt{c+dx^2}}{2d}-\frac{c\int\frac{1}{\sqrt{dx^2+c}}dx}{2d}\right)+\frac{1}{4}x^3\sqrt{c+dx^2}\right)}{2d}\right)}{a+bx^2}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{bx^3(c+dx^2)^{3/2}}{6d}-\frac{(bc-2ad)\left(\frac{1}{4}c\left(\frac{x\sqrt{c+dx^2}}{2d}-\frac{c\int\frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{2d}\right)+\frac{1}{4}x^3\sqrt{c+dx^2}\right)}{2d}\right)}{a+bx^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{bx^3(c+dx^2)^{3/2}}{6d}-\frac{(bc-2ad)\left(\frac{1}{4}c\left(\frac{x\sqrt{c+dx^2}}{2d}-\frac{c\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2d^{3/2}}\right)+\frac{1}{4}x^3\sqrt{c+dx^2}\right)}{2d}\right)}{a+bx^2}$$

input
```
Int[x^2*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((b*x^3*(c + d*x^2)^(3/2))/(6*d) - ((b*c
- 2*a*d)*((x^3*Sqrt[c + d*x^2])/4 + (c*((x*Sqrt[c + d*x^2])/(2*d) - (c*Arc
Tanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*d^(3/2))))/4))/(2*d)))/(a + b*x^2)
```

3.267. $\int x^2\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$

3.267.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.267. $\int x^2\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$

3.267.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.53

method	result
risch	$\frac{x(8b\,x^4d^2+12a\,d^2x^2+2bcd\,x^2+6acd-3b\,c^2)\sqrt{d\,x^2+c}\,\sqrt{(b\,x^2+a)^2}}{48d^2(b\,x^2+a)}-\frac{c^2(2da-bc)\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)\sqrt{(b\,x^2+a)^2}}{16d^{\frac{5}{2}}(b\,x^2+a)}$
default	$\frac{\sqrt{(b\,x^2+a)^2}\,\left(8(d\,x^2+c)^{\frac{3}{2}}d^{\frac{3}{2}}b\,x^3+12(d\,x^2+c)^{\frac{3}{2}}d^{\frac{3}{2}}ax-6(d\,x^2+c)^{\frac{3}{2}}\sqrt{d}\,bcx-6\sqrt{d\,x^2+c}\,d^{\frac{3}{2}}acx+3\sqrt{d\,x^2+c}\,\sqrt{d}\,b\,c^2x-6\ln\left(\sqrt{d}\,x+\sqrt{d}\right.\right.}{48(b\,x^2+a)d^{\frac{5}{2}}}$

input
```
int(x^2*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/48*x*(8*b*d^2*x^4+12*a*d^2*x^2+2*b*c*d*x^2+6*a*c*d-3*b*c^2)*(d*x^2+c)^(1
/2)/d^2*((b*x^2+a)^2)^(1/2)/(b*x^2+a)-1/16*c^2*(2*a*d-b*c)/d^(5/2)*ln(d^(1
/2)*x+(d*x^2+c)^(1/2))*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

3.267.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.85

$$\int x^2\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$$
$$=\left[-\frac{3\,(bc^3-2\,ac^2d)\sqrt{d}\log\left(-2\,dx^2+2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)-2\,(8\,bd^3x^5+2\,(bcd^2+6\,ad^3)x^3-3\,(bc^2d-2\right.}{96\,d^3}\right.$$
$$\left.-\frac{3\,(bc^3-2\,ac^2d)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-(8\,bd^3x^5+2\,(bcd^2+6\,ad^3)x^3-3\,(bc^2d-2\,acd^2)x)\sqrt{dx^2+c}}{48\,d^3}\right]$$

input
```
integrate(x^2*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/96*(3*(b*c^3 - 2*a*c^2*d)*sqrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqr
t(d)*x - c) - 2*(8*b*d^3*x^5 + 2*(b*c*d^2 + 6*a*d^3)*x^3 - 3*(b*c^2*d - 2*
a*c*d^2)*x)*sqrt(d*x^2 + c))/d^3, -1/48*(3*(b*c^3 - 2*a*c^2*d)*sqrt(-d)*ar
ctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (8*b*d^3*x^5 + 2*(b*c*d^2 + 6*a*d^3)*x^
3 - 3*(b*c^2*d - 2*a*c*d^2)*x)*sqrt(d*x^2 + c))/d^3]
```

3.267.6 Sympy [F]

$$\int x^2\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx=\int x^2\sqrt{c+dx^2}\sqrt{(a+bx^2)^2}\,dx$$

input
```
integrate(x**2*(d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2),x)
```

output
```
Integral(x**2*sqrt(c + d*x**2)*sqrt((a + b*x**2)**2), x)
```

3.267.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.51

$$\int x^2\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx=\frac{(dx^2+c)^{\frac{3}{2}}bx^3}{6\,d}-\frac{(dx^2+c)^{\frac{3}{2}}bcx}{8\,d^2}+\frac{\sqrt{dx^2+c}bc^2x}{16\,d^2}+\frac{(dx^2+c)^{\frac{3}{2}}ax}{4\,d}-\frac{\sqrt{dx^2+c}acx}{8\,d}+\frac{bc^3\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{16\,d^{\frac{5}{2}}}-\frac{ac^2\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8\,d^{\frac{3}{2}}}$$

input
```
integrate(x^2*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/6*(d*x^2 + c)^(3/2)*b*x^3/d - 1/8*(d*x^2 + c)^(3/2)*b*c*x/d^2 + 1/16*sqr
t(d*x^2 + c)*b*c^2*x/d^2 + 1/4*(d*x^2 + c)^(3/2)*a*x/d - 1/8*sqrt(d*x^2 +
c)*a*c*x/d + 1/16*b*c^3*arcsinh(d*x/sqrt(c*d))/d^(5/2) - 1/8*a*c^2*arcsinh
(d*x/sqrt(c*d))/d^(3/2)
```

3.267.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.64

$$\int x^2\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$$

$$=\frac{1}{48}\left(2\left(4\,bx^2\mathrm{sgn}\left(bx^2+a\right)+\frac{bcd^3\mathrm{sgn}(bx^2+a)+6\,ad^4\mathrm{sgn}(bx^2+a)}{d^4}\right)x^2-\frac{3\,(bc^2d^2\mathrm{sgn}(bx^2+a)-2\,acd^3\mathrm{sg}}{d^4}\right.$$

$$-\frac{(bc^3\mathrm{sgn}(bx^2+a)-2\,ac^2d\mathrm{sgn}(bx^2+a))\log\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)}{16\,d^{\frac{5}{2}}}$$

input
```
integrate(x^2*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/48*(2*(4*b*x^2*sgn(b*x^2 + a) + (b*c*d^3*sgn(b*x^2 + a) + 6*a*d^4*sgn(b*
x^2 + a))/d^4)*x^2 - 3*(b*c^2*d^2*sgn(b*x^2 + a) - 2*a*c*d^3*sgn(b*x^2 + a
))/d^4)*sqrt(d*x^2 + c)*x - 1/16*(b*c^3*sgn(b*x^2 + a) - 2*a*c^2*d*sgn(b*x
^2 + a))*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(5/2)
```

3.267.9 Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \int x^2\,\sqrt{d\,x^2+c}\,\sqrt{\left(b\,x^2+a\right)^2}\,dx$$

input
```
int(x^2*(c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2),x)
```

output
```
int(x^2*(c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2), x)
```

3.268 $\int x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$

3.268.1 Optimal result

Integrand size = 35, antiderivative size = 108

$$\int x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = -\frac{(bc-ad)\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^2\left(a+bx^2\right)} + \frac{b\left(c+dx^2\right)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^2\left(a+bx^2\right)}$$

output
```
-1/3*(-a*d+b*c)*(d*x^2+c)^(3/2)*((b*x^2+a)^2)^(1/2)/d^2/(b*x^2+a)+1/5*b*(d
*x^2+c)^(5/2)*((b*x^2+a)^2)^(1/2)/d^2/(b*x^2+a)
```

3.268.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.52

$$\int x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{\sqrt{\left(a+bx^2\right)^2}\left(c+dx^2\right)^{3/2}\left(-2bc+5ad+3bdx^2\right)}{15d^2\left(a+bx^2\right)}$$

input
```
Integrate[x*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
(Sqrt[(a + b*x^2)^2]*(c + d*x^2)^(3/2)*(-2*b*c + 5*a*d + 3*b*d*x^2))/(15*d
^2*(a + b*x^2))
```

3.268.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.75, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1384, 27, 353, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int bx(bx^2+a)\sqrt{dx^2+c}dx}{b(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int x(bx^2+a)\sqrt{dx^2+c}dx}{a+bx^2}$$

$$\Big\downarrow \text{353}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int (bx^2+a)\sqrt{dx^2+c}dx^2}{2(a+bx^2)}$$

$$\Big\downarrow \text{53}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{b(dx^2+c)^{3/2}}{d}+\frac{(ad-bc)\sqrt{dx^2+c}}{d}\right)dx^2}{2(a+bx^2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{2b(c+dx^2)^{5/2}}{5d^2}-\frac{2(c+dx^2)^{3/2}(bc-ad)}{3d^2}\right)}{2(a+bx^2)}$$

input
```
Int[x*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((-2*(b*c - a*d)*(c + d*x^2)^(3/2))/(3*d^
2) + (2*b*(c + d*x^2)^(5/2))/(5*d^2)))/(2*(a + b*x^2))
```

3.268.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.268.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.47

method	result	size
gosper	$\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}\left(3bd\,x^{2}+5da-2bc\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{15d^{2}\left(b\,x^{2}+a\right)}$	51
default	$\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}\left(3bd\,x^{2}+5da-2bc\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{15d^{2}\left(b\,x^{2}+a\right)}$	51
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(3b\,x^{4}d^{2}+5a\,d^{2}x^{2}+bcd\,x^{2}+5acd-2b\,c^{2}\right)\sqrt{d\,x^{2}+c}}{15\left(b\,x^{2}+a\right)d^{2}}$	72

input
```
int(x*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(d*x^2+c)^(3/2)*(3*b*d*x^2+5*a*d-2*b*c)*((b*x^2+a)^2)^(1/2)/d^2/(b*x^
2+a)
```

3.268.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.46

$$\int x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx=\frac{(3\,bd^2x^4-2\,bc^2+5\,acd+(bcd+5\,ad^2)x^2)\sqrt{dx^2+c}}{15\,d^2}$$

input
```
integrate(x*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(3*b*d^2*x^4 - 2*b*c^2 + 5*a*c*d + (b*c*d + 5*a*d^2)*x^2)*sqrt(d*x^2
+ c)/d^2
```

3.268.6 Sympy [F]

$$\int x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx=\int x\sqrt{c+dx^2}\sqrt{(a+bx^2)^2}\,dx$$

input
```
integrate(x*(d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2),x)
```

output
```
Integral(x*sqrt(c + d*x**2)*sqrt((a + b*x**2)**2), x)
```

3.268.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.46

$$\int x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx=\frac{(dx^2+c)^{\frac{3}{2}}bx^2}{5\,d}-\frac{2\,(dx^2+c)^{\frac{3}{2}}bc}{15\,d^2}+\frac{(dx^2+c)^{\frac{3}{2}}a}{3\,d}$$

input
```
integrate(x*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/5*(d*x^2 + c)^(3/2)*b*x^2/d - 2/15*(d*x^2 + c)^(3/2)*b*c/d^2 + 1/3*(d*x^
2 + c)^(3/2)*a/d
```

3.268.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.63

$$\int x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$$
$$=\frac{3\,(dx^2+c)^{\frac{5}{2}}b\mathrm{sgn}(bx^2+a)-5\,(dx^2+c)^{\frac{3}{2}}bc\mathrm{sgn}(bx^2+a)+5\,(dx^2+c)^{\frac{3}{2}}ad\mathrm{sgn}(bx^2+a)}{15\,d^2}$$

input
```
integrate(x*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/15*(3*(d*x^2 + c)^(5/2)*b*sgn(b*x^2 + a) - 5*(d*x^2 + c)^(3/2)*b*c*sgn(b
*x^2 + a) + 5*(d*x^2 + c)^(3/2)*a*d*sgn(b*x^2 + a))/d^2
```

3.268.9 Mupad [F(-1)]

Timed out.

$$\int x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx=\int x\,\sqrt{d\,x^2+c}\,\sqrt{{\left(b\,x^2+a\right)}^2}\,dx$$

input
```
int(x*(c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2),x)
```

output
```
int(x*(c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2), x)
```

3.269 $\int \sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$

3.269.1 Optimal result

Integrand size = 34, antiderivative size = 178

$$\int \sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = -\frac{(bc-4ad)x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{8d(a+bx^2)} + \frac{bx(c+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{4d(a+bx^2)} - \frac{c(bc-4ad)\sqrt{a^2+2abx^2+b^2x^4}\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}(a+bx^2)}$$

output
```
1/4*b*x*(d*x^2+c)^(3/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)-1/8*c*(-4*a*d+b*c)
*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))*((b*x^2+a)^2)^(1/2)/d^(3/2)/(b*x^2+a)-
1/8*(-4*a*d+b*c)*x*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)
```

3.269.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.54

$$\int \sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{\sqrt{(a+bx^2)^2}\left(\sqrt{d}x\sqrt{c+dx^2}(4ad+b(c+2dx^2))+c(bc-4ad)\log\left(-\sqrt{d}x+\sqrt{c+dx^2}\right)\right)}{8d^{3/2}(a+bx^2)}$$

input
```
Integrate[Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
(Sqrt[(a + b*x^2)^2]*(Sqrt[d]*x*Sqrt[c + d*x^2]*(4*a*d + b*(c + 2*d*x^2))
+ c*(b*c - 4*a*d)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]]))/(8*d^(3/2)*(a + b*
x^2))
```

3.269.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.64, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1384, 27, 299, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int b\left(bx^2+a\right)\sqrt{dx^2+c}dx}{b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(bx^2+a\right)\sqrt{dx^2+c}dx}{a+bx^2}$$

$$\Big\downarrow \text{299}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{bx\left(c+dx^2\right)^{3/2}}{4d}-\frac{(bc-4ad)\int \sqrt{dx^2+c}dx}{4d}\right)}{a+bx^2}$$

$$\Big\downarrow \text{211}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{bx\left(c+dx^2\right)^{3/2}}{4d}-\frac{(bc-4ad)\left(\frac{1}{2}c\int \frac{1}{\sqrt{dx^2+c}}dx+\frac{1}{2}x\sqrt{c+dx^2}\right)}{4d}\right)}{a+bx^2}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{bx\left(c+dx^2\right)^{3/2}}{4d}-\frac{(bc-4ad)\left(\frac{1}{2}c\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}+\frac{1}{2}x\sqrt{c+dx^2}\right)}{4d}\right)}{a+bx^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{bx\left(c+dx^2\right)^{3/2}}{4d}-\frac{(bc-4ad)\left(\frac{c\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}}+\frac{1}{2}x\sqrt{c+dx^2}\right)}{4d}\right)}{a+bx^2}$$

input
```
Int[Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((b*x*(c + d*x^2)^(3/2))/(4*d) - ((b*c -
4*a*d)*((x*Sqrt[c + d*x^2])/2 + (c*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(
2*Sqrt[d])))/(4*d)))/(a + b*x^2)
```

3.269.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.269.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.58

method	result	size
risch	$\frac{x\left(2bd\,x^{2}+4da+bc\right)\sqrt{d\,x^{2}+c}\,\sqrt{\left(b\,x^{2}+a\right)^{2}}}{8d\left(b\,x^{2}+a\right)}+\frac{c\left(4da-bc\right)\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{8d^{\frac{3}{2}}\left(b\,x^{2}+a\right)}$	103
default	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\left(2\sqrt{d}\,\left(d\,x^{2}+c\right)^{\frac{3}{2}}bx+4d^{\frac{3}{2}}\sqrt{d\,x^{2}+c}\,ax-\sqrt{d}\,\sqrt{d\,x^{2}+c}\,bcx+4\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)acd-\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)b\,c^{2}\right)}{8\left(b\,x^{2}+a\right)d^{\frac{3}{2}}}$	119

input
```
int((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*x*(2*b*d*x^2+4*a*d+b*c)*(d*x^2+c)^(1/2)/d*((b*x^2+a)^2)^(1/2)/(b*x^2+a
)+1/8*c*(4*a*d-b*c)/d^(3/2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))*((b*x^2+a)^2)^(1
/2)/(b*x^2+a)
```

3.269.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.87

$$\int \sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$$
$$=\left[-\frac{(bc^2-4\,acd)\sqrt{d}\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)-2\left(2\,bd^2x^3+(bcd+4\,ad^2)x\right)\sqrt{dx^2+c}}{16\,d^2},\frac{(bc^2-4}{}\right.$$

input
```
integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*((b*c^2 - 4*a*c*d)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)
*x - c) - 2*(2*b*d^2*x^3 + (b*c*d + 4*a*d^2)*x)*sqrt(d*x^2 + c))/d^2, 1/8*
((b*c^2 - 4*a*c*d)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (2*b*d^2*
x^3 + (b*c*d + 4*a*d^2)*x)*sqrt(d*x^2 + c))/d^2]
```

3.269.6 Sympy [F]

$$\int \sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \int \sqrt{c+dx^2}\sqrt{(a+bx^2)^2}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x**2)*sqrt((a + b*x**2)**2), x)
```

3.269.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.46

$$\int \sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{1}{2}\sqrt{dx^2+c}ax + \frac{(dx^2+c)^{\frac{3}{2}}bx}{4\,d} - \frac{\sqrt{dx^2+c}bcx}{8\,d} - \frac{bc^2\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8\,d^{\frac{3}{2}}} + \frac{ac\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{d}}$$

input
```
integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(d*x^2 + c)*a*x + 1/4*(d*x^2 + c)^(3/2)*b*x/d - 1/8*sqrt(d*x^2 + c
)*b*c*x/d - 1/8*b*c^2*arcsinh(d*x/sqrt(c*d))/d^(3/2) + 1/2*a*c*arcsinh(d*x
/sqrt(c*d))/sqrt(d)
```

3.269.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.61

$$\int \sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$$
$$= \frac{1}{8}\left(2\,bx^2\mathrm{sgn}\left(bx^2+a\right)+\frac{bcd\mathrm{sgn}(bx^2+a)+4\,ad^2\mathrm{sgn}(bx^2+a)}{d^2}\right)\sqrt{dx^2+c}x$$
$$+\frac{(bc^2\mathrm{sgn}(bx^2+a)-4\,acd\mathrm{sgn}(bx^2+a))\log\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)}{8\,d^{\frac{3}{2}}}$$

input
```
integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/8*(2*b*x^2*sgn(b*x^2 + a) + (b*c*d*sgn(b*x^2 + a) + 4*a*d^2*sgn(b*x^2 +
a))/d^2)*sqrt(d*x^2 + c)*x + 1/8*(b*c^2*sgn(b*x^2 + a) - 4*a*c*d*sgn(b*x^2
 + a))*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(3/2)
```

3.269.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \int \sqrt{d\,x^2+c}\,\sqrt{{\left(b\,x^2+a\right)}^2}\,dx$$

input
```
int((c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2),x)
```

output
```
int((c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2), x)
```

3.270 $\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx$

3.270.1 Optimal result

Integrand size = 37, antiderivative size = 152

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx = \frac{a\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2} + \frac{b(c+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d(a+bx^2)} - \frac{a\sqrt{c}\sqrt{a^2+2abx^2+b^2x^4}\,\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a+bx^2}$$

output
```
1/3*b*(d*x^2+c)^(3/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)-a*arctanh((d*x^2+c)^
(1/2)/c^(1/2))*c^(1/2)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+a*(d*x^2+c)^(1/2)*((b
*x^2+a)^2)^(1/2)/(b*x^2+a)
```

3.270.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx = \frac{\sqrt{(a+bx^2)^2}\left(\sqrt{c+dx^2}(3ad+b(c+dx^2)) - 3a\sqrt{c}d\,\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)\right)}{3d(a+bx^2)}$$

input
```
Integrate[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x,x]
```

output
```
(Sqrt[(a + b*x^2)^2]*(Sqrt[c + d*x^2]*(3*a*d + b*(c + d*x^2)) - 3*a*Sqrt[c
]*d*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]))/(3*d*(a + b*x^2))
```

3.270.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.63, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {1384, 27, 354, 90, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}}{x}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b(bx^2+a)\sqrt{dx^2+c}}{x}dx}{b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)\sqrt{dx^2+c}}{x}dx}{a+bx^2}$$

$$\Big\downarrow \text{354}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)\sqrt{dx^2+c}}{x^2}dx^2}{2\left(a+bx^2\right)}$$

$$\Big\downarrow \text{90}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(a\int \frac{\sqrt{dx^2+c}}{x^2}dx^2+\frac{2b(c+dx^2)^{3/2}}{3d}\right)}{2\left(a+bx^2\right)}$$

$$\Big\downarrow \text{60}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(a\left(c\int \frac{1}{x^2\sqrt{dx^2+c}}dx^2+2\sqrt{c+dx^2}\right)+\frac{2b(c+dx^2)^{3/2}}{3d}\right)}{2\left(a+bx^2\right)}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(a\left(\frac{2c\int \frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{d}+2\sqrt{c+dx^2}\right)+\frac{2b(c+dx^2)^{3/2}}{3d}\right)}{2\left(a+bx^2\right)}$$

$$\Big\downarrow \; 221$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(a\left(2\sqrt{c+dx^2}-2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)\right)+\frac{2b(c+dx^2)^{3/2}}{3d}\right)}{2\left(a+bx^2\right)}$$

input
```
Int[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((2*b*(c + d*x^2)^(3/2))/(3*d) + a*(2*Sqr
t[c + d*x^2] - 2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])))/(2*(a + b*x^2
))
```

3.270.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.270.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.53

method	result	size
default	$-\frac{\sqrt{(b\,x^{2}+a)^{2}}\,\left(3\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^{2}+c}}{x}\right)ad-b\left(d\,x^{2}+c\right)^{\frac{3}{2}}-3\sqrt{d\,x^{2}+c}\,ad\right)}{3(b\,x^{2}+a)d}$	80

input
```
int((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/3*((b*x^2+a)^2)^(1/2)*(3*c^(1/2)*ln(2*(c^(1/2)*(d*x^2+c)^(1/2)+c)/x)*a*
d-b*(d*x^2+c)^(3/2)-3*(d*x^2+c)^(1/2)*a*d)/(b*x^2+a)/d
```

3.270.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx$$
$$= \left[\frac{3\,a\sqrt{c}d\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)+2\,(bdx^2+bc+3\,ad)\sqrt{dx^2+c}}{6\,d}, \frac{3\,a\sqrt{-c}d\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+(bdx^2+}{3\,d}\right.$$

input
```
integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x,x, algorithm="fricas")
```

output
```
[1/6*(3*a*sqrt(c)*d*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) +
2*(b*d*x^2 + b*c + 3*a*d)*sqrt(d*x^2 + c))/d, 1/3*(3*a*sqrt(-c)*d*arctan(s
qrt(-c)/sqrt(d*x^2 + c)) + (b*d*x^2 + b*c + 3*a*d)*sqrt(d*x^2 + c))/d]
```

3.270.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx = \int \frac{\sqrt{c+dx^2}\sqrt{(a+bx^2)^2}}{x}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2)/x,x)
```

output
```
Integral(sqrt(c + d*x**2)*sqrt((a + b*x**2)**2)/x, x)
```

3.270.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.30

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx = -a\sqrt{c}\,\mathrm{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right)+\sqrt{dx^2+c}a+\frac{(dx^2+c)^{\frac{3}{2}}b}{3\,d}$$

input
```
integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x,x, algorithm="maxima")
```

output
```
-a*sqrt(c)*arcsinh(c/(sqrt(c*d)*abs(x))) + sqrt(d*x^2 + c)*a + 1/3*(d*x^2
+ c)^(3/2)*b/d
```

3.270.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx = \frac{ac\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)\operatorname{sgn}(bx^2+a)}{\sqrt{-c}} + \frac{(dx^2+c)^{\frac{3}{2}}bd^2\operatorname{sgn}(bx^2+a)+3\sqrt{dx^2+c}\,cad^3\operatorname{sgn}(bx^2+a)}{3\,d^3}$$

input
```
integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x,x, algorithm="giac")
```

output
```
a*c*arctan(sqrt(d*x^2 + c)/sqrt(-c))*sgn(b*x^2 + a)/sqrt(-c) + 1/3*((d*x^2
 + c)^(3/2)*b*d^2*sgn(b*x^2 + a) + 3*sqrt(d*x^2 + c)*a*d^3*sgn(b*x^2 + a))
/d^3
```

3.270.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{{\left(b\,x^2+a\right)}^2}}{x}\,dx$$

input
```
int(((c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2))/x,x)
```

output
```
int(((c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2))/x, x)
```

3.271 $\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx$

3.271.1 Optimal result

Integrand size = 37, antiderivative size = 177

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx = \frac{(bc+2ad)x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{2c\,(a+bx^2)} - \frac{a(c+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{cx\,(a+bx^2)} + \frac{(bc+2ad)\sqrt{a^2+2abx^2+b^2x^4}\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}\,(a+bx^2)}$$

output
```
-a*(d*x^2+c)^(3/2)*((b*x^2+a)^2)^(1/2)/c/x/(b*x^2+a)+1/2*(2*a*d+b*c)*arcta
nh(x*d^(1/2)/(d*x^2+c)^(1/2))*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/d^(1/2)+1/2*(2
*a*d+b*c)*x*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/c/(b*x^2+a)
```

3.271.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx = \frac{\sqrt{(a+bx^2)^2}\left(\sqrt{d}(-2a+bx^2)\sqrt{c+dx^2}+2(bc+2ad)x\,\text{arctanh}\left(\frac{\sqrt{d}x}{-\sqrt{c}+\sqrt{c+dx^2}}\right)\right)}{2\sqrt{d}x\,(a+bx^2)}$$

input
```
Integrate[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^2,x]
```

output
```
(Sqrt[(a + b*x^2)^2]*(Sqrt[d]*(-2*a + b*x^2)*Sqrt[c + d*x^2] + 2*(b*c + 2*
a*d)*x*ArcTanh[(Sqrt[d]*x)/(-Sqrt[c] + Sqrt[c + d*x^2])]))/(2*Sqrt[d]*x*(a
 + b*x^2))
```

3.271.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.63, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {1384, 27, 359, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}}{x^2}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b(bx^2+a)\sqrt{dx^2+c}}{x^2}dx}{b(a+bx^2)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)\sqrt{dx^2+c}}{x^2}dx}{a+bx^2}$$

$$\Big\downarrow \text{ 359}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{(2ad+bc)\int \sqrt{dx^2+c}dx}{c}-\frac{a(c+dx^2)^{3/2}}{cx}\right)}{a+bx^2}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{(2ad+bc)\left(\frac{1}{2}c\int \frac{1}{\sqrt{dx^2+c}}dx+\frac{1}{2}x\sqrt{c+dx^2}\right)}{c}-\frac{a(c+dx^2)^{3/2}}{cx}\right)}{a+bx^2}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{(2ad+bc)\left(\frac{1}{2}c\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}+\frac{1}{2}x\sqrt{c+dx^2}\right)}{c}-\frac{a\left(c+dx^2\right)^{3/2}}{cx}\right)}{a+bx^2}$$

$\downarrow$ 219

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{(2ad+bc)\left(\frac{c\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}}+\frac{1}{2}x\sqrt{c+dx^2}\right)}{c}-\frac{a\left(c+dx^2\right)^{3/2}}{cx}\right)}{a+bx^2}$$

input
```
Int[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^2,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-((a*(c + d*x^2)^(3/2))/(c*x)) + ((b*c +
 2*a*d)*((x*Sqrt[c + d*x^2])/2 + (c*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/
(2*Sqrt[d])))/c))/(a + b*x^2)
```

3.271.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.271.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.53

method	result
risch	$-\frac{\sqrt{d\,x^{2}+c}\,\left(-b\,x^{2}+2a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{2x\left(b\,x^{2}+a\right)}+\frac{\left(da+\frac{bc}{2}\right)\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{\sqrt{d}\,\left(b\,x^{2}+a\right)}$
default	$-\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(-2\sqrt{d\,x^{2}+c}\,d^{\frac{3}{2}}\,a\,x^{2}-\sqrt{d\,x^{2}+c}\,\sqrt{d}\,bc\,x^{2}+2\left(d\,x^{2}+c\right)^{\frac{3}{2}}\sqrt{d}\,a-2\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)acdx-\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)b\,c^{2}x\right)}{2\left(b\,x^{2}+a\right)cx\sqrt{d}}$

input
```
int((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(d*x^2+c)^(1/2)*(-b*x^2+2*a)/x*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+(d*a+1/2
*b*c)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))/d^(1/2)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

3.271.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx$$
$$= \left[\frac{(bc+2\,ad)\sqrt{d}x\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+2\,(bdx^2-2\,ad)\sqrt{dx^2+c}}{4\,dx},\right.$$
$$\left.-\frac{(bc+2\,ad)\sqrt{-d}x\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-(bdx^2-2\,ad)\sqrt{dx^2+c}}{2\,dx}\right]$$

input
```
integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
[1/4*((b*c + 2*a*d)*sqrt(d)*x*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x -
 c) + 2*(b*d*x^2 - 2*a*d)*sqrt(d*x^2 + c))/(d*x), -1/2*((b*c + 2*a*d)*sqrt
(-d)*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (b*d*x^2 - 2*a*d)*sqrt(d*x^2 +
 c))/(d*x)]
```

3.271.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx = \int \frac{\sqrt{c+dx^2}\sqrt{(a+bx^2)^2}}{x^2}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(c + d*x**2)*sqrt((a + b*x**2)**2)/x**2, x)
```

3.271.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.33

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx = \frac{1}{2}\sqrt{dx^2+c}bx + \frac{bc\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{d}} + a\sqrt{d}\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right) - \frac{\sqrt{dx^2+c}a}{x}$$

input
```
integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
1/2*sqrt(d*x^2 + c)*b*x + 1/2*b*c*arcsinh(d*x/sqrt(c*d))/sqrt(d) + a*sqrt(
d)*arcsinh(d*x/sqrt(c*d)) - sqrt(d*x^2 + c)*a/x
```

3.271.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx = \frac{1}{2}\sqrt{dx^2+c}bx\mathrm{sgn}\left(bx^2+a\right) + \frac{2\,ac\sqrt{d}\mathrm{sgn}(bx^2+a)}{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c} - \frac{(bc\mathrm{sgn}(bx^2+a)+2\,ad\mathrm{sgn}(bx^2+a))\log\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{4\sqrt{d}}$$

input
```
integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^2,x, algorithm="giac")
```

output
```
1/2*sqrt(d*x^2 + c)*b*x*sgn(b*x^2 + a) + 2*a*c*sqrt(d)*sgn(b*x^2 + a)/((sq
rt(d)*x - sqrt(d*x^2 + c))^2 - c) - 1/4*(b*c*sgn(b*x^2 + a) + 2*a*d*sgn(b*
x^2 + a))*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/sqrt(d)
```

3.271.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{\left(b\,x^2+a\right)^2}}{x^2}\,dx$$

input
```
int(((c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2))/x^2,x)
```

output
```
int(((c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2))/x^2, x)
```

3.272 $\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx$

3.272.1 Optimal result

Integrand size = 37, antiderivative size = 177

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx = \frac{(2bc+ad)\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{2c(a+bx^2)} - \frac{a(c+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{2cx^2(a+bx^2)} - \frac{(2bc+ad)\sqrt{a^2+2abx^2+b^2x^4}\,\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2\sqrt{c}(a+bx^2)}$$

output
```
-1/2*a*(d*x^2+c)^(3/2)*((b*x^2+a)^2)^(1/2)/c/x^2/(b*x^2+a)-1/2*(a*d+2*b*c)
*arctanh((d*x^2+c)^(1/2)/c^(1/2))*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/c^(1/2)+1/
2*(a*d+2*b*c)*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/c/(b*x^2+a)
```

3.272.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.51

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx = -\frac{\sqrt{(a+bx^2)^2}\left(\sqrt{c}(a-2bx^2)\sqrt{c+dx^2}+(2bc+ad)x^2\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)\right)}{2\sqrt{c}x^2(a+bx^2)}$$

input
```
Integrate[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^3,x]
```

output
```
-1/2*(Sqrt[(a + b*x^2)^2]*(Sqrt[c]*(a - 2*b*x^2)*Sqrt[c + d*x^2] + (2*b*c
+ a*d)*x^2*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]))/(Sqrt[c]*x^2*(a + b*x^2))
```

3.272.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.62, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {1384, 27, 354, 87, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}}{x^3}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b(bx^2+a)\sqrt{dx^2+c}}{x^3}dx}{b(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)\sqrt{dx^2+c}}{x^3}dx}{a+bx^2}$$

$$\Big\downarrow \text{354}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)\sqrt{dx^2+c}}{x^4}dx^2}{2(a+bx^2)}$$

$$\Big\downarrow \text{87}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{(ad+2bc)\int \frac{\sqrt{dx^2+c}}{x^2}dx^2}{2c}-\frac{a(c+dx^2)^{3/2}}{cx^2}\right)}{2(a+bx^2)}$$

$$\Big\downarrow \text{60}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{(ad+2bc)\left(c\int \frac{1}{x^2\sqrt{dx^2+c}}dx^2+2\sqrt{c+dx^2}\right)}{2c}-\frac{a(c+dx^2)^{3/2}}{cx^2}\right)}{2(a+bx^2)}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{(ad+2bc)\left(\frac{2c\int \frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{d}+2\sqrt{c+dx^2}\right)}{2c}-\frac{a(c+dx^2)^{3/2}}{cx^2}\right)}{2(a+bx^2)}$$

$$\Big\downarrow \; 221$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{(ad+2bc)\left(2\sqrt{c+dx^2}-2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)\right)}{2c}-\frac{a(c+dx^2)^{3/2}}{cx^2}\right)}{2(a+bx^2)}$$

input
```
Int[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^3,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-((a*(c + d*x^2)^(3/2))/(c*x^2)) + ((2*b
*c + a*d)*(2*Sqrt[c + d*x^2] - 2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])
)/(2*c)))/(2*(a + b*x^2))
```

3.272.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

3.272.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.60

method	result
risch	$-\frac{a\sqrt{d\,x^{2}+c}\,\sqrt{\left(b\,x^{2}+a\right)^{2}}}{2x^{2}\left(b\,x^{2}+a\right)}+\frac{\left(b\sqrt{d\,x^{2}+c}-\frac{\left(da+2bc\right)\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^{2}+c}}{x}\right)}{2\sqrt{c}}\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{b\,x^{2}+a}$
default	$-\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^{2}+c}}{x}\right)a d\,x^{2}+2c^{\frac{3}{2}}\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^{2}+c}}{x}\right)b\,x^{2}-\sqrt{d\,x^{2}+c}\,a d\,x^{2}-2\sqrt{d\,x^{2}+c}\,b c\,x^{2}+\left(d\,x^{2}+c\right)^{\frac{3}{2}}a\right)}{2\left(b\,x^{2}+a\right)c\,x^{2}}$

input
```
int((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*(d*x^2+c)^(1/2)/x^2*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+(b*(d*x^2+c)^(1/2
)-1/2*(a*d+2*b*c)/c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x))*((b*x^2+a
)^2)^(1/2)/(b*x^2+a)
```

3.272.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx$$
$$= \left[\frac{(2\,bc+ad)\sqrt{c}x^2\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)+2\,(2\,bcx^2-ac)\sqrt{dx^2+c}}{4\,cx^2}, \frac{(2\,bc+ad)\sqrt{-c}x^2\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right.}{2\,cx^2}\right.$$

input
```
integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
[1/4*((2*b*c + a*d)*sqrt(c)*x^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) +
2*c)/x^2) + 2*(2*b*c*x^2 - a*c)*sqrt(d*x^2 + c))/(c*x^2), 1/2*((2*b*c + a*
d)*sqrt(-c)*x^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (2*b*c*x^2 - a*c)*sqrt(
d*x^2 + c))/(c*x^2)]
```

3.272.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx = \int \frac{\sqrt{c+dx^2}\sqrt{(a+bx^2)^2}}{x^3}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2)/x**3,x)
```

output
```
Integral(sqrt(c + d*x**2)*sqrt((a + b*x**2)**2)/x**3, x)
```

3.272.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx = -b\sqrt{c}\,\mathrm{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right) - \frac{ad\,\mathrm{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right)}{2\sqrt{c}} + \sqrt{dx^2+c}b + \frac{\sqrt{dx^2+c}ad}{2c} - \frac{(dx^2+c)^{\frac{3}{2}}a}{2cx^2}$$

input
```
integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
-b*sqrt(c)*arcsinh(c/(sqrt(c*d)*abs(x))) - 1/2*a*d*arcsinh(c/(sqrt(c*d)*ab
s(x)))/sqrt(c) + sqrt(d*x^2 + c)*b + 1/2*sqrt(d*x^2 + c)*a*d/c - 1/2*(d*x^
2 + c)^(3/2)*a/(c*x^2)
```

3.272.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx = \frac{2\sqrt{dx^2+c}bd\,\mathrm{sgn}(bx^2+a) + \frac{(2\,bcd\,\mathrm{sgn}(bx^2+a)+ad^2\,\mathrm{sgn}(bx^2+a))\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{\sqrt{dx^2+c}ad\,\mathrm{sgn}(bx^2+a)}{x^2}}{2\,d}$$

input
```
integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^3,x, algorithm="giac")
```

output
```
1/2*(2*sqrt(d*x^2 + c)*b*d*sgn(b*x^2 + a) + (2*b*c*d*sgn(b*x^2 + a) + a*d^
2*sgn(b*x^2 + a))*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqrt(-c) - sqrt(d*x^2 +
 c)*a*d*sgn(b*x^2 + a)/x^2)/d
```

3.272.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{\left(b\,x^2+a\right)^2}}{x^3}\,dx$$

input
```
int(((c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2))/x^3,x)
```

output
```
int(((c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2))/x^3, x)
```

3.273 $\int x^3(d+ex^2)^2(a+bx^2+cx^4)\,dx$

3.273.1 Optimal result

Integrand size = 25, antiderivative size = 78

$$\int x^3(d+ex^2)^2(a+bx^2+cx^4)\,dx=\frac{1}{4}ad^2x^4+\frac{1}{6}d(bd+2ae)x^6+\frac{1}{8}\left(cd^2+e(2bd+ae)\right)x^8+\frac{1}{10}e(2cd+be)x^{10}+\frac{1}{12}ce^2x^{12}$$

output
```
1/4*a*d^2*x^4+1/6*d*(2*a*e+b*d)*x^6+1/8*(c*d^2+e*(a*e+2*b*d))*x^8+1/10*e*(
b*e+2*c*d)*x^10+1/12*c*e^2*x^12
```

3.273.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.92

$$\int x^3(d+ex^2)^2(a+bx^2+cx^4)\,dx=\frac{1}{120}x^4\left(30ad^2+20d(bd+2ae)x^2+15\left(cd^2+e(2bd+ae)\right)x^4+12e(2cd+be)x^6+10ce^2x^8\right)$$

input
```
Integrate[x^3*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

output
```
(x^4*(30*a*d^2 + 20*d*(b*d + 2*a*e)*x^2 + 15*(c*d^2 + e*(2*b*d + a*e))*x^4
 + 12*e*(2*c*d + b*e)*x^6 + 10*c*e^2*x^8))/120
```

3.273.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (d+ex^2)^2 (a+bx^2+cx^4)\, dx$$

$$\Big\downarrow \text{ 1578}$$

$$\frac{1}{2}\int x^2 (ex^2+d)^2 (cx^4+bx^2+a)\, dx^2$$

$$\Big\downarrow \text{ 1195}$$

$$\frac{1}{2}\int \left(ce^2x^{10}+e(2cd+be)x^8+\left(cd^2+e(2bd+ae)\right)x^6+d(bd+2ae)x^4+ad^2x^2\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{1}{4}x^8\left(e(ae+2bd)+cd^2\right)+\frac{1}{3}dx^6(2ae+bd)+\frac{1}{2}ad^2x^4+\frac{1}{5}ex^{10}(be+2cd)+\frac{1}{6}ce^2x^{12}\right)$$

input
```
Int[x^3*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

output
```
((a*d^2*x^4)/2 + (d*(b*d + 2*a*e)*x^6)/3 + ((c*d^2 + e*(2*b*d + a*e))*x^8)
/4 + (e*(2*c*d + b*e)*x^10)/5 + (c*e^2*x^12)/6)/2
```

3.273.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.273.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

method	result
default	$\frac{c\,e^{2} x^{12}}{12}+\frac{\left(b\,e^{2}+2 d c e \right) x^{10}}{10}+\frac{\left(a\,e^{2}+2 b d e +c\,d^{2}\right) x^{8}}{8}+\frac{\left(2 e d a +b\,d^{2}\right) x^{6}}{6}+\frac{a\,d^{2} x^{4}}{4}$
norman	$\frac{c\,e^{2} x^{12}}{12}+\left(\frac{1}{10} b\,e^{2}+\frac{1}{5} d c e \right) x^{10}+\left(\frac{1}{8} a\,e^{2}+\frac{1}{4} b d e +\frac{1}{8} c\,d^{2}\right) x^{8}+\left(\frac{1}{3} e d a +\frac{1}{6} b\,d^{2}\right) x^{6}+\frac{a\,d^{2} x^{4}}{4}$
gosper	$\frac{1}{12} c\,e^{2} x^{12}+\frac{1}{10} x^{10} b\,e^{2}+\frac{1}{5} x^{10} d c e +\frac{1}{8} x^{8} a\,e^{2}+\frac{1}{4} x^{8} b d e +\frac{1}{8} x^{8} c\,d^{2}+\frac{1}{3} x^{6} e d a +\frac{1}{6} x^{6} b\,d^{2}+\frac{1}{4} a\,d^{2} x$
risch	$\frac{1}{12} c\,e^{2} x^{12}+\frac{1}{10} x^{10} b\,e^{2}+\frac{1}{5} x^{10} d c e +\frac{1}{8} x^{8} a\,e^{2}+\frac{1}{4} x^{8} b d e +\frac{1}{8} x^{8} c\,d^{2}+\frac{1}{3} x^{6} e d a +\frac{1}{6} x^{6} b\,d^{2}+\frac{1}{4} a\,d^{2} x$
parallelrisch	$\frac{1}{12} c\,e^{2} x^{12}+\frac{1}{10} x^{10} b\,e^{2}+\frac{1}{5} x^{10} d c e +\frac{1}{8} x^{8} a\,e^{2}+\frac{1}{4} x^{8} b d e +\frac{1}{8} x^{8} c\,d^{2}+\frac{1}{3} x^{6} e d a +\frac{1}{6} x^{6} b\,d^{2}+\frac{1}{4} a\,d^{2} x$

input
```
int(x^3*(e*x^2+d)^2*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/12*c*e^2*x^12+1/10*(b*e^2+2*c*d*e)*x^10+1/8*(a*e^2+2*b*d*e+c*d^2)*x^8+1/
6*(2*a*d*e+b*d^2)*x^6+1/4*a*d^2*x^4
```

3.273.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.92

$$\int x^3 \left(d+e x^2\right)^2 \left(a+b x^2+c x^4\right) \, dx = \frac{1}{12} \, c e^{2} x^{12} + \frac{1}{10} \, {\left(2 \, c d e + b e^{2}\right)} x^{10} + \frac{1}{8} \, {\left(c d^{2} + 2 \, b d e + a e^{2}\right)} x^{8} + \frac{1}{4} \, a d^{2} x^{4} + \frac{1}{6} \, {\left(b d^{2} + 2 \, a d e\right)} x^{6}$$

input
```
integrate(x^3*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
1/12*c*e^2*x^12 + 1/10*(2*c*d*e + b*e^2)*x^10 + 1/8*(c*d^2 + 2*b*d*e + a*e
^2)*x^8 + 1/4*a*d^2*x^4 + 1/6*(b*d^2 + 2*a*d*e)*x^6
```

3.273.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.97

$$\int x^3(d+ex^2)^2\,(a+bx^2+cx^4)\,dx=\frac{ad^2x^4}{4}+\frac{ce^2x^{12}}{12}+x^{10}\left(\frac{be^2}{10}+\frac{cde}{5}\right)+x^8\left(\frac{ae^2}{8}+\frac{bde}{4}+\frac{cd^2}{8}\right)+x^6\left(\frac{ade}{3}+\frac{bd^2}{6}\right)$$

input
```
integrate(x**3*(e*x**2+d)**2*(c*x**4+b*x**2+a),x)
```

output
```
a*d**2*x**4/4 + c*e**2*x**12/12 + x**10*(b*e**2/10 + c*d*e/5) + x**8*(a*e*
*2/8 + b*d*e/4 + c*d**2/8) + x**6*(a*d*e/3 + b*d**2/6)
```

3.273.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.92

$$\int x^3(d+ex^2)^2\,(a+bx^2+cx^4)\,dx=\frac{1}{12}\,ce^2x^{12}+\frac{1}{10}\,(2\,cde+be^2)x^{10}+\frac{1}{8}\,(cd^2+2\,bde+ae^2)x^8+\frac{1}{4}\,ad^2x^4+\frac{1}{6}\,(bd^2+2\,ade)x^6$$

input
```
integrate(x^3*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
1/12*c*e^2*x^12 + 1/10*(2*c*d*e + b*e^2)*x^10 + 1/8*(c*d^2 + 2*b*d*e + a*e
^2)*x^8 + 1/4*a*d^2*x^4 + 1/6*(b*d^2 + 2*a*d*e)*x^6
```

3.273.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01

$$\int x^3(d+ex^2)^2\,(a+bx^2+cx^4)\,dx=\frac{1}{12}\,ce^2x^{12}+\frac{1}{5}\,cdex^{10}+\frac{1}{10}\,be^2x^{10}+\frac{1}{8}\,cd^2x^8+\frac{1}{4}\,bdex^8+\frac{1}{8}\,ae^2x^8+\frac{1}{6}\,bd^2x^6+\frac{1}{3}\,adex^6+\frac{1}{4}\,ad^2x^4$$

input
```
integrate(x^3*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/12*c*e^2*x^12 + 1/5*c*d*e*x^10 + 1/10*b*e^2*x^10 + 1/8*c*d^2*x^8 + 1/4*b
*d*e*x^8 + 1/8*a*e^2*x^8 + 1/6*b*d^2*x^6 + 1/3*a*d*e*x^6 + 1/4*a*d^2*x^4
```

3.273.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

$$\int x^3(d+ex^2)^2\left(a+bx^2+cx^4\right)\,dx = x^8\left(\frac{c\,d^2}{8}+\frac{b\,d\,e}{4}+\frac{a\,e^2}{8}\right)+x^6\left(\frac{b\,d^2}{6}+\frac{a\,e\,d}{3}\right) + x^{10}\left(\frac{b\,e^2}{10}+\frac{c\,d\,e}{5}\right)+\frac{a\,d^2\,x^4}{4}+\frac{c\,e^2\,x^{12}}{12}$$

input
```
int(x^3*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x)
```

output
```
x^8*((a*e^2)/8 + (c*d^2)/8 + (b*d*e)/4) + x^6*((b*d^2)/6 + (a*d*e)/3) + x^
10*((b*e^2)/10 + (c*d*e)/5) + (a*d^2*x^4)/4 + (c*e^2*x^12)/12
```

3.274 $\int x^2(d+ex^2)^2\left(a+bx^2+cx^4\right)\,dx$

3.274.1 Optimal result

Integrand size = 25, antiderivative size = 78

$$\int x^2\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)\,dx=\frac{1}{3}ad^2x^3+\frac{1}{5}d(bd+2ae)x^5+\frac{1}{7}\left(cd^2+e(2bd+ae)\right)x^7+\frac{1}{9}e(2cd+be)x^9+\frac{1}{11}ce^2x^{11}$$

output
```
1/3*a*d^2*x^3+1/5*d*(2*a*e+b*d)*x^5+1/7*(c*d^2+e*(a*e+2*b*d))*x^7+1/9*e*(b
*e+2*c*d)*x^9+1/11*c*e^2*x^11
```

3.274.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

$$\int x^2\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)\,dx=\frac{1}{3}ad^2x^3+\frac{1}{5}d(bd+2ae)x^5+\frac{1}{7}\left(cd^2+2bde+ae^2\right)x^7+\frac{1}{9}e(2cd+be)x^9+\frac{1}{11}ce^2x^{11}$$

input
```
Integrate[x^2*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

output
```
(a*d^2*x^3)/3 + (d*(b*d + 2*a*e)*x^5)/5 + ((c*d^2 + 2*b*d*e + a*e^2)*x^7)/
7 + (e*(2*c*d + b*e)*x^9)/9 + (c*e^2*x^11)/11
```

3.274.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right) \, dx$$

$$\Big\downarrow \text{1584}$$

$$\int \left(x^6 \left(e(ae+2bd)+cd^2\right)+dx^4(2ae+bd)+ad^2x^2+ex^8(be+2cd)+ce^2x^{10}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{7}x^7 \left(e(ae+2bd)+cd^2\right)+\frac{1}{5}dx^5(2ae+bd)+\frac{1}{3}ad^2x^3+\frac{1}{9}ex^9(be+2cd)+\frac{1}{11}ce^2x^{11}$$

input
```
Int[x^2*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

output
```
(a*d^2*x^3)/3 + (d*(b*d + 2*a*e)*x^5)/5 + ((c*d^2 + e*(2*b*d + a*e))*x^7)/
7 + (e*(2*c*d + b*e)*x^9)/9 + (c*e^2*x^11)/11
```

3.274.3.1 Defintions of rubi rules used

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.274.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

method	result
default	$\frac{c e^2 x^{11}}{11}+\frac{(b e^2+2dce)x^9}{9}+\frac{(a e^2+2bde+c d^2)x^7}{7}+\frac{(2eda+b d^2)x^5}{5}+\frac{a d^2 x^3}{3}$
norman	$\frac{c e^2 x^{11}}{11}+\left(\frac{1}{9}b e^2+\frac{2}{9}dce\right)x^9+\left(\frac{1}{7}a e^2+\frac{2}{7}bde+\frac{1}{7}c d^2\right)x^7+\left(\frac{2}{5}eda+\frac{1}{5}b d^2\right)x^5+\frac{a d^2 x^3}{3}$
gosper	$\frac{1}{11}c e^2 x^{11}+\frac{1}{9}x^9 b e^2+\frac{2}{9}x^9 dce+\frac{1}{7}x^7 a e^2+\frac{2}{7}x^7 bde+\frac{1}{7}x^7 c d^2+\frac{2}{5}x^5 eda+\frac{1}{5}x^5 b d^2+\frac{1}{3}a d^2 x^3$
risch	$\frac{1}{11}c e^2 x^{11}+\frac{1}{9}x^9 b e^2+\frac{2}{9}x^9 dce+\frac{1}{7}x^7 a e^2+\frac{2}{7}x^7 bde+\frac{1}{7}x^7 c d^2+\frac{2}{5}x^5 eda+\frac{1}{5}x^5 b d^2+\frac{1}{3}a d^2 x^3$
parallelrisch	$\frac{1}{11}c e^2 x^{11}+\frac{1}{9}x^9 b e^2+\frac{2}{9}x^9 dce+\frac{1}{7}x^7 a e^2+\frac{2}{7}x^7 bde+\frac{1}{7}x^7 c d^2+\frac{2}{5}x^5 eda+\frac{1}{5}x^5 b d^2+\frac{1}{3}a d^2 x^3$

input
```
int(x^2*(e*x^2+d)^2*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/11*c*e^2*x^11+1/9*(b*e^2+2*c*d*e)*x^9+1/7*(a*e^2+2*b*d*e+c*d^2)*x^7+1/5*
(2*a*d*e+b*d^2)*x^5+1/3*a*d^2*x^3
```

3.274.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.92

$$\int x^2(d+ex^2)^2\left(a+bx^2+cx^4\right)\,dx=\frac{1}{11}\,ce^2x^{11}+\frac{1}{9}\left(2\,cde+be^2\right)x^9+\frac{1}{7}\left(cd^2+2\,bde+ae^2\right)x^7+\frac{1}{3}\,ad^2x^3+\frac{1}{5}\left(bd^2+2\,ade\right)x^5$$

input
```
integrate(x^2*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
1/11*c*e^2*x^11 + 1/9*(2*c*d*e + b*e^2)*x^9 + 1/7*(c*d^2 + 2*b*d*e + a*e^2
)*x^7 + 1/3*a*d^2*x^3 + 1/5*(b*d^2 + 2*a*d*e)*x^5
```

3.274.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.05

$$\int x^2(d+ex^2)^2(a+bx^2+cx^4)\,dx = \frac{ad^2x^3}{3} + \frac{ce^2x^{11}}{11} + x^9\left(\frac{be^2}{9} + \frac{2cde}{9}\right) + x^7\left(\frac{ae^2}{7} + \frac{2bde}{7} + \frac{cd^2}{7}\right) + x^5\cdot\left(\frac{2ade}{5} + \frac{bd^2}{5}\right)$$

input
```
integrate(x**2*(e*x**2+d)**2*(c*x**4+b*x**2+a),x)
```

output
```
a*d**2*x**3/3 + c*e**2*x**11/11 + x**9*(b*e**2/9 + 2*c*d*e/9) + x**7*(a*e*
*2/7 + 2*b*d*e/7 + c*d**2/7) + x**5*(2*a*d*e/5 + b*d**2/5)
```

3.274.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.92

$$\int x^2(d+ex^2)^2(a+bx^2+cx^4)\,dx = \frac{1}{11}\,ce^2x^{11} + \frac{1}{9}\,(2\,cde + be^2)x^9 + \frac{1}{7}\,(cd^2 + 2\,bde + ae^2)x^7 + \frac{1}{3}\,ad^2x^3 + \frac{1}{5}\,(bd^2 + 2\,ade)x^5$$

input
```
integrate(x^2*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
1/11*c*e^2*x^11 + 1/9*(2*c*d*e + b*e^2)*x^9 + 1/7*(c*d^2 + 2*b*d*e + a*e^2
)*x^7 + 1/3*a*d^2*x^3 + 1/5*(b*d^2 + 2*a*d*e)*x^5
```

3.274.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01

$$\int x^2(d+ex^2)^2(a+bx^2+cx^4)\,dx = \frac{1}{11}\,ce^2x^{11} + \frac{2}{9}\,cdex^9 + \frac{1}{9}\,be^2x^9 + \frac{1}{7}\,cd^2x^7 + \frac{2}{7}\,bdex^7 + \frac{1}{7}\,ae^2x^7 + \frac{1}{5}\,bd^2x^5 + \frac{2}{5}\,adex^5 + \frac{1}{3}\,ad^2x^3$$

input
```
integrate(x^2*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/11*c*e^2*x^11 + 2/9*c*d*e*x^9 + 1/9*b*e^2*x^9 + 1/7*c*d^2*x^7 + 2/7*b*d*
e*x^7 + 1/7*a*e^2*x^7 + 1/5*b*d^2*x^5 + 2/5*a*d*e*x^5 + 1/3*a*d^2*x^3
```

3.274.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

$$\int x^2 (d + ex^2)^2 \left(a + bx^2 + cx^4\right) \, dx = x^7 \left(\frac{c\,d^2}{7} + \frac{2\,b\,d\,e}{7} + \frac{a\,e^2}{7}\right) + x^5 \left(\frac{b\,d^2}{5} + \frac{2\,a\,e\,d}{5}\right) + x^9 \left(\frac{b\,e^2}{9} + \frac{2\,c\,d\,e}{9}\right) + \frac{a\,d^2\,x^3}{3} + \frac{c\,e^2\,x^{11}}{11}$$

input
```
int(x^2*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x)
```

output
```
x^7*((a*e^2)/7 + (c*d^2)/7 + (2*b*d*e)/7) + x^5*((b*d^2)/5 + (2*a*d*e)/5)
+ x^9*((b*e^2)/9 + (2*c*d*e)/9) + (a*d^2*x^3)/3 + (c*e^2*x^11)/11
```

3.275 $\int x(d+ex^2)^2 (a+bx^2+cx^4)\,dx$

3.275.1 Optimal result

Integrand size = 23, antiderivative size = 75

$$\int x(d+ex^2)^2 (a+bx^2+cx^4)\,dx = \frac{(cd^2-bde+ae^2)(d+ex^2)^3}{6e^3} - \frac{(2cd-be)(d+ex^2)^4}{8e^3} + \frac{c(d+ex^2)^5}{10e^3}$$

output
```
1/6*(a*e^2-b*d*e+c*d^2)*(e*x^2+d)^3/e^3-1/8*(-b*e+2*c*d)*(e*x^2+d)^4/e^3+1
/10*c*(e*x^2+d)^5/e^3
```

3.275.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.96

$$\int x(d+ex^2)^2 (a+bx^2+cx^4)\,dx = \frac{1}{120}x^2\left(60ad^2+30d(bd+2ae)x^2+20\left(cd^2+e(2bd+ae)\right)x^4 + 15e(2cd+be)x^6+12ce^2x^8\right)$$

input
```
Integrate[x*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

output
```
(x^2*(60*a*d^2 + 30*d*(b*d + 2*a*e)*x^2 + 20*(c*d^2 + e*(2*b*d + a*e))*x^4
 + 15*e*(2*c*d + b*e)*x^6 + 12*c*e^2*x^8))/120
```

3.275.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1576, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)dx$$

$$\Big\downarrow \; 1576$$

$$\frac{1}{2}\int \left(ex^2+d\right)^2\left(cx^4+bx^2+a\right)dx^2$$

$$\Big\downarrow \; 1140$$

$$\frac{1}{2}\int \left(\frac{c\left(ex^2+d\right)^4}{e^2}+\frac{(be-2cd)\left(ex^2+d\right)^3}{e^2}+\frac{\left(cd^2-bed+ae^2\right)\left(ex^2+d\right)^2}{e^2}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{\left(d+ex^2\right)^3\left(ae^2-bde+cd^2\right)}{3e^3}-\frac{\left(d+ex^2\right)^4(2cd-be)}{4e^3}+\frac{c\left(d+ex^2\right)^5}{5e^3}\right)$$

input
```
Int[x*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

output
```
(((c*d^2 - b*d*e + a*e^2)*(d + e*x^2)^3)/(3*e^3) - ((2*c*d - b*e)*(d + e*x
^2)^4)/(4*e^3) + (c*(d + e*x^2)^5)/(5*e^3))/2
```

3.275.3.1 Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.275.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

method	result
default	$\frac{c\,e^{2} x^{10}}{10}+\frac{\left(b\,e^{2}+2dce\right) x^{8}}{8}+\frac{\left(a\,e^{2}+2bde+c\,d^{2}\right) x^{6}}{6}+\frac{\left(2eda+b\,d^{2}\right) x^{4}}{4}+\frac{a\,d^{2} x^{2}}{2}$
norman	$\frac{c\,e^{2} x^{10}}{10}+\left(\frac{1}{8}b\,e^{2}+\frac{1}{4}dce\right) x^{8}+\left(\frac{1}{6}a\,e^{2}+\frac{1}{3}bde+\frac{1}{6}c\,d^{2}\right) x^{6}+\left(\frac{1}{2}eda+\frac{1}{4}b\,d^{2}\right) x^{4}+\frac{a\,d^{2} x^{2}}{2}$
gosper	$\frac{1}{10}c\,e^{2} x^{10}+\frac{1}{8}x^{8} b\,e^{2}+\frac{1}{4}x^{8} dce+\frac{1}{6}x^{6} a\,e^{2}+\frac{1}{3}x^{6} bde+\frac{1}{6}x^{6} c\,d^{2}+\frac{1}{2}x^{4} eda+\frac{1}{4}b\,x^{4} d^{2}+\frac{1}{2}a\,d^{2} x^{2}$
risch	$\frac{1}{10}c\,e^{2} x^{10}+\frac{1}{8}x^{8} b\,e^{2}+\frac{1}{4}x^{8} dce+\frac{1}{6}x^{6} a\,e^{2}+\frac{1}{3}x^{6} bde+\frac{1}{6}x^{6} c\,d^{2}+\frac{1}{2}x^{4} eda+\frac{1}{4}b\,x^{4} d^{2}+\frac{1}{2}a\,d^{2} x^{2}$
parallelrisch	$\frac{1}{10}c\,e^{2} x^{10}+\frac{1}{8}x^{8} b\,e^{2}+\frac{1}{4}x^{8} dce+\frac{1}{6}x^{6} a\,e^{2}+\frac{1}{3}x^{6} bde+\frac{1}{6}x^{6} c\,d^{2}+\frac{1}{2}x^{4} eda+\frac{1}{4}b\,x^{4} d^{2}+\frac{1}{2}a\,d^{2} x^{2}$

input
```
int(x*(e*x^2+d)^2*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/10*c*e^2*x^10+1/8*(b*e^2+2*c*d*e)*x^8+1/6*(a*e^2+2*b*d*e+c*d^2)*x^6+1/4*
(2*a*d*e+b*d^2)*x^4+1/2*a*d^2*x^2
```

3.275.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.96

$$\int x(d+ex^2)^2\left(a+bx^2+cx^4\right)\,dx=\frac{1}{10}\,ce^2x^{10}+\frac{1}{8}\left(2\,cde+be^2\right)x^8+\frac{1}{6}\left(cd^2+2\,bde+ae^2\right)x^6+\frac{1}{2}\,ad^2x^2+\frac{1}{4}\left(bd^2+2\,ade\right)x^4$$

input
```
integrate(x*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
1/10*c*e^2*x^10 + 1/8*(2*c*d*e + b*e^2)*x^8 + 1/6*(c*d^2 + 2*b*d*e + a*e^2
)*x^6 + 1/2*a*d^2*x^2 + 1/4*(b*d^2 + 2*a*d*e)*x^4
```

3.275.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.01

$$\int x(d+ex^2)^2\,(a+bx^2+cx^4)\;dx=\frac{ad^2x^2}{2}+\frac{ce^2x^{10}}{10}+x^8\left(\frac{be^2}{8}+\frac{cde}{4}\right)+x^6\left(\frac{ae^2}{6}+\frac{bde}{3}+\frac{cd^2}{6}\right)+x^4\left(\frac{ade}{2}+\frac{bd^2}{4}\right)$$

input
```
integrate(x*(e*x**2+d)**2*(c*x**4+b*x**2+a),x)
```

output
```
a*d**2*x**2/2 + c*e**2*x**10/10 + x**8*(b*e**2/8 + c*d*e/4) + x**6*(a*e**2
/6 + b*d*e/3 + c*d**2/6) + x**4*(a*d*e/2 + b*d**2/4)
```

3.275.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.96

$$\int x(d+ex^2)^2\,(a+bx^2+cx^4)\;dx=\frac{1}{10}\,ce^2x^{10}+\frac{1}{8}\,(2\,cde+be^2)x^8+\frac{1}{6}\,(cd^2+2\,bde+ae^2)x^6+\frac{1}{2}\,ad^2x^2+\frac{1}{4}\,(bd^2+2\,ade)x^4$$

input
```
integrate(x*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
1/10*c*e^2*x^10 + 1/8*(2*c*d*e + b*e^2)*x^8 + 1/6*(c*d^2 + 2*b*d*e + a*e^2
)*x^6 + 1/2*a*d^2*x^2 + 1/4*(b*d^2 + 2*a*d*e)*x^4
```

3.275.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05

$$\int x(d+ex^2)^2\,(a+bx^2+cx^4)\;dx=\frac{1}{10}\,ce^2x^{10}+\frac{1}{4}\,cdex^8+\frac{1}{8}\,be^2x^8+\frac{1}{6}\,cd^2x^6+\frac{1}{3}\,bdex^6+\frac{1}{6}\,ae^2x^6+\frac{1}{4}\,bd^2x^4+\frac{1}{2}\,adex^4+\frac{1}{2}\,ad^2x^2$$

input
```
integrate(x*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/10*c*e^2*x^10 + 1/4*c*d*e*x^8 + 1/8*b*e^2*x^8 + 1/6*c*d^2*x^6 + 1/3*b*d*
e*x^6 + 1/6*a*e^2*x^6 + 1/4*b*d^2*x^4 + 1/2*a*d*e*x^4 + 1/2*a*d^2*x^2
```

3.275.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int x(d+ex^2)^2\left(a+bx^2+cx^4\right)\,dx = x^6\left(\frac{c\,d^2}{6}+\frac{b\,d\,e}{3}+\frac{a\,e^2}{6}\right)+x^4\left(\frac{b\,d^2}{4}+\frac{a\,e\,d}{2}\right) + x^8\left(\frac{b\,e^2}{8}+\frac{c\,d\,e}{4}\right)+\frac{a\,d^2\,x^2}{2}+\frac{c\,e^2\,x^{10}}{10}$$

input
```
int(x*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x)
```

output
```
x^6*((a*e^2)/6 + (c*d^2)/6 + (b*d*e)/3) + x^4*((b*d^2)/4 + (a*d*e)/2) + x^
8*((b*e^2)/8 + (c*d*e)/4) + (a*d^2*x^2)/2 + (c*e^2*x^10)/10
```

3.276 $\int (d+ex^2)^2 (a+bx^2+cx^4) \, dx$

3.276.1 Optimal result

Integrand size = 22, antiderivative size = 73

$$\int (d+ex^2)^2 (a+bx^2+cx^4) \, dx = ad^2x + \frac{1}{3}d(bd+2ae)x^3 + \frac{1}{5}\left(cd^2+e(2bd+ae)\right)x^5 + \frac{1}{7}e(2cd+be)x^7 + \frac{1}{9}ce^2x^9$$

output
```
a*d^2*x+1/3*d*(2*a*e+b*d)*x^3+1/5*(c*d^2+e*(a*e+2*b*d))*x^5+1/7*e*(b*e+2*c
*d)*x^7+1/9*c*e^2*x^9
```

3.276.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int (d+ex^2)^2 (a+bx^2+cx^4) \, dx = ad^2x + \frac{1}{3}d(bd+2ae)x^3 + \frac{1}{5}\left(cd^2+2bde+ae^2\right)x^5 + \frac{1}{7}e(2cd+be)x^7 + \frac{1}{9}ce^2x^9$$

input
```
Integrate[(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

output
```
a*d^2*x + (d*(b*d + 2*a*e)*x^3)/3 + ((c*d^2 + 2*b*d*e + a*e^2)*x^5)/5 + (e
*(2*c*d + b*e)*x^7)/7 + (c*e^2*x^9)/9
```

3.276.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right) \, dx$$

$$\Big\downarrow \text{ 1467}$$

$$\int \left(x^4 \left(e(ae+2bd)+cd^2\right)+dx^2(2ae+bd)+ad^2+ex^6(be+2cd)+ce^2x^8\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{5}x^5 \left(e(ae+2bd)+cd^2\right)+\frac{1}{3}dx^3(2ae+bd)+ad^2x+\frac{1}{7}ex^7(be+2cd)+\frac{1}{9}ce^2x^9$$

input
```
Int[(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

output
```
a*d^2*x + (d*(b*d + 2*a*e)*x^3)/3 + ((c*d^2 + e*(2*b*d + a*e))*x^5)/5 + (e
*(2*c*d + b*e)*x^7)/7 + (c*e^2*x^9)/9
```

3.276.3.1 Defintions of rubi rules used

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.276.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

method	result	siz
default	$\frac{c\,e^{2} x^{9}}{9}+\frac{\left(b\,e^{2}+2 d c e \right) x^{7}}{7}+\frac{\left(a\,e^{2}+2 b d e +c\,d^{2}\right) x^{5}}{5}+\frac{\left(2 e d a +b\,d^{2}\right) x^{3}}{3}+a\,d^{2} x$	70
norman	$\frac{c\,e^{2} x^{9}}{9}+\left(\frac{1}{7} b\,e^{2}+\frac{2}{7} d c e \right) x^{7}+\left(\frac{1}{5} a\,e^{2}+\frac{2}{5} b d e +\frac{1}{5} c\,d^{2}\right) x^{5}+\left(\frac{2}{3} e d a +\frac{1}{3} b\,d^{2}\right) x^{3}+a\,d^{2} x$	71
gosper	$\frac{1}{9} c\,e^{2} x^{9}+\frac{1}{7} x^{7} b\,e^{2}+\frac{2}{7} c d e\,x^{7}+\frac{1}{5} x^{5} a\,e^{2}+\frac{2}{5} x^{5} b d e +\frac{1}{5} x^{5} c\,d^{2}+\frac{2}{3} a d e\,x^{3}+\frac{1}{3} x^{3} b\,d^{2}+a\,d^{2} x$	77
risch	$\frac{1}{9} c\,e^{2} x^{9}+\frac{1}{7} x^{7} b\,e^{2}+\frac{2}{7} c d e\,x^{7}+\frac{1}{5} x^{5} a\,e^{2}+\frac{2}{5} x^{5} b d e +\frac{1}{5} x^{5} c\,d^{2}+\frac{2}{3} a d e\,x^{3}+\frac{1}{3} x^{3} b\,d^{2}+a\,d^{2} x$	77
parallelrisch	$\frac{1}{9} c\,e^{2} x^{9}+\frac{1}{7} x^{7} b\,e^{2}+\frac{2}{7} c d e\,x^{7}+\frac{1}{5} x^{5} a\,e^{2}+\frac{2}{5} x^{5} b d e +\frac{1}{5} x^{5} c\,d^{2}+\frac{2}{3} a d e\,x^{3}+\frac{1}{3} x^{3} b\,d^{2}+a\,d^{2} x$	77

input
```
int((e*x^2+d)^2*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/9*c*e^2*x^9+1/7*(b*e^2+2*c*d*e)*x^7+1/5*(a*e^2+2*b*d*e+c*d^2)*x^5+1/3*(2
*a*d*e+b*d^2)*x^3+a*d^2*x
```

3.276.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

$$\int \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right)\,dx = \frac{1}{9}\,ce^2x^9+\frac{1}{7}\,(2\,cde+be^2)x^7+\frac{1}{5}\,(cd^2+2\,bde+ae^2)x^5 + ad^2x+\frac{1}{3}\,(bd^2+2\,ade)x^3$$

input
```
integrate((e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
1/9*c*e^2*x^9 + 1/7*(2*c*d*e + b*e^2)*x^7 + 1/5*(c*d^2 + 2*b*d*e + a*e^2)*
x^5 + a*d^2*x + 1/3*(b*d^2 + 2*a*d*e)*x^3
```

3.276.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.07

$$\int (d+ex^2)^2 (a+bx^2+cx^4)\ dx = ad^2x + \frac{ce^2x^9}{9} + x^7\left(\frac{be^2}{7}+\frac{2cde}{7}\right) + x^5\left(\frac{ae^2}{5}+\frac{2bde}{5}+\frac{cd^2}{5}\right) + x^3\cdot\left(\frac{2ade}{3}+\frac{bd^2}{3}\right)$$

input
```
integrate((e*x**2+d)**2*(c*x**4+b*x**2+a),x)
```

output
```
a*d**2*x + c*e**2*x**9/9 + x**7*(b*e**2/7 + 2*c*d*e/7) + x**5*(a*e**2/5 +
2*b*d*e/5 + c*d**2/5) + x**3*(2*a*d*e/3 + b*d**2/3)
```

3.276.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

$$\int (d+ex^2)^2 (a+bx^2+cx^4)\ dx = \frac{1}{9}\,ce^2x^9 + \frac{1}{7}\,(2\,cde+be^2)x^7 + \frac{1}{5}\,(cd^2+2\,bde+ae^2)x^5 + ad^2x + \frac{1}{3}\,(bd^2+2\,ade)x^3$$

input
```
integrate((e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
1/9*c*e^2*x^9 + 1/7*(2*c*d*e + b*e^2)*x^7 + 1/5*(c*d^2 + 2*b*d*e + a*e^2)*
x^5 + a*d^2*x + 1/3*(b*d^2 + 2*a*d*e)*x^3
```

3.276.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.04

$$\int (d+ex^2)^2 (a+bx^2+cx^4)\ dx = \frac{1}{9}\,ce^2x^9 + \frac{2}{7}\,cdex^7 + \frac{1}{7}\,be^2x^7 + \frac{1}{5}\,cd^2x^5 + \frac{2}{5}\,bdex^5 + \frac{1}{5}\,ae^2x^5 + \frac{1}{3}\,bd^2x^3 + \frac{2}{3}\,adex^3 + ad^2x$$

input
```
integrate((e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/9*c*e^2*x^9 + 2/7*c*d*e*x^7 + 1/7*b*e^2*x^7 + 1/5*c*d^2*x^5 + 2/5*b*d*e*
x^5 + 1/5*a*e^2*x^5 + 1/3*b*d^2*x^3 + 2/3*a*d*e*x^3 + a*d^2*x
```

3.276.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

$$\int \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right)\,dx = x^5\left(\frac{c\,d^2}{5}+\frac{2\,b\,d\,e}{5}+\frac{a\,e^2}{5}\right)+x^3\left(\frac{b\,d^2}{3}+\frac{2\,a\,e\,d}{3}\right)+x^7\left(\frac{b\,e^2}{7}+\frac{2\,c\,d\,e}{7}\right)+\frac{c\,e^2\,x^9}{9}+a\,d^2\,x$$

input
```
int((d + e*x^2)^2*(a + b*x^2 + c*x^4),x)
```

output
```
x^5*((a*e^2)/5 + (c*d^2)/5 + (2*b*d*e)/5) + x^3*((b*d^2)/3 + (2*a*d*e)/3)
+ x^7*((b*e^2)/7 + (2*c*d*e)/7) + (c*e^2*x^9)/9 + a*d^2*x
```

3.277 $\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x} dx$

3.277.1 Optimal result

Integrand size = 25, antiderivative size = 74

$$\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x} dx = \frac{1}{2}d(bd+2ae)x^2 + \frac{1}{4}\left(cd^2+e(2bd+ae)\right)x^4 + \frac{1}{6}e(2cd+be)x^6 + \frac{1}{8}ce^2x^8 + ad^2\log(x)$$

output
```
1/2*d*(2*a*e+b*d)*x^2+1/4*(c*d^2+e*(a*e+2*b*d))*x^4+1/6*e*(b*e+2*c*d)*x^6+
1/8*c*e^2*x^8+a*d^2*ln(x)
```

3.277.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x} dx = \frac{1}{2}d(bd+2ae)x^2 + \frac{1}{4}\left(cd^2+2bde+ae^2\right)x^4 + \frac{1}{6}e(2cd+be)x^6 + \frac{1}{8}ce^2x^8 + ad^2\log(x)$$

input
```
Integrate[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x,x]
```

output
```
(d*(b*d + 2*a*e)*x^2)/2 + ((c*d^2 + 2*b*d*e + a*e^2)*x^4)/4 + (e*(2*c*d +
b*e)*x^6)/6 + (c*e^2*x^8)/8 + a*d^2*Log[x]
```

3.277.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2 (a+bx^2+cx^4)}{x} \, dx$$

$$\Big\downarrow \text{ 1578}$$

$$\frac{1}{2}\int \frac{(ex^2+d)^2 (cx^4+bx^2+a)}{x^2}dx^2$$

$$\Big\downarrow \text{ 1195}$$

$$\frac{1}{2}\int \left(ce^2x^6+e(2cd+be)x^4+\left(cd^2+e(2bd+ae)\right)x^2+d(bd+2ae)+\frac{ad^2}{x^2}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{1}{2}x^4\left(e(ae+2bd)+cd^2\right)+dx^2(2ae+bd)+ad^2\log\left(x^2\right)+\frac{1}{3}ex^6(be+2cd)+\frac{1}{4}ce^2x^8\right)$$

input
```
Int[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x,x]
```

output
```
(d*(b*d + 2*a*e)*x^2 + ((c*d^2 + e*(2*b*d + a*e))*x^4)/2 + (e*(2*c*d + b*e
)*x^6)/3 + (c*e^2*x^8)/4 + a*d^2*Log[x^2])/2
```

3.277.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.277.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.96

method	result	size
norman	$\left(\frac{1}{6}b\,e^{2}+\frac{1}{3}dce\right)x^{6}+\left(eda+\frac{1}{2}b\,d^{2}\right)x^{2}+\left(\frac{1}{4}a\,e^{2}+\frac{1}{2}bde+\frac{1}{4}c\,d^{2}\right)x^{4}+\frac{c\,e^{2}x^{8}}{8}+a\,d^{2}\ln\left(x\right)$	71
default	$\frac{c\,e^{2}x^{8}}{8}+\frac{b\,e^{2}x^{6}}{6}+\frac{cde\,x^{6}}{3}+\frac{a\,e^{2}x^{4}}{4}+\frac{eb\,x^{4}d}{2}+\frac{c\,d^{2}x^{4}}{4}+ade\,x^{2}+\frac{b\,d^{2}x^{2}}{2}+a\,d^{2}\ln\left(x\right)$	77
risch	$\frac{c\,e^{2}x^{8}}{8}+\frac{b\,e^{2}x^{6}}{6}+\frac{cde\,x^{6}}{3}+\frac{a\,e^{2}x^{4}}{4}+\frac{eb\,x^{4}d}{2}+\frac{c\,d^{2}x^{4}}{4}+ade\,x^{2}+\frac{b\,d^{2}x^{2}}{2}+a\,d^{2}\ln\left(x\right)$	77
parallelrisch	$\frac{c\,e^{2}x^{8}}{8}+\frac{b\,e^{2}x^{6}}{6}+\frac{cde\,x^{6}}{3}+\frac{a\,e^{2}x^{4}}{4}+\frac{eb\,x^{4}d}{2}+\frac{c\,d^{2}x^{4}}{4}+ade\,x^{2}+\frac{b\,d^{2}x^{2}}{2}+a\,d^{2}\ln\left(x\right)$	77

input
```
int((e*x^2+d)^2*(c*x^4+b*x^2+a)/x,x,method=_RETURNVERBOSE)
```

output
```
(1/6*b*e^2+1/3*d*c*e)*x^6+(e*d*a+1/2*b*d^2)*x^2+(1/4*a*e^2+1/2*b*d*e+1/4*c
*d^2)*x^4+1/8*c*e^2*x^8+a*d^2*ln(x)
```

3.277.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex^2)^2\,(a+bx^2+cx^4)}{x}\,dx = \frac{1}{8}\,ce^2x^8+\frac{1}{6}\,(2\,cde+be^2)x^6+\frac{1}{4}\,(cd^2+2\,bde+ae^2)x^4 + ad^2\log\left(x\right)+\frac{1}{2}\,(bd^2+2\,ade)x^2$$

input
```
integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x,x, algorithm="fricas")
```

output
```
1/8*c*e^2*x^8 + 1/6*(2*c*d*e + b*e^2)*x^6 + 1/4*(c*d^2 + 2*b*d*e + a*e^2)*
x^4 + a*d^2*log(x) + 1/2*(b*d^2 + 2*a*d*e)*x^2
```

3.277.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x}\,dx = ad^2\log(x) + \frac{ce^2x^8}{8} + x^6\left(\frac{be^2}{6} + \frac{cde}{3}\right) + x^4\left(\frac{ae^2}{4} + \frac{bde}{2} + \frac{cd^2}{4}\right) + x^2\left(ade + \frac{bd^2}{2}\right)$$

input
```
integrate((e*x**2+d)**2*(c*x**4+b*x**2+a)/x,x)
```

output
```
a*d**2*log(x) + c*e**2*x**8/8 + x**6*(b*e**2/6 + c*d*e/3) + x**4*(a*e**2/4
 + b*d*e/2 + c*d**2/4) + x**2*(a*d*e + b*d**2/2)
```

3.277.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x}\,dx = \frac{1}{8}ce^2x^8 + \frac{1}{6}(2\,cde + be^2)x^6 + \frac{1}{4}(cd^2 + 2\,bde + ae^2)x^4 + \frac{1}{2}ad^2\log(x^2) + \frac{1}{2}(bd^2 + 2\,ade)x^2$$

input
```
integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x,x, algorithm="maxima")
```

output
```
1/8*c*e^2*x^8 + 1/6*(2*c*d*e + b*e^2)*x^6 + 1/4*(c*d^2 + 2*b*d*e + a*e^2)*
x^4 + 1/2*a*d^2*log(x^2) + 1/2*(b*d^2 + 2*a*d*e)*x^2
```

3.277.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x}\,dx = \frac{1}{8}ce^2x^8 + \frac{1}{3}cdex^6 + \frac{1}{6}be^2x^6 + \frac{1}{4}cd^2x^4 + \frac{1}{2}bdex^4 + \frac{1}{4}ae^2x^4 + \frac{1}{2}bd^2x^2 + adex^2 + \frac{1}{2}ad^2\log(x^2)$$

input
```
integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x,x, algorithm="giac")
```

output
```
1/8*c*e^2*x^8 + 1/3*c*d*e*x^6 + 1/6*b*e^2*x^6 + 1/4*c*d^2*x^4 + 1/2*b*d*e*
x^4 + 1/4*a*e^2*x^4 + 1/2*b*d^2*x^2 + a*d*e*x^2 + 1/2*a*d^2*log(x^2)
```

3.277.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x}\,dx = x^4\left(\frac{c\,d^2}{4}+\frac{b\,d\,e}{2}+\frac{a\,e^2}{4}\right)+x^2\left(\frac{b\,d^2}{2}+a\,e\,d\right) + x^6\left(\frac{b\,e^2}{6}+\frac{c\,d\,e}{3}\right)+\frac{c\,e^2\,x^8}{8}+a\,d^2\,\ln(x)$$

input
```
int(((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x,x)
```

output
```
x^4*((a*e^2)/4 + (c*d^2)/4 + (b*d*e)/2) + x^2*((b*d^2)/2 + a*d*e) + x^6*((
b*e^2)/6 + (c*d*e)/3) + (c*e^2*x^8)/8 + a*d^2*log(x)
```

3.278 $\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x^2} dx$

3.278.1 Optimal result

Integrand size = 25, antiderivative size = 71

$$\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x^2} dx = -\frac{ad^2}{x} + d(bd+2ae)x + \frac{1}{3}\left(cd^2+e(2bd+ae)\right)x^3 + \frac{1}{5}e(2cd+be)x^5 + \frac{1}{7}ce^2x^7$$

output
```
-a*d^2/x+d*(2*a*e+b*d)*x+1/3*(c*d^2+e*(a*e+2*b*d))*x^3+1/5*e*(b*e+2*c*d)*x
^5+1/7*c*e^2*x^7
```

3.278.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x^2} dx = -\frac{ad^2}{x} + d(bd+2ae)x + \frac{1}{3}\left(cd^2+2bde+ae^2\right)x^3 + \frac{1}{5}e(2cd+be)x^5 + \frac{1}{7}ce^2x^7$$

input
```
Integrate[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^2,x]
```

output
```
-((a*d^2)/x) + d*(b*d + 2*a*e)*x + ((c*d^2 + 2*b*d*e + a*e^2)*x^3)/3 + (e*
(2*c*d + b*e)*x^5)/5 + (c*e^2*x^7)/7
```

3.278.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x^2}dx$$

$$\Big\downarrow \text{ 1584}$$

$$\int \left(x^2(e(ae+2bd)+cd^2)+d(2ae+bd)+\frac{ad^2}{x^2}+ex^4(be+2cd)+ce^2x^6\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}x^3(e(ae+2bd)+cd^2)+dx(2ae+bd)-\frac{ad^2}{x}+\frac{1}{5}ex^5(be+2cd)+\frac{1}{7}ce^2x^7$$

input
```
Int[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^2,x]
```

output
```
-((a*d^2)/x) + d*(b*d + 2*a*e)*x + ((c*d^2 + e*(2*b*d + a*e))*x^3)/3 + (e*
(2*c*d + b*e)*x^5)/5 + (c*e^2*x^7)/7
```

3.278.3.1 Defintions of rubi rules used

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.278.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.04

method	result	size
norman	$\frac{\frac{c\,e^2 x^8}{7}+\left(\frac{1}{5} b\,e^2+\frac{2}{5} dce\right)x^6+\left(\frac{1}{3} a\,e^2+\frac{2}{3} bde+\frac{1}{3} c\,d^2\right)x^4+\left(2eda+b\,d^2\right)x^2-a\,d^2}{x}$	74
default	$\frac{c\,e^2 x^7}{7}+\frac{b\,e^2 x^5}{5}+\frac{2cde\,x^5}{5}+\frac{a\,e^2 x^3}{3}+\frac{2eb\,x^3 d}{3}+\frac{c\,d^2 x^3}{3}+2edax+b\,d^2 x-\frac{a\,d^2}{x}$	75
risch	$\frac{c\,e^2 x^7}{7}+\frac{b\,e^2 x^5}{5}+\frac{2cde\,x^5}{5}+\frac{a\,e^2 x^3}{3}+\frac{2eb\,x^3 d}{3}+\frac{c\,d^2 x^3}{3}+2edax+b\,d^2 x-\frac{a\,d^2}{x}$	75
gosper	$-\frac{-15c\,e^2 x^8-21b\,e^2 x^6-42cde\,x^6-35a\,e^2 x^4-70eb\,x^4 d-35c\,d^2 x^4-210ade\,x^2-105b\,d^2 x^2+105a\,d^2}{105x}$	82
parallelrisch	$\frac{15c\,e^2 x^8+21b\,e^2 x^6+42cde\,x^6+35a\,e^2 x^4+70eb\,x^4 d+35c\,d^2 x^4+210ade\,x^2+105b\,d^2 x^2-105a\,d^2}{105x}$	82

input
```
int((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/x*(1/7*c*e^2*x^8+(1/5*b*e^2+2/5*d*c*e)*x^6+(1/3*a*e^2+2/3*b*d*e+1/3*c*d^
2)*x^4+(2*a*d*e+b*d^2)*x^2-a*d^2)
```

3.278.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.04

$$\int \frac{(d+ex^2)^2\,(a+bx^2+cx^4)}{x^2}\,dx = \frac{15\,ce^2x^8+21\,(2\,cde+be^2)x^6+35\,(cd^2+2\,bde+ae^2)x^4-105\,ad^2+105\,(bd^2+2\,ade)x^2}{105\,x}$$

input
```
integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^2,x, algorithm="fricas")
```

output
```
1/105*(15*c*e^2*x^8 + 21*(2*c*d*e + b*e^2)*x^6 + 35*(c*d^2 + 2*b*d*e + a*e
^2)*x^4 - 105*a*d^2 + 105*(b*d^2 + 2*a*d*e)*x^2)/x
```

3.278.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03

$$\int \frac{(d+ex^2)^2\,(a+bx^2+cx^4)}{x^2}\,dx = -\frac{ad^2}{x} + \frac{ce^2x^7}{7} + x^5\left(\frac{be^2}{5} + \frac{2cde}{5}\right) + x^3\left(\frac{ae^2}{3} + \frac{2bde}{3} + \frac{cd^2}{3}\right) + x\left(2ade + bd^2\right)$$

input
```
integrate((e*x**2+d)**2*(c*x**4+b*x**2+a)/x**2,x)
```

output
```
-a*d**2/x + c*e**2*x**7/7 + x**5*(b*e**2/5 + 2*c*d*e/5) + x**3*(a*e**2/3 +
 2*b*d*e/3 + c*d**2/3) + x*(2*a*d*e + b*d**2)
```

3.278.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex^2)^2\,(a+bx^2+cx^4)}{x^2}\,dx = \frac{1}{7}\,ce^2x^7 + \frac{1}{5}\,\left(2\,cde + be^2\right)x^5 + \frac{1}{3}\,\left(cd^2 + 2\,bde + ae^2\right)x^3 - \frac{ad^2}{x} + \left(bd^2 + 2\,ade\right)x$$

input
```
integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^2,x, algorithm="maxima")
```

output
```
1/7*c*e^2*x^7 + 1/5*(2*c*d*e + b*e^2)*x^5 + 1/3*(c*d^2 + 2*b*d*e + a*e^2)*
x^3 - a*d^2/x + (b*d^2 + 2*a*d*e)*x
```

3.278.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.04

$$\int \frac{(d+ex^2)^2\,(a+bx^2+cx^4)}{x^2}\,dx = \frac{1}{7}\,ce^2x^7 + \frac{2}{5}\,cdex^5 + \frac{1}{5}\,be^2x^5 + \frac{1}{3}\,cd^2x^3 + \frac{2}{3}\,bdex^3 + \frac{1}{3}\,ae^2x^3 + bd^2x + 2\,adex - \frac{ad^2}{x}$$

input
```
integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^2,x, algorithm="giac")
```

output
```
1/7*c*e^2*x^7 + 2/5*c*d*e*x^5 + 1/5*b*e^2*x^5 + 1/3*c*d^2*x^3 + 2/3*b*d*e*
x^3 + 1/3*a*e^2*x^3 + b*d^2*x + 2*a*d*e*x - a*d^2/x
```

3.278.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex^2)^2 (a+bx^2+cx^4)}{x^2} dx = x^3 \left(\frac{c d^2}{3} + \frac{2 b d e}{3} + \frac{a e^2}{3}\right) + x \left(b d^2 + 2 a e d\right) + x^5 \left(\frac{b e^2}{5} + \frac{2 c d e}{5}\right) - \frac{a d^2}{x} + \frac{c e^2 x^7}{7}$$

input
```
int(((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^2,x)
```

output
```
x^3*((a*e^2)/3 + (c*d^2)/3 + (2*b*d*e)/3) + x*(b*d^2 + 2*a*d*e) + x^5*((b*
e^2)/5 + (2*c*d*e)/5) - (a*d^2)/x + (c*e^2*x^7)/7
```

3.279 $\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x^3} dx$

3.279.1 Optimal result

Integrand size = 25, antiderivative size = 74

$$\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x^3} dx = -\frac{ad^2}{2x^2} + \frac{1}{2}\left(cd^2+e(2bd+ae)\right)x^2 + \frac{1}{4}e(2cd+be)x^4 + \frac{1}{6}ce^2x^6 + d(bd+2ae)\log(x)$$

output
```
-1/2*a*d^2/x^2+1/2*(c*d^2+e*(a*e+2*b*d))*x^2+1/4*e*(b*e+2*c*d)*x^4+1/6*c*e
^2*x^6+d*(2*a*e+b*d)*ln(x)
```

3.279.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x^3} dx = \frac{1}{12}\left(-\frac{6ad^2}{x^2} + 6\left(cd^2+e(2bd+ae)\right)x^2 + 3e(2cd+be)x^4 + 2ce^2x^6 + 12d(bd+2ae)\log(x)\right)$$

input
```
Integrate[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^3,x]
```

output
```
((-6*a*d^2)/x^2 + 6*(c*d^2 + e*(2*b*d + a*e))*x^2 + 3*e*(2*c*d + b*e)*x^4
+ 2*c*e^2*x^6 + 12*d*(b*d + 2*a*e)*Log[x])/12
```

3.279.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x^3}dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{(ex^2+d)^2(cx^4+bx^2+a)}{x^4}dx^2$$

$$\Big\downarrow \text{1195}$$

$$\frac{1}{2}\int \left(ce^2x^4+e(2cd+be)x^2+cd^2\left(\frac{e(2bd+ae)}{cd^2}+1\right)+\frac{d(bd+2ae)}{x^2}+\frac{ad^2}{x^4}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(x^2(e(ae+2bd)+cd^2)+d\log(x^2)(2ae+bd)-\frac{ad^2}{x^2}+\frac{1}{2}ex^4(be+2cd)+\frac{1}{3}ce^2x^6\right)$$

input
```
Int[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^3,x]
```

output
```
(-((a*d^2)/x^2) + (c*d^2 + e*(2*b*d + a*e))*x^2 + (e*(2*c*d + b*e)*x^4)/2
+ (c*e^2*x^6)/3 + d*(b*d + 2*a*e)*Log[x^2])/2
```

3.279.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.279.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

method	result	size
norman	$\frac{\left(\frac{1}{4} b\, e^{2}+\frac{1}{2} d c e\right) x^{6}+\left(\frac{1}{2} a\, e^{2}+b d e+\frac{1}{2} c\, d^{2}\right) x^{4}-\frac{a\, d^{2}}{2}+\frac{c\, e^{2} x^{8}}{6}}{x^{2}}+\left(2 e d a+b\, d^{2}\right) \ln \left(x\right)$	73
default	$\frac{c\, e^{2} x^{6}}{6}+\frac{b\, e^{2} x^{4}}{4}+\frac{d c\, x^{4} e}{2}+\frac{a\, e^{2} x^{2}}{2}+b d e\, x^{2}+\frac{c\, d^{2} x^{2}}{2}+d \left(2 a e+b d\right) \ln \left(x\right)-\frac{a\, d^{2}}{2 x^{2}}$	74
risch	$\frac{c\, e^{2} x^{6}}{6}+\frac{b\, e^{2} x^{4}}{4}+\frac{d c\, x^{4} e}{2}+\frac{a\, e^{2} x^{2}}{2}+b d e\, x^{2}+\frac{c\, d^{2} x^{2}}{2}-\frac{a\, d^{2}}{2 x^{2}}+2 \ln \left(x\right) a d e+\ln \left(x\right) b\, d^{2}$	76
parallelrisch	$\frac{2 c\, e^{2} x^{8}+3 b\, e^{2} x^{6}+6 c d e\, x^{6}+6 a\, e^{2} x^{4}+12 e b\, x^{4} d+6 c\, d^{2} x^{4}+24 \ln \left(x\right) x^{2} a d e+12 \ln \left(x\right) x^{2} b\, d^{2}-6 a\, d^{2}}{12 x^{2}}$	86

input
```
int((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^3,x,method=_RETURNVERBOSE)
```

output
```
((1/4*b*e^2+1/2*d*c*e)*x^6+(1/2*a*e^2+b*d*e+1/2*c*d^2)*x^4-1/2*a*d^2+1/6*c
*e^2*x^8)/x^2+(2*a*d*e+b*d^2)*ln(x)
```

3.279.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.03

$$\int \frac{\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)}{x^3}\,dx = \frac{2\,ce^2x^8+3\,(2\,cde+be^2)x^6+6\,(cd^2+2\,bde+ae^2)x^4+12\,(bd^2+2\,ade)x^2\log\left(x\right)-6\,ad^2}{12\,x^2}$$

input
```
integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^3,x, algorithm="fricas")
```

output
```
1/12*(2*c*e^2*x^8 + 3*(2*c*d*e + b*e^2)*x^6 + 6*(c*d^2 + 2*b*d*e + a*e^2)*
x^4 + 12*(b*d^2 + 2*a*d*e)*x^2*log(x) - 6*a*d^2)/x^2
```

3.279.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x^3}\,dx = -\frac{ad^2}{2x^2}+\frac{ce^2x^6}{6}+d(2ae+bd)\log(x)+x^4\left(\frac{be^2}{4}+\frac{cde}{2}\right)+x^2\left(\frac{ae^2}{2}+bde+\frac{cd^2}{2}\right)$$

input
```
integrate((e*x**2+d)**2*(c*x**4+b*x**2+a)/x**3,x)
```

output
```
-a*d**2/(2*x**2) + c*e**2*x**6/6 + d*(2*a*e + b*d)*log(x) + x**4*(b*e**2/4
 + c*d*e/2) + x**2*(a*e**2/2 + b*d*e + c*d**2/2)
```

3.279.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x^3}\,dx = \frac{1}{6}\,ce^2x^6+\frac{1}{4}\,(2\,cde+be^2)x^4+\frac{1}{2}\,(cd^2+2\,bde+ae^2)x^2+\frac{1}{2}\,(bd^2+2\,ade)\log\left(x^2\right)-\frac{ad^2}{2\,x^2}$$

input
```
integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^3,x, algorithm="maxima")
```

output
```
1/6*c*e^2*x^6 + 1/4*(2*c*d*e + b*e^2)*x^4 + 1/2*(c*d^2 + 2*b*d*e + a*e^2)*
x^2 + 1/2*(b*d^2 + 2*a*d*e)*log(x^2) - 1/2*a*d^2/x^2
```

3.279.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.30

$$\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x^3}\,dx = \frac{1}{6}\,ce^2x^6+\frac{1}{2}\,cdex^4+\frac{1}{4}\,be^2x^4+\frac{1}{2}\,cd^2x^2+bdex^2+\frac{1}{2}\,ae^2x^2+\frac{1}{2}\,(bd^2+2\,ade)\log\left(x^2\right)-\frac{bd^2x^2+2\,adex^2+ad^2}{2\,x^2}$$

input
```
integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^3,x, algorithm="giac")
```

output
```
1/6*c*e^2*x^6 + 1/2*c*d*e*x^4 + 1/4*b*e^2*x^4 + 1/2*c*d^2*x^2 + b*d*e*x^2
+ 1/2*a*e^2*x^2 + 1/2*(b*d^2 + 2*a*d*e)*log(x^2) - 1/2*(b*d^2*x^2 + 2*a*d*
e*x^2 + a*d^2)/x^2
```

3.279.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex^2)^2\,(a+bx^2+cx^4)}{x^3}\,dx = x^2\left(\frac{c\,d^2}{2}+b\,d\,e+\frac{a\,e^2}{2}\right)+x^4\left(\frac{b\,e^2}{4}+\frac{c\,d\,e}{2}\right) + \ln(x)\,\left(b\,d^2+2\,a\,e\,d\right)-\frac{a\,d^2}{2\,x^2}+\frac{c\,e^2\,x^6}{6}$$

input
```
int(((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^3,x)
```

output
```
x^2*((a*e^2)/2 + (c*d^2)/2 + b*d*e) + x^4*((b*e^2)/4 + (c*d*e)/2) + log(x)
*(b*d^2 + 2*a*d*e) - (a*d^2)/(2*x^2) + (c*e^2*x^6)/6
```

3.280 $\int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^2} dx$

3.280.1 Optimal result

Integrand size = 25, antiderivative size = 168

$$\int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^2} dx = -\frac{d(4cd^2-e(3bd-2ae))x}{e^5}+\frac{(3cd^2-e(2bd-ae))x^3}{3e^4} - \frac{(2cd-be)x^5}{5e^3}+\frac{cx^7}{7e^2}-\frac{d^2(cd^2-bde+ae^2)x}{2e^5(d+ex^2)} + \frac{d^{3/2}(9cd^2-e(7bd-5ae))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2e^{11/2}}$$

output
```
-d*(4*c*d^2-e*(-2*a*e+3*b*d))*x/e^5+1/3*(3*c*d^2-e*(-a*e+2*b*d))*x^3/e^4-1
/5*(-b*e+2*c*d)*x^5/e^3+1/7*c*x^7/e^2-1/2*d^2*(a*e^2-b*d*e+c*d^2)*x/e^5/(e
*x^2+d)+1/2*d^(3/2)*(9*c*d^2-e*(-5*a*e+7*b*d))*arctan(x*e^(1/2)/d^(1/2))/e
^(11/2)
```

3.280.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.98

$$\int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^2} dx = -\frac{d(4cd^2-3bde+2ae^2)x}{e^5}+\frac{(3cd^2-2bde+ae^2)x^3}{3e^4} + \frac{(-2cd+be)x^5}{5e^3}+\frac{cx^7}{7e^2}-\frac{(cd^4-bd^3e+ad^2e^2)x}{2e^5(d+ex^2)} + \frac{d^{3/2}(9cd^2-7bde+5ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2e^{11/2}}$$

input
```
Integrate[(x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]
```

output
```
-((d*(4*c*d^2 - 3*b*d*e + 2*a*e^2)*x)/e^5) + ((3*c*d^2 - 2*b*d*e + a*e^2)*
x^3)/(3*e^4) + ((-2*c*d + b*e)*x^5)/(5*e^3) + (c*x^7)/(7*e^2) - ((c*d^4 -
b*d^3*e + a*d^2*e^2)*x)/(2*e^5*(d + e*x^2)) + (d^(3/2)*(9*c*d^2 - 7*b*d*e
+ 5*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*e^(11/2))
```

3.280.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1580, 25, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6 \left(a+b x^2+c x^4\right)}{\left(d+e x^2\right)^2} \, dx$$

$$\Big\downarrow \text{1580}$$

$$-\frac{\int -\frac{2 c e^4 x^8-2 e^3 (c d-b e) x^6+2 e^2 \left(c d^2-b e d+a e^2\right) x^4-2 d e \left(c d^2-b e d+a e^2\right) x^2+d^2 \left(c d^2-b e d+a e^2\right)}{e x^2+d}dx}{2 e^5}-\frac{d^2 x \left(a e^2-b d e+c d^2\right)}{2 e^5 \left(d+e x^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2 c e^4 x^8-2 e^3 (c d-b e) x^6+2 e^2 \left(c d^2-b e d+a e^2\right) x^4-2 d e \left(c d^2-b e d+a e^2\right) x^2+d^2 \left(c d^2-b e d+a e^2\right)}{e x^2+d}dx}{2 e^5}-\frac{d^2 x \left(a e^2-b d e+c d^2\right)}{2 e^5 \left(d+e x^2\right)}$$

$$\Big\downarrow \text{2341}$$

$$\frac{\int \left(2 c e^3 x^6-2 e^2 (2 c d-b e) x^4+2 e \left(3 c d^2-e (2 b d-a e)\right) x^2-2 d \left(4 c d^2-e (3 b d-2 a e)\right)+\frac{9 c d^4-7 b e d^3+5 a e^2 d^2}{e x^2+d}\right)dx}{2 e^5}-\frac{d^2 x \left(a e^2-b d e+c d^2\right)}{2 e^5 \left(d+e x^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{d^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(9cd^2-e(7bd-5ae))}{\sqrt{e}}+\frac{2}{3}ex^3(3cd^2-e(2bd-ae))-2dx(4cd^2-e(3bd-2ae))-\frac{2}{5}e^2x^5(2cd-be)+\frac{2}{7}ce}{2e^5}$$
$$\frac{d^2x(ae^2-bde+cd^2)}{2e^5(d+ex^2)}$$

input
```
Int[(x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]
```

output
```
-1/2*(d^2*(c*d^2 - b*d*e + a*e^2)*x)/(e^5*(d + e*x^2)) + (-2*d*(4*c*d^2 -
e*(3*b*d - 2*a*e))*x + (2*e*(3*c*d^2 - e*(2*b*d - a*e))*x^3)/3 - (2*e^2*(2
*c*d - b*e)*x^5)/5 + (2*c*e^3*x^7)/7 + (d^(3/2)*(9*c*d^2 - e*(7*b*d - 5*a*
e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e])/(2*e^5)
```

3.280.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1580
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[1/(2*e^(2*p + m/2)*
(q + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*
e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b
*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.280. $\int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx$

3.280.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.95

method	result
default	$-\frac{-\frac{1}{7}c\,x^{7}e^{3}-\frac{1}{5}b\,e^{3}x^{5}+\frac{2}{5}cd\,x^{5}e^{2}-\frac{1}{3}a\,e^{3}x^{3}+\frac{2}{3}bd\,e^{2}x^{3}-c\,d^{2}e\,x^{3}+2d\,e^{2}ax-3b\,d^{2}ex+4d^{3}cx}{e^{5}}+\frac{d^{2}\left(\frac{\left(-\frac{1}{2}a\,e^{2}+\frac{1}{2}bde-\frac{1}{2}c\,d^{2}\right)x}{e\,x^{2}+d}+\frac{\left(5a\,e^{2}-\right.}{}\right.}{e^{5}}$
risch	$\frac{c\,x^{7}}{7e^{2}}+\frac{b\,x^{5}}{5e^{2}}-\frac{2cd\,x^{5}}{5e^{3}}+\frac{a\,x^{3}}{3e^{2}}-\frac{2bd\,x^{3}}{3e^{3}}+\frac{c\,d^{2}x^{3}}{e^{4}}-\frac{2dax}{e^{3}}+\frac{3b\,d^{2}x}{e^{4}}-\frac{4d^{3}cx}{e^{5}}+\frac{\left(-\frac{1}{2}e^{2}d^{2}a+\frac{1}{2}d^{3}eb-\frac{1}{2}d^{4}c\right)x}{e^{5}\left(e\,x^{2}+d\right)}+\frac{5\sqrt{-ed}\,d}{}$

input
```
int(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/e^5*(-1/7*c*x^7*e^3-1/5*b*e^3*x^5+2/5*c*d*x^5*e^2-1/3*a*e^3*x^3+2/3*b*d
*e^2*x^3-c*d^2*e*x^3+2*d*e^2*a*x-3*b*d^2*e*x+4*d^3*c*x)+d^2/e^5*((-1/2*a*e
^2+1/2*b*d*e-1/2*c*d^2)*x/(e*x^2+d)+1/2*(5*a*e^2-7*b*d*e+9*c*d^2)/(e*d)^(1
/2)*arctan(e*x/(e*d)^(1/2)))
```

3.280.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 426, normalized size of antiderivative = 2.54

$$\int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx$$

$$=\left[\frac{60\,ce^4x^9-12\,(9\,cde^3-7\,be^4)x^7+28\,(9\,cd^2e^2-7\,bde^3+5\,ae^4)x^5-140\,(9\,cd^3e-7\,bd^2e^2+5\,ade^3)x^3+}{}\right.$$

input
```
integrate(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
[1/420*(60*c*e^4*x^9 - 12*(9*c*d*e^3 - 7*b*e^4)*x^7 + 28*(9*c*d^2*e^2 - 7*
b*d*e^3 + 5*a*e^4)*x^5 - 140*(9*c*d^3*e - 7*b*d^2*e^2 + 5*a*d*e^3)*x^3 + 1
05*(9*c*d^4 - 7*b*d^3*e + 5*a*d^2*e^2 + (9*c*d^3*e - 7*b*d^2*e^2 + 5*a*d*e
^3)*x^2)*sqrt(-d/e)*log((e*x^2 + 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d)) - 210*
(9*c*d^4 - 7*b*d^3*e + 5*a*d^2*e^2)*x)/(e^6*x^2 + d*e^5), 1/210*(30*c*e^4*
x^9 - 6*(9*c*d*e^3 - 7*b*e^4)*x^7 + 14*(9*c*d^2*e^2 - 7*b*d*e^3 + 5*a*e^4)
*x^5 - 70*(9*c*d^3*e - 7*b*d^2*e^2 + 5*a*d*e^3)*x^3 + 105*(9*c*d^4 - 7*b*d
^3*e + 5*a*d^2*e^2 + (9*c*d^3*e - 7*b*d^2*e^2 + 5*a*d*e^3)*x^2)*sqrt(d/e)*
arctan(e*x*sqrt(d/e)/d) - 105*(9*c*d^4 - 7*b*d^3*e + 5*a*d^2*e^2)*x)/(e^6*
x^2 + d*e^5)]
```

3.280.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(153) = 306$.

Time = 0.65 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.90

$$\begin{aligned}
&\int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx \\
&= \frac{cx^7}{7e^2} + x^5\left(\frac{b}{5e^2} - \frac{2cd}{5e^3}\right) + x^3\left(\frac{a}{3e^2} - \frac{2bd}{3e^3} + \frac{cd^2}{e^4}\right) \\
&\quad + x\left(-\frac{2ad}{e^3} + \frac{3bd^2}{e^4} - \frac{4cd^3}{e^5}\right) + \frac{x(-ad^2e^2 + bd^3e - cd^4)}{2de^5 + 2e^6x^2} \\
&\quad - \frac{\sqrt{-\frac{d^3}{e^{11}}}\cdot(5ae^2 - 7bde + 9cd^2)\log\left(-\frac{e^5\sqrt{-\frac{d^3}{e^{11}}}\cdot(5ae^2-7bde+9cd^2)}{5ade^2-7bd^2e+9cd^3} + x\right)}{4} \\
&\quad + \frac{\sqrt{-\frac{d^3}{e^{11}}}\cdot(5ae^2 - 7bde + 9cd^2)\log\left(\frac{e^5\sqrt{-\frac{d^3}{e^{11}}}\cdot(5ae^2-7bde+9cd^2)}{5ade^2-7bd^2e+9cd^3} + x\right)}{4}
\end{aligned}$$

input
```
integrate(x**6*(c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

output
```
c*x**7/(7*e**2) + x**5*(b/(5*e**2) - 2*c*d/(5*e**3)) + x**3*(a/(3*e**2) -
2*b*d/(3*e**3) + c*d**2/e**4) + x*(-2*a*d/e**3 + 3*b*d**2/e**4 - 4*c*d**3/
e**5) + x*(-a*d**2*e**2 + b*d**3*e - c*d**4)/(2*d*e**5 + 2*e**6*x**2) - sq
rt(-d**3/e**11)*(5*a*e**2 - 7*b*d*e + 9*c*d**2)*log(-e**5*sqrt(-d**3/e**11
)*(5*a*e**2 - 7*b*d*e + 9*c*d**2)/(5*a*d*e**2 - 7*b*d**2*e + 9*c*d**3) + x
)/4 + sqrt(-d**3/e**11)*(5*a*e**2 - 7*b*d*e + 9*c*d**2)*log(e**5*sqrt(-d**
3/e**11)*(5*a*e**2 - 7*b*d*e + 9*c*d**2)/(5*a*d*e**2 - 7*b*d**2*e + 9*c*d*
*3) + x)/4
```

3.280.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.280.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.04

$$\int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx = \frac{(9\,cd^4-7\,bd^3e+5\,ad^2e^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}e^5} - \frac{cd^4x-bd^3ex+ad^2e^2x}{2\,(ex^2+d)e^5} + \frac{15\,ce^{12}x^7-42\,cde^{11}x^5+21\,be^{12}x^5+105\,cd^2e^{10}x^3-70\,bde^{11}x^3+35\,ae^{12}x^3-420\,cd^3e^9x+315\,bd^2e^{10}x}{105\,e^{14}}$$

input
```
integrate(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
1/2*(9*c*d^4 - 7*b*d^3*e + 5*a*d^2*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e
^5) - 1/2*(c*d^4*x - b*d^3*e*x + a*d^2*e^2*x)/((e*x^2 + d)*e^5) + 1/105*(1
5*c*e^12*x^7 - 42*c*d*e^11*x^5 + 21*b*e^12*x^5 + 105*c*d^2*e^10*x^3 - 70*b
*d*e^11*x^3 + 35*a*e^12*x^3 - 420*c*d^3*e^9*x + 315*b*d^2*e^10*x - 210*a*d
*e^11*x)/e^14
```

3.280.9 Mupad [B] (verification not implemented)

Time = 7.70 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.49

$$\int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx = x^5\left(\frac{b}{5\,e^2}-\frac{2\,c\,d}{5\,e^3}\right)-x^3\left(\frac{c\,d^2}{3\,e^4}-\frac{a}{3\,e^2}+\frac{2\,d\,\left(\frac{b}{e^2}-\frac{2\,c\,d}{e^3}\right)}{3\,e}\right)+x\left(\frac{2\,d\,\left(\frac{c\,d^2}{e^4}-\frac{a}{e^2}+\frac{2\,d\,\left(\frac{b}{e^2}-\frac{2\,c\,d}{e^3}\right)}{e}\right)}{e}-\frac{d^2\,\left(\frac{b}{e^2}-\frac{2\,c\,d}{e^3}\right)}{e^2}\right)-\frac{x\,\left(\frac{c\,d^4}{2}-\frac{b\,d^3\,e}{2}+\frac{a\,d^2\,e^2}{2}\right)}{e^6\,x^2+d\,e^5}+\frac{c\,x^7}{7\,e^2}+\frac{d^{3/2}\,\mathrm{atan}\left(\frac{d^{3/2}\,\sqrt{e}\,x\,\left(9\,c\,d^2-7\,b\,d\,e+5\,a\,e^2\right)}{9\,c\,d^4-7\,b\,d^3\,e+5\,a\,d^2\,e^2}\right)\,\left(9\,c\,d^2-7\,b\,d\,e+5\,a\,e^2\right)}{2\,e^{11/2}}$$

input
```
int((x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x)
```

output
```
x^5*(b/(5*e^2) - (2*c*d)/(5*e^3)) - x^3*((c*d^2)/(3*e^4) - a/(3*e^2) + (2*
d*(b/e^2 - (2*c*d)/e^3))/(3*e)) + x*((2*d*((c*d^2)/e^4 - a/e^2 + (2*d*(b/e
^2 - (2*c*d)/e^3))/e))/e - (d^2*(b/e^2 - (2*c*d)/e^3))/e^2) - (x*((c*d^4)/
2 + (a*d^2*e^2)/2 - (b*d^3*e)/2))/(d*e^5 + e^6*x^2) + (c*x^7)/(7*e^2) + (d
^(3/2)*atan((d^(3/2)*e^(1/2)*x*(5*a*e^2 + 9*c*d^2 - 7*b*d*e))/(9*c*d^4 + 5
*a*d^2*e^2 - 7*b*d^3*e))*(5*a*e^2 + 9*c*d^2 - 7*b*d*e))/(2*e^(11/2))
```

3.281 $\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^2} dx$

3.281.1 Optimal result

Integrand size = 25, antiderivative size = 135

$$\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^2} dx = \frac{(3cd^2-e(2bd-ae))\,x}{e^4} - \frac{(2cd-be)x^3}{3e^3} + \frac{cx^5}{5e^2} + \frac{d(cd^2-bde+ae^2)\,x}{2e^4\,(d+ex^2)} - \frac{\sqrt{d}(7cd^2-e(5bd-3ae))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2e^{9/2}}$$

output
```
(3*c*d^2-e*(-a*e+2*b*d))*x/e^4-1/3*(-b*e+2*c*d)*x^3/e^3+1/5*c*x^5/e^2+1/2*
d*(a*e^2-b*d*e+c*d^2)*x/e^4/(e*x^2+d)-1/2*(7*c*d^2-e*(-3*a*e+5*b*d))*arcta
n(x*e^(1/2)/d^(1/2))*d^(1/2)/e^(9/2)
```

3.281.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.99

$$\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^2} dx = \frac{(3cd^2-2bde+ae^2)\,x}{e^4} + \frac{(-2cd+be)x^3}{3e^3} + \frac{cx^5}{5e^2} + \frac{(cd^3-bd^2e+ade^2)\,x}{2e^4\,(d+ex^2)} - \frac{\sqrt{d}(7cd^2-5bde+3ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2e^{9/2}}$$

input
```
Integrate[(x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]
```

output
```
((3*c*d^2 - 2*b*d*e + a*e^2)*x)/e^4 + ((-2*c*d + b*e)*x^3)/(3*e^3) + (c*x^
5)/(5*e^2) + ((c*d^3 - b*d^2*e + a*d*e^2)*x)/(2*e^4*(d + e*x^2)) - (Sqrt[d
]*(7*c*d^2 - 5*b*d*e + 3*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*e^(9/2))
```

3.281.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1580, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\left(a+bx^2+cx^4\right)}{\left(d+ex^2\right)^2}dx$$

$$\Big\downarrow \text{1580}$$

$$\frac{dx\left(ae^2-bde+cd^2\right)}{2e^4\left(d+ex^2\right)}-\frac{\int \frac{-2ce^3x^6+2e^2(cd-be)x^4-2e\left(cd^2-bed+ae^2\right)x^2+d\left(cd^2-bed+ae^2\right)}{ex^2+d}dx}{2e^4}$$

$$\Big\downarrow \text{2341}$$

$$\frac{dx\left(ae^2-bde+cd^2\right)}{2e^4\left(d+ex^2\right)}-\frac{\int \left(-2ce^2x^4+2e(2cd-be)x^2-2\left(3cd^2-2bed+ae^2\right)+\frac{7cd^3-5bed^2+3ae^2d}{ex^2+d}\right)dx}{2e^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{dx\left(ae^2-bde+cd^2\right)}{2e^4\left(d+ex^2\right)}-\frac{\frac{\sqrt{d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(7cd^2-e(5bd-3ae)\right)}{\sqrt{e}}-2x\left(3cd^2-e(2bd-ae)\right)+\frac{2}{3}ex^3(2cd-be)-\frac{2}{5}ce^2x^5}{2e^4}$$

input
```
Int[(x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]
```

output
```
(d*(c*d^2 - b*d*e + a*e^2)*x)/(2*e^4*(d + e*x^2)) - (-2*(3*c*d^2 - e*(2*b*
d - a*e))*x + (2*e*(2*c*d - b*e)*x^3)/3 - (2*c*e^2*x^5)/5 + (Sqrt[d]*(7*c*
d^2 - e*(5*b*d - 3*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e])/(2*e^4)
```

3.281.3.1 Defintions of rubi rules used

rule 1580
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[1/(2*e^(2*p + m/2)*
(q + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*
e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b
*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.281.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.90

method	result
default	$\frac{\frac{1}{5} c\,x^{5} e^{2}+\frac{1}{3} b\,e^{2} x^{3}-\frac{2}{3} d c\,x^{3} e+a\,e^{2} x-2 b d e x+3 c\,d^{2} x}{e^{4}}-\frac{d \left(\frac{\left(-\frac{1}{2} a\,e^{2}+\frac{1}{2} b d e-\frac{1}{2} c\,d^{2}\right) x}{e\,x^{2}+d}+\frac{\left(3 a\,e^{2}-5 b d e+7 c\,d^{2}\right) \arctan\left(\frac{e x}{\sqrt{e d}}\right)}{2 \sqrt{e d}}\right)}{e^{4}}$
risch	$\frac{c\,x^{5}}{5 e^{2}}+\frac{b\,x^{3}}{3 e^{2}}-\frac{2 d c\,x^{3}}{3 e^{3}}+\frac{a x}{e^{2}}-\frac{2 b d x}{e^{3}}+\frac{3 c\,d^{2} x}{e^{4}}+\frac{\left(\frac{1}{2} d\,e^{2} a-\frac{1}{2} b\,d^{2} e+\frac{1}{2} d^{3} c\right) x}{e^{4} \left(e\,x^{2}+d\right)}+\frac{3 \sqrt{-e d}\,\ln\left(-\sqrt{-e d}\,x-d\right) a}{4 e^{3}}-\frac{5 \sqrt{-e d}\,\ln(-\sqrt{}}{4 e^{4}}$

input
```
int(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/e^4*(1/5*c*x^5*e^2+1/3*b*e^2*x^3-2/3*d*c*x^3*e+a*e^2*x-2*b*d*e*x+3*c*d^2
*x)-d/e^4*((-1/2*a*e^2+1/2*b*d*e-1/2*c*d^2)*x/(e*x^2+d)+1/2*(3*a*e^2-5*b*d
*e+7*c*d^2)/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2)))
```

3.281.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.59

$$\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx$$
$$= \left[\frac{12\,ce^3x^7 - 4\,(7\,cde^2 - 5\,be^3)x^5 + 20\,(7\,cd^2e - 5\,bde^2 + 3\,ae^3)x^3 + 15\,(7\,cd^3 - 5\,bd^2e + 3\,ade^2 + (7\,cd^2e - }{60\,(e^5x^2 + de^4)} \right.$$

input
```
integrate(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
[1/60*(12*c*e^3*x^7 - 4*(7*c*d*e^2 - 5*b*e^3)*x^5 + 20*(7*c*d^2*e - 5*b*d*
e^2 + 3*a*e^3)*x^3 + 15*(7*c*d^3 - 5*b*d^2*e + 3*a*d*e^2 + (7*c*d^2*e - 5*
b*d*e^2 + 3*a*e^3)*x^2)*sqrt(-d/e)*log((e*x^2 - 2*e*x*sqrt(-d/e) - d)/(e*x
^2 + d)) + 30*(7*c*d^3 - 5*b*d^2*e + 3*a*d*e^2)*x)/(e^5*x^2 + d*e^4), 1/30
*(6*c*e^3*x^7 - 2*(7*c*d*e^2 - 5*b*e^3)*x^5 + 10*(7*c*d^2*e - 5*b*d*e^2 +
3*a*e^3)*x^3 - 15*(7*c*d^3 - 5*b*d^2*e + 3*a*d*e^2 + (7*c*d^2*e - 5*b*d*e^
2 + 3*a*e^3)*x^2)*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d) + 15*(7*c*d^3 - 5*b*d^
2*e + 3*a*d*e^2)*x)/(e^5*x^2 + d*e^4)]
```

3.281.6 Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.40

$$\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx = \frac{cx^5}{5e^2} + x^3\left(\frac{b}{3e^2} - \frac{2cd}{3e^3}\right) + x\left(\frac{a}{e^2} - \frac{2bd}{e^3} + \frac{3cd^2}{e^4}\right) + \frac{x(ade^2 - bd^2e + cd^3)}{2de^4 + 2e^5x^2} + \frac{\sqrt{-\frac{d}{e^9}}\cdot(3ae^2 - 5bde + 7cd^2)\log\left(-e^4\sqrt{-\frac{d}{e^9}} + x\right)}{4} - \frac{\sqrt{-\frac{d}{e^9}}\cdot(3ae^2 - 5bde + 7cd^2)\log\left(e^4\sqrt{-\frac{d}{e^9}} + x\right)}{4}$$

input
```
integrate(x**4*(c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

output
```
c*x**5/(5*e**2) + x**3*(b/(3*e**2) - 2*c*d/(3*e**3)) + x*(a/e**2 - 2*b*d/e
**3 + 3*c*d**2/e**4) + x*(a*d*e**2 - b*d**2*e + c*d**3)/(2*d*e**4 + 2*e**5
*x**2) + sqrt(-d/e**9)*(3*a*e**2 - 5*b*d*e + 7*c*d**2)*log(-e**4*sqrt(-d/e
**9) + x)/4 - sqrt(-d/e**9)*(3*a*e**2 - 5*b*d*e + 7*c*d**2)*log(e**4*sqrt(
-d/e**9) + x)/4
```

3.281.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.281.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.01

$$\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx = -\frac{(7\,cd^3-5\,bd^2e+3\,ade^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}e^4}+\frac{cd^3x-bd^2ex+ade^2x}{2\,(ex^2+d)e^4}+\frac{3\,ce^8x^5-10\,cde^7x^3+5\,be^8x^3+45\,cd^2e^6x-30\,bde^7x+15\,ae^8x}{15\,e^{10}}$$

input
```
integrate(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
-1/2*(7*c*d^3 - 5*b*d^2*e + 3*a*d*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e^
4) + 1/2*(c*d^3*x - b*d^2*e*x + a*d*e^2*x)/((e*x^2 + d)*e^4) + 1/15*(3*c*e
^8*x^5 - 10*c*d*e^7*x^3 + 5*b*e^8*x^3 + 45*c*d^2*e^6*x - 30*b*d*e^7*x + 15
*a*e^8*x)/e^10
```

3.281.9 Mupad [B] (verification not implemented)

Time = 7.68 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.33

$$\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx = x^3\left(\frac{b}{3\,e^2}-\frac{2\,c\,d}{3\,e^3}\right)-x\left(\frac{c\,d^2}{e^4}-\frac{a}{e^2}+\frac{2\,d\,\left(\frac{b}{e^2}-\frac{2\,c\,d}{e^3}\right)}{e}\right)+\frac{c\,x^5}{5\,e^2}+\frac{x\,\left(\frac{c\,d^3}{2}-\frac{b\,d^2\,e}{2}+\frac{a\,d\,e^2}{2}\right)}{e^5\,x^2+d\,e^4}-\frac{\sqrt{d}\,\mathrm{atan}\left(\frac{\sqrt{d}\,\sqrt{e}\,x\,(7\,c\,d^2-5\,b\,d\,e+3\,a\,e^2)}{7\,c\,d^3-5\,b\,d^2\,e+3\,a\,d\,e^2}\right)\,(7\,c\,d^2-5\,b\,d\,e+3\,a\,e^2)}{2\,e^{9/2}}$$

input
```
int((x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x)
```

output
```
x^3*(b/(3*e^2) - (2*c*d)/(3*e^3)) - x*((c*d^2)/e^4 - a/e^2 + (2*d*(b/e^2 -
 (2*c*d)/e^3))/e) + (c*x^5)/(5*e^2) + (x*((c*d^3)/2 + (a*d*e^2)/2 - (b*d^2
*e)/2))/(d*e^4 + e^5*x^2) - (d^(1/2)*atan((d^(1/2)*e^(1/2)*x*(3*a*e^2 + 7*
c*d^2 - 5*b*d*e))/(7*c*d^3 + 3*a*d*e^2 - 5*b*d^2*e))*(3*a*e^2 + 7*c*d^2 -
5*b*d*e))/(2*e^(9/2))
```

3.282 $\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx$

3.282.1 Optimal result

Integrand size = 25, antiderivative size = 106

$$\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx = -\frac{(2cd-be)x}{e^3}+\frac{cx^3}{3e^2}-\frac{(cd^2-bde+ae^2)\,x}{2e^3\,(d+ex^2)} + \frac{(5cd^2-e(3bd-ae))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}e^{7/2}}$$

output
```
-(-b*e+2*c*d)*x/e^3+1/3*c*x^3/e^2-1/2*(a*e^2-b*d*e+c*d^2)*x/e^3/(e*x^2+d)+
1/2*(5*c*d^2-e*(-a*e+3*b*d))*arctan(x*e^(1/2)/d^(1/2))/e^(7/2)/d^(1/2)
```

3.282.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.96

$$\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx = \frac{(-2cd+be)x}{e^3}+\frac{cx^3}{3e^2}-\frac{(cd^2-bde+ae^2)\,x}{2e^3\,(d+ex^2)} + \frac{(5cd^2-3bde+ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}e^{7/2}}$$

input
```
Integrate[(x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]
```

output
```
((-2*c*d + b*e)*x)/e^3 + (c*x^3)/(3*e^2) - ((c*d^2 - b*d*e + a*e^2)*x)/(2*
e^3*(d + e*x^2)) + ((5*c*d^2 - 3*b*d*e + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]
])/(2*Sqrt[d]*e^(7/2))
```

3.282.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1580, 25, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (a+bx^2+cx^4)}{(d+ex^2)^2} \, dx$$

$$\Big\downarrow \ 1580$$

$$-\frac{\int -\frac{2ce^2x^4-2e(cd-be)x^2+cd^2+ae^2-bde}{ex^2+d}dx}{2e^3}-\frac{x(ae^2-bde+cd^2)}{2e^3(d+ex^2)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{2ce^2x^4-2e(cd-be)x^2+cd^2+ae^2-bde}{ex^2+d}dx}{2e^3}-\frac{x(ae^2-bde+cd^2)}{2e^3(d+ex^2)}$$

$$\Big\downarrow \ 1467$$

$$\frac{\int \left(2cex^2-2(2cd-be)+\frac{5cd^2-3bed+ae^2}{ex^2+d}\right)dx}{2e^3}-\frac{x(ae^2-bde+cd^2)}{2e^3(d+ex^2)}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(5cd^2-e(3bd-ae))}{\sqrt{d}\sqrt{e}}-2x(2cd-be)+\frac{2}{3}cex^3}{2e^3}-\frac{x(ae^2-bde+cd^2)}{2e^3(d+ex^2)}$$

input
```
Int[(x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]
```

output
```
-1/2*((c*d^2 - b*d*e + a*e^2)*x)/(e^3*(d + e*x^2)) + (-2*(2*c*d - b*e)*x +
 (2*c*e*x^3)/3 + ((5*c*d^2 - e*(3*b*d - a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])
/(Sqrt[d]*Sqrt[e]))/(2*e^3)
```

3.282.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 1580
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[1/(2*e^(2*p + m/2)*
(q + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*
e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b
*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.282.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.85

method	result
default	$\frac{\frac{1}{3} c\,x^{3} e+bex-2cdx}{e^{3}}+\frac{\frac{\left(-\frac{1}{2} a\,e^{2}+\frac{1}{2} bde-\frac{1}{2} c\,d^{2}\right) x}{e\,x^{2}+d}+\frac{\left(a\,e^{2}-3bde+5c\,d^{2}\right) \arctan\left(\frac{ex}{\sqrt{ed}}\right)}{2\sqrt{ed}}}{e^{3}}$
risch	$\frac{c\,x^{3}}{3e^{2}}+\frac{bx}{e^{2}}-\frac{2cdx}{e^{3}}+\frac{\left(-\frac{1}{2} a\,e^{2}+\frac{1}{2} bde-\frac{1}{2} c\,d^{2}\right) x}{e^{3} \left(e\,x^{2}+d\right)}-\frac{\ln\left(ex+\sqrt{-ed}\right) a}{4e\sqrt{-ed}}+\frac{3\ln\left(ex+\sqrt{-ed}\right) bd}{4e^{2}\sqrt{-ed}}-\frac{5\ln\left(ex+\sqrt{-ed}\right) c\,d^{2}}{4e^{3}\sqrt{-ed}}+\frac{\ln(-ex+}{4e\sqrt{}}$

input
```
int(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/e^3*(1/3*c*x^3*e+b*e*x-2*c*d*x)+1/e^3*((-1/2*a*e^2+1/2*b*d*e-1/2*c*d^2)*
x/(e*x^2+d)+1/2*(a*e^2-3*b*d*e+5*c*d^2)/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2)
))
```

3.282.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 302, normalized size of antiderivative = 2.85

$$\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx$$
$$= \left[\frac{4\,cde^3x^5 - 4\,(5\,cd^2e^2 - 3\,bde^3)x^3 - 3\,(5\,cd^3 - 3\,bd^2e + ade^2 + (5\,cd^2e - 3\,bde^2 + ae^3)x^2)\sqrt{-de}\log\left(\frac{ex^2-}{}\right.}{12\,(de^5x^2 + d^2e^4)}\right.$$

input
```
integrate(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
[1/12*(4*c*d*e^3*x^5 - 4*(5*c*d^2*e^2 - 3*b*d*e^3)*x^3 - 3*(5*c*d^3 - 3*b*
d^2*e + a*d*e^2 + (5*c*d^2*e - 3*b*d*e^2 + a*e^3)*x^2)*sqrt(-d*e)*log((e*x
^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) - 6*(5*c*d^3*e - 3*b*d^2*e^2 + a*d*e
^3)*x)/(d*e^5*x^2 + d^2*e^4), 1/6*(2*c*d*e^3*x^5 - 2*(5*c*d^2*e^2 - 3*b*d*
e^3)*x^3 + 3*(5*c*d^3 - 3*b*d^2*e + a*d*e^2 + (5*c*d^2*e - 3*b*d*e^2 + a*e
^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) - 3*(5*c*d^3*e - 3*b*d^2*e^2 + a*
d*e^3)*x)/(d*e^5*x^2 + d^2*e^4)]
```

3.282.6 Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.53

$$\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx = \frac{cx^3}{3e^2} + x\left(\frac{b}{e^2} - \frac{2cd}{e^3}\right) + \frac{x(-ae^2+bde-cd^2)}{2de^3+2e^4x^2} - \frac{\sqrt{-\frac{1}{de^7}}(ae^2-3bde+5cd^2)\log\left(-de^3\sqrt{-\frac{1}{de^7}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{de^7}}(ae^2-3bde+5cd^2)\log\left(de^3\sqrt{-\frac{1}{de^7}}+x\right)}{4}$$

input
```
integrate(x**2*(c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

output
```
c*x**3/(3*e**2) + x*(b/e**2 - 2*c*d/e**3) + x*(-a*e**2 + b*d*e - c*d**2)/(
2*d*e**3 + 2*e**4*x**2) - sqrt(-1/(d*e**7))*(a*e**2 - 3*b*d*e + 5*c*d**2)*
log(-d*e**3*sqrt(-1/(d*e**7)) + x)/4 + sqrt(-1/(d*e**7))*(a*e**2 - 3*b*d*e
 + 5*c*d**2)*log(d*e**3*sqrt(-1/(d*e**7)) + x)/4
```

3.282.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.282.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93

$$\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx = \frac{(5\,cd^2-3\,bde+ae^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}e^3} - \frac{cd^2x-bdex+ae^2x}{2\,(ex^2+d)e^3} + \frac{ce^4x^3-6\,cde^3x+3\,be^4x}{3\,e^6}$$

input
```
integrate(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
1/2*(5*c*d^2 - 3*b*d*e + a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e^3) - 1/
2*(c*d^2*x - b*d*e*x + a*e^2*x)/((e*x^2 + d)*e^3) + 1/3*(c*e^4*x^3 - 6*c*d
*e^3*x + 3*b*e^4*x)/e^6
```

3.282.9 Mupad [B] (verification not implemented)

Time = 7.79 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.90

$$\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx = x\left(\frac{b}{e^2}-\frac{2\,c\,d}{e^3}\right) - \frac{x\left(\frac{c\,d^2}{2}-\frac{b\,d\,e}{2}+\frac{a\,e^2}{2}\right)}{e^4\,x^2+d\,e^3} + \frac{c\,x^3}{3\,e^2} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,(5\,c\,d^2-3\,b\,d\,e+a\,e^2)}{2\,\sqrt{d}\,e^{7/2}}$$

input
```
int((x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x)
```

output
```
x*(b/e^2 - (2*c*d)/e^3) - (x*((a*e^2)/2 + (c*d^2)/2 - (b*d*e)/2))/(d*e^3 +
 e^4*x^2) + (c*x^3)/(3*e^2) + (atan((e^(1/2)*x)/d^(1/2))*(a*e^2 + 5*c*d^2
- 3*b*d*e))/(2*d^(1/2)*e^(7/2))
```

3.282. $\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx$

3.283 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^2} dx$

3.283.1 Optimal result

Integrand size = 22, antiderivative size = 83

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^2} dx = \frac{cx}{e^2} + \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d(d+ex^2)} - \frac{(3cd^2-e(bd+ae))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}$$

output
```
c*x/e^2+1/2*(a+d*(-b*e+c*d)/e^2)*x/d/(e*x^2+d)-1/2*(3*c*d^2-e*(a*e+b*d))*a
rctan(x*e^(1/2)/d^(1/2))/d^(3/2)/e^(5/2)
```

3.283.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.06

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^2} dx = \frac{cx}{e^2} + \frac{(cd^2-bde+ae^2)x}{2de^2(d+ex^2)} - \frac{(3cd^2-bde-ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^2,x]
```

output
```
(c*x)/e^2 + ((c*d^2 - b*d*e + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - ((3*c*d^2
- b*d*e - a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

3.283.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1471, 27, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^2}\,dx$$

$$\Big\downarrow \; 1471$$

$$\frac{x(ae^2-bde+cd^2)}{2de^2(d+ex^2)}-\frac{\int \frac{cd^2-2cex^2d-e(bd+ae)}{e^2(ex^2+d)}dx}{2d}$$

$$\Big\downarrow \; 27$$

$$\frac{x(ae^2-bde+cd^2)}{2de^2(d+ex^2)}-\frac{\int \frac{cd^2-2cex^2d-e(bd+ae)}{ex^2+d}dx}{2de^2}$$

$$\Big\downarrow \; 299$$

$$\frac{x(ae^2-bde+cd^2)}{2de^2(d+ex^2)}-\frac{(3cd^2-e(ae+bd))\int \frac{1}{ex^2+d}dx-2cdx}{2de^2}$$

$$\Big\downarrow \; 218$$

$$\frac{x(ae^2-bde+cd^2)}{2de^2(d+ex^2)}-\frac{\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(3cd^2-e(ae+bd))}{\sqrt{d}\sqrt{e}}-2cdx}{2de^2}$$

input
```
Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^2,x]
```

output
```
((c*d^2 - b*d*e + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - (-2*c*d*x + ((3*c*d^2
- e*(b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*Sqrt[e]))/(2*d*e^2)
```

3.283.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

3.283.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95

method	result
default	$\frac{cx}{e^2}+\frac{\frac{(a\,e^2-bde+c\,d^2)x}{2d(e\,x^2+d)}+\frac{(a\,e^2+bde-3c\,d^2)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{2d\sqrt{ed}}}{e^2}$
risch	$\frac{cx}{e^2}+\frac{(a\,e^2-bde+c\,d^2)x}{2d\,e^2(e\,x^2+d)}-\frac{\ln(ex+\sqrt{-ed})a}{4\sqrt{-ed}\,d}-\frac{\ln(ex+\sqrt{-ed})b}{4e\sqrt{-ed}}+\frac{3d\ln(ex+\sqrt{-ed})c}{4e^2\sqrt{-ed}}+\frac{\ln(-ex+\sqrt{-ed})a}{4\sqrt{-ed}\,d}+\frac{\ln(-ex+\sqrt{-ed})}{4e\sqrt{-ed}}$

input
```
int((c*x^4+b*x^2+a)/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
c*x/e^2+1/e^2*(1/2*(a*e^2-b*d*e+c*d^2)/d*x/(e*x^2+d)+1/2*(a*e^2+b*d*e-3*c*
d^2)/d/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2)))
```

3.283.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 268, normalized size of antiderivative = 3.23

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^2}\,dx$$
$$= \left[\frac{4\,cd^2e^2x^3+(3\,cd^3-bd^2e-ade^2+(3\,cd^2e-bde^2-ae^3)x^2)\sqrt{-de}\log\left(\frac{ex^2-2\,\sqrt{-de}x-d}{ex^2+d}\right)+2\,(3\,cd^3e-bd^2}{4\,(d^2e^4x^2+d^3e^3)}\right.$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*c*d^2*e^2*x^3 + (3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^
2 - a*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) +
 2*(3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*x)/(d^2*e^4*x^2 + d^3*e^3), 1/2*(2*c*
d^2*e^2*x^3 - (3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^2 - a*e^3)
*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*
x)/(d^2*e^4*x^2 + d^3*e^3)]
```

3.283.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(75) = 150$.

Time = 0.42 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.84

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^2}\,dx = \frac{cx}{e^2}+\frac{x(ae^2-bde+cd^2)}{2d^2e^2+2de^3x^2} - \frac{\sqrt{-\frac{1}{d^3e^5}}(ae^2+bde-3cd^2)\log\left(-d^2e^2\sqrt{-\frac{1}{d^3e^5}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{d^3e^5}}(ae^2+bde-3cd^2)\log\left(d^2e^2\sqrt{-\frac{1}{d^3e^5}}+x\right)}{4}$$

input
```
integrate((c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

output
```
c*x/e**2 + x*(a*e**2 - b*d*e + c*d**2)/(2*d**2*e**2 + 2*d*e**3*x**2) - sqr
t(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(-d**2*e**2*sqrt(-1/(d**3
*e**5)) + x)/4 + sqrt(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(d**2
*e**2*sqrt(-1/(d**3*e**5)) + x)/4
```

3.283.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.283.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^2}\,dx = \frac{cx}{e^2} - \frac{(3\,cd^2-bde-ae^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}de^2} + \frac{cd^2x-bdex+ae^2x}{2\,(ex^2+d)de^2}$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
c*x/e^2 - 1/2*(3*c*d^2 - b*d*e - a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d
*e^2) + 1/2*(c*d^2*x - b*d*e*x + a*e^2*x)/((e*x^2 + d)*d*e^2)
```

3.283.9 Mupad [B] (verification not implemented)

Time = 7.72 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.93

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^2}\,dx = \frac{c\,x}{e^2} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,(-3\,c\,d^2+b\,d\,e+a\,e^2)}{2\,d^{3/2}\,e^{5/2}} + \frac{x\,(c\,d^2-b\,d\,e+a\,e^2)}{2\,d\,\left(e^3\,x^2+d\,e^2\right)}$$

input
```
int((a + b*x^2 + c*x^4)/(d + e*x^2)^2,x)
```

output
```
(c*x)/e^2 + (atan((e^(1/2)*x)/d^(1/2))*(a*e^2 - 3*c*d^2 + b*d*e))/(2*d^(3/
2)*e^(5/2)) + (x*(a*e^2 + c*d^2 - b*d*e))/(2*d*(d*e^2 + e^3*x^2))
```

3.284 $\int \frac{a+bx^2+cx^4}{x^2(d+ex^2)^2} dx$

3.284.1 Optimal result

Integrand size = 25, antiderivative size = 89

$$\int \frac{a+bx^2+cx^4}{x^2(d+ex^2)^2} dx = -\frac{a}{d^2x} - \frac{(cd^2-bde+ae^2)x}{2d^2e(d+ex^2)} + \frac{(cd^2+e(bd-3ae))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}e^{3/2}}$$

output
```
-a/d^2/x-1/2*(a*e^2-b*d*e+c*d^2)*x/d^2/e/(e*x^2+d)+1/2*(c*d^2+e*(-3*a*e+b*
d))*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)/e^(3/2)
```

3.284.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2+cx^4}{x^2(d+ex^2)^2} dx = -\frac{a}{d^2x} - \frac{(cd^2-bde+ae^2)x}{2d^2e(d+ex^2)} + \frac{(cd^2+bde-3ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}e^{3/2}}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^2),x]
```

output
```
-(a/(d^2*x)) - ((c*d^2 - b*d*e + a*e^2)*x)/(2*d^2*e*(d + e*x^2)) + ((c*d^2
 + b*d*e - 3*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(5/2)*e^(3/2))
```

3.284.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1582, 25, 27, 359, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{x^2\left(d+ex^2\right)^2}dx$$

$$\Big\downarrow \text{1582}$$

$$-\frac{\int -\frac{e\left(\left(cd^2+e(bd-ae)\right)x^2+2ade\right)}{x^2(ex^2+d)}dx}{2d^2e^2}-\frac{x\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)}{2\left(d+ex^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{e\left(\left(cd^2+e(bd-ae)\right)x^2+2ade\right)}{x^2(ex^2+d)}dx}{2d^2e^2}-\frac{x\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)}{2\left(d+ex^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(cd^2+e(bd-ae)\right)x^2+2ade}{x^2(ex^2+d)}dx}{2d^2e}-\frac{x\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)}{2\left(d+ex^2\right)}$$

$$\Big\downarrow \text{359}$$

$$\frac{\left(e(bd-3ae)+cd^2\right)\int \frac{1}{ex^2+d}dx-\frac{2ae}{x}}{2d^2e}-\frac{x\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)}{2\left(d+ex^2\right)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e(bd-3ae)+cd^2\right)}{\sqrt{d}\sqrt{e}}-\frac{2ae}{x}}{2d^2e}-\frac{x\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)}{2\left(d+ex^2\right)}$$

input
```
Int[(a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^2),x]
```

output
```
-1/2*((c/e - (b*d - a*e)/d^2)*x)/(d + e*x^2) + ((-2*a*e)/x + ((c*d^2 + e*(
b*d - 3*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*Sqrt[e]))/(2*d^2*e)
```

3.284.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

3.284.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96

method	result
default	$-\frac{a}{d^2 x}-\frac{\frac{\left(a\,e^2-bde+c\,d^2\right)x}{2e\left(e\,x^2+d\right)}+\frac{\left(3a\,e^2-bde-c\,d^2\right)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{2e\sqrt{ed}}}{d^2}$
risch	$\frac{-\frac{\left(3a\,e^2-bde+c\,d^2\right)x^2}{2d^2 e}-\frac{a}{d}}{x(e\,x^2+d)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(d^5e^3\_Z^2+9a^2e^4-6abd\,e^3-6ac\,d^2e^2+b^2d^2e^2+2bc\,d^3e+c^2d^4\right)}\_R\ln\left(\left(3\_R^2d^5e^3+18a^2e^4\right.\right.\right.}{4}$

input
```
int((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-a/d^2/x-1/d^2*(1/2*(a*e^2-b*d*e+c*d^2)/e*x/(e*x^2+d)+1/2*(3*a*e^2-b*d*e-c
*d^2)/e/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2)))
```

3.284.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 267, normalized size of antiderivative = 3.00

$$\int \frac{a+bx^2+cx^4}{x^2\left(d+ex^2\right)^2}\,dx$$
$$= \left[-\frac{4\,ad^2e^2+2\,(cd^3e-bd^2e^2+3\,ade^3)x^2-((cd^2e+bde^2-3\,ae^3)x^3+(cd^3+bd^2e-3\,ade^2)x)\sqrt{-de}\log\left(\right.}{4\,(d^3e^3x^3+d^4e^2x)}\right.$$
$$-\frac{2\,ad^2e^2+(cd^3e-bd^2e^2+3\,ade^3)x^2-((cd^2e+bde^2-3\,ae^3)x^3+(cd^3+bd^2e-3\,ade^2)x)\sqrt{de}\arctan\left(\right.}{2\,(d^3e^3x^3+d^4e^2x)}$$

input
```
integrate((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
[-1/4*(4*a*d^2*e^2 + 2*(c*d^3*e - b*d^2*e^2 + 3*a*d*e^3)*x^2 - ((c*d^2*e +
 b*d*e^2 - 3*a*e^3)*x^3 + (c*d^3 + b*d^2*e - 3*a*d*e^2)*x)*sqrt(-d*e)*log(
(e*x^2 + 2*sqrt(-d*e)*x - d)/(e*x^2 + d)))/(d^3*e^3*x^3 + d^4*e^2*x), -1/2
*(2*a*d^2*e^2 + (c*d^3*e - b*d^2*e^2 + 3*a*d*e^3)*x^2 - ((c*d^2*e + b*d*e^
2 - 3*a*e^3)*x^3 + (c*d^3 + b*d^2*e - 3*a*d*e^2)*x)*sqrt(d*e)*arctan(sqrt(
d*e)*x/d))/(d^3*e^3*x^3 + d^4*e^2*x)]
```

3.284.6 Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.74

$$\int \frac{a+bx^2+cx^4}{x^2\left(d+ex^2\right)^2}\,dx = \frac{\sqrt{-\frac{1}{d^5e^3}}\cdot(3ae^2-bde-cd^2)\log\left(-d^3e\sqrt{-\frac{1}{d^5e^3}}+x\right)}{4} - \frac{\sqrt{-\frac{1}{d^5e^3}}\cdot(3ae^2-bde-cd^2)\log\left(d^3e\sqrt{-\frac{1}{d^5e^3}}+x\right)}{4} + \frac{-2ade+x^2(-3ae^2+bde-cd^2)}{2d^3ex+2d^2e^2x^3}$$

input
```
integrate((c*x**4+b*x**2+a)/x**2/(e*x**2+d)**2,x)
```

output
```
sqrt(-1/(d**5*e**3))*(3*a*e**2 - b*d*e - c*d**2)*log(-d**3*e*sqrt(-1/(d**5
*e**3)) + x)/4 - sqrt(-1/(d**5*e**3))*(3*a*e**2 - b*d*e - c*d**2)*log(d**3
*e*sqrt(-1/(d**5*e**3)) + x)/4 + (-2*a*d*e + x**2*(-3*a*e**2 + b*d*e - c*d
**2))/(2*d**3*e*x + 2*d**2*e**2*x**3)
```

3.284.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^2+cx^4}{x^2\left(d+ex^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.284.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2+cx^4}{x^2\left(d+ex^2\right)^2}\,dx = \frac{\left(cd^2+bde-3\,ae^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}d^2e} - \frac{cd^2x^2-bdex^2+3\,ae^2x^2+2\,ade}{2\,(ex^3+dx)d^2e}$$

input
```
integrate((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
1/2*(c*d^2 + b*d*e - 3*a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^2*e) - 1/
2*(c*d^2*x^2 - b*d*e*x^2 + 3*a*e^2*x^2 + 2*a*d*e)/((e*x^3 + d*x)*d^2*e)
```

3.284.9 Mupad [B] (verification not implemented)

Time = 7.63 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91

$$\int \frac{a+bx^2+cx^4}{x^2\left(d+ex^2\right)^2}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(c\,d^2+b\,d\,e-3\,a\,e^2\right)}{2\,d^{5/2}\,e^{3/2}} - \frac{\frac{a}{d}+\frac{x^2\,\left(c\,d^2-b\,d\,e+3\,a\,e^2\right)}{2\,d^2\,e}}{e\,x^3+d\,x}$$

input
```
int((a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^2),x)
```

output
```
(atan((e^(1/2)*x)/d^(1/2))*(c*d^2 - 3*a*e^2 + b*d*e))/(2*d^(5/2)*e^(3/2))
- (a/d + (x^2*(3*a*e^2 + c*d^2 - b*d*e))/(2*d^2*e))/(d*x + e*x^3)
```

3.285 $\int \frac{a+bx^2+cx^4}{x^4(d+ex^2)^2}\,dx$

3.285.1 Optimal result

Integrand size = 25, antiderivative size = 106

$$\int \frac{a+bx^2+cx^4}{x^4(d+ex^2)^2}\,dx=-\frac{a}{3d^2x^3}-\frac{bd-2ae}{d^3x}+\frac{(cd^2-bde+ae^2)\,x}{2d^3(d+ex^2)}+\frac{(cd^2-e(3bd-5ae))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{7/2}\sqrt{e}}$$

output
```
-1/3*a/d^2/x^3+(2*a*e-b*d)/d^3/x+1/2*(a*e^2-b*d*e+c*d^2)*x/d^3/(e*x^2+d)+1
/2*(c*d^2-e*(-5*a*e+3*b*d))*arctan(x*e^(1/2)/d^(1/2))/d^(7/2)/e^(1/2)
```

3.285.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.99

$$\int \frac{a+bx^2+cx^4}{x^4(d+ex^2)^2}\,dx=-\frac{a}{3d^2x^3}+\frac{-bd+2ae}{d^3x}+\frac{(cd^2-bde+ae^2)\,x}{2d^3(d+ex^2)}+\frac{(cd^2-3bde+5ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{7/2}\sqrt{e}}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^2),x]
```

output
```
-1/3*a/(d^2*x^3) + (-(b*d) + 2*a*e)/(d^3*x) + ((c*d^2 - b*d*e + a*e^2)*x)/
(2*d^3*(d + e*x^2)) + ((c*d^2 - 3*b*d*e + 5*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt
[d]])/(2*d^(7/2)*Sqrt[e])
```

3.285.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1582, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{x^4\left(d+ex^2\right)^2}dx$$

$$\Big\downarrow \text{ 1582}$$

$$\frac{\int \frac{e^2(cd^2-bed+ae^2)x^4+2de^2(bd-ae)x^2+2ad^2e^2}{x^4(ex^2+d)}dx}{2d^3e^2}+\frac{x\left(ae^2-bde+cd^2\right)}{2d^3\left(d+ex^2\right)}$$

$$\Big\downarrow \text{ 1584}$$

$$\frac{\int \left(\frac{(cd^2-e(3bd-5ae))e^2}{ex^2+d}-\frac{2(2ae-bd)e^2}{x^2}+\frac{2ade^2}{x^4}\right)dx}{2d^3e^2}+\frac{x\left(ae^2-bde+cd^2\right)}{2d^3\left(d+ex^2\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(cd^2-e(3bd-5ae))}{\sqrt{d}}-\frac{2e^2(bd-2ae)}{x}-\frac{2ade^2}{3x^3}}{2d^3e^2}+\frac{x\left(ae^2-bde+cd^2\right)}{2d^3\left(d+ex^2\right)}$$

input
```
Int[(a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^2),x]
```

output
```
((c*d^2 - b*d*e + a*e^2)*x)/(2*d^3*(d + e*x^2)) + ((-2*a*d*e^2)/(3*x^3) -
(2*e^2*(b*d - 2*a*e))/x + (e^(3/2)*(c*d^2 - e*(3*b*d - 5*a*e))*ArcTan[(Sqr
t[e]*x)/Sqrt[d]])/Sqrt[d])/(2*d^3*e^2)
```

3.285.3.1 Defintions of rubi rules used

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.285.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.89

method	result
default	$-\frac{a}{3d^2x^3}-\frac{-2ae+bd}{d^3x}+\frac{\frac{\left(\frac{1}{2}a\,e^2-\frac{1}{2}bde+\frac{1}{2}c\,d^2\right)x}{e\,x^2+d}+\frac{\left(5a\,e^2-3bde+c\,d^2\right)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{2\sqrt{ed}}}{d^3}$
risch	$\frac{\frac{\left(5a\,e^2-3bde+c\,d^2\right)x^4}{2d^3}+\frac{(5ae-3bd)x^2}{3d^2}-\frac{a}{3d}}{x^3(e\,x^2+d)}+\frac{\left(\sum_{\_R=\text{RootOf}\left(d^7\_Z^2e+25a^2e^4-30abd\,e^3+10ac\,d^2e^2+9b^2d^2e^2-6bc\,d^3e+c^2d^4\right)}\_R\ln\left(\left(3\_\right.\right.\right.}{}$

input
```
int((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/d^2/x^3-(-2*a*e+b*d)/d^3/x+1/d^3*((1/2*a*e^2-1/2*b*d*e+1/2*c*d^2)*x
/(e*x^2+d)+1/2*(5*a*e^2-3*b*d*e+c*d^2)/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2))
)
```

3.285.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.98

$$\int \frac{a+bx^2+cx^4}{x^4\left(d+ex^2\right)^2}\,dx$$
$$= \left[-\frac{4\,ad^3e-6\,(cd^3e-3\,bd^2e^2+5\,ade^3)x^4+4\,(3\,bd^3e-5\,ad^2e^2)x^2+3\,((cd^2e-3\,bde^2+5\,ae^3)x^5+(cd^3-\right.}{12\,(d^4e^2x^5+d^5ex^3)}$$
$$-\frac{2\,ad^3e-3\,(cd^3e-3\,bd^2e^2+5\,ade^3)x^4+2\,(3\,bd^3e-5\,ad^2e^2)x^2-3\,((cd^2e-3\,bde^2+5\,ae^3)x^5+(cd^3-}{6\,(d^4e^2x^5+d^5ex^3)}$$

input
```
integrate((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
[-1/12*(4*a*d^3*e - 6*(c*d^3*e - 3*b*d^2*e^2 + 5*a*d*e^3)*x^4 + 4*(3*b*d^3
*e - 5*a*d^2*e^2)*x^2 + 3*((c*d^2*e - 3*b*d*e^2 + 5*a*e^3)*x^5 + (c*d^3 -
3*b*d^2*e + 5*a*d*e^2)*x^3)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e
*x^2 + d)))/(d^4*e^2*x^5 + d^5*e*x^3), -1/6*(2*a*d^3*e - 3*(c*d^3*e - 3*b*
d^2*e^2 + 5*a*d*e^3)*x^4 + 2*(3*b*d^3*e - 5*a*d^2*e^2)*x^2 - 3*((c*d^2*e -
 3*b*d*e^2 + 5*a*e^3)*x^5 + (c*d^3 - 3*b*d^2*e + 5*a*d*e^2)*x^3)*sqrt(d*e)
*arctan(sqrt(d*e)*x/d))/(d^4*e^2*x^5 + d^5*e*x^3)]
```

3.285.6 Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.58

$$\int \frac{a+bx^2+cx^4}{x^4\left(d+ex^2\right)^2}\,dx = -\frac{\sqrt{-\frac{1}{d^7e}}(5ae^2-3bde+cd^2)\log\left(-d^4\sqrt{-\frac{1}{d^7e}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{d^7e}}(5ae^2-3bde+cd^2)\log\left(d^4\sqrt{-\frac{1}{d^7e}}+x\right)}{4} + \frac{-2ad^2+x^4\cdot(15ae^2-9bde+3cd^2)+x^2\cdot(10ade-6bd^2)}{6d^4x^3+6d^3ex^5}$$

input
```
integrate((c*x**4+b*x**2+a)/x**4/(e*x**2+d)**2,x)
```

output
```
-sqrt(-1/(d**7*e))*(5*a*e**2 - 3*b*d*e + c*d**2)*log(-d**4*sqrt(-1/(d**7*e
)) + x)/4 + sqrt(-1/(d**7*e))*(5*a*e**2 - 3*b*d*e + c*d**2)*log(d**4*sqrt(
-1/(d**7*e)) + x)/4 + (-2*a*d**2 + x**4*(15*a*e**2 - 9*b*d*e + 3*c*d**2) +
 x**2*(10*a*d*e - 6*b*d**2))/(6*d**4*x**3 + 6*d**3*e*x**5)
```

3.285.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^2+cx^4}{x^4\left(d+ex^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.285.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.91

$$\int \frac{a+bx^2+cx^4}{x^4\left(d+ex^2\right)^2}\,dx = \frac{\left(cd^2-3\,bde+5\,ae^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}d^3} + \frac{cd^2x-bdex+ae^2x}{2\,(ex^2+d)d^3} - \frac{3\,bdx^2-6\,aex^2+ad}{3\,d^3x^3}$$

input
```
integrate((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
1/2*(c*d^2 - 3*b*d*e + 5*a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^3) + 1/
2*(c*d^2*x - b*d*e*x + a*e^2*x)/((e*x^2 + d)*d^3) - 1/3*(3*b*d*x^2 - 6*a*e
*x^2 + a*d)/(d^3*x^3)
```

3.285.9 Mupad [B] (verification not implemented)

Time = 7.64 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.92

$$\int \frac{a+bx^2+cx^4}{x^4\left(d+ex^2\right)^2}\,dx = \frac{\frac{x^2\,(5\,a\,e-3\,b\,d)}{3\,d^2}-\frac{a}{3\,d}+\frac{x^4\,(c\,d^2-3\,b\,d\,e+5\,a\,e^2)}{2\,d^3}}{e\,x^5+d\,x^3}+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,(c\,d^2-3\,b\,d\,e+5\,a\,e^2)}{2\,d^{7/2}\,\sqrt{e}}$$

input
```
int((a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^2),x)
```

output
```
((x^2*(5*a*e - 3*b*d))/(3*d^2) - a/(3*d) + (x^4*(5*a*e^2 + c*d^2 - 3*b*d*e
))/(2*d^3))/(d*x^3 + e*x^5) + (atan((e^(1/2)*x)/d^(1/2))*(5*a*e^2 + c*d^2
- 3*b*d*e))/(2*d^(7/2)*e^(1/2))
```

3.286 $\int \frac{a+bx^2+cx^4}{x^6(d+ex^2)^2}\,dx$

3.286.1 Optimal result

Integrand size = 25, antiderivative size = 136

$$\int \frac{a+bx^2+cx^4}{x^6(d+ex^2)^2}\,dx = -\frac{a}{5d^2x^5}-\frac{bd-2ae}{3d^3x^3}-\frac{cd^2-e(2bd-3ae)}{d^4x} - \frac{e(cd^2-bde+ae^2)\,x}{2d^4(d+ex^2)}-\frac{\sqrt{e}(3cd^2-e(5bd-7ae))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{9/2}}$$

output
```
-1/5*a/d^2/x^5+1/3*(2*a*e-b*d)/d^3/x^3+(-c*d^2+e*(-3*a*e+2*b*d))/d^4/x-1/2
*e*(a*e^2-b*d*e+c*d^2)*x/d^4/(e*x^2+d)-1/2*(3*c*d^2-e*(-7*a*e+5*b*d))*arct
an(x*e^(1/2)/d^(1/2))*e^(1/2)/d^(9/2)
```

3.286.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.99

$$\int \frac{a+bx^2+cx^4}{x^6(d+ex^2)^2}\,dx = -\frac{a}{5d^2x^5}+\frac{-bd+2ae}{3d^3x^3}+\frac{-cd^2+2bde-3ae^2}{d^4x} - \frac{e(cd^2-bde+ae^2)\,x}{2d^4(d+ex^2)}-\frac{\sqrt{e}(3cd^2-5bde+7ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{9/2}}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^2),x]
```

output
```
-1/5*a/(d^2*x^5) + (-(b*d) + 2*a*e)/(3*d^3*x^3) + (-(c*d^2) + 2*b*d*e - 3*
a*e^2)/(d^4*x) - (e*(c*d^2 - b*d*e + a*e^2)*x)/(2*d^4*(d + e*x^2)) - (Sqrt
[e]*(3*c*d^2 - 5*b*d*e + 7*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(9/2))
```

3.286.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1582, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{x^6 \left(d+ex^2\right)^2} \, dx$$

$$\Big\downarrow \text{1582}$$

$$-\frac{\int -\frac{-e^3\left(cd^2-bed+ae^2\right)x^6+2de^2\left(cd^2-bed+ae^2\right)x^4+2d^2e^2(bd-ae)x^2+2ad^3e^2}{x^6\left(ex^2+d\right)}dx}{2d^4e^2}-\frac{ex\left(ae^2-bde+cd^2\right)}{2d^4\left(d+ex^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-e^3\left(cd^2-bed+ae^2\right)x^6+2de^2\left(cd^2-bed+ae^2\right)x^4+2d^2e^2(bd-ae)x^2+2ad^3e^2}{x^6\left(ex^2+d\right)}dx}{2d^4e^2}-\frac{ex\left(ae^2-bde+cd^2\right)}{2d^4\left(d+ex^2\right)}$$

$$\Big\downarrow \text{2333}$$

$$\frac{\int \left(\frac{\left(e(5bd-7ae)-3cd^2\right)e^3}{ex^2+d}+\frac{2\left(cd^2-e(2bd-3ae)\right)e^2}{x^2}+\frac{2d(bd-2ae)e^2}{x^4}+\frac{2ad^2e^2}{x^6}\right)dx}{2d^4e^2}-\frac{ex\left(ae^2-bde+cd^2\right)}{2d^4\left(d+ex^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{e^{5/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3cd^2-e(5bd-7ae)\right)}{\sqrt{d}}-\frac{2e^2\left(cd^2-e(2bd-3ae)\right)}{x}-\frac{2de^2(bd-2ae)}{3x^3}-\frac{2ad^2e^2}{5x^5}}{2d^4e^2}-\frac{ex\left(ae^2-bde+cd^2\right)}{2d^4\left(d+ex^2\right)}$$

input
```
Int[(a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^2),x]
```

output
```
-1/2*(e*(c*d^2 - b*d*e + a*e^2)*x)/(d^4*(d + e*x^2)) + ((-2*a*d^2*e^2)/(5*
x^5) - (2*d*e^2*(b*d - 2*a*e))/(3*x^3) - (2*e^2*(c*d^2 - e*(2*b*d - 3*a*e)
))/x - (e^(5/2)*(3*c*d^2 - e*(5*b*d - 7*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])
/Sqrt[d])/(2*d^4*e^2)
```

3.286.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.286.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.90

method	result
default	$-\frac{a}{5d^2x^5}-\frac{-2ae+bd}{3d^3x^3}-\frac{3a\,e^2-2bde+c\,d^2}{d^4x}-\frac{e\left(\frac{\left(\frac{1}{2}a\,e^2-\frac{1}{2}bde+\frac{1}{2}c\,d^2\right)x}{e\,x^2+d}+\frac{\left(7a\,e^2-5bde+3c\,d^2\right)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{2\sqrt{ed}}\right)}{d^4}$
risch	$\frac{-\frac{e\left(7a\,e^2-5bde+3c\,d^2\right)x^6}{2d^4}-\frac{\left(7a\,e^2-5bde+3c\,d^2\right)x^4}{3d^3}+\frac{(7ae-5bd)x^2}{15d^2}-\frac{a}{5d}}{x^5\left(e\,x^2+d\right)}+\frac{\left(\sum_{\_R=\text{RootOf}\left(d^9\_Z^2+49a^2e^5-70abd\,e^4+42ac\,d^2e^3+25b^2d^2e^3-\right.}\right.}{}$

input
```
int((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a/d^2/x^5-1/3*(-2*a*e+b*d)/d^3/x^3-(3*a*e^2-2*b*d*e+c*d^2)/d^4/x-e/d^
4*((1/2*a*e^2-1/2*b*d*e+1/2*c*d^2)*x/(e*x^2+d)+1/2*(7*a*e^2-5*b*d*e+3*c*d^
2)/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2)))
```

3.286.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.65

$$\int \frac{a+bx^2+cx^4}{x^6(d+ex^2)^2}\,dx$$

$$= \left[-\frac{30\,(3\,cd^2e-5\,bde^2+7\,ae^3)x^6+20\,(3\,cd^3-5\,bd^2e+7\,ade^2)x^4+12\,ad^3+4\,(5\,bd^3-7\,ad^2e)x^2-15\,((3}{60\,(d^4ex^7+d^5x^5)}\right.$$

$$-\frac{15\,(3\,cd^2e-5\,bde^2+7\,ae^3)x^6+10\,(3\,cd^3-5\,bd^2e+7\,ade^2)x^4+6\,ad^3+2\,(5\,bd^3-7\,ad^2e)x^2+15\,((3\,c}{30\,(d^4ex^7+d^5x^5)}$$

input
```
integrate((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
[-1/60*(30*(3*c*d^2*e - 5*b*d*e^2 + 7*a*e^3)*x^6 + 20*(3*c*d^3 - 5*b*d^2*e
 + 7*a*d*e^2)*x^4 + 12*a*d^3 + 4*(5*b*d^3 - 7*a*d^2*e)*x^2 - 15*((3*c*d^2*
e - 5*b*d*e^2 + 7*a*e^3)*x^7 + (3*c*d^3 - 5*b*d^2*e + 7*a*d*e^2)*x^5)*sqrt
(-e/d)*log((e*x^2 - 2*d*x*sqrt(-e/d) - d)/(e*x^2 + d)))/(d^4*e*x^7 + d^5*x
^5), -1/30*(15*(3*c*d^2*e - 5*b*d*e^2 + 7*a*e^3)*x^6 + 10*(3*c*d^3 - 5*b*d
^2*e + 7*a*d*e^2)*x^4 + 6*a*d^3 + 2*(5*b*d^3 - 7*a*d^2*e)*x^2 + 15*((3*c*d
^2*e - 5*b*d*e^2 + 7*a*e^3)*x^7 + (3*c*d^3 - 5*b*d^2*e + 7*a*d*e^2)*x^5)*s
qrt(e/d)*arctan(x*sqrt(e/d)))/(d^4*e*x^7 + d^5*x^5)]
```

3.286.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(126) = 252$.

Time = 1.05 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.09

$$\int \frac{a+bx^2+cx^4}{x^6\left(d+ex^2\right)^2}\,dx = \frac{\sqrt{-\frac{e}{d^9}}\cdot(7ae^2-5bde+3cd^2)\log\left(-\frac{d^5\sqrt{-\frac{e}{d^9}}\cdot(7ae^2-5bde+3cd^2)}{7ae^3-5bde^2+3cd^2e}+x\right)}{4} - \frac{\sqrt{-\frac{e}{d^9}}\cdot(7ae^2-5bde+3cd^2)\log\left(\frac{d^5\sqrt{-\frac{e}{d^9}}\cdot(7ae^2-5bde+3cd^2)}{7ae^3-5bde^2+3cd^2e}+x\right)}{4} + \frac{-6ad^3+x^6(-105ae^3+75bde^2-45cd^2e)+x^4(-70ade^2+50bd^2e-30cd^3)+x^2\cdot(14ad^2e-10bd^3)}{30d^5x^5+30d^4ex^7}$$

input
```
integrate((c*x**4+b*x**2+a)/x**6/(e*x**2+d)**2,x)
```

output
```
sqrt(-e/d**9)*(7*a*e**2 - 5*b*d*e + 3*c*d**2)*log(-d**5*sqrt(-e/d**9)*(7*a
*e**2 - 5*b*d*e + 3*c*d**2)/(7*a*e**3 - 5*b*d*e**2 + 3*c*d**2*e) + x)/4 -
sqrt(-e/d**9)*(7*a*e**2 - 5*b*d*e + 3*c*d**2)*log(d**5*sqrt(-e/d**9)*(7*a*
e**2 - 5*b*d*e + 3*c*d**2)/(7*a*e**3 - 5*b*d*e**2 + 3*c*d**2*e) + x)/4 + (
-6*a*d**3 + x**6*(-105*a*e**3 + 75*b*d*e**2 - 45*c*d**2*e) + x**4*(-70*a*d
*e**2 + 50*b*d**2*e - 30*c*d**3) + x**2*(14*a*d**2*e - 10*b*d**3))/(30*d**
5*x**5 + 30*d**4*e*x**7)
```

3.286.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^2+cx^4}{x^6\left(d+ex^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.286.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.99

$$\int \frac{a+bx^2+cx^4}{x^6\left(d+ex^2\right)^2}\,dx = -\frac{\left(3\,cd^2e-5\,bde^2+7\,ae^3\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}d^4} - \frac{cd^2ex-bde^2x+ae^3x}{2\,(ex^2+d)d^4} - \frac{15\,cd^2x^4-30\,bdex^4+45\,ae^2x^4+5\,bd^2x^2-10\,adex^2+3\,ad^2}{15\,d^4x^5}$$

input
```
integrate((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
-1/2*(3*c*d^2*e - 5*b*d*e^2 + 7*a*e^3)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^
4) - 1/2*(c*d^2*e*x - b*d*e^2*x + a*e^3*x)/((e*x^2 + d)*d^4) - 1/15*(15*c*
d^2*x^4 - 30*b*d*e*x^4 + 45*a*e^2*x^4 + 5*b*d^2*x^2 - 10*a*d*e*x^2 + 3*a*d
^2)/(d^4*x^5)
```

3.286.9 Mupad [B] (verification not implemented)

Time = 7.70 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.94

$$\int \frac{a+bx^2+cx^4}{x^6\left(d+ex^2\right)^2}\,dx = -\frac{\frac{a}{5\,d}-\frac{x^2\,(7\,a\,e-5\,b\,d)}{15\,d^2}+\frac{x^4\,(3\,c\,d^2-5\,b\,d\,e+7\,a\,e^2)}{3\,d^3}+\frac{e\,x^6\,(3\,c\,d^2-5\,b\,d\,e+7\,a\,e^2)}{2\,d^4}}{e\,x^7+d\,x^5} - \frac{\sqrt{e}\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(3\,c\,d^2-5\,b\,d\,e+7\,a\,e^2\right)}{2\,d^{9/2}}$$

input
```
int((a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^2),x)
```

output
```
- (a/(5*d) - (x^2*(7*a*e - 5*b*d))/(15*d^2) + (x^4*(7*a*e^2 + 3*c*d^2 - 5*
b*d*e))/(3*d^3) + (e*x^6*(7*a*e^2 + 3*c*d^2 - 5*b*d*e))/(2*d^4))/(d*x^5 +
e*x^7) - (e^(1/2)*atan((e^(1/2)*x)/d^(1/2))*(7*a*e^2 + 3*c*d^2 - 5*b*d*e))
/(2*d^(9/2))
```

3.287 $\int \frac{a+bx^2+cx^4}{x^8\left(d+ex^2\right)^2}\,dx$

3.287.1 Optimal result

Integrand size = 25, antiderivative size = 167

$$\int \frac{a+bx^2+cx^4}{x^8\left(d+ex^2\right)^2}\,dx = -\frac{a}{7d^2x^7}-\frac{bd-2ae}{5d^3x^5}-\frac{cd^2-e(2bd-3ae)}{3d^4x^3}+\frac{e(2cd^2-e(3bd-4ae))}{d^5x}+\frac{e^2(cd^2-bde+ae^2)\,x}{2d^5\left(d+ex^2\right)}+\frac{e^{3/2}(5cd^2-e(7bd-9ae))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{11/2}}$$

output
```
-1/7*a/d^2/x^7+1/5*(2*a*e-b*d)/d^3/x^5+1/3*(-c*d^2+e*(-3*a*e+2*b*d))/d^4/x
^3+e*(2*c*d^2-e*(-4*a*e+3*b*d))/d^5/x+1/2*e^2*(a*e^2-b*d*e+c*d^2)*x/d^5/(e
*x^2+d)+1/2*e^(3/2)*(5*c*d^2-e*(-9*a*e+7*b*d))*arctan(x*e^(1/2)/d^(1/2))/d
^(11/2)
```

3.287.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.99

$$\int \frac{a+bx^2+cx^4}{x^8\left(d+ex^2\right)^2}\,dx = -\frac{a}{7d^2x^7}+\frac{-bd+2ae}{5d^3x^5}+\frac{-cd^2+2bde-3ae^2}{3d^4x^3}+\frac{e(2cd^2-3bde+4ae^2)}{d^5x}+\frac{e^2(cd^2-bde+ae^2)\,x}{2d^5\left(d+ex^2\right)}+\frac{e^{3/2}(5cd^2-7bde+9ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{11/2}}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/(x^8*(d + e*x^2)^2),x]
```

output
```
-1/7*a/(d^2*x^7) + (-(b*d) + 2*a*e)/(5*d^3*x^5) + (-(c*d^2) + 2*b*d*e - 3*
a*e^2)/(3*d^4*x^3) + (e*(2*c*d^2 - 3*b*d*e + 4*a*e^2))/(d^5*x) + (e^2*(c*d
^2 - b*d*e + a*e^2)*x)/(2*d^5*(d + e*x^2)) + (e^(3/2)*(5*c*d^2 - 7*b*d*e +
 9*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(11/2))
```

3.287.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1582, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{x^8 \left(d+ex^2\right)^2} \, dx$$

$$\Big\downarrow \text{1582}$$

$$\frac{\int \frac{e^4\left(cd^2-bed+ae^2\right)x^8-2de^3\left(cd^2-bed+ae^2\right)x^6+2d^2e^2\left(cd^2-bed+ae^2\right)x^4+2d^3e^2(bd-ae)x^2+2ad^4e^2}{x^8\left(ex^2+d\right)}dx}{2d^5e^2}+\frac{e^2x\left(ae^2-bde+cd^2\right)}{2d^5\left(d+ex^2\right)}$$

$$\Big\downarrow \text{2333}$$

$$\frac{\int \left(\frac{\left(5cd^2-e(7bd-9ae)\right)e^4}{ex^2+d}+\frac{2\left(e(3bd-4ae)-2cd^2\right)e^3}{x^2}+\frac{2d\left(cd^2-e(2bd-3ae)\right)e^2}{x^4}+\frac{2d^2(bd-2ae)e^2}{x^6}+\frac{2ad^3e^2}{x^8}\right)dx}{2d^5e^2}+\frac{e^2x\left(ae^2-bde+cd^2\right)}{2d^5\left(d+ex^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{e^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(5cd^2-e(7bd-9ae)\right)}{\sqrt{d}}+\frac{2e^3\left(2cd^2-e(3bd-4ae)\right)}{x}-\frac{2de^2\left(cd^2-e(2bd-3ae)\right)}{3x^3}-\frac{2d^2e^2(bd-2ae)}{5x^5}-\frac{2ad^3e^2}{7x^7}}{2d^5e^2}+\frac{e^2x\left(ae^2-bde+cd^2\right)}{2d^5\left(d+ex^2\right)}$$

input
```
Int[(a + b*x^2 + c*x^4)/(x^8*(d + e*x^2)^2),x]
```

output
```
(e^2*(c*d^2 - b*d*e + a*e^2)*x)/(2*d^5*(d + e*x^2)) + ((-2*a*d^3*e^2)/(7*x
^7) - (2*d^2*e^2*(b*d - 2*a*e))/(5*x^5) - (2*d*e^2*(c*d^2 - e*(2*b*d - 3*a
*e)))/(3*x^3) + (2*e^3*(2*c*d^2 - e*(3*b*d - 4*a*e)))/x + (e^(7/2)*(5*c*d^
2 - e*(7*b*d - 9*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d])/(2*d^5*e^2)
```

3.287.3.1 Defintions of rubi rules used

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.287.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.89

method	result
default	$-\frac{a}{7d^2x^7}-\frac{-2ae+bd}{5d^3x^5}-\frac{3a\,e^2-2bde+c\,d^2}{3d^4x^3}+\frac{e\left(4a\,e^2-3bde+2c\,d^2\right)}{d^5x}+\frac{e^2\left(\frac{\left(\frac{1}{2}a\,e^2-\frac{1}{2}bde+\frac{1}{2}c\,d^2\right)x}{e\,x^2+d}+\frac{\left(9a\,e^2-7bde+5c\,d^2\right)\arctan\left(\frac{e}{\sqrt{}}\right.}{2\sqrt{ed}}\right.}{d^5}$
risch	$\frac{\frac{e^2\left(9a\,e^2-7bde+5c\,d^2\right)x^8}{2d^5}+\frac{e\left(9a\,e^2-7bde+5c\,d^2\right)x^6}{3d^4}-\frac{\left(9a\,e^2-7bde+5c\,d^2\right)x^4}{15d^3}+\frac{(9ae-7bd)x^2}{35d^2}-\frac{a}{7d}}{x^7(e\,x^2+d)}+\left(\_R=\text{RootOf}\left(d^{11}\_Z^2+81a^2e^7-126ab\right.\right.$

input
```
int((c*x^4+b*x^2+a)/x^8/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/7*a/d^2/x^7-1/5*(-2*a*e+b*d)/d^3/x^5-1/3*(3*a*e^2-2*b*d*e+c*d^2)/d^4/x^
3+e*(4*a*e^2-3*b*d*e+2*c*d^2)/d^5/x+e^2/d^5*((1/2*a*e^2-1/2*b*d*e+1/2*c*d^
2)*x/(e*x^2+d)+1/2*(9*a*e^2-7*b*d*e+5*c*d^2)/(e*d)^(1/2)*arctan(e*x/(e*d)^
(1/2)))
```

3.287.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 436, normalized size of antiderivative = 2.61

$$\int \frac{a+bx^2+cx^4}{x^8\left(d+ex^2\right)^2}\,dx$$
$$= \left[\frac{210\left(5\,cd^2e^2-7\,bde^3+9\,ae^4\right)x^8+140\left(5\,cd^3e-7\,bd^2e^2+9\,ade^3\right)x^6-60\,ad^4-28\left(5\,cd^4-7\,bd^3e+9\,ad\right.}{}\right.$$

input
```
integrate((c*x^4+b*x^2+a)/x^8/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
[1/420*(210*(5*c*d^2*e^2 - 7*b*d*e^3 + 9*a*e^4)*x^8 + 140*(5*c*d^3*e - 7*b
*d^2*e^2 + 9*a*d*e^3)*x^6 - 60*a*d^4 - 28*(5*c*d^4 - 7*b*d^3*e + 9*a*d^2*e
^2)*x^4 - 12*(7*b*d^4 - 9*a*d^3*e)*x^2 + 105*((5*c*d^2*e^2 - 7*b*d*e^3 + 9
*a*e^4)*x^9 + (5*c*d^3*e - 7*b*d^2*e^2 + 9*a*d*e^3)*x^7)*sqrt(-e/d)*log((e
*x^2 + 2*d*x*sqrt(-e/d) - d)/(e*x^2 + d)))/(d^5*e*x^9 + d^6*x^7), 1/210*(1
05*(5*c*d^2*e^2 - 7*b*d*e^3 + 9*a*e^4)*x^8 + 70*(5*c*d^3*e - 7*b*d^2*e^2 +
 9*a*d*e^3)*x^6 - 30*a*d^4 - 14*(5*c*d^4 - 7*b*d^3*e + 9*a*d^2*e^2)*x^4 -
6*(7*b*d^4 - 9*a*d^3*e)*x^2 + 105*((5*c*d^2*e^2 - 7*b*d*e^3 + 9*a*e^4)*x^9
 + (5*c*d^3*e - 7*b*d^2*e^2 + 9*a*d*e^3)*x^7)*sqrt(e/d)*arctan(x*sqrt(e/d)
))/(d^5*e*x^9 + d^6*x^7)]
```

3.287.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. 2(156) = 312.

Time = 1.25 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.96

$$\int \frac{a+bx^2+cx^4}{x^8\left(d+ex^2\right)^2}\,dx = -\frac{\sqrt{-\frac{e^3}{d^{11}}}\cdot\left(9ae^2-7bde+5cd^2\right)\log\left(-\frac{d^6\sqrt{-\frac{e^3}{d^{11}}}\cdot\left(9ae^2-7bde+5cd^2\right)}{9ae^4-7bde^3+5cd^2e^2}+x\right)}{4} + \frac{\sqrt{-\frac{e^3}{d^{11}}}\cdot\left(9ae^2-7bde+5cd^2\right)\log\left(\frac{d^6\sqrt{-\frac{e^3}{d^{11}}}\cdot\left(9ae^2-7bde+5cd^2\right)}{9ae^4-7bde^3+5cd^2e^2}+x\right)}{4} + \frac{-30ad^4+x^8\cdot\left(945ae^4-735bde^3+525cd^2e^2\right)+x^6\cdot\left(630ade^3-490bd^2e^2+350cd^3e\right)+x^4(-126ad^2e^2}{210d^6x^7+210d^5ex^9}$$

input
```
integrate((c*x**4+b*x**2+a)/x**8/(e*x**2+d)**2,x)
```

output
```
-sqrt(-e**3/d**11)*(9*a*e**2 - 7*b*d*e + 5*c*d**2)*log(-d**6*sqrt(-e**3/d*
*11)*(9*a*e**2 - 7*b*d*e + 5*c*d**2)/(9*a*e**4 - 7*b*d*e**3 + 5*c*d**2*e**
2) + x)/4 + sqrt(-e**3/d**11)*(9*a*e**2 - 7*b*d*e + 5*c*d**2)*log(d**6*sqr
t(-e**3/d**11)*(9*a*e**2 - 7*b*d*e + 5*c*d**2)/(9*a*e**4 - 7*b*d*e**3 + 5*
c*d**2*e**2) + x)/4 + (-30*a*d**4 + x**8*(945*a*e**4 - 735*b*d*e**3 + 525*
c*d**2*e**2) + x**6*(630*a*d*e**3 - 490*b*d**2*e**2 + 350*c*d**3*e) + x**4
*(-126*a*d**2*e**2 + 98*b*d**3*e - 70*c*d**4) + x**2*(54*a*d**3*e - 42*b*d
**4))/(210*d**6*x**7 + 210*d**5*e*x**9)
```

3.287.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^2+cx^4}{x^8\left(d+ex^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+b*x^2+a)/x^8/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.287.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.04

$$\int \frac{a+bx^2+cx^4}{x^8\left(d+ex^2\right)^2}\,dx = \frac{\left(5\,cd^2e^2-7\,bde^3+9\,ae^4\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}d^5}+\frac{cd^2e^2x-bde^3x+ae^4x}{2\,(ex^2+d)d^5} + \frac{210\,cd^2ex^6-315\,bde^2x^6+420\,ae^3x^6-35\,cd^3x^4+70\,bd^2ex^4-105\,ade^2x^4-21\,bd^3x^2+42\,ad^2ex^2-15}{105\,d^5x^7}$$

input
```
integrate((c*x^4+b*x^2+a)/x^8/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
1/2*(5*c*d^2*e^2 - 7*b*d*e^3 + 9*a*e^4)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d
^5) + 1/2*(c*d^2*e^2*x - b*d*e^3*x + a*e^4*x)/((e*x^2 + d)*d^5) + 1/105*(2
10*c*d^2*e*x^6 - 315*b*d*e^2*x^6 + 420*a*e^3*x^6 - 35*c*d^3*x^4 + 70*b*d^2
*e*x^4 - 105*a*d*e^2*x^4 - 21*b*d^3*x^2 + 42*a*d^2*e*x^2 - 15*a*d^3)/(d^5*
x^7)
```

3.287.9 Mupad [B] (verification not implemented)

Time = 7.80 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.93

$$\int \frac{a+bx^2+cx^4}{x^8\left(d+ex^2\right)^2}\,dx = \frac{\frac{x^2\,(9\,a\,e-7\,b\,d)}{35\,d^2}-\frac{a}{7\,d}-\frac{x^4\,(5\,c\,d^2-7\,b\,d\,e+9\,a\,e^2)}{15\,d^3}+\frac{e\,x^6\,(5\,c\,d^2-7\,b\,d\,e+9\,a\,e^2)}{3\,d^4}+\frac{e^2\,x^8\,(5\,c\,d^2-7\,b\,d\,e+9\,a\,e^2)}{2\,d^5}}{e\,x^9+d\,x^7} + \frac{e^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(5\,c\,d^2-7\,b\,d\,e+9\,a\,e^2\right)}{2\,d^{11/2}}$$

input
```
int((a + b*x^2 + c*x^4)/(x^8*(d + e*x^2)^2),x)
```

output
```
((x^2*(9*a*e - 7*b*d))/(35*d^2) - a/(7*d) - (x^4*(9*a*e^2 + 5*c*d^2 - 7*b*
d*e))/(15*d^3) + (e*x^6*(9*a*e^2 + 5*c*d^2 - 7*b*d*e))/(3*d^4) + (e^2*x^8*
(9*a*e^2 + 5*c*d^2 - 7*b*d*e))/(2*d^5))/(d*x^7 + e*x^9) + (e^(3/2)*atan((e
^(1/2)*x)/d^(1/2))*(9*a*e^2 + 5*c*d^2 - 7*b*d*e))/(2*d^(11/2))
```

3.288 $\int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx$

3.288.1 Optimal result

Integrand size = 25, antiderivative size = 173

$$\int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx = \frac{(6cd^2-e(3bd-ae))\,x}{e^5}-\frac{(3cd-be)x^3}{3e^4}+\frac{cx^5}{5e^3}-\frac{d^2(cd^2-bde+ae^2)\,x}{4e^5(d+ex^2)^2}+\frac{d(17cd^2-e(13bd-9ae))\,x}{8e^5(d+ex^2)}-\frac{\sqrt{d}(63cd^2-35bde+15ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8e^{11/2}}$$

output
```
(6*c*d^2-e*(-a*e+3*b*d))*x/e^5-1/3*(-b*e+3*c*d)*x^3/e^4+1/5*c*x^5/e^3-1/4*
d^2*(a*e^2-b*d*e+c*d^2)*x/e^5/(e*x^2+d)^2+1/8*d*(17*c*d^2-e*(-9*a*e+13*b*d
))*x/e^5/(e*x^2+d)-1/8*(15*a*e^2-35*b*d*e+63*c*d^2)*arctan(x*e^(1/2)/d^(1/
2))*d^(1/2)/e^(11/2)
```

3.288.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.98

$$\int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx = \frac{(6cd^2+e(-3bd+ae))\,x}{e^5}+\frac{(-3cd+be)x^3}{3e^4}+\frac{cx^5}{5e^3}-\frac{(cd^4+d^2e(-bd+ae))\,x}{4e^5(d+ex^2)^2}+\frac{(17cd^3+de(-13bd+9ae))\,x}{8e^5(d+ex^2)}-\frac{\sqrt{d}(63cd^2+5e(-7bd+3ae))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8e^{11/2}}$$

input
```
Integrate[(x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]
```

output
```
((6*c*d^2 + e*(-3*b*d + a*e))*x)/e^5 + ((-3*c*d + b*e)*x^3)/(3*e^4) + (c*x
^5)/(5*e^3) - ((c*d^4 + d^2*e*(-(b*d) + a*e))*x)/(4*e^5*(d + e*x^2)^2) + (
(17*c*d^3 + d*e*(-13*b*d + 9*a*e))*x)/(8*e^5*(d + e*x^2)) - (Sqrt[d]*(63*c
*d^2 + 5*e*(-7*b*d + 3*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*e^(11/2))
```

3.288.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1580, 25, 2345, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6 \left(a+b x^2+c x^4\right)}{\left(d+e x^2\right)^3} \, dx$$

$$\Big\downarrow \text{1580}$$

$$-\frac{\int -\frac{4 c e^4 x^8-4 e^3 (c d-b e) x^6+4 e^2 \left(c d^2-b e d+a e^2\right) x^4-4 d e \left(c d^2-b e d+a e^2\right) x^2+d^2 \left(c d^2-b e d+a e^2\right)}{\left(e x^2+d\right)^2}dx}{4 e^5}-\frac{d^2 x \left(a e^2-b d e+c d^2\right)}{4 e^5 \left(d+e x^2\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{4 c e^4 x^8-4 e^3 (c d-b e) x^6+4 e^2 \left(c d^2-b e d+a e^2\right) x^4-4 d e \left(c d^2-b e d+a e^2\right) x^2+d^2 \left(c d^2-b e d+a e^2\right)}{\left(e x^2+d\right)^2}dx}{4 e^5}-\frac{d^2 x \left(a e^2-b d e+c d^2\right)}{4 e^5 \left(d+e x^2\right)^2}$$

$$\Big\downarrow \text{2345}$$

$$\frac{\frac{d x \left(17 c d^2-e (13 b d-9 a e)\right)}{2 \left(d+e x^2\right)}-\frac{\int \frac{-8 c d e^3 x^6+8 d e^2 (2 c d-b e) x^4-8 d e \left(3 c d^2-e (2 b d-a e)\right) x^2+d^2 \left(15 c d^2-e (11 b d-7 a e)\right)}{e x^2+d}dx}{2 d}}{4 e^5}-\frac{d^2 x \left(a e^2-b d e+c d^2\right)}{4 e^5 \left(d+e x^2\right)^2}$$

$$\Big\downarrow \text{2341}$$

$$\frac{\frac{dx(17cd^2-e(13bd-9ae))}{2(d+ex^2)}-\frac{\int\left(-8cde^2x^4+8de(3cd-be)x^2-8d(6cd^2-e(3bd-ae))+\frac{63cd^4-35bed^3+15ae^2d^2}{ex^2+d}\right)dx}{2d}}{4e^5}-\frac{d^2x\left(ae^2-bde+cd^2\right)}{4e^5\left(d+ex^2\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{dx(17cd^2-e(13bd-9ae))}{2(d+ex^2)}-\frac{\frac{d^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(15ae^2-35bde+63cd^2\right)}{\sqrt{e}}-8dx\left(6cd^2-e(3bd-ae)\right)+\frac{8}{3}dex^3(3cd-be)-\frac{8}{5}cde^2x^5}{2d}}{4e^5}-\frac{d^2x\left(ae^2-bde+cd^2\right)}{4e^5\left(d+ex^2\right)^2}$$

input
```
Int[(x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]
```

output
```
-1/4*(d^2*(c*d^2 - b*d*e + a*e^2)*x)/(e^5*(d + e*x^2)^2) + ((d*(17*c*d^2 -
 e*(13*b*d - 9*a*e))*x)/(2*(d + e*x^2)) - (-8*d*(6*c*d^2 - e*(3*b*d - a*e)
)*x + (8*d*e*(3*c*d - b*e)*x^3)/3 - (8*c*d*e^2*x^5)/5 + (d^(3/2)*(63*c*d^2
 - 35*b*d*e + 15*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e])/(2*d))/(4*e^
5)
```

3.288.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1580
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[1/(2*e^(2*p + m/2)*
(q + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*
e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b
*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.288. $\int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx$

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.288.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.87

method	result
default	$\frac{\frac{1}{5}c\,x^5e^2+\frac{1}{3}b\,e^2x^3-dc\,x^3e+a\,e^2x-3bdex+6c\,d^2x}{e^5}-\frac{d\left(\frac{\left(-\frac{9}{8}a\,e^3+\frac{13}{8}d\,e^2b-\frac{17}{8}c\,d^2e\right)x^3-\frac{d\left(7a\,e^2-11bde+15c\,d^2\right)x}{8}}{\left(e\,x^2+d\right)^2}+\frac{\left(15a\,e^2-35bde+63c\,\ldots\right.}{8\sqrt{e\ldots}}\right.}{e^5}$
risch	$\frac{c\,x^5}{5e^3}+\frac{b\,x^3}{3e^3}-\frac{dc\,x^3}{e^4}+\frac{ax}{e^3}-\frac{3bdx}{e^4}+\frac{6c\,d^2x}{e^5}+\frac{\left(\frac{9}{8}d\,e^3a-\frac{13}{8}e^2d^2b+\frac{17}{8}d^3ec\right)x^3+\frac{d^2\left(7a\,e^2-11bde+15c\,d^2\right)x}{8}}{e^5\left(e\,x^2+d\right)^2}+\frac{15\sqrt{-ed}\,\ln(-\ldots}{16e^{4}\ldots}$

input
```
int(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/e^5*(1/5*c*x^5*e^2+1/3*b*e^2*x^3-d*c*x^3*e+a*e^2*x-3*b*d*e*x+6*c*d^2*x)-
d/e^5*(((-9/8*a*e^3+13/8*d*e^2*b-17/8*c*d^2*e)*x^3-1/8*d*(7*a*e^2-11*b*d*e
+15*c*d^2)*x)/(e*x^2+d)^2+1/8*(15*a*e^2-35*b*d*e+63*c*d^2)/(e*d)^(1/2)*arc
tan(e*x/(e*d)^(1/2)))
```

3.288.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 504, normalized size of antiderivative = 2.91

$$\int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx$$

$$=\left[\frac{48\,ce^4x^9-16\,(9\,cde^3-5\,be^4)x^7+16\,(63\,cd^2e^2-35\,bde^3+15\,ae^4)x^5+50\,(63\,cd^3e-35\,bd^2e^2+15\,ade^3)\ldots}{\ldots}\right.$$

input
```
integrate(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
[1/240*(48*c*e^4*x^9 - 16*(9*c*d*e^3 - 5*b*e^4)*x^7 + 16*(63*c*d^2*e^2 - 3
5*b*d*e^3 + 15*a*e^4)*x^5 + 50*(63*c*d^3*e - 35*b*d^2*e^2 + 15*a*d*e^3)*x^
3 + 15*(63*c*d^4 - 35*b*d^3*e + 15*a*d^2*e^2 + (63*c*d^2*e^2 - 35*b*d*e^3
+ 15*a*e^4)*x^4 + 2*(63*c*d^3*e - 35*b*d^2*e^2 + 15*a*d*e^3)*x^2)*sqrt(-d/
e)*log((e*x^2 - 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d)) + 30*(63*c*d^4 - 35*b*d
^3*e + 15*a*d^2*e^2)*x)/(e^7*x^4 + 2*d*e^6*x^2 + d^2*e^5), 1/120*(24*c*e^4
*x^9 - 8*(9*c*d*e^3 - 5*b*e^4)*x^7 + 8*(63*c*d^2*e^2 - 35*b*d*e^3 + 15*a*e
^4)*x^5 + 25*(63*c*d^3*e - 35*b*d^2*e^2 + 15*a*d*e^3)*x^3 - 15*(63*c*d^4 -
 35*b*d^3*e + 15*a*d^2*e^2 + (63*c*d^2*e^2 - 35*b*d*e^3 + 15*a*e^4)*x^4 +
2*(63*c*d^3*e - 35*b*d^2*e^2 + 15*a*d*e^3)*x^2)*sqrt(d/e)*arctan(e*x*sqrt(
d/e)/d) + 15*(63*c*d^4 - 35*b*d^3*e + 15*a*d^2*e^2)*x)/(e^7*x^4 + 2*d*e^6*
x^2 + d^2*e^5)]
```

3.288.6 Sympy [A] (verification not implemented)

Time = 1.92 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.36

$$\int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx = \frac{cx^5}{5e^3}+x^3\left(\frac{b}{3e^3}-\frac{cd}{e^4}\right)+x\left(\frac{a}{e^3}-\frac{3bd}{e^4}+\frac{6cd^2}{e^5}\right) + \frac{\sqrt{-\frac{d}{e^{11}}}\cdot(15ae^2-35bde+63cd^2)\log\left(-e^5\sqrt{-\frac{d}{e^{11}}}+x\right)}{16} - \frac{\sqrt{-\frac{d}{e^{11}}}\cdot(15ae^2-35bde+63cd^2)\log\left(e^5\sqrt{-\frac{d}{e^{11}}}+x\right)}{16} + \frac{x^3\cdot(9ade^3-13bd^2e^2+17cd^3e)+x(7ad^2e^2-11bd^3e+15cd^4)}{8d^2e^5+16de^6x^2+8e^7x^4}$$

input
```
integrate(x**6*(c*x**4+b*x**2+a)/(e*x**2+d)**3,x)
```

output
```
c*x**5/(5*e**3) + x**3*(b/(3*e**3) - c*d/e**4) + x*(a/e**3 - 3*b*d/e**4 +
6*c*d**2/e**5) + sqrt(-d/e**11)*(15*a*e**2 - 35*b*d*e + 63*c*d**2)*log(-e*
*5*sqrt(-d/e**11) + x)/16 - sqrt(-d/e**11)*(15*a*e**2 - 35*b*d*e + 63*c*d*
*2)*log(e**5*sqrt(-d/e**11) + x)/16 + (x**3*(9*a*d*e**3 - 13*b*d**2*e**2 +
 17*c*d**3*e) + x*(7*a*d**2*e**2 - 11*b*d**3*e + 15*c*d**4))/(8*d**2*e**5
+ 16*d*e**6*x**2 + 8*e**7*x**4)
```

3.288.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.288.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.99

$$\begin{aligned}\int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx &= -\frac{(63\,cd^3-35\,bd^2e+15\,ade^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}e^5} \\ &\quad+\frac{17\,cd^3ex^3-13\,bd^2e^2x^3+9\,ade^3x^3+15\,cd^4x-11\,bd^3ex+7\,ad^2e^2x}{8\,(ex^2+d)^2e^5} \\ &\quad+\frac{3\,ce^{12}x^5-15\,cde^{11}x^3+5\,be^{12}x^3+90\,cd^2e^{10}x-45\,bde^{11}x+15\,ae^{12}x}{15\,e^{15}}\end{aligned}$$

input
```
integrate(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
-1/8*(63*c*d^3 - 35*b*d^2*e + 15*a*d*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)
*e^5) + 1/8*(17*c*d^3*e*x^3 - 13*b*d^2*e^2*x^3 + 9*a*d*e^3*x^3 + 15*c*d^4*
x - 11*b*d^3*e*x + 7*a*d^2*e^2*x)/((e*x^2 + d)^2*e^5) + 1/15*(3*c*e^12*x^5
 - 15*c*d*e^11*x^3 + 5*b*e^12*x^3 + 90*c*d^2*e^10*x - 45*b*d*e^11*x + 15*a
*e^12*x)/e^15
```

3.288.9 Mupad [B] (verification not implemented)

Time = 7.75 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.29

$$\int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx$$
$$= x^3\left(\frac{b}{3\,e^3}-\frac{c\,d}{e^4}\right)-x\left(\frac{3\,c\,d^2}{e^5}-\frac{a}{e^3}+\frac{3\,d\left(\frac{b}{e^3}-\frac{3\,c\,d}{e^4}\right)}{e}\right)$$
$$+\frac{\left(\frac{17\,c\,d^3\,e}{8}-\frac{13\,b\,d^2\,e^2}{8}+\frac{9\,a\,d\,e^3}{8}\right)x^3+\left(\frac{15\,c\,d^4}{8}-\frac{11\,b\,d^3\,e}{8}+\frac{7\,a\,d^2\,e^2}{8}\right)x}{d^2\,e^5+2\,d\,e^6\,x^2+e^7\,x^4}+\frac{c\,x^5}{5\,e^3}$$
$$-\frac{\sqrt{d}\,\mathrm{atan}\left(\frac{\sqrt{d}\,\sqrt{e}\,x\,(63\,c\,d^2-35\,b\,d\,e+15\,a\,e^2)}{63\,c\,d^3-35\,b\,d^2\,e+15\,a\,d\,e^2}\right)(63\,c\,d^2-35\,b\,d\,e+15\,a\,e^2)}{8\,e^{11/2}}$$

input
```
int((x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x)
```

output
```
x^3*(b/(3*e^3) - (c*d)/e^4) - x*((3*c*d^2)/e^5 - a/e^3 + (3*d*(b/e^3 - (3*
c*d)/e^4))/e) + (x^3*((9*a*d*e^3)/8 - (13*b*d^2*e^2)/8 + (17*c*d^3*e)/8) +
 x*((15*c*d^4)/8 + (7*a*d^2*e^2)/8 - (11*b*d^3*e)/8))/(d^2*e^5 + e^7*x^4 +
 2*d*e^6*x^2) + (c*x^5)/(5*e^3) - (d^(1/2)*atan((d^(1/2)*e^(1/2)*x*(15*a*e
^2 + 63*c*d^2 - 35*b*d*e))/(63*c*d^3 + 15*a*d*e^2 - 35*b*d^2*e))*(15*a*e^2
 + 63*c*d^2 - 35*b*d*e))/(8*e^(11/2))
```

3.289 $\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx$

3.289.1 Optimal result

Integrand size = 25, antiderivative size = 143

$$\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx = -\frac{(3cd-be)x}{e^4}+\frac{cx^3}{3e^3}+\frac{d(cd^2-bde+ae^2)\,x}{4e^4(d+ex^2)^2} - \frac{(13cd^2-e(9bd-5ae))\,x}{8e^4(d+ex^2)} + \frac{(35cd^2-3e(5bd-ae))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{d}e^{9/2}}$$

output
```
-(-b*e+3*c*d)*x/e^4+1/3*c*x^3/e^3+1/4*d*(a*e^2-b*d*e+c*d^2)*x/e^4/(e*x^2+d
)^2-1/8*(13*c*d^2-e*(-5*a*e+9*b*d))*x/e^4/(e*x^2+d)+1/8*(35*c*d^2-3*e*(-a*
e+5*b*d))*arctan(x*e^(1/2)/d^(1/2))/e^(9/2)/d^(1/2)
```

3.289.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.99

$$\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx = \frac{(-3cd+be)x}{e^4}+\frac{cx^3}{3e^3}+\frac{(cd^3-bd^2e+ade^2)\,x}{4e^4(d+ex^2)^2} - \frac{(13cd^2-9bde+5ae^2)\,x}{8e^4(d+ex^2)} + \frac{(35cd^2-15bde+3ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{d}e^{9/2}}$$

input
```
Integrate[(x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]
```

output
```
((-3*c*d + b*e)*x)/e^4 + (c*x^3)/(3*e^3) + ((c*d^3 - b*d^2*e + a*d*e^2)*x)
/(4*e^4*(d + e*x^2)^2) - ((13*c*d^2 - 9*b*d*e + 5*a*e^2)*x)/(8*e^4*(d + e*
x^2)) + ((35*c*d^2 - 15*b*d*e + 3*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*S
qrt[d]*e^(9/2))
```

3.289.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1580, 2345, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 (a+bx^2+cx^4)}{(d+ex^2)^3} \, dx$$

$$\Big\downarrow \text{1580}$$

$$\frac{dx(ae^2-bde+cd^2)}{4e^4(d+ex^2)^2}-\frac{\int \frac{-4ce^3x^6+4e^2(cd-be)x^4-4e(cd^2-bed+ae^2)x^2+d(cd^2-bed+ae^2)}{(ex^2+d)^2}dx}{4e^4}$$

$$\Big\downarrow \text{2345}$$

$$\frac{dx(ae^2-bde+cd^2)}{4e^4(d+ex^2)^2}-\frac{\frac{x(13cd^2-e(9bd-5ae))}{2(d+ex^2)}-\frac{\int \frac{8cde^2x^4-8de(2cd-be)x^2+d(11cd^2-e(7bd-3ae))}{ex^2+d}dx}{2d}}{4e^4}$$

$$\Big\downarrow \text{1467}$$

$$\frac{dx(ae^2-bde+cd^2)}{4e^4(d+ex^2)^2}-\frac{\frac{x(13cd^2-e(9bd-5ae))}{2(d+ex^2)}-\frac{\int \left(8cdex^2-8d(3cd-be)+\frac{35cd^3-15bed^2+3ae^2d}{ex^2+d}\right)dx}{2d}}{4e^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{dx(ae^2-bde+cd^2)}{4e^4(d+ex^2)^2}-\frac{\frac{x(13cd^2-e(9bd-5ae))}{2(d+ex^2)}-\frac{\frac{\sqrt{d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(35cd^2-3e(5bd-ae))}{\sqrt{e}}-8dx(3cd-be)+\frac{8}{3}cdex^3}{2d}}{4e^4}$$

input
```
Int[(x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]
```

output
```
(d*(c*d^2 - b*d*e + a*e^2)*x)/(4*e^4*(d + e*x^2)^2) - (((13*c*d^2 - e*(9*b
*d - 5*a*e))*x)/(2*(d + e*x^2)) - (-8*d*(3*c*d - b*e)*x + (8*c*d*e*x^3)/3
+ (Sqrt[d]*(35*c*d^2 - 3*e*(5*b*d - a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqr
t[e])/(2*d))/(4*e^4)
```

3.289.3.1 Defintions of rubi rules used

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 1580
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[1/(2*e^(2*p + m/2)*
(q + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*
e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b
*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.289. $\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx$

3.289.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.84

method	result
default	$\frac{\frac{1}{3}c\,x^{3}e+bex-3cdx}{e^{4}}+\frac{\frac{\left(-\frac{5}{8}a\,e^{3}+\frac{9}{8}d\,e^{2}b-\frac{13}{8}c\,d^{2}e\right)x^{3}-\frac{d\left(3a\,e^{2}-7bde+11c\,d^{2}\right)x}{8}}{\left(e\,x^{2}+d\right)^{2}}+\frac{\left(3a\,e^{2}-15bde+35c\,d^{2}\right)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{8\sqrt{ed}}}{e^{4}}$
risch	$\frac{c\,x^{3}}{3e^{3}}+\frac{bx}{e^{3}}-\frac{3cdx}{e^{4}}+\frac{\left(-\frac{5}{8}a\,e^{3}+\frac{9}{8}d\,e^{2}b-\frac{13}{8}c\,d^{2}e\right)x^{3}-\frac{d\left(3a\,e^{2}-7bde+11c\,d^{2}\right)x}{8}}{e^{4}\left(e\,x^{2}+d\right)^{2}}-\frac{3\ln\left(ex+\sqrt{-ed}\right)a}{16e^{2}\sqrt{-ed}}+\frac{15\ln\left(ex+\sqrt{-ed}\right)bd}{16e^{3}\sqrt{-ed}}-$ 35

input
```
int(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/e^4*(1/3*c*x^3*e+b*e*x-3*c*d*x)+1/e^4*(((-5/8*a*e^3+9/8*d*e^2*b-13/8*c*d
^2*e)*x^3-1/8*d*(3*a*e^2-7*b*d*e+11*c*d^2)*x)/(e*x^2+d)^2+1/8*(3*a*e^2-15*
b*d*e+35*c*d^2)/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2)))
```

3.289.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 462, normalized size of antiderivative = 3.23

$$\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx$$
$$=\left[\frac{16\,cde^4x^7-16\,(7\,cd^2e^3-3\,bde^4)x^5-10\,(35\,cd^3e^2-15\,bd^2e^3+3\,ade^4)x^3-3\,(35\,cd^4-15\,bd^3e+3\,ad^2e^2\right.$$

input
```
integrate(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
[1/48*(16*c*d*e^4*x^7 - 16*(7*c*d^2*e^3 - 3*b*d*e^4)*x^5 - 10*(35*c*d^3*e^
2 - 15*b*d^2*e^3 + 3*a*d*e^4)*x^3 - 3*(35*c*d^4 - 15*b*d^3*e + 3*a*d^2*e^2
 + (35*c*d^2*e^2 - 15*b*d*e^3 + 3*a*e^4)*x^4 + 2*(35*c*d^3*e - 15*b*d^2*e^
2 + 3*a*d*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d
)) - 6*(35*c*d^4*e - 15*b*d^3*e^2 + 3*a*d^2*e^3)*x)/(d*e^7*x^4 + 2*d^2*e^6
*x^2 + d^3*e^5), 1/24*(8*c*d*e^4*x^7 - 8*(7*c*d^2*e^3 - 3*b*d*e^4)*x^5 - 5
*(35*c*d^3*e^2 - 15*b*d^2*e^3 + 3*a*d*e^4)*x^3 + 3*(35*c*d^4 - 15*b*d^3*e
+ 3*a*d^2*e^2 + (35*c*d^2*e^2 - 15*b*d*e^3 + 3*a*e^4)*x^4 + 2*(35*c*d^3*e
- 15*b*d^2*e^2 + 3*a*d*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) - 3*(35*c
*d^4*e - 15*b*d^3*e^2 + 3*a*d^2*e^3)*x)/(d*e^7*x^4 + 2*d^2*e^6*x^2 + d^3*e
^5)]
```

3.289.6 Sympy [A] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.48

$$\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx = \frac{cx^3}{3e^3}+x\left(\frac{b}{e^3}-\frac{3cd}{e^4}\right) - \frac{\sqrt{-\frac{1}{de^9}}\cdot(3ae^2-15bde+35cd^2)\log\left(-de^4\sqrt{-\frac{1}{de^9}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{de^9}}\cdot(3ae^2-15bde+35cd^2)\log\left(de^4\sqrt{-\frac{1}{de^9}}+x\right)}{16} + \frac{x^3(-5ae^3+9bde^2-13cd^2e)+x(-3ade^2+7bd^2e-11cd^3)}{8d^2e^4+16de^5x^2+8e^6x^4}$$

input
```
integrate(x**4*(c*x**4+b*x**2+a)/(e*x**2+d)**3,x)
```

output
```
c*x**3/(3*e**3) + x*(b/e**3 - 3*c*d/e**4) - sqrt(-1/(d*e**9))*(3*a*e**2 -
15*b*d*e + 35*c*d**2)*log(-d*e**4*sqrt(-1/(d*e**9)) + x)/16 + sqrt(-1/(d*e
**9))*(3*a*e**2 - 15*b*d*e + 35*c*d**2)*log(d*e**4*sqrt(-1/(d*e**9)) + x)/
16 + (x**3*(-5*a*e**3 + 9*b*d*e**2 - 13*c*d**2*e) + x*(-3*a*d*e**2 + 7*b*d
**2*e - 11*c*d**3))/(8*d**2*e**4 + 16*d*e**5*x**2 + 8*e**6*x**4)
```

3.289.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.289.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.94

$$\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx = \frac{(35\,cd^2-15\,bde+3\,ae^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\,\sqrt{de}e^4} - \frac{13\,cd^2ex^3-9\,bde^2x^3+5\,ae^3x^3+11\,cd^3x-7\,bd^2ex+3\,ade^2x}{8\,(ex^2+d)^2e^4} + \frac{ce^6x^3-9\,cde^5x+3\,be^6x}{3\,e^9}$$

input
```
integrate(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
1/8*(35*c*d^2 - 15*b*d*e + 3*a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e^4)
- 1/8*(13*c*d^2*e*x^3 - 9*b*d*e^2*x^3 + 5*a*e^3*x^3 + 11*c*d^3*x - 7*b*d^2
*e*x + 3*a*d*e^2*x)/((e*x^2 + d)^2*e^4) + 1/3*(c*e^6*x^3 - 9*c*d*e^5*x + 3
*b*e^6*x)/e^9
```

3.289.9 Mupad [B] (verification not implemented)

Time = 7.71 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.96

$$\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx = x\left(\frac{b}{e^3}-\frac{3\,c\,d}{e^4}\right) - \frac{\left(\frac{13\,c\,d^2\,e}{8}-\frac{9\,b\,d\,e^2}{8}+\frac{5\,a\,e^3}{8}\right)x^3+\left(\frac{11\,c\,d^3}{8}-\frac{7\,b\,d^2\,e}{8}+\frac{3\,a\,d\,e^2}{8}\right)x}{d^2\,e^4+2\,d\,e^5\,x^2+e^6\,x^4} + \frac{c\,x^3}{3\,e^3} + \frac{\operatorname{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)(35\,c\,d^2-15\,b\,d\,e+3\,a\,e^2)}{8\,\sqrt{d}\,e^{9/2}}$$

input
```
int((x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x)
```

output
```
x*(b/e^3 - (3*c*d)/e^4) - (x*((11*c*d^3)/8 + (3*a*d*e^2)/8 - (7*b*d^2*e)/8
) + x^3*((5*a*e^3)/8 - (9*b*d*e^2)/8 + (13*c*d^2*e)/8))/(d^2*e^4 + e^6*x^4
 + 2*d*e^5*x^2) + (c*x^3)/(3*e^3) + (atan((e^(1/2)*x)/d^(1/2))*(3*a*e^2 +
35*c*d^2 - 15*b*d*e))/(8*d^(1/2)*e^(9/2))
```

3.290 $\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^3} dx$

3.290.1 Optimal result

Integrand size = 25, antiderivative size = 124

$$\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^3} dx = \frac{cx}{e^3} - \frac{(cd^2-bde+ae^2)\,x}{4e^3\,(d+ex^2)^2} + \frac{(9cd^2-e(5bd-ae))\,x}{8de^3\,(d+ex^2)} - \frac{(15cd^2-e(3bd+ae))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{3/2}e^{7/2}}$$

output
```
c*x/e^3-1/4*(a*e^2-b*d*e+c*d^2)*x/e^3/(e*x^2+d)^2+1/8*(9*c*d^2-e*(-a*e+5*b
*d))*x/d/e^3/(e*x^2+d)-1/8*(15*c*d^2-e*(a*e+3*b*d))*arctan(x*e^(1/2)/d^(1/
2))/d^(3/2)/e^(7/2)
```

3.290.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.98

$$\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^3} dx = \frac{cx}{e^3} - \frac{(cd^2-bde+ae^2)\,x}{4e^3\,(d+ex^2)^2} + \frac{(9cd^2-5bde+ae^2)\,x}{8de^3\,(d+ex^2)} - \frac{(15cd^2-3bde-ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{3/2}e^{7/2}}$$

input
```
Integrate[(x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]
```

output
```
(c*x)/e^3 - ((c*d^2 - b*d*e + a*e^2)*x)/(4*e^3*(d + e*x^2)^2) + ((9*c*d^2
- 5*b*d*e + a*e^2)*x)/(8*d*e^3*(d + e*x^2)) - ((15*c*d^2 - 3*b*d*e - a*e^2
)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(3/2)*e^(7/2))
```

3.290.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1580, 25, 1471, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(a+b x^2+c x^4\right)}{\left(d+e x^2\right)^3} \, dx$$

$$\Big\downarrow \text{1580}$$

$$-\frac{\int -\frac{4ce^2x^4-4e(cd-be)x^2+cd^2+ae^2-bde}{(ex^2+d)^2}dx}{4e^3}-\frac{x\left(ae^2-bde+cd^2\right)}{4e^3\left(d+ex^2\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{4ce^2x^4-4e(cd-be)x^2+cd^2+ae^2-bde}{(ex^2+d)^2}dx}{4e^3}-\frac{x\left(ae^2-bde+cd^2\right)}{4e^3\left(d+ex^2\right)^2}$$

$$\Big\downarrow \text{1471}$$

$$\frac{\frac{x(9cd^2-e(5bd-ae))}{2d(d+ex^2)}-\frac{\int \frac{7cd^2-8cex^2d-e(3bd+ae)}{ex^2+d}dx}{2d}}{4e^3}-\frac{x\left(ae^2-bde+cd^2\right)}{4e^3\left(d+ex^2\right)^2}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{x(9cd^2-e(5bd-ae))}{2d(d+ex^2)}-\frac{(15cd^2-e(ae+3bd))\int \frac{1}{ex^2+d}dx-8cdx}{2d}}{4e^3}-\frac{x\left(ae^2-bde+cd^2\right)}{4e^3\left(d+ex^2\right)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{x(9cd^2-e(5bd-ae))}{2d(d+ex^2)}-\frac{\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(15cd^2-e(ae+3bd))}{\sqrt{d}\sqrt{e}}-8cdx}{2d}}{4e^3}-\frac{x\left(ae^2-bde+cd^2\right)}{4e^3\left(d+ex^2\right)^2}$$

input
```
Int[(x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]
```

3.290. $\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx$

output
```
-1/4*((c*d^2 - b*d*e + a*e^2)*x)/(e^3*(d + e*x^2)^2) + (((9*c*d^2 - e*(5*b
*d - a*e))*x)/(2*d*(d + e*x^2)) - (-8*c*d*x + ((15*c*d^2 - e*(3*b*d + a*e)
)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*Sqrt[e]))/(2*d))/(4*e^3)
```

3.290.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 1580
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[1/(2*e^(2*p + m/2)*
(q + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*
e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b
*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

3.290.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.85

method	result
default	$\frac{cx}{e^3}+\frac{\frac{\frac{e\left(a\,e^2-5bde+9c\,d^2\right)x^3}{8d}+\left(-\frac{1}{8}a\,e^2-\frac{3}{8}bde+\frac{7}{8}c\,d^2\right)x}{\left(e\,x^2+d\right)^2}+\frac{\left(a\,e^2+3bde-15c\,d^2\right)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{8d\sqrt{ed}}}{e^3}$
risch	$\frac{cx}{e^3}+\frac{\frac{e\left(a\,e^2-5bde+9c\,d^2\right)x^3}{8d}+\left(-\frac{1}{8}a\,e^2-\frac{3}{8}bde+\frac{7}{8}c\,d^2\right)x}{e^3(e\,x^2+d)^2}-\frac{\ln(ex+\sqrt{-ed})a}{16e\sqrt{-ed}\,d}-\frac{3\ln(ex+\sqrt{-ed})b}{16e^2\sqrt{-ed}}+\frac{15d\ln(ex+\sqrt{-ed})c}{16e^3\sqrt{-ed}}+\frac{\ln(-e}{16}$

input
```
int(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
c*x/e^3+1/e^3*((1/8*e*(a*e^2-5*b*d*e+9*c*d^2)/d*x^3+(-1/8*a*e^2-3/8*b*d*e+
7/8*c*d^2)*x)/(e*x^2+d)^2+1/8*(a*e^2+3*b*d*e-15*c*d^2)/d/(e*d)^(1/2)*arcta
n(e*x/(e*d)^(1/2)))
```

3.290.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 421, normalized size of antiderivative = 3.40

$$\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx$$
$$=\left[\frac{16\,cd^2e^3x^5+2\,(25\,cd^3e^2-5\,bd^2e^3+ade^4)x^3+(15\,cd^4-3\,bd^3e-ad^2e^2+(15\,cd^2e^2-3\,bde^3-ae^4)x^4+}{16\,(d^2e^6x^4+2\,d^3e^5x^2}\right.$$

input
```
integrate(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
[1/16*(16*c*d^2*e^3*x^5 + 2*(25*c*d^3*e^2 - 5*b*d^2*e^3 + a*d*e^4)*x^3 + (
15*c*d^4 - 3*b*d^3*e - a*d^2*e^2 + (15*c*d^2*e^2 - 3*b*d*e^3 - a*e^4)*x^4
+ 2*(15*c*d^3*e - 3*b*d^2*e^2 - a*d*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sq
rt(-d*e)*x - d)/(e*x^2 + d)) + 2*(15*c*d^4*e - 3*b*d^3*e^2 - a*d^2*e^3)*x)
/(d^2*e^6*x^4 + 2*d^3*e^5*x^2 + d^4*e^4), 1/8*(8*c*d^2*e^3*x^5 + (25*c*d^3
*e^2 - 5*b*d^2*e^3 + a*d*e^4)*x^3 - (15*c*d^4 - 3*b*d^3*e - a*d^2*e^2 + (1
5*c*d^2*e^2 - 3*b*d*e^3 - a*e^4)*x^4 + 2*(15*c*d^3*e - 3*b*d^2*e^2 - a*d*e
^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (15*c*d^4*e - 3*b*d^3*e^2 - a*d
^2*e^3)*x)/(d^2*e^6*x^4 + 2*d^3*e^5*x^2 + d^4*e^4)]
```

3.290.6 Sympy [A] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.62

$$\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx = \frac{cx}{e^3} - \frac{\sqrt{-\frac{1}{d^3e^7}}(ae^2+3bde-15cd^2)\log\left(-d^2e^3\sqrt{-\frac{1}{d^3e^7}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{d^3e^7}}(ae^2+3bde-15cd^2)\log\left(d^2e^3\sqrt{-\frac{1}{d^3e^7}}+x\right)}{16} + \frac{x^3(ae^3-5bde^2+9cd^2e)+x(-ade^2-3bd^2e+7cd^3)}{8d^3e^3+16d^2e^4x^2+8de^5x^4}$$

input
```
integrate(x**2*(c*x**4+b*x**2+a)/(e*x**2+d)**3,x)
```

output
```
c*x/e**3 - sqrt(-1/(d**3*e**7))*(a*e**2 + 3*b*d*e - 15*c*d**2)*log(-d**2*e
**3*sqrt(-1/(d**3*e**7)) + x)/16 + sqrt(-1/(d**3*e**7))*(a*e**2 + 3*b*d*e
- 15*c*d**2)*log(d**2*e**3*sqrt(-1/(d**3*e**7)) + x)/16 + (x**3*(a*e**3 -
5*b*d*e**2 + 9*c*d**2*e) + x*(-a*d*e**2 - 3*b*d**2*e + 7*c*d**3))/(8*d**3*
e**3 + 16*d**2*e**4*x**2 + 8*d*e**5*x**4)
```

3.290.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.290.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.94

$$\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx = \frac{cx}{e^3} - \frac{(15\,cd^2-3\,bde-ae^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\,\sqrt{de}de^3} + \frac{9\,cd^2ex^3-5\,bde^2x^3+ae^3x^3+7\,cd^3x-3\,bd^2ex-ade^2x}{8\,(ex^2+d)^2de^3}$$

input
```
integrate(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
c*x/e^3 - 1/8*(15*c*d^2 - 3*b*d*e - a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e
)*d*e^3) + 1/8*(9*c*d^2*e*x^3 - 5*b*d*e^2*x^3 + a*e^3*x^3 + 7*c*d^3*x - 3*
b*d^2*e*x - a*d*e^2*x)/((e*x^2 + d)^2*d*e^3)
```

3.290.9 Mupad [B] (verification not implemented)

Time = 7.76 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.95

$$\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx = \frac{c\,x}{e^3} - \frac{x\left(-\frac{7\,c\,d^2}{8}+\frac{3\,b\,d\,e}{8}+\frac{a\,e^2}{8}\right)-\frac{x^3\,(9\,c\,d^2\,e-5\,b\,d\,e^2+a\,e^3)}{8\,d}}{d^2\,e^3+2\,d\,e^4\,x^2+e^5\,x^4} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,(-15\,c\,d^2+3\,b\,d\,e+a\,e^2)}{8\,d^{3/2}\,e^{7/2}}$$

input
```
int((x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x)
```

output
```
(c*x)/e^3 - (x*((a*e^2)/8 - (7*c*d^2)/8 + (3*b*d*e)/8) - (x^3*(a*e^3 - 5*b
*d*e^2 + 9*c*d^2*e))/(8*d))/(d^2*e^3 + e^5*x^4 + 2*d*e^4*x^2) + (atan((e^(
1/2)*x)/d^(1/2))*(a*e^2 - 15*c*d^2 + 3*b*d*e))/(8*d^(3/2)*e^(7/2))
```

3.291 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^3}\,dx$

3.291.1 Optimal result

Integrand size = 22, antiderivative size = 115

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^3}\,dx = \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{4d(d+ex^2)^2} - \frac{(5cd^2-e(bd+3ae))x}{8d^2e^2(d+ex^2)} + \frac{(3cd^2+e(bd+3ae))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}$$

output
```
1/4*(a+d*(-b*e+c*d)/e^2)*x/d/(e*x^2+d)^2-1/8*(5*c*d^2-e*(3*a*e+b*d))*x/d^2
/e^2/(e*x^2+d)+1/8*(3*c*d^2+e*(3*a*e+b*d))*arctan(x*e^(1/2)/d^(1/2))/d^(5/
2)/e^(5/2)
```

3.291.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.96

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^3}\,dx = \frac{x(-cd^2(3d+5ex^2)+e(bd(-d+ex^2)+ae(5d+3ex^2)))}{8d^2e^2(d+ex^2)^2} + \frac{(3cd^2+e(bd+3ae))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^3,x]
```

output
```
(x*(-(c*d^2*(3*d + 5*e*x^2)) + e*(b*d*(-d + e*x^2) + a*e*(5*d + 3*e*x^2)))
)/(8*d^2*e^2*(d + e*x^2)^2) + ((3*c*d^2 + e*(b*d + 3*a*e))*ArcTan[(Sqrt[e]
*x)/Sqrt[d]])/(8*d^(5/2)*e^(5/2))
```

3.291.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1471, 25, 27, 298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^3} \, dx$$

$$\Big\downarrow \text{1471}$$

$$\frac{x(ae^2-bde+cd^2)}{4de^2(d+ex^2)^2}-\frac{\int -\frac{4cdx^2+e\left(3a-\frac{d(cd-be)}{e^2}\right)}{e(ex^2+d)^2}dx}{4d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-\frac{cd^2}{e}+4cx^2d+bd+3ae}{e(ex^2+d)^2}dx}{4d}+\frac{x(ae^2-bde+cd^2)}{4de^2(d+ex^2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{-\frac{cd^2}{e}+4cx^2d+bd+3ae}{(ex^2+d)^2}dx}{4de}+\frac{x(ae^2-bde+cd^2)}{4de^2(d+ex^2)^2}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{1}{2}\left(\frac{3ae}{d}+b+\frac{3cd}{e}\right)\int \frac{1}{ex^2+d}dx+\frac{x\left(3ae+bd-\frac{5cd^2}{e}\right)}{2d(d+ex^2)}}{4de}+\frac{x(ae^2-bde+cd^2)}{4de^2(d+ex^2)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(\frac{3ae}{d}+b+\frac{3cd}{e}\right)}{2\sqrt{d}\sqrt{e}}+\frac{x\left(3ae+bd-\frac{5cd^2}{e}\right)}{2d(d+ex^2)}}{4de}+\frac{x(ae^2-bde+cd^2)}{4de^2(d+ex^2)^2}$$

input
```
Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^3,x]
```

output
```
((c*d^2 - b*d*e + a*e^2)*x)/(4*d*e^2*(d + e*x^2)^2) + (((b*d - (5*c*d^2)/e
 + 3*a*e)*x)/(2*d*(d + e*x^2)) + ((b + (3*c*d)/e + (3*a*e)/d)*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(2*Sqrt[d]*Sqrt[e]))/(4*d*e)
```

3.291.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

3.291.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.93

method	result
default	$\frac{\frac{\left(3a\,e^{2}+bde-5c\,d^{2}\right)x^{3}}{8d^{2}e}+\frac{\left(5a\,e^{2}-bde-3c\,d^{2}\right)x}{8d\,e^{2}}}{\left(e\,x^{2}+d\right)^{2}}+\frac{\left(3a\,e^{2}+bde+3c\,d^{2}\right)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{8e^{2}d^{2}\sqrt{ed}}$
risch	$\frac{\frac{\left(3a\,e^{2}+bde-5c\,d^{2}\right)x^{3}}{8d^{2}e}+\frac{\left(5a\,e^{2}-bde-3c\,d^{2}\right)x}{8d\,e^{2}}}{\left(e\,x^{2}+d\right)^{2}}-\frac{3\ln\left(ex+\sqrt{-ed}\right)a}{16\sqrt{-ed}\,d^{2}}-\frac{\ln\left(ex+\sqrt{-ed}\right)b}{16\sqrt{-ed}\,ed}-\frac{3\ln\left(ex+\sqrt{-ed}\right)c}{16\sqrt{-ed}\,e^{2}}+\frac{3\ln\left(-ex+\sqrt{-ed}\right)a}{16\sqrt{-ed}\,d^{2}}-$

input
```
int((c*x^4+b*x^2+a)/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8*(3*a*e^2+b*d*e-5*c*d^2)/d^2/e*x^3+1/8*(5*a*e^2-b*d*e-3*c*d^2)/d/e^2*x
)/(e*x^2+d)^2+1/8*(3*a*e^2+b*d*e+3*c*d^2)/e^2/d^2/(e*d)^(1/2)*arctan(e*x/(
e*d)^(1/2))
```

3.291.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 391, normalized size of antiderivative = 3.40

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^3}\,dx$$
$$=\left[-\frac{2\,(5\,cd^3e^2-bd^2e^3-3\,ade^4)x^3+(3\,cd^4+bd^3e+3\,ad^2e^2+(3\,cd^2e^2+bde^3+3\,ae^4)x^4+2\,(3\,cd^3e+bd^2e^{2}}{16\,(d^3e^5x^4+2\,d^4e^4x^2+d^5e^3)}\right.$$
$$-\frac{(5\,cd^3e^2-bd^2e^3-3\,ade^4)x^3-(3\,cd^4+bd^3e+3\,ad^2e^2+(3\,cd^2e^2+bde^3+3\,ae^4)x^4+2\,(3\,cd^3e+bd^2e^{2}}{8\,(d^3e^5x^4+2\,d^4e^4x^2+d^5e^3)}$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
[-1/16*(2*(5*c*d^3*e^2 - b*d^2*e^3 - 3*a*d*e^4)*x^3 + (3*c*d^4 + b*d^3*e +
 3*a*d^2*e^2 + (3*c*d^2*e^2 + b*d*e^3 + 3*a*e^4)*x^4 + 2*(3*c*d^3*e + b*d^
2*e^2 + 3*a*d*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2
 + d)) + 2*(3*c*d^4*e + b*d^3*e^2 - 5*a*d^2*e^3)*x)/(d^3*e^5*x^4 + 2*d^4*e
^4*x^2 + d^5*e^3), -1/8*((5*c*d^3*e^2 - b*d^2*e^3 - 3*a*d*e^4)*x^3 - (3*c*
d^4 + b*d^3*e + 3*a*d^2*e^2 + (3*c*d^2*e^2 + b*d*e^3 + 3*a*e^4)*x^4 + 2*(3
*c*d^3*e + b*d^2*e^2 + 3*a*d*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (
3*c*d^4*e + b*d^3*e^2 - 5*a*d^2*e^3)*x)/(d^3*e^5*x^4 + 2*d^4*e^4*x^2 + d^5
*e^3)]
```

3.291.6 Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.70

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^3}\,dx = -\frac{\sqrt{-\frac{1}{d^5e^5}}\cdot(3ae^2+bde+3cd^2)\log\left(-d^3e^2\sqrt{-\frac{1}{d^5e^5}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{d^5e^5}}\cdot(3ae^2+bde+3cd^2)\log\left(d^3e^2\sqrt{-\frac{1}{d^5e^5}}+x\right)}{16} + \frac{x^3\cdot(3ae^3+bde^2-5cd^2e)+x(5ade^2-bd^2e-3cd^3)}{8d^4e^2+16d^3e^3x^2+8d^2e^4x^4}$$

input
```
integrate((c*x**4+b*x**2+a)/(e*x**2+d)**3,x)
```

output
```
-sqrt(-1/(d**5*e**5))*(3*a*e**2 + b*d*e + 3*c*d**2)*log(-d**3*e**2*sqrt(-1
/(d**5*e**5)) + x)/16 + sqrt(-1/(d**5*e**5))*(3*a*e**2 + b*d*e + 3*c*d**2)
*log(d**3*e**2*sqrt(-1/(d**5*e**5)) + x)/16 + (x**3*(3*a*e**3 + b*d*e**2 -
 5*c*d**2*e) + x*(5*a*d*e**2 - b*d**2*e - 3*c*d**3))/(8*d**4*e**2 + 16*d**
3*e**3*x**2 + 8*d**2*e**4*x**4)
```

3.291.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.291.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.95

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^3}\,dx = \frac{(3\,cd^2+bde+3\,ae^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\,\sqrt{de}d^2e^2} - \frac{5\,cd^2ex^3-bde^2x^3-3\,ae^3x^3+3\,cd^3x+bd^2ex-5\,ade^2x}{8\,(ex^2+d)^2d^2e^2}$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
1/8*(3*c*d^2 + b*d*e + 3*a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^2*e^2)
- 1/8*(5*c*d^2*e*x^3 - b*d*e^2*x^3 - 3*a*e^3*x^3 + 3*c*d^3*x + b*d^2*e*x -
 5*a*d*e^2*x)/((e*x^2 + d)^2*d^2*e^2)
```

3.291.9 Mupad [B] (verification not implemented)

Time = 7.71 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^3}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(3\,c\,d^2+b\,d\,e+3\,a\,e^2\right)}{8\,d^{5/2}\,e^{5/2}} - \frac{\frac{x\,\left(3\,c\,d^2+b\,d\,e-5\,a\,e^2\right)}{8\,d\,e^2}-\frac{x^3\,\left(-5\,c\,d^2+b\,d\,e+3\,a\,e^2\right)}{8\,d^2\,e}}{d^2+2\,d\,e\,x^2+e^2\,x^4}$$

input
```
int((a + b*x^2 + c*x^4)/(d + e*x^2)^3,x)
```

output
```
(atan((e^(1/2)*x)/d^(1/2))*(3*a*e^2 + 3*c*d^2 + b*d*e))/(8*d^(5/2)*e^(5/2)
) - ((x*(3*c*d^2 - 5*a*e^2 + b*d*e))/(8*d*e^2) - (x^3*(3*a*e^2 - 5*c*d^2 +
 b*d*e))/(8*d^2*e))/(d^2 + e^2*x^4 + 2*d*e*x^2)
```

3.292 $\int \frac{a+bx^2+cx^4}{x^2(d+ex^2)^3}\,dx$

3.292.1 Optimal result

Integrand size = 25, antiderivative size = 127

$$\int \frac{a+bx^2+cx^4}{x^2(d+ex^2)^3}\,dx = -\frac{a}{d^3x} - \frac{(cd^2-bde+ae^2)\,x}{4d^2e\,(d+ex^2)^2} + \frac{(cd^2+e(3bd-7ae))\,x}{8d^3e\,(d+ex^2)} + \frac{(cd^2+3e(bd-5ae))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{7/2}e^{3/2}}$$

output
```
-a/d^3/x-1/4*(a*e^2-b*d*e+c*d^2)*x/d^2/e/(e*x^2+d)^2+1/8*(c*d^2+e*(-7*a*e+
3*b*d))*x/d^3/e/(e*x^2+d)+1/8*(c*d^2+3*e*(-5*a*e+b*d))*arctan(x*e^(1/2)/d^
(1/2))/d^(7/2)/e^(3/2)
```

3.292.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.98

$$\int \frac{a+bx^2+cx^4}{x^2(d+ex^2)^3}\,dx = \frac{\frac{\sqrt{d}(-ae(8d^2+25dex^2+15e^2x^4)+dx^2(cd(-d+ex^2)+be(5d+3ex^2)))}{ex(d+ex^2)^2} + \frac{(cd^2+3e(bd-5ae))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}}}{8d^{7/2}}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^3),x]
```

output
```
((Sqrt[d]*(-(a*e*(8*d^2 + 25*d*e*x^2 + 15*e^2*x^4)) + d*x^2*(c*d*(-d + e*x
^2) + b*e*(5*d + 3*e*x^2))))/(e*x*(d + e*x^2)^2) + ((c*d^2 + 3*e*(b*d - 5*
a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/e^(3/2))/(8*d^(7/2))
```

3.292.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1582, 25, 27, 361, 25, 359, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{x^2 (d+ex^2)^3} \, dx$$

$$\Big\downarrow \text{1582}$$

$$-\frac{\int -\frac{e\left(\left(cd^2+3e(bd-ae)\right)x^2+4ade\right)}{x^2(ex^2+d)^2}dx}{4d^2e^2}-\frac{x\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)}{4\left(d+ex^2\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{e\left(\left(cd^2+3e(bd-ae)\right)x^2+4ade\right)}{x^2(ex^2+d)^2}dx}{4d^2e^2}-\frac{x\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)}{4\left(d+ex^2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(cd^2+3e(bd-ae)\right)x^2+4ade}{x^2(ex^2+d)^2}dx}{4d^2e}-\frac{x\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)}{4\left(d+ex^2\right)^2}$$

$$\Big\downarrow \text{361}$$

$$\frac{\frac{x\left(e(3bd-7ae)+cd^2\right)}{2d(d+ex^2)}-\frac{1}{2}\int -\frac{\left(cd+e\left(3b-\frac{7ae}{d}\right)\right)x^2+8ae}{x^2(ex^2+d)}dx}{4d^2e}-\frac{x\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)}{4\left(d+ex^2\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{2}\int \frac{\left(cd+e\left(3b-\frac{7ae}{d}\right)\right)x^2+8ae}{x^2(ex^2+d)}dx+\frac{x\left(e(3bd-7ae)+cd^2\right)}{2d(d+ex^2)}}{4d^2e}-\frac{x\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)}{4\left(d+ex^2\right)^2}$$

$$\Big\downarrow \text{359}$$

$$\frac{\frac{1}{2}\left(\frac{(3e(bd-5ae)+cd^2)\int\frac{1}{ex^2+d}dx}{d}-\frac{8ae}{dx}\right)+\frac{x(e(3bd-7ae)+cd^2)}{2d(d+ex^2)}}{4d^2e}-\frac{x\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)}{4\left(d+ex^2\right)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(3e(bd-5ae)+cd^2)}{d^{3/2}\sqrt{e}}-\frac{8ae}{dx}\right)+\frac{x(e(3bd-7ae)+cd^2)}{2d(d+ex^2)}}{4d^2e}-\frac{x\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)}{4\left(d+ex^2\right)^2}$$

input
```
Int[(a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^3),x]
```

output
```
-1/4*((c/e - (b*d - a*e)/d^2)*x)/(d + e*x^2)^2 + (((c*d^2 + e*(3*b*d - 7*a
*e))*x)/(2*d*(d + e*x^2)) + ((-8*a*e)/(d*x) + ((c*d^2 + 3*e*(b*d - 5*a*e))
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*Sqrt[e]))/2)/(4*d^2*e)
```

3.292.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

3.292.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.87

method	result
default	$-\frac{a}{d^{3}x}-\frac{\frac{\left(\frac{7}{8}a\,e^{2}-\frac{3}{8}bde-\frac{1}{8}c\,d^{2}\right)x^{3}+\frac{d\left(9a\,e^{2}-5bde+c\,d^{2}\right)x}{8e}}{\left(e\,x^{2}+d\right)^{2}}+\frac{\left(15a\,e^{2}-3bde-c\,d^{2}\right)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{8e\sqrt{ed}}}{d^{3}}$
risch	$\frac{-\frac{\left(15a\,e^{2}-3bde-c\,d^{2}\right)x^{4}}{8d^{3}}-\frac{\left(25a\,e^{2}-5bde+c\,d^{2}\right)x^{2}}{8d^{2}e}-\frac{a}{d}}{x\left(e\,x^{2}+d\right)^{2}}-\frac{15e\ln\left(-\sqrt{-ed}\,x-d\right)a}{16\sqrt{-ed}\,d^{3}}+\frac{3\ln\left(-\sqrt{-ed}\,x-d\right)b}{16\sqrt{-ed}\,d^{2}}+\frac{\ln\left(-\sqrt{-ed}\,x-d\right)c}{16\sqrt{-ed}\,ed}+15e$

input
```
int((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-a/d^3/x-1/d^3*(((7/8*a*e^2-3/8*b*d*e-1/8*c*d^2)*x^3+1/8*d*(9*a*e^2-5*b*d*
e+c*d^2)/e*x)/(e*x^2+d)^2+1/8*(15*a*e^2-3*b*d*e-c*d^2)/e/(e*d)^(1/2)*arcta
n(e*x/(e*d)^(1/2)))
```

3.292.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 421, normalized size of antiderivative = 3.31

$$\int \frac{a+bx^2+cx^4}{x^2(d+ex^2)^3}\,dx$$
$$= \left[-\frac{16\,ad^3e^2-2\,(cd^3e^2+3\,bd^2e^3-15\,ade^4)x^4+2\,(cd^4e-5\,bd^3e^2+25\,ad^2e^3)x^2-((cd^2e^2+3\,bde^3-15\,a}{16\,(d^4e^4x^5+2\,d^5e^3}\right.$$
$$-\frac{8\,ad^3e^2-(cd^3e^2+3\,bd^2e^3-15\,ade^4)x^4+(cd^4e-5\,bd^3e^2+25\,ad^2e^3)x^2-((cd^2e^2+3\,bde^3-15\,ae^4)x^5}{8\,(d^4e^4x^5+2\,d^5e^3x^3+d^6}$$

input
```
integrate((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
[-1/16*(16*a*d^3*e^2 - 2*(c*d^3*e^2 + 3*b*d^2*e^3 - 15*a*d*e^4)*x^4 + 2*(c
*d^4*e - 5*b*d^3*e^2 + 25*a*d^2*e^3)*x^2 - ((c*d^2*e^2 + 3*b*d*e^3 - 15*a*
e^4)*x^5 + 2*(c*d^3*e + 3*b*d^2*e^2 - 15*a*d*e^3)*x^3 + (c*d^4 + 3*b*d^3*e
 - 15*a*d^2*e^2)*x)*sqrt(-d*e)*log((e*x^2 + 2*sqrt(-d*e)*x - d)/(e*x^2 + d
)))/(d^4*e^4*x^5 + 2*d^5*e^3*x^3 + d^6*e^2*x), -1/8*(8*a*d^3*e^2 - (c*d^3*
e^2 + 3*b*d^2*e^3 - 15*a*d*e^4)*x^4 + (c*d^4*e - 5*b*d^3*e^2 + 25*a*d^2*e^
3)*x^2 - ((c*d^2*e^2 + 3*b*d*e^3 - 15*a*e^4)*x^5 + 2*(c*d^3*e + 3*b*d^2*e^
2 - 15*a*d*e^3)*x^3 + (c*d^4 + 3*b*d^3*e - 15*a*d^2*e^2)*x)*sqrt(d*e)*arct
an(sqrt(d*e)*x/d))/(d^4*e^4*x^5 + 2*d^5*e^3*x^3 + d^6*e^2*x)]
```

3.292.6 Sympy [A] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.59

$$\int \frac{a+bx^2+cx^4}{x^2(d+ex^2)^3}\,dx = \frac{\sqrt{-\frac{1}{d^7e^3}}\cdot(15ae^2-3bde-cd^2)\log\left(-d^4e\sqrt{-\frac{1}{d^7e^3}}+x\right)}{16} - \frac{\sqrt{-\frac{1}{d^7e^3}}\cdot(15ae^2-3bde-cd^2)\log\left(d^4e\sqrt{-\frac{1}{d^7e^3}}+x\right)}{16} + \frac{-8ad^2e+x^4(-15ae^3+3bde^2+cd^2e)+x^2(-25ade^2+5bd^2e-cd^3)}{8d^5ex+16d^4e^2x^3+8d^3e^3x^5}$$

input
```
integrate((c*x**4+b*x**2+a)/x**2/(e*x**2+d)**3,x)
```

output
```
sqrt(-1/(d**7*e**3))*(15*a*e**2 - 3*b*d*e - c*d**2)*log(-d**4*e*sqrt(-1/(d
**7*e**3)) + x)/16 - sqrt(-1/(d**7*e**3))*(15*a*e**2 - 3*b*d*e - c*d**2)*l
og(d**4*e*sqrt(-1/(d**7*e**3)) + x)/16 + (-8*a*d**2*e + x**4*(-15*a*e**3 +
 3*b*d*e**2 + c*d**2*e) + x**2*(-25*a*d*e**2 + 5*b*d**2*e - c*d**3))/(8*d*
*5*e*x + 16*d**4*e**2*x**3 + 8*d**3*e**3*x**5)
```

3.292.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^2+cx^4}{x^2\left(d+ex^2\right)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.292.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.93

$$\int \frac{a+bx^2+cx^4}{x^2\left(d+ex^2\right)^3}\,dx = \frac{\left(cd^2+3\,bde-15\,ae^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\,\sqrt{de}d^3e} - \frac{a}{d^3x} + \frac{cd^2ex^3+3\,bde^2x^3-7\,ae^3x^3-cd^3x+5\,bd^2ex-9\,ade^2x}{8\,\left(ex^2+d\right)^2d^3e}$$

input
```
integrate((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
1/8*(c*d^2 + 3*b*d*e - 15*a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^3*e) -
 a/(d^3*x) + 1/8*(c*d^2*e*x^3 + 3*b*d*e^2*x^3 - 7*a*e^3*x^3 - c*d^3*x + 5*
b*d^2*e*x - 9*a*d*e^2*x)/((e*x^2 + d)^2*d^3*e)
```

3.292.9 Mupad [B] (verification not implemented)

Time = 7.70 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.93

$$\int \frac{a+bx^2+cx^4}{x^2\left(d+ex^2\right)^3}\,dx=\frac{\operatorname{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(c\,d^2+3\,b\,d\,e-15\,a\,e^2\right)}{8\,d^{7/2}\,e^{3/2}}-\frac{\frac{a}{d}-\frac{x^4\,\left(c\,d^2+3\,b\,d\,e-15\,a\,e^2\right)}{8\,d^3}+\frac{x^2\,\left(c\,d^2-5\,b\,d\,e+25\,a\,e^2\right)}{8\,d^2\,e}}{d^2\,x+2\,d\,e\,x^3+e^2\,x^5}$$

input
```
int((a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^3),x)
```

output
```
(atan((e^(1/2)*x)/d^(1/2))*(c*d^2 - 15*a*e^2 + 3*b*d*e))/(8*d^(7/2)*e^(3/2
)) - (a/d - (x^4*(c*d^2 - 15*a*e^2 + 3*b*d*e))/(8*d^3) + (x^2*(25*a*e^2 +
c*d^2 - 5*b*d*e))/(8*d^2*e))/(d^2*x + e^2*x^5 + 2*d*e*x^3)
```

3.293 $\int \frac{a+bx^2+cx^4}{x^4(d+ex^2)^3}\,dx$

3.293.1 Optimal result

Integrand size = 25, antiderivative size = 142

$$\int \frac{a+bx^2+cx^4}{x^4(d+ex^2)^3}\,dx = -\frac{a}{3d^3x^3}-\frac{bd-3ae}{d^4x}+\frac{(cd^2-bde+ae^2)\,x}{4d^3(d+ex^2)^2}+\frac{(3cd^2-e(7bd-11ae))\,x}{8d^4(d+ex^2)}+\frac{(3cd^2-15bde+35ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{9/2}\sqrt{e}}$$

output
```
-1/3*a/d^3/x^3+(3*a*e-b*d)/d^4/x+1/4*(a*e^2-b*d*e+c*d^2)*x/d^3/(e*x^2+d)^2
+1/8*(3*c*d^2-e*(-11*a*e+7*b*d))*x/d^4/(e*x^2+d)+1/8*(35*a*e^2-15*b*d*e+3*
c*d^2)*arctan(x*e^(1/2)/d^(1/2))/d^(9/2)/e^(1/2)
```

3.293.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.99

$$\int \frac{a+bx^2+cx^4}{x^4(d+ex^2)^3}\,dx = -\frac{a}{3d^3x^3}+\frac{-bd+3ae}{d^4x}+\frac{(cd^2-bde+ae^2)\,x}{4d^3(d+ex^2)^2}+\frac{(3cd^2-7bde+11ae^2)\,x}{8d^4(d+ex^2)}+\frac{(3cd^2-15bde+35ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{9/2}\sqrt{e}}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^3),x]
```

output
```
-1/3*a/(d^3*x^3) + (-(b*d) + 3*a*e)/(d^4*x) + ((c*d^2 - b*d*e + a*e^2)*x)/
(4*d^3*(d + e*x^2)^2) + ((3*c*d^2 - 7*b*d*e + 11*a*e^2)*x)/(8*d^4*(d + e*x
^2)) + ((3*c*d^2 - 15*b*d*e + 35*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^
(9/2)*Sqrt[e])
```

3.293.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.20, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1582, 1582, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{x^4 (d+ex^2)^3} \, dx$$

$$\Big\downarrow \text{ 1582}$$

$$\frac{\int \frac{3e^2(cd^2-bed+ae^2)x^4+4de^2(bd-ae)x^2+4ad^2e^2}{x^4(ex^2+d)^2}dx}{4d^3e^2}+\frac{x(ae^2-bde+cd^2)}{4d^3(d+ex^2)^2}$$

$$\Big\downarrow \text{ 1582}$$

$$\frac{\frac{\int \frac{8ad^4e^4+d^2\left(3cd^2-e(7bd-11ae)\right)x^4e^4+8d^3(bd-2ae)x^2e^4}{x^4\left(ex^2+d\right)}dx}{2d^3e^2}+\frac{e^2x(3cd^2-e(7bd-11ae))}{2d(d+ex^2)}}{4d^3e^2}+\frac{x(ae^2-bde+cd^2)}{4d^3(d+ex^2)^2}$$

$$\Big\downarrow \text{ 1584}$$

$$\frac{\frac{\int \left(\frac{d^2\left(3cd^2-15bed+35ae^2\right)e^4}{ex^2+d}+\frac{8d^2(bd-3ae)e^4}{x^2}+\frac{8ad^3e^4}{x^4}\right)dx}{2d^3e^2}+\frac{e^2x(3cd^2-e(7bd-11ae))}{2d(d+ex^2)}}{4d^3e^2}+\frac{x(ae^2-bde+cd^2)}{4d^3(d+ex^2)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{d^{3/2}e^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(35ae^2-15bde+3cd^2)-\frac{8d^2e^4(bd-3ae)}{x}-\frac{8ad^3e^4}{3x^3}}{2d^3e^2}+\frac{e^2x(3cd^2-e(7bd-11ae))}{2d(d+ex^2)}}{4d^3e^2}+\frac{x(ae^2-bde+cd^2)}{4d^3(d+ex^2)^2}$$

input
```
Int[(a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^3),x]
```

output
```
((c*d^2 - b*d*e + a*e^2)*x)/(4*d^3*(d + e*x^2)^2) + ((e^2*(3*c*d^2 - e*(7*
b*d - 11*a*e))*x)/(2*d*(d + e*x^2)) + ((-8*a*d^3*e^4)/(3*x^3) - (8*d^2*e^4
*(b*d - 3*a*e))/x + d^(3/2)*e^(7/2)*(3*c*d^2 - 15*b*d*e + 35*a*e^2)*ArcTan
[(Sqrt[e]*x)/Sqrt[d]])/(2*d^3*e^2))/(4*d^3*e^2)
```

3.293.3.1 Defintions of rubi rules used

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.293.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.87

method	result
default	$-\frac{a}{3d^{3}x^{3}}-\frac{-3ae+bd}{d^{4}x}+\frac{\frac{\left(\frac{11}{8}a\,e^{3}-\frac{7}{8}d\,e^{2}b+\frac{3}{8}c\,d^{2}e\right)x^{3}+\frac{d\left(13a\,e^{2}-9bde+5c\,d^{2}\right)x}{8}}{\left(e\,x^{2}+d\right)^{2}}+\frac{\left(35a\,e^{2}-15bde+3c\,d^{2}\right)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{8\sqrt{ed}}}{d^{4}}$
risch	$\frac{\frac{e\left(35a\,e^{2}-15bde+3c\,d^{2}\right)x^{6}}{8d^{4}}+\frac{5\left(35a\,e^{2}-15bde+3c\,d^{2}\right)x^{4}}{24d^{3}}+\frac{(7ae-3bd)x^{2}}{3d^{2}}-\frac{a}{3d}}{x^{3}\left(e\,x^{2}+d\right)^{2}}-\frac{35\ln\left(-\sqrt{-ed}\,x+d\right)a\,e^{2}}{16\sqrt{-ed}\,d^{4}}+\frac{15\ln\left(-\sqrt{-ed}\,x+d\right)be}{16\sqrt{-ed}\,d^{3}}-$ 3 ln

input
```
int((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/d^3/x^3-(-3*a*e+b*d)/d^4/x+1/d^4*(((11/8*a*e^3-7/8*d*e^2*b+3/8*c*d^
2*e)*x^3+1/8*d*(13*a*e^2-9*b*d*e+5*c*d^2)*x)/(e*x^2+d)^2+1/8*(35*a*e^2-15*
b*d*e+3*c*d^2)/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2)))
```

3.293.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 476, normalized size of antiderivative = 3.35

$$\int \frac{a+bx^2+cx^4}{x^4(d+ex^2)^3}\,dx$$
$$= \left[\frac{6\,(3\,cd^3e^2 - 15\,bd^2e^3 + 35\,ade^4)x^6 - 16\,ad^4e + 10\,(3\,cd^4e - 15\,bd^3e^2 + 35\,ad^2e^3)x^4 - 16\,(3\,bd^4e - 7\,ad^3e^{2}}{} \right.$$

input
```
integrate((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
[1/48*(6*(3*c*d^3*e^2 - 15*b*d^2*e^3 + 35*a*d*e^4)*x^6 - 16*a*d^4*e + 10*(
3*c*d^4*e - 15*b*d^3*e^2 + 35*a*d^2*e^3)*x^4 - 16*(3*b*d^4*e - 7*a*d^3*e^2
)*x^2 - 3*((3*c*d^2*e^2 - 15*b*d*e^3 + 35*a*e^4)*x^7 + 2*(3*c*d^3*e - 15*b
*d^2*e^2 + 35*a*d*e^3)*x^5 + (3*c*d^4 - 15*b*d^3*e + 35*a*d^2*e^2)*x^3)*sq
rt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)))/(d^5*e^3*x^7 + 2*d
^6*e^2*x^5 + d^7*e*x^3), 1/24*(3*(3*c*d^3*e^2 - 15*b*d^2*e^3 + 35*a*d*e^4)
*x^6 - 8*a*d^4*e + 5*(3*c*d^4*e - 15*b*d^3*e^2 + 35*a*d^2*e^3)*x^4 - 8*(3*
b*d^4*e - 7*a*d^3*e^2)*x^2 + 3*((3*c*d^2*e^2 - 15*b*d*e^3 + 35*a*e^4)*x^7
+ 2*(3*c*d^3*e - 15*b*d^2*e^2 + 35*a*d*e^3)*x^5 + (3*c*d^4 - 15*b*d^3*e +
35*a*d^2*e^2)*x^3)*sqrt(d*e)*arctan(sqrt(d*e)*x/d))/(d^5*e^3*x^7 + 2*d^6*e
^2*x^5 + d^7*e*x^3)]
```

3.293.6 Sympy [A] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.51

$$\int \frac{a+bx^2+cx^4}{x^4(d+ex^2)^3}\,dx = -\frac{\sqrt{-\frac{1}{d^9e}}(35ae^2-15bde+3cd^2)\log\left(-d^5\sqrt{-\frac{1}{d^9e}}+x\right)}{16}$$
$$+\frac{\sqrt{-\frac{1}{d^9e}}(35ae^2-15bde+3cd^2)\log\left(d^5\sqrt{-\frac{1}{d^9e}}+x\right)}{16}$$
$$+\frac{-8ad^3+x^6\cdot(105ae^3-45bde^2+9cd^2e)+x^4\cdot(175ade^2-75bd^2e+15cd^3)+x^2\cdot(56ad^2e-24bd^3)}{24d^6x^3+48d^5ex^5+24d^4e^2x^7}$$

input
```
integrate((c*x**4+b*x**2+a)/x**4/(e*x**2+d)**3,x)
```

output
```
-sqrt(-1/(d**9*e))*(35*a*e**2 - 15*b*d*e + 3*c*d**2)*log(-d**5*sqrt(-1/(d*
*9*e)) + x)/16 + sqrt(-1/(d**9*e))*(35*a*e**2 - 15*b*d*e + 3*c*d**2)*log(d
**5*sqrt(-1/(d**9*e)) + x)/16 + (-8*a*d**3 + x**6*(105*a*e**3 - 45*b*d*e**
2 + 9*c*d**2*e) + x**4*(175*a*d*e**2 - 75*b*d**2*e + 15*c*d**3) + x**2*(56
*a*d**2*e - 24*b*d**3))/(24*d**6*x**3 + 48*d**5*e*x**5 + 24*d**4*e**2*x**7
)
```

3.293.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^2+cx^4}{x^4\left(d+ex^2\right)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.293.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.92

$$\int \frac{a+bx^2+cx^4}{x^4\left(d+ex^2\right)^3}\,dx = \frac{\left(3\,cd^2-15\,bde+35\,ae^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\,\sqrt{de}d^4} + \frac{3\,cd^2ex^3-7\,bde^2x^3+11\,ae^3x^3+5\,cd^3x-9\,bd^2ex+13\,ade^2x}{8\left(ex^2+d\right)^2d^4} - \frac{3\,bdx^2-9\,aex^2+ad}{3\,d^4x^3}$$

input
```
integrate((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
1/8*(3*c*d^2 - 15*b*d*e + 35*a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^4)
+ 1/8*(3*c*d^2*e*x^3 - 7*b*d*e^2*x^3 + 11*a*e^3*x^3 + 5*c*d^3*x - 9*b*d^2*
e*x + 13*a*d*e^2*x)/((e*x^2 + d)^2*d^4) - 1/3*(3*b*d*x^2 - 9*a*e*x^2 + a*d
)/(d^4*x^3)
```

3.293.9 Mupad [B] (verification not implemented)

Time = 7.74 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.97

$$\int \frac{a+bx^2+cx^4}{x^4(d+ex^2)^3}\,dx = \frac{\frac{x^2\,(7\,a\,e-3\,b\,d)}{3\,d^2}-\frac{a}{3\,d}+\frac{5\,x^4\,(3\,c\,d^2-15\,b\,d\,e+35\,a\,e^2)}{24\,d^3}+\frac{e\,x^6\,(3\,c\,d^2-15\,b\,d\,e+35\,a\,e^2)}{8\,d^4}}{d^2\,x^3+2\,d\,e\,x^5+e^2\,x^7} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,(3\,c\,d^2-15\,b\,d\,e+35\,a\,e^2)}{8\,d^{9/2}\,\sqrt{e}}$$

input
```
int((a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^3),x)
```

output
```
((x^2*(7*a*e - 3*b*d))/(3*d^2) - a/(3*d) + (5*x^4*(35*a*e^2 + 3*c*d^2 - 15
*b*d*e))/(24*d^3) + (e*x^6*(35*a*e^2 + 3*c*d^2 - 15*b*d*e))/(8*d^4))/(d^2*
x^3 + e^2*x^7 + 2*d*e*x^5) + (atan((e^(1/2)*x)/d^(1/2))*(35*a*e^2 + 3*c*d^
2 - 15*b*d*e))/(8*d^(9/2)*e^(1/2))
```

3.294 $\int \frac{a+bx^2+cx^4}{x^6\left(d+ex^2\right)^3}\,dx$

3.294.1 Optimal result

Integrand size = 25, antiderivative size = 171

$$\int \frac{a+bx^2+cx^4}{x^6\left(d+ex^2\right)^3}\,dx=-\frac{a}{5d^3x^5}-\frac{bd-3ae}{3d^4x^3}-\frac{cd^2-3bde+6ae^2}{d^5x}-\frac{e(cd^2-bde+ae^2)\,x}{4d^4\left(d+ex^2\right)^2}-\frac{e(7cd^2-e(11bd-15ae))\,x}{8d^5\left(d+ex^2\right)}-\frac{\sqrt{e}(15cd^2-35bde+63ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{11/2}}$$

output
```
-1/5*a/d^3/x^5+1/3*(3*a*e-b*d)/d^4/x^3+(-6*a*e^2+3*b*d*e-c*d^2)/d^5/x-1/4*
e*(a*e^2-b*d*e+c*d^2)*x/d^4/(e*x^2+d)^2-1/8*e*(7*c*d^2-e*(-15*a*e+11*b*d))
*x/d^5/(e*x^2+d)-1/8*(63*a*e^2-35*b*d*e+15*c*d^2)*arctan(x*e^(1/2)/d^(1/2)
)*e^(1/2)/d^(11/2)
```

3.294.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.01

$$\int \frac{a+bx^2+cx^4}{x^6\left(d+ex^2\right)^3}\,dx=-\frac{a}{5d^3x^5}+\frac{-bd+3ae}{3d^4x^3}+\frac{-cd^2+3bde-6ae^2}{d^5x}-\frac{e(cd^2-bde+ae^2)\,x}{4d^4\left(d+ex^2\right)^2}-\frac{(7cd^2e-11bde^2+15ae^3)\,x}{8d^5\left(d+ex^2\right)}-\frac{\sqrt{e}(15cd^2-35bde+63ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{11/2}}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^3),x]
```

output
```
-1/5*a/(d^3*x^5) + (-(b*d) + 3*a*e)/(3*d^4*x^3) + (-(c*d^2) + 3*b*d*e - 6*
a*e^2)/(d^5*x) - (e*(c*d^2 - b*d*e + a*e^2)*x)/(4*d^4*(d + e*x^2)^2) - ((7
*c*d^2*e - 11*b*d*e^2 + 15*a*e^3)*x)/(8*d^5*(d + e*x^2)) - (Sqrt[e]*(15*c*
d^2 - 35*b*d*e + 63*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(11/2))
```

3.294.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1582, 25, 2336, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{x^6 \left(d+ex^2\right)^3} \, dx$$

$$\Big\downarrow \; 1582$$

$$-\frac{\int -\frac{-3e^3\left(cd^2-bed+ae^2\right)x^6+4de^2\left(cd^2-bed+ae^2\right)x^4+4d^2e^2(bd-ae)x^2+4ad^3e^2}{x^6\left(ex^2+d\right)^2}dx}{4d^4e^2}-\frac{ex\left(ae^2-bde+cd^2\right)}{4d^4\left(d+ex^2\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{-3e^3\left(cd^2-bed+ae^2\right)x^6+4de^2\left(cd^2-bed+ae^2\right)x^4+4d^2e^2(bd-ae)x^2+4ad^3e^2}{x^6\left(ex^2+d\right)^2}dx}{4d^4e^2}-\frac{ex\left(ae^2-bde+cd^2\right)}{4d^4\left(d+ex^2\right)^2}$$

$$\Big\downarrow \; 2336$$

$$\frac{-\frac{\int -\frac{-e^3\left(7cd^2-e(11bd-15ae)\right)x^6+8de^2\left(cd^2-e(2bd-3ae)\right)x^4+8d^2e^2(bd-2ae)x^2+8ad^3e^2}{x^6\left(ex^2+d\right)}dx}{2d}-\frac{e^3x\left(7cd^2-e(11bd-15ae)\right)}{2d\left(d+ex^2\right)}}{4d^4e^2}-\frac{ex\left(ae^2-bde+cd^2\right)}{4d^4\left(d+ex^2\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{-e^3\left(7cd^2-e(11bd-15ae)\right)x^6+8de^2\left(cd^2-e(2bd-3ae)\right)x^4+8d^2e^2(bd-2ae)x^2+8ad^3e^2}{x^6\left(ex^2+d\right)}dx}{2d}-\frac{e^3x\left(7cd^2-e(11bd-15ae)\right)}{2d\left(d+ex^2\right)}}{4d^4e^2}-\frac{ex\left(ae^2-bde+cd^2\right)}{4d^4\left(d+ex^2\right)^2}$$

$$\Big\downarrow \; 2333$$

$$\frac{\frac{\int \left(-\frac{(15cd^2-35bed+63ae^2)e^3}{ex^2+d}+\frac{8(cd^2-3bed+6ae^2)e^2}{x^2}+\frac{8d(bd-3ae)e^2}{x^4}+\frac{8ad^2e^2}{x^6}\right)dx}{2d}-\frac{e^3x(7cd^2-e(11bd-15ae))}{2d(d+ex^2)}}{4d^4e^2}-\frac{ex\left(ae^2-bde+cd^2\right)}{4d^4\left(d+ex^2\right)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{-\frac{e^{5/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(63ae^2-35bde+15cd^2)}{\sqrt{d}}-\frac{8e^2(6ae^2-3bde+cd^2)}{x}-\frac{8de^2(bd-3ae)}{3x^3}-\frac{8ad^2e^2}{5x^5}}{2d}-\frac{e^3x(7cd^2-e(11bd-15ae))}{2d(d+ex^2)}}{4d^4e^2}-\frac{ex\left(ae^2-bde+cd^2\right)}{4d^4\left(d+ex^2\right)^2}$$

input
```
Int[(a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^3),x]
```

output
```
-1/4*(e*(c*d^2 - b*d*e + a*e^2)*x)/(d^4*(d + e*x^2)^2) + (-1/2*(e^3*(7*c*d
^2 - e*(11*b*d - 15*a*e))*x)/(d*(d + e*x^2)) + ((-8*a*d^2*e^2)/(5*x^5) - (
8*d*e^2*(b*d - 3*a*e))/(3*x^3) - (8*e^2*(c*d^2 - 3*b*d*e + 6*a*e^2))/x - (
e^(5/2)*(15*c*d^2 - 35*b*d*e + 63*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt
[d])/(2*d))/(4*d^4*e^2)
```

3.294.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

3.294.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.88

method	result
default	$-\frac{a}{5d^3x^5}-\frac{-3ae+bd}{3d^4x^3}-\frac{6a\,e^2-3bde+c\,d^2}{d^5x}-\frac{e\left(\frac{\left(\frac{15}{8}a\,e^3-\frac{11}{8}d\,e^2b+\frac{7}{8}c\,d^2e\right)x^3+\frac{d\left(17a\,e^2-13bde+9c\,d^2\right)x}{8}}{\left(e\,x^2+d\right)^2}+\frac{\left(63a\,e^2-35bde+15c\,d^2\right)\mathrm{arc}}{8\sqrt{ed}}\right.}{d^5}$
risch	$\frac{-\frac{e^2\left(63a\,e^2-35bde+15c\,d^2\right)x^8}{8d^5}-\frac{5e\left(63a\,e^2-35bde+15c\,d^2\right)x^6}{24d^4}-\frac{\left(63a\,e^2-35bde+15c\,d^2\right)x^4}{15d^3}+\frac{(9ae-5bd)x^2}{15d^2}-\frac{a}{5d}}{x^5(e\,x^2+d)^2}+\frac{63\sqrt{-ed}\,\ln\left(-ex+\sqrt{-ed}\right)}{16d^6}$

input
```
int((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a/d^3/x^5-1/3*(-3*a*e+b*d)/d^4/x^3-(6*a*e^2-3*b*d*e+c*d^2)/d^5/x-1/d^
5*e*(((15/8*a*e^3-11/8*d*e^2*b+7/8*c*d^2*e)*x^3+1/8*d*(17*a*e^2-13*b*d*e+9
*c*d^2)*x)/(e*x^2+d)^2+1/8*(63*a*e^2-35*b*d*e+15*c*d^2)/(e*d)^(1/2)*arctan
(e*x/(e*d)^(1/2)))
```

3.294.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 514, normalized size of antiderivative = 3.01

$$\int \frac{a+bx^2+cx^4}{x^6 (d+ex^2)^3}\,dx$$

$$= \left[-\frac{30\,(15\,cd^2e^2-35\,bde^3+63\,ae^4)x^8+50\,(15\,cd^3e-35\,bd^2e^2+63\,ade^3)x^6+48\,ad^4+16\,(15\,cd^4-35\,bd\right.$$

$$-\frac{15\,(15\,cd^2e^2-35\,bde^3+63\,ae^4)x^8+25\,(15\,cd^3e-35\,bd^2e^2+63\,ade^3)x^6+24\,ad^4+8\,(15\,cd^4-35\,bd^3e}{}$$

input
```
integrate((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
[-1/240*(30*(15*c*d^2*e^2 - 35*b*d*e^3 + 63*a*e^4)*x^8 + 50*(15*c*d^3*e -
35*b*d^2*e^2 + 63*a*d*e^3)*x^6 + 48*a*d^4 + 16*(15*c*d^4 - 35*b*d^3*e + 63
*a*d^2*e^2)*x^4 + 16*(5*b*d^4 - 9*a*d^3*e)*x^2 - 15*((15*c*d^2*e^2 - 35*b*
d*e^3 + 63*a*e^4)*x^9 + 2*(15*c*d^3*e - 35*b*d^2*e^2 + 63*a*d*e^3)*x^7 + (
15*c*d^4 - 35*b*d^3*e + 63*a*d^2*e^2)*x^5)*sqrt(-e/d)*log((e*x^2 - 2*d*x*s
qrt(-e/d) - d)/(e*x^2 + d)))/(d^5*e^2*x^9 + 2*d^6*e*x^7 + d^7*x^5), -1/120
*(15*(15*c*d^2*e^2 - 35*b*d*e^3 + 63*a*e^4)*x^8 + 25*(15*c*d^3*e - 35*b*d^
2*e^2 + 63*a*d*e^3)*x^6 + 24*a*d^4 + 8*(15*c*d^4 - 35*b*d^3*e + 63*a*d^2*e
^2)*x^4 + 8*(5*b*d^4 - 9*a*d^3*e)*x^2 + 15*((15*c*d^2*e^2 - 35*b*d*e^3 + 6
3*a*e^4)*x^9 + 2*(15*c*d^3*e - 35*b*d^2*e^2 + 63*a*d*e^3)*x^7 + (15*c*d^4
- 35*b*d^3*e + 63*a*d^2*e^2)*x^5)*sqrt(e/d)*arctan(x*sqrt(e/d)))/(d^5*e^2*
x^9 + 2*d^6*e*x^7 + d^7*x^5)]
```

3.294.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(163) = 326$.

Time = 1.79 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.93

$$\int \frac{a+bx^2+cx^4}{x^6(d+ex^2)^3}\,dx$$
$$= \frac{\sqrt{-\frac{e}{d^{11}}}\cdot(63ae^2-35bde+15cd^2)\log\left(-\frac{d^6\sqrt{-\frac{e}{d^{11}}}\cdot(63ae^2-35bde+15cd^2)}{63ae^3-35bde^2+15cd^2e}+x\right)}{16}$$
$$-\frac{\sqrt{-\frac{e}{d^{11}}}\cdot(63ae^2-35bde+15cd^2)\log\left(\frac{d^6\sqrt{-\frac{e}{d^{11}}}\cdot(63ae^2-35bde+15cd^2)}{63ae^3-35bde^2+15cd^2e}+x\right)}{16}$$
$$+\frac{-24ad^4+x^8(-945ae^4+525bde^3-225cd^2e^2)+x^6(-1575ade^3+875bd^2e^2-375cd^3e)+x^4(-504ad^2e}{120d^7x^5+240d^6ex^7+120d^5e^2x^9}$$

input
```
integrate((c*x**4+b*x**2+a)/x**6/(e*x**2+d)**3,x)
```

output
```
sqrt(-e/d**11)*(63*a*e**2 - 35*b*d*e + 15*c*d**2)*log(-d**6*sqrt(-e/d**11)
*(63*a*e**2 - 35*b*d*e + 15*c*d**2)/(63*a*e**3 - 35*b*d*e**2 + 15*c*d**2*e
) + x)/16 - sqrt(-e/d**11)*(63*a*e**2 - 35*b*d*e + 15*c*d**2)*log(d**6*sqr
t(-e/d**11)*(63*a*e**2 - 35*b*d*e + 15*c*d**2)/(63*a*e**3 - 35*b*d*e**2 +
15*c*d**2*e) + x)/16 + (-24*a*d**4 + x**8*(-945*a*e**4 + 525*b*d*e**3 - 22
5*c*d**2*e**2) + x**6*(-1575*a*d*e**3 + 875*b*d**2*e**2 - 375*c*d**3*e) +
x**4*(-504*a*d**2*e**2 + 280*b*d**3*e - 120*c*d**4) + x**2*(72*a*d**3*e -
40*b*d**4))/(120*d**7*x**5 + 240*d**6*e*x**7 + 120*d**5*e**2*x**9)
```

3.294.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^2+cx^4}{x^6(d+ex^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.294.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2+cx^4}{x^6\left(d+ex^2\right)^3}\,dx = -\frac{\left(15\,cd^2e-35\,bde^2+63\,ae^3\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\,\sqrt{de}d^5} - \frac{7\,cd^2e^2x^3-11\,bde^3x^3+15\,ae^4x^3+9\,cd^3ex-13\,bd^2e^2x+17\,ade^3x}{8\,\left(ex^2+d\right)^2d^5} - \frac{15\,cd^2x^4-45\,bdex^4+90\,ae^2x^4+5\,bd^2x^2-15\,adex^2+3\,ad^2}{15\,d^5x^5}$$

input
```
integrate((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
-1/8*(15*c*d^2*e - 35*b*d*e^2 + 63*a*e^3)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)
*d^5) - 1/8*(7*c*d^2*e^2*x^3 - 11*b*d*e^3*x^3 + 15*a*e^4*x^3 + 9*c*d^3*e*x
 - 13*b*d^2*e^2*x + 17*a*d*e^3*x)/((e*x^2 + d)^2*d^5) - 1/15*(15*c*d^2*x^4
 - 45*b*d*e*x^4 + 90*a*e^2*x^4 + 5*b*d^2*x^2 - 15*a*d*e*x^2 + 3*a*d^2)/(d^
5*x^5)
```

3.294.9 Mupad [B] (verification not implemented)

Time = 7.73 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.98

$$\int \frac{a+bx^2+cx^4}{x^6\left(d+ex^2\right)^3}\,dx = -\frac{\frac{a}{5\,d}-\frac{x^2\,\left(9\,a\,e-5\,b\,d\right)}{15\,d^2}+\frac{x^4\,\left(15\,c\,d^2-35\,b\,d\,e+63\,a\,e^2\right)}{15\,d^3}+\frac{5\,e\,x^6\,\left(15\,c\,d^2-35\,b\,d\,e+63\,a\,e^2\right)}{24\,d^4}+\frac{e^2\,x^8\,\left(15\,c\,d^2-35\,b\,d\,e+63\,a\,e^2\right)}{8\,d^5}}{d^2\,x^5+2\,d\,e\,x^7+e^2\,x^9} - \frac{\sqrt{e}\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(15\,c\,d^2-35\,b\,d\,e+63\,a\,e^2\right)}{8\,d^{11/2}}$$

input
```
int((a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^3),x)
```

output
```
- (a/(5*d) - (x^2*(9*a*e - 5*b*d))/(15*d^2) + (x^4*(63*a*e^2 + 15*c*d^2 -
35*b*d*e))/(15*d^3) + (5*e*x^6*(63*a*e^2 + 15*c*d^2 - 35*b*d*e))/(24*d^4)
+ (e^2*x^8*(63*a*e^2 + 15*c*d^2 - 35*b*d*e))/(8*d^5))/(d^2*x^5 + e^2*x^9 +
 2*d*e*x^7) - (e^(1/2)*atan((e^(1/2)*x)/d^(1/2))*(63*a*e^2 + 15*c*d^2 - 35
*b*d*e))/(8*d^(11/2))
```

3.295 $\int \frac{x^9}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

3.295.1 Optimal result

Integrand size = 27, antiderivative size = 230

$$\int \frac{x^9}{(d+ex^2)(a+bx^2+cx^4)}\,dx = -\frac{(cd+be)x^2}{2c^2e^2}+\frac{x^4}{4ce}-\frac{(b^4d-4ab^2cd+2a^2c^2d-ab^3e+3a^2bce)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)} + \frac{d^4\log(d+ex^2)}{2e^3(cd^2-bde+ae^2)}-\frac{(b^3d-2abcd-ab^2e+a^2ce)\log(a+bx^2+cx^4)}{4c^3(cd^2-bde+ae^2)}$$

output
```
-1/2*(b*e+c*d)*x^2/c^2/e^2+1/4*x^4/c/e+1/2*d^4*ln(e*x^2+d)/e^3/(a*e^2-b*d*
e+c*d^2)-1/4*(a^2*c*e-a*b^2*e-2*a*b*c*d+b^3*d)*ln(c*x^4+b*x^2+a)/c^3/(a*e^
2-b*d*e+c*d^2)-1/2*(3*a^2*b*c*e+2*a^2*c^2*d-a*b^3*e-4*a*b^2*c*d+b^4*d)*arc
tanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^3/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^
(1/2)
```

3.295.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.99

$$\int \frac{x^9}{(d+ex^2)(a+bx^2+cx^4)}\,dx$$
$$= \frac{1}{4}\left(-\frac{2(cd+be)x^2}{c^2e^2}+\frac{x^4}{ce}-\frac{2(b^4d-4ab^2cd+2a^2c^2d-ab^3e+3a^2bce)\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{c^3\sqrt{-b^2+4ac}\,(-cd^2+e(bd-ae))}\right.$$
$$\left.+\frac{2d^4\log(d+ex^2)}{e^3(cd^2+e(-bd+ae))}+\frac{(-b^3d+2abcd+ab^2e-a^2ce)\log(a+bx^2+cx^4)}{c^3(cd^2+e(-bd+ae))}\right)$$

input
```
Integrate[x^9/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

output
```
((-2*(c*d + b*e)*x^2)/(c^2*e^2) + x^4/(c*e) - (2*(b^4*d - 4*a*b^2*c*d + 2*
a^2*c^2*d - a*b^3*e + 3*a^2*b*c*e)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]
])/(c^3*Sqrt[-b^2 + 4*a*c]*(-(c*d^2) + e*(b*d - a*e))) + (2*d^4*Log[d + e*
x^2])/(e^3*(c*d^2 + e*(-(b*d) + a*e))) + ((-(b^3*d) + 2*a*b*c*d + a*b^2*e
- a^2*c*e)*Log[a + b*x^2 + c*x^4])/(c^3*(c*d^2 + e*(-(b*d) + a*e))))/4
```

3.295.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1578, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{(d+ex^2)(a+bx^2+cx^4)}\,dx$$
$$\Big\downarrow \text{1578}$$
$$\frac{1}{2}\int \frac{x^8}{(ex^2+d)(cx^4+bx^2+a)}dx^2$$
$$\Big\downarrow \text{1200}$$
$$\frac{1}{2}\int \left(\frac{d^4}{e^2(cd^2-bed+ae^2)(ex^2+d)}+\frac{x^2}{ce}+\frac{-cd-be}{c^2e^2}+\frac{-((db^3-aeb^2-2acdb+a^2ce)x^2)-a(db^2-aeb-ac}{c^2(cd^2-bed+ae^2)(cx^4+bx^2+a)}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d)}{c^3\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)}-\frac{(a^2ce-ab^2e-2abcd+b^3d)\log\left(a+bx^2+c\right.}{2c^3\,(ae^2-bde+cd^2)}\right.$$

input
```
Int[x^9/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

output
```
(-(((c*d + b*e)*x^2)/(c^2*e^2)) + x^4/(2*c*e) - ((b^4*d - 4*a*b^2*c*d + 2*
a^2*c^2*d - a*b^3*e + 3*a^2*b*c*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]
])/(c^3*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) + (d^4*Log[d + e*x^2])/
(e^3*(c*d^2 - b*d*e + a*e^2)) - ((b^3*d - 2*a*b*c*d - a*b^2*e + a^2*c*e)*L
og[a + b*x^2 + c*x^4])/(2*c^3*(c*d^2 - b*d*e + a*e^2)))/2
```

3.295.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.295.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.94

method	result
default	$\frac{\left(-c\,x^{2} e+b e+c d\right)^{2}}{4 e^{3} c^{3}}+\frac{\frac{\left(-a^{2} c e+a\,b^{2} e+2 a b c d-b^{3} d\right) \ln \left(c\,x^{4}+b\,x^{2}+a\right)}{2 c}+\frac{2 \left(a^{2} b e+a^{2} c d-a\,b^{2} d-\frac{\left(-a^{2} c e+a\,b^{2} e+2 a b c d-b^{3} d\right) b}{2 c}\right) \arctan \left(\frac{2 c\,x^{2}+b}{\sqrt{4 a c-b^{2}}}\right)}{\sqrt{4 a c-b^{2}}}}{2 (a\,e^{2}-b d e+c\,d^{2}) c^{2}}$
risch	$\frac{x^{4}}{4 c e}-\frac{b\,x^{2}}{2 e\,c^{2}}-\frac{d\,x^{2}}{2 c\,e^{2}}+\frac{b^{2}}{4 e\,c^{3}}+\frac{b d}{2 e^{2} c^{2}}+\frac{d^{2}}{4 c\,e^{3}}+\frac{d^{4} \ln \left(e\,x^{2}+d\right)}{2 e^{3} (a\,e^{2}-b d e+c\,d^{2})}+\frac{\_R=\operatorname{RootOf}\left(\left(4 a^{2} c^{2} e^{2}-a\,b^{2} c\,e^{2}-4 a b\,c^{2} d e+4 a\,c^{3} d^{2}+b^{3}\right.\right.}{}$

input
```
int(x^9/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-c*e*x^2+b*e+c*d)^2/e^3/c^3+1/2/(a*e^2-b*d*e+c*d^2)/c^2*(1/2*(-a^2*c*
e+a*b^2*e+2*a*b*c*d-b^3*d)/c*ln(c*x^4+b*x^2+a)+2*(a^2*b*e+a^2*c*d-a*b^2*d-
1/2*(-a^2*c*e+a*b^2*e+2*a*b*c*d-b^3*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*
x^2+b)/(4*a*c-b^2)^(1/2)))+1/2*d^4*ln(e*x^2+d)/e^3/(a*e^2-b*d*e+c*d^2)
```

3.295.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^9}{(d+ex^2)\left(a+bx^2+cx^4\right)}\,dx=\text{Timed out}$$

input
```
integrate(x^9/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.295.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^9}{(d+ex^2)\left(a+bx^2+cx^4\right)}\,dx=\text{Timed out}$$

input
```
integrate(x**9/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.295.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^9}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^9/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.295.8 Giac [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.01

$$\begin{aligned}\int \frac{x^9}{(d+ex^2)(a+bx^2+cx^4)}\,dx &= \frac{d^4\log(|ex^2+d|)}{2(cd^2e^3-bde^4+ae^5)} - \frac{(b^3d-2\,abcd-ab^2e+a^2ce)\log(cx^4+bx^2+a)}{4(c^4d^2-bc^3de+ac^3e^2)} \\ &\quad + \frac{(b^4d-4\,ab^2cd+2\,a^2c^2d-ab^3e+3\,a^2bce)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2(c^4d^2-bc^3de+ac^3e^2)\sqrt{-b^2+4\,ac}} + \frac{cex^4-2\,cdx^2-2\,bex^2}{4\,c^2e^2}\end{aligned}$$

input
```
integrate(x^9/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/2*d^4*log(abs(e*x^2 + d))/(c*d^2*e^3 - b*d*e^4 + a*e^5) - 1/4*(b^3*d - 2
*a*b*c*d - a*b^2*e + a^2*c*e)*log(c*x^4 + b*x^2 + a)/(c^4*d^2 - b*c^3*d*e
+ a*c^3*e^2) + 1/2*(b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d - a*b^3*e + 3*a^2*b*
c*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((c^4*d^2 - b*c^3*d*e + a*c^
3*e^2)*sqrt(-b^2 + 4*a*c)) + 1/4*(c*e*x^4 - 2*c*d*x^2 - 2*b*e*x^2)/(c^2*e^
2)
```

3.295.9 Mupad [B] (verification not implemented)

Time = 73.40 (sec) , antiderivative size = 7024, normalized size of antiderivative = 30.54

$$\int \frac{x^9}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
int(x^9/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

output
```
(d^4*log(d + e*x^2))/(2*a*e^5 + 2*c*d^2*e^3 - 2*b*d*e^4) + (log((x^2*(a^7*
e^7 + b^7*d^7 - 2*a^3*b*c^3*d^7 - a^4*c^3*d^6*e - 2*a^6*c*d^2*e^5 + 7*a^2*
b^3*c^2*d^7 + 3*a^2*b^5*d^5*e^2 + 4*a^3*b^4*d^4*e^3 + 4*a^4*b^3*d^3*e^4 +
3*a^5*b^2*d^2*e^5 + 2*a^5*c^2*d^4*e^3 - 5*a*b^5*c*d^7 + 2*a*b^6*d^6*e + 2*
a^6*b*d*e^6 - 8*a^2*b^4*c*d^6*e - 6*a^5*b*c*d^3*e^4 + 8*a^3*b^2*c^2*d^6*e
- 9*a^3*b^3*c*d^5*e^2 + 5*a^4*b*c^2*d^5*e^2 - 9*a^4*b^2*c*d^4*e^3))/(c^4*e
^4) + (a*d*(a^3*e^3 + b^3*d^3 - 2*a*b*c*d^3 + a*b^2*d^2*e + a^2*b*d*e^2 -
a^2*c*d^2*e)^2)/(c^4*e^4) + (((x^2*(4*a^2*c^6*d^8 + 6*a^4*b^4*e^8 + 18*a^6
*c^2*e^8 + 6*b^4*c^4*d^8 + 6*b^8*d^4*e^4 - 16*a*b^2*c^5*d^8 - 26*a^5*b^2*c
*e^8 + 8*a*b^7*d^3*e^5 + 8*a^3*b^5*d*e^7 - 2*b^5*c^3*d^7*e - 2*b^7*c*d^5*e
^3 + 8*a^2*b^6*d^2*e^6 - 20*a^3*c^5*d^6*e^2 + 40*a^4*c^4*d^4*e^4 - 36*a^5*
c^3*d^2*e^6 + 2*b^6*c^2*d^6*e^2 + 42*a^2*b^2*c^4*d^6*e^2 - 28*a^2*b^3*c^3*
d^5*e^3 + 80*a^2*b^4*c^2*d^4*e^4 - 64*a^3*b^2*c^3*d^4*e^4 + 80*a^3*b^3*c^2
*d^3*e^5 + 48*a^4*b^2*c^2*d^2*e^6 + 18*a*b^3*c^4*d^7*e - 40*a*b^6*c*d^4*e^
4 - 26*a^2*b*c^5*d^7*e - 32*a^4*b^3*c*d*e^7 + 12*a^5*b*c^2*d*e^7 - 16*a*b^
4*c^3*d^6*e^2 + 10*a*b^5*c^2*d^5*e^3 - 48*a^2*b^5*c*d^3*e^5 + 46*a^3*b*c^4
*d^5*e^3 - 40*a^3*b^4*c*d^2*e^6 - 48*a^4*b*c^3*d^3*e^5))/(c^4*e^4) + (((x^
2*(8*a*b^8*e^9 + 8*b*c^8*d^9 + 8*b^9*d*e^8 + 120*a^5*c^4*e^9 - 72*a^2*b^6*
c*e^9 - 8*b^2*c^7*d^8*e - 8*b^8*c*d^2*e^7 + 212*a^3*b^4*c^2*e^9 - 240*a^4*
b^2*c^3*e^9 - 112*a^2*c^7*d^6*e^3 + 240*a^3*c^6*d^4*e^5 - 228*a^4*c^5*d...
```

3.296 $\int \frac{x^7}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

3.296.1 Optimal result

Integrand size = 27, antiderivative size = 189

$$\int \frac{x^7}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \frac{x^2}{2ce}+\frac{(b^3d-3abcd-ab^2e+2a^2ce)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)} - \frac{d^3\log(d+ex^2)}{2e^2(cd^2-bde+ae^2)} + \frac{(b^2d-acd-abe)\log(a+bx^2+cx^4)}{4c^2(cd^2-bde+ae^2)}$$

output
```
1/2*x^2/c/e-1/2*d^3*ln(e*x^2+d)/e^2/(a*e^2-b*d*e+c*d^2)+1/4*(-a*b*e-a*c*d+
b^2*d)*ln(c*x^4+b*x^2+a)/c^2/(a*e^2-b*d*e+c*d^2)+1/2*(2*a^2*c*e-a*b^2*e-3*
a*b*c*d+b^3*d)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^2/(a*e^2-b*d*e+c*
d^2)/(-4*a*c+b^2)^(1/2)
```

3.296.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.98

$$\int \frac{x^7}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \frac{2e^2(b^3d-3abcd-ab^2e+2a^2ce)\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)+\sqrt{-b^2+4ac}(2c^2d^3\log(d+ex^2)+e(-2c(cd^2-bde}{4c^2\sqrt{-b^2+4ac}e^2(-cd^2+e(bd-ae))}$$

input
```
Integrate[x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

output
```
(2*e^2*(b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)*ArcTan[(b + 2*c*x^2)/Sqrt
[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*a*c]*(2*c^2*d^3*Log[d + e*x^2] + e*(-2*c*(
c*d^2 - b*d*e + a*e^2)*x^2 + e*(-(b^2*d) + a*c*d + a*b*e)*Log[a + b*x^2 +
c*x^4])))/(4*c^2*Sqrt[-b^2 + 4*a*c]*e^2*(-(c*d^2) + e*(b*d - a*e)))
```

3.296.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1578, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(d+ex^2)(a+bx^2+cx^4)}\,dx$$

$$\Big\downarrow \ 1578$$

$$\frac{1}{2}\int \frac{x^6}{(ex^2+d)(cx^4+bx^2+a)}dx^2$$

$$\Big\downarrow \ 1200$$

$$\frac{1}{2}\int \left(-\frac{d^3}{e(cd^2-bed+ae^2)(ex^2+d)}+\frac{1}{ce}+\frac{(db^2-aeb-acd)x^2+a(bd-ae)}{c(cd^2-bed+ae^2)(cx^4+bx^2+a)}\right)dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(2a^2ce-ab^2e-3abcd+b^3d)}{c^2\sqrt{b^2-4ac}(ae^2-bde+cd^2)}+\frac{(-abe-acd+b^2d)\log(a+bx^2+cx^4)}{2c^2(ae^2-bde+cd^2)}-\frac{d^3\log(d+e\ldots}{e^2(ae^2-bde+\ldots}\right.$$

input
```
Int[x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

output
```
(x^2/(c*e) + ((b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)*ArcTanh[(b + 2*c*x
^2)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) -
(d^3*Log[d + e*x^2])/(e^2*(c*d^2 - b*d*e + a*e^2)) + ((b^2*d - a*c*d - a*b
*e)*Log[a + b*x^2 + c*x^4])/(2*c^2*(c*d^2 - b*d*e + a*e^2)))/2
```

3.296.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.296.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{x^2}{2ce}-\frac{\frac{\left(abe+acd-b^2d\right)\ln\left(c\,x^4+b\,x^2+a\right)}{2c}+\frac{2\left(e\,a^2-dab-\frac{\left(abe+acd-b^2d\right)b}{2c}\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2\left(a\,e^2-bde+c\,d^2\right)c}-\frac{d^3\ln\left(e\,x^2+d\right)}{2e^2\left(a\,e^2-bde+c\,d^2\right)}$	172
risch	Expression too large to display	15081

input
```
int(x^7/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/c/e-1/2/(a*e^2-b*d*e+c*d^2)/c*(1/2*(a*b*e+a*c*d-b^2*d)/c*ln(c*x^4+
b*x^2+a)+2*(e*a^2-d*a*b-1/2*(a*b*e+a*c*d-b^2*d)*b/c)/(4*a*c-b^2)^(1/2)*arc
tan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))-1/2*d^3*ln(e*x^2+d)/e^2/(a*e^2-b*d*e+c
*d^2)
```

3.296.5 Fricas [A] (verification not implemented)

Time = 113.05 (sec) , antiderivative size = 616, normalized size of antiderivative = 3.26

$$\int \frac{x^7}{(d+ex^2)(a+bx^2+cx^4)}\,dx$$
$$= \left[-\frac{2\,(b^2c^2-4\,ac^3)d^3\log\left(ex^2+d\right)-2\left((b^2c^2-4\,ac^3)d^2e-(b^3c-4\,abc^2)de^2+(ab^2c-4\,a^2c^2)e^3\right)x^2-\left((b^3\right.}{4\left((b^2c^3\right.}\right.$$
$$-\frac{2\,(b^2c^2-4\,ac^3)d^3\log\left(ex^2+d\right)-2\left((b^2c^2-4\,ac^3)d^2e-(b^3c-4\,abc^2)de^2+(ab^2c-4\,a^2c^2)e^3\right)x^2-2\left((b\right.}{4\left((b^2c^3-4\,\right.}$$

input
```
integrate(x^7/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
[-1/4*(2*(b^2*c^2 - 4*a*c^3)*d^3*log(e*x^2 + d) - 2*((b^2*c^2 - 4*a*c^3)*d
^2*e - (b^3*c - 4*a*b*c^2)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3)*x^2 - ((b^3
- 3*a*b*c)*d*e^2 - (a*b^2 - 2*a^2*c)*e^3)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4
 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x
^2 + a)) - ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d*e^2 - (a*b^3 - 4*a^2*b*c)*e^3)
*log(c*x^4 + b*x^2 + a))/((b^2*c^3 - 4*a*c^4)*d^2*e^2 - (b^3*c^2 - 4*a*b*c
^3)*d*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*e^4), -1/4*(2*(b^2*c^2 - 4*a*c^3)*d^3*
log(e*x^2 + d) - 2*((b^2*c^2 - 4*a*c^3)*d^2*e - (b^3*c - 4*a*b*c^2)*d*e^2
+ (a*b^2*c - 4*a^2*c^2)*e^3)*x^2 - 2*((b^3 - 3*a*b*c)*d*e^2 - (a*b^2 - 2*a
^2*c)*e^3)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^
2 - 4*a*c)) - ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d*e^2 - (a*b^3 - 4*a^2*b*c)*e
^3)*log(c*x^4 + b*x^2 + a))/((b^2*c^3 - 4*a*c^4)*d^2*e^2 - (b^3*c^2 - 4*a*
b*c^3)*d*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*e^4)]
```

3.296.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^7}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x**7/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.296.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^7/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.296.8 Giac [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.03

$$\int \frac{x^7}{(d+ex^2)(a+bx^2+cx^4)}\,dx = -\frac{d^3\log(|ex^2+d|)}{2(cd^2e^2-bde^3+ae^4)} + \frac{(b^2d-acd-abe)\log(cx^4+bx^2+a)}{4(c^3d^2-bc^2de+ac^2e^2)} - \frac{(b^3d-3abcd-ab^2e+2a^2ce)\arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{2(c^3d^2-bc^2de+ac^2e^2)\sqrt{-b^2+4ac}} + \frac{x^2}{2ce}$$

input
```
integrate(x^7/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*d^3*log(abs(e*x^2 + d))/(c*d^2*e^2 - b*d*e^3 + a*e^4) + 1/4*(b^2*d -
a*c*d - a*b*e)*log(c*x^4 + b*x^2 + a)/(c^3*d^2 - b*c^2*d*e + a*c^2*e^2) -
1/2*(b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)*arctan((2*c*x^2 + b)/sqrt(-b
^2 + 4*a*c))/((c^3*d^2 - b*c^2*d*e + a*c^2*e^2)*sqrt(-b^2 + 4*a*c)) + 1/2*
x^2/(c*e)
```

3.296.9 Mupad [B] (verification not implemented)

Time = 18.41 (sec) , antiderivative size = 2304, normalized size of antiderivative = 12.19

$$\int \frac{x^7}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
int(x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

output
```
x^2/(2*c*e) - (d^3*log(d + e*x^2))/(2*(a*e^4 + c*d^2*e^2 - b*d*e^3)) - (lo
g(a*e^5*x^2*(b^2 - 4*a*c)^(7/2) - 128*a^5*c^3*e^5 - 8*c^3*d^5*(b^2 - 4*a*c
)^(5/2) - 512*a^3*c^5*d^4*e + 8*b^2*c^3*d^5*(b^2 - 4*a*c)^(3/2) + 6*b^3*d^
2*e^3*(b^2 - 4*a*c)^(5/2) - 3*b^5*d^2*e^3*(b^2 - 4*a*c)^(3/2) + 32*a^4*b^2
*c^2*e^5 + 384*a^4*c^4*d^2*e^3 + 256*a^2*c^6*d^5*x^2 + 16*b^4*c^4*d^5*x^2
+ 3*a*d*e^4*(b^2 - 4*a*c)^(7/2) - 3*b*d^2*e^3*(b^2 - 4*a*c)^(7/2) - 3*c*d^
3*e^2*(b^2 - 4*a*c)^(7/2) - 16*a^2*b^3*c^3*d^3*e^2 + 48*a^2*b^4*c^2*d^2*e^
3 - 288*a^3*b^2*c^3*d^2*e^3 + 16*a^3*b^3*c^2*e^5*x^2 - 384*a^3*c^5*d^3*e^2
*x^2 + 16*b^6*c^2*d^3*e^2*x^2 - 6*a*b^2*d*e^4*(b^2 - 4*a*c)^(5/2) + 3*a*b^
4*d*e^4*(b^2 - 4*a*c)^(3/2) + 8*b*c^2*d^4*e*(b^2 - 4*a*c)^(5/2) - 32*a*b^4
*c^3*d^4*e + 192*a^4*b*c^3*d*e^4 - 2*b^2*c*d^3*e^2*(b^2 - 4*a*c)^(5/2) - 8
*b^3*c^2*d^4*e*(b^2 - 4*a*c)^(3/2) + 5*b^4*c*d^3*e^2*(b^2 - 4*a*c)^(3/2) -
 2*a*b^2*e^5*x^2*(b^2 - 4*a*c)^(5/2) + a*b^4*e^5*x^2*(b^2 - 4*a*c)^(3/2) +
 16*b*c^4*d^5*x^2*(b^2 - 4*a*c)^(3/2) - 3*c*d^2*e^3*x^2*(b^2 - 4*a*c)^(7/2
) - 16*c^3*d^4*e*x^2*(b^2 - 4*a*c)^(5/2) + 256*a^2*b^2*c^4*d^4*e + 64*a^3*
b*c^4*d^3*e^2 - 48*a^3*b^3*c^2*d*e^4 - 128*a*b^2*c^5*d^5*x^2 - 64*a^4*b*c^
3*e^5*x^2 + 384*a^4*c^4*d*e^4*x^2 - 32*b^5*c^3*d^4*e*x^2 + 480*a^2*b^2*c^4
*d^3*e^2*x^2 + 48*a^2*b^3*c^3*d^2*e^3*x^2 + 256*a*b^3*c^4*d^4*e*x^2 - 512*
a^2*b*c^5*d^4*e*x^2 + 8*b*c^2*d^3*e^2*x^2*(b^2 - 4*a*c)^(5/2) + 6*b^2*c*d^
2*e^3*x^2*(b^2 - 4*a*c)^(5/2) - 16*b^2*c^3*d^4*e*x^2*(b^2 - 4*a*c)^(3/2...
```

3.297 $\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

3.297.1 Optimal result

Integrand size = 27, antiderivative size = 158

$$\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)}\,dx = -\frac{(b^2d-2acd-abe)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)} + \frac{d^2\log(d+ex^2)}{2e\,(cd^2-bde+ae^2)} - \frac{(bd-ae)\log(a+bx^2+cx^4)}{4c\,(cd^2-bde+ae^2)}$$

output
```
1/2*d^2*ln(e*x^2+d)/e/(a*e^2-b*d*e+c*d^2)-1/4*(-a*e+b*d)*ln(c*x^4+b*x^2+a)
/c/(a*e^2-b*d*e+c*d^2)-1/2*(-a*b*e-2*a*c*d+b^2*d)*arctanh((2*c*x^2+b)/(-4*
a*c+b^2)^(1/2))/c/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)
```

3.297.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.88

$$\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)}\,dx = -\frac{2e(-b^2d+2acd+abe)\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)+\sqrt{-b^2+4ac}\left(-2cd^2\log(d+ex^2)+e(bd-ae)\log(a+bx^2\right.}{4c\sqrt{-b^2+4ac}\,e\,(cd^2+e(-bd+ae))}$$

input
```
Integrate[x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

output
```
-1/4*(2*e*(-(b^2*d) + 2*a*c*d + a*b*e)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*
a*c]] + Sqrt[-b^2 + 4*a*c]*(-2*c*d^2*Log[d + e*x^2] + e*(b*d - a*e)*Log[a
+ b*x^2 + c*x^4]))/(c*Sqrt[-b^2 + 4*a*c]*e*(c*d^2 + e*(-(b*d) + a*e)))
```

3.297.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1578, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)}\,dx$$

$$\Big\downarrow \text{ 1578}$$

$$\frac{1}{2}\int \frac{x^4}{(ex^2+d)(cx^4+bx^2+a)}dx^2$$

$$\Big\downarrow \text{ 1200}$$

$$\frac{1}{2}\int \left(\frac{d^2}{(cd^2-bed+ae^2)(ex^2+d)}+\frac{-((bd-ae)x^2)-ad}{(cd^2-bed+ae^2)(cx^4+bx^2+a)}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(-abe-2acd+b^2d)}{c\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)}+\frac{d^2\log(d+ex^2)}{e(ae^2-bde+cd^2)}-\frac{(bd-ae)\log(a+bx^2+cx^4)}{2c(ae^2-bde+cd^2)}\right)$$

input
```
Int[x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

output
```
(-(((b^2*d - 2*a*c*d - a*b*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(c
*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2))) + (d^2*Log[d + e*x^2])/(e*(c*
d^2 - b*d*e + a*e^2)) - ((b*d - a*e)*Log[a + b*x^2 + c*x^4])/(2*c*(c*d^2 -
 b*d*e + a*e^2)))/2
```

3.297.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.297.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.87

method	result
default	$-\frac{\frac{(-ae+bd)\ln\left(c\,x^4+b\,x^2+a\right)}{2c}+\frac{2\left(da-\frac{(-ae+bd)b}{2c}\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2(a\,e^2-bde+c\,d^2)}+\frac{d^2\ln\left(e\,x^2+d\right)}{2e(a\,e^2-bde+c\,d^2)}$
risch	$\frac{d^2\ln\left(e\,x^2+d\right)}{2e(a\,e^2-bde+c\,d^2)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^2c^2e^2-a\,b^2c\,e^2-4ab\,c^2de+4a\,c^3d^2+b^3cde-b^2c^2d^2\right)\_Z^2+\left(-4a^2ce+a\,b^2e+4abcd-b^3d\right)\_Z+a^2\right)} \ldots\right.}{\ldots}$

input
```
int(x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(a*e^2-b*d*e+c*d^2)*(1/2*(-a*e+b*d)/c*ln(c*x^4+b*x^2+a)+2*(d*a-1/2*(-
a*e+b*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))+1/2
*d^2*ln(e*x^2+d)/e/(a*e^2-b*d*e+c*d^2)
```

3.297.5 Fricas [A] (verification not implemented)

Time = 32.44 (sec) , antiderivative size = 421, normalized size of antiderivative = 2.66

$$\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)}\,dx$$
$$= \left[\frac{2\,(b^2c-4\,ac^2)d^2\log\left(ex^2+d\right)+(abe^2-(b^2-2\,ac)de)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right.}{4\left((b^2c^2-4\,ac^3)d^2e-(b^3c-4\,abc^2)de^2+(ab^2c-4\,a\right.} \right.$$

input
```
integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
[1/4*(2*(b^2*c - 4*a*c^2)*d^2*log(e*x^2 + d) + (a*b*e^2 - (b^2 - 2*a*c)*d*
e)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 +
 b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - ((b^3 - 4*a*b*c)*d*e - (a*b^
2 - 4*a^2*c)*e^2)*log(c*x^4 + b*x^2 + a))/((b^2*c^2 - 4*a*c^3)*d^2*e - (b^
3*c - 4*a*b*c^2)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3), 1/4*(2*(b^2*c - 4*a*c
^2)*d^2*log(e*x^2 + d) + 2*(a*b*e^2 - (b^2 - 2*a*c)*d*e)*sqrt(-b^2 + 4*a*c
)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^3 - 4*a*b*
c)*d*e - (a*b^2 - 4*a^2*c)*e^2)*log(c*x^4 + b*x^2 + a))/((b^2*c^2 - 4*a*c^
3)*d^2*e - (b^3*c - 4*a*b*c^2)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3)]
```

3.297.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x**5/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.297.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.297.8 Giac [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.98

$$\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \frac{d^2\log(|ex^2+d|)}{2(cd^2e-bde^2+ae^3)} - \frac{(bd-ae)\log(cx^4+bx^2+a)}{4(c^2d^2-bcde+ace^2)} + \frac{(b^2d-2\,acd-abe)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2(c^2d^2-bcde+ace^2)\sqrt{-b^2+4\,ac}}$$

input
```
integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/2*d^2*log(abs(e*x^2 + d))/(c*d^2*e - b*d*e^2 + a*e^3) - 1/4*(b*d - a*e)*
log(c*x^4 + b*x^2 + a)/(c^2*d^2 - b*c*d*e + a*c*e^2) + 1/2*(b^2*d - 2*a*c*
d - a*b*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((c^2*d^2 - b*c*d*e +
a*c*e^2)*sqrt(-b^2 + 4*a*c))
```

3.297.9 Mupad [B] (verification not implemented)

Time = 15.42 (sec) , antiderivative size = 1853, normalized size of antiderivative = 11.73

$$\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
int(x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

output
```
(d^2*log(d + e*x^2))/(2*a*e^3 - 2*b*d*e^2 + 2*c*d^2*e) + (log(4*a^2*e^4*(b
^2 - 4*a*c)^(5/2) + 8*c^2*d^4*(b^2 - 4*a*c)^(5/2) + 5*d^2*e^2*(b^2 - 4*a*c
)^(7/2) + 3*d*e^3*x^2*(b^2 - 4*a*c)^(7/2) - 16*a^3*b^3*c*e^4 + 64*a^4*b*c^
2*e^4 + 640*a^3*c^4*d^3*e - 384*a^4*c^3*d*e^3 - 4*a^2*b^2*e^4*(b^2 - 4*a*c
)^(3/2) - 8*b^2*c^2*d^4*(b^2 - 4*a*c)^(3/2) - 6*b^2*d^2*e^2*(b^2 - 4*a*c)^
(5/2) + b^4*d^2*e^2*(b^2 - 4*a*c)^(3/2) - 256*a^2*c^5*d^4*x^2 - 128*a^4*c^
3*e^4*x^2 - 16*b^4*c^3*d^4*x^2 + 80*a^2*b^3*c^2*d^2*e^2 + 96*a^3*b^2*c^2*e
^4*x^2 + 640*a^3*c^4*d^2*e^2*x^2 + 4*b^3*c*d^3*e*(b^2 - 4*a*c)^(3/2) + 4*a
*b*e^4*x^2*(b^2 - 4*a*c)^(5/2) + 48*a*b^4*c^2*d^3*e - 16*a*b^5*c*d^2*e^2 -
 4*a*b^3*e^4*x^2*(b^2 - 4*a*c)^(3/2) - 16*b*c^3*d^4*x^2*(b^2 - 4*a*c)^(3/2
) - 6*b^2*d*e^3*x^2*(b^2 - 4*a*c)^(5/2) + 3*b^4*d*e^3*x^2*(b^2 - 4*a*c)^(3
/2) + 20*c^2*d^3*e*x^2*(b^2 - 4*a*c)^(5/2) - 352*a^2*b^2*c^3*d^3*e - 64*a^
3*b*c^3*d^2*e^2 + 96*a^3*b^2*c^2*d*e^3 + 128*a*b^2*c^4*d^4*x^2 - 16*a^2*b^
4*c*e^4*x^2 + 32*b^5*c^2*d^3*e*x^2 - 16*b^6*c*d^2*e^2*x^2 - 4*b*c*d^3*e*(b
^2 - 4*a*c)^(5/2) - 480*a^2*b^2*c^3*d^2*e^2*x^2 - 12*b*c*d^2*e^2*x^2*(b^2
- 4*a*c)^(5/2) - 240*a*b^3*c^3*d^3*e*x^2 + 448*a^2*b*c^4*d^3*e*x^2 - 192*a
^3*b*c^3*d*e^3*x^2 + 12*b^2*c^2*d^3*e*x^2*(b^2 - 4*a*c)^(3/2) - 4*b^3*c*d^
2*e^2*x^2*(b^2 - 4*a*c)^(3/2) + 144*a*b^4*c^2*d^2*e^2*x^2 + 48*a^2*b^3*c^2
*d*e^3*x^2)*((b^3*d)/4 + e*(a^2*c - (a*b^2)/4 + (a*b*(b^2 - 4*a*c)^(1/2))/
4) - (b^2*d*(b^2 - 4*a*c)^(1/2))/4 + (a*c*d*(b^2 - 4*a*c)^(1/2))/2 - a*...
```

3.298 $\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

3.298.1 Optimal result

Integrand size = 27, antiderivative size = 132

$$\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \frac{(bd-2ae)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)} - \frac{d\log(d+ex^2)}{2(cd^2-bde+ae^2)} + \frac{d\log(a+bx^2+cx^4)}{4(cd^2-bde+ae^2)}$$

output
```
-1/2*d*ln(e*x^2+d)/(a*e^2-b*d*e+c*d^2)+1/4*d*ln(c*x^4+b*x^2+a)/(a*e^2-b*d*
e+c*d^2)+1/2*(-2*a*e+b*d)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(a*e^2-b
*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)
```

3.298.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.86

$$\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \frac{2(bd-2ae)\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)+\sqrt{-b^2+4ac}\,d(2\log(d+ex^2)-\log(a+bx^2+cx^4))}{4\sqrt{-b^2+4ac}\,(-cd^2+e(bd-ae))}$$

input
```
Integrate[x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

output
```
(2*(b*d - 2*a*e)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*
a*c]*d*(2*Log[d + e*x^2] - Log[a + b*x^2 + c*x^4]))/(4*Sqrt[-b^2 + 4*a*c]*
(-(c*d^2) + e*(b*d - a*e)))
```

3.298.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1578, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)}dx$$

$$\Big\downarrow \text{ 1578}$$

$$\frac{1}{2}\int \frac{x^2}{(ex^2+d)(cx^4+bx^2+a)}dx^2$$

$$\Big\downarrow \text{ 1200}$$

$$\frac{1}{2}\int \left(\frac{cdx^2+ae}{(cd^2-bed+ae^2)(cx^4+bx^2+a)}-\frac{de}{(cd^2-bed+ae^2)(ex^2+d)}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{(bd-2ae)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}(ae^2-bde+cd^2)}-\frac{d\log(d+ex^2)}{ae^2-bde+cd^2}+\frac{d\log(a+bx^2+cx^4)}{2(ae^2-bde+cd^2)}\right)$$

input
```
Int[x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

output
```
(((b*d - 2*a*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*
c]*(c*d^2 - b*d*e + a*e^2)) - (d*Log[d + e*x^2])/(c*d^2 - b*d*e + a*e^2) +
 (d*Log[a + b*x^2 + c*x^4])/(2*(c*d^2 - b*d*e + a*e^2)))/2
```

3.298.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.298.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{\frac{d \ln\left(c\,x^{4}+b\,x^{2}+a\right)}{2}+\frac{2\left(a e-\frac{b d}{2}\right) \arctan\left(\frac{2 c\,x^{2}+b}{\sqrt{4 a c-b^{2}}}\right)}{\sqrt{4 a c-b^{2}}}}{2 a\,e^{2}-2 b d e+2 c\,d^{2}}-\frac{d \ln\left(e\,x^{2}+d\right)}{2\left(a\,e^{2}-b d e+c\,d^{2}\right)}$	112
risch	Expression too large to display	4521

input
```
int(x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/(a*e^2-b*d*e+c*d^2)*(1/2*d*ln(c*x^4+b*x^2+a)+2*(a*e-1/2*b*d)/(4*a*c-b^
2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))-1/2*d*ln(e*x^2+d)/(a*e^2-b
*d*e+c*d^2)
```

3.298.5 Fricas [A] (verification not implemented)

Time = 11.07 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.43

$$\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)}\,dx$$
$$= \left[\frac{(b^2-4ac)d\log(cx^4+bx^2+a) - 2(b^2-4ac)d\log(ex^2+d) - \sqrt{b^2-4ac}(bd-2ae)\log\left(\frac{2c^2x^4+2bcx^2+b}{}\right.}{4((b^2c-4ac^2)d^2-(b^3-4abc)de+(ab^2-4a^2c)e^2)}\right.$$

input
```
integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
[1/4*((b^2 - 4*a*c)*d*log(c*x^4 + b*x^2 + a) - 2*(b^2 - 4*a*c)*d*log(e*x^2
 + d) - sqrt(b^2 - 4*a*c)*(b*d - 2*a*e)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 -
 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/((b^2*c -
4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2), 1/4*((b^2 - 4
*a*c)*d*log(c*x^4 + b*x^2 + a) - 2*(b^2 - 4*a*c)*d*log(e*x^2 + d) + 2*sqrt
(-b^2 + 4*a*c)*(b*d - 2*a*e)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2
 - 4*a*c)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*
c)*e^2)]
```

3.298.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x**3/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.298.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.298.8 Giac [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.99

$$\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)}\,dx = -\frac{de\log(|ex^2+d|)}{2(cd^2e-bde^2+ae^3)} + \frac{d\log(cx^4+bx^2+a)}{4(cd^2-bde+ae^2)} - \frac{(bd-2ae)\arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{2(cd^2-bde+ae^2)\sqrt{-b^2+4ac}}$$

input
```
integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*d*e*log(abs(e*x^2 + d))/(c*d^2*e - b*d*e^2 + a*e^3) + 1/4*d*log(c*x^4
 + b*x^2 + a)/(c*d^2 - b*d*e + a*e^2) - 1/2*(b*d - 2*a*e)*arctan((2*c*x^2
+ b)/sqrt(-b^2 + 4*a*c))/((c*d^2 - b*d*e + a*e^2)*sqrt(-b^2 + 4*a*c))
```

3.298.9 Mupad [B] (verification not implemented)

Time = 14.13 (sec) , antiderivative size = 3704, normalized size of antiderivative = 28.06

$$\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
int(x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

output
```
(log(76*d^3*e^3*(b^2 - 4*a*c)^(9/2) - 64*a^3*b^6*e^6 - 4608*a^3*c^6*d^6 +
512*a^6*c^3*e^6 - 320*a*b^4*c^4*d^6 + 512*a^4*b^4*c*e^6 - 64*a*b^8*d^2*e^4
 - 128*a^2*b^7*d*e^5 + 32*a^3*b^3*e^6*(b^2 - 4*a*c)^(3/2) - 48*b^3*c^3*d^6
*(b^2 - 4*a*c)^(3/2) - 68*b^2*d^3*e^3*(b^2 - 4*a*c)^(7/2) - 28*b^4*d^3*e^3
*(b^2 - 4*a*c)^(5/2) + 20*b^6*d^3*e^3*(b^2 - 4*a*c)^(3/2) + 4*a^2*e^6*x^2*
(b^2 - 4*a*c)^(7/2) + 144*c^4*d^6*x^2*(b^2 - 4*a*c)^(5/2) + 39*d^2*e^4*x^2
*(b^2 - 4*a*c)^(9/2) + 2432*a^2*b^2*c^5*d^6 - 1152*a^5*b^2*c^2*e^6 + 40448
*a^4*c^5*d^4*e^2 - 19968*a^5*c^4*d^2*e^4 - 64*a^2*b^7*e^6*x^2 - 64*b^5*c^4
*d^6*x^2 - 64*b^9*d^2*e^4*x^2 + 32*a^3*b*e^6*(b^2 - 4*a*c)^(5/2) + 48*b*c^
3*d^6*(b^2 - 4*a*c)^(5/2) + 40*a^2*d*e^5*(b^2 - 4*a*c)^(7/2) + 168*c^2*d^5
*e*(b^2 - 4*a*c)^(7/2) + 40*a^2*b^2*e^6*x^2*(b^2 - 4*a*c)^(5/2) + 20*a^2*b
^4*e^6*x^2*(b^2 - 4*a*c)^(3/2) - 80*b^2*c^4*d^6*x^2*(b^2 - 4*a*c)^(3/2) +
155*b^2*d^2*e^4*x^2*(b^2 - 4*a*c)^(7/2) - 155*b^4*d^2*e^4*x^2*(b^2 - 4*a*c
)^(5/2) + 25*b^6*d^2*e^4*x^2*(b^2 - 4*a*c)^(3/2) + 316*c^2*d^4*e^2*x^2*(b^
2 - 4*a*c)^(7/2) + 5120*a^2*b^4*c^3*d^4*e^2 - 4096*a^2*b^5*c^2*d^3*e^3 - 2
4448*a^3*b^2*c^4*d^4*e^2 + 21760*a^3*b^3*c^3*d^3*e^3 - 9920*a^3*b^4*c^2*d^
2*e^4 + 26240*a^4*b^2*c^3*d^2*e^4 - 1600*a^4*b^3*c^2*e^6*x^2 + 38912*a^4*c
^5*d^3*e^3*x^2 - 384*b^7*c^2*d^4*e^2*x^2 + 212*a*b*d^2*e^4*(b^2 - 4*a*c)^(
7/2) - 176*b*c*d^4*e^2*(b^2 - 4*a*c)^(7/2) + 256*a*b^5*c^3*d^5*e + 256*a*b
^7*c*d^3*e^3 + 2560*a^3*b*c^5*d^5*e + 1664*a^3*b^5*c*d*e^5 + 8704*a^5*b...
```

3.298. $\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

3.299 $\int \frac{x}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

3.299.1 Optimal result

Integrand size = 25, antiderivative size = 133

$$\int \frac{x}{(d+ex^2)(a+bx^2+cx^4)}\,dx = -\frac{(2cd-be)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)} + \frac{e\log(d+ex^2)}{2(cd^2-bde+ae^2)} - \frac{e\log(a+bx^2+cx^4)}{4(cd^2-bde+ae^2)}$$

output
```
1/2*e*ln(e*x^2+d)/(a*e^2-b*d*e+c*d^2)-1/4*e*ln(c*x^4+b*x^2+a)/(a*e^2-b*d*e
+c*d^2)-1/2*(-b*e+2*c*d)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*
d*e+c*d^2)/(-4*a*c+b^2)^(1/2)
```

3.299.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.84

$$\int \frac{x}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \frac{(-4cd+2be)\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)+\sqrt{-b^2+4ac}\,e(-2\log(d+ex^2)+\log(a+bx^2+cx^4))}{4\sqrt{-b^2+4ac}\,(-cd^2+e(bd-ae))}$$

input
```
Integrate[x/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

output
```
((-4*c*d + 2*b*e)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4
*a*c]*e*(-2*Log[d + e*x^2] + Log[a + b*x^2 + c*x^4]))/(4*Sqrt[-b^2 + 4*a*c
]*(-(c*d^2) + e*(b*d - a*e)))
```

3.299.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1576, 1144, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(d+ex^2)(a+bx^2+cx^4)}dx$$

$$\Big\downarrow \text{1576}$$

$$\frac{1}{2}\int \frac{1}{(ex^2+d)(cx^4+bx^2+a)}dx^2$$

$$\Big\downarrow \text{1144}$$

$$\frac{1}{2}\left(\frac{\int \frac{-cex^2+cd-be}{cx^4+bx^2+a}dx^2}{ae^2-bde+cd^2}+\frac{e\log(d+ex^2)}{ae^2-bde+cd^2}\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{\frac{1}{2}(2cd-be)\int \frac{1}{cx^4+bx^2+a}dx^2-\frac{1}{2}e\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{ae^2-bde+cd^2}+\frac{e\log(d+ex^2)}{ae^2-bde+cd^2}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{-\left((2cd-be)\int \frac{1}{-x^4+b^2-4ac}d(2cx^2+b)\right)-\frac{1}{2}e\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{ae^2-bde+cd^2}+\frac{e\log(d+ex^2)}{ae^2-bde+cd^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{-\frac{1}{2}e\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2-\frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{ae^2-bde+cd^2}+\frac{e\log(d+ex^2)}{ae^2-bde+cd^2}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{-\frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}-\frac{1}{2}e\log\left(a+bx^2+cx^4\right)}{ae^2-bde+cd^2}+\frac{e\log\left(d+ex^2\right)}{ae^2-bde+cd^2}\right)$$

input
```
Int[x/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

output
```
((e*Log[d + e*x^2])/(c*d^2 - b*d*e + a*e^2) + (-(((2*c*d - b*e)*ArcTanh[(b
 + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]) - (e*Log[a + b*x^2 + c*
x^4])/2)/(c*d^2 - b*d*e + a*e^2))/2
```

3.299.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

3.299.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.85

method	result
default	$-\frac{\frac{e\ln\left(c\,x^4+b\,x^2+a\right)}{2}+\frac{2\left(\frac{be}{2}-cd\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2(a\,e^2-bde+c\,d^2)}+\frac{e\ln\left(e\,x^2+d\right)}{2a\,e^2-2bde+2c\,d^2}$
risch	$\frac{e\ln\left(e\,x^2+d\right)}{2a\,e^2-2bde+2c\,d^2}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(4a^2c\,e^2-a\,b^2e^2-4abcde+4a\,c^2d^2+b^3de-b^2c\,d^2\right)\_Z^2+\left(4ace-b^2e\right)\_Z+c\right)}\_R\ln\left(\left(\left(4a^2c\,e^3-a\,b^2e\right.\right.\right.\right.}{}$

input
```
int(x/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(a*e^2-b*d*e+c*d^2)*(1/2*e*ln(c*x^4+b*x^2+a)+2*(1/2*b*e-c*d)/(4*a*c-b
^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))+1/2*e*ln(e*x^2+d)/(a*e^2-
b*d*e+c*d^2)
```

3.299.5 Fricas [A] (verification not implemented)

Time = 7.67 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.41

$$\int \frac{x}{(d+ex^2)(a+bx^2+cx^4)}\,dx$$
$$=\left[-\frac{(b^2-4\,ac)e\log\left(cx^4+bx^2+a\right)-2\,(b^2-4\,ac)e\log\left(ex^2+d\right)+\sqrt{b^2-4\,ac}(2\,cd-be)\log\left(\frac{2\,c^2x^4+2\,bcx^2+}{}\right.}{4\left((b^2c-4\,ac^2)d^2-(b^3-4\,abc)de+(ab^2-4\,a^2c)e^2\right)}\right.$$
$$-\frac{(b^2-4\,ac)e\log\left(cx^4+bx^2+a\right)-2\,(b^2-4\,ac)e\log\left(ex^2+d\right)+2\sqrt{-b^2+4\,ac}(2\,cd-be)\arctan\left(-\frac{(2\,c}{}\right.}{4\left((b^2c-4\,ac^2)d^2-(b^3-4\,abc)de+(ab^2-4\,a^2c)e^2\right)}$$

input
```
integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
[-1/4*((b^2 - 4*a*c)*e*log(c*x^4 + b*x^2 + a) - 2*(b^2 - 4*a*c)*e*log(e*x^
2 + d) + sqrt(b^2 - 4*a*c)*(2*c*d - b*e)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2
- 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/((b^2*c -
 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2), -1/4*((b^2 -
 4*a*c)*e*log(c*x^4 + b*x^2 + a) - 2*(b^2 - 4*a*c)*e*log(e*x^2 + d) + 2*sq
rt(-b^2 + 4*a*c)*(2*c*d - b*e)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b
^2 - 4*a*c)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^
2*c)*e^2)]
```

3.299.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.299.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.299.8 Giac [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00

$$\int \frac{x}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \frac{e^2\log(|ex^2+d|)}{2(cd^2e-bde^2+ae^3)} - \frac{e\log(cx^4+bx^2+a)}{4(cd^2-bde+ae^2)} + \frac{(2\,cd-be)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2(cd^2-bde+ae^2)\sqrt{-b^2+4\,ac}}$$

input
```
integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/2*e^2*log(abs(e*x^2 + d))/(c*d^2*e - b*d*e^2 + a*e^3) - 1/4*e*log(c*x^4
+ b*x^2 + a)/(c*d^2 - b*d*e + a*e^2) + 1/2*(2*c*d - b*e)*arctan((2*c*x^2 +
 b)/sqrt(-b^2 + 4*a*c))/((c*d^2 - b*d*e + a*e^2)*sqrt(-b^2 + 4*a*c))
```

3.299.9 Mupad [B] (verification not implemented)

Time = 13.23 (sec) , antiderivative size = 2434, normalized size of antiderivative = 18.30

$$\int \frac{x}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
int(x/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

output
```
(e*log(d + e*x^2))/(2*a*e^2 + 2*c*d^2 - 2*b*d*e) - (log(36*a^4*c^3*e^5 - 4
*a*b^6*e^5 - 4*b^7*e^5*x^2 + 32*a^2*b^4*c*e^5 + 36*a^2*c^5*d^4*e - 4*a*c^6
*d^5*x^2 - 4*b^6*e^5*x^2*(b^2 - 4*a*c)^(1/2) - 73*a^3*b^2*c^2*e^5 - 184*a^
3*c^4*d^2*e^3 + b^2*c^5*d^5*x^2 - 4*a*b^5*e^5*(b^2 - 4*a*c)^(1/2) + 2*a*c^
5*d^5*(b^2 - 4*a*c)^(1/2) + 16*a*b^5*c*d*e^4 - 60*a^2*c^4*d^3*e^2*(b^2 - 4
*a*c)^(1/2) + 18*a^3*c^3*e^5*x^2*(b^2 - 4*a*c)^(1/2) + 146*a^2*b^2*c^3*d^2
*e^3 - 101*a^2*b^3*c^2*e^5*x^2 + 120*a^2*c^5*d^3*e^2*x^2 + 19*b^4*c^3*d^3*
e^2*x^2 - 25*b^5*c^2*d^2*e^3*x^2 - 9*a*b^2*c^4*d^4*e + 184*a^3*b*c^3*d*e^4
 + 36*a*b^5*c*e^5*x^2 + 16*b^6*c*d*e^4*x^2 + 24*a^2*b^3*c*e^5*(b^2 - 4*a*c
)^(1/2) - 33*a^3*b*c^2*e^5*(b^2 - 4*a*c)^(1/2) + 66*a^3*c^3*d*e^4*(b^2 - 4
*a*c)^(1/2) + b*c^5*d^5*x^2*(b^2 - 4*a*c)^(1/2) + 18*a*b^3*c^3*d^3*e^2 - 2
5*a*b^4*c^2*d^2*e^3 - 72*a^2*b*c^4*d^3*e^2 - 110*a^2*b^3*c^2*d*e^4 + 84*a^
3*b*c^3*e^5*x^2 - 132*a^3*c^4*d*e^4*x^2 - 7*b^3*c^4*d^4*e*x^2 + 28*a*b^4*c
*e^5*x^2*(b^2 - 4*a*c)^(1/2) + 18*a*c^5*d^4*e*x^2*(b^2 - 4*a*c)^(1/2) + 16
*b^5*c*d*e^4*x^2*(b^2 - 4*a*c)^(1/2) - 126*a*b^4*c^2*d*e^4*x^2 + 20*a*b^2*
c^3*d^3*e^2*(b^2 - 4*a*c)^(1/2) - 25*a*b^3*c^2*d^2*e^3*(b^2 - 4*a*c)^(1/2)
 + 90*a^2*b*c^3*d^2*e^3*(b^2 - 4*a*c)^(1/2) - 78*a^2*b^2*c^2*d*e^4*(b^2 -
4*a*c)^(1/2) - 7*b^2*c^4*d^4*e*x^2*(b^2 - 4*a*c)^(1/2) - 106*a*b^2*c^4*d^3
*e^2*x^2 + 168*a*b^3*c^3*d^2*e^3*x^2 - 272*a^2*b*c^4*d^2*e^3*x^2 + 281*a^2
*b^2*c^3*d*e^4*x^2 - 5*a*b*c^4*d^4*e*(b^2 - 4*a*c)^(1/2) + 16*a*b^4*c*d...
```

3.299. $\int \frac{x}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

3.300 $\int \frac{1}{x(d+ex^2)(a+bx^2+cx^4)}\,dx$

3.300.1 Optimal result

Integrand size = 27, antiderivative size = 167

$$\int \frac{1}{x(d+ex^2)(a+bx^2+cx^4)}\,dx = \frac{(bcd-b^2e+2ace)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)} + \frac{\log(x)}{ad} - \frac{e^2\log(d+ex^2)}{2d\,(cd^2-bde+ae^2)} - \frac{(cd-be)\log(a+bx^2+cx^4)}{4a\,(cd^2-bde+ae^2)}$$

output
```
ln(x)/a/d-1/2*e^2*ln(e*x^2+d)/d/(a*e^2-b*d*e+c*d^2)-1/4*(-b*e+c*d)*ln(c*x^
4+b*x^2+a)/a/(a*e^2-b*d*e+c*d^2)+1/2*(2*a*c*e-b^2*e+b*c*d)*arctanh((2*c*x^
2+b)/(-4*a*c+b^2)^(1/2))/a/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)
```

3.300.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.45

$$\int \frac{1}{x(d+ex^2)(a+bx^2+cx^4)}\,dx = \frac{4\sqrt{b^2-4ac}(cd^2+e(-bd+ae))\log(x) - d\left(bcd+c\sqrt{b^2-4ac}d-b^2e+2ace-b\sqrt{b^2-4ac}e\right)\log\left(b-\sqrt{b^2}\right.}{4a\sqrt{b^2}}$$

input
```
Integrate[1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

output
```
(4*Sqrt[b^2 - 4*a*c]*(c*d^2 + e*(-(b*d) + a*e))*Log[x] - d*(b*c*d + c*Sqrt
[b^2 - 4*a*c]*d - b^2*e + 2*a*c*e - b*Sqrt[b^2 - 4*a*c]*e)*Log[b - Sqrt[b^
2 - 4*a*c] + 2*c*x^2] + d*(b*c*d - c*Sqrt[b^2 - 4*a*c]*d - b^2*e + 2*a*c*e
 + b*Sqrt[b^2 - 4*a*c]*e)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2] - 2*a*Sqrt[
b^2 - 4*a*c]*e^2*Log[d + e*x^2])/(4*a*Sqrt[b^2 - 4*a*c]*d*(c*d^2 + e*(-(b*
d) + a*e)))
```

3.300.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1578, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (d+e x^2) (a+b x^2+c x^4)} dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2} \int \frac{1}{x^2 (e x^2+d) (c x^4+b x^2+a)} dx^2$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{2} \int \left(-\frac{e^3}{d (c d^2-b e d+a e^2) (e x^2+d)}+\frac{e b^2-c d b-c (c d-b e) x^2-a c e}{a (c d^2-b e d+a e^2) (c x^4+b x^2+a)}+\frac{1}{a d x^2}\right) dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(\frac{\text{arctanh}\left(\frac{b+2 c x^2}{\sqrt{b^2-4 a c}}\right) (2 a c e+b^2 (-e)+b c d)}{a \sqrt{b^2-4 a c} (a e^2-b d e+c d^2)}-\frac{e^2 \log (d+e x^2)}{d (a e^2-b d e+c d^2)}-\frac{(c d-b e) \log (a+b x^2+c x^4)}{2 a (a e^2-b d e+c d^2)}+\frac{\log (x^2)}{a d}\right)$$

input
```
Int[1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

output
```
(((b*c*d - b^2*e + 2*a*c*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(a*S
qrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) + Log[x^2]/(a*d) - (e^2*Log[d +
e*x^2])/(d*(c*d^2 - b*d*e + a*e^2)) - ((c*d - b*e)*Log[a + b*x^2 + c*x^4])
/(2*a*(c*d^2 - b*d*e + a*e^2)))/2
```

3.300.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.300.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.99

method	result
default	$\frac{\ln(x)}{ad}-\frac{\frac{\left(-ebc+c^2d\right)\ln\left(c\,x^4+b\,x^2+a\right)}{2c}+\frac{2\left(ace-b^2e+bcd-\frac{\left(-ebc+c^2d\right)b}{2c}\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2(a\,e^2-bde+c\,d^2)a}-\frac{e^2\ln(e\,x^2+d)}{2d(a\,e^2-bde+c\,d^2)}$
risch	$\frac{\ln(x)}{ad}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(4a^3c\,e^2-a^2b^2e^2-4a^2bcde+4a^2c^2d^2+a\,b^3de-a\,b^2c\,d^2\right)\_Z^2+\left(-4abce+4a\,c^2d+b^3e-b^2cd\right)\_Z+c^2\right)}\_R\ln\left(\left(\left(-12e\right.\right.\right.$

input
```
int(1/x/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
ln(x)/a/d-1/2/(a*e^2-b*d*e+c*d^2)/a*(1/2*(-b*c*e+c^2*d)/c*ln(c*x^4+b*x^2+a
)+2*(a*c*e-b^2*e+b*c*d-1/2*(-b*c*e+c^2*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2
*c*x^2+b)/(4*a*c-b^2)^(1/2)))-1/2*e^2*ln(e*x^2+d)/d/(a*e^2-b*d*e+c*d^2)
```

3.300.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.300.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/x/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.300.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.300.8 Giac [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.02

$$\int \frac{1}{x(d+ex^2)(a+bx^2+cx^4)}\,dx = -\frac{e^3\log(|ex^2+d|)}{2(cd^3e-bd^2e^2+ade^3)} - \frac{(cd-be)\log(cx^4+bx^2+a)}{4(acd^2-abde+a^2e^2)} - \frac{(bcd-b^2e+2\,ace)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2(acd^2-abde+a^2e^2)\sqrt{-b^2+4\,ac}} + \frac{\log(x^2)}{2\,ad}$$

input
```
integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*e^3*log(abs(e*x^2 + d))/(c*d^3*e - b*d^2*e^2 + a*d*e^3) - 1/4*(c*d -
b*e)*log(c*x^4 + b*x^2 + a)/(a*c*d^2 - a*b*d*e + a^2*e^2) - 1/2*(b*c*d - b
^2*e + 2*a*c*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a*c*d^2 - a*b*d
*e + a^2*e^2)*sqrt(-b^2 + 4*a*c)) + 1/2*log(x^2)/(a*d)
```

3.300.9 Mupad [B] (verification not implemented)

Time = 19.84 (sec) , antiderivative size = 6285, normalized size of antiderivative = 37.63

$$\int \frac{1}{x(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
int(1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

output
```
(log(256*a^4*e^8*(4*a*c - b^2)^4 - 80*c^4*d^8*(4*a*c - b^2)^4 - 61*d^4*e^4
*(4*a*c - b^2)^6 + 160*b^3*c^4*d^8*(b^2 - 4*a*c)^(5/2) + 16*b^5*c^4*d^8*(b
^2 - 4*a*c)^(3/2) - 184*b^3*d^4*e^4*(b^2 - 4*a*c)^(9/2) + 370*b^5*d^4*e^4*
(b^2 - 4*a*c)^(7/2) + 128*b^7*d^4*e^4*(b^2 - 4*a*c)^(5/2) + 5*b^9*d^4*e^4*
(b^2 - 4*a*c)^(3/2) + 128*a^3*e^8*x^2*(b^2 - 4*a*c)^(9/2) + 160*c^5*d^8*x^
2*(b^2 - 4*a*c)^(7/2) - 256*a^4*b^2*e^8*(4*a*c - b^2)^3 + 32*b^2*c^4*d^8*(
4*a*c - b^2)^3 + 112*b^4*c^4*d^8*(4*a*c - b^2)^2 - 144*a^2*d^2*e^6*(4*a*c
- b^2)^5 + 544*b^2*d^4*e^4*(4*a*c - b^2)^5 + 382*b^4*d^4*e^4*(4*a*c - b^2)
^4 - 152*b^6*d^4*e^4*(4*a*c - b^2)^3 + 71*b^8*d^4*e^4*(4*a*c - b^2)^2 + 20
0*c^2*d^6*e^2*(4*a*c - b^2)^5 - 13*d^3*e^5*x^2*(4*a*c - b^2)^6 + 512*a^4*b
*e^8*(b^2 - 4*a*c)^(7/2) - 176*b*c^4*d^8*(b^2 - 4*a*c)^(7/2) - 26*a*d^3*e^
5*(b^2 - 4*a*c)^(11/2) + 352*a^3*d*e^7*(b^2 - 4*a*c)^(9/2) - 319*b*d^4*e^4
*(b^2 - 4*a*c)^(11/2) + 148*c*d^5*e^3*(b^2 - 4*a*c)^(11/2) + 168*c^3*d^7*e
*(b^2 - 4*a*c)^(9/2) - 768*a*b^3*d^3*e^5*(4*a*c - b^2)^4 - 368*a*b^5*d^3*e
^5*(4*a*c - b^2)^3 + 128*a^3*b^3*d*e^7*(4*a*c - b^2)^3 - 32*a*b^7*d^3*e^5*
(4*a*c - b^2)^2 - 672*a^2*b^3*d^2*e^6*(b^2 - 4*a*c)^(7/2) - 272*a^2*b^5*d^
2*e^6*(b^2 - 4*a*c)^(5/2) + 408*b^3*c*d^5*e^3*(4*a*c - b^2)^4 + 256*b^3*c^
3*d^7*e*(4*a*c - b^2)^3 + 792*b^5*c*d^5*e^3*(4*a*c - b^2)^3 - 352*b^5*c^3*
d^7*e*(4*a*c - b^2)^2 - 248*b^7*c*d^5*e^3*(4*a*c - b^2)^2 - 328*b^3*c^2*d^
6*e^2*(b^2 - 4*a*c)^(7/2) + 1064*b^5*c^2*d^6*e^2*(b^2 - 4*a*c)^(5/2) + ...
```

3.300. $\int \frac{1}{x(d+ex^2)(a+bx^2+cx^4)}\,dx$

3.301 $\int \frac{1}{x^3(d+ex^2)(a+bx^2+cx^4)}\,dx$

3.301.1 Optimal result

Integrand size = 27, antiderivative size = 205

$$\int \frac{1}{x^3\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx = -\frac{1}{2adx^2} - \frac{\left(b^2cd-2ac^2d-b^3e+3abce\right)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}\left(cd^2-bde+ae^2\right)} - \frac{(bd+ae)\log(x)}{a^2d^2} + \frac{e^3\log\left(d+ex^2\right)}{2d^2\left(cd^2-bde+ae^2\right)} + \frac{\left(bcd-b^2e+ace\right)\log\left(a+bx^2+cx^4\right)}{4a^2\left(cd^2-bde+ae^2\right)}$$

output
```
-1/2/a/d/x^2-(a*e+b*d)*ln(x)/a^2/d^2+1/2*e^3*ln(e*x^2+d)/d^2/(a*e^2-b*d*e+
c*d^2)+1/4*(a*c*e-b^2*e+b*c*d)*ln(c*x^4+b*x^2+a)/a^2/(a*e^2-b*d*e+c*d^2)-1
/2*(3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1
/2))/a^2/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)
```

3.301.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.61

$$\int \frac{1}{x^3 (d+ex^2) (a+bx^2+cx^4)} \, dx = \frac{1}{4}\left(-\frac{2}{adx^2} - \frac{4(bd+ae)\log(x)}{a^2d^2} + \frac{\left(b^3e - bc\left(\sqrt{b^2-4ac}d+3ae\right) + ac\left(2cd-\sqrt{b^2-4ac}e\right) + b^2\left(-cd+\sqrt{b^2-4ac}e\right)\right)\log\left(b-\sqrt{b^2-4ac}+\right.}{a^2\sqrt{b^2-4ac}\,(-cd^2+e(bd-ae))} + \frac{\left(-b^3e + bc\left(-\sqrt{b^2-4ac}d+3ae\right) + b^2\left(cd+\sqrt{b^2-4ac}e\right) - ac\left(2cd+\sqrt{b^2-4ac}e\right)\right)\log\left(b+\sqrt{b^2-4ac}\right.}{a^2\sqrt{b^2-4ac}\,(-cd^2+e(bd-ae))} + \frac{2e^3\log(d+ex^2)}{cd^4+d^2e(-bd+ae)}\right)$$

input
```
Integrate[1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

output
```
(-2/(a*d*x^2) - (4*(b*d + a*e)*Log[x])/(a^2*d^2) + ((b^3*e - b*c*(Sqrt[b^2
 - 4*a*c]*d + 3*a*e) + a*c*(2*c*d - Sqrt[b^2 - 4*a*c]*e) + b^2*(-(c*d) + S
qrt[b^2 - 4*a*c]*e))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(a^2*Sqrt[b^2 -
 4*a*c]*(-(c*d^2) + e*(b*d - a*e))) + ((-(b^3*e) + b*c*(-(Sqrt[b^2 - 4*a*c
]*d) + 3*a*e) + b^2*(c*d + Sqrt[b^2 - 4*a*c]*e) - a*c*(2*c*d + Sqrt[b^2 -
4*a*c]*e))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(a^2*Sqrt[b^2 - 4*a*c]*(-
(c*d^2) + e*(b*d - a*e))) + (2*e^3*Log[d + e*x^2])/(c*d^4 + d^2*e*(-(b*d)
+ a*e)))/4
```

3.301.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1578, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (d+ex^2) (a+bx^2+cx^4)} \, dx$$

$$\Big\downarrow \text{ 1578}$$

$$\frac{1}{2}\int \frac{1}{x^4 (ex^2+d) (cx^4+bx^2+a)}dx^2$$

$$\Big\downarrow \ 1200$$

$$\frac{1}{2}\int \left(\frac{e^4}{d^2\left(cd^2-bed+ae^2\right)\left(ex^2+d\right)}+\frac{-eb^3+cdb^2+2aceb+c\left(-eb^2+cdb+ace\right)x^2-ac^2d}{a^2\left(cd^2-bed+ae^2\right)\left(cx^4+bx^2+a\right)}+\frac{-bd-ae}{a^2d^2x^2}+\frac{1}{adx\ldots}\right.$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(-\frac{\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(3abce-2ac^2d+b^3(-e)+b^2cd\right)}{a^2\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}+\frac{\left(ace+b^2(-e)+bcd\right)\log\left(a+bx^2+cx^4\right)}{2a^2\left(ae^2-bde+cd^2\right)}-\frac{\log\left(x^2\right)\ldots}{a\ldots}\right.$$

input
```
Int[1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

output
```
(-(1/(a*d*x^2)) - ((b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)*ArcTanh[(b +
2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2
)) - ((b*d + a*e)*Log[x^2])/(a^2*d^2) + (e^3*Log[d + e*x^2])/(d^2*(c*d^2 -
 b*d*e + a*e^2)) + ((b*c*d - b^2*e + a*c*e)*Log[a + b*x^2 + c*x^4])/(2*a^2
*(c*d^2 - b*d*e + a*e^2)))/2
```

3.301.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.301.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.05

method	result
default	$-\frac{1}{2ad\,x^2}+\frac{(-ae-bd)\ln(x)}{a^2d^2}+\frac{\frac{\left(a\,c^2e-b^2ce+b\,c^2d\right)\ln\left(c\,x^4+b\,x^2+a\right)}{2c}+\frac{2\left(2abce-a\,c^2d-b^3e+b^2cd-\frac{\left(a\,c^2e-b^2ce+b\,c^2d\right)b}{2c}\right)\arctan\left(\frac{2c\,x^2\ldots}{\sqrt{4ac-\ldots}}\right.}{\sqrt{4ac-b^2}}}{2(a\,e^2-bde+c\,d^2)a^2}$
risch	$-\frac{1}{2ad\,x^2}-\frac{e\ln(x)}{a\,d^2}-\frac{\ln(x)b}{a^2d}+\frac{e^3\ln(-e\,x^2-d)}{2d^2(a\,e^2-bde+c\,d^2)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^4c\,e^2-a^3b^2e^2-4a^3bcde+4a^3c^2d^2+a^2b^3de-a^2b^2c\,d^2\right)\_Z^2\ldots\right.}\right.}{\ldots}$

input
```
int(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/d/x^2+1/a^2/d^2*(-a*e-b*d)*ln(x)+1/2/(a*e^2-b*d*e+c*d^2)/a^2*(1/2*(
a*c^2*e-b^2*c*e+b*c^2*d)/c*ln(c*x^4+b*x^2+a)+2*(2*a*b*c*e-a*c^2*d-b^3*e+b^
2*c*d-1/2*(a*c^2*e-b^2*c*e+b*c^2*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2
+b)/(4*a*c-b^2)^(1/2)))+1/2*e^3*ln(e*x^2+d)/d^2/(a*e^2-b*d*e+c*d^2)
```

3.301.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.301.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/x**3/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.301.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (d + ex^2) (a + bx^2 + cx^4)} dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.301.8 Giac [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.13

$$\begin{aligned}\int \frac{1}{x^3 (d + ex^2) (a + bx^2 + cx^4)} dx &= \frac{e^4 \log(|ex^2 + d|)}{2 (cd^4e - bd^3e^2 + ad^2e^3)} \\ &+ \frac{(bcd - b^2e + ace) \log(cx^4 + bx^2 + a)}{4 (a^2cd^2 - a^2bde + a^3e^2)} \\ &+ \frac{(b^2cd - 2 ac^2d - b^3e + 3 abce) \arctan\left(\frac{2 cx^2 + b}{\sqrt{-b^2 + 4 ac}}\right)}{2 (a^2cd^2 - a^2bde + a^3e^2)\sqrt{-b^2 + 4 ac}} \\ &- \frac{(bd + ae) \log(x^2)}{2 a^2d^2} + \frac{bdx^2 + aex^2 - ad}{2 a^2d^2x^2}\end{aligned}$$

input
```
integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/2*e^4*log(abs(e*x^2 + d))/(c*d^4*e - b*d^3*e^2 + a*d^2*e^3) + 1/4*(b*c*d
 - b^2*e + a*c*e)*log(c*x^4 + b*x^2 + a)/(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)
 + 1/2*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)*arctan((2*c*x^2 + b)/sqrt
(-b^2 + 4*a*c))/((a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*sqrt(-b^2 + 4*a*c)) - 1
/2*(b*d + a*e)*log(x^2)/(a^2*d^2) + 1/2*(b*d*x^2 + a*e*x^2 - a*d)/(a^2*d^2
*x^2)
```

3.301.9 Mupad [B] (verification not implemented)

Time = 65.89 (sec) , antiderivative size = 5368, normalized size of antiderivative = 26.19

$$\int \frac{1}{x^3 (d + ex^2) (a + bx^2 + cx^4)} dx = \text{Too large to display}$$

input
```
int(1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

output
```
(log((((((4*c^2*e^2*(a*c^6*d^7 - 4*a^2*b^5*e^7 - 4*b^2*c^5*d^7 - 4*b^7*d^2
*e^5 + 28*a^3*b^3*c*e^7 - 48*a^4*b*c^2*e^7 + 8*b^3*c^4*d^6*e + 8*b^6*c*d^3
*e^4 - 16*a^2*c^5*d^5*e^2 + 16*a^3*c^4*d^3*e^4 - 4*b^4*c^3*d^5*e^2 - 4*b^5
*c^2*d^4*e^3 - 7*a*b^6*d*e^6 - 20*a*b*c^5*d^6*e + 56*a^2*b^2*c^3*d^3*e^4 -
 76*a^2*b^3*c^2*d^2*e^5 + 32*a*b^5*c*d^2*e^5 + 46*a^2*b^4*c*d*e^6 + 20*a*b
^2*c^4*d^5*e^2 + 6*a*b^3*c^3*d^4*e^3 - 44*a*b^4*c^2*d^3*e^4 + 22*a^2*b*c^4
*d^4*e^3 + 48*a^3*b*c^3*d^2*e^5 - 75*a^3*b^2*c^2*d*e^6))/(a^2*d^2) + (((16
*c^2*e^2*(a^3*b^4*e^7 + 16*a^5*c^2*e^7 + b^3*c^4*d^7 + b^7*d^3*e^4 - 8*a^4
*b^2*c*e^7 + 2*a*b^6*d^2*e^5 + 2*a^2*b^5*d*e^6 - 4*a^2*c^5*d^6*e - 4*b^4*c
^3*d^6*e - 4*b^6*c*d^4*e^3 + 20*a^3*c^4*d^4*e^3 - 32*a^4*c^3*d^2*e^5 + 6*b
^5*c^2*d^5*e^2 - a*b*c^5*d^7 - 52*a^2*b^2*c^3*d^4*e^3 + 45*a^2*b^3*c^2*d^3
*e^4 + 48*a^3*b^2*c^2*d^2*e^5 + 11*a*b^2*c^4*d^6*e - 12*a*b^5*c*d^3*e^4 -
15*a^3*b^3*c*d*e^6 + 28*a^4*b*c^2*d*e^6 - 27*a*b^3*c^3*d^5*e^2 + 27*a*b^4*
c^2*d^4*e^3 + 27*a^2*b*c^4*d^5*e^2 - 18*a^2*b^4*c*d^2*e^5 - 52*a^3*b*c^3*d
^3*e^4))/(a*d) + (8*c^2*e^2*x^2*(10*a*c^6*d^7 + a^2*b^5*e^7 + b^2*c^5*d^7
+ b^7*d^2*e^5 - 11*a^3*b^3*c*e^7 + 28*a^4*b*c^2*e^7 - 88*a^4*c^3*d*e^6 - 6
*b^3*c^4*d^6*e - 6*b^6*c*d^3*e^4 + 26*a^2*c^5*d^5*e^2 + 88*a^3*c^4*d^3*e^4
 + 5*b^4*c^3*d^5*e^2 + 5*b^5*c^2*d^4*e^3 + 12*a*b^6*d*e^6 - 3*a*b*c^5*d^6*
e - 110*a^2*b^2*c^3*d^3*e^4 + 155*a^2*b^3*c^2*d^2*e^5 - 28*a*b^5*c*d^2*e^5
 - 93*a^2*b^4*c*d*e^6 - 10*a*b^2*c^4*d^5*e^2 - 27*a*b^3*c^3*d^4*e^3 + 4...
```

3.302 $\int \frac{1}{x^5(d+ex^2)(a+bx^2+cx^4)}\,dx$

3.302.1 Optimal result

Integrand size = 27, antiderivative size = 268

$$\begin{aligned}\int \frac{1}{x^5(d+ex^2)(a+bx^2+cx^4)}\,dx &= -\frac{1}{4adx^4}+\frac{bd+ae}{2a^2d^2x^2}+\frac{(b^3cd-3abc^2d-b^4e+4ab^2ce-2a^2c^2e)\,\mathrm{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)}\\&\quad+\frac{(b^2d^2+abde-a(cd^2-ae^2))\log(x)}{a^3d^3}-\frac{e^4\log(d+ex^2)}{2d^3(cd^2-bde+ae^2)}\\&\quad-\frac{(b^2cd-ac^2d-b^3e+2abce)\log(a+bx^2+cx^4)}{4a^3(cd^2-bde+ae^2)}\end{aligned}$$

output
```
-1/4/a/d/x^4+1/2*(a*e+b*d)/a^2/d^2/x^2+(b^2*d^2+a*b*d*e-a*(-a*e^2+c*d^2))*
ln(x)/a^3/d^3-1/2*e^4*ln(e*x^2+d)/d^3/(a*e^2-b*d*e+c*d^2)-1/4*(2*a*b*c*e-a
*c^2*d-b^3*e+b^2*c*d)*ln(c*x^4+b*x^2+a)/a^3/(a*e^2-b*d*e+c*d^2)+1/2*(-2*a^
2*c^2*e+4*a*b^2*c*e-3*a*b*c^2*d-b^4*e+b^3*c*d)*arctanh((2*c*x^2+b)/(-4*a*c
+b^2)^(1/2))/a^3/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)
```

3.302.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.59

$$\int \frac{1}{x^5 (d+ex^2) (a+bx^2+cx^4)} \, dx$$
$$= \frac{1}{4}\left(-\frac{1}{adx^4}+\frac{2(bd+ae)}{a^2d^2x^2}+\frac{4(b^2d^2+abde+a(-cd^2+ae^2))\log(x)}{a^3d^3}\right.$$
$$-\frac{(b^4e+ac^2(\sqrt{b^2-4ac}d+2ae)-b^2c(\sqrt{b^2-4ac}d+4ae)+abc(3cd-2\sqrt{b^2-4ac}e)+b^3(-cd+\sqrt{b^2-}}{a^3\sqrt{b^2-4ac}(-cd^2+e(bd-ae))}$$
$$-\frac{(-b^4e+ac^2(\sqrt{b^2-4ac}d-2ae)+b^2c(-\sqrt{b^2-4ac}d+4ae)+b^3(cd+\sqrt{b^2-4ac}e)-abc(3cd+2\sqrt{b^2}}{a^3\sqrt{b^2-4ac}(-cd^2+e(bd-ae))}$$
$$\left.-\frac{2e^4\log(d+ex^2)}{cd^5+d^3e(-bd+ae)}\right)$$

input
```
Integrate[1/(x^5*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

output
```
(-(1/(a*d*x^4)) + (2*(b*d + a*e))/(a^2*d^2*x^2) + (4*(b^2*d^2 + a*b*d*e +
a*(-(c*d^2) + a*e^2))*Log[x])/(a^3*d^3) - ((b^4*e + a*c^2*(Sqrt[b^2 - 4*a*
c]*d + 2*a*e) - b^2*c*(Sqrt[b^2 - 4*a*c]*d + 4*a*e) + a*b*c*(3*c*d - 2*Sqr
t[b^2 - 4*a*c]*e) + b^3*(-(c*d) + Sqrt[b^2 - 4*a*c]*e))*Log[b - Sqrt[b^2 -
 4*a*c] + 2*c*x^2])/(a^3*Sqrt[b^2 - 4*a*c]*(-(c*d^2) + e*(b*d - a*e))) - (
(-(b^4*e) + a*c^2*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + b^2*c*(-(Sqrt[b^2 - 4*a*
c]*d) + 4*a*e) + b^3*(c*d + Sqrt[b^2 - 4*a*c]*e) - a*b*c*(3*c*d + 2*Sqrt[b
^2 - 4*a*c]*e))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(a^3*Sqrt[b^2 - 4*a*
c]*(-(c*d^2) + e*(b*d - a*e))) - (2*e^4*Log[d + e*x^2])/(c*d^5 + d^3*e*(-(
b*d) + a*e)))/4
```

3.302.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1578, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (d+ex^2) (a+bx^2+cx^4)} \, dx$$

$$\Big\downarrow \; 1578$$

$$\frac{1}{2}\int \frac{1}{x^6\left(ex^2+d\right)\left(cx^4+bx^2+a\right)}dx^2$$

$$\Big\downarrow \; 1200$$

$$\frac{1}{2}\int \left(-\frac{e^5}{d^3\left(cd^2-bed+ae^2\right)\left(ex^2+d\right)}+\frac{eb^4-cdb^3-3aceb^2+2ac^2db-c\left(-eb^3+cdb^2+2aceb-ac^2d\right)x^2+a^2c\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{\log\left(x^2\right)\left(abde-a\left(cd^2-ae^2\right)+b^2d^2\right)}{a^3d^3}-\frac{\left(2abce-ac^2d+b^3(-e)+b^2cd\right)\log\left(a+bx^2+cx^4\right)}{2a^3\left(ae^2-bde+cd^2\right)}+\frac{ae+bd}{a^2d^2x^2}+\frac{\text{ar}}{}\right.$$

input
```
Int[1/(x^5*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

output
```
(-1/2*1/(a*d*x^4) + (b*d + a*e)/(a^2*d^2*x^2) + ((b^3*c*d - 3*a*b*c^2*d -
b^4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]
])/(a^3*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) + ((b^2*d^2 + a*b*d*e -
 a*(c*d^2 - a*e^2))*Log[x^2])/(a^3*d^3) - (e^4*Log[d + e*x^2])/(d^3*(c*d^2
 - b*d*e + a*e^2)) - ((b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*e)*Log[a + b*x^
2 + c*x^4])/(2*a^3*(c*d^2 - b*d*e + a*e^2)))/2
```

3.302.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.302.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.06

method	result
default	$-\frac{1}{4ad\,x^{4}}-\frac{-ae-bd}{2a^{2}d^{2}x^{2}}+\frac{\left(e^{2}a^{2}+abde-d^{2}ac+b^{2}d^{2}\right)\ln \left(x \right)}{d^{3}a^{3}}+\frac{\frac{\left(-2ab\,c^{2}e+a\,c^{3}d+b^{3}ce-b^{2}c^{2}d\right)\ln \left(c\,x^{4}+b\,x^{2}+a\right)}{2c}+\frac{2\left(a^{2}c^{2}e-3a\,b^{2}ce+2ab \right.}{}}{2(a\,e^{2}-bde-}$
risch	Expression too large to display

input
```
int(1/x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4/a/d/x^4-1/2*(-a*e-b*d)/a^2/d^2/x^2+(a^2*e^2+a*b*d*e-a*c*d^2+b^2*d^2)/
d^3/a^3*ln(x)+1/2/(a*e^2-b*d*e+c*d^2)/a^3*(1/2*(-2*a*b*c^2*e+a*c^3*d+b^3*c
*e-b^2*c^2*d)/c*ln(c*x^4+b*x^2+a)+2*(a^2*c^2*e-3*a*b^2*c*e+2*a*b*c^2*d+b^4
*e-b^3*c*d-1/2*(-2*a*b*c^2*e+a*c^3*d+b^3*c*e-b^2*c^2*d)*b/c)/(4*a*c-b^2)^(
1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))-1/2*e^4*ln(e*x^2+d)/d^3/(a*e^2
-b*d*e+c*d^2)
```

3.302.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^5\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.302.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^5\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/x**5/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.302.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^5(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.302.8 Giac [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.22

$$\begin{aligned}\int \frac{1}{x^5(d+ex^2)(a+bx^2+cx^4)}\,dx &= -\frac{e^5\log(|ex^2+d|)}{2(cd^5e-bd^4e^2+ad^3e^3)} - \frac{(b^2cd-ac^2d-b^3e+2\,abce)\log(cx^4+bx^2+a)}{4(a^3cd^2-a^3bde+a^4e^2)} \\ &\quad - \frac{(b^3cd-3\,abc^2d-b^4e+4\,ab^2ce-2\,a^2c^2e)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2(a^3cd^2-a^3bde+a^4e^2)\sqrt{-b^2+4\,ac}} \\ &\quad + \frac{(b^2d^2-acd^2+abde+a^2e^2)\log(x^2)}{2\,a^3d^3} \\ &\quad - \frac{3\,b^2d^2x^4-3\,acd^2x^4+3\,abdex^4+3\,a^2e^2x^4-2\,abd^2x^2-2\,a^2dex^2+a^2d^2}{4\,a^3d^3x^4}\end{aligned}$$

input
```
integrate(1/x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*e^5*log(abs(e*x^2 + d))/(c*d^5*e - b*d^4*e^2 + a*d^3*e^3) - 1/4*(b^2*
c*d - a*c^2*d - b^3*e + 2*a*b*c*e)*log(c*x^4 + b*x^2 + a)/(a^3*c*d^2 - a^3
*b*d*e + a^4*e^2) - 1/2*(b^3*c*d - 3*a*b*c^2*d - b^4*e + 4*a*b^2*c*e - 2*a
^2*c^2*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a^3*c*d^2 - a^3*b*d*e
 + a^4*e^2)*sqrt(-b^2 + 4*a*c)) + 1/2*(b^2*d^2 - a*c*d^2 + a*b*d*e + a^2*e
^2)*log(x^2)/(a^3*d^3) - 1/4*(3*b^2*d^2*x^4 - 3*a*c*d^2*x^4 + 3*a*b*d*e*x^
4 + 3*a^2*e^2*x^4 - 2*a*b*d^2*x^2 - 2*a^2*d*e*x^2 + a^2*d^2)/(a^3*d^3*x^4)
```

3.302.9 Mupad [B] (verification not implemented)

Time = 142.00 (sec) , antiderivative size = 10300, normalized size of antiderivative = 38.43

$$\int \frac{1}{x^5 (d + ex^2) (a + bx^2 + cx^4)} dx = \text{Too large to display}$$

input
```
int(1/(x^5*(d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

output
```
(log((c^8*e^8*(a^2*e^2 + b^2*d^2 - a*c*d^2 + a*b*d*e))/(a^6*d^6) - (c^9*e^
9*x^2)/(a^5*d^5) - (((c^5*e^5*(4*a^3*b^3*e^6 + 4*b^3*c^3*d^6 + 4*b^6*d^3*e
^3 + 8*a*b^5*d^2*e^4 + 8*a^2*b^4*d*e^5 + 4*a^2*c^4*d^5*e + 16*a^4*c^2*d*e^
5 - 19*a^3*c^3*d^3*e^3 - 4*a*b*c^4*d^6 - 12*a^4*b*c*e^6 + 36*a^2*b^2*c^2*d
^3*e^3 - 24*a*b^4*c*d^3*e^3 - 32*a^3*b^2*c*d*e^5 - 36*a^2*b^3*c*d^2*e^4 +
28*a^3*b*c^2*d^2*e^4))/(a^6*d^6) - (((4*a^4*b^6*c^2*e^12 - 24*a^5*b^4*c^3*
e^12 + 36*a^6*b^2*c^4*e^12 - 4*a^3*c^9*d^8*e^4 + 64*a^4*c^8*d^6*e^6 - 144*
a^5*c^7*d^4*e^8 + 96*a^6*c^6*d^2*e^10 + 4*b^4*c^8*d^10*e^2 + 8*b^7*c^5*d^7
*e^5 + 4*b^10*c^2*d^4*e^8 + 64*a^2*b^3*c^7*d^7*e^5 - 8*a^2*b^4*c^6*d^6*e^6
 - 8*a^2*b^5*c^5*d^5*e^7 + 172*a^2*b^6*c^4*d^4*e^8 - 112*a^2*b^7*c^3*d^3*e
^9 + 16*a^2*b^8*c^2*d^2*e^10 - 72*a^3*b^2*c^7*d^6*e^6 + 56*a^3*b^3*c^6*d^5
*e^7 - 312*a^3*b^4*c^5*d^4*e^8 + 348*a^3*b^5*c^4*d^3*e^9 - 132*a^3*b^6*c^3
*d^2*e^10 + 324*a^4*b^2*c^6*d^4*e^8 - 428*a^4*b^3*c^5*d^3*e^9 + 344*a^4*b^
4*c^4*d^2*e^10 - 300*a^5*b^2*c^5*d^2*e^10 - 96*a^6*b*c^5*d*e^11 - 4*a*b^2*
c^9*d^10*e^2 - 4*a*b^3*c^8*d^9*e^3 - 48*a*b^5*c^6*d^7*e^5 + 8*a*b^6*c^5*d^
6*e^6 - 44*a*b^8*c^3*d^4*e^8 + 12*a*b^9*c^2*d^3*e^9 + 8*a^2*b*c^9*d^9*e^3
- 24*a^3*b*c^8*d^7*e^5 + 12*a^3*b^7*c^2*d*e^11 - 88*a^4*b*c^7*d^5*e^7 - 88
*a^4*b^5*c^3*d*e^11 + 228*a^5*b*c^6*d^3*e^9 + 188*a^5*b^3*c^4*d*e^11)/(a^6
*d^6) + (x^2*(32*a^6*c^6*d*e^11 - 24*a^6*b*c^5*e^12 + 4*a^3*b^7*c^2*e^12 -
 28*a^4*b^5*c^3*e^12 + 56*a^5*b^3*c^4*e^12 + 2*a^3*c^9*d^7*e^5 + 104*a^...
```

3.303 $\int \frac{x^8}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

3.303.1 Optimal result

Integrand size = 27, antiderivative size = 387

$$\int \frac{x^8}{(d+ex^2)(a+bx^2+cx^4)}\,dx = -\frac{(cd+be)x}{c^2e^2}+\frac{x^3}{3ce} - \frac{\left(b^3d-2abcd-ab^2e+a^2ce-\frac{b^4d-4ab^2cd+2a^2c^2d-ab^3e+3a^2bce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} - \frac{\left(b^3d-2abcd-ab^2e+a^2ce+\frac{b^4d-4ab^2cd+2a^2c^2d-ab^3e+3a^2bce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} + \frac{d^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{5/2}(cd^2-bde+ae^2)}$$

output
```
-(b*e+c*d)*x/c^2/e^2+1/3*x^3/c/e+d^(7/2)*arctan(x*e^(1/2)/d^(1/2))/e^(5/2)
/(a*e^2-b*d*e+c*d^2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(
1/2))*(b^3*d-2*a*b*c*d-a*b^2*e+a^2*c*e+(-3*a^2*b*c*e-2*a^2*c^2*d+a*b^3*e+4
*a*b^2*c*d-b^4*d)/(-4*a*c+b^2)^(1/2))/c^(5/2)/(a*e^2-b*d*e+c*d^2)*2^(1/2)/
(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^
(1/2))^(1/2))*(b^3*d-2*a*b*c*d-a*b^2*e+a^2*c*e+(3*a^2*b*c*e+2*a^2*c^2*d-a*
b^3*e-4*a*b^2*c*d+b^4*d)/(-4*a*c+b^2)^(1/2))/c^(5/2)/(a*e^2-b*d*e+c*d^2)*2
^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.303.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.20

$$\int \frac{x^8}{(d+ex^2)(a+bx^2+cx^4)}\,dx = -\frac{(cd+be)x}{c^2e^2}+\frac{x^3}{3ce}$$
$$+\frac{\left(-b^4d+b^3\left(\sqrt{b^2-4ac}d+ae\right)-abc\left(2\sqrt{b^2-4ac}d+3ae\right)+ab^2\left(4cd-\sqrt{b^2-4ac}e\right)+a^2c\left(-2cd+\sqrt{b^2\right.\right.}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\,(-cd^2+e(bd-ae))}$$
$$+\frac{\left(b^4d+b^3\left(\sqrt{b^2-4ac}d-ae\right)+abc\left(-2\sqrt{b^2-4ac}d+3ae\right)+a^2c\left(2cd+\sqrt{b^2-4ac}e\right)-ab^2\left(4cd+\sqrt{b^2-\right.\right.}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}\,(-cd^2+e(bd-ae))}$$
$$+\frac{d^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{5/2}\left(cd^2-bde+ae^2\right)}$$

input
```
Integrate[x^8/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

output
```
-(((c*d + b*e)*x)/(c^2*e^2)) + x^3/(3*c*e) + ((-(b^4*d) + b^3*(Sqrt[b^2 -
4*a*c]*d + a*e) - a*b*c*(2*Sqrt[b^2 - 4*a*c]*d + 3*a*e) + a*b^2*(4*c*d - S
qrt[b^2 - 4*a*c]*e) + a^2*c*(-2*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a
*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + e*(b*d - a*e))) + ((b^4*d + b^
3*(Sqrt[b^2 - 4*a*c]*d - a*e) + a*b*c*(-2*Sqrt[b^2 - 4*a*c]*d + 3*a*e) + a
^2*c*(2*c*d + Sqrt[b^2 - 4*a*c]*e) - a*b^2*(4*c*d + Sqrt[b^2 - 4*a*c]*e))*
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*
Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + e*(b*d - a*e)))
+ (d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(5/2)*(c*d^2 - b*d*e + a*e^2))
```

3.303.3 Rubi [A] (verified)

Time = 2.60 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1610, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(d+ex^2)(a+bx^2+cx^4)}\,dx$$
$$\Big\downarrow \text{1610}$$

$$\int \left(\frac{-\left(x^2\left(a^2ce-ab^2e-2abcd+b^3d\right)\right)-a\left(-abe-acd+b^2d\right)}{c^2\left(a+bx^2+cx^4\right)\left(ae^2-bde+cd^2\right)}+\frac{d^4}{e^2\left(d+ex^2\right)\left(ae^2-bde+cd^2\right)}+\frac{-be-cd}{c^2e^2}+\frac{x}{c}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d}{\sqrt{b^2-4ac}}+a^2ce-ab^2e-2abcd+b^3d\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}\left(ae^2-bde+cd^2\right)}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d}{\sqrt{b^2-4ac}}+a^2ce-ab^2e-2abcd+b^3d\right)}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}\left(ae^2-bde+cd^2\right)}+\frac{d^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{5/2}\left(ae^2-bde+cd^2\right)}-\frac{x(be+cd)}{c^2e^2}+\frac{x^3}{3ce}$$

input
```
Int[x^8/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

output
```
-(((c*d + b*e)*x)/(c^2*e^2)) + x^3/(3*c*e) - ((b^3*d - 2*a*b*c*d - a*b^2*e
 + a^2*c*e - (b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d - a*b^3*e + 3*a^2*b*c*e)/S
qrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/(Sqrt[2]*c^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - (
(b^3*d - 2*a*b*c*d - a*b^2*e + a^2*c*e + (b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*
d - a*b^3*e + 3*a^2*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*
(c*d^2 - b*d*e + a*e^2)) + (d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(5/2)*
(c*d^2 - b*d*e + a*e^2))
```

3.303.3.1 Defintions of rubi rules used

rule 1610
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.303.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.07

method	result
default	$-\frac{-\frac{1}{3}c x^{3} e+bex+cdx}{e^{2} c^{2}}+\frac{\frac{\left(-a^{2} c e \sqrt{-4ac+b^{2}}+a b^{2} e \sqrt{-4ac+b^{2}}+2abcd \sqrt{-4ac+b^{2}}-b^{3} d \sqrt{-4ac+b^{2}}-3a^{2} bce-2a^{2} c^{2} d+a b^{3} e+4a b^{2} cd-d b^{4}\right)\sqrt{2}\, a}{2c\sqrt{-4ac+b^{2}}\,\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}}{}$
risch	Expression too large to display

input
```
int(x^8/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/e^2/c^2*(-1/3*c*x^3*e+b*e*x+c*d*x)+4/(a*e^2-b*d*e+c*d^2)/c*(1/8*(-a^2*c
*e*(-4*a*c+b^2)^(1/2)+a*b^2*e*(-4*a*c+b^2)^(1/2)+2*a*b*c*d*(-4*a*c+b^2)^(1
/2)-b^3*d*(-4*a*c+b^2)^(1/2)-3*a^2*b*c*e-2*a^2*c^2*d+a*b^3*e+4*a*b^2*c*d-d
*b^4)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/8*(-a^2*c*e*(-4*a*c+b^2)^
(1/2)+a*b^2*e*(-4*a*c+b^2)^(1/2)+2*a*b*c*d*(-4*a*c+b^2)^(1/2)-b^3*d*(-4*a*
c+b^2)^(1/2)+3*a^2*b*c*e+2*a^2*c^2*d-a*b^3*e-4*a*b^2*c*d+d*b^4)/c/(-4*a*c+
b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))+1/e^2*d^4/(a*e^2-b*d*e+c*d^2)/(e*d)^(1/
2)*arctan(e*x/(e*d)^(1/2))
```

3.303.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12250 vs. 2(341) = 682.

Time = 164.88 (sec) , antiderivative size = 24520, normalized size of antiderivative = 63.36

$$\int \frac{x^8}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(x^8/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.303.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^8}{(d+ex^2)\,(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x**8/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.303.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8}{(d+ex^2)\,(a+bx^2+cx^4)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^8/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.303.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12525 vs. 2(341) = 682.

Time = 2.85 (sec) , antiderivative size = 12525, normalized size of antiderivative = 32.36

$$\int \frac{x^8}{(d+ex^2)\,(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(x^8/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
d^4*arctan(e*x/sqrt(d*e))/((c*d^2*e^2 - b*d*e^3 + a*e^4)*sqrt(d*e)) + 1/8*
((2*b^7*c^8 - 16*a*b^5*c^9 + 36*a^2*b^3*c^10 - 16*a^3*b*c^11 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^6 + 8*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^7 - 18*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 - 8*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^8 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^9 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^9 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^9 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^10 - 2*(b^2 - 4*a*c)*b^5*c^8 + 8*(b^2 -
4*a*c)*a*b^3*c^9 - 4*(b^2 - 4*a*c)*a^2*b*c^10)*d^5 - (4*b^8*c^7 - 30*a*b^6
*c^8 + 58*a^2*b^4*c^9 - 8*a^3*b^2*c^10 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*b^8*c^5 + 15*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^6 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^7*c^6 - 29*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*b^4*c^7 - 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^5*c^7 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*b^6*c^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt...
```

3.303.9 Mupad [B] (verification not implemented)

Time = 11.97 (sec) , antiderivative size = 41755, normalized size of antiderivative = 107.89

$$\int \frac{x^8}{(d+ex^2)\,(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
int(x^8/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

output
```
atan(((((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 + 192*a^5*c^6*d^2*e^9
 - 48*a^2*b^2*c^7*d^6*e^5 + 96*a^2*b^3*c^6*d^5*e^6 - 48*a^2*b^4*c^5*d^4*e^
7 + 96*a^3*b^2*c^6*d^4*e^7 + 96*a^3*b^3*c^5*d^3*e^8 - 48*a^4*b^2*c^5*d^2*e
^9 - 384*a^3*b*c^7*d^5*e^6 - 384*a^4*b*c^6*d^3*e^8)/(c^3*e^3) - (2*x*(-(b^
9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2
 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 -
63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-
(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^
3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3
)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) -
5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c
^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a
*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*
c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e
 + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*
e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3
*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2)*(128*a^
4*b^2*c^6*e^12 - 16*a^3*b^4*c^5*e^12 - 256*a^5*c^7*e^12 + 256*a^2*c^10*d^6
*e^6 + 256*a^3*c^9*d^4*e^8 - 256*a^4*c^8*d^2*e^10 - 16*b^3*c^9*d^7*e^5 + 6
4*b^4*c^8*d^6*e^6 - 96*b^5*c^7*d^5*e^7 + 64*b^6*c^6*d^4*e^8 - 16*b^7*c^...
```

3.303. $\int \frac{x^8}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

3.304 $\int \frac{x^6}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

3.304.1 Optimal result

Integrand size = 27, antiderivative size = 323

$$\int \frac{x^6}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \frac{x}{ce}+\frac{\left(b^2d-acd-abe-\frac{b^3d-3abcd-ab^2e+2a^2ce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} + \frac{\left(b^2d-acd-abe+\frac{b^3d-3abcd-ab^2e+2a^2ce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} - \frac{d^{5/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}\,(cd^2-bde+ae^2)}$$

output
```
x/c/e-d^(5/2)*arctan(x*e^(1/2)/d^(1/2))/e^(3/2)/(a*e^2-b*d*e+c*d^2)+1/2*ar
ctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b^2*d-a*c*d-a*b*e+(-
2*a^2*c*e+a*b^2*e+3*a*b*c*d-b^3*d)/(-4*a*c+b^2)^(1/2))/c^(3/2)/(a*e^2-b*d*
e+c*d^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/2)
/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2*d-a*c*d-a*b*e+(2*a^2*c*e-a*b^2*e-3*a*b
*c*d+b^3*d)/(-4*a*c+b^2)^(1/2))/c^(3/2)/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(b+(-4
*a*c+b^2)^(1/2))^(1/2)
```

3.304.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.19

$$\int \frac{x^6}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \frac{x}{ce} + \frac{\left(b^3d-b^2\left(\sqrt{b^2-4ac}d+ae\right)+ac\left(\sqrt{b^2-4ac}d+2ae\right)+ab\left(-3cd+\sqrt{b^2-4ac}e\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\left(-cd^2+e(bd-ae)\right)} + \frac{\left(b^3d+b^2\left(\sqrt{b^2-4ac}d-ae\right)+ac\left(-\sqrt{b^2-4ac}d+2ae\right)-ab\left(3cd+\sqrt{b^2-4ac}e\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}\left(cd^2+e(-bd+ae)\right)} - \frac{d^{5/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}\left(cd^2-bde+ae^2\right)}$$

input
```
Integrate[x^6/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

output
```
x/(c*e) + ((b^3*d - b^2*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*c*(Sqrt[b^2 - 4*a*
c]*d + 2*a*e) + a*b*(-3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt
[b - Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + e*(b*d - a*e))) + ((b^3*d + b^2*(Sqrt[
b^2 - 4*a*c]*d - a*e) + a*c*(-(Sqrt[b^2 - 4*a*c]*d) + 2*a*e) - a*b*(3*c*d
+ Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a
*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d
^2 + e*(-(b*d) + a*e))) - (d^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(3/2)*(
c*d^2 - b*d*e + a*e^2))
```

3.304.3 Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1610, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(d+ex^2)(a+bx^2+cx^4)}\,dx$$

$$\Big\downarrow \text{ 1610}$$

$$\int \left(\frac{x^2(-abe-acd+b^2d)+a(bd-ae)}{c\,(a+bx^2+cx^4)\,(ae^2-bde+cd^2)}-\frac{d^3}{e\,(d+ex^2)\,(ae^2-bde+cd^2)}+\frac{1}{ce}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)}-\frac{d^{5/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}\,(ae^2-bde+cd^2)}+\frac{x}{ce}$$

input
```
Int[x^6/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

output
```
x/(c*e) + ((b^2*d - a*c*d - a*b*e - (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c
*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*
c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)
) + ((b^2*d - a*c*d - a*b*e + (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)/Sq
rt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/
(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - (d
^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(3/2)*(c*d^2 - b*d*e + a*e^2))
```

3.304.3.1 Defintions of rubi rules used

rule 1610
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.304.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.02

method	result
default	$\frac{x}{ce}+\frac{\frac{\left(-abe\sqrt{-4ac+b^2}-acd\sqrt{-4ac+b^2}+b^2d\sqrt{-4ac+b^2}+2a^2ce-a\,b^2e-3abcd+b^3d\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2c\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(-abe\sqrt{-4ac+b^2}-acd\right.}{}}{a\,e^2-bde+c\,d^2}$
risch	Expression too large to display

input
```
int(x^6/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
x/c/e+4/(a*e^2-b*d*e+c*d^2)*(1/8*(-a*b*e*(-4*a*c+b^2)^(1/2)-a*c*d*(-4*a*c+
b^2)^(1/2)+b^2*d*(-4*a*c+b^2)^(1/2)+2*a^2*c*e-a*b^2*e-3*a*b*c*d+b^3*d)/c/(
-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/8*(-a*b*e*(-4*a*c+b^2)^(1/2)-a*c*d
*(-4*a*c+b^2)^(1/2)+b^2*d*(-4*a*c+b^2)^(1/2)-2*a^2*c*e+a*b^2*e+3*a*b*c*d-b
^3*d)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
nh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))-1/e*d^3/(a*e^2-b*d*e+c*
d^2)/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2))
```

3.304.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10064 vs. 2(281) = 562.

Time = 25.69 (sec) , antiderivative size = 20147, normalized size of antiderivative = 62.37

$$\int \frac{x^6}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(x^6/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.304.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^6}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x**6/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.304.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^6}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^6/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.304.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11046 vs. 2(281) = 562.

Time = 2.44 (sec) , antiderivative size = 11046, normalized size of antiderivative = 34.20

$$\int \frac{x^6}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(x^6/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
-d^3*arctan(e*x/sqrt(d*e))/((c*d^2*e - b*d*e^2 + a*e^3)*sqrt(d*e)) - 1/8*(
(2*b^6*c^6 - 14*a*b^4*c^7 + 24*a^2*b^2*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^4 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*b^5*c^5 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^3*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*b^4*c^6 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b^2*c^7 - 2*(b^2 - 4*a*c)*b^4*c^6 + 6*(b^2 - 4*a*c)*a*b^2*c^
7)*d^5 - (4*b^7*c^5 - 26*a*b^5*c^6 + 36*a^2*b^3*c^7 + 16*a^3*b*c^8 - 2*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^3 + 13*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 + 4*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^4 - 18*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 - 10*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 2*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^5 - 8*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 - 4*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 + 5*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^6 + 2*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^7 - 4*(b^2 - 4*a*c)*b^5*c...
```

3.304.9 Mupad [B] (verification not implemented)

Time = 11.48 (sec) , antiderivative size = 33892, normalized size of antiderivative = 104.93

$$\int \frac{x^6}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
int(x^6/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

output
```
x/(c*e) - atan(((((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d
^3*e^6 - 144*a^2*b^2*c^5*d^5*e^4 + 64*a^2*b^3*c^4*d^4*e^5 + 16*a^2*b^4*c^3
*d^3*e^6 - 96*a^3*b^2*c^4*d^3*e^6 + 16*a^3*b^3*c^3*d^2*e^7 - 16*a*b^3*c^5*
d^6*e^3 + 32*a*b^4*c^4*d^5*e^4 - 16*a*b^5*c^3*d^4*e^5 + 64*a^2*b*c^6*d^6*e
^3 - 64*a^4*b*c^4*d^2*e^7 - 16*a^4*b^2*c^3*d*e^8)/(c*e) - (2*x*(-(b^7*d^2
+ a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^
3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*
b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*
c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^
3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c
 - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(
1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4
- 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e
^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c
^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3
)))^(1/2)*(128*a^4*b^2*c^4*e^10 - 16*a^3*b^4*c^3*e^10 - 256*a^5*c^5*e^10 +
 256*a^2*c^8*d^6*e^4 + 256*a^3*c^7*d^4*e^6 - 256*a^4*c^6*d^2*e^8 - 16*b^3*
c^7*d^7*e^3 + 64*b^4*c^6*d^6*e^4 - 96*b^5*c^5*d^5*e^5 + 64*b^6*c^4*d^4*e^6
 - 16*b^7*c^3*d^3*e^7 + 256*a^2*b^2*c^6*d^4*e^6 + 144*a^2*b^3*c^5*d^3*e^7
- 96*a^2*b^4*c^4*d^2*e^8 + 192*a^3*b^2*c^5*d^2*e^8 + 64*a*b*c^8*d^7*e^3...
```

3.304. $\int \frac{x^6}{(d+ex^2)(a+bx^2+cx^4)} \, dx$

3.305 $\int \frac{x^4}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

3.305.1 Optimal result

Integrand size = 27, antiderivative size = 280

$$\int \frac{x^4}{(d+ex^2)(a+bx^2+cx^4)}\,dx = -\frac{\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} - \frac{\left(bd-ae+\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} + \frac{d^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}\,(cd^2-bde+ae^2)}$$

output
```
d^(3/2)*arctan(x*e^(1/2)/d^(1/2))/(a*e^2-b*d*e+c*d^2)/e^(1/2)-1/2*arctan(x
*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b*d-a*e+(a*b*e+2*a*c*d-b^2
*d)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2
)^(1/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*
(b*d-a*e+(-a*b*e-2*a*c*d+b^2*d)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)*2^
(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.305.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.15

$$\int \frac{x^4}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \frac{\left(-b^2d+2acd+b\sqrt{b^2-4ac}d+abe-a\sqrt{b^2-4ac}e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\,(-cd^2+bde-ae^2)} + \frac{\left(b^2d-2acd+b\sqrt{b^2-4ac}d-abe-a\sqrt{b^2-4ac}e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}\,(-cd^2+bde-ae^2)} + \frac{d^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}\,(cd^2-bde+ae^2)}$$

input
```
Integrate[x^4/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

output
```
((-(b^2*d) + 2*a*c*d + b*Sqrt[b^2 - 4*a*c]*d + a*b*e - a*Sqrt[b^2 - 4*a*c]
*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt
[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + b*d*e - a*e^
2)) + ((b^2*d - 2*a*c*d + b*Sqrt[b^2 - 4*a*c]*d - a*b*e - a*Sqrt[b^2 - 4*a
*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*S
qrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + b*d*e - a
*e^2)) + (d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*(c*d^2 - b*d*e + a
*e^2))
```

3.305.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1610, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(d+ex^2)(a+bx^2+cx^4)}\,dx$$

$$\Big\downarrow \; 1610$$

$$\int \left(\frac{d^2}{(d+ex^2)(ae^2-bde+cd^2)} + \frac{-(x^2(bd-ae))-ad}{(a+bx^2+cx^4)(ae^2-bde+cd^2)}\right) dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)} - \frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)} + \frac{d^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}\,(ae^2-bde+cd^2)}$$

input
```
Int[x^4/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

output
```
-(((b*d - a*e - (b^2*d - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt
[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2))) - ((b*d - a*e + (b^2*d - 2*a*c*d
- a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a
*e^2)) + (d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*(c*d^2 - b*d*e + a
*e^2))
```

3.305.3.1 Defintions of rubi rules used

rule 1610
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.305.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.94

method	result
default	$\frac{4c\left(\frac{\left(ae\sqrt{-4ac+b^2}-bd\sqrt{-4ac+b^2}+abe+2acd-b^2d\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8c\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(ae\sqrt{-4ac+b^2}-bd\sqrt{-4ac+b^2}-abe-2acd+b^2d\right)\sqrt{2}\,\mathrm{arcta}}{8c\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b\right.}}}\right.}{a\,e^2-bde+c\,d^2}$
risch	Expression too large to display

input
```
int(x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
4/(a*e^2-b*d*e+c*d^2)*c*(1/8*(a*e*(-4*a*c+b^2)^(1/2)-b*d*(-4*a*c+b^2)^(1/2
)+a*b*e+2*a*c*d-b^2*d)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/8*(a*e*(
-4*a*c+b^2)^(1/2)-b*d*(-4*a*c+b^2)^(1/2)-a*b*e-2*a*c*d+b^2*d)/c/(-4*a*c+b^
2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))+d^2/(a*e^2-b*d*e+c*d^2)/(e*d)^(1/2)*arcta
n(e*x/(e*d)^(1/2))
```

3.305.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7767 vs. 2(236) = 472.

Time = 2.57 (sec) , antiderivative size = 15553, normalized size of antiderivative = 55.55

$$\int \frac{x^4}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.305.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x**4/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.305.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.305.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8670 vs. $2(236) = 472$.

Time = 2.27 (sec) , antiderivative size = 8670, normalized size of antiderivative = 30.96

$$\int \frac{x^4}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
d^2*arctan(e*x/sqrt(d*e))/((c*d^2 - b*d*e + a*e^2)*sqrt(d*e)) + 1/8*((2*b^
5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5)*d^5 - (4*b^
6*c^3 - 22*a*b^4*c^4 + 24*a^2*b^2*c^5 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 11*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^5*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*b^3*c^3 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*b^4*c^3 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b^2*c^4 - 4*(b^2 - 4*a*c)*b^4*c^3 + 6*(b^2 - 4*a*c)*a*b^2*c^4)
*d^4*e + (2*b^7*c^2 - 4*a*b^5*c^3 - 24*a^2*b^3*c^4 + 32*a^3*b*c^5 - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 2*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^...
```

3.305.9 Mupad [B] (verification not implemented)

Time = 10.98 (sec) , antiderivative size = 25202, normalized size of antiderivative = 90.01

$$\int \frac{x^4}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
int(x^4/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

output
```
atan((((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d
^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d
^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 1
2*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 +
 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^
2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5
*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 -
6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*((x*(8*a^3*b^3*c*e^7 -
 32*a^4*b*c^2*e^7 - 112*a^4*c^3*d*e^6 + 8*b^3*c^4*d^6*e + 8*b^6*c*d^3*e^4
- 112*a^2*c^5*d^5*e^2 + 32*a^3*c^4*d^3*e^4 - 8*b^4*c^3*d^5*e^2 - 8*b^5*c^2
*d^4*e^3 - 32*a*b*c^5*d^6*e - 48*a^2*b^2*c^3*d^3*e^4 + 8*a^2*b^3*c^2*d^2*e
^5 - 8*a*b^5*c*d^2*e^5 - 8*a^2*b^4*c*d*e^6 + 64*a*b^2*c^4*d^5*e^2 + 8*a*b^
3*c^3*d^4*e^3 - 16*a*b^4*c^2*d^3*e^4 + 64*a^2*b*c^4*d^4*e^3 + 64*a^3*b*c^3
*d^2*e^5 + 64*a^3*b^2*c^2*d*e^6) + (-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4
*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2
 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*
b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)
^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^
4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 +
32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^...
```

3.305. $\int \frac{x^4}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

3.306 $\int \frac{x^2}{(d+ex^2)(a+bx^2+cx^4)} \, dx$

3.306.1 Optimal result

Integrand size = 27, antiderivative size = 251

$$\int \frac{x^2}{(d+ex^2)(a+bx^2+cx^4)} \, dx = \frac{\sqrt{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} + \frac{\sqrt{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} - \frac{\sqrt{d}\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{cd^2-bde+ae^2}$$

output
```
-arctan(x*e^(1/2)/d^(1/2))*d^(1/2)*e^(1/2)/(a*e^2-b*d*e+c*d^2)+1/2*arctan(
x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(d+(2*a*e-b*d)/(-4
*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+
1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(d+(-2*
a*e+b*d)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(b+(-4*a*c+b^2)^(
1/2))^(1/2)
```

3.306.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.10

$$\int \frac{x^2}{(d+ex^2)(a+bx^2+cx^4)}\,dx = -\frac{\sqrt{c}\left(-bd+\sqrt{b^2-4ac}d+2ae\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\left(-cd^2+bde-ae^2\right)} - \frac{\sqrt{c}\left(bd+\sqrt{b^2-4ac}d-2ae\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}\left(-cd^2+bde-ae^2\right)} - \frac{\sqrt{d}\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{cd^2-bde+ae^2}$$

input
```
Integrate[x^2/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

output
```
-((Sqrt[c]*(-(b*d) + Sqrt[b^2 - 4*a*c]*d + 2*a*e)*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[
b^2 - 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2))) - (Sqrt[c]*(b*d + Sqrt[b^2 - 4*
a*c]*d - 2*a*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(
Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + b*d*e -
a*e^2)) - (Sqrt[d]*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 - b*d*e + a
*e^2)
```

3.306.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1610, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(d+ex^2)(a+bx^2+cx^4)}\,dx$$

$$\Big\downarrow \text{1610}$$

$$\int \left(\frac{ae+cdx^2}{(a+bx^2+cx^4)(ae^2-bde+cd^2)}-\frac{de}{(d+ex^2)(ae^2-bde+cd^2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)}+\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)}-\frac{\sqrt{d}\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{ae^2-bde+cd^2}$$

input
```
Int[x^2/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

output
```
(Sqrt[c]*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2
- b*d*e + a*e^2)) + (Sqrt[c]*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b
^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - (Sqrt[d]*Sqrt[e]*ArcTan[(Sqrt[e]*x
)/Sqrt[d]])/(c*d^2 - b*d*e + a*e^2)
```

3.306.3.1 Defintions of rubi rules used

rule 1610
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.306.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.85

method	result
default	$\frac{4c\left(\frac{\left(d\sqrt{-4ac+b^2}-2ae+bd\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(d\sqrt{-4ac+b^2}+2ae-bd\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{a\,e^2-bde+c\,d^2}-\frac{de\,\text{arcta}}{(a\,e^2-bde}$
risch	Expression too large to display

input
```
int(x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
4/(a*e^2-b*d*e+c*d^2)*c*(1/8*(d*(-4*a*c+b^2)^(1/2)-2*a*e+b*d)/(-4*a*c+b^2)
^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2))-1/8*(d*(-4*a*c+b^2)^(1/2)+2*a*e-b*d)/(-4*a*c+b^
2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))-d*e/(a*e^2-b*d*e+c*d^2)/(e*d)^(1/2)*arcta
n(e*x/(e*d)^(1/2))
```

3.306.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6125 vs. 2(209) = 418.

Time = 1.12 (sec) , antiderivative size = 12269, normalized size of antiderivative = 48.88

$$\int \frac{x^2}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.306.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x**2/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.306.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.306.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6936 vs. 2(209) = 418.

Time = 1.78 (sec) , antiderivative size = 6936, normalized size of antiderivative = 27.63

$$\int \frac{x^2}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
-d*e*arctan(e*x/sqrt(d*e))/((c*d^2 - b*d*e + a*e^2)*sqrt(d*e)) - 1/8*((2*b
^4*c^4 - 8*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c
^4 - 2*(b^2 - 4*a*c)*b^2*c^4)*d^5 - 2*(2*b^5*c^3 - 6*a*b^3*c^4 - 8*a^2*b*c
^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 3*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 2*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 -
 2*(b^2 - 4*a*c)*a*b*c^4)*d^4*e + (2*b^6*c^2 + 4*a*b^4*c^3 - 48*a^2*b^2*c^
4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 2*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 24*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 12*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 6*sqrt(2)*sqrt(b^2 ...
```

3.306.9 Mupad [B] (verification not implemented)

Time = 10.55 (sec) , antiderivative size = 19401, normalized size of antiderivative = 77.29

$$\int \frac{x^2}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
int(x^2/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

output
```
(log(b^4*d^3*e^5 - a*b^3*d^2*e^6 + a*c^3*d^5*e^3 - b^3*c*d^4*e^4 + 2*a^2*c
^2*d^3*e^5 + a^3*c*d*e^7 + b^4*e^3*x*(-d*e)^(5/2) + a*b^3*e^5*x*(-d*e)^(3/
2) + a^3*c*e^7*x*(-d*e)^(1/2) + 2*a*b*c^2*d^4*e^4 - 3*a*b^2*c*d^3*e^5 + 2*
a^2*b*c*d^2*e^6 + 2*a^2*c^2*e^3*x*(-d*e)^(5/2) - a*c^3*d*x*(-d*e)^(7/2) +
b^3*c*e*x*(-d*e)^(7/2) - 2*a*b*c^2*e*x*(-d*e)^(7/2) - 3*a*b^2*c*e^3*x*(-d*
e)^(5/2) - 2*a^2*b*c*e^5*x*(-d*e)^(3/2))*(-d*e)^(1/2))/(2*a*e^2 + 2*c*d^2
- 2*b*d*e) - atan(((x*(2*a^2*c^3*e^5 - 4*a*c^4*d^2*e^3 + 2*b^2*c^3*d^2*e^3
) - (-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4
*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*
a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d
^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2
- 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 -
 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*((x
*(32*a^3*b*c^3*e^7 + 16*a*c^6*d^5*e^2 - 112*a^3*c^4*d*e^6 - 8*a^2*b^3*c^2*
e^7 + 160*a^2*c^5*d^3*e^4 - 8*b^2*c^5*d^5*e^2 + 8*b^3*c^4*d^4*e^3 + 8*b^4*
c^3*d^3*e^4 - 8*b^5*c^2*d^2*e^5 - 96*a*b^2*c^4*d^3*e^4 + 64*a*b^3*c^3*d^2*
e^5 - 96*a^2*b*c^4*d^2*e^5 + 24*a^2*b^2*c^3*d*e^6) + (-(a*b^3*e^2 - a*e^2*
(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*
a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*
e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a...
```

3.306. $\int \frac{x^2}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

3.307 $\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

3.307.1 Optimal result

Integrand size = 24, antiderivative size = 254

$$\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx = -\frac{\sqrt{c}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} - \frac{\sqrt{c}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} + \frac{e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2-bde+ae^2)}$$

output
```
e^(3/2)*arctan(x*e^(1/2)/d^(1/2))/(a*e^2-b*d*e+c*d^2)/d^(1/2)-1/2*arctan(x
*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(e+(b*e-2*c*d)/(-4*
a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1
/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(e+(-b*e
+2*c*d)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1
/2))^(1/2)
```

3.307.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.08

$$\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \frac{\sqrt{c}\left(-2cd+be+\sqrt{b^2-4ace}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\,(-cd^2+bde-ae^2)} + \frac{\sqrt{c}\left(2cd-be+\sqrt{b^2-4ace}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}\,(-cd^2+bde-ae^2)} + \frac{e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2-bde+ae^2)}$$

input
```
Integrate[1/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

output
```
(Sqrt[c]*(-2*c*d + b*e + Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2
- 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2)) + (Sqrt[c]*(2*c*d - b*e + Sqrt[b^2 -
 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[
2]*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2
)) + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2
))
```

3.307.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1484, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx$$

$$\Big\downarrow \ 1484$$

$$\int \left(\frac{e^2}{(d+ex^2)(ae^2-bde+cd^2)} + \frac{-be+cd-cex^2}{(a+bx^2+cx^4)(ae^2-bde+cd^2)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)}-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)}+\frac{e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\,(ae^2-bde+cd^2)}$$

input

```
Int[1/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

output

```
-((Sqrt[c]*(e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^
2 - b*d*e + a*e^2))) - (Sqrt[c]*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcT
an[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqr
t[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqr
t[d]])/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2))
```

3.307.3.1 Defintions of rubi rules used

rule 1484

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.307.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.85

method	result
default	$\frac{4c\left(\frac{\left(be-2cd-e\sqrt{-4ac+b^2}\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(-e\sqrt{-4ac+b^2}-be+2cd\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{a\,e^2-bde+c\,d^2}+\frac{e^2\,\mathrm{arct}}{(a\,e^2-bd}$
risch	Expression too large to display

input
```
int(1/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
4/(a*e^2-b*d*e+c*d^2)*c*(1/8*(b*e-2*c*d-e*(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2))-1/8*(-e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d)/(-4*a*c+b
^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))+e^2/(a*e^2-b*d*e+c*d^2)/(e*d)^(1/2)*arct
an(e*x/(e*d)^(1/2))
```

3.307.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7995 vs. 2(210) = 420.

Time = 13.24 (sec) , antiderivative size = 16013, normalized size of antiderivative = 63.04

$$\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.307.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.307.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.307.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7664 vs. $2(210) = 420$.

Time = 1.78 (sec) , antiderivative size = 7664, normalized size of antiderivative = 30.17

$$\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
e^2*arctan(e*x/sqrt(d*e))/((c*d^2 - b*d*e + a*e^2)*sqrt(d*e)) + 1/8*(2*(2*
b^3*c^5 - 8*a*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^
2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^5 -
2*(b^2 - 4*a*c)*b*c^5)*d^5 - 5*(2*b^4*c^4 - 8*a*b^2*c^5 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - 2*(b^2 - 4*a*c)*b^2*c^4)*d^4*e + 4
*(2*b^5*c^3 - 6*a*b^3*c^4 - 8*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*b*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 - 2*(b^2 - 4*a*c)*a*b*c^4)*d^3*e^2 - (
2*b^6*c^2 + 4*a*b^4*c^3 - 48*a^2*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +...
```

3.307.9 Mupad [B] (verification not implemented)

Time = 11.03 (sec) , antiderivative size = 23640, normalized size of antiderivative = 93.07

$$\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
int(1/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

output
```
atan((((-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d
^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d
^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 1
2*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16
*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^
2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5
*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 +
16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*((x*(16*b^5*c^2*e^7 +
16*c^7*d^5*e^2 - 112*a*b^3*c^3*e^7 + 192*a^2*b*c^4*e^7 + 32*a*c^6*d^3*e^4
- 240*a^2*c^5*d*e^6 - 32*b*c^6*d^4*e^3 - 32*b^4*c^3*d*e^6 + 16*b^2*c^5*d^3
*e^4 + 16*b^3*c^4*d^2*e^5 - 96*a*b*c^5*d^2*e^5 + 192*a*b^2*c^4*d*e^6) - (-
(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a
*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*
b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c
^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*
d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2
*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e
- 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^
3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(x*(-(b^5*e^2 + b^3*c^2*d^2 + b
^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12...
```

3.307. $\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

3.308 $\int \frac{1}{x^2(d+ex^2)(a+bx^2+cx^4)}\,dx$

3.308.1 Optimal result

Integrand size = 27, antiderivative size = 298

$$\int \frac{1}{x^2(d+ex^2)(a+bx^2+cx^4)}\,dx = -\frac{1}{adx} - \frac{\sqrt{c}\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} - \frac{\sqrt{c}\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} - \frac{e^{5/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}(cd^2-bde+ae^2)}$$

output
```
-1/a/d/x-e^(5/2)*arctan(x*e^(1/2)/d^(1/2))/d^(3/2)/(a*e^2-b*d*e+c*d^2)-1/2
*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(c*d-b*e+(
2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))/a/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(b-
(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/
2))^(1/2))*c^(1/2)*(c*d-b*e+(-2*a*c*e+b^2*e-b*c*d)/(-4*a*c+b^2)^(1/2))/a/(
a*e^2-b*d*e+c*d^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.308.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^2(d+ex^2)(a+bx^2+cx^4)}\,dx$$
$$= -\frac{1}{adx} - \frac{\sqrt{c}\left(bcd+c\sqrt{b^2-4ac}d-b^2e+2ace-b\sqrt{b^2-4ac}e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2+e(-bd+ae))}$$
$$+\frac{\sqrt{c}\left(bcd-c\sqrt{b^2-4ac}d-b^2e+2ace+b\sqrt{b^2-4ac}e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2+e(-bd+ae))}$$
$$-\frac{e^{5/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}(cd^2-bde+ae^2)}$$

input
```
Integrate[1/(x^2*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

output
```
-(1/(a*d*x)) - (Sqrt[c]*(b*c*d + c*Sqrt[b^2 - 4*a*c]*d - b^2*e + 2*a*c*e -
 b*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a
*c]]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 + e
*(-(b*d) + a*e))) + (Sqrt[c]*(b*c*d - c*Sqrt[b^2 - 4*a*c]*d - b^2*e + 2*a*
c*e + b*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
- 4*a*c]]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^
2 + e*(-(b*d) + a*e))) - (e^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*(c
*d^2 - b*d*e + a*e^2))
```

3.308.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1610, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(d+ex^2)(a+bx^2+cx^4)}\,dx$$
$$\Big\downarrow \text{1610}$$
$$\int \left(\frac{-ace+b^2e-cx^2(cd-be)-bcd}{a(a+bx^2+cx^4)(ae^2-bde+cd^2)} - \frac{e^3}{d(d+ex^2)(ae^2-bde+cd^2)} + \frac{1}{adx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)}-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)}{\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)}-\frac{e^{5/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\,(ae^2-bde+cd^2)}-\frac{1}{adx}$$

input
```
Int[1/(x^2*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

output
```
-(1/(a*d*x)) - (Sqrt[c]*(c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*
a*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - (Sqrt[c]*(c*d - b
*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c
*d^2 - b*d*e + a*e^2)) - (e^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*(c
*d^2 - b*d*e + a*e^2))
```

3.308.3.1 Defintions of rubi rules used

rule 1610
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.308.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.93

method	result
default	$\frac{4c\left(\frac{\left(be\sqrt{-4ac+b^2}-cd\sqrt{-4ac+b^2}+2ace-b^2e+bcd\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(be\sqrt{-4ac+b^2}-cd\sqrt{-4ac+b^2}-2ace+b^2e-bcd\right)\sqrt{2}\,\mathrm{arctan}}{8\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right.}}\right.}{\left(a\,e^2-bde+c\,d^2\right)a}$
risch	Expression too large to display

input
```
int(1/x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
4/(a*e^2-b*d*e+c*d^2)/a*c*(1/8*(b*e*(-4*a*c+b^2)^(1/2)-c*d*(-4*a*c+b^2)^(1
/2)+2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/8*(b*e*(
-4*a*c+b^2)^(1/2)-c*d*(-4*a*c+b^2)^(1/2)-2*a*c*e+b^2*e-b*c*d)/(-4*a*c+b^2)
^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)))-1/d*e^3/(a*e^2-b*d*e+c*d^2)/(e*d)^(1/2)*arc
tan(e*x/(e*d)^(1/2))-1/a/d/x
```

3.308.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10286 vs. 2(254) = 508.

Time = 166.67 (sec) , antiderivative size = 20595, normalized size of antiderivative = 69.11

$$\int \frac{1}{x^2\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.308.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (d + ex^2) (a + bx^2 + cx^4)} \, dx = \text{Timed out}$$

input
```
integrate(1/x**2/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.308.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (d + ex^2) (a + bx^2 + cx^4)} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.308.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10072 vs. $2(254) = 508$.

Time = 2.51 (sec) , antiderivative size = 10072, normalized size of antiderivative = 33.80

$$\int \frac{1}{x^2 (d + ex^2) (a + bx^2 + cx^4)} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
-e^3*arctan(e*x/sqrt(d*e))/((c*d^3 - b*d^2*e + a*d*e^2)*sqrt(d*e)) - 1/8*(
(2*a^2*b^4*c^5 - 8*a^3*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^3*b^2*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b^2*c^5 - 2*(b^2 - 4*a*c)*a^2*b^2*c^5)*d^5 - (6*a^2*b^5
*c^4 - 28*a^3*b^3*c^5 + 16*a^4*b*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^5 - 6*(b^2 - 4*a*c)*a^2*b^3*c^4 + 4*(b^2 -
 4*a*c)*a^3*b*c^5)*d^4*e + (6*a^2*b^6*c^3 - 28*a^3*b^4*c^4 + 16*a^4*b^2*c^
5 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c
+ 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2
 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2
 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3
 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*...
```

3.308.9 Mupad [B] (verification not implemented)

Time = 11.33 (sec) , antiderivative size = 33644, normalized size of antiderivative = 112.90

$$\int \frac{1}{x^2 (d + ex^2) (a + bx^2 + cx^4)} \, dx = \text{Too large to display}$$

input
```
int(1/(x^2*(d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

output
```
atan((((-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b
^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^
3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^
2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^
3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*
b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-
(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4
- 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4
+ a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*
d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6
*a^4*b^4*c*d^2*e^2)))^(1/2)*(((-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c
- b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c
- b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^
2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2)
- 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c
 - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3
*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^
4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d
^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*
c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 ...
```

3.308. $\int \frac{1}{x^2(d+ex^2)(a+bx^2+cx^4)}\,dx$

3.309 $\int \frac{1}{x^4(d+ex^2)(a+bx^2+cx^4)}\,dx$

3.309.1 Optimal result

Integrand size = 27, antiderivative size = 348

$$\begin{aligned}\int \frac{1}{x^4(d+ex^2)(a+bx^2+cx^4)}\,dx &= -\frac{1}{3adx^3}+\frac{bd+ae}{a^2d^2x}+\frac{\sqrt{c}\left(bcd-b^2e+ace+\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)}\\&\quad+\frac{\sqrt{c}\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)}\\&\quad+\frac{e^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}(cd^2-bde+ae^2)}\end{aligned}$$

output
```
-1/3/a/d/x^3+(a*e+b*d)/a^2/d^2/x+e^(7/2)*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)
/(a*e^2-b*d*e+c*d^2)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(
1/2))*c^(1/2)*(b*c*d-b^2*e+a*c*e+(3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)/(-4*a
*c+b^2)^(1/2))/a^2/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2
)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b*c*
d-b^2*e+a*c*e+(-3*a*b*c*e+2*a*c^2*d+b^3*e-b^2*c*d)/(-4*a*c+b^2)^(1/2))/a^2
/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.309.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.18

$$\int \frac{1}{x^4(d+ex^2)(a+bx^2+cx^4)}\,dx = -\frac{1}{3adx^3}+\frac{bd+ae}{a^2d^2x}$$
$$+\frac{\sqrt{c}\left(-b^3e+bc\left(\sqrt{b^2-4ac}d+3ae\right)+b^2\left(cd-\sqrt{b^2-4ac}e\right)+ac\left(-2cd+\sqrt{b^2-4ac}e\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2+e(-bd+ae))}$$
$$+\frac{\sqrt{c}\left(b^3e+bc\left(\sqrt{b^2-4ac}d-3ae\right)-b^2\left(cd+\sqrt{b^2-4ac}e\right)+ac\left(2cd+\sqrt{b^2-4ac}e\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2+e(-bd+ae))}$$
$$+\frac{e^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}(cd^2-bde+ae^2)}$$

input
```
Integrate[1/(x^4*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

output
```
-1/3*1/(a*d*x^3) + (b*d + a*e)/(a^2*d^2*x) + (Sqrt[c]*(-(b^3*e) + b*c*(Sqr
t[b^2 - 4*a*c]*d + 3*a*e) + b^2*(c*d - Sqrt[b^2 - 4*a*c]*e) + a*c*(-2*c*d
+ Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a
*c]]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 +
 e*(-(b*d) + a*e))) + (Sqrt[c]*(b^3*e + b*c*(Sqrt[b^2 - 4*a*c]*d - 3*a*e)
- b^2*(c*d + Sqrt[b^2 - 4*a*c]*e) + a*c*(2*c*d + Sqrt[b^2 - 4*a*c]*e))*Arc
Tan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b^
2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 + e*(-(b*d) + a*e))) + (e^(7
/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(5/2)*(c*d^2 - b*d*e + a*e^2))
```

3.309.3 Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1610, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4(d+ex^2)(a+bx^2+cx^4)}\,dx$$
$$\Big\downarrow \text{1610}$$

$$\int \left(\frac{cx^2\left(ace+b^2(-e)+bcd\right)+2abce-ac^2d+b^3(-e)+b^2cd}{a^2\left(a+bx^2+cx^4\right)\left(ae^2-bde+cd^2\right)}+\frac{-ae-bd}{a^2d^2x^2}+\frac{e^4}{d^2\left(d+ex^2\right)\left(ae^2-bde+cd^2\right)}+\frac{}{ad}$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}\left(ae^2-bde+cd^2\right)}+\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)}{\sqrt{2}a^2\sqrt{\sqrt{b^2-4ac}+b}\left(ae^2-bde+cd^2\right)}+\frac{ae+bd}{a^2d^2x}+\frac{e^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\left(ae^2-bde+cd^2\right)}-\frac{1}{3adx^3}$$

input
```
Int[1/(x^4*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

output
```
-1/3*1/(a*d*x^3) + (b*d + a*e)/(a^2*d^2*x) + (Sqrt[c]*(b*c*d - b^2*e + a*c
*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b - Sqr
t[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) + (Sqrt[c]*(b*c*d - b^2*e + a*c*e
 - (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b + Sqrt[
b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) + (e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[
d]])/(d^(5/2)*(c*d^2 - b*d*e + a*e^2))
```

3.309.3.1 Defintions of rubi rules used

rule 1610
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.309.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.00

method	result
default	$-\frac{1}{3ad\,x^3}-\frac{-ae-bd}{x\,a^2d^2}+\frac{4c\left(\frac{\left(ace\sqrt{-4ac+b^2}-b^2e\sqrt{-4ac+b^2}+bcd\sqrt{-4ac+b^2}-3abce+2a\,c^2d+b^3e-b^2cd\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right.}{(a\,e^2-bd}$
risch	Expression too large to display

input
```
int(1/x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3/a/d/x^3-(-a*e-b*d)/x/a^2/d^2+4/(a*e^2-b*d*e+c*d^2)/a^2*c*(1/8*(a*c*e*
(-4*a*c+b^2)^(1/2)-b^2*e*(-4*a*c+b^2)^(1/2)+b*c*d*(-4*a*c+b^2)^(1/2)-3*a*b
*c*e+2*a*c^2*d+b^3*e-b^2*c*d)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/8*(
a*c*e*(-4*a*c+b^2)^(1/2)-b^2*e*(-4*a*c+b^2)^(1/2)+b*c*d*(-4*a*c+b^2)^(1/2)
+3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)))+1/d^2*e^4/(a*e^2-b*d*e+c*d^2)/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2))
```

3.309.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12482 vs. 2(304) = 608.

Time = 293.42 (sec) , antiderivative size = 24988, normalized size of antiderivative = 71.80

$$\int \frac{1}{x^4\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.309.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (d + ex^2) (a + bx^2 + cx^4)} dx = \text{Timed out}$$

input
```
integrate(1/x**4/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.309.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^4 (d + ex^2) (a + bx^2 + cx^4)} dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.309.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12281 vs. $2(304) = 608$.

Time = 2.65 (sec) , antiderivative size = 12281, normalized size of antiderivative = 35.29

$$\int \frac{1}{x^4 (d + ex^2) (a + bx^2 + cx^4)} dx = \text{Too large to display}$$

input
```
integrate(1/x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
e^4*arctan(e*x/sqrt(d*e))/((c*d^4 - b*d^3*e + a*d^2*e^2)*sqrt(d*e)) - 1/8*
((2*a^4*b^5*c^5 - 12*a^5*b^3*c^6 + 16*a^6*b*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^5 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^6 - 2*(b^2 - 4*a*c)*a^4*b^3*c^5 + 4*
(b^2 - 4*a*c)*a^5*b*c^6)*d^5 - (6*a^4*b^6*c^4 - 38*a^5*b^4*c^5 + 56*a^6*b^
2*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^
6*c^2 + 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b
^4*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b
^5*c^3 - 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*
b^2*c^4 - 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5
*b^3*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4
*b^4*c^4 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5
*b^2*c^5 - 6*(b^2 - 4*a*c)*a^4*b^4*c^4 + 14*(b^2 - 4*a*c)*a^5*b^2*c^5)*d^4
*e + (6*a^4*b^7*c^3 - 36*a^5*b^5*c^4 + 40*a^6*b^3*c^5 + 32*a^7*b*c^6 - 3*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c + 18...
```

3.309.9 Mupad [B] (verification not implemented)

Time = 11.88 (sec) , antiderivative size = 42882, normalized size of antiderivative = 123.22

$$\int \frac{1}{x^4 (d + ex^2) (a + bx^2 + cx^4)} \, dx = \text{Too large to display}$$

input
```
int(1/(x^4*(d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

output
```
(log(c^9*d^27*e^6 - b^9*d^18*e^15 + 2*a*c^8*d^25*e^8 - 2*b*c^8*d^26*e^7 +
2*b^8*c*d^19*e^14 + a^5*b^4*d^13*e^20 + a^2*c^7*d^23*e^10 + 16*a^4*c^5*d^1
9*e^14 + 16*a^7*c^2*d^13*e^20 + b^2*c^7*d^25*e^8 - b^7*c^2*d^20*e^13 - 25*
a^2*b^3*c^4*d^20*e^13 + 66*a^2*b^4*c^3*d^19*e^14 - 42*a^2*b^5*c^2*d^18*e^1
5 - 76*a^3*b^2*c^4*d^19*e^14 + 63*a^3*b^3*c^3*d^18*e^15 - a^5*b^4*e^3*x*(-
d^5*e^7)^(5/2) + a^2*c^7*d^15*x*(-d^5*e^7)^(3/2) - 16*a^7*c^2*e^3*x*(-d^5*
e^7)^(5/2) - b^9*d^10*e^5*x*(-d^5*e^7)^(3/2) - c^9*d^24*e^3*x*(-d^5*e^7)^(
1/2) - 2*a*b*c^7*d^24*e^9 + 11*a*b^7*c*d^18*e^15 + 9*a*b^5*c^3*d^20*e^13 -
 20*a*b^6*c^2*d^19*e^14 + 20*a^3*b*c^5*d^20*e^13 - 28*a^4*b*c^4*d^18*e^15
- 8*a^6*b^2*c*d^13*e^20 + 16*a^4*c^5*d^11*e^4*x*(-d^5*e^7)^(3/2) - b^7*c^2
*d^12*e^3*x*(-d^5*e^7)^(3/2) - b^2*c^7*d^22*e^5*x*(-d^5*e^7)^(1/2) + 8*a^6
*b^2*c*e^3*x*(-d^5*e^7)^(5/2) - 2*a*c^8*d^22*e^5*x*(-d^5*e^7)^(1/2) + 2*b^
8*c*d^11*e^4*x*(-d^5*e^7)^(3/2) + 2*b*c^8*d^23*e^4*x*(-d^5*e^7)^(1/2) + 11
*a*b^7*c*d^10*e^5*x*(-d^5*e^7)^(3/2) + 2*a*b*c^7*d^21*e^6*x*(-d^5*e^7)^(1/
2) + 9*a*b^5*c^3*d^12*e^3*x*(-d^5*e^7)^(3/2) - 20*a*b^6*c^2*d^11*e^4*x*(-d
^5*e^7)^(3/2) + 20*a^3*b*c^5*d^12*e^3*x*(-d^5*e^7)^(3/2) - 28*a^4*b*c^4*d^
10*e^5*x*(-d^5*e^7)^(3/2) - 25*a^2*b^3*c^4*d^12*e^3*x*(-d^5*e^7)^(3/2) + 6
6*a^2*b^4*c^3*d^11*e^4*x*(-d^5*e^7)^(3/2) - 42*a^2*b^5*c^2*d^10*e^5*x*(-d^
5*e^7)^(3/2) - 76*a^3*b^2*c^4*d^11*e^4*x*(-d^5*e^7)^(3/2) + 63*a^3*b^3*c^3
*d^10*e^5*x*(-d^5*e^7)^(3/2))*(-d^5*e^7)^(1/2))/(2*c*d^7 + 2*a*d^5*e^2 ...
```

3.309. $\int \frac{1}{x^4(d+ex^2)(a+bx^2+cx^4)}\,dx$

3.310 $\int \frac{1}{\sqrt{fx}(d+ex^2)(a+bx^2+cx^4)}\,dx$

3.310.1 Optimal result

Integrand size = 31, antiderivative size = 866

$$\int \frac{1}{\sqrt{fx}\,(d+ex^2)\,(a+bx^2+cx^4)}\,dx$$
$$= \frac{c^{3/4}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{fx}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}\sqrt{f}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}(cd^2-bde+ae^2)\sqrt{f}}$$
$$-\frac{c^{3/4}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{fx}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}\sqrt{f}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}(cd^2-bde+ae^2)\sqrt{f}}$$
$$-\frac{e^{7/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{fx}}{\sqrt[4]{d}\sqrt{f}}\right)}{\sqrt{2}d^{3/4}(cd^2-bde+ae^2)\sqrt{f}}+\frac{e^{7/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{fx}}{\sqrt[4]{d}\sqrt{f}}\right)}{\sqrt{2}d^{3/4}(cd^2-bde+ae^2)\sqrt{f}}$$
$$+\frac{c^{3/4}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{fx}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}\sqrt{f}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}(cd^2-bde+ae^2)\sqrt{f}}$$
$$-\frac{c^{3/4}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{fx}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}\sqrt{f}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}(cd^2-bde+ae^2)\sqrt{f}}$$
$$-\frac{e^{7/4}\log\left(\sqrt{d}\sqrt{f}+\sqrt{e}\sqrt{f}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{fx}\right)}{2\sqrt{2}d^{3/4}(cd^2-bde+ae^2)\sqrt{f}}$$
$$+\frac{e^{7/4}\log\left(\sqrt{d}\sqrt{f}+\sqrt{e}\sqrt{f}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{fx}\right)}{2\sqrt{2}d^{3/4}(cd^2-bde+ae^2)\sqrt{f}}$$

output
```
-1/2*e^(7/4)*arctan(1-e^(1/4)*2^(1/2)*(f*x)^(1/2)/d^(1/4)/f^(1/2))/d^(3/4)
/(a*e^2-b*d*e+c*d^2)*2^(1/2)/f^(1/2)+1/2*e^(7/4)*arctan(1+e^(1/4)*2^(1/2)*
(f*x)^(1/2)/d^(1/4)/f^(1/2))/d^(3/4)/(a*e^2-b*d*e+c*d^2)*2^(1/2)/f^(1/2)-1
/4*e^(7/4)*ln(d^(1/2)*f^(1/2)+x*e^(1/2)*f^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(f
*x)^(1/2))/d^(3/4)/(a*e^2-b*d*e+c*d^2)*2^(1/2)/f^(1/2)+1/4*e^(7/4)*ln(d^(1
/2)*f^(1/2)+x*e^(1/2)*f^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(f*x)^(1/2))/d^(3/4)
/(a*e^2-b*d*e+c*d^2)*2^(1/2)/f^(1/2)+1/2*c^(3/4)*arctan(2^(1/4)*c^(1/4)*(f
*x)^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)/f^(1/2))*(2*c*d-e*(b-(-4*a*c+b^2)^
(1/2)))*2^(3/4)/(a*e^2-b*d*e+c*d^2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)/(-4*a*c+
b^2)^(1/2)/f^(1/2)+1/2*c^(3/4)*arctanh(2^(1/4)*c^(1/4)*(f*x)^(1/2)/(-b-(-4
*a*c+b^2)^(1/2))^(1/4)/f^(1/2))*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))*2^(3/4)/(
a*e^2-b*d*e+c*d^2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)/(-4*a*c+b^2)^(1/2)/f^(1/2
)-1/2*c^(3/4)*arctan(2^(1/4)*c^(1/4)*(f*x)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(
1/4)/f^(1/2))*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))*2^(3/4)/(a*e^2-b*d*e+c*d^2)
/(-4*a*c+b^2)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)/f^(1/2)-1/2*c^(3/4)*arct
anh(2^(1/4)*c^(1/4)*(f*x)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)/f^(1/2))*(2*
c*d-e*(b+(-4*a*c+b^2)^(1/2)))*2^(3/4)/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/
2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)/f^(1/2)
```

3.310.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.30 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.25

$$\int \frac{1}{\sqrt{fx}\,(d+ex^2)\,(a+bx^2+cx^4)}\,dx = -\frac{\sqrt{x}\left(\sqrt{2}e^{7/4}\left(\arctan\left(\frac{\sqrt{d}-\sqrt{e}x}{\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{x}}\right)-\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{x}}{\sqrt{d}+\sqrt{e}x}\right)\right)+d^{3/4}\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\right.\right.}{2d^{3/4}\,(cd^2+e(-bd+ae))\,\sqrt{fx}}$$

input
```
Integrate[1/(Sqrt[f*x]*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

output
```
-1/2*(Sqrt[x]*(Sqrt[2]*e^(7/4)*(ArcTan[(Sqrt[d] - Sqrt[e]*x)/(Sqrt[2]*d^(1
/4)*e^(1/4)*Sqrt[x])] - ArcTanh[(Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x])/(Sqrt[d]
 + Sqrt[e]*x)]) + d^(3/4)*RootSum[a + b*#1^4 + c*#1^8 & , (-(c*d*Log[Sqrt[
x] - #1]) + b*e*Log[Sqrt[x] - #1] + c*e*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 +
2*c*#1^7) & ]))/(d^(3/4)*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[f*x])
```

3.310.3 Rubi [A] (verified)

Time = 1.83 (sec) , antiderivative size = 878, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1592, 27, 1754, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{fx}\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}dx$$

$$\Big\downarrow \text{ 1592}$$

$$\frac{2\int \frac{f^2}{(ex^2f^2+df^2)(cx^4+bx^2+a)}d\sqrt{fx}}{f}$$

$$\Big\downarrow \text{ 27}$$

$$2f\int \frac{1}{\left(ex^2f^2+df^2\right)\left(cx^4+bx^2+a\right)}d\sqrt{fx}$$

$$\Big\downarrow \text{ 1754}$$

$$2f\int \left(\frac{e^2}{\left(cd^2-bed+ae^2\right)\left(ex^2f^2+df^2\right)}+\frac{-cex^2f^2+cdf^2-bef^2}{\left(cd^2-bed+ae^2\right)\left(cx^4f^4+bx^2f^4+af^4\right)}\right)d\sqrt{fx}$$

$$\Big\downarrow \text{ 2009}$$

$$2f\left(-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{fx}}{\sqrt[4]{d}\sqrt{f}}\right)e^{7/4}}{2\sqrt{2}d^{3/4}\left(cd^2-bed+ae^2\right)f^{3/2}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{fx}}{\sqrt[4]{d}\sqrt{f}}+1\right)e^{7/4}}{2\sqrt{2}d^{3/4}\left(cd^2-bed+ae^2\right)f^{3/2}}-\frac{\log\left(\sqrt{e}xf+\sqrt{d}f-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{fx}\sqrt{f}\right.}{4\sqrt{2}d^{3/4}\left(cd^2-bed+ae^2\right)f^{3/2}}\right.$$

input
```
Int[1/(Sqrt[f*x]*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

output
```
2*f*((c^(3/4)*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*ArcTan[(2^(1/4)*c^(1/4)*
Sqrt[f*x])/((-b - Sqrt[b^2 - 4*a*c])^(1/4)*Sqrt[f])])/(2*2^(1/4)*Sqrt[b^2
- 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e + a*e^2)*f^(3/2)) -
 (c^(3/4)*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt
[f*x])/((-b + Sqrt[b^2 - 4*a*c])^(1/4)*Sqrt[f])])/(2*2^(1/4)*Sqrt[b^2 - 4*
a*c]*(-b + Sqrt[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e + a*e^2)*f^(3/2)) - (e^
(7/4)*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[f*x])/(d^(1/4)*Sqrt[f])])/(2*Sqrt[2
]*d^(3/4)*(c*d^2 - b*d*e + a*e^2)*f^(3/2)) + (e^(7/4)*ArcTan[1 + (Sqrt[2]*
e^(1/4)*Sqrt[f*x])/(d^(1/4)*Sqrt[f])])/(2*Sqrt[2]*d^(3/4)*(c*d^2 - b*d*e +
 a*e^2)*f^(3/2)) + (c^(3/4)*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*ArcTanh[(2
^(1/4)*c^(1/4)*Sqrt[f*x])/((-b - Sqrt[b^2 - 4*a*c])^(1/4)*Sqrt[f])])/(2*2^
(1/4)*Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e + a*
e^2)*f^(3/2)) - (c^(3/4)*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*ArcTanh[(2^(1
/4)*c^(1/4)*Sqrt[f*x])/((-b + Sqrt[b^2 - 4*a*c])^(1/4)*Sqrt[f])])/(2*2^(1/
4)*Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e + a*e^2
)*f^(3/2)) - (e^(7/4)*Log[Sqrt[d]*f + Sqrt[e]*f*x - Sqrt[2]*d^(1/4)*e^(1/4
)*Sqrt[f]*Sqrt[f*x]])/(4*Sqrt[2]*d^(3/4)*(c*d^2 - b*d*e + a*e^2)*f^(3/2))
+ (e^(7/4)*Log[Sqrt[d]*f + Sqrt[e]*f*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[f]*S
qrt[f*x]])/(4*Sqrt[2]*d^(3/4)*(c*d^2 - b*d*e + a*e^2)*f^(3/2)))
```

3.310.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1592
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k/f   Subst[
Int[x^(k*(m + 1) - 1)*(d + e*(x^(2*k)/f^2))^q*(a + b*(x^(2*k)/f^k) + c*(x^(
4*k)/f^4))^p, x], x, (f*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, e, f, p, q}, x
] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

rule 1754
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)),
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.310.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.99 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.30

method	result
derivativedivides	$2f^5\left(\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+bf^2\_Z^4+af^4\right)}\frac{\left(-\_R^4ce-bef^2+cdf^2\right)\ln\left(\sqrt{fx}-\_R\right)}{2\_R^7c+\_R^3bf^2}}{4f^4(ae^2-bde+cd^2)}+\frac{e^2\left(\frac{df^2}{e}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{fx+\left(\frac{df^2}{e}\right.}{fx-\left(\frac{df^2}{e}\right.}\right.\right.}{}\right.$
default	$\frac{\left(\frac{df^2}{e}\right)^{\frac{1}{4}}e^2\left(2\arctan\left(\frac{\sqrt{2}\sqrt{fx}+\left(\frac{df^2}{e}\right)^{\frac{1}{4}}}{\left(\frac{df^2}{e}\right)^{\frac{1}{4}}}\right)+\ln\left(\frac{fx+\left(\frac{df^2}{e}\right)^{\frac{1}{4}}\sqrt{fx}\sqrt{2}+\sqrt{\frac{df^2}{e}}}{fx-\left(\frac{df^2}{e}\right)^{\frac{1}{4}}\sqrt{fx}\sqrt{2}+\sqrt{\frac{df^2}{e}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{fx}-\left(\frac{df^2}{e}\right)^{\frac{1}{4}}}{\left(\frac{df^2}{e}\right)^{\frac{1}{4}}}\right)\right)\sqrt{2}}{4f(ae^2-bde+cd^2)d}$
pseudoelliptic	$\frac{\left(\frac{df^2}{e}\right)^{\frac{1}{4}}e^2\left(2\arctan\left(\frac{\sqrt{2}\sqrt{fx}+\left(\frac{df^2}{e}\right)^{\frac{1}{4}}}{\left(\frac{df^2}{e}\right)^{\frac{1}{4}}}\right)+\ln\left(\frac{fx+\left(\frac{df^2}{e}\right)^{\frac{1}{4}}\sqrt{fx}\sqrt{2}+\sqrt{\frac{df^2}{e}}}{fx-\left(\frac{df^2}{e}\right)^{\frac{1}{4}}\sqrt{fx}\sqrt{2}+\sqrt{\frac{df^2}{e}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{fx}-\left(\frac{df^2}{e}\right)^{\frac{1}{4}}}{\left(\frac{df^2}{e}\right)^{\frac{1}{4}}}\right)\right)\sqrt{2}}{4f(ae^2-bde+cd^2)d}$

input
```
int(1/(e*x^2+d)/(c*x^4+b*x^2+a)/(f*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*f^5*(1/4/f^4/(a*e^2-b*d*e+c*d^2)*sum((-_R^4*c*e-b*e*f^2+c*d*f^2)/(2*_R^7
*c+_R^3*b*f^2)*ln((f*x)^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b*f^2+a*f^4))+1/8*
e^2/f^6/(a*e^2-b*d*e+c*d^2)*(d*f^2/e)^(1/4)/d*2^(1/2)*(ln((f*x+(d*f^2/e)^(
1/4)*(f*x)^(1/2)*2^(1/2)+(d*f^2/e)^(1/2))/(f*x-(d*f^2/e)^(1/4)*(f*x)^(1/2)
*2^(1/2)+(d*f^2/e)^(1/2)))+2*arctan(2^(1/2)/(d*f^2/e)^(1/4)*(f*x)^(1/2)+1)
+2*arctan(2^(1/2)/(d*f^2/e)^(1/4)*(f*x)^(1/2)-1)))
```

3.310.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{fx}\,(d+ex^2)\,(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)/(f*x)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.310.6 Sympy [F]

$$\int \frac{1}{\sqrt{fx}\,(d+ex^2)\,(a+bx^2+cx^4)}\,dx = \int \frac{1}{\sqrt{fx}\,(d+ex^2)\,(a+bx^2+cx^4)}\,dx$$

input
```
integrate(1/(e*x**2+d)/(c*x**4+b*x**2+a)/(f*x)**(1/2),x)
```

output
```
Integral(1/(sqrt(f*x)*(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

3.310.7 Maxima [F]

$$\int \frac{1}{\sqrt{fx}\,(d+ex^2)\,(a+bx^2+cx^4)}\,dx = \int \frac{1}{(cx^4+bx^2+a)(ex^2+d)\sqrt{fx}}\,dx$$

input
```
integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)/(f*x)^(1/2),x, algorithm="maxima")
```

output
```
-2*e^2*sqrt(x)/(c*d^3*sqrt(f) - b*d^2*e*sqrt(f) + a*d*e^2*sqrt(f)) + 1/4*(
2*sqrt(2)*e^2*arctan(1/2*sqrt(2)*(sqrt(2)*d^(1/4)*e^(1/4) + 2*sqrt(e)*sqrt
(x))/sqrt(sqrt(d)*sqrt(e)))/(sqrt(d)*sqrt(sqrt(d)*sqrt(e))) + 2*sqrt(2)*e^
2*arctan(-1/2*sqrt(2)*(sqrt(2)*d^(1/4)*e^(1/4) - 2*sqrt(e)*sqrt(x))/sqrt(s
qrt(d)*sqrt(e)))/(sqrt(d)*sqrt(sqrt(d)*sqrt(e))) + sqrt(2)*e^(7/4)*log(sqr
t(2)*d^(1/4)*e^(1/4)*sqrt(x) + sqrt(e)*x + sqrt(d))/d^(3/4) - sqrt(2)*e^(7
/4)*log(-sqrt(2)*d^(1/4)*e^(1/4)*sqrt(x) + sqrt(e)*x + sqrt(d))/d^(3/4))/(
c*d^2*sqrt(f) - b*d*e*sqrt(f) + a*e^2*sqrt(f)) + 2*sqrt(x)/(a*d*sqrt(f)) +
 integrate(-((c^2*d - b*c*e)*x^(7/2) + (b*c*d - b^2*e + a*c*e)*x^(3/2))/(a
^3*e^2*sqrt(f) + (a^2*c*e^2*sqrt(f) + (c^2*d^2*sqrt(f) - b*c*d*e*sqrt(f))*
a)*x^4 + (c*d^2*sqrt(f) - b*d*e*sqrt(f))*a^2 + (a^2*b*e^2*sqrt(f) + (b*c*d
^2*sqrt(f) - b^2*d*e*sqrt(f))*a)*x^2), x)
```

3.310.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{fx}\,(d+ex^2)\,(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)/(f*x)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.310.9 Mupad [B] (verification not implemented)

Time = 12.01 (sec) , antiderivative size = 43112, normalized size of antiderivative = 49.78

$$\int \frac{1}{\sqrt{fx}\,(d+ex^2)\,(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
int(1/((f*x)^(1/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

output
```
symsum(log(-root(8388608*a^7*b*c^11*d^18*e*f^6*h^12 - 513802240*a^10*b^2*c
^7*d^11*e^8*f^6*h^12 - 381681664*a^11*b^2*c^6*d^9*e^10*f^6*h^12 - 38168166
4*a^9*b^2*c^8*d^13*e^6*f^6*h^12 - 300941312*a^9*b^5*c^5*d^10*e^9*f^6*h^12
- 300941312*a^8*b^5*c^6*d^12*e^7*f^6*h^12 + 293601280*a^10*b^3*c^6*d^10*e^
9*f^6*h^12 + 293601280*a^9*b^3*c^7*d^12*e^7*f^6*h^12 - 168820736*a^10*b^5*
c^4*d^8*e^11*f^6*h^12 - 168820736*a^7*b^5*c^7*d^14*e^5*f^6*h^12 + 16606822
4*a^8*b^6*c^5*d^11*e^8*f^6*h^12 - 146800640*a^12*b^2*c^5*d^7*e^12*f^6*h^12
 - 146800640*a^8*b^2*c^9*d^15*e^4*f^6*h^12 + 124780544*a^10*b^4*c^5*d^9*e^
10*f^6*h^12 + 124780544*a^8*b^4*c^7*d^13*e^6*f^6*h^12 + 119275520*a^9*b^4*
c^6*d^11*e^8*f^6*h^12 + 117440512*a^11*b^3*c^5*d^8*e^11*f^6*h^12 + 1174405
12*a^8*b^3*c^8*d^14*e^5*f^6*h^12 + 102760448*a^9*b^6*c^4*d^9*e^10*f^6*h^12
 + 102760448*a^7*b^6*c^6*d^13*e^6*f^6*h^12 + 91750400*a^11*b^4*c^4*d^7*e^1
2*f^6*h^12 + 91750400*a^7*b^4*c^8*d^15*e^4*f^6*h^12 - 71065600*a^7*b^8*c^4
*d^11*e^8*f^6*h^12 - 53444608*a^8*b^8*c^3*d^9*e^10*f^6*h^12 - 53444608*a^6
*b^8*c^5*d^13*e^6*f^6*h^12 + 40370176*a^9*b^7*c^3*d^8*e^11*f^6*h^12 + 4037
0176*a^6*b^7*c^6*d^14*e^5*f^6*h^12 - 36700160*a^11*b^5*c^3*d^6*e^13*f^6*h^
12 - 36700160*a^6*b^5*c^8*d^16*e^3*f^6*h^12 + 34078720*a^8*b^7*c^4*d^10*e^
9*f^6*h^12 + 34078720*a^7*b^7*c^5*d^12*e^7*f^6*h^12 + 26214400*a^12*b^4*c^
3*d^5*e^14*f^6*h^12 + 26214400*a^6*b^4*c^9*d^17*e^2*f^6*h^12 + 22118400*a^
7*b^9*c^3*d^10*e^9*f^6*h^12 + 22118400*a^6*b^9*c^4*d^12*e^7*f^6*h^12 - ...
```

3.311 $\int \frac{x^5\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$

3.311.1 Optimal result

Integrand size = 29, antiderivative size = 272

$$\begin{aligned}\int \frac{x^5\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx &= \frac{((2cd-be)(4cd+be)-2ce(2cd+be)x^2)\sqrt{a+bx^2+cx^4}}{16c^2e^3}+\frac{(a+bx^2+cx^4)^{3/2}}{6ce}\\ &\quad-\frac{(16c^3d^3-b^3e^3-2bce^2(bd-2ae)-8c^2de(bd-ae))\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{5/2}e^4}\\ &\quad+\frac{d^2\sqrt{cd^2-bde+ae^2}\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2e^4}\end{aligned}$$

output
```
1/6*(c*x^4+b*x^2+a)^(3/2)/c/e-1/32*(16*c^3*d^3-b^3*e^3-2*b*c*e^2*(-2*a*e+b
*d)-8*c^2*d*e*(-a*e+b*d))*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^
(1/2))/c^(5/2)/e^4+1/2*d^2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2
-b*d*e+c*d^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2))*(a*e^2-b*d*e+c*d^2)^(1/2)/e^4+1
/16*((-b*e+2*c*d)*(b*e+4*c*d)-2*c*e*(b*e+2*c*d)*x^2)*(c*x^4+b*x^2+a)^(1/2)
/c^2/e^3
```

3.311.2 Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.95

$$\int \frac{x^5\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$$
$$= \frac{2\sqrt{c}e\sqrt{a+bx^2+cx^4}(-3b^2e^2+2ce(-3bd+4ae+bex^2)+4c^2(6d^2-3dex^2+2e^2x^4))+96c^{5/2}d^2\sqrt{-cd^2+}}{}$$

input
```
Integrate[(x^5*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2),x]
```

output
```
(2*Sqrt[c]*e*Sqrt[a + b*x^2 + c*x^4]*(-3*b^2*e^2 + 2*c*e*(-3*b*d + 4*a*e +
 b*e*x^2) + 4*c^2*(6*d^2 - 3*d*e*x^2 + 2*e^2*x^4)) + 96*c^(5/2)*d^2*Sqrt[-
(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x^2) - e*Sqrt[a + b*x^2 +
c*x^4])/Sqrt[-(c*d^2) + e*(b*d - a*e)]] + 3*(16*c^3*d^3 - b^3*e^3 - 2*b*c*
e^2*(b*d - 2*a*e) + 8*c^2*d*e*(-(b*d) + a*e))*Log[b + 2*c*x^2 - 2*Sqrt[c]*
Sqrt[a + b*x^2 + c*x^4]])/(96*c^(5/2)*e^4)
```

3.311.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {1578, 1267, 27, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{x^4\sqrt{cx^4+bx^2+a}}{ex^2+d}dx^2$$

$$\Big\downarrow \text{1267}$$

$$\frac{1}{2}\left(\frac{\int -\frac{3e((2cd+be)x^2+bd)\sqrt{cx^4+bx^2+a}}{2(ex^2+d)}dx^2}{3ce^2}+\frac{\left(a+bx^2+cx^4\right)^{3/2}}{3ce}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\left(a+bx^2+cx^4\right)^{3/2}}{3ce}-\frac{\int \frac{((2cd+be)x^2+bd)\sqrt{cx^4+bx^2+a}}{ex^2+d}dx^2}{2ce}\right)$$

$$\Big\downarrow \text{1231}$$

$$\frac{1}{2}\left(\frac{\left(a+bx^2+cx^4\right)^{3/2}}{3ce}-\frac{-\frac{\int -\frac{\left(16c^3d^3-8c^2e(bd-ae)d-b^3e^3-2bce^2(bd-2ae)\right)x^2+d(2cd-be)\left(eb^2+4cdb-4ace\right)}{2\left(ex^2+d\right)\sqrt{cx^4+bx^2+a}}dx^2}{4ce^2}-\frac{\sqrt{a+bx^2+cx^4}((2cd-be)(}{4}}{2ce}\right.$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\left(a+bx^2+cx^4\right)^{3/2}}{3ce}-\frac{\frac{\int \frac{\left(16c^3d^3-8c^2e(bd-ae)d-b^3e^3-2bce^2(bd-2ae)\right)x^2+d(2cd-be)\left(eb^2+4cdb-4ace\right)}{\left(ex^2+d\right)\sqrt{cx^4+bx^2+a}}dx^2}{8ce^2}-\frac{\sqrt{a+bx^2+cx^4}((2cd-be)(be+}{4ce^2}}{2ce}\right.$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{2}\left(\frac{\left(a+bx^2+cx^4\right)^{3/2}}{3ce}-\frac{\frac{\frac{\left(-8c^2de(bd-ae)-2bce^2(bd-2ae)-b^3e^3+16c^3d^3\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx^2}{e}-\frac{16c^2d^2\left(ae^2-bde+cd^2\right)\int \frac{1}{\left(ex^2+d\right)\sqrt{cx^4+bx^2+a}}d}{e}}{8ce^2}}{2ce}\right.$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}\left(\frac{\left(a+bx^2+cx^4\right)^{3/2}}{3ce}-\frac{\frac{\frac{2\left(-8c^2de(bd-ae)-2bce^2(bd-2ae)-b^3e^3+16c^3d^3\right)\int \frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}}{e}-\frac{16c^2d^2\left(ae^2-bde+cd^2\right)\int \frac{1}{\left(ex^2+d\right)\sqrt{cx^4+bx^2}}}{e}}{8ce^2}}{2ce}\right.$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{\left(a+bx^2+cx^4\right)^{3/2}}{3ce}-\frac{\frac{\frac{\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)\left(-8c^2de(bd-ae)-2bce^2(bd-2ae)-b^3e^3+16c^3d^3\right)}{\sqrt{c}e}-\frac{16c^2d^2\left(ae^2-bde+cd^2\right)\int \frac{1}{\left(ex^2+d\right)\sqrt{c}}}{e}}{8ce^2}}{2ce}\right.$$

$$\Big\downarrow \text{1154}$$

3.311. $\int \frac{x^5\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$

$$\frac{1}{2}\left(\frac{\left(a+bx^2+cx^4\right)^{3/2}}{3ce}-\frac{\frac{\frac{32c^2d^2\left(ae^2-bde+cd^2\right)\int \frac{1}{4\left(cd^2-bed+ae^2\right)-x^4}d\left(-\frac{(2cd-be)x^2+bd-2ae}{\sqrt{cx^4+bx^2+a}}\right)}{e}+\frac{\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)\left(-8c^2de(bd\right.}{\sqrt{ce}}}{8ce^2}}{2ce}\right.$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}\left(\frac{\left(a+bx^2+cx^4\right)^{3/2}}{3ce}-\frac{\frac{\frac{\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)\left(-8c^2de(bd-ae)-2bce^2(bd-2ae)-b^3e^3+16c^3d^3\right)}{\sqrt{ce}}-\frac{16c^2d^2\sqrt{ae^2-bde+cd^2}\,\text{arctanh}\left(\frac{}{2\sqrt{}}\right.}{e}}{8ce^2}}{2ce}\right.$$

input
```
Int[(x^5*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2),x]
```

output
```
((a + b*x^2 + c*x^4)^(3/2)/(3*c*e) - (-1/4*(((2*c*d - b*e)*(4*c*d + b*e) -
 2*c*e*(2*c*d + b*e)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(c*e^2) + (((16*c^3*d^3
 - b^3*e^3 - 2*b*c*e^2*(b*d - 2*a*e) - 8*c^2*d*e*(b*d - a*e))*ArcTanh[(b +
 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(Sqrt[c]*e) - (16*c^2*d^2*
Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*S
qrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/e)/(8*c*e^2))/(2*c*e
))/2
```

3.311.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1267
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - g^n*(d
+ e*x)^(n - 2)*(b*d*e*(p + 1) + a*e^2*(m + n - 1) - c*d^2*(m + n + 2*p + 1)
 - e*(2*c*d - b*e)*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && IGtQ[n, 1] && IntegerQ[m] && NeQ[m + n + 2*p + 1, 0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.311. $\int \frac{x^5\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$

3.311.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.36

method	result
pseudoelliptic	$\frac{\left(\left(-3e^2d^2a+3d^3eb\right)c^{\frac{5}{2}}-3c^{\frac{7}{2}}d^4\right)\ln\left(\frac{2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,e+\left(b\,x^2+2a\right)e-d\left(2c\,x^2+b\right)}{e\,x^2+d}\right)+e\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\left(\left(\left(-\frac{3}{4}\cdots\right.\right.\right.}{\cdots}$
risch	$\frac{(8e^2c^2x^4+2bc\,e^2x^2-12c^2de\,x^2+8e^2ac-3b^2e^2-6bcde+24c^2d^2)\sqrt{c\,x^4+b\,x^2+a}}{48c^2e^3}-\frac{8d^2\left(a\,e^2-bde+c\,d^2\right)c^2\ln\left(\frac{2a\,e^2-2bde+2c\,d\cdots}{e^2}\cdots\right.}{\cdots}$
default	$\frac{\frac{\left(c\,x^4+b\,x^2+a\right)^{\frac{3}{2}}}{6c}-\frac{b\left(\frac{\left(2c\,x^2+b\right)\sqrt{c\,x^4+b\,x^2+a}}{4c}+\frac{\left(4ac-b^2\right)\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{8c^{\frac{3}{2}}}\right)}{4c}}{e}-\frac{d\left(\frac{\left(2c\,x^2+b\right)\sqrt{c\,x^4+b\,x^2+a}}{8c}+\frac{\left(4ac-\cdots\right.}{\cdots}\right.}{e\cdots}$
elliptic	$\frac{-d\left(\frac{\left(2c\,x^2+b\right)\sqrt{c\,x^4+b\,x^2+a}}{4c}+\frac{\left(4ac-b^2\right)\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{8c^{\frac{3}{2}}}\right)+e\left(\frac{\left(c\,x^4+b\,x^2+a\right)^{\frac{3}{2}}}{3c}-\frac{b\left(\frac{\left(2c\,x^2+b\right)\sqrt{c\,x^4+b\,x^2+a}}{4c}+\frac{\left(4c\cdots\right.}{\cdots}\right.}{\cdots}\cdots\right.}{2e^2}$

input
```
int(x^5*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/6/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)/c^(5/2)*(((-3*a*d^2*e^2+3*b*d^3*e)*c^(
5/2)-3*c^(7/2)*d^4)*ln((2*(c*x^4+b*x^2+a)^(1/2)*((a*e^2-b*d*e+c*d^2)/e^2)^
(1/2)*e+(b*x^2+2*a)*e-d*(2*c*x^2+b))/(e*x^2+d))+e*((a*e^2-b*d*e+c*d^2)/e^2
)^(1/2)*(((-3/4*a*b*c+3/16*b^3)*e^3-3/2*c*d*(a*c-1/4*b^2)*e^2+3/2*b*c^2*d^
2*e-3*c^3*d^3)*ln((2*c*x^2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c^(1/2))+e*(
(e^2*x^4-3/2*e*d*x^2+3*d^2)*c^(5/2)+e*(((1/4*b*x^2+a)*e-3/4*b*d)*c^(3/2)-3
/8*b^2*e*c^(1/2)))*(c*x^4+b*x^2+a)^(1/2)+3/4*((a*b*c-1/4*b^3)*e^3+2*c*d*(a
*c-1/4*b^2)*e^2-2*b*c^2*d^2*e+4*c^3*d^3)*ln(2)))/e^5
```

3.311.5 Fricas [A] (verification not implemented)

Time = 94.12 (sec) , antiderivative size = 1525, normalized size of antiderivative = 5.61

$$\int \frac{x^5\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^5*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="fricas")
```

output
```
[1/192*(48*sqrt(c*d^2 - b*d*e + a*e^2)*c^3*d^2*log(-((8*c^2*d^2 - 8*b*c*d*
e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2
*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2
+ a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x
^4 + 2*d*e*x^2 + d^2)) + 3*(16*c^3*d^3 - 8*b*c^2*d^2*e - 2*(b^2*c - 4*a*c^
2)*d*e^2 - (b^3 - 4*a*b*c)*e^3)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 +
 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*(8*c^3*e^3*x
^4 + 24*c^3*d^2*e - 6*b*c^2*d*e^2 - (3*b^2*c - 8*a*c^2)*e^3 - 2*(6*c^3*d*e
^2 - b*c^2*e^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(c^3*e^4), 1/192*(96*sqrt(-c
*d^2 + b*d*e - a*e^2)*c^3*d^2*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*
d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e
 + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e
^2)*x^2)) + 3*(16*c^3*d^3 - 8*b*c^2*d^2*e - 2*(b^2*c - 4*a*c^2)*d*e^2 - (b
^3 - 4*a*b*c)*e^3)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c*x^4
 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*(8*c^3*e^3*x^4 + 24*c^3*d
^2*e - 6*b*c^2*d*e^2 - (3*b^2*c - 8*a*c^2)*e^3 - 2*(6*c^3*d*e^2 - b*c^2*e^
3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(c^3*e^4), 1/96*(24*sqrt(c*d^2 - b*d*e +
a*e^2)*c^3*d^2*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a
*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2
 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e...
```

3.311.6 Sympy [F]

$$\int \frac{x^5\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx = \int \frac{x^5\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$$

input
```
integrate(x**5*(c*x**4+b*x**2+a)**(1/2)/(e*x**2+d),x)
```

output
```
Integral(x**5*sqrt(a + b*x**2 + c*x**4)/(d + e*x**2), x)
```

3.311.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.311.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^5\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.311.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx = \int \frac{x^5\,\sqrt{c\,x^4+b\,x^2+a}}{e\,x^2+d}\,dx$$

input
```
int((x^5*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2),x)
```

output
```
int((x^5*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2), x)
```

3.312 $\int \frac{x^3\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$

3.312.1 Optimal result

Integrand size = 29, antiderivative size = 208

$$\int \frac{x^3\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx = -\frac{(4cd-be-2cex^2)\sqrt{a+bx^2+cx^4}}{8ce^2} + \frac{(8c^2d^2-b^2e^2-4ce(bd-ae))\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}e^3} - \frac{d\sqrt{cd^2-bde+ae^2}\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2e^3}$$

output
```
1/16*(8*c^2*d^2-b^2*e^2-4*c*e*(-a*e+b*d))*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/
(c*x^4+b*x^2+a)^(1/2))/c^(3/2)/e^3-1/2*d*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*
d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2))*(a*e^2-b*d*e+c*d^
2)^(1/2)/e^3-1/8*(-2*c*e*x^2-b*e+4*c*d)*(c*x^4+b*x^2+a)^(1/2)/c/e^2
```

3.312.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.96

$$\int \frac{x^3\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx = \frac{2\sqrt{c}\left(e(-4cd+be+2cex^2)\sqrt{a+bx^2+cx^4} - 8cd\sqrt{-cd^2+bde-ae^2}\arctan\left(\frac{\sqrt{c}(d+ex^2)-e\sqrt{a+bx^2+cx^4}}{\sqrt{-cd^2+bde-ae^2}}\right)\right) +}{16c^{3/2}e^3}$$

input
```
Integrate[(x^3*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2),x]
```

output
```
(2*Sqrt[c]*(e*(-4*c*d + b*e + 2*c*e*x^2)*Sqrt[a + b*x^2 + c*x^4] - 8*c*d*S
qrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x^2) - e*Sqrt[a + b*x
^2 + c*x^4])/Sqrt[-(c*d^2) + b*d*e - a*e^2]]) + (8*c^2*d^2 - b^2*e^2 + 4*c
*e*(-(b*d) + a*e))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4
])])/(16*c^(3/2)*e^3)
```

3.312.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {1578, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{a+bx^2+cx^4}}{d+ex^2}dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{x^2\sqrt{cx^4+bx^2+a}}{ex^2+d}dx^2$$

$$\Big\downarrow \text{1231}$$

$$\frac{1}{2}\left(-\frac{\int -\frac{(8c^2d^2-b^2e^2-4ce(bd-ae))x^2+d(-eb^2+4cdb-4ace)}{2(ex^2+d)\sqrt{cx^4+bx^2+a}}dx^2}{4ce^2}-\frac{\sqrt{a+bx^2+cx^4}(-be+4cd-2cex^2)}{4ce^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\int \frac{(8c^2d^2-b^2e^2-4ce(bd-ae))x^2+d(-eb^2+4cdb-4ace)}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx^2}{8ce^2}-\frac{\sqrt{a+bx^2+cx^4}(-be+4cd-2cex^2)}{4ce^2}\right)$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{2}\left(\frac{\frac{(-4ce(bd-ae)-b^2e^2+8c^2d^2)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx^2}{e}-\frac{8cd(ae^2-bde+cd^2)\int \frac{1}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx^2}{e}}{8ce^2}-\frac{\sqrt{a+bx^2+cx^4}(-be+4cd-}{4ce^2}\right.$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}\left(\frac{\frac{2(-4ce(bd-ae)-b^2e^2+8c^2d^2)\int \frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}}{e}-\frac{8cd(ae^2-bde+cd^2)\int \frac{1}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx^2}{e}}{8ce^2}-\frac{\sqrt{a+bx^2+cx^4}(-be+4}{4ce^2}\right.$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{\frac{\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)(-4ce(bd-ae)-b^2e^2+8c^2d^2)}{\sqrt{c}e}-\frac{8cd(ae^2-bde+cd^2)\int \frac{1}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx^2}{e}}{8ce^2}-\frac{\sqrt{a+bx^2+cx^4}(-b}{4ce^2}\right.$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{2}\left(\frac{\frac{16cd(ae^2-bde+cd^2)\int \frac{1}{4(cd^2-bed+ae^2)-x^4}d\left(-\frac{(2cd-be)x^2+bd-2ae}{\sqrt{cx^4+bx^2+a}}\right)}{e}+\frac{\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)(-4ce(bd-ae)-b^2e^2+8c^2d^2)}{\sqrt{c}e}}{8ce^2}-\frac{\sqrt{}}{}\right.$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{\frac{\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)(-4ce(bd-ae)-b^2e^2+8c^2d^2)}{\sqrt{c}e}-\frac{8cd\sqrt{ae^2-bde+cd^2}\,\text{arctanh}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{e}}{8ce^2}-\frac{\sqrt{a+bx}}{}\right.$$

input
```
Int[(x^3*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2),x]
```

output
```
(-1/4*((4*c*d - b*e - 2*c*e*x^2)*Sqrt[a + b*x^2 + c*x^4])/(c*e^2) + (((8*c
^2*d^2 - b^2*e^2 - 4*c*e*(b*d - a*e))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqr
t[a + b*x^2 + c*x^4])])/(Sqrt[c]*e) - (8*c*d*Sqrt[c*d^2 - b*d*e + a*e^2]*A
rcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sq
rt[a + b*x^2 + c*x^4])])/e)/(8*c*e^2))/2
```

3.312. $\int \frac{x^3\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$

3.312.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.312. $\int \frac{x^3\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.312.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.38

method	result
pseudoelliptic	$\frac{(a\,e^{2}-bde+c\,d^{2})\,d\,c^{\frac{5}{2}}\ln\left(\frac{2\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}\,e+\left(b\,x^{2}+2a\right)e-d\left(2c\,x^{2}+b\right)}{e\,x^{2}+d}\right)-\left(-\frac{\left(2c^{2}d^{2}+\left(a\,e^{2}-bde\right)c-\frac{b^{2}e^{2}}{4}\right)c\ln\left(\cdots\right.}{2}\cdots\right.}{2\sqrt{\frac{a\,e^{2}-bd\cdots}{e^{2}}}\cdots}$
risch	$\frac{(2c\,x^{2}e+be-4cd)\sqrt{c\,x^{4}+b\,x^{2}+a}}{8c\,e^{2}}+\frac{\frac{\left(4e^{2}ac-b^{2}e^{2}-4bcde+8c^{2}d^{2}\right)\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{2e\sqrt{c}}+\frac{4d\left(a\,e^{2}-bde+c\,d^{2}\right)c\ln\left(\frac{2a\,e^{2}-\cdots}{\cdots}\right.}{\cdots}}{\cdots}$
default	$\frac{\frac{\left(2c\,x^{2}+b\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}{8c}+\frac{\left(4ac-b^{2}\right)\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{16c^{\frac{3}{2}}}}{e}-\frac{d\left(\sqrt{c\left(x^{2}+\frac{d}{e}\right)^{2}+\frac{(be-2cd)\left(x^{2}+\frac{d}{e}\right)}{e}+\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}+\frac{(be-\cdots}{\cdots}\right.}{\cdots}$
elliptic	$\frac{\frac{\left(2c\,x^{2}+b\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}{4c}+\frac{\left(4ac-b^{2}\right)\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{8c^{\frac{3}{2}}}}{2e}-\frac{d\left(\sqrt{c\left(x^{2}+\frac{d}{e}\right)^{2}+\frac{(be-2cd)\left(x^{2}+\frac{d}{e}\right)}{e}+\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}+\frac{(be-\cdots}{\cdots}\right.}{\cdots}$

input
```
int(x^3*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((a*e^2-b*d*e+c*d^2)*d*c^(5/2)*ln((2*(
c*x^4+b*x^2+a)^(1/2)*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*e+(b*x^2+2*a)*e-d*(2*
c*x^2+b))/(e*x^2+d))-(-1/2*(2*c^2*d^2+(a*e^2-b*d*e)*c-1/4*b^2*e^2)*c*ln((2
*c*x^2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c^(1/2))+e*c^(3/2)*((-1/2*e*x^2+
d)*c-1/4*b*e)*(c*x^4+b*x^2+a)^(1/2)+1/2*(2*c^2*d^2+(a*e^2-b*d*e)*c-1/4*b^2
*e^2)*ln(2)*c)*e*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(5/2)/e^4
```

3.312.5 Fricas [A] (verification not implemented)

Time = 10.36 (sec) , antiderivative size = 1231, normalized size of antiderivative = 5.92

$$\int \frac{x^3\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="fricas")
```

output
```
[1/32*(8*sqrt(c*d^2 - b*d*e + a*e^2)*c^2*d*log(-((8*c^2*d^2 - 8*b*c*d*e +
(b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*
b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)
*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 +
 2*d*e*x^2 + d^2)) + (8*c^2*d^2 - 4*b*c*d*e - (b^2 - 4*a*c)*e^2)*sqrt(c)*l
og(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*
sqrt(c) - 4*a*c) + 4*(2*c^2*e^2*x^2 - 4*c^2*d*e + b*c*e^2)*sqrt(c*x^4 + b*
x^2 + a))/(c^2*e^3), -1/32*(16*sqrt(-c*d^2 + b*d*e - a*e^2)*c^2*d*arctan(-
1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^
2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e +
a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - (8*c^2*d^2 - 4*b*c*d*e - (
b^2 - 4*a*c)*e^2)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4
+ b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 4*(2*c^2*e^2*x^2 - 4*c^2*d*e
 + b*c*e^2)*sqrt(c*x^4 + b*x^2 + a))/(c^2*e^3), 1/16*(4*sqrt(c*d^2 - b*d*e
 + a*e^2)*c^2*d*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*
a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^
2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2
)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)) - (8*c^2
*d^2 - 4*b*c*d*e - (b^2 - 4*a*c)*e^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x
^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(2*c^2*e^...
```

3.312.6 Sympy [F]

$$\int \frac{x^3\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx = \int \frac{x^3\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$$

input
```
integrate(x**3*(c*x**4+b*x**2+a)**(1/2)/(e*x**2+d),x)
```

output
```
Integral(x**3*sqrt(a + b*x**2 + c*x**4)/(d + e*x**2), x)
```

3.312.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.312.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.312.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx = \int \frac{x^3\,\sqrt{c\,x^4+b\,x^2+a}}{e\,x^2+d}\,dx$$

input
```
int((x^3*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2),x)
```

output
```
int((x^3*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2), x)
```

3.313 $\int \frac{x\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$

3.313.1 Optimal result

Integrand size = 27, antiderivative size = 168

$$\int \frac{x\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx=\frac{\sqrt{a+bx^2+cx^4}}{2e}-\frac{(2cd-be)\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}e^2}+\frac{\sqrt{cd^2-bde+ae^2}\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2e^2}$$

output
```
-1/4*(-b*e+2*c*d)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/e
^2/c^(1/2)+1/2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2
)^(1/2)/(c*x^4+b*x^2+a)^(1/2))*(a*e^2-b*d*e+c*d^2)^(1/2)/e^2+1/2*(c*x^4+b*
x^2+a)^(1/2)/e
```

3.313.2 Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.95

$$\int \frac{x\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$$
$$=\frac{e\sqrt{a+bx^2+cx^4}+2\sqrt{-cd^2+bde-ae^2}\arctan\left(\frac{\sqrt{-cd^2+bde-ae^2}x^2}{\sqrt{a}(d+ex^2)-d\sqrt{a+bx^2+cx^4}}\right)+\frac{(-2cd+be)\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{-\sqrt{a}+\sqrt{a+bx^2+cx}}\right.}{\sqrt{c}}}{2e^2}$$

input
```
Integrate[(x*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2),x]
```

output
```
(e*Sqrt[a + b*x^2 + c*x^4] + 2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt
[-(c*d^2) + b*d*e - a*e^2]*x^2)/(Sqrt[a]*(d + e*x^2) - d*Sqrt[a + b*x^2 +
c*x^4])] + ((-2*c*d + b*e)*ArcTanh[(Sqrt[c]*x^2)/(-Sqrt[a] + Sqrt[a + b*x^
2 + c*x^4])])/Sqrt[c])/(2*e^2)
```

3.313.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {1576, 1162, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{a+bx^2+cx^4}}{d+ex^2}dx$$

$$\Big\downarrow \text{1576}$$

$$\frac{1}{2}\int \frac{\sqrt{cx^4+bx^2+a}}{ex^2+d}dx^2$$

$$\Big\downarrow \text{1162}$$

$$\frac{1}{2}\left(\frac{\sqrt{a+bx^2+cx^4}}{e}-\frac{\int \frac{(2cd-be)x^2+bd-2ae}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx^2}{2e}\right)$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{2}\left(\frac{\sqrt{a+bx^2+cx^4}}{e}-\frac{\frac{(2cd-be)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx^2}{e}-\frac{2(ae^2-bde+cd^2)\int \frac{1}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx^2}{e}}{2e}\right)$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}\left(\frac{\sqrt{a+bx^2+cx^4}}{e}-\frac{\frac{2(2cd-be)\int \frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}}{e}-\frac{2(ae^2-bde+cd^2)\int \frac{1}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx^2}{e}}{2e}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{\sqrt{a+bx^2+cx^4}}{e}-\frac{\frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{c}e}-\frac{2(ae^2-bde+cd^2)\int\frac{1}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx^2}{e}}{2e}\right)$$

$\downarrow$ 1154

$$\frac{1}{2}\left(\frac{\sqrt{a+bx^2+cx^4}}{e}-\frac{\frac{4(ae^2-bde+cd^2)\int\frac{1}{4(cd^2-bed+ae^2)-x^4}d\left(-\frac{(2cd-be)x^2+bd-2ae}{\sqrt{cx^4+bx^2+a}}\right)}{e}+\frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{c}e}}{2e}\right)$$

$\downarrow$ 219

$$\frac{1}{2}\left(\frac{\sqrt{a+bx^2+cx^4}}{e}-\frac{\frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{c}e}-\frac{2\sqrt{ae^2-bde+cd^2}\text{arctanh}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{e}}{2e}\right)$$

input
```
Int[(x*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2),x]
```

output
```
(Sqrt[a + b*x^2 + c*x^4]/e - (((2*c*d - b*e)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt
[c]*Sqrt[a + b*x^2 + c*x^4])])/(Sqrt[c]*e) - (2*Sqrt[c*d^2 - b*d*e + a*e^2
]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]
*Sqrt[a + b*x^2 + c*x^4])])/e)/(2*e))/2
```

3.313.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

3.313. $\int\frac{x\sqrt{a+bx^2+cx^4}}{d+ex^2}dx$

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

3.313.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.13

method	result
pseudoelliptic	$\frac{\sqrt{c\,x^4+b\,x^2+a}-\frac{(be-2cd)\left(\ln(2)-\ln\left(\frac{2c\,x^2+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{c}+b}{\sqrt{c}}\right)\right)}{2e\sqrt{c}}-\frac{\left(a\,e^2-bde+c\,d^2\right)\ln\left(\frac{2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,e+(b}{e\,x^2+d}\right)}{\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,e^2}}{2e}$
risch	$\frac{\sqrt{c\,x^4+b\,x^2+a}}{2e}+\frac{\frac{(be-2cd)\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{2e\sqrt{c}}-\frac{\left(a\,e^2-bde+c\,d^2\right)\ln\left(\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+2\sqrt{a\,e^2-b}\right)}{e^2\sqrt{\frac{a\,e^2-bde+}{e^2}}}}{2e}$
default	$\frac{\sqrt{c\left(x^2+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a\,e^2-bde+c\,d^2}{e^2}}+\frac{(be-2cd)\ln\left(\frac{\frac{be-2cd}{2e}+c\left(x^2+\frac{d}{e}\right)}{\sqrt{c}}+\sqrt{c\left(x^2+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a\,e^2-bde+}{e^2}}\right)}{2e\sqrt{c}}}{}$
elliptic	$\frac{\sqrt{c\left(x^2+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a\,e^2-bde+c\,d^2}{e^2}}+\frac{(be-2cd)\ln\left(\frac{\frac{be-2cd}{2e}+c\left(x^2+\frac{d}{e}\right)}{\sqrt{c}}+\sqrt{c\left(x^2+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a\,e^2-bde+}{e^2}}\right)}{2e\sqrt{c}}}{}$

input
```
int(x*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/2*((c*x^4+b*x^2+a)^(1/2)-1/2*(b*e-2*c*d)*(ln(2)-ln((2*c*x^2+2*(c*x^4+b*x
^2+a)^(1/2)*c^(1/2)+b)/c^(1/2)))/e/c^(1/2)-(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d
*e+c*d^2)/e^2)^(1/2)*ln((2*(c*x^4+b*x^2+a)^(1/2)*((a*e^2-b*d*e+c*d^2)/e^2)
^(1/2)*e+(b*x^2+2*a)*e-d*(2*c*x^2+b))/(e*x^2+d))/e^2)/e
```

3.313.5 Fricas [A] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 1050, normalized size of antiderivative = 6.25

$$\int \frac{x\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$$

$$= \left[\frac{4\sqrt{cx^4+bx^2+a}ce-(2\,cd-be)\sqrt{c}\log\left(-8\,c^2x^4-8\,bcx^2-b^2-4\sqrt{cx^4+bx^2+a}(2\,cx^2+b)\sqrt{c}-4\,ac\right)}{}\right.$$

input
```
integrate(x*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="fricas")
```

output
```
[1/8*(4*sqrt(c*x^4 + b*x^2 + a)*c*e - (2*c*d - b*e)*sqrt(c)*log(-8*c^2*x^4
 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a
*c) + 2*sqrt(c*d^2 - b*d*e + a*e^2)*c*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2
+ 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d
^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt
(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*
e*x^2 + d^2)))/(c*e^2), 1/4*(2*sqrt(c*x^4 + b*x^2 + a)*c*e + (2*c*d - b*e)
*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x
^4 + b*c*x^2 + a*c)) + sqrt(c*d^2 - b*d*e + a*e^2)*c*log(-((8*c^2*d^2 - 8*
b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d
^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 +
b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/
(e^2*x^4 + 2*d*e*x^2 + d^2)))/(c*e^2), 1/8*(4*sqrt(c*x^4 + b*x^2 + a)*c*e
+ 4*sqrt(-c*d^2 + b*d*e - a*e^2)*c*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqr
t(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*
c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e +
a*b*e^2)*x^2)) - (2*c*d - b*e)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 -
4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c))/(c*e^2), 1/4*(2*
sqrt(c*x^4 + b*x^2 + a)*c*e + 2*sqrt(-c*d^2 + b*d*e - a*e^2)*c*arctan(-1/2
*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^...
```

3.313.6 Sympy [F]

$$\int \frac{x\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx = \int \frac{x\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$$

input
```
integrate(x*(c*x**4+b*x**2+a)**(1/2)/(e*x**2+d),x)
```

output
```
Integral(x*sqrt(a + b*x**2 + c*x**4)/(d + e*x**2), x)
```

3.313.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-b*d*e>0)', see `assume?` f
or more de
```

3.313.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.313.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx = \int \frac{x\,\sqrt{c\,x^4+b\,x^2+a}}{e\,x^2+d}\,dx$$

input
```
int((x*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2),x)
```

output
```
int((x*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2), x)
```

3.314 $\int \frac{\sqrt{a+bx^2+cx^4}}{x(d+ex^2)}\,dx$

3.314.1 Optimal result

Integrand size = 29, antiderivative size = 186

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x(d+ex^2)}\,dx = -\frac{\sqrt{a}\,\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d}+\frac{\sqrt{c}\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2e} - \frac{\sqrt{cd^2-bde+ae^2}\,\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2de}$$

output
```
-1/2*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))*a^(1/2)/d+1/2*
arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))*c^(1/2)/e-1/2*arcta
nh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^4+b*x^2
+a)^(1/2))*(a*e^2-b*d*e+c*d^2)^(1/2)/d/e
```

3.314.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x(d+ex^2)}\,dx = -\frac{2\sqrt{-cd^2+bde-ae^2}\arctan\left(\frac{\sqrt{c}(d+ex^2)-e\sqrt{a+bx^2+cx^4}}{\sqrt{-cd^2+bde-ae^2}}\right)-2\sqrt{a}e\,\text{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)+\sqrt{c}d\log\left(e(b\right.}{2de}$$

input
```
Integrate[Sqrt[a + b*x^2 + c*x^4]/(x*(d + e*x^2)),x]
```

output
```
-1/2*(2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x^2) - e*Sqr
t[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + b*d*e - a*e^2]] - 2*Sqrt[a]*e*ArcTan
h[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]] + Sqrt[c]*d*Log[e*(b +
2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(d*e)
```

3.314.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {1578, 1270, 25, 1154, 219, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x\left(d+ex^2\right)}dx$$

$$\Big\downarrow \text{ 1578}$$

$$\frac{1}{2}\int \frac{\sqrt{cx^4+bx^2+a}}{x^2\left(ex^2+d\right)}dx^2$$

$$\Big\downarrow \text{ 1270}$$

$$\frac{1}{2}\left(\frac{a\int \frac{1}{x^2\sqrt{cx^4+bx^2+a}}dx^2}{d}-\frac{\int -\frac{cdx^2+bd-ae}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx^2}{d}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\left(\frac{\int \frac{cdx^2+bd-ae}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx^2}{d}+\frac{a\int \frac{1}{x^2\sqrt{cx^4+bx^2+a}}dx^2}{d}\right)$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{1}{2}\left(\frac{\int \frac{cdx^2+bd-ae}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx^2}{d}-\frac{2a\int \frac{1}{4a-x^4}d\frac{bx^2+2a}{\sqrt{cx^4+bx^2+a}}}{d}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}\left(\frac{\int \frac{cdx^2+bd-ae}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx^2}{d}-\frac{\sqrt{a}\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{d}\right)$$

$$\Big\downarrow \text{ 1269}$$

$$\frac{1}{2}\left(\frac{\frac{cd\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx^2}{e}-\frac{\left(ae^2-bde+cd^2\right)\int \frac{1}{\left(ex^2+d\right)\sqrt{cx^4+bx^2+a}}dx^2}{e}}{d}-\frac{\sqrt{a}\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{d}\right)$$

$\Big\downarrow$ 1092

$$\frac{1}{2}\left(\frac{\frac{2cd\int \frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}}{e}-\frac{\left(ae^2-bde+cd^2\right)\int \frac{1}{\left(ex^2+d\right)\sqrt{cx^4+bx^2+a}}dx^2}{e}}{d}-\frac{\sqrt{a}\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{d}\right)$$

$\Big\downarrow$ 219

$$\frac{1}{2}\left(\frac{\frac{\sqrt{c}d\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{e}-\frac{\left(ae^2-bde+cd^2\right)\int \frac{1}{\left(ex^2+d\right)\sqrt{cx^4+bx^2+a}}dx^2}{e}}{d}-\frac{\sqrt{a}\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{d}\right)$$

$\Big\downarrow$ 1154

$$\frac{1}{2}\left(\frac{\frac{2\left(ae^2-bde+cd^2\right)\int \frac{1}{4\left(cd^2-bed+ae^2\right)-x^4}d\left(-\frac{(2cd-be)x^2+bd-2ae}{\sqrt{cx^4+bx^2+a}}\right)}{e}+\frac{\sqrt{c}d\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{e}}{d}-\frac{\sqrt{a}\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{d}\right)$$

$\Big\downarrow$ 219

$$\frac{1}{2}\left(\frac{\frac{\sqrt{c}d\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{e}-\frac{\sqrt{ae^2-bde+cd^2}\text{arctanh}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{e}}{d}-\frac{\sqrt{a}\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{d}\right)$$

input
```
Int[Sqrt[a + b*x^2 + c*x^4]/(x*(d + e*x^2)),x]
```

output
```
(-((Sqrt[a]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/d)
 + ((Sqrt[c]*d*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])
/e - (Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2
)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/e)/d)/2
```

3.314. $\int \frac{\sqrt{a+bx^2+cx^4}}{x(d+ex^2)}\,dx$

3.314.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1270
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) +
 (g_.)*(x_))), x_Symbol] :> Simp[(c*d^2 - b*d*e + a*e^2)/(e*(e*f - d*g))
Int[(a + b*x + c*x^2)^(p - 1)/(d + e*x), x], x] - Simp[1/(e*(e*f - d*g))
Int[Simp[c*d*f - b*e*f + a*e*g - c*(e*f - d*g)*x, x]*((a + b*x + c*x^2)^(p
- 1)/(f + g*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && FractionQ[p]
&& GtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.314. $\int \frac{\sqrt{a+bx^2+cx^4}}{x(d+ex^2)}\,dx$

3.314.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.29

method	result
pseudoelliptic	$\frac{\left(\left(a\,e^{2}-bde\right)\sqrt{c}+c^{\frac{3}{2}} d^{2}\right) \ln\left(\frac{2 \sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}\,e+\left(b\,x^{2}+2 a\right) e-d \left(2 c\,x^{2}+b\right)}{e\,x^{2}+d}\right)-\sqrt{\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}\,e \left(\sqrt{c}\,\ln\left(\frac{2 a+b\,x^{2}\cdots}{}\right.\right.}{2 \sqrt{\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}\,\sqrt{c}\,d\,e^{2}}$
elliptic	$\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}+\frac{b \ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{2 \sqrt{c}}-\sqrt{a}\,\ln\left(\frac{2 a+b\,x^{2}+2 \sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{2 d}-\frac{\sqrt{c \left(x^{2}+\frac{d}{e}\right)^{2}+\frac{\left(be-2cd\right) \left(x^{2}+\frac{d}{e}\right)}{e}+\cdots}}{}$
default	$\frac{\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}}{2}+\frac{b \ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{4 \sqrt{c}}-\frac{\sqrt{a}\,\ln\left(\frac{2 a+b\,x^{2}+2 \sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{2}}{d}-\frac{\sqrt{c \left(x^{2}+\frac{d}{e}\right)^{2}+\frac{\left(be-2cd\right) \left(x^{2}+\frac{d}{e}\right)}{e}+\cdots}}{}$

input
```
int((c*x^4+b*x^2+a)^(1/2)/x/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(((a*e^2-b*d*e)*c^(1/2)+c^(3/2)*d^2)*l
n((2*(c*x^4+b*x^2+a)^(1/2)*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*e+(b*x^2+2*a)*e
-d*(2*c*x^2+b))/(e*x^2+d))-((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*e*(c^(1/2)*ln((
2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)*e*a^(1/2)-ln((2*c*x^2+2*(c
*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c^(1/2))*c*d+ln(2)*c*d))/c^(1/2)/d/e^2
```

3.314.5 Fricas [A] (verification not implemented)

Time = 42.12 (sec) , antiderivative size = 2367, normalized size of antiderivative = 12.73

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x\,(d+ex^2)}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/x/(e*x^2+d),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(c)*d*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 +
a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + sqrt(a)*e*log(-((b^2 + 4*a*c)*x^4 + 8*
a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) +
sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e
^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b
*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b
*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^
2)))/(d*e), -1/4*(2*sqrt(-c)*d*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2
 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - sqrt(a)*e*log(-((b^2 + 4*a*c)*
x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)
/x^4) - sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 +
4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2
 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c
*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*
x^2 + d^2)))/(d*e), 1/4*(sqrt(c)*d*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sq
rt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + sqrt(a)*e*log(-((b^
2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(
a) + 8*a^2)/x^4) - 2*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(c*x^4 +
 b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)
/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*...
```

3.314.6 Sympy [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x\,(d+ex^2)}\,dx = \int \frac{\sqrt{a+bx^2+cx^4}}{x\,(d+ex^2)}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**(1/2)/x/(e*x**2+d),x)
```

output
```
Integral(sqrt(a + b*x**2 + c*x**4)/(x*(d + e*x**2)), x)
```

3.314.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x\left(d+ex^2\right)}\,dx=\int \frac{\sqrt{cx^4+bx^2+a}}{(ex^2+d)x}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/x/(e*x^2+d),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)/((e*x^2 + d)*x), x)
```

3.314.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x\left(d+ex^2\right)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/x/(e*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.314.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x\left(d+ex^2\right)}\,dx=\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{x\,\left(e\,x^2+d\right)}\,dx$$

input
```
int((a + b*x^2 + c*x^4)^(1/2)/(x*(d + e*x^2)),x)
```

output
```
int((a + b*x^2 + c*x^4)^(1/2)/(x*(d + e*x^2)), x)
```

3.315 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^3(d+ex^2)}\,dx$

3.315.1 Optimal result

Integrand size = 29, antiderivative size = 361

$$\begin{aligned}\int \frac{\sqrt{a+bx^2+cx^4}}{x^3(d+ex^2)}\,dx =& -\frac{\sqrt{a+bx^2+cx^4}}{2dx^2} - \frac{b\,\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}d} \\ &+ \frac{\sqrt{a}e\,\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d^2} + \frac{\sqrt{c}\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2d} \\ &- \frac{be\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}d^2} - \frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}d^2} \\ &+ \frac{\sqrt{cd^2-bde+ae^2}\,\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2d^2}\end{aligned}$$

output
```
-1/4*b*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d/a^(1/2)+1/
2*e*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))*a^(1/2)/d^2-1/4
*b*e*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d^2/c^(1/2)-1/
4*(-b*e+2*c*d)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d^2/
c^(1/2)+1/2*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))*c^(1/2)
/d+1/2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/
(c*x^4+b*x^2+a)^(1/2))*(a*e^2-b*d*e+c*d^2)^(1/2)/d^2-1/2*(c*x^4+b*x^2+a)^(
1/2)/d/x^2
```

3.315.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^3(d+ex^2)}\,dx$$
$$= \frac{-\frac{d\sqrt{a+bx^2+cx^4}}{x^2}+2\sqrt{-cd^2+bde-ae^2}\arctan\left(\frac{\sqrt{c}(d+ex^2)-e\sqrt{a+bx^2+cx^4}}{\sqrt{-cd^2+bde-ae^2}}\right)+\frac{(bd-2ae)\text{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{\sqrt{a}}}{2d^2}$$

input
```
Integrate[Sqrt[a + b*x^2 + c*x^4]/(x^3*(d + e*x^2)),x]
```

output
```
(-((d*Sqrt[a + b*x^2 + c*x^4])/x^2) + 2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*Arc
Tan[(Sqrt[c]*(d + e*x^2) - e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + b*d*
e - a*e^2]] + ((b*d - 2*a*e)*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4
])/Sqrt[a]])/Sqrt[a])/(2*d^2)
```

3.315.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 354, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1578, 1289, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^3(d+ex^2)}\,dx$$
$$\Big\downarrow \; 1578$$
$$\frac{1}{2}\int \frac{\sqrt{cx^4+bx^2+a}}{x^4(ex^2+d)}dx^2$$
$$\Big\downarrow \; 1289$$
$$\frac{1}{2}\int \left(\frac{\sqrt{cx^4+bx^2+a}e^2}{d^2(ex^2+d)}-\frac{\sqrt{cx^4+bx^2+a}e}{d^2x^2}+\frac{\sqrt{cx^4+bx^2+a}}{dx^4}\right)dx^2$$
$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{\sqrt{ae^2-bde+cd^2}\operatorname{arctanh}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{d^2}+\frac{\sqrt{a}e\operatorname{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{d^2}-\frac{be\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx}}\right.}{2\sqrt{c}d^2}\right.$$

input
```
Int[Sqrt[a + b*x^2 + c*x^4]/(x^3*(d + e*x^2)),x]
```

output
```
(-(Sqrt[a + b*x^2 + c*x^4]/(d*x^2)) - (b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*
Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[a]*d) + (Sqrt[a]*e*ArcTanh[(2*a + b*x^2
)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/d^2 + (Sqrt[c]*ArcTanh[(b + 2*c*x^
2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/d - (b*e*ArcTanh[(b + 2*c*x^2)/(2
*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[c]*d^2) - ((2*c*d - b*e)*ArcTa
nh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[c]*d^2) + (
Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*S
qrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/d^2)/2
```

3.315.3.1 Defintions of rubi rules used

rule 1289
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0]))
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.315.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.62

method	result
pseudoelliptic	$-\frac{a x^{2}\left(a e^{2}-b d e+c d^{2}\right) \ln\left(\frac{2 \sqrt{c x^{4}+b x^{2}+a}\, \sqrt{\frac{a e^{2}-b d e+c d^{2}}{e^{2}}}\, e+\left(b x^{2}+2 a\right) e-d\left(2 c x^{2}+b\right)}{e x^{2}+d}\right)+e\left(\frac{x^{2}\left(b \sqrt{a}\, d-2 a^{\frac{3}{2}} e\right) \ln\left(\frac{2 a+b x^{2}+2 \cdots}{\cdots}\right)}{2} \cdots\right)}{2 \sqrt{\frac{a e^{2}-b d e+c d^{2}}{e^{2}}}\, e a x^{2} d^{2}}$
risch	$-\frac{\sqrt{c x^{4}+b x^{2}+a}}{2 d x^{2}}-\frac{-\frac{(2 a e-b d) \ln\left(\frac{2 a+b x^{2}+2 \sqrt{a}\, \sqrt{c x^{4}+b x^{2}+a}}{x^{2}}\right)}{2 d \sqrt{a}}+\frac{\left(a e^{2}-b d e+c d^{2}\right) \ln\left(\frac{\frac{2 a e^{2}-2 b d e+2 c d^{2}}{e^{2}}+\frac{(b e-2 c d)\left(x^{2}+\frac{d}{e}\right)}{e}+\cdots}{\cdots}\right)}{d e \sqrt{\cdots}}}{2 d}$
default	$\frac{-\frac{\left(c x^{4}+b x^{2}+a\right)^{\frac{3}{2}}}{2 a x^{2}}+\frac{b \sqrt{c x^{4}+b x^{2}+a}}{2 a}-\frac{b \ln\left(\frac{2 a+b x^{2}+2 \sqrt{a}\, \sqrt{c x^{4}+b x^{2}+a}}{x^{2}}\right)}{4 \sqrt{a}}+\frac{c \sqrt{c x^{4}+b x^{2}+a}\, x^{2}}{2 a}+\frac{\sqrt{c}\, \ln\left(\frac{\frac{b}{2}+c x^{2}}{\sqrt{c}}+\sqrt{c x^{4}+b x^{2}+a}\right)}{2}}{d}$
elliptic	$\frac{-\frac{\left(c x^{4}+b x^{2}+a\right)^{\frac{3}{2}}}{a x^{2}}+\frac{b\left(\sqrt{c x^{4}+b x^{2}+a}+\frac{b \ln\left(\frac{\frac{b}{2}+c x^{2}}{\sqrt{c}}+\sqrt{c x^{4}+b x^{2}+a}\right)}{2 \sqrt{c}}-\sqrt{a}\, \ln\left(\frac{2 a+b x^{2}+2 \sqrt{a}\, \sqrt{c x^{4}+b x^{2}+a}}{x^{2}}\right)\right)}{2 a}+\frac{2 c\left(\frac{\left(2 c x^{2}+b\right) \sqrt{\cdots}}{4 \cdots} \cdots\right)}{\cdots}}{2 d}$

input
```
int((c*x^4+b*x^2+a)^(1/2)/x^3/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(a*x^2*(a*e^2-b*d*e+c*d^2)*ln((2*(c*x
^4+b*x^2+a)^(1/2)*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*e+(b*x^2+2*a)*e-d*(2*c*x
^2+b))/(e*x^2+d))+e*(1/2*x^2*(b*a^(1/2)*d-2*a^(3/2)*e)*ln((2*a+b*x^2+2*a^(
1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)+a*d*(c*x^4+b*x^2+a)^(1/2))*((a*e^2-b*d*e+
c*d^2)/e^2)^(1/2))/e/a/x^2/d^2
```

3.315.5 Fricas [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 1094, normalized size of antiderivative = 3.03

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^3\left(d+ex^2\right)}\,dx$$
$$= \left[\frac{2\sqrt{cd^2-bde+ae^2}ax^2\log\left(-\frac{\left(8c^2d^2-8bcde+\left(b^2+4ac\right)e^2\right)x^4-8abde+8a^2e^2+\left(b^2+4ac\right)d^2+2\left(4bcd^2+4abe^2-\left(3b^2+4ac\right)de\right)x^2-}{e^2x^4+2dex^2+d^2}\right.}{}\right.$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/x^3/(e*x^2+d),x, algorithm="fricas")
```

output
```
[1/8*(2*sqrt(c*d^2 - b*d*e + a*e^2)*a*x^2*log(-((8*c^2*d^2 - 8*b*c*d*e + (
b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b
*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*
sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 +
2*d*e*x^2 + d^2)) - (b*d - 2*a*e)*sqrt(a)*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*
a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) -
4*sqrt(c*x^4 + b*x^2 + a)*a*d)/(a*d^2*x^2), 1/8*(4*sqrt(-c*d^2 + b*d*e - a
*e^2)*a*x^2*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^
2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 +
a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - (b*d -
 2*a*e)*sqrt(a)*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b
*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*sqrt(c*x^4 + b*x^2 + a)*
a*d)/(a*d^2*x^2), 1/4*((b*d - 2*a*e)*sqrt(-a)*x^2*arctan(1/2*sqrt(c*x^4 +
b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + sqrt(c*d^2
- b*d*e + a*e^2)*a*x^2*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x
^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2
- (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e
+ a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)) -
 2*sqrt(c*x^4 + b*x^2 + a)*a*d)/(a*d^2*x^2), 1/4*(2*sqrt(-c*d^2 + b*d*e -
a*e^2)*a*x^2*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - ...
```

3.315.6 Sympy [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^3(d+ex^2)}\,dx = \int \frac{\sqrt{a+bx^2+cx^4}}{x^3(d+ex^2)}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**(1/2)/x**3/(e*x**2+d),x)
```

output
```
Integral(sqrt(a + b*x**2 + c*x**4)/(x**3*(d + e*x**2)), x)
```

3.315.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^3(d+ex^2)}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}}{(ex^2+d)x^3}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/x^3/(e*x^2+d),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)/((e*x^2 + d)*x^3), x)
```

3.315.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^3(d+ex^2)}\,dx = \frac{(cd^2-bde+ae^2)\arctan\left(-\frac{\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{\sqrt{-cd^2+bde-ae^2}\,d^2} + \frac{(bd-2\,ae)\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}\,d^2} + \frac{\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)b+2\,a\sqrt{c}}{2\left(\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^2-a\right)d}$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/x^3/(e*x^2+d),x, algorithm="giac")
```

output
```
(c*d^2 - b*d*e + a*e^2)*arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*e
 + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))/(sqrt(-c*d^2 + b*d*e - a*e^2)*
d^2) + 1/2*(b*d - 2*a*e)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/s
qrt(-a))/(sqrt(-a)*d^2) + 1/2*((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*b +
 2*a*sqrt(c))/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)*d)
```

3.315.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^3(d+ex^2)}\,dx = \int \frac{\sqrt{c\,x^4+b\,x^2+a}}{x^3\,\left(e\,x^2+d\right)}\,dx$$

input
```
int((a + b*x^2 + c*x^4)^(1/2)/(x^3*(d + e*x^2)),x)
```

output
```
int((a + b*x^2 + c*x^4)^(1/2)/(x^3*(d + e*x^2)), x)
```

3.316 $\int \frac{x^4\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx$

3.316.1 Optimal result

Integrand size = 29, antiderivative size = 424

$$\int \frac{x^4\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx = -\frac{1}{60}x\left(13-6x^2\right)\sqrt{1+2x^2+2x^4} + \frac{109x\sqrt{1+2x^2+2x^4}}{60\sqrt{2}\left(1+\sqrt{2}x^2\right)} + \frac{3}{16}\sqrt{15}\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) - \frac{109\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{60\ 2^{3/4}\sqrt{1+2x^2+2x^4}} + \frac{\left(-70+263\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{60\ 2^{3/4}\left(-2+3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}} + \frac{15\left(3+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{1}{24}\left(12-11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{16\ 2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}$$

output
```
3/16*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)-1/60*x*(-6*x^2+
13)*(2*x^4+2*x^2+1)^(1/2)+109/120*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2
^(1/2))-109/120*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x)
)*EllipticE(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2)
)*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)+
15/32*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*Elliptic
Pi(sin(2*arctan(2^(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^
(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(
2-3*2^(1/2))/(2*x^4+2*x^2+1)^(1/2)+1/120*(cos(2*arctan(2^(1/4)*x))^2)^(1/2
)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/
2))^(1/2))*(-70+263*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/
2))^2)^(1/2)*2^(1/4)/(-2+3*2^(1/2))/(2*x^4+2*x^2+1)^(1/2)
```

3.316.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.77 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.49

$$\int \frac{x^4\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx$$
$$= \frac{-52x-80x^3-56x^5+48x^7-218i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}E\left(i\text{arcsinh}\left(\sqrt{1-i}x\right)\middle|i\right)-(19}{}$$

input
```
Integrate[(x^4*Sqrt[1 + 2*x^2 + 2*x^4])/(3 + 2*x^2),x]
```

output
```
(-52*x - 80*x^3 - 56*x^5 + 48*x^7 - (218*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x
^2]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] - (199 -
417*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I
*ArcSinh[Sqrt[1 - I]*x], I] + 225*(1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqrt
[1 + (1 + I)*x^2]*EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(240
*Sqrt[1 + 2*x^2 + 2*x^4])
```

3.316.3 Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {1630, 27, 2207, 27, 2207, 27, 1511, 1416, 1509, 2220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{2x^4+2x^2+1}}{2x^2+3}dx$$

$$\Big\downarrow \text{1630}$$

$$\frac{1}{28}\int \frac{56x^6-28x^4+70x^2+15(2+3\sqrt{2})}{2\sqrt{2x^4+2x^2+1}}dx-\frac{45}{56}\int \frac{2(3+\sqrt{2})x^2+3\sqrt{2}+2}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{56}\int \frac{56x^6-28x^4+70x^2+15(2+3\sqrt{2})}{\sqrt{2x^4+2x^2+1}}dx-\frac{45}{56}\int \frac{2(3+\sqrt{2})x^2+3\sqrt{2}+2}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{56}\left(\frac{1}{10}\int \frac{2(-364x^4+266x^2+75(2+3\sqrt{2}))}{\sqrt{2x^4+2x^2+1}}dx+\frac{28}{5}\sqrt{2x^4+2x^2+1}x^3\right)-\frac{45}{56}\int \frac{2(3+\sqrt{2})x^2+3\sqrt{2}+2}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{56}\left(\frac{1}{5}\int \frac{-364x^4+266x^2+75(2+3\sqrt{2})}{\sqrt{2x^4+2x^2+1}}dx+\frac{28}{5}\sqrt{2x^4+2x^2+1}x^3\right)-\frac{45}{56}\int \frac{2(3+\sqrt{2})x^2+3\sqrt{2}+2}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{56}\left(\frac{1}{5}\left(\frac{1}{6}\int \frac{2(1526x^2+675\sqrt{2}+632)}{\sqrt{2x^4+2x^2+1}}dx-\frac{182}{3}x\sqrt{2x^4+2x^2+1}\right)+\frac{28}{5}\sqrt{2x^4+2x^2+1}x^3\right)-\frac{45}{56}\int \frac{2(3+\sqrt{2})x^2+3\sqrt{2}+2}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{56}\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{1526x^2+675\sqrt{2}+632}{\sqrt{2x^4+2x^2+1}}dx-\frac{182}{3}x\sqrt{2x^4+2x^2+1}\right)+\frac{28}{5}\sqrt{2x^4+2x^2+1}x^3\right)-\frac{45}{56}\int \frac{2\left(3+\sqrt{2}\right)x^2+3\sqrt{2}+2}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \; 1511$$

$$\frac{1}{56}\left(\frac{1}{5}\left(\frac{1}{3}\left(2\left(316+719\sqrt{2}\right)\int \frac{1}{\sqrt{2x^4+2x^2+1}}dx-763\sqrt{2}\int \frac{1-\sqrt{2}x^2}{\sqrt{2x^4+2x^2+1}}dx\right)-\frac{182}{3}x\sqrt{2x^4+2x^2+1}\right)+\right.$$
$$\frac{45}{56}\int \frac{2\left(3+\sqrt{2}\right)x^2+3\sqrt{2}+2}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \; 1416$$

$$\frac{1}{56}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(316+719\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-763\sqrt{2}\int \frac{1-}{\sqrt{2x^4-}}\right.\right.\right.$$
$$\frac{45}{56}\int \frac{2\left(3+\sqrt{2}\right)x^2+3\sqrt{2}+2}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \; 1509$$

$$\frac{1}{56}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(316+719\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-763\sqrt{2}\left(\frac{\left(\sqrt{2}x^2-\right.}{}\right.\right.\right.\right.$$
$$\frac{45}{56}\int \frac{2\left(3+\sqrt{2}\right)x^2+3\sqrt{2}+2}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \; 2220$$

$$\frac{1}{56}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(316+719\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-763\sqrt{2}\left(\frac{\left(\sqrt{2}x^2-\right.}{}\right.\right.\right.\right.$$
$$\frac{45}{56}\left(\frac{\left(3+\sqrt{2}\right)^2\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticPi}\left(\frac{1}{24}\left(12-11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{12\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-\frac{7\arctan\left(\right.}{2\sqrt{}}\right.$$

3.316. $\int \frac{x^4\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx$

input
```
Int[(x^4*Sqrt[1 + 2*x^2 + 2*x^4])/(3 + 2*x^2),x]
```

output
```
((28*x^3*Sqrt[1 + 2*x^2 + 2*x^4])/5 + ((-182*x*Sqrt[1 + 2*x^2 + 2*x^4])/3
+ (-763*Sqrt[2]*(-((x*Sqrt[1 + 2*x^2 + 2*x^4])/(1 + Sqrt[2]*x^2)) + ((1 +
Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*Arc
Tan[2^(1/4)*x], (2 - Sqrt[2])/4])/(2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])) + ((3
16 + 719*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*
x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(2^(1/4)*Sqrt[1 +
 2*x^2 + 2*x^4]))/3)/5)/56 - (45*((-7*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2
+ 2*x^4]])/(2*Sqrt[15]) + ((3 + Sqrt[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x
^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan
[2^(1/4)*x], (2 - Sqrt[2])/4])/(12*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])))/56
```

3.316.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1630
```
Int[((x_)^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[(-(-d/e)^(m/2))*((c*d^2 - b*d*e + a*e^2)^(p + 1/2)
/(e^(2*p)*(c*d^2 - a*e^2)))   Int[(a*d*Rt[c/a, 2] + a*e + (c*d + a*e*Rt[c/a
, 2])*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] + Simp[1/(e^(2*p)*
(c*d^2 - a*e^2))   Int[(1/Sqrt[a + b*x^2 + c*x^4])*ExpandToSum[(e^(2*p)*(c*
d^2 - a*e^2)*x^m*(a + b*x^2 + c*x^4)^(p + 1/2) + (-d/e)^(m/2)*(c*d^2 - b*d*
e + a*e^2)^(p + 1/2)*(a*d*Rt[c/a, 2] + a*e + (c*d + a*e*Rt[c/a, 2])*x^2))/(
d + e*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& IGtQ[p + 1/2, 0] && IGtQ[m/2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

rule 2220
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a
+ b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*El
lipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &
& EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[-b + c*(d/e) + a*(e/d)]
```

3.316.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.10 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.58

method	result
risch	$\frac{x(6x^2-13)\sqrt{2x^4+2x^2+1}}{60}-\frac{199\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{120\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{109}{120}+\frac{109i}{120}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,(}{\sqrt{-1+}}$
elliptic	$\frac{x^3\sqrt{2x^4+2x^2+1}}{10}-\frac{13x\sqrt{2x^4+2x^2+1}}{60}-\frac{77\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{30\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{109i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+}}{120\sqrt{-1+i}\,\sqrt{2}}$
default	$-\frac{13x\sqrt{2x^4+2x^2+1}}{60}-\frac{8\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{15\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(\frac{13}{60}-\frac{13i}{60}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1-}\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2}}$

input
```
int(x^4*(2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x,method=_RETURNVERBOSE)
```

output
```
1/60*x*(6*x^2-13)*(2*x^4+2*x^2+1)^(1/2)-199/120/(-1+I)^(1/2)*(1+(1-I)*x^2)
^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),
1/2*2^(1/2)+1/2*I*2^(1/2))+(-109/120+109/120*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)
^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2)
,1/2*2^(1/2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(
1/2)))+15/8/(-1+I)^(1/2)*(1-I*x^2+x^2)^(1/2)*(1+I*x^2+x^2)^(1/2)/(2*x^4+2*
x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2)
)
```

3.316.5 Fricas [F]

$$\int \frac{x^4\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx=\int \frac{\sqrt{2\,x^4+2\,x^2+1}x^4}{2\,x^2+3}\,dx$$

input
```
integrate(x^4*(2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x, algorithm="fricas")
```

output
```
integral(sqrt(2*x^4 + 2*x^2 + 1)*x^4/(2*x^2 + 3), x)
```

3.316.6 Sympy [F]

$$\int \frac{x^4\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx=\int \frac{x^4\sqrt{2x^4+2x^2+1}}{2x^2+3}\,dx$$

input
```
integrate(x**4*(2*x**4+2*x**2+1)**(1/2)/(2*x**2+3),x)
```

output
```
Integral(x**4*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**2 + 3), x)
```

3.316.7 Maxima [F]

$$\int \frac{x^4\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx = \int \frac{\sqrt{2\,x^4+2\,x^2+1}x^4}{2\,x^2+3}\,dx$$

input
```
integrate(x^4*(2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x, algorithm="maxima")
```

output
```
integrate(sqrt(2*x^4 + 2*x^2 + 1)*x^4/(2*x^2 + 3), x)
```

3.316.8 Giac [F]

$$\int \frac{x^4\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx = \int \frac{\sqrt{2\,x^4+2\,x^2+1}x^4}{2\,x^2+3}\,dx$$

input
```
integrate(x^4*(2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x, algorithm="giac")
```

output
```
integrate(sqrt(2*x^4 + 2*x^2 + 1)*x^4/(2*x^2 + 3), x)
```

3.316.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx = \int \frac{x^4\,\sqrt{2\,x^4+2\,x^2+1}}{2\,x^2+3}\,dx$$

input
```
int((x^4*(2*x^2 + 2*x^4 + 1)^(1/2))/(2*x^2 + 3),x)
```

output
```
int((x^4*(2*x^2 + 2*x^4 + 1)^(1/2))/(2*x^2 + 3), x)
```

3.317 $\int \frac{x^2\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx$

3.317.1 Optimal result

Integrand size = 29, antiderivative size = 417

$$\int \frac{x^2\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx$$
$$= \frac{1}{6}x\sqrt{1+2x^2+2x^4} - \frac{7x\sqrt{1+2x^2+2x^4}}{6\sqrt{2}\left(1+\sqrt{2}x^2\right)} - \frac{1}{8}\sqrt{15}\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)$$
$$+ \frac{7\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{6\ 2^{3/4}\sqrt{1+2x^2+2x^4}}$$
$$- \frac{\left(-4+17\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{6\ 2^{3/4}\left(-2+3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}$$
$$- \frac{5\left(3+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{1}{24}\left(12-11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{8\ 2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}$$

output
```
-1/8*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)+1/6*x*(2*x^4+2*
x^2+1)^(1/2)-7/12*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))+7/12*(co
s(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticE(sin(2*a
rctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/
(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)-5/16*(cos(2*arctan(
2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(2^(1
/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*(1+x^2*2^(1/2
))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2-3*2^(1/2))/(2*x^4+
2*x^2+1)^(1/2)-1/12*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4
)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(-4+17*2^(
1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(-
2+3*2^(1/2))/(2*x^4+2*x^2+1)^(1/2)
```

3.317.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.06 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.49

$$\int \frac{x^2\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx$$
$$= \frac{4x+8x^3+8x^5+14i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}E\left(i\operatorname{arcsinh}\left(\sqrt{1-i}x\right)\middle|i\right)+(13-27i)\sqrt{1-i}\cdots}{}$$

input
```
Integrate[(x^2*Sqrt[1 + 2*x^2 + 2*x^4])/(3 + 2*x^2),x]
```

output
```
(4*x + 8*x^3 + 8*x^5 + (14*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (
1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] + (13 - 27*I)*Sqrt[1 -
I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1
- I]*x], I] - 15*(1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]
*EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(24*Sqrt[1 + 2*x^2 +
2*x^4])
```

3.317.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 408, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {1630, 25, 2207, 27, 1511, 1416, 1509, 2220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{2x^4+2x^2+1}}{2x^2+3}dx$$

$$\Big\downarrow \text{1630}$$

$$\frac{15}{28}\int \frac{2\left(3+\sqrt{2}\right)x^2+3\sqrt{2}+2}{\left(2x^2+3\right)\sqrt{2x^4+2x^2+1}}dx+\frac{1}{28}\int -\frac{-28x^4+14x^2+5\left(2+3\sqrt{2}\right)}{\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{15}{28}\int \frac{2\left(3+\sqrt{2}\right)x^2+3\sqrt{2}+2}{\left(2x^2+3\right)\sqrt{2x^4+2x^2+1}}dx-\frac{1}{28}\int \frac{-28x^4+14x^2+5\left(2+3\sqrt{2}\right)}{\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{28}\left(\frac{14}{3}x\sqrt{2x^4+2x^2+1}-\frac{1}{6}\int \frac{2\left(98x^2+45\sqrt{2}+44\right)}{\sqrt{2x^4+2x^2+1}}dx\right)+\frac{15}{28}\int \frac{2\left(3+\sqrt{2}\right)x^2+3\sqrt{2}+2}{\left(2x^2+3\right)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{28}\left(\frac{14}{3}x\sqrt{2x^4+2x^2+1}-\frac{1}{3}\int \frac{98x^2+45\sqrt{2}+44}{\sqrt{2x^4+2x^2+1}}dx\right)+\frac{15}{28}\int \frac{2\left(3+\sqrt{2}\right)x^2+3\sqrt{2}+2}{\left(2x^2+3\right)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \text{1511}$$

$$\frac{1}{28}\left(\frac{1}{3}\left(49\sqrt{2}\int \frac{1-\sqrt{2}x^2}{\sqrt{2x^4+2x^2+1}}dx-2\left(22+47\sqrt{2}\right)\int \frac{1}{\sqrt{2x^4+2x^2+1}}dx\right)+\frac{14}{3}\sqrt{2x^4+2x^2+1}x\right)+\frac{15}{28}\int \frac{2\left(3+\sqrt{2}\right)x^2+3\sqrt{2}+2}{\left(2x^2+3\right)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \text{1416}$$

$$\frac{1}{28}\left(\frac{1}{3}\left(49\sqrt{2}\int \frac{1-\sqrt{2}x^2}{\sqrt{2x^4+2x^2+1}}dx-\frac{\left(22+47\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\right.\right.}{\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}\right.\right.$$

$$\frac{15}{28}\int \frac{2\left(3+\sqrt{2}\right)x^2+3\sqrt{2}+2}{\left(2x^2+3\right)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow 1509$$

$$\frac{15}{28}\int \frac{2(3+\sqrt{2})x^2+3\sqrt{2}+2}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx+ \frac{1}{28}\left(\frac{1}{3}\left(49\sqrt{2}\left(\frac{(\sqrt{2}x^2+1)\sqrt{\frac{2x^4+2x^2+1}{(\sqrt{2}x^2+1)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-\frac{x\sqrt{2x^4+2x^2+1}}{\sqrt{2}x^2+1}\right)-\frac{(22+47\sqrt{2})(}{}\right.\right.$$

$$\Big\downarrow 2220$$

$$\frac{15}{28}\left(\frac{(3+\sqrt{2})^2(\sqrt{2}x^2+1)\sqrt{\frac{2x^4+2x^2+1}{(\sqrt{2}x^2+1)^2}}\text{EllipticPi}\left(\frac{1}{24}(12-11\sqrt{2}),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{12\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-\frac{7\arctan\left(\right.}{2\sqrt{}}\right.$$

$$\frac{1}{28}\left(\frac{1}{3}\left(49\sqrt{2}\left(\frac{(\sqrt{2}x^2+1)\sqrt{\frac{2x^4+2x^2+1}{(\sqrt{2}x^2+1)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-\frac{x\sqrt{2x^4+2x^2+1}}{\sqrt{2}x^2+1}\right)-\frac{(22+47\sqrt{2})(}{}\right.\right.$$

input
```
Int[(x^2*Sqrt[1 + 2*x^2 + 2*x^4])/(3 + 2*x^2),x]
```

output
```
((14*x*Sqrt[1 + 2*x^2 + 2*x^4])/3 + (49*Sqrt[2]*(-((x*Sqrt[1 + 2*x^2 + 2*x
^4])/(1 + Sqrt[2]*x^2)) + ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 +
 Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(2^(1/4)
*Sqrt[1 + 2*x^2 + 2*x^4])) - ((22 + 47*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1
+ 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 -
Sqrt[2])/4])/(2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]))/3)/28 + (15*((-7*ArcTan[(S
qrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/(2*Sqrt[15]) + ((3 + Sqrt[2])^2*(1 +
 Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12
 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(12*2^(1/4)*Sqrt
[1 + 2*x^2 + 2*x^4])))/28
```

3.317. $\int \frac{x^2\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx$

3.317.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1630
```
Int[((x_)^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[(-(-d/e)^(m/2))*((c*d^2 - b*d*e + a*e^2)^(p + 1/2)
/(e^(2*p)*(c*d^2 - a*e^2)))   Int[(a*d*Rt[c/a, 2] + a*e + (c*d + a*e*Rt[c/a
, 2])*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] + Simp[1/(e^(2*p)*
(c*d^2 - a*e^2))   Int[(1/Sqrt[a + b*x^2 + c*x^4])*ExpandToSum[(e^(2*p)*(c*
d^2 - a*e^2)*x^m*(a + b*x^2 + c*x^4)^(p + 1/2) + (-d/e)^(m/2)*(c*d^2 - b*d*
e + a*e^2)^(p + 1/2)*(a*d*Rt[c/a, 2] + a*e + (c*d + a*e*Rt[c/a, 2])*x^2))/(
d + e*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& IGtQ[p + 1/2, 0] && IGtQ[m/2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

3.317. $\int \frac{x^2\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx$

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

rule 2220
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a
+ b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*El
lipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &
& EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[-b + c*(d/e) + a*(e/d)]
```

3.317.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.37 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.57

method	result
risch	$\frac{x\sqrt{2x^4+2x^2+1}}{6}+\frac{13\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{12\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(\frac{7}{12}-\frac{7i}{12}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1+i},\cdots\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+\cdots}}$
elliptic	$\frac{x\sqrt{2x^4+2x^2+1}}{6}+\frac{5\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{3\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}-\frac{7i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{12\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}\cdots$
default	$\frac{x\sqrt{2x^4+2x^2+1}}{6}+\frac{\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{3\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{1}{6}+\frac{i}{6}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\cdots\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$

input
```
int(x^2*(2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x,method=_RETURNVERBOSE)
```

output
```
1/6*x*(2*x^4+2*x^2+1)^(1/2)+13/12/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I
)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/
2*I*2^(1/2))+(7/12-7/12*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^
(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^
(1/2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))-5/4/(-1+I)^(1/
2)*(1-I*x^2+x^2)^(1/2)*(1+I*x^2+x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticP
i(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

3.317.5 Fricas [F]

$$\int \frac{x^2\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx = \int \frac{\sqrt{2\,x^4+2\,x^2+1}x^2}{2\,x^2+3}\,dx$$

input
```
integrate(x^2*(2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x, algorithm="fricas")
```

output
```
integral(sqrt(2*x^4 + 2*x^2 + 1)*x^2/(2*x^2 + 3), x)
```

3.317.6 Sympy [F]

$$\int \frac{x^2\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx = \int \frac{x^2\sqrt{2x^4+2x^2+1}}{2x^2+3}\,dx$$

input
```
integrate(x**2*(2*x**4+2*x**2+1)**(1/2)/(2*x**2+3),x)
```

output
```
Integral(x**2*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**2 + 3), x)
```

3.317.7 Maxima [F]

$$\int \frac{x^2\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx = \int \frac{\sqrt{2\,x^4+2\,x^2+1}x^2}{2\,x^2+3}\,dx$$

input
```
integrate(x^2*(2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x, algorithm="maxima")
```

output
```
integrate(sqrt(2*x^4 + 2*x^2 + 1)*x^2/(2*x^2 + 3), x)
```

3.317.8 Giac [F]

$$\int \frac{x^2\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx = \int \frac{\sqrt{2\,x^4+2\,x^2+1}x^2}{2\,x^2+3}\,dx$$

input
```
integrate(x^2*(2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x, algorithm="giac")
```

output
```
integrate(sqrt(2*x^4 + 2*x^2 + 1)*x^2/(2*x^2 + 3), x)
```

3.317.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx = \int \frac{x^2\,\sqrt{2\,x^4+2\,x^2+1}}{2\,x^2+3}\,dx$$

input
```
int((x^2*(2*x^2 + 2*x^4 + 1)^(1/2))/(2*x^2 + 3),x)
```

output
```
int((x^2*(2*x^2 + 2*x^4 + 1)^(1/2))/(2*x^2 + 3), x)
```

3.318 $\int \frac{\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx$

3.318.1 Optimal result

Integrand size = 26, antiderivative size = 381

$$\int \frac{\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx = \frac{x\sqrt{1+2x^2+2x^4}}{\sqrt{2}\left(1+\sqrt{2}x^2\right)}+\frac{1}{4}\sqrt{\frac{5}{3}}\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)$$
$$-\frac{\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{2^{3/4}\sqrt{1+2x^2+2x^4}}$$
$$+\frac{2^{3/4}\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{\left(-2+3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}$$
$$+\frac{5\left(3+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{1}{24}\left(12-11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{12\ 2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}$$

output
```
1/12*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)+1/2*x*(2*x^4+2*
x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))-1/2*(cos(2*arctan(2^(1/4)*x))^2)^(1/2
)/cos(2*arctan(2^(1/4)*x))*EllipticE(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/
2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/
4)/(2*x^4+2*x^2+1)^(1/2)+5/24*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arc
tan(2^(1/4)*x))*EllipticPi(sin(2*arctan(2^(1/4)*x)),1/2-11/24*2^(1/2),1/2*
(2-2^(1/2))^(1/2))*(3+2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(
1/2))^2)^(1/2)*2^(1/4)/(2-3*2^(1/2))/(2*x^4+2*x^2+1)^(1/2)+2^(3/4)*(cos(2*
arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arcta
n(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x
^2*2^(1/2))^2)^(1/2)/(-2+3*2^(1/2))/(2*x^4+2*x^2+1)^(1/2)
```

3.318.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.62 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.33

$$\int \frac{\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx = -\frac{\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\left((3+3i)E\left(i\text{arcsinh}\left(\sqrt{1-i}x\right)\middle|i\right)-(3+6i)\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{1-\right.\right.}{6\sqrt{1-i}\sqrt{1+2x^2+2x^4}}$$

input
```
Integrate[Sqrt[1 + 2*x^2 + 2*x^4]/(3 + 2*x^2),x]
```

output
```
-1/6*(Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*((3 + 3*I)*EllipticE[I*A
rcSinh[Sqrt[1 - I]*x], I] - (3 + 6*I)*EllipticF[I*ArcSinh[Sqrt[1 - I]*x],
I] + (5*I)*EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I]))/(Sqrt[1 -
I]*Sqrt[1 + 2*x^2 + 2*x^4])
```

3.318.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1523, 27, 1511, 1416, 1509, 2220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2x^4+2x^2+1}}{2x^2+3}dx$$

$\big\downarrow$ 1523

$$\frac{5\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{2-3\sqrt{2}}-\frac{\int \frac{2\left(-\left(\left(2-3\sqrt{2}\right)x^2\right)+\sqrt{2}+1\right)}{\sqrt{2x^4+2x^2+1}}dx}{2\left(2-3\sqrt{2}\right)}$$

$\big\downarrow$ 27

$$\frac{5\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{2-3\sqrt{2}}-\frac{\int \frac{-\left(\left(2-3\sqrt{2}\right)x^2\right)+\sqrt{2}+1}{\sqrt{2x^4+2x^2+1}}dx}{2-3\sqrt{2}}$$

$\big\downarrow$ 1511

$$\frac{5\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{2-3\sqrt{2}}-\frac{4\int \frac{1}{\sqrt{2x^4+2x^2+1}}dx-\left(3-\sqrt{2}\right)\int \frac{1-\sqrt{2}x^2}{\sqrt{2x^4+2x^2+1}}dx}{2-3\sqrt{2}}$$

$\big\downarrow$ 1416

$$\frac{5\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{2-3\sqrt{2}}-\frac{\frac{2^{3/4}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{\sqrt{2x^4+2x^2+1}}-\left(3-\sqrt{2}\right)\int \frac{1-\sqrt{2}x^2}{\sqrt{2x^4+2x^2+1}}dx}{2-3\sqrt{2}}$$

$\big\downarrow$ 1509

$$\frac{5\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{2-3\sqrt{2}}-\frac{\frac{2^{3/4}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{\sqrt{2x^4+2x^2+1}}-\left(3-\sqrt{2}\right)\left(\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{\right.\right.}{\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}\right.}{2-3\sqrt{2}}$$

$\big\downarrow$ 2220

3.318. $\int \frac{\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx$

$$\frac{5\left(\frac{(3+\sqrt{2})(\sqrt{2}x^2+1)\sqrt{\frac{2x^4+2x^2+1}{(\sqrt{2}x^2+1)^2}}\text{EllipticPi}\left(\frac{1}{24}(12-11\sqrt{2}),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{12\ 2^{3/4}\sqrt{2x^4+2x^2+1}}-\frac{(3-\sqrt{2})\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{2\sqrt{30}}\right)}{2-3\sqrt{2}}-$$

$$\frac{\frac{2^{3/4}(\sqrt{2}x^2+1)\sqrt{\frac{2x^4+2x^2+1}{(\sqrt{2}x^2+1)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{\sqrt{2x^4+2x^2+1}}-(3-\sqrt{2})\left(\frac{(\sqrt{2}x^2+1)\sqrt{\frac{2x^4+2x^2+1}{(\sqrt{2}x^2+1)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{\right.}{\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}\right.}{2-3\sqrt{2}}$$

input
```
Int[Sqrt[1 + 2*x^2 + 2*x^4]/(3 + 2*x^2),x]
```

output
```
-((-((3 - Sqrt[2])*(-((x*Sqrt[1 + 2*x^2 + 2*x^4])/(1 + Sqrt[2]*x^2)) + ((1
 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*
ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]))) +
 (2^(3/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*
EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/Sqrt[1 + 2*x^2 + 2*x^4])/
(2 - 3*Sqrt[2])) + (5*(-1/2*((3 - Sqrt[2])*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2
*x^2 + 2*x^4]])/Sqrt[30] + ((3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^
2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[
2^(1/4)*x], (2 - Sqrt[2])/4])/(12*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4])))/(2 -
3*Sqrt[2])
```

3.318.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1523
```
Int[Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(c*d^2 - b*d*e + a*e^2)/(e*(e - d*q))   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] - Simp[1/(e*(e
 - d*q))   Int[(c*d - b*e + a*e*q - (c*e - a*d*q^3)*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2220
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a
+ b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*El
lipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &
& EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[-b + c*(d/e) + a*(e/d)]
```

3.318. $\int \frac{\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx$

3.318.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.90

method	result
default	$-\frac{\sqrt{-i x^2+x^2+1}\,\sqrt{i x^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{i\sqrt{-i x^2+x^2+1}\,\sqrt{i x^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{2\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\sqrt{-i x^2+x^2+1}\,\sqrt{}}{2\sqrt{}}$
elliptic	$-\frac{\sqrt{-i x^2+x^2+1}\,\sqrt{i x^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{i\sqrt{-i x^2+x^2+1}\,\sqrt{i x^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{2\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\sqrt{-i x^2+x^2+1}\,\sqrt{}}{2\sqrt{}}$

input
```
int((2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x,method=_RETURNVERBOSE)
```

output
```
-1/(-1+I)^(1/2)*(1-I*x^2+x^2)^(1/2)*(1+I*x^2+x^2)^(1/2)/(2*x^4+2*x^2+1)^(1
/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+1/2*I/(-1+I)^(1/2)
*(1-I*x^2+x^2)^(1/2)*(1+I*x^2+x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x
*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+1/2/(-1+I)^(1/2)*(1-I*x^2+x^2)^(1
/2)*(1+I*x^2+x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2
*2^(1/2)+1/2*I*2^(1/2))-1/2*I/(-1+I)^(1/2)*(1-I*x^2+x^2)^(1/2)*(1+I*x^2+x^
2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*
2^(1/2))+5/6/(-1+I)^(1/2)*(1-I*x^2+x^2)^(1/2)*(1+I*x^2+x^2)^(1/2)/(2*x^4+2
*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2
))
```

3.318.5 Fricas [F]

$$\int \frac{\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx=\int \frac{\sqrt{2\,x^4+2\,x^2+1}}{2\,x^2+3}\,dx$$

input
```
integrate((2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x, algorithm="fricas")
```

output
```
integral(sqrt(2*x^4 + 2*x^2 + 1)/(2*x^2 + 3), x)
```

3.318.6 Sympy [F]

$$\int \frac{\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx=\int \frac{\sqrt{2x^4+2x^2+1}}{2x^2+3}\,dx$$

input
```
integrate((2*x**4+2*x**2+1)**(1/2)/(2*x**2+3),x)
```

output
```
Integral(sqrt(2*x**4 + 2*x**2 + 1)/(2*x**2 + 3), x)
```

3.318.7 Maxima [F]

$$\int \frac{\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx=\int \frac{\sqrt{2\,x^4+2\,x^2+1}}{2\,x^2+3}\,dx$$

input
```
integrate((2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x, algorithm="maxima")
```

output
```
integrate(sqrt(2*x^4 + 2*x^2 + 1)/(2*x^2 + 3), x)
```

3.318.8 Giac [F]

$$\int \frac{\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx=\int \frac{\sqrt{2\,x^4+2\,x^2+1}}{2\,x^2+3}\,dx$$

input
```
integrate((2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x, algorithm="giac")
```

output
```
integrate(sqrt(2*x^4 + 2*x^2 + 1)/(2*x^2 + 3), x)
```

3.318.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx=\int \frac{\sqrt{2\,x^4+2\,x^2+1}}{2\,x^2+3}\,dx$$

input
```
int((2*x^2 + 2*x^4 + 1)^(1/2)/(2*x^2 + 3),x)
```

output
```
int((2*x^2 + 2*x^4 + 1)^(1/2)/(2*x^2 + 3), x)
```

3.319 $\int \frac{\sqrt{1+2x^2+2x^4}}{x^2(3+2x^2)}\,dx$

3.319.1 Optimal result

Integrand size = 29, antiderivative size = 399

$$\begin{aligned}
&\int \frac{\sqrt{1+2x^2+2x^4}}{x^2\left(3+2x^2\right)}\,dx \\
&= -\frac{\sqrt{1+2x^2+2x^4}}{3x}+\frac{\sqrt{2}x\sqrt{1+2x^2+2x^4}}{3\left(1+\sqrt{2}x^2\right)}-\frac{1}{6}\sqrt{\frac{5}{3}}\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) \\
&\quad -\frac{\sqrt[4]{2}\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{3\sqrt{1+2x^2+2x^4}} \\
&\quad +\frac{\left(3+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{21\sqrt[4]{2}\sqrt{1+2x^2+2x^4}} \\
&\quad +\frac{5\left(3+\sqrt{2}\right)^2\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{1}{24}\left(12-11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{252\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}
\end{aligned}$$

output
```
-1/18*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)-1/3*(2*x^4+2*x
^2+1)^(1/2)/x+1/3*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))-1/3*(cos
(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticE(sin(2*ar
ctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(
1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)+1/42*(cos(2*arctan(2
^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4
)*x)),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/
(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)+5/504*(cos(2*arctan
(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(2^(
1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))^2*(1+x^2*2^(
1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1
/2)
```

3.319.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.18 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^2(3+2x^2)}\,dx$$
$$= \frac{-6-12x^2-12x^4-6i\sqrt{1-i}x\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}E\left(i\operatorname{arcsinh}\left(\sqrt{1-i}x\right)\middle|i\right)+(9-3i)\sqrt{1-i}}{}$$

input
```
Integrate[Sqrt[1 + 2*x^2 + 2*x^4]/(x^2*(3 + 2*x^2)),x]
```

output
```
(-6 - 12*x^2 - 12*x^4 - (6*I)*Sqrt[1 - I]*x*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 +
 (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] + (9 - 3*I)*Sqrt[1 -
I]*x*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[
1 - I]*x], I] - 5*(1 - I)^(3/2)*x*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x
^2]*EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(18*x*Sqrt[1 + 2*x
^2 + 2*x^4])
```

3.319.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {1634, 27, 1604, 25, 1511, 1416, 1509, 2220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2x^4+2x^2+1}}{x^2\left(2x^2+3\right)}dx$$

$$\Big\downarrow \text{ 1634}$$

$$\frac{5}{21}\int \frac{2\left(3+\sqrt{2}\right)x^2+3\sqrt{2}+2}{\left(2x^2+3\right)\sqrt{2x^4+2x^2+1}}dx-\frac{1}{42}\int -\frac{2\left(\left(6-5\sqrt{2}\right)x^2+7\right)}{x^2\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{21}\int \frac{\left(6-5\sqrt{2}\right)x^2+7}{x^2\sqrt{2x^4+2x^2+1}}dx+\frac{5}{21}\int \frac{2\left(3+\sqrt{2}\right)x^2+3\sqrt{2}+2}{\left(2x^2+3\right)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \text{ 1604}$$

$$\frac{1}{21}\left(-\int -\frac{14x^2-5\sqrt{2}+6}{\sqrt{2x^4+2x^2+1}}dx-\frac{7\sqrt{2x^4+2x^2+1}}{x}\right)+\frac{5}{21}\int \frac{2\left(3+\sqrt{2}\right)x^2+3\sqrt{2}+2}{\left(2x^2+3\right)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{21}\left(\int \frac{14x^2-5\sqrt{2}+6}{\sqrt{2x^4+2x^2+1}}dx-\frac{7\sqrt{2x^4+2x^2+1}}{x}\right)+\frac{5}{21}\int \frac{2\left(3+\sqrt{2}\right)x^2+3\sqrt{2}+2}{\left(2x^2+3\right)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \text{ 1511}$$

$$\frac{1}{21}\left(2\left(3+\sqrt{2}\right)\int \frac{1}{\sqrt{2x^4+2x^2+1}}dx-7\sqrt{2}\int \frac{1-\sqrt{2}x^2}{\sqrt{2x^4+2x^2+1}}dx-\frac{7\sqrt{2x^4+2x^2+1}}{x}\right)+\frac{5}{21}\int \frac{2\left(3+\sqrt{2}\right)x^2+3\sqrt{2}+2}{\left(2x^2+3\right)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \text{ 1416}$$

$$\frac{1}{21}\left(-7\sqrt{2}\int \frac{1-\sqrt{2}x^2}{\sqrt{2x^4+2x^2+1}}dx+\frac{\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{\sqrt[4]{2}\sqrt{2x^4+2x^2+1}} -\right.$$
$$\frac{5}{21}\int \frac{2\left(3+\sqrt{2}\right)x^2+3\sqrt{2}+2}{\left(2x^2+3\right)\sqrt{2x^4+2x^2+1}}dx$$

$\downarrow$ 1509

$$\frac{5}{21}\int \frac{2(3+\sqrt{2})x^2+3\sqrt{2}+2}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx+$$

$$\frac{1}{21}\left(\frac{(3+\sqrt{2})(\sqrt{2}x^2+1)\sqrt{\frac{2x^4+2x^2+1}{(\sqrt{2}x^2+1)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-7\sqrt{2}\left(\frac{(\sqrt{2}x^2+1)\sqrt{\frac{2x^4+2x^2+}{(\sqrt{2}x^2+1)}}}{\sqrt[4]{2}\sqrt{}}\right.\right.$$

$\downarrow$ 2220

$$\frac{5}{21}\left(\frac{(3+\sqrt{2})^2(\sqrt{2}x^2+1)\sqrt{\frac{2x^4+2x^2+1}{(\sqrt{2}x^2+1)^2}}\text{EllipticPi}\left(\frac{1}{24}(12-11\sqrt{2}),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{12\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-\frac{7\arctan\left(\right.}{2\sqrt{}}\right.$$

$$\frac{1}{21}\left(\frac{(3+\sqrt{2})(\sqrt{2}x^2+1)\sqrt{\frac{2x^4+2x^2+1}{(\sqrt{2}x^2+1)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-7\sqrt{2}\left(\frac{(\sqrt{2}x^2+1)\sqrt{\frac{2x^4+2x^2+}{(\sqrt{2}x^2+1)}}}{\sqrt[4]{2}\sqrt{}}\right.\right.$$

input
```
Int[Sqrt[1 + 2*x^2 + 2*x^4]/(x^2*(3 + 2*x^2)),x]
```

output
```
((-7*Sqrt[1 + 2*x^2 + 2*x^4])/x - 7*Sqrt[2]*(-((x*Sqrt[1 + 2*x^2 + 2*x^4])
/(1 + Sqrt[2]*x^2)) + ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqr
t[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(2^(1/4)*Sqr
t[1 + 2*x^2 + 2*x^4])) + ((3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2
+ 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])
/4])/(2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]))/21 + (5*((-7*ArcTan[(Sqrt[5/3]*x)/
Sqrt[1 + 2*x^2 + 2*x^4]])/(2*Sqrt[15]) + ((3 + Sqrt[2])^2*(1 + Sqrt[2]*x^2
)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2
])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(12*2^(1/4)*Sqrt[1 + 2*x^2 +
 2*x^4])))/21
```

3.319. $\int \frac{\sqrt{1+2x^2+2x^4}}{x^2(3+2x^2)}\,dx$

3.319.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.319. $\int \frac{\sqrt{1+2x^2+2x^4}}{x^2(3+2x^2)}\,dx$

rule 1634
```
Int[((x_)^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[(-(-d/e)^(m/2))*((c*d^2 - b*d*e + a*e^2)^(p + 1/2)
/(e^(2*p)*(c*d^2 - a*e^2)))   Int[(a*d*Rt[c/a, 2] + a*e + (c*d + a*e*Rt[c/a
, 2])*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] + Simp[(-d/e)^(m/2
)/(e^(2*p)*(c*d^2 - a*e^2))   Int[(x^m/Sqrt[a + b*x^2 + c*x^4])*ExpandToSum
[((e^(2*p)*(c*d^2 - a*e^2)*(a + b*x^2 + c*x^4)^(p + 1/2))/(-d/e)^(m/2) + ((
a*d*Rt[c/a, 2] + a*e + (c*d + a*e*Rt[c/a, 2])*x^2)*(c*d^2 - b*d*e + a*e^2)^
(p + 1/2))/x^m)/(d + e*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p + 1/2, 0] && ILtQ[m/2, 0] && NeQ[c*d^2 - a*e^2,
0] && PosQ[c/a]
```

rule 2220
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a
+ b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*El
lipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &
& EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[-b + c*(d/e) + a*(e/d)]
```

3.319.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.90 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.60

method	result
risch	$-\frac{\sqrt{2x^4+2x^2+1}}{3x}+\frac{\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{1}{3}+\frac{i}{3}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$
elliptic	$-\frac{\sqrt{2x^4+2x^2+1}}{3x}+\frac{2\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{3\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{3\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$
default	$-\frac{\sqrt{2x^4+2x^2+1}}{3x}+\frac{2\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{3\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{2}{3}+\frac{2i}{3}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1+i},\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2}}$

input
```
int((2*x^4+2*x^2+1)^(1/2)/x^2/(2*x^2+3),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(2*x^4+2*x^2+1)^(1/2)/x+1/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x
^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I
*2^(1/2))+(-1/3+1/3*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2
)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2
))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))-5/9/(-1+I)^(1/2)*(
1-I*x^2+x^2)^(1/2)*(1+I*x^2+x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi(x*
(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

3.319.5 Fricas [F]

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^2(3+2x^2)}\,dx = \int \frac{\sqrt{2\,x^4+2\,x^2+1}}{(2\,x^2+3)x^2}\,dx$$

input
```
integrate((2*x^4+2*x^2+1)^(1/2)/x^2/(2*x^2+3),x, algorithm="fricas")
```

output
```
integral(sqrt(2*x^4 + 2*x^2 + 1)/(2*x^4 + 3*x^2), x)
```

3.319.6 Sympy [F]

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^2(3+2x^2)}\,dx = \int \frac{\sqrt{2x^4+2x^2+1}}{x^2\cdot(2x^2+3)}\,dx$$

input
```
integrate((2*x**4+2*x**2+1)**(1/2)/x**2/(2*x**2+3),x)
```

output
```
Integral(sqrt(2*x**4 + 2*x**2 + 1)/(x**2*(2*x**2 + 3)), x)
```

3.319.7 Maxima [F]

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^2(3+2x^2)}\,dx = \int \frac{\sqrt{2\,x^4+2\,x^2+1}}{(2\,x^2+3)x^2}\,dx$$

input
```
integrate((2*x^4+2*x^2+1)^(1/2)/x^2/(2*x^2+3),x, algorithm="maxima")
```

output
```
integrate(sqrt(2*x^4 + 2*x^2 + 1)/((2*x^2 + 3)*x^2), x)
```

3.319.8 Giac [F]

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^2\,(3+2x^2)}\,dx = \int \frac{\sqrt{2\,x^4+2\,x^2+1}}{(2\,x^2+3)x^2}\,dx$$

input
```
integrate((2*x^4+2*x^2+1)^(1/2)/x^2/(2*x^2+3),x, algorithm="giac")
```

output
```
integrate(sqrt(2*x^4 + 2*x^2 + 1)/((2*x^2 + 3)*x^2), x)
```

3.319.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^2\,(3+2x^2)}\,dx = \int \frac{\sqrt{2\,x^4+2\,x^2+1}}{x^2\,(2\,x^2+3)}\,dx$$

input
```
int((2*x^2 + 2*x^4 + 1)^(1/2)/(x^2*(2*x^2 + 3)),x)
```

output
```
int((2*x^2 + 2*x^4 + 1)^(1/2)/(x^2*(2*x^2 + 3)), x)
```

3.320 $\int \frac{\sqrt{1+2x^2+2x^4}}{x^4(3+2x^2)}\,dx$

3.320.1 Optimal result

Integrand size = 29, antiderivative size = 360

$$
\begin{aligned}
\int \frac{\sqrt{1+2x^2+2x^4}}{x^4(3+2x^2)}\,dx &= -\frac{\sqrt{1+2x^2+2x^4}}{9x^3}+\frac{1}{9}\sqrt{\frac{5}{3}}\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) \\
&- \frac{\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{9\sqrt[4]{2}\sqrt{1+2x^2+2x^4}} \\
&+ \frac{5\left(3+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{63\sqrt[4]{2}\sqrt{1+2x^2+2x^4}} \\
&- \frac{5\left(3+\sqrt{2}\right)^2\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{1}{24}\left(12-11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{378\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}
\end{aligned}
$$

output
```
1/27*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)-1/9*(2*x^4+2*x^
2+1)^(1/2)/x^3-1/18*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4
)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(
1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1
/2)+5/126*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*Elli
pticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*(1+x^2*2
^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^
(1/2)-5/756*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*El
lipticPi(sin(2*arctan(2^(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))
*(3+2^(1/2))^2*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2
^(3/4)/(2*x^4+2*x^2+1)^(1/2)
```

3.320.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.17 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.43

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^4(3+2x^2)}\,dx = -\frac{3+6x^2+6x^4+3(1-i)^{3/2}x^3\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{1-i}x\right),i\right)-5(1-}{27x^3\sqrt{1+2x^2+2x^4}}$$

input
```
Integrate[Sqrt[1 + 2*x^2 + 2*x^4]/(x^4*(3 + 2*x^2)),x]
```

output
```
-1/27*(3 + 6*x^2 + 6*x^4 + 3*(1 - I)^(3/2)*x^3*Sqrt[1 + (1 - I)*x^2]*Sqrt[
1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] - 5*(1 - I)^(3/2)*
x^3*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[1/3 + I/3, I*Ar
cSinh[Sqrt[1 - I]*x], I])/(x^3*Sqrt[1 + 2*x^2 + 2*x^4])
```

3.320.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.86, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1634, 2199, 1604, 27, 1416, 2220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2x^4+2x^2+1}}{x^4\left(2x^2+3\right)}dx$$

$$\Big\downarrow \text{1634}$$

$$\frac{1}{63}\int \frac{10\left(3+\sqrt{2}\right)x^4+28x^2+21}{x^4\sqrt{2x^4+2x^2+1}}dx-\frac{10}{63}\int \frac{2\left(3+\sqrt{2}\right)x^2+3\sqrt{2}+2}{\left(2x^2+3\right)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \text{2199}$$

$$\frac{1}{63}\left(\int \frac{-4\left(8+5\sqrt{2}\right)x^2-3\left(8+5\sqrt{2}\right)}{x^4\sqrt{2x^4+2x^2+1}}dx-\frac{5\left(3+\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}{x^3}\right)-\frac{10}{63}\int \frac{2\left(3+\sqrt{2}\right)x^2+3\sqrt{2}+2}{\left(2x^2+3\right)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \text{1604}$$

$$\frac{1}{63}\left(-\frac{1}{3}\int -\frac{6\left(8+5\sqrt{2}\right)}{\sqrt{2x^4+2x^2+1}}dx+\frac{\left(8+5\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}{x^3}-\frac{5\left(3+\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}{x^3}\right)-\frac{10}{63}\int \frac{2\left(3+\sqrt{2}\right)x^2+3\sqrt{2}+2}{\left(2x^2+3\right)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{63}\left(2\left(8+5\sqrt{2}\right)\int \frac{1}{\sqrt{2x^4+2x^2+1}}dx+\frac{\left(8+5\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}{x^3}-\frac{5\left(3+\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}{x^3}\right)-\frac{10}{63}\int \frac{2\left(3+\sqrt{2}\right)x^2+3\sqrt{2}+2}{\left(2x^2+3\right)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \text{1416}$$

$$\frac{1}{63}\left(\frac{\left(8+5\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}+\frac{\left(8+5\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}{x^3}-\right.$$
$$\frac{10}{63}\int \frac{2\left(3+\sqrt{2}\right)x^2+3\sqrt{2}+2}{\left(2x^2+3\right)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \text{2220}$$

3.320. $\int \frac{\sqrt{1+2x^2+2x^4}}{x^4\left(3+2x^2\right)}dx$

$$\frac{1}{63}\left(\frac{(8+5\sqrt{2})\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}+\frac{(8+5\sqrt{2})\sqrt{2x^4+2x^2+1}}{x^3}\right.$$

$$\frac{10}{63}\left(\frac{(3+\sqrt{2})^2\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\mathrm{EllipticPi}\left(\frac{1}{24}(12-11\sqrt{2}),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{12\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-\frac{7\arctan\left(\right.}{2\sqrt{}}\right.$$

input
```
Int[Sqrt[1 + 2*x^2 + 2*x^4]/(x^4*(3 + 2*x^2)),x]
```

output
```
((-5*(3 + Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])/x^3 + ((8 + 5*Sqrt[2])*Sqrt[1
+ 2*x^2 + 2*x^4])/x^3 + ((8 + 5*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2
 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2]
)/4])/(2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]))/63 - (10*((-7*ArcTan[(Sqrt[5/3]*x
)/Sqrt[1 + 2*x^2 + 2*x^4]])/(2*Sqrt[15]) + ((3 + Sqrt[2])^2*(1 + Sqrt[2]*x
^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt
[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(12*2^(1/4)*Sqrt[1 + 2*x^2
 + 2*x^4])))/63
```

3.320.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1634
```
Int[((x_)^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[(-(-d/e)^(m/2))*((c*d^2 - b*d*e + a*e^2)^(p + 1/2)
/(e^(2*p)*(c*d^2 - a*e^2)))   Int[(a*d*Rt[c/a, 2] + a*e + (c*d + a*e*Rt[c/a
, 2])*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] + Simp[(-d/e)^(m/2
)/(e^(2*p)*(c*d^2 - a*e^2))   Int[(x^m/Sqrt[a + b*x^2 + c*x^4])*ExpandToSum
[((e^(2*p)*(c*d^2 - a*e^2)*(a + b*x^2 + c*x^4)^(p + 1/2))/(-d/e)^(m/2) + ((
a*d*Rt[c/a, 2] + a*e + (c*d + a*e*Rt[c/a, 2])*x^2)*(c*d^2 - b*d*e + a*e^2)^
(p + 1/2))/x^m)/(d + e*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p + 1/2, 0] && ILtQ[m/2, 0] && NeQ[c*d^2 - a*e^2,
0] && PosQ[c/a]
```

rule 2199
```
Int[(Px_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> With[{q = Expon[Px, x^2]}, Simp[Coeff[Px, x^2, q]*(d*x)^(m + 2*q
- 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*d^(2*q - 3)*(m + 4*p + 2*q + 1))), x]
+ Int[(d*x)^m*(a + b*x^2 + c*x^4)^p*ExpandToSum[Px - Coeff[Px, x^2, q]*x^(2
*q) - Coeff[Px, x^2, q]*((a*(m + 2*q - 3)*x^(2*(q - 2)) + b*(m + 2*p + 2*q
- 1)*x^(2*(q - 1)))/(c*(m + 4*p + 2*q + 1))), x], x] /; GtQ[q, 1] && NeQ[m
+ 4*p + 2*q + 1, 0]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Px, x^2] && N
eQ[b^2 - 4*a*c, 0]
```

rule 2220
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a
+ b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*El
lipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &
& EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[-b + c*(d/e) + a*(e/d)]
```

3.320. $\int \frac{\sqrt{1+2x^2+2x^4}}{x^4(3+2x^2)}\,dx$

3.320.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.19 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.42

method	result
risch	$-\frac{\sqrt{2x^4+2x^2+1}}{9x^3}-\frac{2\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{9\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{10\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,\Pi\left(x\sqrt{-1+i},\frac{1}{3}+\frac{i}{3},\frac{\sqrt{-1}}{\sqrt{-1}}\right.}{27\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$
elliptic	$-\frac{\sqrt{2x^4+2x^2+1}}{9x^3}-\frac{2\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{9\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{10\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,\Pi\left(x\sqrt{-1+i},\frac{1}{3}+\frac{i}{3},\frac{\sqrt{-1}}{\sqrt{-1}}\right.}{27\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$
default	$\frac{\left(\frac{2}{9}-\frac{2i}{9}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)-E\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\right)}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}-\frac{\sqrt{2x^4+2x^2+1}}{9x^3}-\frac{4\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2}}{9\sqrt{-1}}$

input
```
int((2*x^4+2*x^2+1)^(1/2)/x^4/(2*x^2+3),x,method=_RETURNVERBOSE)
```

output
```
-1/9*(2*x^4+2*x^2+1)^(1/2)/x^3-2/9/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+
I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1
/2*I*2^(1/2))+10/27/(-1+I)^(1/2)*(1-I*x^2+x^2)^(1/2)*(1+I*x^2+x^2)^(1/2)/(
2*x^4+2*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+
I)^(1/2))
```

3.320.5 Fricas [F]

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^4\left(3+2x^2\right)}\,dx=\int \frac{\sqrt{2\,x^4+2\,x^2+1}}{(2\,x^2+3)x^4}\,dx$$

input
```
integrate((2*x^4+2*x^2+1)^(1/2)/x^4/(2*x^2+3),x, algorithm="fricas")
```

output
```
integral(sqrt(2*x^4 + 2*x^2 + 1)/(2*x^6 + 3*x^4), x)
```

3.320.6 Sympy [F]

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^4\left(3+2x^2\right)}\,dx=\int \frac{\sqrt{2x^4+2x^2+1}}{x^4\cdot(2x^2+3)}\,dx$$

input
```
integrate((2*x**4+2*x**2+1)**(1/2)/x**4/(2*x**2+3),x)
```

output
```
Integral(sqrt(2*x**4 + 2*x**2 + 1)/(x**4*(2*x**2 + 3)), x)
```

3.320.7 Maxima [F]

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^4\left(3+2x^2\right)}\,dx=\int \frac{\sqrt{2\,x^4+2\,x^2+1}}{(2\,x^2+3)x^4}\,dx$$

input
```
integrate((2*x^4+2*x^2+1)^(1/2)/x^4/(2*x^2+3),x, algorithm="maxima")
```

output
```
integrate(sqrt(2*x^4 + 2*x^2 + 1)/((2*x^2 + 3)*x^4), x)
```

3.320.8 Giac [F]

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^4\left(3+2x^2\right)}\,dx=\int \frac{\sqrt{2\,x^4+2\,x^2+1}}{(2\,x^2+3)x^4}\,dx$$

input
```
integrate((2*x^4+2*x^2+1)^(1/2)/x^4/(2*x^2+3),x, algorithm="giac")
```

output
```
integrate(sqrt(2*x^4 + 2*x^2 + 1)/((2*x^2 + 3)*x^4), x)
```

3.320.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^4\left(3+2x^2\right)}\,dx=\int \frac{\sqrt{2\,x^4+2\,x^2+1}}{x^4\,\left(2\,x^2+3\right)}\,dx$$

input
```
int((2*x^2 + 2*x^4 + 1)^(1/2)/(x^4*(2*x^2 + 3)),x)
```

output
```
int((2*x^2 + 2*x^4 + 1)^(1/2)/(x^4*(2*x^2 + 3)), x)
```

3.321 $\int \frac{\sqrt{1+2x^2+2x^4}}{x^6(3+2x^2)}\,dx$

3.321.1 Optimal result

Integrand size = 29, antiderivative size = 546

$$
\begin{aligned}
\int \frac{\sqrt{1+2x^2+2x^4}}{x^6\left(3+2x^2\right)}\,dx = &-\frac{\sqrt{1+2x^2+2x^4}}{15x^5}+\frac{4\sqrt{1+2x^2+2x^4}}{135x^3}-\frac{4\sqrt{1+2x^2+2x^4}}{45x} \\
&+\frac{4\sqrt{2}x\sqrt{1+2x^2+2x^4}}{45\left(1+\sqrt{2}x^2\right)}-\frac{2}{27}\sqrt{\frac{5}{3}}\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) \\
&-\frac{4\sqrt[4]{2}\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{45\sqrt{1+2x^2+2x^4}} \\
&+\frac{5\sqrt[4]{2}\left(5-3\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{189\sqrt{1+2x^2+2x^4}} \\
&-\frac{\sqrt[4]{2}\left(19-2\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{135\sqrt{1+2x^2+2x^4}} \\
&+\frac{5\left(3+\sqrt{2}\right)^2\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{1}{24}\left(12-11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{567\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}
\end{aligned}
$$

output
```
-2/81*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)-1/15*(2*x^4+2*
x^2+1)^(1/2)/x^5+4/135*(2*x^4+2*x^2+1)^(1/2)/x^3-4/45*(2*x^4+2*x^2+1)^(1/2
)/x+4/45*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))-4/45*(cos(2*arcta
n(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticE(sin(2*arctan(2^(
1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^
(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)+5/189*2^(1/4)*(cos(2*arctan(
2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/
4)*x)),1/2*(2-2^(1/2))^(1/2))*(5-3*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+
1)/(1+x^2*2^(1/2))^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)-1/135*2^(1/4)*(cos(2*arc
tan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2
^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(19-2*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2
*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)+5/1134*(cos(2*arcta
n(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(2^
(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))^2*(1+x^2*2^
(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(
1/2)
```

3.321.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.22 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.41

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^6(3+2x^2)}\,dx =$$
$$-\frac{27+42x^2+66x^4+48x^6+72x^8+36i\sqrt{1-i}x^5\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}E\left(i\text{arcsinh}\left(\sqrt{1-i}x\right)\middle|\,i\right.}{}$$

input
```
Integrate[Sqrt[1 + 2*x^2 + 2*x^4]/(x^6*(3 + 2*x^2)),x]
```

output
```
-1/405*(27 + 42*x^2 + 66*x^4 + 48*x^6 + 72*x^8 + (36*I)*Sqrt[1 - I]*x^5*Sq
rt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*
x], I] - (12 + 24*I)*Sqrt[1 - I]*x^5*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I
)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] + 50*(1 - I)^(3/2)*x^5*Sqrt[
1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[1/3 + I/3, I*ArcSinh[Sqr
t[1 - I]*x], I])/(x^5*Sqrt[1 + 2*x^2 + 2*x^4])
```

3.321.3 Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 535, normalized size of antiderivative = 0.98, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.448, Rules used = {1634, 27, 2199, 2199, 1604, 27, 1604, 27, 1604, 1511, 1416, 1509, 2220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2x^4+2x^2+1}}{x^6\left(2x^2+3\right)}dx$$

$$\Big\downarrow \text{1634}$$

$$\frac{20}{189}\int \frac{2\left(3+\sqrt{2}\right)x^2+3\sqrt{2}+2}{\left(2x^2+3\right)\sqrt{2x^4+2x^2+1}}dx-\frac{2}{189}\int -\frac{-20\left(3+\sqrt{2}\right)x^6+70x^4+84x^2+63}{2x^6\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{20}{189}\int \frac{2\left(3+\sqrt{2}\right)x^2+3\sqrt{2}+2}{\left(2x^2+3\right)\sqrt{2x^4+2x^2+1}}dx+\frac{1}{189}\int \frac{-20\left(3+\sqrt{2}\right)x^6+70x^4+84x^2+63}{x^6\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \text{2199}$$

$$\frac{20}{189}\int \frac{2\left(3+\sqrt{2}\right)x^2+3\sqrt{2}+2}{\left(2x^2+3\right)\sqrt{2x^4+2x^2+1}}dx+$$
$$\frac{1}{189}\left(\int \frac{10\left(19+4\sqrt{2}\right)x^4+6\left(29+5\sqrt{2}\right)x^2+63}{x^6\sqrt{2x^4+2x^2+1}}dx+\frac{10\left(3+\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}{x^3}\right)$$

$$\Big\downarrow \text{2199}$$

$$\frac{20}{189}\int \frac{2\left(3+\sqrt{2}\right)x^2+3\sqrt{2}+2}{\left(2x^2+3\right)\sqrt{2x^4+2x^2+1}}dx+$$
$$\frac{1}{189}\left(\int \frac{-\frac{14}{3}\left(17+5\sqrt{2}\right)x^2-\frac{2}{3}\left(143+50\sqrt{2}\right)}{x^6\sqrt{2x^4+2x^2+1}}dx-\frac{5\left(19+4\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}{3x^5}+\frac{10\left(3+\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}{x^3}\right)$$

$$\Big\downarrow \text{1604}$$

$$\frac{20}{189}\int \frac{2\left(3+\sqrt{2}\right)x^2+3\sqrt{2}+2}{\left(2x^2+3\right)\sqrt{2x^4+2x^2+1}}dx+$$
$$\frac{1}{189}\left(-\frac{1}{5}\int -\frac{2\left(2\left(143+50\sqrt{2}\right)x^2+3\left(61+25\sqrt{2}\right)\right)}{x^4\sqrt{2x^4+2x^2+1}}dx+\frac{2\left(143+50\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}{15x^5}-\frac{5\left(19+4\sqrt{2}\right)\sqrt{2x^4-}}{3x^5}\right.$$

$$\Big\downarrow \text{27}$$

$$\frac{20}{189}\int \frac{2\left(3+\sqrt{2}\right)x^2+3\sqrt{2}+2}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx+$$
$$\frac{1}{189}\left(\frac{2}{5}\int \frac{2\left(143+50\sqrt{2}\right)x^2+3\left(61+25\sqrt{2}\right)}{x^4\sqrt{2x^4+2x^2+1}}dx+\frac{2\left(143+50\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}{15x^5}-\frac{5\left(19+4\sqrt{2}\right)\sqrt{2x^4+2x^2+}}{3x^5}\right.$$

$\downarrow$ 1604

$$\frac{20}{189}\int \frac{2\left(3+\sqrt{2}\right)x^2+3\sqrt{2}+2}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx+$$
$$\frac{1}{189}\left(\frac{2}{5}\left(-\frac{1}{3}\int -\frac{6\left(21-\left(61+25\sqrt{2}\right)x^2\right)}{x^2\sqrt{2x^4+2x^2+1}}dx-\frac{\left(61+25\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}{x^3}\right)+\frac{2\left(143+50\sqrt{2}\right)\sqrt{2x^4+2x^2+}}{15x^5}\right.$$

$\downarrow$ 27

$$\frac{20}{189}\int \frac{2\left(3+\sqrt{2}\right)x^2+3\sqrt{2}+2}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx+$$
$$\frac{1}{189}\left(\frac{2}{5}\left(2\int \frac{21-\left(61+25\sqrt{2}\right)x^2}{x^2\sqrt{2x^4+2x^2+1}}dx-\frac{\left(61+25\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}{x^3}\right)+\frac{2\left(143+50\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}{15x^5}-\frac{5(1}{}\right.$$

$\downarrow$ 1604

$$\frac{20}{189}\int \frac{2\left(3+\sqrt{2}\right)x^2+3\sqrt{2}+2}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx+$$
$$\frac{1}{189}\left(\frac{2}{5}\left(2\left(-\int \frac{-42x^2+25\sqrt{2}+61}{\sqrt{2x^4+2x^2+1}}dx-\frac{21\sqrt{2x^4+2x^2+1}}{x}\right)-\frac{\left(61+25\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}{x^3}\right)+\frac{2(143+50}{}\right.$$

$\downarrow$ 1511

$$\frac{20}{189}\int \frac{2\left(3+\sqrt{2}\right)x^2+3\sqrt{2}+2}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx+$$
$$\frac{1}{189}\left(\frac{2}{5}\left(2\left(-\left(\left(61+4\sqrt{2}\right)\int \frac{1}{\sqrt{2x^4+2x^2+1}}dx\right)-21\sqrt{2}\int \frac{1-\sqrt{2}x^2}{\sqrt{2x^4+2x^2+1}}dx-\frac{21\sqrt{2x^4+2x^2+1}}{x}\right)-\frac{(61}{}\right.\right.$$

$\downarrow$ 1416

$$\frac{1}{189}\left(\frac{2}{5}\left(2\left(-21\sqrt{2}\int \frac{1-\sqrt{2}x^2}{\sqrt{2x^4+2x^2+1}}dx-\frac{\left(61+4\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(\right.}{2\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}\right.\right.\right.$$
$$\frac{20}{189}\int \frac{2\left(3+\sqrt{2}\right)x^2+3\sqrt{2}+2}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx$$

$\downarrow$ 1509

3.321. $\int \frac{\sqrt{1+2x^2+2x^4}}{x^6(3+2x^2)}\,dx$

$$\frac{20}{189}\int \frac{2\left(3+\sqrt{2}\right)x^2+3\sqrt{2}+2}{\left(2x^2+3\right)\sqrt{2x^4+2x^2+1}}dx+$$

$$\frac{1}{189}\left(\frac{2}{5}\left(2\left(-\frac{\left(61+4\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{2\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-21\sqrt{2}\left(\frac{\left(\sqrt{2}x^2+1\right.}{}\right.\right.\right.\right.$$

$\downarrow$ 2220

$$\frac{20}{189}\left(\frac{\left(3+\sqrt{2}\right)^2\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticPi}\left(\frac{1}{24}\left(12-11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{12\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-\frac{7\arctan\left(\right.}{2}\right.$$

$$\frac{1}{189}\left(\frac{2}{5}\left(2\left(-\frac{\left(61+4\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{2\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-21\sqrt{2}\left(\frac{\left(\sqrt{2}x^2+1\right.}{}\right.\right.\right.\right.$$

input
```
Int[Sqrt[1 + 2*x^2 + 2*x^4]/(x^6*(3 + 2*x^2)),x]
```

output
```
((-5*(19 + 4*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])/(3*x^5) + (2*(143 + 50*Sqrt
[2])*Sqrt[1 + 2*x^2 + 2*x^4])/(15*x^5) + (10*(3 + Sqrt[2])*Sqrt[1 + 2*x^2
+ 2*x^4])/x^3 + (2*(-(((61 + 25*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])/x^3) + 2
*((-21*Sqrt[1 + 2*x^2 + 2*x^4])/x - 21*Sqrt[2]*(-((x*Sqrt[1 + 2*x^2 + 2*x^
4])/(1 + Sqrt[2]*x^2)) + ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 +
Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(2^(1/4)*
Sqrt[1 + 2*x^2 + 2*x^4])) - ((61 + 4*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 +
2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sq
rt[2])/4])/(2*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]))))/5)/189 + (20*((-7*ArcTan
[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/(2*Sqrt[15]) + ((3 + Sqrt[2])^2*(
1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[
(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(12*2^(1/4)*S
qrt[1 + 2*x^2 + 2*x^4])))/189
```

3.321. $\int \frac{\sqrt{1+2x^2+2x^4}}{x^6\left(3+2x^2\right)}\,dx$

3.321.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.321. $\int \frac{\sqrt{1+2x^2+2x^4}}{x^6(3+2x^2)}\,dx$

rule 1634
```
Int[((x_)^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[(-(-d/e)^(m/2))*((c*d^2 - b*d*e + a*e^2)^(p + 1/2)
/(e^(2*p)*(c*d^2 - a*e^2)))   Int[(a*d*Rt[c/a, 2] + a*e + (c*d + a*e*Rt[c/a
, 2])*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] + Simp[(-d/e)^(m/2
)/(e^(2*p)*(c*d^2 - a*e^2))   Int[(x^m/Sqrt[a + b*x^2 + c*x^4])*ExpandToSum
[((e^(2*p)*(c*d^2 - a*e^2)*(a + b*x^2 + c*x^4)^(p + 1/2))/(-d/e)^(m/2) + ((
a*d*Rt[c/a, 2] + a*e + (c*d + a*e*Rt[c/a, 2])*x^2)*(c*d^2 - b*d*e + a*e^2)^
(p + 1/2))/x^m)/(d + e*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p + 1/2, 0] && ILtQ[m/2, 0] && NeQ[c*d^2 - a*e^2,
0] && PosQ[c/a]
```

rule 2199
```
Int[(Px_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> With[{q = Expon[Px, x^2]}, Simp[Coeff[Px, x^2, q]*(d*x)^(m + 2*q
- 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*d^(2*q - 3)*(m + 4*p + 2*q + 1))), x]
+ Int[(d*x)^m*(a + b*x^2 + c*x^4)^p*ExpandToSum[Px - Coeff[Px, x^2, q]*x^(2
*q) - Coeff[Px, x^2, q]*((a*(m + 2*q - 3)*x^(2*(q - 2)) + b*(m + 2*p + 2*q
- 1)*x^(2*(q - 1)))/(c*(m + 4*p + 2*q + 1))), x], x] /; GtQ[q, 1] && NeQ[m
+ 4*p + 2*q + 1, 0]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Px, x^2] && N
eQ[b^2 - 4*a*c, 0]
```

rule 2220
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a
+ b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*El
lipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &
& EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[-b + c*(d/e) + a*(e/d)]
```

3.321. $\int \frac{\sqrt{1+2x^2+2x^4}}{x^6(3+2x^2)}\,dx$

3.321.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.10 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.48

method	result
risch	$-\frac{24x^8+16x^6+22x^4+14x^2+9}{135x^5\sqrt{2x^4+2x^2+1}}+\frac{8\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{135\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{4}{45}+\frac{4i}{45}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}}{\sqrt{-1}}$
elliptic	$-\frac{\sqrt{2x^4+2x^2+1}}{15x^5}+\frac{4\sqrt{2x^4+2x^2+1}}{135x^3}-\frac{4\sqrt{2x^4+2x^2+1}}{45x}-\frac{4\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{135\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{4i\sqrt{-ix^2+x^2}}{}$
default	$-\frac{\sqrt{2x^4+2x^2+1}}{15x^5}+\frac{4\sqrt{2x^4+2x^2+1}}{135x^3}-\frac{4\sqrt{2x^4+2x^2+1}}{45x}-\frac{4\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{45\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{32}{135}+\frac{32i}{135}\right)}{}$

input
```
int((2*x^4+2*x^2+1)^(1/2)/x^6/(2*x^2+3),x,method=_RETURNVERBOSE)
```

output
```
-1/135*(24*x^8+16*x^6+22*x^4+14*x^2+9)/x^5/(2*x^4+2*x^2+1)^(1/2)+8/135/(-1
+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*El
lipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+(-4/45+4/45*I)/(-1+I)^(1
/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(Ellipti
cF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/2*
2^(1/2)+1/2*I*2^(1/2)))-20/81/(-1+I)^(1/2)*(1-I*x^2+x^2)^(1/2)*(1+I*x^2+x^
2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^
(1/2)/(-1+I)^(1/2))
```

3.321.5 Fricas [F]

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^6\left(3+2x^2\right)}\,dx=\int \frac{\sqrt{2\,x^4+2\,x^2+1}}{\left(2\,x^2+3\right)x^6}\,dx$$

input
```
integrate((2*x^4+2*x^2+1)^(1/2)/x^6/(2*x^2+3),x, algorithm="fricas")
```

output
```
integral(sqrt(2*x^4 + 2*x^2 + 1)/(2*x^8 + 3*x^6), x)
```

3.321.6 Sympy [F]

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^6\,(3+2x^2)}\,dx = \int \frac{\sqrt{2x^4+2x^2+1}}{x^6\cdot(2x^2+3)}\,dx$$

input
```
integrate((2*x**4+2*x**2+1)**(1/2)/x**6/(2*x**2+3),x)
```

output
```
Integral(sqrt(2*x**4 + 2*x**2 + 1)/(x**6*(2*x**2 + 3)), x)
```

3.321.7 Maxima [F]

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^6\,(3+2x^2)}\,dx = \int \frac{\sqrt{2\,x^4+2\,x^2+1}}{(2\,x^2+3)x^6}\,dx$$

input
```
integrate((2*x^4+2*x^2+1)^(1/2)/x^6/(2*x^2+3),x, algorithm="maxima")
```

output
```
integrate(sqrt(2*x^4 + 2*x^2 + 1)/((2*x^2 + 3)*x^6), x)
```

3.321.8 Giac [F]

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^6\,(3+2x^2)}\,dx = \int \frac{\sqrt{2\,x^4+2\,x^2+1}}{(2\,x^2+3)x^6}\,dx$$

input
```
integrate((2*x^4+2*x^2+1)^(1/2)/x^6/(2*x^2+3),x, algorithm="giac")
```

output
```
integrate(sqrt(2*x^4 + 2*x^2 + 1)/((2*x^2 + 3)*x^6), x)
```

3.321.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^6(3+2x^2)}\,dx = \int \frac{\sqrt{2\,x^4+2\,x^2+1}}{x^6\,\left(2\,x^2+3\right)}\,dx$$

input
```
int((2*x^2 + 2*x^4 + 1)^(1/2)/(x^6*(2*x^2 + 3)),x)
```

output
```
int((2*x^2 + 2*x^4 + 1)^(1/2)/(x^6*(2*x^2 + 3)), x)
```

3.322 $\int \frac{x^5 (a+bx^2+cx^4)^{3/2}}{d+ex^2} \, dx$

3.322.1 Optimal result

Integrand size = 29, antiderivative size = 482

$$\int \frac{x^5(a+bx^2+cx^4)^{3/2}}{d+ex^2} \, dx = \frac{(128c^4d^4+3b^4e^4-32c^3d^2e(5bd-4ae)+8bc^2de^2(2bd-3ae)+6b^2ce^3(bd-2a\ldots}{2\ldots}$$
$$+\frac{(16c^2d^2-6bcde-3b^2e^2-6ce(2cd+be)x^2)(a+bx^2+cx^4)^{3/2}}{96c^2e^3}+\frac{(a+bx^2+cx^4)^{5/2}}{10ce}$$
$$-\frac{(256c^5d^5+3b^5e^5+6b^3ce^4(bd-4ae)-384c^4d^3e(bd-ae)+96c^3de^2(bd-ae)^2+16bc^2e^3(b^2d^2-3abde+\ldots}{512c^{7/2}e^6}$$
$$+\frac{d^2(cd^2-bde+ae^2)^{3/2}\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2e^6}$$

output
```
1/96*(16*c^2*d^2-6*b*c*d*e-3*b^2*e^2-6*c*e*(b*e+2*c*d)*x^2)*(c*x^4+b*x^2+a
)^(3/2)/c^2/e^3+1/10*(c*x^4+b*x^2+a)^(5/2)/c/e-1/512*(256*c^5*d^5+3*b^5*e^
5+6*b^3*c*e^4*(-4*a*e+b*d)-384*c^4*d^3*e*(-a*e+b*d)+96*c^3*d*e^2*(-a*e+b*d
)^2+16*b*c^2*e^3*(3*a^2*e^2-3*a*b*d*e+b^2*d^2))*arctanh(1/2*(2*c*x^2+b)/c^
(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(7/2)/e^6+1/2*d^2*(a*e^2-b*d*e+c*d^2)^(3/2)
*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^4
+b*x^2+a)^(1/2))/e^6+1/256*(128*c^4*d^4+3*b^4*e^4-32*c^3*d^2*e*(-4*a*e+5*b
*d)+8*b*c^2*d*e^2*(-3*a*e+2*b*d)+6*b^2*c*e^3*(-2*a*e+b*d)-2*c*e*(32*c^3*d^
3-3*b^3*e^3-8*c^2*d*e*(-3*a*e+2*b*d)-6*b*c*e^2*(-2*a*e+b*d))*x^2)*(c*x^4+b
*x^2+a)^(1/2)/c^3/e^5
```

3.322.2 Mathematica [A] (verified)

Time = 10.67 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.13

$$\int \frac{x^5(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx = \frac{1280d^2(a+bx^2+cx^4)^{3/2} - \frac{480de(b+2cx^2)(a+bx^2+cx^4)^{3/2}}{c} + \frac{768e^2(a+bx^2+cx^4)^{5/2}}{c} - \frac{90(\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[(x^5*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]
```

output
```
(1280*d^2*(a + b*x^2 + c*x^4)^(3/2) - (480*d*e*(b + 2*c*x^2)*(a + b*x^2 +
c*x^4)^(3/2))/c + (768*e^2*(a + b*x^2 + c*x^4)^(5/2))/c - (90*(b^2 - 4*a*c
)*d*e*(-2*Sqrt[c]*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4] + (b^2 - 4*a*c)*Ar
cTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]))/c^(5/2) + (15*b
*e^2*(-16*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(3/2) + 3*(b^2 - 4*a*c)*((2*(b
 + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/c + ((-b^2 + 4*a*c)*ArcTanh[(b + 2*c*
x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/c^(3/2))))/c^2 - (240*d^2*((2*c
*d - b*e)*(8*c^2*d^2 - b^2*e^2 + 4*c*e*(-2*b*d + 3*a*e))*ArcTanh[(b + 2*c*
x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])] + 2*Sqrt[c]*(e*Sqrt[a + b*x^2 +
c*x^4]*(-(b^2*e^2) + 4*c^2*d*(-2*d + e*x^2) - 2*c*e*(-5*b*d + 4*a*e + b*e*
x^2)) + 8*c*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*ArcTanh[(-(b*d) + 2*a*e - 2*c
*d*x^2 + b*e*x^2)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + b*x^2 + c*x^4
])])))/(c^(3/2)*e^3))/(7680*e^3)
```

3.322.3 Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 517, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {1578, 1267, 27, 1231, 27, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\left(a+bx^2+cx^4\right)^{3/2}}{d+ex^2}dx$$

$$\Big\downarrow \text{ 1578}$$

$$\frac{1}{2}\int \frac{x^4\left(cx^4+bx^2+a\right)^{3/2}}{ex^2+d}dx^2$$

$$\Big\downarrow \ 1267$$

$$\frac{1}{2}\left(\frac{\int -\frac{5e((2cd+be)x^2+bd)(cx^4+bx^2+a)^{3/2}}{2(ex^2+d)}dx^2}{5ce^2}+\frac{(a+bx^2+cx^4)^{5/2}}{5ce}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{2}\left(\frac{(a+bx^2+cx^4)^{5/2}}{5ce}-\frac{\int \frac{((2cd+be)x^2+bd)(cx^4+bx^2+a)^{3/2}}{ex^2+d}dx^2}{2ce}\right)$$

$$\Big\downarrow \ 1231$$

$$\frac{1}{2}\left(\frac{(a+bx^2+cx^4)^{5/2}}{5ce}-\frac{-\frac{\int \frac{\left(d\left(3e^2b^3+6cdeb^2-4c\left(4cd^2+3ae^2\right)b+8ac^2de\right)-\left(32c^3d^3-8c^2e(2bd-3ae)d-3b^3e^3-6bce^2(bd-2ae)\right)x^2\right)\sqrt{cx^4+bx^2+a}}{2\left(ex^2+d\right)}d}{8ce^2}}{2ce}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{2}\left(\frac{(a+bx^2+cx^4)^{5/2}}{5ce}-\frac{-\frac{\int \frac{\left(d\left(3e^2b^3+6cdeb^2-4c\left(4cd^2+3ae^2\right)b+8ac^2de\right)-\left(32c^3d^3-8c^2e(2bd-3ae)d-3b^3e^3-6bce^2(bd-2ae)\right)x^2\right)\sqrt{cx^4+bx^2+a}}{ex^2+d}d}{16ce^2}}{2ce}\right)$$

$$\Big\downarrow \ 1231$$

$$\frac{1}{2}\left(\frac{(a+bx^2+cx^4)^{5/2}}{5ce}-\frac{-\frac{\sqrt{a+bx^2+cx^4}\left(-2cex^2\left(-8c^2de(2bd-3ae)-6bce^2(bd-2ae)-3b^3e^3+32c^3d^3\right)+6b^2ce^3(bd-2ae)-32c^3d^2e(5bd-4ae)+8bc^2d\right)}{4ce^2}}{}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{2}\left(\frac{(a+bx^2+cx^4)^{5/2}}{5ce}-\frac{-\frac{\sqrt{a+bx^2+cx^4}\left(-2cex^2\left(-8c^2de(2bd-3ae)-6bce^2(bd-2ae)-3b^3e^3+32c^3d^3\right)+6b^2ce^3(bd-2ae)-32c^3d^2e(5bd-4ae)+8bc^2d\right)}{4ce^2}}{}\right)$$

$$\Big\downarrow \ 1269$$

3.322. $\int \frac{x^5(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx$

$$\frac{1}{2}\left(\frac{\left(a+bx^2+cx^4\right)^{5/2}}{5ce}-\frac{-\frac{\sqrt{a+bx^2+cx^4}\left(-2cex^2\left(-8c^2de(2bd-3ae)-6bce^2(bd-2ae)-3b^3e^3+32c^3d^3\right)+6b^2ce^3(bd-2ae)-32c^3d^2e(5bd-4ae)+8bc^2de\right.}{4ce^2}}{}\right.$$

$\downarrow$ 1092

$$\frac{1}{2}\left(\frac{\left(a+bx^2+cx^4\right)^{5/2}}{5ce}-\frac{-\frac{\sqrt{a+bx^2+cx^4}\left(-2cex^2\left(-8c^2de(2bd-3ae)-6bce^2(bd-2ae)-3b^3e^3+32c^3d^3\right)+6b^2ce^3(bd-2ae)-32c^3d^2e(5bd-4ae)+8bc^2de\right.}{4ce^2}}{}\right.$$

$\downarrow$ 219

$$\frac{1}{2}\left(\frac{\left(a+bx^2+cx^4\right)^{5/2}}{5ce}-\frac{-\frac{\sqrt{a+bx^2+cx^4}\left(-2cex^2\left(-8c^2de(2bd-3ae)-6bce^2(bd-2ae)-3b^3e^3+32c^3d^3\right)+6b^2ce^3(bd-2ae)-32c^3d^2e(5bd-4ae)+8bc^2de\right.}{4ce^2}}{}\right.$$

$\downarrow$ 1154

$$\frac{1}{2}\left(\frac{\left(a+bx^2+cx^4\right)^{5/2}}{5ce}-\frac{-\frac{\sqrt{a+bx^2+cx^4}\left(-2cex^2\left(-8c^2de(2bd-3ae)-6bce^2(bd-2ae)-3b^3e^3+32c^3d^3\right)+6b^2ce^3(bd-2ae)-32c^3d^2e(5bd-4ae)+8bc^2de\right.}{4ce^2}}{}\right.$$

$\downarrow$ 219

$$\frac{1}{2}\left(\frac{\left(a+bx^2+cx^4\right)^{5/2}}{5ce}-\frac{-\frac{\sqrt{a+bx^2+cx^4}\left(-2cex^2\left(-8c^2de(2bd-3ae)-6bce^2(bd-2ae)-3b^3e^3+32c^3d^3\right)+6b^2ce^3(bd-2ae)-32c^3d^2e(5bd-4ae)+8bc^2de\right.}{4ce^2}}{}\right.$$

input
```
Int[(x^5*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]
```

3.322. $\int \frac{x^5\left(a+bx^2+cx^4\right)^{3/2}}{d+ex^2}\,dx$

output
```
((a + b*x^2 + c*x^4)^(5/2)/(5*c*e) - (-1/24*((16*c^2*d^2 - 6*b*c*d*e - 3*b
^2*e^2 - 6*c*e*(2*c*d + b*e)*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(c*e^2) - (((
128*c^4*d^4 + 3*b^4*e^4 - 32*c^3*d^2*e*(5*b*d - 4*a*e) + 8*b*c^2*d*e^2*(2*
b*d - 3*a*e) + 6*b^2*c*e^3*(b*d - 2*a*e) - 2*c*e*(32*c^3*d^3 - 3*b^3*e^3 -
 8*c^2*d*e*(2*b*d - 3*a*e) - 6*b*c*e^2*(b*d - 2*a*e))*x^2)*Sqrt[a + b*x^2
+ c*x^4])/(4*c*e^2) - (((256*c^5*d^5 + 3*b^5*e^5 + 6*b^3*c*e^4*(b*d - 4*a*
e) - 384*c^4*d^3*e*(b*d - a*e) + 96*c^3*d*e^2*(b*d - a*e)^2 + 16*b*c^2*e^3
*(b^2*d^2 - 3*a*b*d*e + 3*a^2*e^2))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[
a + b*x^2 + c*x^4])])/(Sqrt[c]*e) - (256*c^3*d^2*(c*d^2 - b*d*e + a*e^2)^(
3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e
^2]*Sqrt[a + b*x^2 + c*x^4])])/e)/(8*c*e^2))/(16*c*e^2))/(2*c*e))/2
```

3.322.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1267
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - g^n*(d
+ e*x)^(n - 2)*(b*d*e*(p + 1) + a*e^2*(m + n - 1) - c*d^2*(m + n + 2*p + 1)
 - e*(2*c*d - b*e)*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && IGtQ[n, 1] && IntegerQ[m] && NeQ[m + n + 2*p + 1, 0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.322. $\int \frac{x^5 \left(a+b x^2+c x^4\right)^{3/2}}{d+e x^2} \, dx$

3.322.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 620, normalized size of antiderivative = 1.29

method	result
pseudoelliptic	$\frac{-5\left(e^2(ae-bd)^2c^{\frac{7}{2}}+(2e^2d^2a-2d^3eb)c^{\frac{9}{2}}+c^{\frac{11}{2}}d^4\right)d^2\ln\left(\frac{2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,e+\left(b\,x^2+2a\right)e-d\left(2c\,x^2+b\right)}{e\,x^2+d}\right)+\left(\left(-\right.\right.}{}$
risch	$\frac{(384c^4e^4x^8+528b\,c^3e^4x^6-480c^4d\,e^3x^6+768a\,c^3e^4x^4+24b^2c^2e^4x^4-720b\,c^3d\,e^3x^4+640c^4d^2e^2x^4+168ab\,c^2e^4x^2-1200a\,c^3d}{}$
default	Expression too large to display
elliptic	Expression too large to display

input
```
int(x^5*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/10/c^(7/2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(-5*(e^2*(a*e-b*d)^2*c^(7/2)+
(2*a*d^2*e^2-2*b*d^3*e)*c^(9/2)+c^(11/2)*d^4)*d^2*ln((2*(c*x^4+b*x^2+a)^(1
/2)*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*e+(b*x^2+2*a)*e-d*(2*c*x^2+b))/(e*x^2+
d))+((-15/16*b*(a*c-1/4*b^2)^2*e^5-15/8*c*d*(a*c-1/4*b^2)^2*e^4+15/4*c^2*d
^2*(a*c-1/12*b^2)*b*e^3-15/2*(a*c+1/4*b^2)*c^3*d^3*e^2+15/2*b*c^4*d^4*e-5*
c^5*d^5)*ln((2*c*x^2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c^(1/2))+e*(20/3*e
*(3/10*x^4*(11/16*b*x^2+a)*e^3-15/32*d*(3/5*b*x^2+a)*x^2*e^2+d^2*(7/16*b*x
^2+a)*e-15/16*b*d^3)*c^(7/2)+(5/3*d^2*e^2*x^4-5/2*d^3*e*x^2-5/4*d*e^3*x^6+
e^4*x^8+5*d^4)*c^(9/2)+(((1/16*b^2*x^4+a^2+7/16*a*b*x^2)*e^2-25/16*d*b*(1/
10*b*x^2+a)*e+5/8*b^2*d^2)*c^(5/2)-25/32*(((1/10*b*x^2+a)*e-3/10*b*d)*c^(3
/2)-3/20*b^2*e*c^(1/2))*e*b^2)*e^2)*(c*x^4+b*x^2+a)^(1/2)+15/16*(b*(a*c-1/
4*b^2)^2*e^5+2*c*d*(a*c-1/4*b^2)^2*e^4-4*c^2*d^2*(a*c-1/12*b^2)*b*e^3+(8*a
*c^4+2*b^2*c^3)*d^3*e^2-8*b*c^4*d^4*e+16/3*c^5*d^5)*ln(2))*e*((a*e^2-b*d*e
+c*d^2)/e^2)^(1/2))/e^7
```

3.322.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^5(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx = \text{Timed out}$$

input
```
integrate(x^5*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.322.6 Sympy [F]

$$\int \frac{x^5(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx = \int \frac{x^5(a+bx^2+cx^4)^{\frac{3}{2}}}{d+ex^2}\,dx$$

input
```
integrate(x**5*(c*x**4+b*x**2+a)**(3/2)/(e*x**2+d),x)
```

output
```
Integral(x**5*(a + b*x**2 + c*x**4)**(3/2)/(d + e*x**2), x)
```

3.322.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.322.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^5(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.322.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx = \int \frac{x^5\,\left(c\,x^4+b\,x^2+a\right)^{3/2}}{e\,x^2+d}\,dx$$

input
```
int((x^5*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x)
```

output
```
int((x^5*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2), x)
```

3.323 $\int \frac{x^3(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx$

3.323.1 Optimal result

Integrand size = 29, antiderivative size = 360

$$\int \frac{x^3(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx = -\frac{(64c^3d^3+3b^3e^3-16c^2de(5bd-4ae)+4bce^2(2bd-3ae)-2ce(16c^2d^2-3b^2e^2-4ce(2bd-3ae))\,x^2)\,\sqrt{a}}{128c^2e^4}$$
$$-\frac{(8cd-3be-6cex^2)\,(a+bx^2+cx^4)^{3/2}}{48ce^2}$$
$$+\frac{(128c^4d^4+3b^4e^4+8b^2ce^3(bd-3ae)-192c^3d^2e(bd-ae)+48c^2e^2(bd-ae)^2)\,\mathrm{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{256c^{5/2}e^5}$$
$$-\frac{d(cd^2-bde+ae^2)^{3/2}\,\mathrm{arctanh}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2e^5}$$

output
```
-1/48*(-6*c*e*x^2-3*b*e+8*c*d)*(c*x^4+b*x^2+a)^(3/2)/c/e^2+1/256*(128*c^4*
d^4+3*b^4*e^4+8*b^2*c*e^3*(-3*a*e+b*d)-192*c^3*d^2*e*(-a*e+b*d)+48*c^2*e^2
*(-a*e+b*d)^2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(5
/2)/e^5-1/2*d*(a*e^2-b*d*e+c*d^2)^(3/2)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d
)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2))/e^5-1/128*(64*c^3*
d^3+3*b^3*e^3-16*c^2*d*e*(-4*a*e+5*b*d)+4*b*c*e^2*(-3*a*e+2*b*d)-2*c*e*(16
*c^2*d^2-3*b^2*e^2-4*c*e*(-3*a*e+2*b*d))*x^2)*(c*x^4+b*x^2+a)^(1/2)/c^2/e^
4
```

3.323.2 Mathematica [A] (verified)

Time = 1.94 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.96

$$\int \frac{x^3(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx = \frac{\frac{2e\sqrt{a+bx^2+cx^4}(-9b^3e^3+6bce^2(-4bd+10ae+bex^2)-16c^3(12d^3-6d^2ex^2+4de^2x^4-3e^3x^6)+8c^2e(ae(-32d}{c^2}}{}$$

input
```
Integrate[(x^3*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]
```

output
```
((2*e*Sqrt[a + b*x^2 + c*x^4]*(-9*b^3*e^3 + 6*b*c*e^2*(-4*b*d + 10*a*e + b
*e*x^2) - 16*c^3*(12*d^3 - 6*d^2*e*x^2 + 4*d*e^2*x^4 - 3*e^3*x^6) + 8*c^2*
e*(a*e*(-32*d + 15*e*x^2) + b*(30*d^2 - 14*d*e*x^2 + 9*e^2*x^4))))/c^2 - 7
68*d*Sqrt[-(c*d^2) + b*d*e - a*e^2]*(c*d^2 + e*(-(b*d) + a*e))*ArcTan[(Sqr
t[c]*(d + e*x^2) - e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + e*(b*d - a*e
)]] - (3*(128*c^4*d^4 + 3*b^4*e^4 + 8*b^2*c*e^3*(b*d - 3*a*e) - 192*c^3*d^
2*e*(b*d - a*e) + 48*c^2*e^2*(b*d - a*e)^2)*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sq
rt[a + b*x^2 + c*x^4]])/c^(5/2))/(768*e^5)
```

3.323.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {1578, 1231, 27, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx$$

$$\Big\downarrow \ 1578$$

$$\frac{1}{2}\int \frac{x^2(cx^4+bx^2+a)^{3/2}}{ex^2+d}dx^2$$

$$\Big\downarrow \ 1231$$

$$\frac{1}{2}\left(-\frac{\int -\frac{((16c^2d^2-3b^2e^2-4ce(2bd-3ae))x^2+d(-3eb^2+8cdb-4ace))\sqrt{cx^4+bx^2+a}}{2(ex^2+d)}dx^2}{8ce^2}-\frac{(a+bx^2+cx^4)^{3/2}(-3be+8cd-6cex}{24ce^2}\right.$$

$$\Big\downarrow \ 27$$

$$\frac{1}{2}\left(\frac{\int \frac{((16c^2d^2-3b^2e^2-4ce(2bd-3ae))x^2+d(-3eb^2+8cdb-4ace))\sqrt{cx^4+bx^2+a}}{ex^2+d}dx^2}{16ce^2}-\frac{(a+bx^2+cx^4)^{3/2}(-3be+8cd-6cex^2)}{24ce^2}\right)$$

$\big\downarrow$ 1231

$$\frac{1}{2}\left(\frac{-\frac{\int -\frac{(128c^4d^4-192c^3e(bd-ae)d^2+3b^4e^4+48c^2e^2(bd-ae)^2+8b^2ce^3(bd-3ae))x^2+d(3e^3b^4+8cde^2b^3-8ce(10cd^2+3ae^2)b^2+32c^2d(2cd^2+5ae^2)b-16ac^2e(}{2(ex^2+d)\sqrt{cx^4+bx^2+a}}}{4ce^2}}{16ce^2}\right.$$

$\big\downarrow$ 27

$$\frac{1}{2}\left(\frac{\frac{\int \frac{(128c^4d^4-192c^3e(bd-ae)d^2+3b^4e^4+48c^2e^2(bd-ae)^2+8b^2ce^3(bd-3ae))x^2+d(3e^3b^4+8cde^2b^3-8ce(10cd^2+3ae^2)b^2+32c^2d(2cd^2+5ae^2)b-16ac^2e(4cd^2}{(ex^2+d)\sqrt{cx^4+bx^2+a}}}{8ce^2}}{16ce^2}\right.$$

$\big\downarrow$ 1269

$$\frac{1}{2}\left(\frac{\frac{\frac{(8b^2ce^3(bd-3ae)-192c^3d^2e(bd-ae)+48c^2e^2(bd-ae)^2+3b^4e^4+128c^4d^4)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx^2}{e}-\frac{128c^2d(ae^2-bde+cd^2)^2\int \frac{1}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx^2}{e}}{8ce^2}}{16ce^2}-\right.$$

$\big\downarrow$ 1092

$$\frac{1}{2}\left(\frac{\frac{\frac{2(8b^2ce^3(bd-3ae)-192c^3d^2e(bd-ae)+48c^2e^2(bd-ae)^2+3b^4e^4+128c^4d^4)\int \frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}}{e}-\frac{128c^2d(ae^2-bde+cd^2)^2\int \frac{1}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx^2}{e}}{8ce^2}}{16ce^2}\right.$$

$\big\downarrow$ 219

$$\frac{1}{2}\left(\frac{\frac{\frac{\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)(8b^2ce^3(bd-3ae)-192c^3d^2e(bd-ae)+48c^2e^2(bd-ae)^2+3b^4e^4+128c^4d^4)}{\sqrt{c}e}-\frac{128c^2d(ae^2-bde+cd^2)^2\int \frac{1}{(ex^2+d)\sqrt{cx^4+bx^2}}}{e}}{8ce^2}}{16ce^2}\right.$$

$\big\downarrow$ 1154

3.323. $\int \frac{x^3(a+bx^2+cx^4)^{3/2}}{d+ex^2}dx$

$$\frac{1}{2}\left(\frac{\frac{\frac{256c^2d\left(ae^2-bde+cd^2\right)^2\int\frac{1}{4\left(cd^2-bed+ae^2\right)-x^4}d\left(-\frac{(2cd-be)x^2+bd-2ae}{\sqrt{cx^4+bx^2+a}}\right)}{e}+\frac{\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)\left(8b^2ce^3(bd-3ae)-192c^3d^2e(bd-ae)+48c^2\right.}{\sqrt{c}e}}{8ce^2}}{16}\right.$$

$\downarrow$ 219

$$\frac{1}{2}\left(\frac{\frac{\frac{\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)\left(8b^2ce^3(bd-3ae)-192c^3d^2e(bd-ae)+48c^2e^2(bd-ae)^2+3b^4e^4+128c^4d^4\right)}{\sqrt{c}e}-\frac{128c^2d\left(ae^2-bde+cd^2\right)^{3/2}\operatorname{arctanh}\left(\frac{-2}{2\sqrt{a+b}}\right.}{e}}{8ce^2}}{16c}\right.$$

input
```
Int[(x^3*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]
```

output
```
(-1/24*((8*c*d - 3*b*e - 6*c*e*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(c*e^2) + (
-1/4*((64*c^3*d^3 + 3*b^3*e^3 - 16*c^2*d*e*(5*b*d - 4*a*e) + 4*b*c*e^2*(2*
b*d - 3*a*e) - 2*c*e*(16*c^2*d^2 - 3*b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*x^2)
*Sqrt[a + b*x^2 + c*x^4])/(c*e^2) + (((128*c^4*d^4 + 3*b^4*e^4 + 8*b^2*c*e
^3*(b*d - 3*a*e) - 192*c^3*d^2*e*(b*d - a*e) + 48*c^2*e^2*(b*d - a*e)^2)*A
rcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(Sqrt[c]*e) - (
128*c^2*d*(c*d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*
e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/e)/(8*c*
e^2))/(16*c*e^2))/2
```

3.323.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

3.323. $\int\frac{x^3\left(a+bx^2+cx^4\right)^{3/2}}{d+ex^2}\,dx$

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.323.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.37

method	result
pseudoelliptic	$\frac{\left(e^{2}(ae-bd)^{2}c^{\frac{7}{2}}+c^{\frac{9}{2}}d^{2}\left(2a\,e^{2}-2bde+c\,d^{2}\right)\right)d\ln\left(\frac{2\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}\,e+\left(b\,x^{2}+2a\right)e-d\left(2c\,x^{2}+b\right)}{e\,x^{2}+d}\right)-e\sqrt{\frac{a\,e^{2}-bde+}{e^{2}}}}{}$
risch	$\frac{(48c^{3}e^{3}x^{6}+72b\,c^{2}e^{3}x^{4}-64c^{3}d\,e^{2}x^{4}+120a\,c^{2}e^{3}x^{2}+6b^{2}c\,e^{3}x^{2}-112b\,c^{2}d\,e^{2}x^{2}+96c^{3}d^{2}e\,x^{2}+60abc\,e^{3}-256a\,c^{2}d\,e^{2}-9b^{3}e^{3}-24}{384c^{2}e^{4}}$
default	Expression too large to display
elliptic	Expression too large to display

input

```
int(x^3*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output

```
1/2/c^(7/2)*((e^2*(a*e-b*d)^2*c^(7/2)+c^(9/2)*d^2*(2*a*e^2-2*b*d*e+c*d^2))
*d*ln((2*(c*x^4+b*x^2+a)^(1/2)*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*e+(b*x^2+2*
a)*e-d*(2*c*x^2+b))/(e*x^2+d))-e*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(-3/8*((a
*c-1/4*b^2)^2*e^4-2*c*d*(a*c-1/12*b^2)*b*e^3+(4*a*c^3+b^2*c^2)*d^2*e^2-4*b
*c^3*e*d^3+8/3*c^4*d^4)*c*ln((2*c*x^2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c
^(1/2))+(4/3*e*(-15/32*(3/5*b*x^2+a)*x^2*e^2+d*(7/16*b*x^2+a)*e-15/16*b*d^
2)*c^(7/2)+3/64*c^(3/2)*b^3*e^3+c^(5/2)*(1/4*(-c^2*x^6-5/4*b*(1/10*b*x^2+a
))*e^3+1/8*d*(8/3*c^2*x^4+b^2)*e^2-1/2*d^2*e*c^2*x^2+c^2*d^3))*e*(c*x^4+b*
x^2+a)^(1/2)+3/8*((a*c-1/4*b^2)^2*e^4-2*c*d*(a*c-1/12*b^2)*b*e^3+(4*a*c^3+
b^2*c^2)*d^2*e^2-4*b*c^3*e*d^3+8/3*c^4*d^4)*ln(2)*c))/((a*e^2-b*d*e+c*d^2)
/e^2)^(1/2)/e^6
```

3.323.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx = \text{Timed out}$$

input

```
integrate(x^3*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="fricas")
```

output

```
Timed out
```

3.323.6 Sympy [F]

$$\int \frac{x^3(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx = \int \frac{x^3(a+bx^2+cx^4)^{\frac{3}{2}}}{d+ex^2}\,dx$$

input
```
integrate(x**3*(c*x**4+b*x**2+a)**(3/2)/(e*x**2+d),x)
```

output
```
Integral(x**3*(a + b*x**2 + c*x**4)**(3/2)/(d + e*x**2), x)
```

3.323.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.323.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.323.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx = \int \frac{x^3\,(c\,x^4+b\,x^2+a)^{3/2}}{e\,x^2+d}\,dx$$

input
```
int((x^3*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x)
```

output
```
int((x^3*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2), x)
```

3.324 $\int \frac{x(a+bx^2+cx^4)^{3/2}}{d+ex^2} dx$

3.324.1 Optimal result

Integrand size = 27, antiderivative size = 269

$$\int \frac{x(a+bx^2+cx^4)^{3/2}}{d+ex^2} dx = \frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x^2)\sqrt{a+bx^2+cx^4}}{16ce^3} + \frac{(a+bx^2+cx^4)^{3/2}}{6e} - \frac{(2cd-be)\left(8c^2d^2-b^2e^2-4ce(2bd-3ae)\right)\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{3/2}e^4} + \frac{(cd^2-bde+ae^2)^{3/2}\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2e^4}$$

output
```
1/6*(c*x^4+b*x^2+a)^(3/2)/e-1/32*(-b*e+2*c*d)*(8*c^2*d^2-b^2*e^2-4*c*e*(-3
*a*e+2*b*d))*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(3/2
)/e^4+1/2*(a*e^2-b*d*e+c*d^2)^(3/2)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^
2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2))/e^4+1/16*(8*c^2*d^2+b^
2*e^2-2*c*e*(-4*a*e+5*b*d)-2*c*e*(-b*e+2*c*d)*x^2)*(c*x^4+b*x^2+a)^(1/2)/c
/e^3
```

3.324.2 Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.99

$$\int \frac{x(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx = \frac{\frac{e\sqrt{a+bx^2+cx^4}(3b^2e^2+2ce(-15bd+16ae+7bex^2)+4c^2(6d^2-3dex^2+2e^2x^4))}{c}+48\sqrt{-cd^2+e(bd-}}{}$$

input
```
Integrate[(x*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]
```

output
```
((e*Sqrt[a + b*x^2 + c*x^4]*(3*b^2*e^2 + 2*c*e*(-15*b*d + 16*a*e + 7*b*e*x
^2) + 4*c^2*(6*d^2 - 3*d*e*x^2 + 2*e^2*x^4)))/c + 48*Sqrt[-(c*d^2) + e*(b*
d - a*e)]*(c*d^2 + e*(-(b*d) + a*e))*ArcTan[(Sqrt[-(c*d^2) + e*(b*d - a*e)
]*x^2)/(Sqrt[a]*(d + e*x^2) - d*Sqrt[a + b*x^2 + c*x^4])] - (3*(2*c*d - b*
e)*(8*c^2*d^2 - b^2*e^2 + 4*c*e*(-2*b*d + 3*a*e))*ArcTanh[(Sqrt[c]*x^2)/(-
Sqrt[a] + Sqrt[a + b*x^2 + c*x^4])])/c^(3/2))/(48*e^4)
```

3.324.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1576, 1162, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx$$

$$\Big\downarrow \text{1576}$$

$$\frac{1}{2}\int \frac{(cx^4+bx^2+a)^{3/2}}{ex^2+d}dx^2$$

$$\Big\downarrow \text{1162}$$

$$\frac{1}{2}\left(\frac{(a+bx^2+cx^4)^{3/2}}{3e}-\frac{\int \frac{((2cd-be)x^2+bd-2ae)\sqrt{cx^4+bx^2+a}}{ex^2+d}dx^2}{2e}\right)$$

$$\Big\downarrow \text{1231}$$

$$\frac{1}{2}\left(\frac{\left(a+bx^2+cx^4\right)^{3/2}}{3e}-\frac{-\frac{\int \frac{4ce(bd-2ae)^2-(2cd-be)\left(8c^2d^2-b^2e^2-4ce(2bd-3ae)\right)x^2-d(2cd-be)\left(-eb^2+4cdb-4ace\right)}{2\left(ex^2+d\right)\sqrt{cx^4+bx^2+a}}dx^2}{4ce^2}-\frac{\sqrt{a+bx^2+cx^4}(-2c}{}}{2e}\right.$$

$\downarrow$ 27

$$\frac{1}{2}\left(\frac{\left(a+bx^2+cx^4\right)^{3/2}}{3e}-\frac{-\frac{\int \frac{4ce(bd-2ae)^2-(2cd-be)\left(8c^2d^2-b^2e^2-4ce(2bd-3ae)\right)x^2-d(2cd-be)\left(-eb^2+4cdb-4ace\right)}{\left(ex^2+d\right)\sqrt{cx^4+bx^2+a}}dx^2}{8ce^2}-\frac{\sqrt{a+bx^2+cx^4}(-2c}{}}{2e}\right.$$

$\downarrow$ 1269

$$\frac{1}{2}\left(\frac{\left(a+bx^2+cx^4\right)^{3/2}}{3e}-\frac{-\frac{\frac{16c\left(ae^2-bde+cd^2\right)^2\int \frac{1}{\left(ex^2+d\right)\sqrt{cx^4+bx^2+a}}dx^2}{e}-\frac{(2cd-be)\left(-4ce(2bd-3ae)-b^2e^2+8c^2d^2\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx^2}{e}}{8ce^2}-}{2e}\right.$$

$\downarrow$ 1092

$$\frac{1}{2}\left(\frac{\left(a+bx^2+cx^4\right)^{3/2}}{3e}-\frac{-\frac{\frac{16c\left(ae^2-bde+cd^2\right)^2\int \frac{1}{\left(ex^2+d\right)\sqrt{cx^4+bx^2+a}}dx^2}{e}-\frac{2(2cd-be)\left(-4ce(2bd-3ae)-b^2e^2+8c^2d^2\right)\int \frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}}{e}}{8ce^2}}{2e}\right.$$

$\downarrow$ 219

$$\frac{1}{2}\left(\frac{\left(a+bx^2+cx^4\right)^{3/2}}{3e}-\frac{-\frac{\frac{16c\left(ae^2-bde+cd^2\right)^2\int \frac{1}{\left(ex^2+d\right)\sqrt{cx^4+bx^2+a}}dx^2}{e}-\frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)\left(-4ce(2bd-3ae)-b^2e^2+8\right.}{\sqrt{ce}}}{8ce^2}}{2e}\right.$$

$\downarrow$ 1154

$$\frac{1}{2}\left(\frac{\left(a+bx^2+cx^4\right)^{3/2}}{3e}-\frac{-\frac{-\frac{32c\left(ae^2-bde+cd^2\right)^2\int \frac{1}{4\left(cd^2-bed+ae^2\right)-x^4}d\left(-\frac{(2cd-be)x^2+bd-2ae}{\sqrt{cx^4+bx^2+a}}\right)}{e}-\frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)\left(\right.}{\sqrt{ce}}}{8ce^2}}{2e}\right.$$

3.324. $\displaystyle\int \frac{x\left(a+bx^2+cx^4\right)^{3/2}}{d+ex^2}\,dx$

$$\downarrow 219$$

$$\frac{1}{2}\left(\frac{(a+bx^2+cx^4)^{3/2}}{3e}-\frac{-\frac{\frac{16c(ae^2-bde+cd^2)^{3/2}\text{arctanh}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{e}-\frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)(-4ce(}{\sqrt{ce}}}{8ce^2}}{2e}\right.$$

input
```
Int[(x*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]
```

output
```
((a + b*x^2 + c*x^4)^(3/2)/(3*e) - (-1/4*((8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*
b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(c*e^2) -
 (-(((2*c*d - b*e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(
b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(Sqrt[c]*e)) + (16*c*(c
*d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*S
qrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/e)/(8*c*e^2))/(2*e))
/2
```

3.324.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

3.324. $\int \frac{x(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx$

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

3.324. $\int \frac{x(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx$

3.324.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.34

method	result
pseudoelliptic	$-\frac{\left(e^{2}(ae-bd)^{2}c^{\frac{3}{2}}+2\left(e^{2}d^{2}a-d^{3}eb\right)c^{\frac{5}{2}}+c^{\frac{7}{2}}d^{4}\right)\ln\left(\frac{2\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}\,e+\left(b\,x^{2}+2a\right)e-d\left(2c\,x^{2}+b\right)}{e\,x^{2}+d}\right)-\frac{4e\left(\frac{9(be-2c\cdots}{\cdots}\right.}{\cdots}}{\cdots}$
risch	$\frac{\left(8e^{2}c^{2}x^{4}+14bc\,e^{2}x^{2}-12c^{2}de\,x^{2}+32e^{2}ac+3b^{2}e^{2}-30bcde+24c^{2}d^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}{48c\,e^{3}}+\frac{\left(12abc\,e^{3}-24a\,c^{2}d\,e^{2}-b^{3}e^{3}-6b^{2}cd\,e^{2}+2\cdots\right.}{\cdots}$
default	$-\frac{\left(a^{2}e^{4}-2abd\,e^{3}+2ac\,d^{2}e^{2}+b^{2}d^{2}e^{2}-2bc\,d^{3}e+c^{2}d^{4}\right)\ln\left(\frac{\frac{2a\,e^{2}-2bde+2c\,d^{2}}{e^{2}}+\frac{(be-2cd)\left(x^{2}+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}\,\sqrt{c\left(x^{2}+\frac{d}{e}\right)^{2}\cdots}}{x^{2}+\frac{d}{e}}\right.\cdots}{2e^{5}\sqrt{\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}}$
elliptic	$-\frac{\left(a^{2}e^{4}-2abd\,e^{3}+2ac\,d^{2}e^{2}+b^{2}d^{2}e^{2}-2bc\,d^{3}e+c^{2}d^{4}\right)\ln\left(\frac{\frac{2a\,e^{2}-2bde+2c\,d^{2}}{e^{2}}+\frac{(be-2cd)\left(x^{2}+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}\,\sqrt{c\left(x^{2}+\frac{d}{e}\right)^{2}\cdots}}{x^{2}+\frac{d}{e}}\right.\cdots}{2e^{5}\sqrt{\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}}$

input
```
int(x*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/2*((e^2*(a*e-b*d)^2*c^(3/2)+2*(a*d^2*e^2-b*d^3*e)*c^(5/2)+c^(7/2)*d^4)*
ln((2*(c*x^4+b*x^2+a)^(1/2)*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*e+(b*x^2+2*a)*
e-d*(2*c*x^2+b))/(e*x^2+d))-4/3*e*(9/16*(b*e-2*c*d)*((a*c-1/12*b^2)*e^2-2/
3*b*c*d*e+2/3*c^2*d^2)*ln((2*c*x^2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c^(1
/2))+e*(e*((7/16*b*x^2+a)*e-15/16*b*d)*c^(3/2)+1/4*(e^2*x^4-3/2*e*d*x^2+3*
d^2)*c^(5/2)+3/32*b^2*e^2*c^(1/2))*(c*x^4+b*x^2+a)^(1/2)-9/16*(b*e-2*c*d)*
((a*c-1/12*b^2)*e^2-2/3*b*c*d*e+2/3*c^2*d^2)*ln(2))*((a*e^2-b*d*e+c*d^2)/e
^2)^(1/2))/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)/c^(3/2)/e^5
```

3.324.5 Fricas [A] (verification not implemented)

Time = 111.85 (sec) , antiderivative size = 1589, normalized size of antiderivative = 5.91

$$\int \frac{x(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx = \text{Too large to display}$$

input
```
integrate(x*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="fricas")
```

output
```
[-1/192*(3*(16*c^3*d^3 - 24*b*c^2*d^2*e + 6*(b^2*c + 4*a*c^2)*d*e^2 + (b^3
 - 12*a*b*c)*e^3)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4
+ b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 48*(c^3*d^2 - b*c^2*d*e + a*
c^2*e^2)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 +
 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^
2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(
c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e
*x^2 + d^2)) - 4*(8*c^3*e^3*x^4 + 24*c^3*d^2*e - 30*b*c^2*d*e^2 + (3*b^2*c
 + 32*a*c^2)*e^3 - 2*(6*c^3*d*e^2 - 7*b*c^2*e^3)*x^2)*sqrt(c*x^4 + b*x^2 +
 a))/(c^2*e^4), 1/96*(3*(16*c^3*d^3 - 24*b*c^2*d^2*e + 6*(b^2*c + 4*a*c^2)
*d*e^2 + (b^3 - 12*a*b*c)*e^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)
*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 24*(c^3*d^2 - b*c^2*d
*e + a*c^2*e^2)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e +
 (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4
*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a
)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4
+ 2*d*e*x^2 + d^2)) + 2*(8*c^3*e^3*x^4 + 24*c^3*d^2*e - 30*b*c^2*d*e^2 + (
3*b^2*c + 32*a*c^2)*e^3 - 2*(6*c^3*d*e^2 - 7*b*c^2*e^3)*x^2)*sqrt(c*x^4 +
b*x^2 + a))/(c^2*e^4), 1/192*(96*(c^3*d^2 - b*c^2*d*e + a*c^2*e^2)*sqrt(-c
*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 +...
```

3.324.6 Sympy [F]

$$\int \frac{x(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx = \int \frac{x(a+bx^2+cx^4)^{\frac{3}{2}}}{d+ex^2}\,dx$$

input
```
integrate(x*(c*x**4+b*x**2+a)**(3/2)/(e*x**2+d),x)
```

output
```
Integral(x*(a + b*x**2 + c*x**4)**(3/2)/(d + e*x**2), x)
```

3.324.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-b*d*e>0)', see `assume?` f
or more de
```

3.324.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.324.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx = \int \frac{x\,\left(c\,x^4+b\,x^2+a\right)^{3/2}}{e\,x^2+d}\,dx$$

input
```
int((x*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x)
```

output
```
int((x*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2), x)
```

3.325 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x(d+ex^2)}\,dx$

3.325.1 Optimal result

Integrand size = 29, antiderivative size = 350

$$\begin{aligned}\int \frac{(a+bx^2+cx^4)^{3/2}}{x(d+ex^2)}\,dx &= \frac{a\sqrt{a+bx^2+cx^4}}{2d} \\ &- \frac{(4cd^2-e(5bd-4ae)-2cdex^2)\sqrt{a+bx^2+cx^4}}{8de^2} \\ &- \frac{a^{3/2}\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d} + \frac{ab\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}d} \\ &+ \frac{(8c^2d^3+be^2(3bd-4ae)-12cde(bd-ae))\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{c}de^3} \\ &- \frac{(cd^2-bde+ae^2)^{3/2}\,\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2de^3}\end{aligned}$$

output
```
-1/2*a^(3/2)*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d-1/2*
(a*e^2-b*d*e+c*d^2)^(3/2)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-
b*d*e+c*d^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d/e^3+1/4*a*b*arctanh(1/2*(2*c*x
^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d/c^(1/2)+1/16*(8*c^2*d^3+b*e^2*(-4*a
*e+3*b*d)-12*c*d*e*(-a*e+b*d))*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^
2+a)^(1/2))/d/e^3/c^(1/2)+1/2*a*(c*x^4+b*x^2+a)^(1/2)/d-1/8*(4*c*d^2-e*(-4
*a*e+5*b*d)-2*c*d*e*x^2)*(c*x^4+b*x^2+a)^(1/2)/d/e^2
```

3.325.2 Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x(d+ex^2)}\,dx = \frac{(-4cd+5be+2cex^2)\sqrt{a+bx^2+cx^4}}{8e^2} - \frac{\sqrt{-cd^2+e(bd-ae)}(cd^2+e(-bd+ae))\arctan\left(\frac{\sqrt{c}(d+ex^2)-e\sqrt{a+bx^2+cx^4}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{de^3} + \frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{d} - \frac{(8c^2d^2+3b^2e^2+12ce(-bd+ae))\log\left(b+2cx^2-2\sqrt{c}\sqrt{a+bx^2+cx^4}\right)}{16\sqrt{c}e^3}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^(3/2)/(x*(d + e*x^2)),x]
```

output
```
((-4*c*d + 5*b*e + 2*c*e*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*e^2) - (Sqrt[-(c
*d^2) + e*(b*d - a*e)]*(c*d^2 + e*(-(b*d) + a*e))*ArcTan[(Sqrt[c]*(d + e*x
^2) - e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(d*e^3)
+ (a^(3/2)*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/d - (
(8*c^2*d^2 + 3*b^2*e^2 + 12*c*e*(-(b*d) + a*e))*Log[b + 2*c*x^2 - 2*Sqrt[c
]*Sqrt[a + b*x^2 + c*x^4]])/(16*Sqrt[c]*e^3)
```

3.325.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.98, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {1578, 1270, 25, 1162, 25, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x(d+ex^2)}\,dx$$

$$\Big\downarrow \text{ 1578}$$

$$\frac{1}{2}\int \frac{(cx^4+bx^2+a)^{3/2}}{x^2(ex^2+d)}dx^2$$

$$\Big\downarrow \text{ 1270}$$

$$\frac{1}{2}\left(\frac{a\int\frac{\sqrt{cx^4+bx^2+a}}{x^2}dx^2}{d}-\frac{\int-\frac{(cdx^2+bd-ae)\sqrt{cx^4+bx^2+a}}{ex^2+d}dx^2}{d}\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\left(\frac{\int\frac{(cdx^2+bd-ae)\sqrt{cx^4+bx^2+a}}{ex^2+d}dx^2}{d}+\frac{a\int\frac{\sqrt{cx^4+bx^2+a}}{x^2}dx^2}{d}\right)$$

$$\Big\downarrow \; 1162$$

$$\frac{1}{2}\left(\frac{\int\frac{(cdx^2+bd-ae)\sqrt{cx^4+bx^2+a}}{ex^2+d}dx^2}{d}+\frac{a\left(\sqrt{a+bx^2+cx^4}-\frac{1}{2}\int-\frac{bx^2+2a}{x^2\sqrt{cx^4+bx^2+a}}dx^2\right)}{d}\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\left(\frac{\int\frac{(cdx^2+bd-ae)\sqrt{cx^4+bx^2+a}}{ex^2+d}dx^2}{d}+\frac{a\left(\frac{1}{2}\int\frac{bx^2+2a}{x^2\sqrt{cx^4+bx^2+a}}dx^2+\sqrt{a+bx^2+cx^4}\right)}{d}\right)$$

$$\Big\downarrow \; 1231$$

$$\frac{1}{2}\Bigg(\frac{-\frac{\int-\frac{c\left(4bcd^3-5b^2ed^2-4aced^2+12abe^2d-8a^2e^3+\left(8c^2d^3-12ce(bd-ae)d+be^2(3bd-4ae)\right)x^2\right)}{2\left(ex^2+d\right)\sqrt{cx^4+bx^2+a}}dx^2}{4ce^2}-\frac{\sqrt{a+bx^2+cx^4}\left(-e(5bd-4ae)+4cd^2-2cdex^2\right)}{4e^2}}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\Bigg(\frac{\frac{\int\frac{4bcd^3-5b^2ed^2-4aced^2+12abe^2d-8a^2e^3+\left(8c^2d^3-12ce(bd-ae)d+be^2(3bd-4ae)\right)x^2}{\left(ex^2+d\right)\sqrt{cx^4+bx^2+a}}dx^2}{8e^2}-\frac{\sqrt{a+bx^2+cx^4}\left(-e(5bd-4ae)+4cd^2-2cdex^2\right)}{4e^2}}{d}+\frac{a\Big(\frac{1}{2}}{}$$

$$\Big\downarrow \; 1269$$

$$\frac{1}{2}\Bigg(\frac{\frac{\frac{\left(-12cde(bd-ae)+be^2(3bd-4ae)+8c^2d^3\right)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx^2}{e}-\frac{8\left(ae^2-bde+cd^2\right)^2\int\frac{1}{\left(ex^2+d\right)\sqrt{cx^4+bx^2+a}}dx^2}{e}}{8e^2}-\frac{\sqrt{a+bx^2+cx^4}\left(-e(5bd-4ae)+4cd^2\right.}{4e^2}}{d}$$

$$\Big\downarrow \; 1092$$

3.325. $\quad \displaystyle\int\frac{\left(a+bx^2+cx^4\right)^{3/2}}{x\left(d+ex^2\right)}\,dx$

$$\frac{1}{2}\left(\frac{\frac{\frac{2\left(-12cde(bd-ae)+be^2(3bd-4ae)+8c^2d^3\right)\int \frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}}{e}-\frac{8\left(ae^2-bde+cd^2\right)^2\int \frac{1}{\left(ex^2+d\right)\sqrt{cx^4+bx^2+a}}dx^2}{e}}{8e^2}-\frac{\sqrt{a+bx^2+cx^4}(-e(5bd-4ae)}{4e^2}}{d}\right.$$

$\Big\downarrow$ 219

$$\frac{1}{2}\left(\frac{\frac{\frac{\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)\left(-12cde(bd-ae)+be^2(3bd-4ae)+8c^2d^3\right)}{\sqrt{c}e}-\frac{8\left(ae^2-bde+cd^2\right)^2\int \frac{1}{\left(ex^2+d\right)\sqrt{cx^4+bx^2+a}}dx^2}{e}}{8e^2}-\frac{\sqrt{a+bx^2+cx^4}(-e(5bd}{4e}}{d}\right.$$

$\Big\downarrow$ 1154

$$\frac{1}{2}\left(\frac{\frac{\frac{16\left(ae^2-bde+cd^2\right)^2\int \frac{1}{4\left(cd^2-bed+ae^2\right)-x^4}d\left(-\frac{(2cd-be)x^2+bd-2ae}{\sqrt{cx^4+bx^2+a}}\right)}{e}+\frac{\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)\left(-12cde(bd-ae)+be^2(3bd-4ae)+8c^2d^3\right)}{\sqrt{c}e}}{8e^2}-}{d}\right.$$

$\Big\downarrow$ 219

$$\frac{1}{2}\left(\frac{\frac{\frac{\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)\left(-12cde(bd-ae)+be^2(3bd-4ae)+8c^2d^3\right)}{\sqrt{c}e}-\frac{8\left(ae^2-bde+cd^2\right)^{3/2}\text{arctanh}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{e}}{8e^2}-\frac{\sqrt{a+b}}{}}{d}\right.$$

input
```
Int[(a + b*x^2 + c*x^4)^(3/2)/(x*(d + e*x^2)),x]
```

output
```
((a*(Sqrt[a + b*x^2 + c*x^4] + (-2*Sqrt[a]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a
]*Sqrt[a + b*x^2 + c*x^4])] + (b*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a +
 b*x^2 + c*x^4])])/Sqrt[c])/2))/d + (-1/4*((4*c*d^2 - e*(5*b*d - 4*a*e) -
2*c*d*e*x^2)*Sqrt[a + b*x^2 + c*x^4])/e^2 + (((8*c^2*d^3 + b*e^2*(3*b*d -
4*a*e) - 12*c*d*e*(b*d - a*e))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b
*x^2 + c*x^4])])/(Sqrt[c]*e) - (8*(c*d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh[(b
*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*
x^2 + c*x^4])])/e)/(8*e^2))/d)/2
```

3.325. $\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x(d+ex^2)}\,dx$

3.325.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1270
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) +
 (g_.)*(x_))), x_Symbol] :> Simp[(c*d^2 - b*d*e + a*e^2)/(e*(e*f - d*g))
Int[(a + b*x + c*x^2)^(p - 1)/(d + e*x), x], x] - Simp[1/(e*(e*f - d*g))
Int[Simp[c*d*f - b*e*f + a*e*g - c*(e*f - d*g)*x, x]*((a + b*x + c*x^2)^(p
- 1)/(f + g*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && FractionQ[p]
&& GtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.325. $\int \frac{(a+bx^2+cx^4)^{3/2}}{x(d+ex^2)}\,dx$

3.325.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$\frac{\left(e^{2}(ae-bd)^{2}\sqrt{c}+2\left(e^{2}d^{2}a-d^{3}eb\right)c^{\frac{3}{2}}+c^{\frac{5}{2}}d^{4}\right)\ln\left(\frac{2\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}\,e+\left(b\,x^{2}+2a\right)e-d\left(2c\,x^{2}+b\right)}{e\,x^{2}+d}\right)+\frac{3e\left(d\left(\left(ac+\frac{b^{2}}{4}\right.\right.\right.}{}}{}$
elliptic	$-\frac{a^{\frac{3}{2}}\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{2d}+\frac{\frac{b^{2}e^{2}\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{\sqrt{c}}+c^{\frac{3}{2}}d^{2}\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)+e^{2}c^{2}\left(x^{2}\right.}{}$
default	Expression too large to display

input
```
int((c*x^4+b*x^2+a)^(3/2)/x/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)/c^(1/2)*((e^2*(a*e-b*d)^2*c^(1/2)+2*(a
*d^2*e^2-b*d^3*e)*c^(3/2)+c^(5/2)*d^4)*ln((2*(c*x^4+b*x^2+a)^(1/2)*((a*e^2
-b*d*e+c*d^2)/e^2)^(1/2)*e+(b*x^2+2*a)*e-d*(2*c*x^2+b))/(e*x^2+d))+3/2*e*(
d*((a*c+1/4*b^2)*e^2-b*c*d*e+2/3*c^2*d^2)*ln((2*c*x^2+2*(c*x^4+b*x^2+a)^(1
/2)*c^(1/2)+b)/c^(1/2))-2/3*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))
/x^2)*a^(3/2)*e^3*c^(1/2)-(-5/6*e*(2/5*(e*x^2-2*d)*c^(3/2)+b*c^(1/2)*e)*(c
*x^4+b*x^2+a)^(1/2)+ln(2)*((a*c+1/4*b^2)*e^2-b*c*d*e+2/3*c^2*d^2))*d)*((a*
e^2-b*d*e+c*d^2)/e^2)^(1/2))/e^4/d
```

3.325.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x\,(d+ex^2)}\,dx=\text{Timed out}$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/x/(e*x^2+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.325.6 Sympy [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x(d+ex^2)}\,dx = \int \frac{(a+bx^2+cx^4)^{\frac{3}{2}}}{x(d+ex^2)}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**(3/2)/x/(e*x**2+d),x)
```

output
```
Integral((a + b*x**2 + c*x**4)**(3/2)/(x*(d + e*x**2)), x)
```

3.325.7 Maxima [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x(d+ex^2)}\,dx = \int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}}{(ex^2+d)x}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/x/(e*x^2+d),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)/((e*x^2 + d)*x), x)
```

3.325.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x(d+ex^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/x/(e*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.325.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x\left(d+ex^2\right)}\,dx=\int \frac{\left(c\,x^4+b\,x^2+a\right)^{3/2}}{x\,\left(e\,x^2+d\right)}\,dx$$

input
```
int((a + b*x^2 + c*x^4)^(3/2)/(x*(d + e*x^2)),x)
```

output
```
int((a + b*x^2 + c*x^4)^(3/2)/(x*(d + e*x^2)), x)
```

3.326 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x^3(d+ex^2)}\,dx$

3.326.1 Optimal result

Integrand size = 29, antiderivative size = 562

$$\begin{aligned}\int \frac{(a+bx^2+cx^4)^{3/2}}{x^3(d+ex^2)}\,dx =& \frac{3(3b+2cx^2)\sqrt{a+bx^2+cx^4}}{8d} \\ &- \frac{e(b^2+8ac+2bcx^2)\sqrt{a+bx^2+cx^4}}{16cd^2} \\ &+ \frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x^2)\sqrt{a+bx^2+cx^4}}{16cd^2e} \\ &- \frac{(a+bx^2+cx^4)^{3/2}}{2dx^2} - \frac{3\sqrt{a}b\,\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4d} \\ &+ \frac{a^{3/2}e\,\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d^2} + \frac{3(b^2+4ac)\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{c}d} \\ &+ \frac{b(b^2-12ac)\,e\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{3/2}d^2} \\ &- \frac{(2cd-be)(8c^2d^2-b^2e^2-4ce(2bd-3ae))\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{3/2}d^2e^2} \\ &+ \frac{(cd^2-bde+ae^2)^{3/2}\,\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2d^2e^2}\end{aligned}$$

output
```
-1/2*(c*x^4+b*x^2+a)^(3/2)/d/x^2+1/2*a^(3/2)*e*arctanh(1/2*(b*x^2+2*a)/a^(
1/2)/(c*x^4+b*x^2+a)^(1/2))/d^2+1/32*b*(-12*a*c+b^2)*e*arctanh(1/2*(2*c*x^
2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(3/2)/d^2-1/32*(-b*e+2*c*d)*(8*c^2*d
^2-b^2*e^2-4*c*e*(-3*a*e+2*b*d))*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*
x^2+a)^(1/2))/c^(3/2)/d^2/e^2+1/2*(a*e^2-b*d*e+c*d^2)^(3/2)*arctanh(1/2*(b
*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
)/d^2/e^2-3/4*b*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))*a^(
1/2)/d+3/16*(4*a*c+b^2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1
/2))/d/c^(1/2)+3/8*(2*c*x^2+3*b)*(c*x^4+b*x^2+a)^(1/2)/d-1/16*e*(2*b*c*x^2
+8*a*c+b^2)*(c*x^4+b*x^2+a)^(1/2)/c/d^2+1/16*(8*c^2*d^2+b^2*e^2-2*c*e*(-4*
a*e+5*b*d)-2*c*e*(-b*e+2*c*d)*x^2)*(c*x^4+b*x^2+a)^(1/2)/c/d^2/e
```

3.326.2 Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.41

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^3(d+ex^2)}\,dx = \frac{-\frac{4\left(-cd^2+e(bd-ae)\right)^{3/2}\arctan\left(\frac{\sqrt{c}\left(d+ex^2\right)-e\sqrt{a+bx^2+cx^4}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{e^2}+2\sqrt{a}(-3bd+2ae)\text{arctanh}\left(\frac{-\sqrt{c}}{}\right.}{4d}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^(3/2)/(x^3*(d + e*x^2)),x]
```

output
```
((-4*(-(c*d^2) + e*(b*d - a*e))^(3/2)*ArcTan[(Sqrt[c]*(d + e*x^2) - e*Sqrt
[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/e^2 + 2*Sqrt[a]*(-3*
b*d + 2*a*e)*ArcTanh[(-(Sqrt[c]*x^2) + Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]] +
 (d*(2*e*(-(a*e) + c*d*x^2)*Sqrt[a + b*x^2 + c*x^4] + Sqrt[c]*d*(2*c*d - 3
*b*e)*x^2*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]]))/(e^2*x^2)
)/(4*d^2)
```

3.326.3 Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 558, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1578, 1289, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^3\left(d+ex^2\right)}\,dx$$

$$\Big\downarrow \; 1578$$

$$\frac{1}{2}\int \frac{\left(cx^4+bx^2+a\right)^{3/2}}{x^4\left(ex^2+d\right)}dx^2$$

$$\Big\downarrow \; 1289$$

$$\frac{1}{2}\int \left(\frac{\left(cx^4+bx^2+a\right)^{3/2}e^2}{d^2\left(ex^2+d\right)}-\frac{\left(cx^4+bx^2+a\right)^{3/2}e}{d^2x^2}+\frac{\left(cx^4+bx^2+a\right)^{3/2}}{dx^4}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{a^{3/2}e\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{d^2}+\frac{be\left(b^2-12ac\right)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}d^2}-\frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c\cdots}\right.$$

input
```
Int[(a + b*x^2 + c*x^4)^(3/2)/(x^3*(d + e*x^2)),x]
```

output
```
((3*(3*b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(4*d) - (e*(b^2 + 8*a*c + 2*b
*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c*d^2) + ((8*c^2*d^2 + b^2*e^2 - 2*c*e
*(5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c*
d^2*e) - (a + b*x^2 + c*x^4)^(3/2)/(d*x^2) - (3*Sqrt[a]*b*ArcTanh[(2*a + b
*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*d) + (a^(3/2)*e*ArcTanh[(2*
a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/d^2 + (3*(b^2 + 4*a*c)*Ar
cTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(8*Sqrt[c]*d) +
(b*(b^2 - 12*a*c)*e*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^
4])])/(16*c^(3/2)*d^2) - ((2*c*d - b*e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*
d - 3*a*e))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(1
6*c^(3/2)*d^2*e^2) + ((c*d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e +
 (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4]
)])/(d^2*e^2))/2
```

3.326. $\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^3\left(d+ex^2\right)}\,dx$

3.326.3.1 Defintions of rubi rules used

rule 1289
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0]))
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.326.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.61

method	result
pseudoelliptic	$-\frac{\left(e^2(ae-bd)^2\sqrt{c}+(2e^2d^2a-2d^3eb)c^{\frac{3}{2}}+c^{\frac{5}{2}}d^4\right)x^2\ln\left(\frac{2\sqrt{cx^4+bx^2+a}\sqrt{\frac{ae^2-bde+cd^2}{e^2}}\,e+\left(bx^2+2a\right)e-d\left(2cx^2+b\right)}{ex^2+d}\right)+e\left(-\frac{3(}{}\right.}{}$
risch	$-\frac{a\sqrt{cx^4+bx^2+a}}{2d\,x^2}+\frac{\frac{\sqrt{a}\,(2ae-3bd)\ln\left(\frac{2a+bx^2+2\sqrt{a}\sqrt{cx^4+bx^2+a}}{x^2}\right)}{2d}+\frac{2cd\left(ec\left(\frac{\sqrt{cx^4+bx^2+a}}{2c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx^2}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)}{4c^{\frac{3}{2}}}\right.\right.}{e^2}}{}$
elliptic	$\frac{a^2\left(-\frac{\sqrt{cx^4+bx^2+a}}{a\,x^2}+\frac{b\ln\left(\frac{2a+bx^2+2\sqrt{a}\sqrt{cx^4+bx^2+a}}{x^2}\right)}{2a^{\frac{3}{2}}}\right)}{2d}+\frac{\sqrt{a}\,(ae-2bd)\ln\left(\frac{2a+bx^2+2\sqrt{a}\sqrt{cx^4+bx^2+a}}{x^2}\right)}{2d^2}+\frac{c\left(\frac{2be\ln(}{}\right.}{}$
default	Expression too large to display

input
```
int((c*x^4+b*x^2+a)^(3/2)/x^3/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/2*((e^2*(a*e-b*d)^2*c^(1/2)+(2*a*d^2*e^2-2*b*d^3*e)*c^(3/2)+c^(5/2)*d^4
)*x^2*ln((2*(c*x^4+b*x^2+a)^(1/2)*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*e+(b*x^2
+2*a)*e-d*(2*c*x^2+b))/(e*x^2+d))+e*(-3/2*(b*e-2/3*c*d)*c*d^2*x^2*ln((2*c*
x^2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c^(1/2))+3/2*e^2*(b*a^(1/2)*d-2/3*a
^(3/2)*e)*c^(1/2)*x^2*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)+
(e*(-c^(3/2)*d*x^2+a*c^(1/2)*e)*(c*x^4+b*x^2+a)^(1/2)+3/2*(b*e-2/3*c*d)*ln
(2)*c*d*x^2)*d)*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/((a*e^2-b*d*e+c*d^2)/e^2)
^(1/2)/c^(1/2)/e^3/x^2/d^2
```

3.326.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^3(d+ex^2)}\,dx = \text{Timed out}$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/x^3/(e*x^2+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.326.6 Sympy [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^3(d+ex^2)}\,dx = \int \frac{(a+bx^2+cx^4)^{\frac{3}{2}}}{x^3(d+ex^2)}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**(3/2)/x**3/(e*x**2+d),x)
```

output
```
Integral((a + b*x**2 + c*x**4)**(3/2)/(x**3*(d + e*x**2)), x)
```

3.326.7 Maxima [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^3(d+ex^2)}\,dx = \int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}}{(ex^2+d)x^3}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/x^3/(e*x^2+d),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)/((e*x^2 + d)*x^3), x)
```

3.326.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^3(d+ex^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/x^3/(e*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.326.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^3(d+ex^2)}\,dx = \int \frac{(c\,x^4+b\,x^2+a)^{3/2}}{x^3\,(e\,x^2+d)}\,dx$$

input
```
int((a + b*x^2 + c*x^4)^(3/2)/(x^3*(d + e*x^2)),x)
```

output
```
int((a + b*x^2 + c*x^4)^(3/2)/(x^3*(d + e*x^2)), x)
```

3.327 $\int \frac{x^2(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx$

3.327.1 Optimal result

Integrand size = 29, antiderivative size = 463

$$\begin{aligned}
\int \frac{x^2(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx &= -\frac{213}{140}x\sqrt{1+2x^2+2x^4}-\frac{27}{70}x^3\sqrt{1+2x^2+2x^4} \\
&- \frac{2211x\sqrt{1+2x^2+2x^4}}{140\sqrt{2}\left(1+\sqrt{2}x^2\right)}-\frac{1}{14}x\left(1+2x^2+2x^4\right)^{3/2}+\frac{17}{16}\sqrt{51}\operatorname{arctanh}\left(\frac{\sqrt{\frac{17}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) \\
&+ \frac{2211\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{140\ 2^{3/4}\sqrt{1+2x^2+2x^4}} \\
&- \frac{3\left(514+2717\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{140\ 2^{3/4}\left(2+3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}} \\
&- \frac{289\left(3-\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{1}{24}\left(12+11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{16\ 2^{3/4}\left(2+3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}
\end{aligned}$$

output
```
-1/14*x*(2*x^4+2*x^2+1)^(3/2)+17/16*arctanh(1/3*x*51^(1/2)/(2*x^4+2*x^2+1)
^(1/2))*51^(1/2)-213/140*x*(2*x^4+2*x^2+1)^(1/2)-27/70*x^3*(2*x^4+2*x^2+1)
^(1/2)-2211/280*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))+2211/280*(
cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticE(sin(2
*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1
)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)-289/32*(cos(2*arc
tan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(
2^(1/4)*x)),1/2+11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3-2^(1/2))*(1+x^2*2^
(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2+3*2^(1/2))/(2*
x^4+2*x^2+1)^(1/2)-3/280*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2
^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(514+
2717*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^
(1/4)/(2+3*2^(1/2))/(2*x^4+2*x^2+1)^(1/2)
```

3.327.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.96 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.46

$$\int \frac{x^2(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx = \frac{-892x-2080x^3-2456x^5-752x^7-160x^9+4422i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{}}{}$$

input
```
Integrate[(x^2*(1 + 2*x^2 + 2*x^4)^(3/2))/(3 - 2*x^2),x]
```

output
```
(-892*x - 2080*x^3 - 2456*x^5 - 752*x^7 - 160*x^9 + (4422*I)*Sqrt[1 - I]*S
qrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]
*x], I] - (9669 - 5247*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 +
I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] + 10115*(1 - I)^(3/2)*Sqrt[
1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[-1/3 - I/3, I*ArcSinh[Sq
rt[1 - I]*x], I])/(560*Sqrt[1 + 2*x^2 + 2*x^4])
```

3.327.3 Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 454, normalized size of antiderivative = 0.98, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.448$, Rules used = {1630, 25, 27, 2207, 27, 2207, 27, 2207, 27, 1511, 1416, 1509, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(2x^4+2x^2+1\right)^{3/2}}{3-2x^2}dx$$

$$\Big\downarrow \text{1630}$$

$$\frac{867}{56}\int -\frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{\left(3-2x^2\right)\sqrt{2x^4+2x^2+1}}dx-\frac{1}{112}\int -\frac{2\left(-112x^8-392x^6-812x^4-1330x^2+289\left(2-3\sqrt{2}\right)\right)}{\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \text{25}$$

$$-\frac{867}{56}\int \frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{\left(3-2x^2\right)\sqrt{2x^4+2x^2+1}}dx-\frac{1}{112}\int -\frac{2\left(-112x^8-392x^6-812x^4-1330x^2+289\left(2-3\sqrt{2}\right)\right)}{\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{56}\int \frac{-112x^8-392x^6-812x^4-1330x^2+289\left(2-3\sqrt{2}\right)}{\sqrt{2x^4+2x^2+1}}dx-\frac{867}{56}\int \frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{\left(3-2x^2\right)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{56}\left(\frac{1}{14}\int \frac{14\left(-296x^6-772x^4-1330x^2+289\left(2-3\sqrt{2}\right)\right)}{\sqrt{2x^4+2x^2+1}}dx-8x^5\sqrt{2x^4+2x^2+1}\right)-\frac{867}{56}\int \frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{\left(3-2x^2\right)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{56}\left(\int \frac{-296x^6-772x^4-1330x^2+289\left(2-3\sqrt{2}\right)}{\sqrt{2x^4+2x^2+1}}dx-8x^5\sqrt{2x^4+2x^2+1}\right)-\frac{867}{56}\int \frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{\left(3-2x^2\right)\sqrt{2x^4+2x^2+1}}dx$$

$$\downarrow 2207$$

$$\frac{1}{56}\left(\frac{1}{10}\int \frac{2\left(-2676x^4-6206x^2+1445\left(2-3\sqrt{2}\right)\right)}{\sqrt{2x^4+2x^2+1}}dx-8\sqrt{2x^4+2x^2+1}x^5-\frac{148}{5}\sqrt{2x^4+2x^2+1}x^3\right)-\frac{867}{56}\int \frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{\left(3-2x^2\right)\sqrt{2x^4+2x^2+1}}dx$$

$$\downarrow 27$$

$$\frac{1}{56}\left(\frac{1}{5}\int \frac{-2676x^4-6206x^2+1445\left(2-3\sqrt{2}\right)}{\sqrt{2x^4+2x^2+1}}dx-8\sqrt{2x^4+2x^2+1}x^5-\frac{148}{5}\sqrt{2x^4+2x^2+1}x^3\right)-\frac{867}{56}\int \frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{\left(3-2x^2\right)\sqrt{2x^4+2x^2+1}}dx$$

$$\downarrow 2207$$

$$\frac{1}{56}\left(\frac{1}{5}\left(\frac{1}{6}\int \frac{18\left(-1474x^2-1445\sqrt{2}+1112\right)}{\sqrt{2x^4+2x^2+1}}dx-446x\sqrt{2x^4+2x^2+1}\right)-8\sqrt{2x^4+2x^2+1}x^5-\frac{148}{5}\sqrt{2x^4+2}\right.$$
$$\frac{867}{56}\int \frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{\left(3-2x^2\right)\sqrt{2x^4+2x^2+1}}dx$$

$$\downarrow 27$$

$$\frac{1}{56}\left(\frac{1}{5}\left(3\int \frac{-1474x^2-1445\sqrt{2}+1112}{\sqrt{2x^4+2x^2+1}}dx-446x\sqrt{2x^4+2x^2+1}\right)-8\sqrt{2x^4+2x^2+1}x^5-\frac{148}{5}\sqrt{2x^4+2x^2+}\right.$$
$$\frac{867}{56}\int \frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{\left(3-2x^2\right)\sqrt{2x^4+2x^2+1}}dx$$

$$\downarrow 1511$$

$$\frac{1}{56}\left(\frac{1}{5}\left(3\left(2\left(556-1091\sqrt{2}\right)\int \frac{1}{\sqrt{2x^4+2x^2+1}}dx+737\sqrt{2}\int \frac{1-\sqrt{2}x^2}{\sqrt{2x^4+2x^2+1}}dx\right)-446x\sqrt{2x^4+2x^2+1}\right)-\right.$$
$$\frac{867}{56}\int \frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{\left(3-2x^2\right)\sqrt{2x^4+2x^2+1}}dx$$

$$\downarrow 1416$$

3.327. $\int \frac{x^2\left(1+2x^2+2x^4\right)^{3/2}}{3-2x^2}dx$

$$\frac{1}{56}\left(\frac{1}{5}\left(3\left(737\sqrt{2}\int\frac{1-\sqrt{2}x^2}{\sqrt{2x^4+2x^2+1}}dx+\frac{(556-1091\sqrt{2})\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\right.}{\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}\right.\right.\right.$$

$$\frac{867}{56}\int\frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{\left(3-2x^2\right)\sqrt{2x^4+2x^2+1}}dx$$

$$\downarrow 1509$$

$$\frac{1}{56}\left(\frac{1}{5}\left(3\left(\frac{(556-1091\sqrt{2})\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}+737\sqrt{2}\left(\frac{\left(\sqrt{2}x^2\right.}{}\right.\right.\right.\right.$$

$$\frac{867}{56}\int\frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{\left(3-2x^2\right)\sqrt{2x^4+2x^2+1}}dx$$

$$\downarrow 2222$$

$$\frac{1}{56}\left(\frac{1}{5}\left(3\left(\frac{(556-1091\sqrt{2})\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}+737\sqrt{2}\left(\frac{\left(\sqrt{2}x^2\right.}{}\right.\right.\right.\right.$$

$$\frac{867}{56}\left(\frac{\left(3-\sqrt{2}\right)^2\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticPi}\left(\frac{1}{24}\left(12+11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{12\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-\frac{7\text{arctanh}\left(\right.}{2}\right.$$

input
```
Int[(x^2*(1 + 2*x^2 + 2*x^4)^(3/2))/(3 - 2*x^2),x]
```

output
```
((-148*x^3*Sqrt[1 + 2*x^2 + 2*x^4])/5 - 8*x^5*Sqrt[1 + 2*x^2 + 2*x^4] + (-
446*x*Sqrt[1 + 2*x^2 + 2*x^4] + 3*(737*Sqrt[2]*(-((x*Sqrt[1 + 2*x^2 + 2*x^
4])/(1 + Sqrt[2]*x^2)) + ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 +
Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(2^(1/4)*
Sqrt[1 + 2*x^2 + 2*x^4])) + ((556 - 1091*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(
1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2
- Sqrt[2])/4])/(2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])))/5)/56 - (867*((-7*ArcTa
nh[(Sqrt[17/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/(2*Sqrt[51]) + ((3 - Sqrt[2])^
2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Elliptic
Pi[(12 + 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(12*2^(1/4
)*Sqrt[1 + 2*x^2 + 2*x^4])))/56
```

3.327. $\int \frac{x^2\left(1+2x^2+2x^4\right)^{3/2}}{3-2x^2}\,dx$

3.327.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1630
```
Int[((x_)^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[(-(-d/e)^(m/2))*((c*d^2 - b*d*e + a*e^2)^(p + 1/2)
/(e^(2*p)*(c*d^2 - a*e^2)))   Int[(a*d*Rt[c/a, 2] + a*e + (c*d + a*e*Rt[c/a
, 2])*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] + Simp[1/(e^(2*p)*
(c*d^2 - a*e^2))   Int[(1/Sqrt[a + b*x^2 + c*x^4])*ExpandToSum[(e^(2*p)*(c*
d^2 - a*e^2)*x^m*(a + b*x^2 + c*x^4)^(p + 1/2) + (-d/e)^(m/2)*(c*d^2 - b*d*
e + a*e^2)^(p + 1/2)*(a*d*Rt[c/a, 2] + a*e + (c*d + a*e*Rt[c/a, 2])*x^2))/(
d + e*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& IGtQ[p + 1/2, 0] && IGtQ[m/2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

3.327. $\int \frac{x^2(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx$

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

3.327.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.84 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.54

method	result
risch	$-\frac{x\left(20x^4+74x^2+223\right)\sqrt{2x^4+2x^2+1}}{140}-\frac{9669\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{280\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(\frac{2211}{280}-\frac{2211i}{280}\right)\sqrt{1+(1-i)x^2}\,\sqrt{}}{}$
elliptic	$-\frac{x^5\sqrt{2x^4+2x^2+1}}{7}-\frac{37x^3\sqrt{2x^4+2x^2+1}}{70}-\frac{223x\sqrt{2x^4+2x^2+1}}{140}-\frac{3729\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{140\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}-$
default	$-\frac{x^5\sqrt{2x^4+2x^2+1}}{7}-\frac{37x^3\sqrt{2x^4+2x^2+1}}{70}-\frac{223x\sqrt{2x^4+2x^2+1}}{140}-\frac{9\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{35\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(\frac{6}{35}\right.}{}$

input
```
int(x^2*(2*x^4+2*x^2+1)^(3/2)/(-2*x^2+3),x,method=_RETURNVERBOSE)
```

output
```
-1/140*x*(20*x^4+74*x^2+223)*(2*x^4+2*x^2+1)^(1/2)-9669/280/(-1+I)^(1/2)*(
1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(
-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+(2211/280-2211/280*I)/(-1+I)^(1/2)*
(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x
*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1
/2)+1/2*I*2^(1/2)))+289/8/(-1+I)^(1/2)*(1-I*x^2+x^2)^(1/2)*(1+I*x^2+x^2)^(
1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),-1/3-1/3*I,(-1-I)^(1/
2)/(-1+I)^(1/2))
```

3.327.5 Fricas [F]

$$\int \frac{x^2(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx=\int -\frac{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}x^2}{2\,x^2-3}\,dx$$

input
```
integrate(x^2*(2*x^4+2*x^2+1)^(3/2)/(-2*x^2+3),x, algorithm="fricas")
```

output
```
integral(-(2*x^6 + 2*x^4 + x^2)*sqrt(2*x^4 + 2*x^2 + 1)/(2*x^2 - 3), x)
```

3.327.6 Sympy [F]

$$\int \frac{x^2(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx=-\int \frac{x^2\sqrt{2x^4+2x^2+1}}{2x^2-3}\,dx-\int \frac{2x^4\sqrt{2x^4+2x^2+1}}{2x^2-3}\,dx-\int \frac{2x^6\sqrt{2x^4+2x^2+1}}{2x^2-3}\,dx$$

input
```
integrate(x**2*(2*x**4+2*x**2+1)**(3/2)/(-2*x**2+3),x)
```

output
```
-Integral(x**2*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**2 - 3), x) - Integral(2*x**
4*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**2 - 3), x) - Integral(2*x**6*sqrt(2*x**4
 + 2*x**2 + 1)/(2*x**2 - 3), x)
```

3.327.7 Maxima [F]

$$\int \frac{x^2(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx = \int -\frac{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}x^2}{2\,x^2-3}\,dx$$

input
```
integrate(x^2*(2*x^4+2*x^2+1)^(3/2)/(-2*x^2+3),x, algorithm="maxima")
```

output
```
-integrate((2*x^4 + 2*x^2 + 1)^(3/2)*x^2/(2*x^2 - 3), x)
```

3.327.8 Giac [F]

$$\int \frac{x^2(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx = \int -\frac{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}x^2}{2\,x^2-3}\,dx$$

input
```
integrate(x^2*(2*x^4+2*x^2+1)^(3/2)/(-2*x^2+3),x, algorithm="giac")
```

output
```
integrate(-(2*x^4 + 2*x^2 + 1)^(3/2)*x^2/(2*x^2 - 3), x)
```

3.327.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx = -\int \frac{x^2\,(2\,x^4+2\,x^2+1)^{3/2}}{2\,x^2-3}\,dx$$

input
```
int(-(x^2*(2*x^2 + 2*x^4 + 1)^(3/2))/(2*x^2 - 3),x)
```

output
```
-int((x^2*(2*x^2 + 2*x^4 + 1)^(3/2))/(2*x^2 - 3), x)
```

3.328 $\int \frac{(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx$

3.328.1 Optimal result

Integrand size = 26, antiderivative size = 428

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx = -\frac{1}{10}x(9+2x^2)\sqrt{1+2x^2+2x^4} - \frac{103x\sqrt{1+2x^2+2x^4}}{10\sqrt{2}\,(1+\sqrt{2}x^2)} + \frac{17}{8}\sqrt{\frac{17}{3}}\operatorname{arctanh}\left(\frac{\sqrt{\frac{17}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) + \frac{103(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{10\,2^{3/4}\sqrt{1+2x^2+2x^4}} - \frac{(66+383\sqrt{2})(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{10\,2^{3/4}(2+3\sqrt{2})\sqrt{1+2x^2+2x^4}} - \frac{289(3-\sqrt{2})(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\operatorname{EllipticPi}\left(\frac{1}{24}(12+11\sqrt{2}),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{24\,2^{3/4}(2+3\sqrt{2})\sqrt{1+2x^2+2x^4}}$$

output
```
17/24*arctanh(1/3*x*51^(1/2)/(2*x^4+2*x^2+1)^(1/2))*51^(1/2)-1/10*x*(2*x^2
+9)*(2*x^4+2*x^2+1)^(1/2)-103/20*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^
(1/2))+103/20*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*
EllipticE(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*
((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)-28
9/48*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticP
i(sin(2*arctan(2^(1/4)*x)),1/2+11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3-2^(
1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2
+3*2^(1/2))/(2*x^4+2*x^2+1)^(1/2)-1/20*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/
cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2)
)^(1/2))*(66+383*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))
^2)^(1/2)*2^(1/4)/(2+3*2^(1/2))/(2*x^4+2*x^2+1)^(1/2)
```

3.328.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.20 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.49

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx = \frac{-108x-240x^3-264x^5-48x^7+618i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}E(}{}$$

input
```
Integrate[(1 + 2*x^2 + 2*x^4)^(3/2)/(3 - 2*x^2),x]
```

output
```
(-108*x - 240*x^3 - 264*x^5 - 48*x^7 + (618*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I
)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] - (137
1 - 753*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*Ellipti
cF[I*ArcSinh[Sqrt[1 - I]*x], I] + 1445*(1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]
*Sqrt[1 + (1 + I)*x^2]*EllipticPi[-1/3 - I/3, I*ArcSinh[Sqrt[1 - I]*x], I]
)/(120*Sqrt[1 + 2*x^2 + 2*x^4])
```

3.328.3 Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {1530, 25, 27, 2207, 27, 2207, 27, 1511, 1416, 1509, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.328. $\int \frac{(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx$

$$\int \frac{\left(2x^4+2x^2+1\right)^{3/2}}{3-2x^2}dx$$

$$\Big\downarrow \ 1530$$

$$\frac{289}{28}\int -\frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{\left(3-2x^2\right)\sqrt{2x^4+2x^2+1}}dx-\frac{1}{112}\int -\frac{4\left(-56x^6-196x^4-406x^2-289\sqrt{2}+202\right)}{\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \ 25$$

$$-\frac{289}{28}\int \frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{\left(3-2x^2\right)\sqrt{2x^4+2x^2+1}}dx-\frac{1}{112}\int -\frac{4\left(-56x^6-196x^4-406x^2-289\sqrt{2}+202\right)}{\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \ 27$$

$$\frac{1}{28}\int \frac{-56x^6-196x^4-406x^2-289\sqrt{2}+202}{\sqrt{2x^4+2x^2+1}}dx-\frac{289}{28}\int \frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{\left(3-2x^2\right)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \ 2207$$

$$\frac{1}{28}\left(\frac{1}{10}\int \frac{2\left(-756x^4-1946x^2+5\left(202-289\sqrt{2}\right)\right)}{\sqrt{2x^4+2x^2+1}}dx-\frac{28}{5}x^3\sqrt{2x^4+2x^2+1}\right)-\frac{289}{28}\int \frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{\left(3-2x^2\right)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \ 27$$

$$\frac{1}{28}\left(\frac{1}{5}\int \frac{-756x^4-1946x^2+5\left(202-289\sqrt{2}\right)}{\sqrt{2x^4+2x^2+1}}dx-\frac{28}{5}x^3\sqrt{2x^4+2x^2+1}\right)-\frac{289}{28}\int \frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{\left(3-2x^2\right)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \ 2207$$

$$\frac{1}{28}\left(\frac{1}{5}\left(\frac{1}{6}\int \frac{6\left(-1442x^2-1445\sqrt{2}+1136\right)}{\sqrt{2x^4+2x^2+1}}dx-126x\sqrt{2x^4+2x^2+1}\right)-\frac{28}{5}x^3\sqrt{2x^4+2x^2+1}\right)-\frac{289}{28}\int \frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{\left(3-2x^2\right)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \ 27$$

$$\frac{1}{28}\left(\frac{1}{5}\left(\int \frac{-1442x^2-1445\sqrt{2}+1136}{\sqrt{2x^4+2x^2+1}}dx-126x\sqrt{2x^4+2x^2+1}\right)-\frac{28}{5}x^3\sqrt{2x^4+2x^2+1}\right)-\frac{289}{28}\int \frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{\left(3-2x^2\right)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \ 1511$$

3.328. $\int \frac{\left(1+2x^2+2x^4\right)^{3/2}}{3-2x^2}dx$

$$\frac{1}{28}\left(\frac{1}{5}\left(2\left(568-1083\sqrt{2}\right)\int \frac{1}{\sqrt{2x^4+2x^2+1}}dx+721\sqrt{2}\int \frac{1-\sqrt{2}x^2}{\sqrt{2x^4+2x^2+1}}dx-126\sqrt{2x^4+2x^2+1}x\right)-\frac{28}{5}x\right.$$
$$\frac{289}{28}\int \frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{\left(3-2x^2\right)\sqrt{2x^4+2x^2+1}}dx$$

$\Big\downarrow$ 1416

$$\frac{1}{28}\left(\frac{1}{5}\left(721\sqrt{2}\int \frac{1-\sqrt{2}x^2}{\sqrt{2x^4+2x^2+1}}dx+\frac{\left(568-1083\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2\right.\right.}{\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}\right.\right.$$
$$\frac{289}{28}\int \frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{\left(3-2x^2\right)\sqrt{2x^4+2x^2+1}}dx$$

$\Big\downarrow$ 1509

$$\frac{1}{28}\left(\frac{1}{5}\left(\frac{\left(568-1083\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}+721\sqrt{2}\left(\frac{\left(\sqrt{2}x^2+1\right.}{}\right.\right.\right.$$
$$\frac{289}{28}\int \frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{\left(3-2x^2\right)\sqrt{2x^4+2x^2+1}}dx$$

$\Big\downarrow$ 2222

$$\frac{1}{28}\left(\frac{1}{5}\left(\frac{\left(568-1083\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}+721\sqrt{2}\left(\frac{\left(\sqrt{2}x^2+1\right.}{}\right.\right.\right.$$
$$\frac{289}{28}\left(\frac{\left(3-\sqrt{2}\right)^2\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticPi}\left(\frac{1}{24}\left(12+11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{12\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-\frac{7\text{arctanh}\left(\right.}{2}\right.$$

input
```
Int[(1 + 2*x^2 + 2*x^4)^(3/2)/(3 - 2*x^2),x]
```

3.328. $\int \frac{\left(1+2x^2+2x^4\right)^{3/2}}{3-2x^2}\,dx$

output
```
((-28*x^3*Sqrt[1 + 2*x^2 + 2*x^4])/5 + (-126*x*Sqrt[1 + 2*x^2 + 2*x^4] + 7
21*Sqrt[2]*(-((x*Sqrt[1 + 2*x^2 + 2*x^4])/(1 + Sqrt[2]*x^2)) + ((1 + Sqrt[
2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2
^(1/4)*x], (2 - Sqrt[2])/4])/(2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])) + ((568 -
1083*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)
^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(2^(1/4)*Sqrt[1 + 2*x
^2 + 2*x^4]))/5)/28 - (289*((-7*ArcTanh[(Sqrt[17/3]*x)/Sqrt[1 + 2*x^2 + 2*
x^4]])/(2*Sqrt[51]) + ((3 - Sqrt[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 +
 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 + 11*Sqrt[2])/24, 2*ArcTan[2^(
1/4)*x], (2 - Sqrt[2])/4])/(12*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])))/28
```

3.328.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

3.328. $\int \frac{(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx$

rule 1530
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[-(c*d^2 - b*d*e + a*e^2)^(p + 1/2)/(e^(2*p)*(c*d^2 - a*e^2))
Int[(a*d*Rt[c/a, 2] + a*e + (c*d + a*e*Rt[c/a, 2])*x^2)/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x] + Simp[1/(e^(2*p)*(c*d^2 - a*e^2))   Int[(1/Sqrt
[a + b*x^2 + c*x^4])*ExpandToSum[(e^(2*p)*(c*d^2 - a*e^2)*(a + b*x^2 + c*x^
4)^(p + 1/2) + (c*d^2 - b*d*e + a*e^2)^(p + 1/2)*(a*d*Rt[c/a, 2] + a*e + (c
*d + a*e*Rt[c/a, 2])*x^2))/(d + e*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p - 1/
2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

3.328.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.35 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.57

method	result
risch	$-\frac{x(2x^2+9)\sqrt{2x^4+2x^2+1}}{10}-\frac{457\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{20\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(\frac{103}{20}-\frac{103i}{20}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,(}{\sqrt{-1+}}$
default	$-\frac{x^3\sqrt{2x^4+2x^2+1}}{5}-\frac{9x\sqrt{2x^4+2x^2+1}}{10}-\frac{177\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{10\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}-\frac{103i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x}}{20\sqrt{-1+i}\,\sqrt{}}$
elliptic	$-\frac{x^3\sqrt{2x^4+2x^2+1}}{5}-\frac{9x\sqrt{2x^4+2x^2+1}}{10}-\frac{177\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{10\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}-\frac{103i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x}}{20\sqrt{-1+i}\,\sqrt{}}$

input
```
int((2*x^4+2*x^2+1)^(3/2)/(-2*x^2+3),x,method=_RETURNVERBOSE)
```

output
```
-1/10*x*(2*x^2+9)*(2*x^4+2*x^2+1)^(1/2)-457/20/(-1+I)^(1/2)*(1+(1-I)*x^2)^
(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1
/2*2^(1/2)+1/2*I*2^(1/2))+(103/20-103/20*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/
2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2
*2^(1/2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)
))+289/12/(-1+I)^(1/2)*(1-I*x^2+x^2)^(1/2)*(1+I*x^2+x^2)^(1/2)/(2*x^4+2*x^
2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),-1/3-1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

3.328.5 Fricas [F]

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx=\int -\frac{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}}{2\,x^2-3}\,dx$$

input
```
integrate((2*x^4+2*x^2+1)^(3/2)/(-2*x^2+3),x, algorithm="fricas")
```

output
```
integral(-(2*x^4 + 2*x^2 + 1)^(3/2)/(2*x^2 - 3), x)
```

3.328.6 Sympy [F]

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx=-\int \frac{\sqrt{2x^4+2x^2+1}}{2x^2-3}\,dx-\int \frac{2x^2\sqrt{2x^4+2x^2+1}}{2x^2-3}\,dx-\int \frac{2x^4\sqrt{2x^4+2x^2+1}}{2x^2-3}\,dx$$

input
```
integrate((2*x**4+2*x**2+1)**(3/2)/(-2*x**2+3),x)
```

output
```
-Integral(sqrt(2*x**4 + 2*x**2 + 1)/(2*x**2 - 3), x) - Integral(2*x**2*sqr
t(2*x**4 + 2*x**2 + 1)/(2*x**2 - 3), x) - Integral(2*x**4*sqrt(2*x**4 + 2*
x**2 + 1)/(2*x**2 - 3), x)
```

3.328.7 Maxima [F]

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx = \int -\frac{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}}{2\,x^2-3}\,dx$$

input
```
integrate((2*x^4+2*x^2+1)^(3/2)/(-2*x^2+3),x, algorithm="maxima")
```

output
```
-integrate((2*x^4 + 2*x^2 + 1)^(3/2)/(2*x^2 - 3), x)
```

3.328.8 Giac [F]

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx = \int -\frac{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}}{2\,x^2-3}\,dx$$

input
```
integrate((2*x^4+2*x^2+1)^(3/2)/(-2*x^2+3),x, algorithm="giac")
```

output
```
integrate(-(2*x^4 + 2*x^2 + 1)^(3/2)/(2*x^2 - 3), x)
```

3.328.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx = -\int \frac{(2\,x^4+2\,x^2+1)^{3/2}}{2\,x^2-3}\,dx$$

input
```
int(-(2*x^2 + 2*x^4 + 1)^(3/2)/(2*x^2 - 3),x)
```

output
```
-int((2*x^2 + 2*x^4 + 1)^(3/2)/(2*x^2 - 3), x)
```

3.329 $\int \frac{\left(1+2x^2+2x^4\right)^{3/2}}{x^2\left(3-2x^2\right)}\,dx$

3.329.1 Optimal result

Integrand size = 29, antiderivative size = 722

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{x^2(3-2x^2)}\,dx = -\frac{(1+x^2)\sqrt{1+2x^2+2x^4}}{3x} - \frac{17x\sqrt{1+2x^2+2x^4}}{3\sqrt{2}\,(1+\sqrt{2}x^2)}$$

$$+ \frac{\sqrt{2}x\sqrt{1+2x^2+2x^4}}{3\,(1+\sqrt{2}x^2)} + \frac{17}{12}\sqrt{\frac{17}{3}}\operatorname{arctanh}\left(\frac{\sqrt{\frac{17}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)$$

$$+ \frac{17(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{3\ 2^{3/4}\sqrt{1+2x^2+2x^4}}$$

$$- \frac{\sqrt[4]{2}(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{3\sqrt{1+2x^2+2x^4}}$$

$$+ \frac{(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{3\ 2^{3/4}\sqrt{1+2x^2+2x^4}}$$

$$+ \frac{289(3-\sqrt{2})\,(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{84\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$

$$- \frac{17(5+\sqrt{2})\,(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{12\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$

$$- \frac{289(11-6\sqrt{2})\,(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\operatorname{EllipticPi}\left(\frac{1}{24}(12+11\sqrt{2}),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{504\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$

output
```
17/36*arctanh(1/3*x*51^(1/2)/(2*x^4+2*x^2+1)^(1/2))*51^(1/2)-1/3*(x^2+1)*(
2*x^4+2*x^2+1)^(1/2)/x-5/2*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))
+5/2*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticE
(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2
*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)+1/6*(cos(2*
arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arcta
n(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x
^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)-289/1008*(cos(2*arctan(
2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(2^(1
/4)*x)),1/2+11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(11-6*2^(1/2))*(1+x^2*2^(
1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1
/2)+289/168*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*El
lipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(3-2^(1/2))*(1+x^2
*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1
)^(1/2)-17/24*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*
EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(5+2^(1/2))*(1+x
^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2
+1)^(1/2)
```

3.329.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.24 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.30

$$\int \frac{\left(1+2x^2+2x^4\right)^{3/2}}{x^2\left(3-2x^2\right)}\,dx = \frac{-12-36x^2-48x^4-24x^6+90i\sqrt{1-i}x\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}E\big(i\mathrm{arcs}}{}$$

input
```
Integrate[(1 + 2*x^2 + 2*x^4)^(3/2)/(x^2*(3 - 2*x^2)),x]
```

output
```
(-12 - 36*x^2 - 48*x^4 - 24*x^6 + (90*I)*Sqrt[1 - I]*x*Sqrt[1 + (1 - I)*x^
2]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] - (255 - 1
65*I)*Sqrt[1 - I]*x*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[
I*ArcSinh[Sqrt[1 - I]*x], I] + 289*(1 - I)^(3/2)*x*Sqrt[1 + (1 - I)*x^2]*S
qrt[1 + (1 + I)*x^2]*EllipticPi[-1/3 - I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/
(36*x*Sqrt[1 + 2*x^2 + 2*x^4])
```

3.329.3 Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 423, normalized size of antiderivative = 0.59, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {1634, 25, 27, 2199, 2199, 1604, 25, 1511, 1416, 1509, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^4+2x^2+1)^{3/2}}{x^2(3-2x^2)}dx$$

$$\Big\downarrow \text{1634}$$

$$\frac{289}{42}\int -\frac{-2(3-\sqrt{2})x^2-3\sqrt{2}+2}{(3-2x^2)\sqrt{2x^4+2x^2+1}}dx-\frac{1}{168}\int -\frac{4(-84x^6-294x^4+(258-289\sqrt{2})x^2+14)}{x^2\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \text{25}$$

$$-\frac{289}{42}\int \frac{-2(3-\sqrt{2})x^2-3\sqrt{2}+2}{(3-2x^2)\sqrt{2x^4+2x^2+1}}dx-\frac{1}{168}\int -\frac{4(-84x^6-294x^4+(258-289\sqrt{2})x^2+14)}{x^2\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{42}\int \frac{-84x^6-294x^4+(258-289\sqrt{2})x^2+14}{x^2\sqrt{2x^4+2x^2+1}}dx-\frac{289}{42}\int \frac{-2(3-\sqrt{2})x^2-3\sqrt{2}+2}{(3-2x^2)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \text{2199}$$

$$\frac{1}{42}\left(\int \frac{-238x^4+17(16-17\sqrt{2})x^2+14}{x^2\sqrt{2x^4+2x^2+1}}dx-14x\sqrt{2x^4+2x^2+1}\right)-\frac{289}{42}\int \frac{-2(3-\sqrt{2})x^2-3\sqrt{2}+2}{(3-2x^2)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \text{2199}$$

$$\frac{1}{42}\left(\int \frac{17(16-17\sqrt{2})x^2-105}{x^2\sqrt{2x^4+2x^2+1}}dx-14\sqrt{2x^4+2x^2+1}x-\frac{119\sqrt{2x^4+2x^2+1}}{x}\right)-\frac{289}{42}\int \frac{-2(3-\sqrt{2})x^2-3\sqrt{2}+2}{(3-2x^2)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \text{1604}$$

$$\frac{1}{42}\left(-\int -\frac{17(16-17\sqrt{2})-210x^2}{\sqrt{2x^4+2x^2+1}}dx-14\sqrt{2x^4+2x^2+1}x-\frac{14\sqrt{2x^4+2x^2+1}}{x}\right)-\frac{289}{42}\int \frac{-2(3-\sqrt{2})x^2-3\sqrt{2}+2}{(3-2x^2)\sqrt{2x^4+2x^2+1}}dx$$

$$\downarrow 25$$

$$\frac{1}{42}\left(\int \frac{17\left(16-17\sqrt{2}\right)-210x^2}{\sqrt{2x^4+2x^2+1}}dx-14\sqrt{2x^4+2x^2+1}x-\frac{14\sqrt{2x^4+2x^2+1}}{x}\right)-\frac{289}{42}\int \frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{\left(3-2x^2\right)\sqrt{2x^4+2x^2+1}}dx$$

$$\downarrow 1511$$

$$\frac{1}{42}\left(2\left(136-197\sqrt{2}\right)\int \frac{1}{\sqrt{2x^4+2x^2+1}}dx+105\sqrt{2}\int \frac{1-\sqrt{2}x^2}{\sqrt{2x^4+2x^2+1}}dx-14\sqrt{2x^4+2x^2+1}x-\frac{14\sqrt{2x^4+2}}{x}\right.$$
$$\frac{289}{42}\int \frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{\left(3-2x^2\right)\sqrt{2x^4+2x^2+1}}dx$$

$$\downarrow 1416$$

$$\frac{1}{42}\left(105\sqrt{2}\int \frac{1-\sqrt{2}x^2}{\sqrt{2x^4+2x^2+1}}dx+\frac{\left(136-197\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{}\right.\right.}{\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}\right.$$
$$\frac{289}{42}\int \frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{\left(3-2x^2\right)\sqrt{2x^4+2x^2+1}}dx$$

$$\downarrow 1509$$

$$\frac{1}{42}\left(\frac{\left(136-197\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}+105\sqrt{2}\left(\frac{\left(\sqrt{2}x^2+1\right)\sqrt{}}{}\right.\right.$$
$$\frac{289}{42}\int \frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{\left(3-2x^2\right)\sqrt{2x^4+2x^2+1}}dx$$

$$\downarrow 2222$$

$$\frac{1}{42}\left(\frac{\left(136-197\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}+105\sqrt{2}\left(\frac{\left(\sqrt{2}x^2+1\right)\sqrt{}}{}\right.\right.$$
$$\frac{289}{42}\left(\frac{\left(3-\sqrt{2}\right)^2\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticPi}\left(\frac{1}{24}\left(12+11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{12\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-\frac{7\text{arctanh}\left(\right.}{2}\right.$$

3.329. $\int \frac{\left(1+2x^2+2x^4\right)^{3/2}}{x^2\left(3-2x^2\right)}\,dx$

input
```
Int[(1 + 2*x^2 + 2*x^4)^(3/2)/(x^2*(3 - 2*x^2)),x]
```

output
```
((-14*Sqrt[1 + 2*x^2 + 2*x^4])/x - 14*x*Sqrt[1 + 2*x^2 + 2*x^4] + 105*Sqrt
[2]*(-((x*Sqrt[1 + 2*x^2 + 2*x^4])/(1 + Sqrt[2]*x^2)) + ((1 + Sqrt[2]*x^2)
*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*
x], (2 - Sqrt[2])/4])/(2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])) + ((136 - 197*Sqr
t[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Elli
pticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(2^(1/4)*Sqrt[1 + 2*x^2 + 2*x
^4]))/42 - (289*((-7*ArcTanh[(Sqrt[17/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/(2*S
qrt[51]) + ((3 - Sqrt[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1
+ Sqrt[2]*x^2)^2]*EllipticPi[(12 + 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2
 - Sqrt[2])/4])/(12*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])))/42
```

3.329.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1634
```
Int[((x_)^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[(-(-d/e)^(m/2))*((c*d^2 - b*d*e + a*e^2)^(p + 1/2)
/(e^(2*p)*(c*d^2 - a*e^2)))   Int[(a*d*Rt[c/a, 2] + a*e + (c*d + a*e*Rt[c/a
, 2])*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] + Simp[(-d/e)^(m/2
)/(e^(2*p)*(c*d^2 - a*e^2))   Int[(x^m/Sqrt[a + b*x^2 + c*x^4])*ExpandToSum
[((e^(2*p)*(c*d^2 - a*e^2)*(a + b*x^2 + c*x^4)^(p + 1/2))/(-d/e)^(m/2) + ((
a*d*Rt[c/a, 2] + a*e + (c*d + a*e*Rt[c/a, 2])*x^2)*(c*d^2 - b*d*e + a*e^2)^
(p + 1/2))/x^m)/(d + e*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p + 1/2, 0] && ILtQ[m/2, 0] && NeQ[c*d^2 - a*e^2,
0] && PosQ[c/a]
```

rule 2199
```
Int[(Px_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> With[{q = Expon[Px, x^2]}, Simp[Coeff[Px, x^2, q]*(d*x)^(m + 2*q
- 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*d^(2*q - 3)*(m + 4*p + 2*q + 1))), x]
+ Int[(d*x)^m*(a + b*x^2 + c*x^4)^p*ExpandToSum[Px - Coeff[Px, x^2, q]*x^(2
*q) - Coeff[Px, x^2, q]*((a*(m + 2*q - 3)*x^(2*(q - 2)) + b*(m + 2*p + 2*q
- 1)*x^(2*(q - 1)))/(c*(m + 4*p + 2*q + 1))), x], x] /; GtQ[q, 1] && NeQ[m
+ 4*p + 2*q + 1, 0]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Px, x^2] && N
eQ[b^2 - 4*a*c, 0]
```

3.329. $\int \frac{(1+2x^2+2x^4)^{3/2}}{x^2(3-2x^2)}\,dx$

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

3.329.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.96 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.36

method	result
risch	$-\frac{2x^6+4x^4+3x^2+1}{3x\sqrt{2x^4+2x^2+1}}-\frac{85\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{6\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(\frac{5}{2}-\frac{5i}{2}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1+\ldots\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2\ldots}}$
elliptic	$-\frac{\sqrt{2x^4+2x^2+1}}{3x}-\frac{x\sqrt{2x^4+2x^2+1}}{3}-\frac{35\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{3\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}-\frac{5i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\ldots}{2\sqrt{-1+i}\,\sqrt{2x^4+\ldots}}$
default	$-\frac{\sqrt{2x^4+2x^2+1}}{3x}-\frac{x\sqrt{2x^4+2x^2+1}}{3}+\frac{16\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{15\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\left(-\frac{14}{15}+\frac{14i}{15}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(\ldots}$

input
```
int((2*x^4+2*x^2+1)^(3/2)/x^2/(-2*x^2+3),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(2*x^6+4*x^4+3*x^2+1)/x/(2*x^4+2*x^2+1)^(1/2)-85/6/(-1+I)^(1/2)*(1+(1
-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I
)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+(5/2-5/2*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^
(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),
1/2*2^(1/2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1
/2)))+289/18/(-1+I)^(1/2)*(1-I*x^2+x^2)^(1/2)*(1+I*x^2+x^2)^(1/2)/(2*x^4+2
*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),-1/3-1/3*I,(-1-I)^(1/2)/(-1+I)^(1/
2))
```

3.329.5 Fricas [F]

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{x^2(3-2x^2)}\,dx = \int -\frac{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}}{(2\,x^2-3)x^2}\,dx$$

input
```
integrate((2*x^4+2*x^2+1)^(3/2)/x^2/(-2*x^2+3),x, algorithm="fricas")
```

output
```
integral(-(2*x^4 + 2*x^2 + 1)^(3/2)/(2*x^4 - 3*x^2), x)
```

3.329.6 Sympy [F]

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{x^2(3-2x^2)}\,dx = -\int \frac{\sqrt{2x^4+2x^2+1}}{2x^4-3x^2}\,dx - \int \frac{2x^2\sqrt{2x^4+2x^2+1}}{2x^4-3x^2}\,dx - \int \frac{2x^4\sqrt{2x^4+2x^2+1}}{2x^4-3x^2}\,dx$$

input
```
integrate((2*x**4+2*x**2+1)**(3/2)/x**2/(-2*x**2+3),x)
```

output
```
-Integral(sqrt(2*x**4 + 2*x**2 + 1)/(2*x**4 - 3*x**2), x) - Integral(2*x**
2*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**4 - 3*x**2), x) - Integral(2*x**4*sqrt(2
*x**4 + 2*x**2 + 1)/(2*x**4 - 3*x**2), x)
```

3.329.7 Maxima [F]

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{x^2(3-2x^2)}\,dx = \int -\frac{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}}{(2\,x^2-3)x^2}\,dx$$

input
```
integrate((2*x^4+2*x^2+1)^(3/2)/x^2/(-2*x^2+3),x, algorithm="maxima")
```

output
```
-integrate((2*x^4 + 2*x^2 + 1)^(3/2)/((2*x^2 - 3)*x^2), x)
```

3.329.8 Giac [F]

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{x^2(3-2x^2)}\,dx = \int -\frac{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}}{(2\,x^2-3)x^2}\,dx$$

input
```
integrate((2*x^4+2*x^2+1)^(3/2)/x^2/(-2*x^2+3),x, algorithm="giac")
```

output
```
integrate(-(2*x^4 + 2*x^2 + 1)^(3/2)/((2*x^2 - 3)*x^2), x)
```

3.329.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{x^2(3-2x^2)}\,dx = -\int \frac{(2\,x^4+2\,x^2+1)^{3/2}}{x^2\,(2\,x^2-3)}\,dx$$

input
```
int(-(2*x^2 + 2*x^4 + 1)^(3/2)/(x^2*(2*x^2 - 3)),x)
```

output
```
-int((2*x^2 + 2*x^4 + 1)^(3/2)/(x^2*(2*x^2 - 3)), x)
```

3.330 $\int \frac{(1+2x^2+2x^4)^{3/2}}{x^4(3-2x^2)} dx$

3.330.1 Optimal result

Integrand size = 29, antiderivative size = 625

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{x^4(3-2x^2)} dx = -\frac{2\sqrt{1+2x^2+2x^4}}{x} - \frac{(1-8x^2)\sqrt{1+2x^2+2x^4}}{9x^3}$$
$$+ \frac{\sqrt{2}x\sqrt{1+2x^2+2x^4}}{9(1+\sqrt{2}x^2)} + \frac{17}{18}\sqrt{\frac{17}{3}}\text{arctanh}\left(\frac{\sqrt{\frac{17}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)$$
$$- \frac{\sqrt[4]{2}(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{9\sqrt{1+2x^2+2x^4}}$$
$$+ \frac{289(3-\sqrt{2})(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{126\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$
$$- \frac{17(5+\sqrt{2})(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{18\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$
$$+ \frac{\sqrt[4]{2}(9+5\sqrt{2})(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{9\sqrt{1+2x^2+2x^4}}$$
$$- \frac{289(11-6\sqrt{2})(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\text{EllipticPi}\left(\frac{1}{24}(12+11\sqrt{2}),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{756\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$

output
```
17/54*arctanh(1/3*x*51^(1/2)/(2*x^4+2*x^2+1)^(1/2))*51^(1/2)-2*(2*x^4+2*x^
2+1)^(1/2)/x-1/9*(-8*x^2+1)*(2*x^4+2*x^2+1)^(1/2)/x^3+1/9*x*(2*x^4+2*x^2+1
)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))-1/9*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos
(2*arctan(2^(1/4)*x))*EllipticE(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(
1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2
*x^4+2*x^2+1)^(1/2)-289/1512*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arct
an(2^(1/4)*x))*EllipticPi(sin(2*arctan(2^(1/4)*x)),1/2+11/24*2^(1/2),1/2*(
2-2^(1/2))^(1/2))*(11-6*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2
^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)+289/252*(cos(2*arctan(2^(1/
4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)
),1/2*(2-2^(1/2))^(1/2))*(3-2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x
^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)-17/36*(cos(2*arctan(2^(
1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*
x)),1/2*(2-2^(1/2))^(1/2))*(5+2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1
+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)+1/9*2^(1/4)*(cos(2*ar
ctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(
2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(9+5*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2
*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)
```

3.330.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.25 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.35

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{x^4(3-2x^2)}\,dx = \frac{-6-72x^2-132x^4-120x^6-6i\sqrt{1-i}x^3\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}E(i\text{ar}$$

input
```
Integrate[(1 + 2*x^2 + 2*x^4)^(3/2)/(x^4*(3 - 2*x^2)),x]
```

output
```
(-6 - 72*x^2 - 132*x^4 - 120*x^6 - (6*I)*Sqrt[1 - I]*x^3*Sqrt[1 + (1 - I)*
x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] - (195 -
 201*I)*Sqrt[1 - I]*x^3*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*Ellipt
icF[I*ArcSinh[Sqrt[1 - I]*x], I] + 289*(1 - I)^(3/2)*x^3*Sqrt[1 + (1 - I)*
x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[-1/3 - I/3, I*ArcSinh[Sqrt[1 - I]*x]
, I])/(54*x^3*Sqrt[1 + 2*x^2 + 2*x^4])
```

3.330.3 Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 468, normalized size of antiderivative = 0.75, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.448, Rules used = {1634, 25, 27, 2199, 2199, 1604, 27, 1604, 25, 1511, 1416, 1509, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(2x^4+2x^2+1\right)^{3/2}}{x^4\left(3-2x^2\right)}dx$$

$$\Big\downarrow \; 1634$$

$$\frac{289}{63}\int -\frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{\left(3-2x^2\right)\sqrt{2x^4+2x^2+1}}dx-\frac{1}{252}\int -\frac{4\left(-126x^6+\left(426-289\sqrt{2}\right)x^4+98x^2+21\right)}{x^4\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \; 25$$

$$-\frac{289}{63}\int \frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{\left(3-2x^2\right)\sqrt{2x^4+2x^2+1}}dx-\frac{1}{252}\int -\frac{4\left(-126x^6+\left(426-289\sqrt{2}\right)x^4+98x^2+21\right)}{x^4\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \; 27$$

$$\frac{1}{63}\int \frac{-126x^6+\left(426-289\sqrt{2}\right)x^4+98x^2+21}{x^4\sqrt{2x^4+2x^2+1}}dx-\frac{289}{63}\int \frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{\left(3-2x^2\right)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \; 2199$$

$$\frac{1}{63}\left(\int \frac{\left(426-289\sqrt{2}\right)x^4+35x^2+21}{x^4\sqrt{2x^4+2x^2+1}}dx-\frac{63\sqrt{2x^4+2x^2+1}}{x}\right)-\frac{289}{63}\int \frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{\left(3-2x^2\right)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \; 2199$$

$$\frac{1}{63}\left(\int \frac{-\left(\left(817-578\sqrt{2}\right)x^2\right)-\frac{3}{2}\left(412-289\sqrt{2}\right)}{x^4\sqrt{2x^4+2x^2+1}}dx-\frac{63\sqrt{2x^4+2x^2+1}}{x}-\frac{\left(426-289\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}{2x^3}\right)-\frac{289}{63}\int \frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{\left(3-2x^2\right)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \; 1604$$

$$\frac{1}{63}\left(-\frac{1}{3}\int -\frac{3\left(\left(412-289\sqrt{2}\right)x^2+7\right)}{x^2\sqrt{2x^4+2x^2+1}}dx-\frac{63\sqrt{2x^4+2x^2+1}}{x}-\frac{\left(426-289\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}{2x^3}+\frac{(412-289\sqrt{}}{}\right.$$
$$\frac{289}{63}\int \frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{(3-2x^2)\sqrt{2x^4+2x^2+1}}dx$$

$\downarrow$ 27

$$\frac{1}{63}\left(\int \frac{\left(412-289\sqrt{2}\right)x^2+7}{x^2\sqrt{2x^4+2x^2+1}}dx-\frac{63\sqrt{2x^4+2x^2+1}}{x}-\frac{\left(426-289\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}{2x^3}+\frac{\left(412-289\sqrt{2}\right)\sqrt{2x^4}}{2x^3}\right.$$
$$\frac{289}{63}\int \frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{(3-2x^2)\sqrt{2x^4+2x^2+1}}dx$$

$\downarrow$ 1604

$$\frac{1}{63}\left(-\int -\frac{14x^2-289\sqrt{2}+412}{\sqrt{2x^4+2x^2+1}}dx-\frac{70\sqrt{2x^4+2x^2+1}}{x}-\frac{\left(426-289\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}{2x^3}+\frac{\left(412-289\sqrt{2}\right)\sqrt{2}}{2x^3}\right.$$
$$\frac{289}{63}\int \frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{(3-2x^2)\sqrt{2x^4+2x^2+1}}dx$$

$\downarrow$ 25

$$\frac{1}{63}\left(\int \frac{14x^2-289\sqrt{2}+412}{\sqrt{2x^4+2x^2+1}}dx-\frac{70\sqrt{2x^4+2x^2+1}}{x}-\frac{\left(426-289\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}{2x^3}+\frac{\left(412-289\sqrt{2}\right)\sqrt{2x^4+}}{2x^3}\right.$$
$$\frac{289}{63}\int \frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{(3-2x^2)\sqrt{2x^4+2x^2+1}}dx$$

$\downarrow$ 1511

$$\frac{1}{63}\left(2\left(206-141\sqrt{2}\right)\int \frac{1}{\sqrt{2x^4+2x^2+1}}dx-7\sqrt{2}\int \frac{1-\sqrt{2}x^2}{\sqrt{2x^4+2x^2+1}}dx-\frac{70\sqrt{2x^4+2x^2+1}}{x}-\frac{\left(426-289\sqrt{2}\right)}{2}\right.$$
$$\frac{289}{63}\int \frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{(3-2x^2)\sqrt{2x^4+2x^2+1}}dx$$

$\downarrow$ 1416

$$\frac{1}{63}\left(-7\sqrt{2}\int \frac{1-\sqrt{2}x^2}{\sqrt{2x^4+2x^2+1}}dx+\frac{\left(206-141\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{}\right.}{\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}\right.$$
$$\frac{289}{63}\int \frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{(3-2x^2)\sqrt{2x^4+2x^2+1}}dx$$

3.330. $\int \frac{\left(1+2x^2+2x^4\right)^{3/2}}{x^4(3-2x^2)}\,dx$

$$\Big\downarrow \text{1509}$$

$$\frac{1}{63}\left(\frac{\left(206-141\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-7\sqrt{2}\left(\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4}{(\sqrt{}}}}{}\right.\right.$$

$$\frac{289}{63}\int\frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{\left(3-2x^2\right)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \text{2222}$$

$$\frac{1}{63}\left(\frac{\left(206-141\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-7\sqrt{2}\left(\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4}{(\sqrt{}}}}{}\right.\right.$$

$$\frac{289}{63}\left(\frac{\left(3-\sqrt{2}\right)^2\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticPi}\left(\frac{1}{24}\left(12+11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{12\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-\frac{7\text{arctanh}\left(\right.}{2}\right.$$

input
```
Int[(1 + 2*x^2 + 2*x^4)^(3/2)/(x^4*(3 - 2*x^2)),x]
```

output
```
(((412 - 289*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])/(2*x^3) - ((426 - 289*Sqrt[
2])*Sqrt[1 + 2*x^2 + 2*x^4])/(2*x^3) - (70*Sqrt[1 + 2*x^2 + 2*x^4])/x - 7*
Sqrt[2]*(-((x*Sqrt[1 + 2*x^2 + 2*x^4])/(1 + Sqrt[2]*x^2)) + ((1 + Sqrt[2]*
x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1
/4)*x], (2 - Sqrt[2])/4])/(2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])) + ((206 - 141
*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*
EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(2^(1/4)*Sqrt[1 + 2*x^2 +
 2*x^4]))/63 - (289*((-7*ArcTanh[(Sqrt[17/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/
(2*Sqrt[51]) + ((3 - Sqrt[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)
/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 + 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x]
, (2 - Sqrt[2])/4])/(12*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])))/63
```

3.330. $\int \frac{\left(1+2x^2+2x^4\right)^{3/2}}{x^4\left(3-2x^2\right)}dx$

3.330.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.330. $\int \frac{(1+2x^2+2x^4)^{3/2}}{x^4(3-2x^2)} \, dx$

rule 1634
```
Int[((x_)^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[(-(-d/e)^(m/2))*((c*d^2 - b*d*e + a*e^2)^(p + 1/2)
/(e^(2*p)*(c*d^2 - a*e^2)))   Int[(a*d*Rt[c/a, 2] + a*e + (c*d + a*e*Rt[c/a
, 2])*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] + Simp[(-d/e)^(m/2
)/(e^(2*p)*(c*d^2 - a*e^2))   Int[(x^m/Sqrt[a + b*x^2 + c*x^4])*ExpandToSum
[((e^(2*p)*(c*d^2 - a*e^2)*(a + b*x^2 + c*x^4)^(p + 1/2))/(-d/e)^(m/2) + ((
a*d*Rt[c/a, 2] + a*e + (c*d + a*e*Rt[c/a, 2])*x^2)*(c*d^2 - b*d*e + a*e^2)^
(p + 1/2))/x^m)/(d + e*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p + 1/2, 0] && ILtQ[m/2, 0] && NeQ[c*d^2 - a*e^2,
0] && PosQ[c/a]
```

rule 2199
```
Int[(Px_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> With[{q = Expon[Px, x^2]}, Simp[Coeff[Px, x^2, q]*(d*x)^(m + 2*q
- 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*d^(2*q - 3)*(m + 4*p + 2*q + 1))), x]
+ Int[(d*x)^m*(a + b*x^2 + c*x^4)^p*ExpandToSum[Px - Coeff[Px, x^2, q]*x^(2
*q) - Coeff[Px, x^2, q]*((a*(m + 2*q - 3)*x^(2*(q - 2)) + b*(m + 2*p + 2*q
- 1)*x^(2*(q - 1)))/(c*(m + 4*p + 2*q + 1))), x], x] /; GtQ[q, 1] && NeQ[m
+ 4*p + 2*q + 1, 0]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Px, x^2] && N
eQ[b^2 - 4*a*c, 0]
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

3.330. $\int \frac{(1+2x^2+2x^4)^{3/2}}{x^4(3-2x^2)}\,dx$

3.330.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.11 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.41

method	result
risch	$-\frac{20x^6+22x^4+12x^2+1}{9x^3\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{1}{9}+\frac{i}{9}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)-E\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\right)}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}-\frac{65\sqrt{1+(1\cdots}{\cdots}$
elliptic	$-\frac{\sqrt{2x^4+2x^2+1}}{9x^3}-\frac{10\sqrt{2x^4+2x^2+1}}{9x}-\frac{22\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{3\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F(\cdots}{9\sqrt{-1+i}\,\sqrt{2x^4+\cdots}}$
default	$-\frac{10\sqrt{2x^4+2x^2+1}}{9x}+\frac{44\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{15\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{12}{5}+\frac{12i}{5}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-\cdots}\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+\cdots}}$

input
```
int((2*x^4+2*x^2+1)^(3/2)/x^4/(-2*x^2+3),x,method=_RETURNVERBOSE)
```

output
```
-1/9*(20*x^6+22*x^4+12*x^2+1)/x^3/(2*x^4+2*x^2+1)^(1/2)+(-1/9+1/9*I)/(-1+I
)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(Ell
ipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),
1/2*2^(1/2)+1/2*I*2^(1/2)))-65/9/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)
*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2
*I*2^(1/2))+289/27/(-1+I)^(1/2)*(1-I*x^2+x^2)^(1/2)*(1+I*x^2+x^2)^(1/2)/(2
*x^4+2*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),-1/3-1/3*I,(-1-I)^(1/2)/(-1+
I)^(1/2))
```

3.330.5 Fricas [F]

$$\int \frac{\left(1+2x^2+2x^4\right)^{3/2}}{x^4\left(3-2x^2\right)}\,dx=\int -\frac{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}}{\left(2\,x^2-3\right)x^4}\,dx$$

input
```
integrate((2*x^4+2*x^2+1)^(3/2)/x^4/(-2*x^2+3),x, algorithm="fricas")
```

output
```
integral(-(2*x^4 + 2*x^2 + 1)^(3/2)/(2*x^6 - 3*x^4), x)
```

3.330.6 Sympy [F]

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{x^4(3-2x^2)}\,dx = -\int \frac{\sqrt{2x^4+2x^2+1}}{2x^6-3x^4}\,dx - \int \frac{2x^2\sqrt{2x^4+2x^2+1}}{2x^6-3x^4}\,dx - \int \frac{2x^4\sqrt{2x^4+2x^2+1}}{2x^6-3x^4}\,dx$$

input
```
integrate((2*x**4+2*x**2+1)**(3/2)/x**4/(-2*x**2+3),x)
```

output
```
-Integral(sqrt(2*x**4 + 2*x**2 + 1)/(2*x**6 - 3*x**4), x) - Integral(2*x**
2*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**6 - 3*x**4), x) - Integral(2*x**4*sqrt(2
*x**4 + 2*x**2 + 1)/(2*x**6 - 3*x**4), x)
```

3.330.7 Maxima [F]

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{x^4(3-2x^2)}\,dx = \int -\frac{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}}{(2\,x^2-3)x^4}\,dx$$

input
```
integrate((2*x^4+2*x^2+1)^(3/2)/x^4/(-2*x^2+3),x, algorithm="maxima")
```

output
```
-integrate((2*x^4 + 2*x^2 + 1)^(3/2)/((2*x^2 - 3)*x^4), x)
```

3.330.8 Giac [F]

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{x^4(3-2x^2)}\,dx = \int -\frac{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}}{(2\,x^2-3)x^4}\,dx$$

input
```
integrate((2*x^4+2*x^2+1)^(3/2)/x^4/(-2*x^2+3),x, algorithm="giac")
```

output
```
integrate(-(2*x^4 + 2*x^2 + 1)^(3/2)/((2*x^2 - 3)*x^4), x)
```

3.330.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{x^4(3-2x^2)}\,dx = -\int \frac{(2\,x^4+2\,x^2+1)^{3/2}}{x^4\,(2\,x^2-3)}\,dx$$

input
```
int(-(2*x^2 + 2*x^4 + 1)^(3/2)/(x^4*(2*x^2 - 3)),x)
```

output
```
-int((2*x^2 + 2*x^4 + 1)^(3/2)/(x^4*(2*x^2 - 3)), x)
```

3.331 $\int \frac{(1+2x^2+2x^4)^{3/2}}{x^6(3-2x^2)}\,dx$

3.331.1 Optimal result

Integrand size = 29, antiderivative size = 553

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{x^6(3-2x^2)}\,dx = \frac{74\sqrt{1+2x^2+2x^4}}{135x^3} - \frac{262\sqrt{1+2x^2+2x^4}}{135x} - \frac{(3+40x^2)\sqrt{1+2x^2+2x^4}}{45x^5} + \frac{262\sqrt{2}x\sqrt{1+2x^2+2x^4}}{135(1+\sqrt{2}x^2)} + \frac{17}{27}\sqrt{\frac{17}{3}}\operatorname{arctanh}\left(\frac{\sqrt{\frac{17}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) - \frac{262\sqrt[4]{2}(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)\middle|\frac{1}{4}(2-\sqrt{2})\right)}{135\sqrt{1+2x^2+2x^4}} + \frac{85\ 2^{3/4}(3-\sqrt{2})(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{189\sqrt{1+2x^2+2x^4}} + \frac{2^{3/4}(37+23\sqrt{2})(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{135\sqrt{1+2x^2+2x^4}} - \frac{289(11-6\sqrt{2})(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\operatorname{EllipticPi}\left(\frac{1}{24}(12+11\sqrt{2}),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{1134\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$

output
```
17/81*arctanh(1/3*x*51^(1/2)/(2*x^4+2*x^2+1)^(1/2))*51^(1/2)+74/135*(2*x^4
+2*x^2+1)^(1/2)/x^3-262/135*(2*x^4+2*x^2+1)^(1/2)/x-1/45*(40*x^2+3)*(2*x^4
+2*x^2+1)^(1/2)/x^5+262/135*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2)
)-262/135*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*Elli
pticE(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*
x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)-289/22
68*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(
sin(2*arctan(2^(1/4)*x)),1/2+11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(11-6*2^
(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(
2*x^4+2*x^2+1)^(1/2)+85/189*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arcta
n(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(3
-2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4
)/(2*x^4+2*x^2+1)^(1/2)+1/135*2^(3/4)*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/c
os(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))
^(1/2))*(37+23*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2
)^(1/2)/(2*x^4+2*x^2+1)^(1/2)
```

3.331.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.27 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.41

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{x^6(3-2x^2)}\,dx =$$

$$-\frac{27+192x^2+1116x^4+1848x^6+1572x^8+786i\sqrt{1-i}x^5\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}E\left(i\operatorname{arcsinh}\left(\sqrt{1-\right.\right.}{}$$

input
```
Integrate[(1 + 2*x^2 + 2*x^4)^(3/2)/(x^6*(3 - 2*x^2)),x]
```

output
```
-1/405*(27 + 192*x^2 + 1116*x^4 + 1848*x^6 + 1572*x^8 + (786*I)*Sqrt[1 - I
]*x^5*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt
[1 - I]*x], I] + (543 - 1329*I)*Sqrt[1 - I]*x^5*Sqrt[1 + (1 - I)*x^2]*Sqrt
[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] - 1445*(1 - I)^(3
/2)*x^5*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[-1/3 - I/3,
 I*ArcSinh[Sqrt[1 - I]*x], I])/(x^5*Sqrt[1 + 2*x^2 + 2*x^4])
```

3.331.3 Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 535, normalized size of antiderivative = 0.97, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.483$, Rules used = {1634, 25, 27, 2199, 2199, 1604, 1604, 27, 1604, 25, 1511, 1416, 1509, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^4+2x^2+1)^{3/2}}{x^6(3-2x^2)}dx$$

$$\Big\downarrow \text{ 1634}$$

$$\frac{578}{189}\int -\frac{-2(3-\sqrt{2})x^2-3\sqrt{2}+2}{(3-2x^2)\sqrt{2x^4+2x^2+1}}dx-\frac{1}{378}\int -\frac{2(2(678-289\sqrt{2})x^6+700x^4+294x^2+63)}{x^6\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{578}{189}\int \frac{-2(3-\sqrt{2})x^2-3\sqrt{2}+2}{(3-2x^2)\sqrt{2x^4+2x^2+1}}dx-\frac{1}{378}\int -\frac{2(2(678-289\sqrt{2})x^6+700x^4+294x^2+63)}{x^6\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{189}\int \frac{2(678-289\sqrt{2})x^6+700x^4+294x^2+63}{x^6\sqrt{2x^4+2x^2+1}}dx-\frac{578}{189}\int \frac{-2(3-\sqrt{2})x^2-3\sqrt{2}+2}{(3-2x^2)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \text{ 2199}$$

$$\frac{1}{189}\left(\int \frac{-4(503-289\sqrt{2})x^4-3(580-289\sqrt{2})x^2+63}{x^6\sqrt{2x^4+2x^2+1}}dx-\frac{(678-289\sqrt{2})\sqrt{2x^4+2x^2+1}}{x^3}\right)-\frac{578}{189}\int \frac{-2(3-\sqrt{2})x^2-3\sqrt{2}+2}{(3-2x^2)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \text{ 2199}$$

$$\frac{1}{189}\left(\int \frac{\frac{7}{3}(404-289\sqrt{2})x^2+\frac{17}{3}(307-170\sqrt{2})}{x^6\sqrt{2x^4+2x^2+1}}dx+\frac{2(503-289\sqrt{2})\sqrt{2x^4+2x^2+1}}{3x^5}-\frac{(678-289\sqrt{2})\sqrt{2x^4+}}{x^3}\right.$$
$$\frac{578}{189}\int \frac{-2(3-\sqrt{2})x^2-3\sqrt{2}+2}{(3-2x^2)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \text{ 1604}$$

$$\frac{1}{189}\left(-\frac{1}{5}\int \frac{34\left(307-170\sqrt{2}\right)x^2+3\left(3068-1445\sqrt{2}\right)}{x^4\sqrt{2x^4+2x^2+1}}dx-\frac{17\left(307-170\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}{15x^5}+\frac{2\left(503-289\sqrt{2}\right.}{3}\right.$$
$$\frac{578}{189}\int \frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{\left(3-2x^2\right)\sqrt{2x^4+2x^2+1}}dx$$

$\big\downarrow$ 1604

$$\frac{1}{189}\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{6\left(\left(3068-1445\sqrt{2}\right)x^2+917\right)}{x^2\sqrt{2x^4+2x^2+1}}dx+\frac{\left(3068-1445\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}{x^3}\right)-\frac{17\left(307-170\sqrt{2}\right)\sqrt{2x^4}}{15x^5}\right.$$
$$\frac{578}{189}\int \frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{\left(3-2x^2\right)\sqrt{2x^4+2x^2+1}}dx$$

$\big\downarrow$ 27

$$\frac{1}{189}\left(\frac{1}{5}\left(2\int \frac{\left(3068-1445\sqrt{2}\right)x^2+917}{x^2\sqrt{2x^4+2x^2+1}}dx+\frac{\left(3068-1445\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}{x^3}\right)-\frac{17\left(307-170\sqrt{2}\right)\sqrt{2x^4+2}}{15x^5}\right.$$
$$\frac{578}{189}\int \frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{\left(3-2x^2\right)\sqrt{2x^4+2x^2+1}}dx$$

$\big\downarrow$ 1604

$$\frac{1}{189}\left(\frac{1}{5}\left(2\left(-\int -\frac{1834x^2-1445\sqrt{2}+3068}{\sqrt{2x^4+2x^2+1}}dx-\frac{917\sqrt{2x^4+2x^2+1}}{x}\right)+\frac{\left(3068-1445\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}{x^3}\right)-\right.$$
$$\frac{578}{189}\int \frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{\left(3-2x^2\right)\sqrt{2x^4+2x^2+1}}dx$$

$\big\downarrow$ 25

$$\frac{1}{189}\left(\frac{1}{5}\left(2\left(\int \frac{1834x^2-1445\sqrt{2}+3068}{\sqrt{2x^4+2x^2+1}}dx-\frac{917\sqrt{2x^4+2x^2+1}}{x}\right)+\frac{\left(3068-1445\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}{x^3}\right)-\frac{17(}{}\right.$$
$$\frac{578}{189}\int \frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{\left(3-2x^2\right)\sqrt{2x^4+2x^2+1}}dx$$

$\big\downarrow$ 1511

$$\frac{1}{189}\left(\frac{1}{5}\left(2\left(4\left(767-132\sqrt{2}\right)\int \frac{1}{\sqrt{2x^4+2x^2+1}}dx-917\sqrt{2}\int \frac{1-\sqrt{2}x^2}{\sqrt{2x^4+2x^2+1}}dx-\frac{917\sqrt{2x^4+2x^2+1}}{x}\right)+\frac{(3}{}\right.\right.$$
$$\frac{578}{189}\int \frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{\left(3-2x^2\right)\sqrt{2x^4+2x^2+1}}dx$$

$\big\downarrow$ 1416

3.331. $\int \frac{\left(1+2x^2+2x^4\right)^{3/2}}{x^6\left(3-2x^2\right)}\,dx$

$$\frac{1}{189}\left(\frac{1}{5}\left(2\left(-917\sqrt{2}\int\frac{1-\sqrt{2}x^2}{\sqrt{2x^4+2x^2+1}}dx+\frac{2^{3/4}\left(767-132\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{\right.\right.}{\sqrt{2x^4+2x^2+1}}\right.\right.\right.$$

$$\frac{578}{189}\int\frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{\left(3-2x^2\right)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \text{1509}$$

$$\frac{1}{189}\left(\frac{1}{5}\left(2\left(\frac{2^{3/4}\left(767-132\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{\sqrt{2x^4+2x^2+1}}-917\sqrt{2}\left(\frac{\left(\sqrt{\right.}}{}\right.\right.\right.\right.$$

$$\frac{578}{189}\int\frac{-2\left(3-\sqrt{2}\right)x^2-3\sqrt{2}+2}{\left(3-2x^2\right)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \text{2222}$$

$$\frac{1}{189}\left(\frac{1}{5}\left(2\left(\frac{2^{3/4}\left(767-132\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{\sqrt{2x^4+2x^2+1}}-917\sqrt{2}\left(\frac{\left(\sqrt{\right.}}{}\right.\right.\right.\right.$$

$$\frac{578}{189}\left(\frac{\left(3-\sqrt{2}\right)^2\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticPi}\left(\frac{1}{24}\left(12+11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{12\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-\frac{7\text{arctanh}\left(\right.}{2}\right.$$

input
```
Int[(1 + 2*x^2 + 2*x^4)^(3/2)/(x^6*(3 - 2*x^2)),x]
```

3.331. $\int\frac{\left(1+2x^2+2x^4\right)^{3/2}}{x^6\left(3-2x^2\right)}dx$

output
```
((2*(503 - 289*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])/(3*x^5) - (17*(307 - 170*
Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])/(15*x^5) - ((678 - 289*Sqrt[2])*Sqrt[1 +
 2*x^2 + 2*x^4])/x^3 + (((3068 - 1445*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])/x^
3 + 2*((-917*Sqrt[1 + 2*x^2 + 2*x^4])/x - 917*Sqrt[2]*(-((x*Sqrt[1 + 2*x^2
 + 2*x^4])/(1 + Sqrt[2]*x^2)) + ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4
)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(2
^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])) + (2^(3/4)*(767 - 132*Sqrt[2])*(1 + Sqrt[
2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2
^(1/4)*x], (2 - Sqrt[2])/4])/Sqrt[1 + 2*x^2 + 2*x^4]))/5)/189 - (578*((-7*
ArcTanh[(Sqrt[17/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/(2*Sqrt[51]) + ((3 - Sqrt
[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Ell
ipticPi[(12 + 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(12*2
^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])))/189
```

3.331.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

3.331. $\int \frac{(1+2x^2+2x^4)^{3/2}}{x^6(3-2x^2)}\,dx$

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1634
```
Int[((x_)^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[(-(-d/e)^(m/2))*((c*d^2 - b*d*e + a*e^2)^(p + 1/2)
/(e^(2*p)*(c*d^2 - a*e^2)))   Int[(a*d*Rt[c/a, 2] + a*e + (c*d + a*e*Rt[c/a
, 2])*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] + Simp[(-d/e)^(m/2
)/(e^(2*p)*(c*d^2 - a*e^2))   Int[(x^m/Sqrt[a + b*x^2 + c*x^4])*ExpandToSum
[((e^(2*p)*(c*d^2 - a*e^2)*(a + b*x^2 + c*x^4)^(p + 1/2))/(-d/e)^(m/2) + ((
a*d*Rt[c/a, 2] + a*e + (c*d + a*e*Rt[c/a, 2])*x^2)*(c*d^2 - b*d*e + a*e^2)^
(p + 1/2))/x^m)/(d + e*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p + 1/2, 0] && ILtQ[m/2, 0] && NeQ[c*d^2 - a*e^2,
0] && PosQ[c/a]
```

rule 2199
```
Int[(Px_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> With[{q = Expon[Px, x^2]}, Simp[Coeff[Px, x^2, q]*(d*x)^(m + 2*q
- 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*d^(2*q - 3)*(m + 4*p + 2*q + 1))), x]
+ Int[(d*x)^m*(a + b*x^2 + c*x^4)^p*ExpandToSum[Px - Coeff[Px, x^2, q]*x^(2
*q) - Coeff[Px, x^2, q]*((a*(m + 2*q - 3)*x^(2*(q - 2)) + b*(m + 2*p + 2*q
- 1)*x^(2*(q - 1)))/(c*(m + 4*p + 2*q + 1))), x], x] /; GtQ[q, 1] && NeQ[m
+ 4*p + 2*q + 1, 0]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Px, x^2] && N
eQ[b^2 - 4*a*c, 0]
```

3.331. $\int \frac{(1+2x^2+2x^4)^{3/2}}{x^6(3-2x^2)}\,dx$

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

3.331.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.11 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.48

method	result
risch	$-\frac{524x^8+616x^6+372x^4+64x^2+9}{135x^5\sqrt{2x^4+2x^2+1}}-\frac{362\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{135\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{262}{135}+\frac{262i}{135}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(}$
elliptic	$-\frac{\sqrt{2x^4+2x^2+1}}{15x^5}-\frac{46\sqrt{2x^4+2x^2+1}}{135x^3}-\frac{262\sqrt{2x^4+2x^2+1}}{135x}-\frac{208\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{45\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{262i\sqrt{-}}{}$
default	$-\frac{\sqrt{2x^4+2x^2+1}}{15x^5}-\frac{46\sqrt{2x^4+2x^2+1}}{135x^3}-\frac{262\sqrt{2x^4+2x^2+1}}{135x}+\frac{184\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{45\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{52}{15}+\right.}{}$

input
```
int((2*x^4+2*x^2+1)^(3/2)/x^6/(-2*x^2+3),x,method=_RETURNVERBOSE)
```

output
```
-1/135*(524*x^8+616*x^6+372*x^4+64*x^2+9)/x^5/(2*x^4+2*x^2+1)^(1/2)-362/13
5/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/
2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+(-262/135+262/135*I
)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/
2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)
^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))+578/81/(-1+I)^(1/2)*(1-I*x^2+x^2)^(1/2)
*(1+I*x^2+x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),-1/3-
1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

3.331.5 Fricas [F]

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{x^6(3-2x^2)}\,dx = \int -\frac{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}}{(2\,x^2-3)x^6}\,dx$$

input
```
integrate((2*x^4+2*x^2+1)^(3/2)/x^6/(-2*x^2+3),x, algorithm="fricas")
```

output
```
integral(-(2*x^4 + 2*x^2 + 1)^(3/2)/(2*x^8 - 3*x^6), x)
```

3.331.6 Sympy [F]

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{x^6(3-2x^2)}\,dx = -\int \frac{\sqrt{2x^4+2x^2+1}}{2x^8-3x^6}\,dx - \int \frac{2x^2\sqrt{2x^4+2x^2+1}}{2x^8-3x^6}\,dx - \int \frac{2x^4\sqrt{2x^4+2x^2+1}}{2x^8-3x^6}\,dx$$

input
```
integrate((2*x**4+2*x**2+1)**(3/2)/x**6/(-2*x**2+3),x)
```

output
```
-Integral(sqrt(2*x**4 + 2*x**2 + 1)/(2*x**8 - 3*x**6), x) - Integral(2*x**
2*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**8 - 3*x**6), x) - Integral(2*x**4*sqrt(2
*x**4 + 2*x**2 + 1)/(2*x**8 - 3*x**6), x)
```

3.331.7 Maxima [F]

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{x^6(3-2x^2)}\,dx = \int -\frac{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}}{(2\,x^2-3)x^6}\,dx$$

input
```
integrate((2*x^4+2*x^2+1)^(3/2)/x^6/(-2*x^2+3),x, algorithm="maxima")
```

output
```
-integrate((2*x^4 + 2*x^2 + 1)^(3/2)/((2*x^2 - 3)*x^6), x)
```

3.331.8 Giac [F]

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{x^6(3-2x^2)}\,dx = \int -\frac{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}}{(2\,x^2-3)x^6}\,dx$$

input
```
integrate((2*x^4+2*x^2+1)^(3/2)/x^6/(-2*x^2+3),x, algorithm="giac")
```

output
```
integrate(-(2*x^4 + 2*x^2 + 1)^(3/2)/((2*x^2 - 3)*x^6), x)
```

3.331.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{x^6(3-2x^2)}\,dx = -\int \frac{(2\,x^4+2\,x^2+1)^{3/2}}{x^6\,(2\,x^2-3)}\,dx$$

input
```
int(-(2*x^2 + 2*x^4 + 1)^(3/2)/(x^6*(2*x^2 - 3)),x)
```

output
```
-int((2*x^2 + 2*x^4 + 1)^(3/2)/(x^6*(2*x^2 - 3)), x)
```

3.332 $\int \frac{x^5}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

3.332.1 Optimal result

Integrand size = 29, antiderivative size = 173

$$\int \frac{x^5}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \frac{\sqrt{a+bx^2+cx^4}}{2ce} - \frac{(2cd+be)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}e^2} + \frac{d^2\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2e^2\sqrt{cd^2-bde+ae^2}}$$

output
```
-1/4*(b*e+2*c*d)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^
(3/2)/e^2+1/2*d^2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*
d^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2))/e^2/(a*e^2-b*d*e+c*d^2)^(1/2)+1/2*(c*x^4
+b*x^2+a)^(1/2)/c/e
```

3.332.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.96

$$\int \frac{x^5}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \frac{2\sqrt{c}\left(e\sqrt{a+bx^2+cx^4} - \frac{2cd^2\arctan\left(\frac{\sqrt{c}(d+ex^2)-e\sqrt{a+bx^2+cx^4}}{\sqrt{-cd^2+bde-ae^2}}\right)}{\sqrt{-cd^2+bde-ae^2}}\right) - (2cd+be)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}e^2}$$

input
```
Integrate[x^5/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
(2*Sqrt[c]*(e*Sqrt[a + b*x^2 + c*x^4] - (2*c*d^2*ArcTan[(Sqrt[c]*(d + e*x^
2) - e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + b*d*e - a*e^2]])/Sqrt[-(c*
d^2) + b*d*e - a*e^2]) - (2*c*d + b*e)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sq
rt[a + b*x^2 + c*x^4])])/(4*c^(3/2)*e^2)
```

3.332.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {1578, 1267, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(d+ex^2)\sqrt{a+bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{x^4}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx^2$$

$$\Big\downarrow \text{1267}$$

$$\frac{1}{2}\left(\frac{\int -\frac{e((2cd+be)x^2+bd)}{2(ex^2+d)\sqrt{cx^4+bx^2+a}}dx^2}{ce^2}+\frac{\sqrt{a+bx^2+cx^4}}{ce}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\sqrt{a+bx^2+cx^4}}{ce}-\frac{\int \frac{(2cd+be)x^2+bd}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx^2}{2ce}\right)$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{2}\left(\frac{\sqrt{a+bx^2+cx^4}}{ce}-\frac{\frac{(be+2cd)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx^2}{e}-\frac{2cd^2\int \frac{1}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx^2}{e}}{2ce}\right)$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}\left(\frac{\sqrt{a+bx^2+cx^4}}{ce}-\frac{\frac{2(be+2cd)\int \frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}}{e}-\frac{2cd^2\int \frac{1}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx^2}{e}}{2ce}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{\sqrt{a+bx^2+cx^4}}{ce}-\frac{\frac{(be+2cd)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{ce}}-\frac{2cd^2\int \frac{1}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx^2}{e}}{2ce}\right)$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{2}\left(\frac{\sqrt{a+bx^2+cx^4}}{ce}-\frac{\frac{4cd^2\int \frac{1}{4(cd^2-bed+ae^2)-x^4}d\left(-\frac{(2cd-be)x^2+bd-2ae}{\sqrt{cx^4+bx^2+a}}\right)}{e}+\frac{(be+2cd)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{ce}}}{2ce}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{\sqrt{a+bx^2+cx^4}}{ce}-\frac{\frac{(be+2cd)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{ce}}-\frac{2cd^2\text{arctanh}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{e\sqrt{ae^2-bde+cd^2}}}{2ce}\right)$$

input
```
Int[x^5/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
(Sqrt[a + b*x^2 + c*x^4]/(c*e) - (((2*c*d + b*e)*ArcTanh[(b + 2*c*x^2)/(2*
Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(Sqrt[c]*e) - (2*c*d^2*ArcTanh[(b*d - 2
*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 +
c*x^4])])/(e*Sqrt[c*d^2 - b*d*e + a*e^2]))/(2*c*e))/2
```

3.332.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1267
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - g^n*(d
+ e*x)^(n - 2)*(b*d*e*(p + 1) + a*e^2*(m + n - 1) - c*d^2*(m + n + 2*p + 1)
 - e*(2*c*d - b*e)*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && IGtQ[n, 1] && IntegerQ[m] && NeQ[m + n + 2*p + 1, 0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.332. $\int \frac{x^5}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

3.332.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.22

method	result
pseudoelliptic	$\frac{-\ln\left(\frac{2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,e+\left(b\,x^2+2a\right)e-d\left(2c\,x^2+b\right)}{e\,x^2+d}\right)c^{\frac{3}{2}}d^2+\left(\left(-cd-\frac{be}{2}\right)\ln\left(\frac{2c\,x^2+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{c}+b}{\sqrt{c}}\right)+\sqrt{c\,x\cdots}{2c^{\frac{3}{2}}\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,e^3}$
risch	$\frac{\sqrt{c\,x^4+b\,x^2+a}}{2ce}-\frac{\frac{(be+2cd)\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{2e\sqrt{c}}+\frac{c\,d^2\ln\left(\frac{\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\sqrt{c(\cdots}{x^2+\frac{d}{e}}\right)}{e^2\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\cdots}}{2ce}$
default	$\frac{\frac{\sqrt{c\,x^4+b\,x^2+a}}{2c}-\frac{b\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{4c^{\frac{3}{2}}}}{e}-\frac{d\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{2e^2\sqrt{c}}-\frac{d^2\ln\left(\frac{\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+\frac{(be-2cd)\left(x^2\cdots}{e}}{\cdots}\right)}{\cdots}$
elliptic	$-\frac{d\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{2e^2\sqrt{c}}+\frac{\sqrt{c\,x^4+b\,x^2+a}}{2ce}-\frac{b\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{4e\,c^{\frac{3}{2}}}-\frac{d^2\ln\left(\frac{\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+\frac{(be-\cdots}{\cdots}\right)}{\cdots}$

input
```
int(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/c^(3/2)*(-ln((2*(c*x^4+b*x^2+a)^(1/2)*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*
e+(b*x^2+2*a)*e-d*(2*c*x^2+b))/(e*x^2+d))*c^(3/2)*d^2+((-c*d-1/2*b*e)*ln((
2*c*x^2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c^(1/2))+(c*x^4+b*x^2+a)^(1/2)*
c^(1/2)*e+1/2*ln(2)*(b*e+2*c*d))*e*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/((a*e^
2-b*d*e+c*d^2)/e^2)^(1/2)/e^3
```

3.332.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 319 vs. $2(149) = 298$.

Time = 12.72 (sec) , antiderivative size = 1364, normalized size of antiderivative = 7.88

$$\int \frac{x^5}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \text{Too large to display}$$

input
```
integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(2*sqrt(c*d^2 - b*d*e + a*e^2)*c^2*d^2*log(-((8*c^2*d^2 - 8*b*c*d*e +
 (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4
*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a
)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4
+ 2*d*e*x^2 + d^2)) + (2*c^2*d^3 - b*c*d^2*e + a*b*e^3 - (b^2 - 2*a*c)*d*e
^2)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(
2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*(c^2*d^2*e - b*c*d*e^2 + a*c*e^3)*sqrt(c
*x^4 + b*x^2 + a))/(c^3*d^2*e^2 - b*c^2*d*e^3 + a*c^2*e^4), 1/8*(4*sqrt(-c
*d^2 + b*d*e - a*e^2)*c^2*d^2*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*
d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e
 + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e
^2)*x^2)) + (2*c^2*d^3 - b*c*d^2*e + a*b*e^3 - (b^2 - 2*a*c)*d*e^2)*sqrt(c
)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 +
b)*sqrt(c) - 4*a*c) + 4*(c^2*d^2*e - b*c*d*e^2 + a*c*e^3)*sqrt(c*x^4 + b*x
^2 + a))/(c^3*d^2*e^2 - b*c^2*d*e^3 + a*c^2*e^4), 1/4*(sqrt(c*d^2 - b*d*e
+ a*e^2)*c^2*d^2*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8
*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b
^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^
2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)) + (2*c^
2*d^3 - b*c*d^2*e + a*b*e^3 - (b^2 - 2*a*c)*d*e^2)*sqrt(-c)*arctan(1/2*...
```

3.332.6 Sympy [F]

$$\int \frac{x^5}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^5}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate(x**5/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(x**5/((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

3.332.7 Maxima [F]

$$\int \frac{x^5}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^5}{\sqrt{cx^4+bx^2+a}(ex^2+d)}\,dx$$

input
```
integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^5/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

3.332.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^5}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.332.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^5}{(e\,x^2+d)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int(x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int(x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.333 $\int \frac{x^3}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

3.333.1 Optimal result

Integrand size = 29, antiderivative size = 137

$$\int \frac{x^3}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \frac{\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}e} - \frac{d\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2e\sqrt{cd^2-bde+ae^2}}$$

output
```
1/2*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/e/c^(1/2)-1/2*d
*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^4
+b*x^2+a)^(1/2))/e/(a*e^2-b*d*e+c*d^2)^(1/2)
```

3.333.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.07

$$\int \frac{x^3}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\frac{2d\sqrt{-cd^2+bde-ae^2}\arctan\left(\frac{\sqrt{c}(d+ex^2)-e\sqrt{a+bx^2+cx^4}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{cd^2+e(-bd+ae)} + \frac{\log\left(e\left(b+2cx^2-2\sqrt{c}\sqrt{a+bx^2+cx^4}\right)\right)}{\sqrt{c}}}{2e}$$

input
```
Integrate[x^3/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
-1/2*((2*d*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x^2) - e*
Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(c*d^2 + e*(-(b*
d) + a*e)) + Log[e*(b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]/Sqrt
[c])/e
```

3.333.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1578, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(d+ex^2)\sqrt{a+bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{x^2}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx^2$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx^2}{e}-\frac{d\int \frac{1}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx^2}{e}\right)$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}\left(\frac{2\int \frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}}{e}-\frac{d\int \frac{1}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx^2}{e}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{c}e}-\frac{d\int \frac{1}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx^2}{e}\right)$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{2}\left(\frac{2d\int \frac{1}{4(cd^2-bed+ae^2)-x^4}d\left(-\frac{(2cd-be)x^2+bd-2ae}{\sqrt{cx^4+bx^2+a}}\right)}{e}+\frac{\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{c}e}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{c}e}-\frac{d\operatorname{arctanh}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{e\sqrt{ae^2-bde+cd^2}}\right)$$

input
```
Int[x^3/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
(ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]/(Sqrt[c]*e) -
(d*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2
]*Sqrt[a + b*x^2 + c*x^4])])/(e*Sqrt[c*d^2 - b*d*e + a*e^2]))/2
```

3.333.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.333.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.25

method	result
pseudoelliptic	$\frac{d \ln \Big(\frac{2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,e+\left(b\,x^2+2a\right)e-d\left(2c\,x^2+b\right)}{e\,x^2+d} \Big)\sqrt{c}+\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,e\Big(-\ln(2)+\ln\Big(\frac{2c\,x^2+2\sqrt{c\,x^4+b\,x^2+a}}{\sqrt{c}}}{2\sqrt{c}\,\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,e^2}$
default	$\frac{\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{2e\sqrt{c}}+\frac{d\ln\Big(\frac{\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,\sqrt{c\left(x^2+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}}}{x^2+\frac{d}{e}}}{2e^2\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}}$
elliptic	$\frac{\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{2e\sqrt{c}}+\frac{d\ln\Big(\frac{\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,\sqrt{c\left(x^2+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}}}{x^2+\frac{d}{e}}}{2e^2\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}}$

input
```
int(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/c^(1/2)*(d*ln((2*(c*x^4+b*x^2+a)^(1/2)*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
*e+(b*x^2+2*a)*e-d*(2*c*x^2+b))/(e*x^2+d))*c^(1/2)+((a*e^2-b*d*e+c*d^2)/e^
2)^(1/2)*e*(-ln(2)+ln((2*c*x^2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c^(1/2))
))/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)/e^2
```

3.333.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(117) = 234$.

Time = 0.96 (sec) , antiderivative size = 1084, normalized size of antiderivative = 7.91

$$\int \frac{x^3}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$$

$$= \Bigg[\frac{\sqrt{cd^2-bde+ae^2}cd\log\Big(-\frac{(8\,c^2d^2-8\,bcde+(b^2+4\,ac)e^2)x^4-8\,abde+8\,a^2e^2+(b^2+4\,ac)d^2+2\,(4\,bcd^2+4\,abe^2-(3\,b^2+4\,ac)de)x^2-4\,\sqrt{}}{e^2x^4+2\,dex^2+d^2}}{}$$

$$-\frac{2\sqrt{-cd^2+bde-ae^2}cd\arctan\Big(-\frac{\sqrt{cx^4+bx^2+a}\sqrt{-cd^2+bde-ae^2}((2\,cd-be)x^2+bd-2\,ae)}{2\,((c^2d^2-bcde+ace^2)x^4+acd^2-abde+a^2e^2+(bcd^2-b^2de+abe^2)x^2)}\Big)-(cd^2-bde+a}{4\,(c^2d^2e-bcde^2+ace^3)}$$

$$-\frac{\sqrt{-cd^2+bde-ae^2}cd\arctan\Big(-\frac{\sqrt{cx^4+bx^2+a}\sqrt{-cd^2+bde-ae^2}((2\,cd-be)x^2+bd-2\,ae)}{2\,((c^2d^2-bcde+ace^2)x^4+acd^2-abde+a^2e^2+(bcd^2-b^2de+abe^2)x^2)}\Big)+(cd^2-bde+ae^2}{2\,(c^2d^2e-bcde^2+ace^3)}$$

input
```
integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(c*d^2 - b*d*e + a*e^2)*c*d*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2
+ 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d
^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt
(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*
e*x^2 + d^2)) + (c*d^2 - b*d*e + a*e^2)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2
 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c))/(c^2*d^
2*e - b*c*d*e^2 + a*c*e^3), -1/4*(2*sqrt(-c*d^2 + b*d*e - a*e^2)*c*d*arcta
n(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)
*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e
 + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - (c*d^2 - b*d*e + a*e^2)
*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c
*x^2 + b)*sqrt(c) - 4*a*c))/(c^2*d^2*e - b*c*d*e^2 + a*c*e^3), 1/4*(sqrt(c
*d^2 - b*d*e + a*e^2)*c*d*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2
)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e
^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d
*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)
) - 2*(c*d^2 - b*d*e + a*e^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*
(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)))/(c^2*d^2*e - b*c*d*e^2
+ a*c*e^3), -1/2*(sqrt(-c*d^2 + b*d*e - a*e^2)*c*d*arctan(-1/2*sqrt(c*x^4
+ b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*...
```

3.333.6 Sympy [F]

$$\int \frac{x^3}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^3}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate(x**3/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(x**3/((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

3.333.7 Maxima [F]

$$\int \frac{x^3}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^3}{\sqrt{cx^4+bx^2+a}(ex^2+d)}\,dx$$

input
```
integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

3.333.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.333.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^3}{(e\,x^2+d)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int(x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int(x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.334 $\int \frac{x}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

3.334.1 Optimal result

Integrand size = 27, antiderivative size = 86

$$\int \frac{x}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx=\frac{\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{cd^2-bde+ae^2}}$$

output
```
1/2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*
x^4+b*x^2+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(1/2)
```

3.334.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.12

$$\int \frac{x}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx=\frac{\sqrt{-cd^2+bde-ae^2}\arctan\left(\frac{\sqrt{c}(d+ex^2)-e\sqrt{a+bx^2+cx^4}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{cd^2+e(-bd+ae)}$$

input
```
Integrate[x/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
(Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x^2) - e*Sqrt[a + b
*x^2 + c*x^4])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(c*d^2 + e*(-(b*d) + a*e))
```

3.334.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1576, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(d+ex^2)\sqrt{a+bx^2+cx^4}}dx$$

$$\Big\downarrow \text{ 1576}$$

$$\frac{1}{2}\int \frac{1}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx^2$$

$$\Big\downarrow \text{ 1154}$$

$$-\int \frac{1}{4(cd^2-bed+ae^2)-x^4}d\left(-\frac{(2cd-be)x^2+bd-2ae}{\sqrt{cx^4+bx^2+a}}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\text{arctanh}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2\sqrt{ae^2-bde+cd^2}}$$

input
```
Int[x/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*S
qrt[a + b*x^2 + c*x^4])]/(2*Sqrt[c*d^2 - b*d*e + a*e^2])
```

3.334.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

3.334.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.17

method	result	size
pseudoelliptic	$-\frac{\ln\left(\frac{2\sqrt{c x^4+b x^2+a}\,\sqrt{\frac{a e^2-bde+c d^2}{e^2}}\, e+\left(b x^2+2a\right)e-d\left(2c x^2+b\right)}{e x^2+d}\right)}{2\sqrt{\frac{a e^2-bde+c d^2}{e^2}}\, e}$	101
default	$-\frac{\ln\left(\frac{\frac{2a e^2-2bde+2c d^2}{e^2}+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a e^2-bde+c d^2}{e^2}}\sqrt{c\left(x^2+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a e^2-bde+c d^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{2e\sqrt{\frac{a e^2-bde+c d^2}{e^2}}}$	165
elliptic	$-\frac{\ln\left(\frac{\frac{2a e^2-2bde+2c d^2}{e^2}+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a e^2-bde+c d^2}{e^2}}\sqrt{c\left(x^2+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a e^2-bde+c d^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{2e\sqrt{\frac{a e^2-bde+c d^2}{e^2}}}$	165

input
```
int(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(c*x^4+b*x^2+a)^(1/2)*((a*e^2-b
*d*e+c*d^2)/e^2)^(1/2)*e+(b*x^2+2*a)*e-d*(2*c*x^2+b))/(e*x^2+d))/e
```

3.334.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(76) = 152$.

Time = 0.34 (sec) , antiderivative size = 357, normalized size of antiderivative = 4.15

$$\int \frac{x}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \left[\frac{\log\left(-\frac{(8\,c^2d^2-8\,bcde+(b^2+4\,ac)e^2)x^4-8\,abde+8\,a^2e^2+(b^2+4\,ac)d^2+2\,(4\,bcd^2+4\,abe^2-(3\,b^2+4\,ac)de)x^2+4\,\sqrt{cx^4+bx^2+a}\sqrt{cd^2-bde+ae^2}}{e^2x^4+2\,dex^2+d^2}\right.}{4\,\sqrt{cd^2-bde+ae^2}}\right.$$

input
```
integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8
*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*
d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d -
 b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2))/sqrt(c*d^2 - b*d*e
+ a*e^2), 1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(c*x^4 + b*x^2
+ a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*
d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^
2*d*e + a*b*e^2)*x^2))/(c*d^2 - b*d*e + a*e^2)]
```

3.334.6 Sympy [F]

$$\int \frac{x}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate(x/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(x/((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

3.334.7 Maxima [F]

$$\int \frac{x}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x}{\sqrt{cx^4+bx^2+a}(ex^2+d)}\,dx$$

input
```
integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

3.334.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.86

$$\int \frac{x}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \frac{\arctan\left(-\frac{\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{\sqrt{-cd^2+bde-ae^2}}$$

input
```
integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^
2 + b*d*e - a*e^2))/sqrt(-c*d^2 + b*d*e - a*e^2)
```

3.334.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x}{(e\,x^2+d)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int(x/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int(x/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.335 $\int \frac{1}{x(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

3.335.1 Optimal result

Integrand size = 29, antiderivative size = 138

$$\int \frac{1}{x(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}d} - \frac{e\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2d\sqrt{cd^2-bde+ae^2}}$$

output
```
-1/2*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d/a^(1/2)-1/2*
e*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^
4+b*x^2+a)^(1/2))/d/(a*e^2-b*d*e+c*d^2)^(1/2)
```

3.335.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.04

$$\int \frac{1}{x(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \frac{-\frac{e\sqrt{-cd^2+bde-ae^2}\arctan\left(\frac{\sqrt{c}(d+ex^2)-e\sqrt{a+bx^2+cx^4}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{cd^2+e(-bd+ae)} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{\sqrt{a}}}{d}$$

input
```
Integrate[1/(x*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
(-((e*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x^2) - e*Sqrt[
a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(c*d^2 + e*(-(b*d) +
a*e))) + ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]]/Sqrt[a])
/d
```

3.335.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1578, 1289, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(d+ex^2)\sqrt{a+bx^2+cx^4}}dx$$

$$\Big\downarrow \text{ 1578}$$

$$\frac{1}{2}\int \frac{1}{x^2(ex^2+d)\sqrt{cx^4+bx^2+a}}dx^2$$

$$\Big\downarrow \text{ 1289}$$

$$\frac{1}{2}\int \left(\frac{1}{dx^2\sqrt{cx^4+bx^2+a}}-\frac{e}{d(ex^2+d)\sqrt{cx^4+bx^2+a}}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{e\,\text{arctanh}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{d\sqrt{ae^2-bde+cd^2}}-\frac{\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{a}d}\right)$$

input
```
Int[1/(x*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
(-(ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/(Sqrt[a]*d))
 - (e*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*
e^2]*Sqrt[a + b*x^2 + c*x^4])])/(d*Sqrt[c*d^2 - b*d*e + a*e^2]))/2
```

3.335.3.1 Defintions of rubi rules used

rule 1289
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0]))
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.335.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.20

method	result
pseudoelliptic	$\frac{\ln\left(\frac{2\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}\,e+\left(b\,x^{2}+2a\right)e-d\left(2c\,x^{2}+b\right)}{e\,x^{2}+d}\right)\sqrt{a}-\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)\sqrt{\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}}{2d\sqrt{a}\,\sqrt{\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}}$
default	$-\frac{\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{2d\sqrt{a}}+\frac{\ln\left(\frac{\frac{2a\,e^{2}-2bde+2c\,d^{2}}{e^{2}}+\frac{(be-2cd)\left(x^{2}+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}\,\sqrt{c\left(x^{2}+\frac{d}{e}\right)^{2}+\frac{(be-2cd)\left(x^{2}\cdots\right.}{e}}}{x^{2}+\frac{d}{e}}\cdots\right.}{2d\sqrt{\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}}$
elliptic	$-\frac{\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{2d\sqrt{a}}+\frac{\ln\left(\frac{\frac{2a\,e^{2}-2bde+2c\,d^{2}}{e^{2}}+\frac{(be-2cd)\left(x^{2}+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}\,\sqrt{c\left(x^{2}+\frac{d}{e}\right)^{2}+\frac{(be-2cd)\left(x^{2}\cdots\right.}{e}}}{x^{2}+\frac{d}{e}}\cdots\right.}{2d\sqrt{\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}}$

input
```
int(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(ln((2*(c*x^4+b*x^2+a)^(1/2)*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*e+(b*x^2+
2*a)*e-d*(2*c*x^2+b))/(e*x^2+d))*a^(1/2)-ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*
x^2+a)^(1/2))/x^2)*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/d/a^(1/2)/((a*e^2-b*d*
e+c*d^2)/e^2)^(1/2)
```

3.335.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(118) = 236$.

Time = 0.46 (sec) , antiderivative size = 1097, normalized size of antiderivative = 7.95

$$\int \frac{1}{x(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \left[\frac{\sqrt{cd^2-bde+ae^2}ae\log\left(-\frac{(8c^2d^2-8bcde+(b^2+4ac)e^2)x^4-8abde+8a^2e^2+(b^2+4ac)d^2+2(4bcd^2+4abe^2-(3b^2+4ac)de)x^2-4\sqrt{}}{e^2x^4+2dex^2+d^2}\right.}{4(acd^3}\right.$$
$$-\frac{2\sqrt{-cd^2+bde-ae^2}ae\arctan\left(-\frac{\sqrt{cx^4+bx^2+a}\sqrt{-cd^2+bde-ae^2}((2cd-be)x^2+bd-2ae)}{2((c^2d^2-bcde+ace^2)x^4+acd^2-abde+a^2e^2+(bcd^2-b^2de+abe^2)x^2)}\right)-(cd^2-bde+a}{4(acd^3-abd^2e+a^2de^2)}$$
$$-\frac{\sqrt{-cd^2+bde-ae^2}ae\arctan\left(-\frac{\sqrt{cx^4+bx^2+a}\sqrt{-cd^2+bde-ae^2}((2cd-be)x^2+bd-2ae)}{2((c^2d^2-bcde+ace^2)x^4+acd^2-abde+a^2e^2+(bcd^2-b^2de+abe^2)x^2)}\right)-(cd^2-bde+ae^2}{2(acd^3-abd^2e+a^2de^2)}$$

input
```
integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(c*d^2 - b*d*e + a*e^2)*a*e*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2
+ 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d
^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt
(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*
e*x^2 + d^2)) + (c*d^2 - b*d*e + a*e^2)*sqrt(a)*log(-((b^2 + 4*a*c)*x^4 +
8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4))
/(a*c*d^3 - a*b*d^2*e + a^2*d*e^2), -1/4*(2*sqrt(-c*d^2 + b*d*e - a*e^2)*a
*e*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*
d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 -
 a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - (c*d^2 - b*d*e
+ a*e^2)*sqrt(a)*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^
2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4))/(a*c*d^3 - a*b*d^2*e + a^2*d*e
^2), 1/4*(sqrt(c*d^2 - b*d*e + a*e^2)*a*e*log(-((8*c^2*d^2 - 8*b*c*d*e + (
b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b
*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*
sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 +
2*d*e*x^2 + d^2)) + 2*(c*d^2 - b*d*e + a*e^2)*sqrt(-a)*arctan(1/2*sqrt(c*x
^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)))/(a*c*d^
3 - a*b*d^2*e + a^2*d*e^2), -1/2*(sqrt(-c*d^2 + b*d*e - a*e^2)*a*e*arctan(
-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e...
```

3.335.6 Sympy [F]

$$\int \frac{1}{x\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{x\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate(1/x/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(1/(x*(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

3.335.7 Maxima [F]

$$\int \frac{1}{x\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^2+a}(ex^2+d)x}\,dx$$

input
```
integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*x), x)
```

3.335.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.335.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{1}{x\,\left(e\,x^2+d\right)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int(1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int(1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.336 $\int \frac{1}{x^3(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

3.336.1 Optimal result

Integrand size = 29, antiderivative size = 218

$$\int \frac{1}{x^3(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\sqrt{a+bx^2+cx^4}}{2adx^2}+\frac{b\,\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}d}+\frac{e\,\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}d^2}+\frac{e^2\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2d^2\sqrt{cd^2-bde+ae^2}}$$

output
```
1/4*b*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(3/2)/d+1/2
*e*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d^2/a^(1/2)+1/2*
e^2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*
x^4+b*x^2+a)^(1/2))/d^2/(a*e^2-b*d*e+c*d^2)^(1/2)-1/2*(c*x^4+b*x^2+a)^(1/2
)/a/d/x^2
```

3.336.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^3 (d+ex^2) \sqrt{a+bx^2+cx^4}}\,dx = \frac{-\frac{2d\sqrt{a+bx^2+cx^4}}{ax^2}+\frac{4e^2\sqrt{-cd^2+bde-ae^2}\arctan\left(\frac{\sqrt{-cd^2+bde-ae^2}x^2}{\sqrt{a}\left(d+ex^2\right)-d\sqrt{a+bx^2+cx^4}}\right)}{cd^2+e(-bd+ae)}+\frac{(bd+2ae)\log(x^2)}{a^{3/2}}-\frac{(bd+2ae)\log\left(ad^2\left(2a+bx^2-2\sqrt{a}\sqrt{\cdots}\right)\right)}{a^{3/2}}}{4d^2}$$

input
```
Integrate[1/(x^3*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
((-2*d*Sqrt[a + b*x^2 + c*x^4])/(a*x^2) + (4*e^2*Sqrt[-(c*d^2) + b*d*e - a
*e^2]*ArcTan[(Sqrt[-(c*d^2) + b*d*e - a*e^2]*x^2)/(Sqrt[a]*(d + e*x^2) - d
*Sqrt[a + b*x^2 + c*x^4])])/(c*d^2 + e*(-(b*d) + a*e)) + ((b*d + 2*a*e)*Lo
g[x^2])/a^(3/2) - ((b*d + 2*a*e)*Log[a*d^2*(2*a + b*x^2 - 2*Sqrt[a]*Sqrt[a
 + b*x^2 + c*x^4])])/a^(3/2))/(4*d^2)
```

3.336.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1578, 1289, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (d+ex^2) \sqrt{a+bx^2+cx^4}}\,dx$$

$$\Big\downarrow \; 1578$$

$$\frac{1}{2}\int \frac{1}{x^4 (ex^2+d) \sqrt{cx^4+bx^2+a}}dx^2$$

$$\Big\downarrow \; 1289$$

$$\frac{1}{2}\int \left(\frac{e^2}{d^2 (ex^2+d) \sqrt{cx^4+bx^2+a}}-\frac{e}{d^2x^2\sqrt{cx^4+bx^2+a}}+\frac{1}{dx^4\sqrt{cx^4+bx^2+a}}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{b\operatorname{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}d}+\frac{e^2\operatorname{arctanh}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{d^2\sqrt{ae^2-bde+cd^2}}+\frac{e\operatorname{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{a}d^2}-\frac{\sqrt{a+bx^2}}{adx}\right.$$

input
```
Int[1/(x^3*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
(-(Sqrt[a + b*x^2 + c*x^4]/(a*d*x^2)) + (b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a
]*Sqrt[a + b*x^2 + c*x^4])])/(2*a^(3/2)*d) + (e*ArcTanh[(2*a + b*x^2)/(2*S
qrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(Sqrt[a]*d^2) + (e^2*ArcTanh[(b*d - 2*a*
e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x
^4])])/(d^2*Sqrt[c*d^2 - b*d*e + a*e^2]))/2
```

3.336.3.1 Defintions of rubi rules used

rule 1289
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0]))
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.336.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.94

method	result
pseudoelliptic	$\frac{-\ln\left(\frac{2\sqrt{c x^4+b x^2+a}\,\sqrt{\frac{a e^2-bde+c d^2}{e^2}}\,e+\left(b x^2+2a\right)e-d\left(2c x^2+b\right)}{e x^2+d}\right)a^{\frac{3}{2}}e x^2+\sqrt{\frac{a e^2-bde+c d^2}{e^2}}\,\left(x^2\left(ae+\frac{bd}{2}\right)\ln\left(\frac{2a+b x^2+2\sqrt{a}\,\sqrt{}}{x^2}\right.\right.}{2a^{\frac{3}{2}}\sqrt{\frac{a e^2-bde+c d^2}{e^2}}\,d^2x^2}$
risch	$-\frac{\sqrt{c x^4+b x^2+a}}{2ad\,x^2}-\frac{-\frac{(2ae+bd)\ln\left(\frac{2a+b x^2+2\sqrt{a}\,\sqrt{c x^4+b x^2+a}}{x^2}\right)}{2d\sqrt{a}}+\frac{ae\ln\left(\frac{\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2-bde+c}{e^2}}}{x^2+}\right.}{d\sqrt{\frac{a\,e^2-bde}{e^2}}}}{2da}$
default	$\frac{-\frac{\sqrt{c x^4+b x^2+a}}{2a\,x^2}+\frac{b\ln\left(\frac{2a+b x^2+2\sqrt{a}\,\sqrt{c x^4+b x^2+a}}{x^2}\right)}{4a^{\frac{3}{2}}}}{d}+\frac{e\ln\left(\frac{2a+b x^2+2\sqrt{a}\,\sqrt{c x^4+b x^2+a}}{x^2}\right)}{2d^2\sqrt{a}}-\frac{e\ln\left(\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+\frac{(be-}{}\right.}{}$
elliptic	$-\frac{\sqrt{c x^4+b x^2+a}}{2ad\,x^2}+\frac{b\ln\left(\frac{2a+b x^2+2\sqrt{a}\,\sqrt{c x^4+b x^2+a}}{x^2}\right)}{4d\,a^{\frac{3}{2}}}+\frac{e\ln\left(\frac{2a+b x^2+2\sqrt{a}\,\sqrt{c x^4+b x^2+a}}{x^2}\right)}{2d^2\sqrt{a}}-\frac{e\ln\left(\frac{\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+}{}\right.}{}$

input
```
int(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/a^(3/2)*(-ln((2*(c*x^4+b*x^2+a)^(1/2)*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*
e+(b*x^2+2*a)*e-d*(2*c*x^2+b))/(e*x^2+d))*a^(3/2)*e*x^2+((a*e^2-b*d*e+c*d^
2)/e^2)^(1/2)*(x^2*(a*e+1/2*b*d)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(
1/2))/x^2)-(c*x^4+b*x^2+a)^(1/2)*a^(1/2)*d))/((a*e^2-b*d*e+c*d^2)/e^2)^(1/
2)/d^2/x^2
```

3.336.5 Fricas [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 1414, normalized size of antiderivative = 6.49

$$\int \frac{1}{x^3\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx=\text{Too large to display}$$

input
```
integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(2*sqrt(c*d^2 - b*d*e + a*e^2)*a^2*e^2*x^2*log(-((8*c^2*d^2 - 8*b*c*d
*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 +
2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2
 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*
x^4 + 2*d*e*x^2 + d^2)) + (b*c*d^3 - a*b*d*e^2 + 2*a^2*e^3 - (b^2 - 2*a*c)
*d^2*e)*sqrt(a)*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b
*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*(a*c*d^3 - a*b*d^2*e + a
^2*d*e^2)*sqrt(c*x^4 + b*x^2 + a))/((a^2*c*d^4 - a^2*b*d^3*e + a^3*d^2*e^2
)*x^2), 1/8*(4*sqrt(-c*d^2 + b*d*e - a*e^2)*a^2*e^2*x^2*arctan(-1/2*sqrt(c
*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d -
2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 +
(b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) + (b*c*d^3 - a*b*d*e^2 + 2*a^2*e^3 - (
b^2 - 2*a*c)*d^2*e)*sqrt(a)*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sq
rt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*(a*c*d^3 - a
*b*d^2*e + a^2*d*e^2)*sqrt(c*x^4 + b*x^2 + a))/((a^2*c*d^4 - a^2*b*d^3*e +
 a^3*d^2*e^2)*x^2), 1/4*(sqrt(c*d^2 - b*d*e + a*e^2)*a^2*e^2*x^2*log(-((8*
c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^
2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*s
qrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*
d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)) - (b*c*d^3 - a*b*d*e^2 + 2*a^2...
```

3.336.6 Sympy [F]

$$\int \frac{1}{x^3\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{1}{x^3\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate(1/x**3/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(1/(x**3*(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

3.336.7 Maxima [F]

$$\int \frac{1}{x^3 (d+ex^2) \sqrt{a+bx^2+cx^4}} \, dx = \int \frac{1}{\sqrt{cx^4+bx^2+a}(ex^2+d)x^3} \, dx$$

input
```
integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*x^3), x)
```

3.336.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^3 (d+ex^2) \sqrt{a+bx^2+cx^4}} \, dx = \frac{e^2 \arctan\left(-\frac{\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{\sqrt{-cd^2+bde-ae^2}d^2} - \frac{(bd+2\,ae) \arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{2\,\sqrt{-a}ad^2} + \frac{\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)b+2\,a\sqrt{c}}{2\left(\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^2-a\right)ad}$$

input
```
integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
e^2*arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*e + sqrt(c)*d)/sqrt(-
c*d^2 + b*d*e - a*e^2))/(sqrt(-c*d^2 + b*d*e - a*e^2)*d^2) - 1/2*(b*d + 2*
a*e)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a
*d^2) + 1/2*((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*b + 2*a*sqrt(c))/(((s
qrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)*a*d)
```

3.336.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (d+ex^2) \sqrt{a+bx^2+cx^4}} \, dx = \int \frac{1}{x^3 \left(e\,x^2+d\right) \sqrt{c\,x^4+b\,x^2+a}} \, dx$$

input
```
int(1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int(1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.337 $\int \frac{x^4}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$

3.337.1 Optimal result

Integrand size = 29, antiderivative size = 418

$$\begin{aligned}\int \frac{x^4}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx &= \frac{x\sqrt{1+2x^2+2x^4}}{2\sqrt{2}\left(1+\sqrt{2}x^2\right)}-\frac{3\sqrt{\frac{3}{10}}\left(3-\sqrt{2}\right)\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)}{4\left(2-3\sqrt{2}\right)}\\
&-\frac{\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{2\,2^{3/4}\sqrt{1+2x^2+2x^4}}\\
&+\frac{\left(1-3\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{2\,2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}\\
&+\frac{3\left(3+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{1}{24}\left(12-11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{8\,2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}\end{aligned}$$

output
```
-3/40*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*30^(1/2)*(3-2^(1/2))/(2
-3*2^(1/2))+1/4*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))-1/4*(cos(2
*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticE(sin(2*arct
an(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+
x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)+1/4*(cos(2*arctan(2^(1
/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x
)),1/2*(2-2^(1/2))^(1/2))*(1-3*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(
1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2-3*2^(1/2))/(2*x^4+2*x^2+1)^(1/2)+3/16*(
cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(
2*arctan(2^(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*
(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2-3*2^(
1/2))/(2*x^4+2*x^2+1)^(1/2)
```

3.337.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.21 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.30

$$\int \frac{x^4}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = -\frac{\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\left((1+i)E\left(i\text{arcsinh}\left(\sqrt{1-i}x\right)\middle|i\right)-(1+4i)\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{1-i}x\right.\right.\right.}{4\sqrt{1-i}\sqrt{1+2x^2+2x^4}}$$

input
```
Integrate[x^4/((3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

output
```
-1/4*(Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*((1 + I)*EllipticE[I*Arc
Sinh[Sqrt[1 - I]*x], I] - (1 + 4*I)*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I]
 + (3*I)*EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I]))/(Sqrt[1 - I]
*Sqrt[1 + 2*x^2 + 2*x^4])
```

3.337.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 410, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1662, 1416, 1509, 2220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \text{1662}$$

$$-\frac{\left(6-\sqrt{2}\right)\int \frac{1}{\sqrt{2x^4+2x^2+1}}dx}{2\left(2-3\sqrt{2}\right)}-\frac{\int \frac{1-\sqrt{2}x^2}{\sqrt{2x^4+2x^2+1}}dx}{2\sqrt{2}}+\frac{9\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{2\left(2-3\sqrt{2}\right)}$$

$$\Big\downarrow \text{1416}$$

$$-\frac{\int \frac{1-\sqrt{2}x^2}{\sqrt{2x^4+2x^2+1}}dx}{2\sqrt{2}}+\frac{9\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{2\left(2-3\sqrt{2}\right)}-\frac{\left(6-\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{4\sqrt[4]{2}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$

$$\Big\downarrow \text{1509}$$

$$\frac{9\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{2\left(2-3\sqrt{2}\right)}-\frac{\left(6-\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{4\sqrt[4]{2}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}-\frac{\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-\frac{x\sqrt{2x^4+2x^2+1}}{\sqrt{2}x^2+1}}{2\sqrt{2}}$$

$$\Big\downarrow \text{2220}$$

$$-\frac{\left(6-\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{4\sqrt[4]{2}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}-\frac{\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-\frac{x\sqrt{2x^4+2x^2+1}}{\sqrt{2}x^2+1}}{2\sqrt{2}}+\frac{9\left(\frac{\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticPi}\left(\frac{1}{24}\left(12-11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{12\ 2^{3/4}\sqrt{2x^4+2x^2+1}}-\frac{\left(3-\sqrt{2}\right)\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{2\sqrt{30}}\right)}{2\left(2-3\sqrt{2}\right)}$$

input
```
Int[x^4/((3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

output
```
-1/2*(-((x*Sqrt[1 + 2*x^2 + 2*x^4])/(1 + Sqrt[2]*x^2)) + ((1 + Sqrt[2]*x^2
)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)
*x], (2 - Sqrt[2])/4])/(2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]))/Sqrt[2] - ((6 -
Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*E
llipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(4*2^(1/4)*(2 - 3*Sqrt[2])
*Sqrt[1 + 2*x^2 + 2*x^4]) + (9*(-1/2*((3 - Sqrt[2])*ArcTan[(Sqrt[5/3]*x)/S
qrt[1 + 2*x^2 + 2*x^4]])/Sqrt[30] + ((3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[
(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24,
2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(12*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]
)))/(2*(2 - 3*Sqrt[2]))
```

3.337.3.1 Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1662
```
Int[(x_)^4/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[-(2*c*d - a*e*q)/(c*e*(e - d*q))
   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + (-Simp[1/(e*q)   Int[(1 - q*x^2)
/Sqrt[a + b*x^2 + c*x^4], x], x] + Simp[d^2/(e*(e - d*q))   Int[(1 + q*x^2)
/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x])] /; FreeQ[{a, b, c, d, e},
x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a] && NeQ[c*d^2 - a*e^2, 0]
```

rule 2220
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a
+ b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*El
lipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &
& EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[-b + c*(d/e) + a*(e/d)]
```

3.337.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.89 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.53

method	result
default	$-\frac{3\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{4\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{1}{4}+\frac{i}{4}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)-E\left(x\sqrt{-\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$
elliptic	$-\frac{\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{4\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\sqrt{-ix^2+x^2+1}\,\sqrt{}}{4\sqrt{}}$

input
```
int(x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-3/4/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^
(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+(-1/4+1/4*I)/(-1
+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(E
llipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2
),1/2*2^(1/2)+1/2*I*2^(1/2)))+3/4/(-1+I)^(1/2)*(1-I*x^2+x^2)^(1/2)*(1+I*x^
2+x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1
-I)^(1/2)/(-1+I)^(1/2))
```

3.337.5 Fricas [F]

$$\int \frac{x^4}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = \int \frac{x^4}{\sqrt{2\,x^4+2\,x^2+1}(2\,x^2+3)}\,dx$$

input
```
integrate(x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(2*x^4 + 2*x^2 + 1)*x^4/(4*x^6 + 10*x^4 + 8*x^2 + 3), x)
```

3.337.6 Sympy [F]

$$\int \frac{x^4}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = \int \frac{x^4}{(2x^2+3)\sqrt{2x^4+2x^2+1}}\,dx$$

input
```
integrate(x**4/(2*x**2+3)/(2*x**4+2*x**2+1)**(1/2),x)
```

output
```
Integral(x**4/((2*x**2 + 3)*sqrt(2*x**4 + 2*x**2 + 1)), x)
```

3.337.7 Maxima [F]

$$\int \frac{x^4}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = \int \frac{x^4}{\sqrt{2\,x^4+2\,x^2+1}(2\,x^2+3)}\,dx$$

input
```
integrate(x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)), x)
```

3.337.8 Giac [F]

$$\int \frac{x^4}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = \int \frac{x^4}{\sqrt{2\,x^4+2\,x^2+1}(2\,x^2+3)}\,dx$$

input
```
integrate(x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)), x)
```

3.337.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = \int \frac{x^4}{(2\,x^2+3)\,\sqrt{2\,x^4+2\,x^2+1}}\,dx$$

input
```
int(x^4/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(1/2)),x)
```

output
```
int(x^4/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(1/2)), x)
```

3.338 $\int \frac{x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$

3.338.1 Optimal result

Integrand size = 29, antiderivative size = 247

$$\int \frac{x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = -\frac{1}{4}\sqrt{\frac{3}{5}}\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) - \frac{(3+\sqrt{2})\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{14\ 2^{3/4}\sqrt{1+2x^2+2x^4}} + \frac{(3+\sqrt{2})^2\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{1}{24}(12-11\sqrt{2}),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{56\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$

output
```
-1/20*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)-1/28*(cos(2*ar
ctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(
2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x
^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)+1/112*(cos(2*
arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arct
an(2^(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))^2*(1+x
^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2
+1)^(1/2)
```

3.338.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.12 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.40

$$\int \frac{x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = \frac{(1-i)^{3/2}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{1-i}x\right),i\right)-\text{EllipticPi}\left(\frac{1}{3}+\frac{i}{3},i\text{arcsinh}\left(\sqrt{\right.\right.\right.}{4\sqrt{1+2x^2+2x^4}}$$

input
```
Integrate[x^2/((3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

output
```
((1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*(EllipticF[I*Ar
cSinh[Sqrt[1 - I]*x], I] - EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x],
 I]))/(4*Sqrt[1 + 2*x^2 + 2*x^4])
```

3.338.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1656, 1416, 2220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \text{1656}$$

$$\frac{3}{14}\left(2+3\sqrt{2}\right)\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx-\frac{1}{14}\left(2+3\sqrt{2}\right)\int \frac{1}{\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \text{1416}$$

$$\frac{3}{14}\left(2+3\sqrt{2}\right)\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx-\frac{\left(2+3\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{28\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}$$

$$\Big\downarrow \text{2220}$$

$$\frac{3}{14}\left(2+3\sqrt{2}\right)\left(\frac{\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\operatorname{EllipticPi}\left(\frac{1}{24}\left(12-11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{12\ 2^{3/4}\sqrt{2x^4+2x^2+1}} - \cdots\right.$$

$$\frac{\left(2+3\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{28\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}$$

input
```
Int[x^2/((3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

output
```
-1/28*((2 + 3*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqr
t[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(2^(1/4)*Sqr
t[1 + 2*x^2 + 2*x^4]) + (3*(2 + 3*Sqrt[2])*(-1/2*((3 - Sqrt[2])*ArcTan[(Sq
rt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/Sqrt[30] + ((3 + Sqrt[2])*(1 + Sqrt[2
]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*S
qrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(12*2^(3/4)*Sqrt[1 + 2*
x^2 + 2*x^4])))/14
```

3.338.3.1 Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1656
```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[(-a)*((e + d*q)/(c*d^2 - a*e^2))
   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Simp[a*d*((e + d*q)/(c*d^2 - a*e
^2))   Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a] && NeQ[c*d^2 -
a*e^2, 0]
```

rule 2220
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a
+ b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*El
lipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &
& EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[-b + c*(d/e) + a*(e/d)]
```

3.338.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.54

method	result	size
default	$\frac{\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{2\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}-\frac{\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,\Pi\left(x\sqrt{-1+i},\frac{1}{3}+\frac{i}{3},\frac{\sqrt{-1-i}}{\sqrt{-1+i}}\right)}{2\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$	134
elliptic	$\frac{\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{2\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}-\frac{\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,\Pi\left(x\sqrt{-1+i},\frac{1}{3}+\frac{i}{3},\frac{\sqrt{-1-i}}{\sqrt{-1+i}}\right)}{2\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$	138

input
```
int(x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(
1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-1/2/(-1+I)^(1/2)*
(1-I*x^2+x^2)^(1/2)*(1+I*x^2+x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi(x
*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

3.338.5 Fricas [F]

$$\int \frac{x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx=\int \frac{x^2}{\sqrt{2\,x^4+2\,x^2+1}(2\,x^2+3)}\,dx$$

input
```
integrate(x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(2*x^4 + 2*x^2 + 1)*x^2/(4*x^6 + 10*x^4 + 8*x^2 + 3), x)
```

3.338.6 Sympy [F]

$$\int \frac{x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = \int \frac{x^2}{(2x^2+3)\sqrt{2x^4+2x^2+1}}\,dx$$

input
```
integrate(x**2/(2*x**2+3)/(2*x**4+2*x**2+1)**(1/2),x)
```

output
```
Integral(x**2/((2*x**2 + 3)*sqrt(2*x**4 + 2*x**2 + 1)), x)
```

3.338.7 Maxima [F]

$$\int \frac{x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = \int \frac{x^2}{\sqrt{2\,x^4+2\,x^2+1}(2\,x^2+3)}\,dx$$

input
```
integrate(x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)), x)
```

3.338.8 Giac [F]

$$\int \frac{x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = \int \frac{x^2}{\sqrt{2\,x^4+2\,x^2+1}(2\,x^2+3)}\,dx$$

input
```
integrate(x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)), x)
```

3.338.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = \int \frac{x^2}{(2\,x^2+3)\,\sqrt{2\,x^4+2\,x^2+1}}\,dx$$

input
```
int(x^2/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(1/2)),x)
```

output
```
int(x^2/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(1/2)), x)
```

3.339 $\int \frac{1}{(3+2x^2)\sqrt{1+2x^2+2x^4}} \, dx$

3.339.1 Optimal result

Integrand size = 26, antiderivative size = 245

$$\int \frac{1}{(3+2x^2)\sqrt{1+2x^2+2x^4}} \, dx = \frac{\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)}{2\sqrt{15}} + \frac{\left(3+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{14\sqrt[4]{2}\sqrt{1+2x^2+2x^4}} - \frac{\left(3+\sqrt{2}\right)^2\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\text{EllipticPi}\left(\frac{1}{24}\left(12-11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{84\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$

output
```
1/30*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)+1/28*(cos(2*arc
tan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2
^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^
2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)-1/168*(cos(2*a
rctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arcta
n(2^(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))^2*(1+x^
2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+
1)^(1/2)
```

3.339.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.33

$$\int \frac{1}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = -\frac{i\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,\text{EllipticPi}\left(\frac{1}{3}+\frac{i}{3},i\text{arcsinh}\left(\sqrt{1-i}x\right),i\right)}{3\sqrt{1-i}\sqrt{1+2x^2+2x^4}}$$

input
```
Integrate[1/((3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

output
```
((-1/3*I)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[1/3 + I/3
, I*ArcSinh[Sqrt[1 - I]*x], I])/(Sqrt[1 - I]*Sqrt[1 + 2*x^2 + 2*x^4])
```

3.339.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1540, 1416, 2220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}\,dx$$

$$\Big\downarrow \text{1540}$$

$$\frac{1}{7}\left(3+\sqrt{2}\right)\int \frac{1}{\sqrt{2x^4+2x^2+1}}dx-\frac{1}{7}\left(2+3\sqrt{2}\right)\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \text{1416}$$

$$\frac{\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{14\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-\frac{1}{7}\left(2+3\sqrt{2}\right)\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \text{2220}$$

$$\frac{(3+\sqrt{2})\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{14\sqrt[4]{2}\sqrt{2x^4+2x^2+1}} -$$

$$\frac{1}{7}\left(2+3\sqrt{2}\right)\left(\frac{(3+\sqrt{2})\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticPi}\left(\frac{1}{24}(12-11\sqrt{2}),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{12\ 2^{3/4}\sqrt{2x^4+2x^2+1}} - \cdots (3\right.$$

input
```
Int[1/((3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

output
```
((3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2
)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(14*2^(1/4)*Sqrt[1 +
 2*x^2 + 2*x^4]) - ((2 + 3*Sqrt[2])*(-1/2*((3 - Sqrt[2])*ArcTan[(Sqrt[5/3]
*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/Sqrt[30] + ((3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*
Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])
/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(12*2^(3/4)*Sqrt[1 + 2*x^2 + 2
*x^4])))/7
```

3.339.3.1 Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2220
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a
+ b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*El
lipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &
& EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[-b + c*(d/e) + a*(e/d)]
```

3.339.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.29

method	result	size
default	$\frac{\sqrt{-i x^{2}+x^{2}+1}\,\sqrt{i x^{2}+x^{2}+1}\,\Pi\left(x\sqrt{-1+i},\frac{1}{3}+\frac{i}{3},\frac{\sqrt{-1-i}}{\sqrt{-1+i}}\right)}{3\sqrt{-1+i}\,\sqrt{2x^{4}+2x^{2}+1}}$	70
elliptic	$\frac{\sqrt{-i x^{2}+x^{2}+1}\,\sqrt{i x^{2}+x^{2}+1}\,\Pi\left(x\sqrt{-1+i},\frac{1}{3}+\frac{i}{3},\frac{\sqrt{-1-i}}{\sqrt{-1+i}}\right)}{3\sqrt{-1+i}\,\sqrt{2x^{4}+2x^{2}+1}}$	70

input
```
int(1/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/(-1+I)^(1/2)*(1-I*x^2+x^2)^(1/2)*(1+I*x^2+x^2)^(1/2)/(2*x^4+2*x^2+1)^(
1/2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

3.339.5 Fricas [F]

$$\int \frac{1}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+2\,x^2+1}(2\,x^2+3)}\,dx$$

input
```
integrate(1/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(2*x^4 + 2*x^2 + 1)/(4*x^6 + 10*x^4 + 8*x^2 + 3), x)
```

3.339.6 Sympy [F]

$$\int \frac{1}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = \int \frac{1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}\,dx$$

input
```
integrate(1/(2*x**2+3)/(2*x**4+2*x**2+1)**(1/2),x)
```

output
```
Integral(1/((2*x**2 + 3)*sqrt(2*x**4 + 2*x**2 + 1)), x)
```

3.339.7 Maxima [F]

$$\int \frac{1}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+2\,x^2+1}(2\,x^2+3)}\,dx$$

input
```
integrate(1/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)), x)
```

3.339.8 Giac [F]

$$\int \frac{1}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2\,x^4+2\,x^2+1}(2\,x^2+3)}\,dx$$

input
```
integrate(1/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)), x)
```

3.339.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = \int \frac{1}{(2\,x^2+3)\,\sqrt{2\,x^4+2\,x^2+1}}\,dx$$

input
```
int(1/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(1/2)),x)
```

output
```
int(1/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(1/2)), x)
```

3.340 $\int \frac{1}{x^2(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$

3.340.1 Optimal result

Integrand size = 29, antiderivative size = 399

$$\begin{aligned}\int \frac{1}{x^2(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx &= -\frac{\sqrt{1+2x^2+2x^4}}{3x}+\frac{\sqrt{2}x\sqrt{1+2x^2+2x^4}}{3(1+\sqrt{2}x^2)}\\
&-\frac{\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)}{3\sqrt{15}}-\frac{\sqrt[4]{2}(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{3\sqrt{1+2x^2+2x^4}}\\
&+\frac{(5-3\sqrt{2})(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{21\ 2^{3/4}\sqrt{1+2x^2+2x^4}}\\
&+\frac{(3+\sqrt{2})^2(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\operatorname{EllipticPi}\left(\frac{1}{24}(12-11\sqrt{2}),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{126\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}\end{aligned}$$

output
```
-1/45*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)-1/3*(2*x^4+2*x
^2+1)^(1/2)/x+1/3*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))-1/3*(cos
(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticE(sin(2*ar
ctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(
1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)+1/42*2^(1/4)*(cos(2*
arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arcta
n(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(5-3*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4
+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)+1/252*(cos(2*arct
an(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(2
^(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))^2*(1+x^2*2
^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^
(1/2)
```

3.340.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.18 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.37

$$\int \frac{1}{x^2(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = -\frac{i\left(-3i(1+2x^2+2x^4)+\sqrt{1-i}x\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\left(3E\left(i\operatorname{arcsinh}\left(\sqrt{1-i}x\right)\middle|i\right)-3\operatorname{Ellipti}\right.\right.}{9x\sqrt{1+2x^2+2x^4}}$$

input
```
Integrate[1/(x^2*(3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

output
```
((-1/9*I)*((-3*I)*(1 + 2*x^2 + 2*x^4) + Sqrt[1 - I]*x*Sqrt[1 + (1 - I)*x^2
]*Sqrt[1 + (1 + I)*x^2]*(3*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] - 3*Elli
pticF[I*ArcSinh[Sqrt[1 - I]*x], I] - (1 + I)*EllipticPi[1/3 + I/3, I*ArcSi
nh[Sqrt[1 - I]*x], I]))/(x*Sqrt[1 + 2*x^2 + 2*x^4])
```

3.340.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {1668, 27, 2232, 27, 1509, 2226, 1416, 2220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(2x^2+3)\sqrt{2x^4+2x^2+1}}dx$$

$$\Big\downarrow \text{1668}$$

$$\frac{1}{3}\int -\frac{2(-2x^4-3x^2+1)}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx-\frac{\sqrt{2x^4+2x^2+1}}{3x}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2}{3}\int \frac{-2x^4-3x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx-\frac{\sqrt{2x^4+2x^2+1}}{3x}$$

$$\Big\downarrow \text{2232}$$

$$-\frac{2}{3}\left(\frac{1}{4}\int \frac{2(-2\sqrt{2}x^2-3\sqrt{2}+2)}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx+\frac{\int \frac{1-\sqrt{2}x^2}{\sqrt{2x^4+2x^2+1}}dx}{\sqrt{2}}\right)-\frac{\sqrt{2x^4+2x^2+1}}{3x}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2}{3}\left(\frac{1}{2}\int \frac{-2\sqrt{2}x^2-3\sqrt{2}+2}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx+\frac{\int \frac{1-\sqrt{2}x^2}{\sqrt{2x^4+2x^2+1}}dx}{\sqrt{2}}\right)-\frac{\sqrt{2x^4+2x^2+1}}{3x}$$

$$\Big\downarrow \text{1509}$$

$$-\frac{2}{3}\left(\frac{1}{2}\int \frac{-2\sqrt{2}x^2-3\sqrt{2}+2}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx+\frac{\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-\frac{x\sqrt{2x^4+2x^2+1}}{\sqrt{2}x^2+1}}{\sqrt{2}}\right)-$$

$$\frac{\sqrt{2x^4+2x^2+1}}{3x}$$

$$\Big\downarrow \text{2226}$$

$$-\frac{2}{3}\left(\frac{1}{2}\left(\frac{1}{7}\left(6-5\sqrt{2}\right)\int\frac{1}{\sqrt{2x^4+2x^2+1}}dx-\frac{2}{7}\left(2+3\sqrt{2}\right)\int\frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx\right)+\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x}{\left(\sqrt{2}x^2+\right.}}}{}\right.$$

$$\frac{\sqrt{2x^4+2x^2+1}}{3x}$$

$\downarrow$ 1416

$$-\frac{2}{3}\left(\frac{1}{2}\left(\frac{\left(6-5\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{14\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-\frac{2}{7}\left(2+3\sqrt{2}\right)\int\frac{}{(2x^2+3}\right.\right.$$

$$\frac{\sqrt{2x^4+2x^2+1}}{3x}$$

$\downarrow$ 2220

$$-\frac{2}{3}\left(\frac{\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-\frac{x\sqrt{2x^4+2x^2+1}}{\sqrt{2}x^2+1}}{\sqrt{2}}+\frac{1}{2}\left(\frac{\left(6-5\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{Ell}}{14\sqrt[4]{2}\sqrt{2x^4}}\right.\right.$$

$$\frac{\sqrt{2x^4+2x^2+1}}{3x}$$

input
```
Int[1/(x^2*(3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

output
```
-1/3*Sqrt[1 + 2*x^2 + 2*x^4]/x - (2*((-((x*Sqrt[1 + 2*x^2 + 2*x^4])/(1 + S
qrt[2]*x^2)) + ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^
2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(2^(1/4)*Sqrt[1 + 2
*x^2 + 2*x^4]))/Sqrt[2] + (((6 - 5*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*
x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt
[2])/4])/(14*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (2*(2 + 3*Sqrt[2])*(-1/2*(
(3 - Sqrt[2])*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/Sqrt[30] + ((
3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^
2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])
/(12*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4])))/7)/2))/3
```

3.340. $\int \frac{1}{x^2(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$

3.340.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1668
```
Int[(x_)^(m_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4]), x_Symbol] :> Simp[x^(m + 1)*(Sqrt[a + b*x^2 + c*x^4]/(a*d*(m + 1))), x
] - Simp[1/(a*d*(m + 1))   Int[(x^(m + 2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x
^4]))*Simp[a*e*(m + 1) + b*d*(m + 2) + (b*e*(m + 2) + c*d*(m + 3))*x^2 + c*
e*(m + 3)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c,
0] && ILtQ[m/2, 0]
```

rule 2220
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a
+ b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*El
lipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &
& EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[-b + c*(d/e) + a*(e/d)]
```

3.340. $\int \frac{1}{x^2(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$

rule 2226
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[(A*(c*d + a*e*q)
- a*B*(e + d*q))/(c*d^2 - a*e^2)   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Simp[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2))   Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

rule 2232
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, Simp[-C/(e*q)   Int[(1 - q*x^2)/Sqrt[a + b*x^2
 + c*x^4], x], x] + Simp[1/(c*e)   Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d -
 a*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
&&  !GtQ[b^2 - 4*a*c, 0]
```

3.340.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.93 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.45

method	result
default	$-\frac{\sqrt{2x^4+2x^2+1}}{3x}+\frac{\left(-\frac{1}{3}+\frac{i}{3}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)-E\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\right)}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}-\frac{2\sqrt{-ix^2+x^2+1}\,\sqrt{\ldots}}{9\ldots}$
risch	$-\frac{\sqrt{2x^4+2x^2+1}}{3x}+\frac{\left(-\frac{1}{3}+\frac{i}{3}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)-E\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\right)}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}-\frac{2\sqrt{-ix^2+x^2+1}\,\sqrt{\ldots}}{9\ldots}$
elliptic	$-\frac{\sqrt{2x^4+2x^2+1}}{3x}-\frac{\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{3\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{3\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\ldots$

input
```
int(1/x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(2*x^4+2*x^2+1)^(1/2)/x+(-1/3+1/3*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)
*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2
^(1/2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))
-2/9/(-1+I)^(1/2)*(1-I*x^2+x^2)^(1/2)*(1+I*x^2+x^2)^(1/2)/(2*x^4+2*x^2+1)^
(1/2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

3.340.5 Fricas [F]

$$\int \frac{1}{x^2 (3 + 2x^2) \sqrt{1 + 2x^2 + 2x^4}} \, dx = \int \frac{1}{\sqrt{2\, x^4 + 2\, x^2 + 1}(2\, x^2 + 3)x^2} \, dx$$

input
```
integrate(1/x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(2*x^4 + 2*x^2 + 1)/(4*x^8 + 10*x^6 + 8*x^4 + 3*x^2), x)
```

3.340.6 Sympy [F]

$$\int \frac{1}{x^2 (3 + 2x^2) \sqrt{1 + 2x^2 + 2x^4}} \, dx = \int \frac{1}{x^2 \cdot (2x^2 + 3) \sqrt{2x^4 + 2x^2 + 1}} \, dx$$

input
```
integrate(1/x**2/(2*x**2+3)/(2*x**4+2*x**2+1)**(1/2),x)
```

output
```
Integral(1/(x**2*(2*x**2 + 3)*sqrt(2*x**4 + 2*x**2 + 1)), x)
```

3.340.7 Maxima [F]

$$\int \frac{1}{x^2 (3 + 2x^2) \sqrt{1 + 2x^2 + 2x^4}} \, dx = \int \frac{1}{\sqrt{2\, x^4 + 2\, x^2 + 1}(2\, x^2 + 3)x^2} \, dx$$

input
```
integrate(1/x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)*x^2), x)
```

3.340.8 Giac [F]

$$\int \frac{1}{x^2(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = \int \frac{1}{\sqrt{2x^4+2x^2+1}(2x^2+3)x^2}\,dx$$

input
```
integrate(1/x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)*x^2), x)
```

3.340.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = \int \frac{1}{x^2\,(2x^2+3)\,\sqrt{2x^4+2x^2+1}}\,dx$$

input
```
int(1/(x^2*(2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(1/2)),x)
```

output
```
int(1/(x^2*(2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(1/2)), x)
```

3.341 $\int \frac{1}{x^4(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$

3.341.1 Optimal result

Integrand size = 29, antiderivative size = 422

$$\int \frac{1}{x^4(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = -\frac{\sqrt{1+2x^2+2x^4}}{9x^3}+\frac{2\sqrt{1+2x^2+2x^4}}{3x}-\frac{2\sqrt{2}x\sqrt{1+2x^2+2x^4}}{3\left(1+\sqrt{2}x^2\right)}+\frac{2\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)}{9\sqrt{15}}$$
$$+\frac{2\sqrt[4]{2}\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{3\sqrt{1+2x^2+2x^4}}$$
$$-\frac{\left(1+19\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{63\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$
$$-\frac{\left(3+\sqrt{2}\right)^2\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{1}{24}\left(12-11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{189\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$

output
```
2/135*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)-1/9*(2*x^4+2*x
^2+1)^(1/2)/x^3+2/3*(2*x^4+2*x^2+1)^(1/2)/x-2/3*x*(2*x^4+2*x^2+1)^(1/2)*2^
(1/2)/(1+x^2*2^(1/2))+2/3*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(
2^(1/4)*x))*EllipticE(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x
^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2
+1)^(1/2)-1/378*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x)
)*EllipticPi(sin(2*arctan(2^(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1
/2))*(3+2^(1/2))^2*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/
2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)-1/126*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/
cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2)
)^(1/2))*(1+19*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2
)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)
```

3.341.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.18 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.52

$$\int \frac{1}{x^4 (3+2x^2) \sqrt{1+2x^2+2x^4}} \, dx$$
$$= \frac{-3+12x^2+30x^4+36x^6+18i\sqrt{1-i}x^3\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}E\left(i\text{arcsinh}\left(\sqrt{1-i}x\right)\middle|i\right)-(3+\cdots}{}$$

input
```
Integrate[1/(x^4*(3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

output
```
(-3 + 12*x^2 + 30*x^4 + 36*x^6 + (18*I)*Sqrt[1 - I]*x^3*Sqrt[1 + (1 - I)*x
^2]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] - (3 + 15
*I)*Sqrt[1 - I]*x^3*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[
I*ArcSinh[Sqrt[1 - I]*x], I] + 2*(1 - I)^(3/2)*x^3*Sqrt[1 + (1 - I)*x^2]*S
qrt[1 + (1 + I)*x^2]*EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(
27*x^3*Sqrt[1 + 2*x^2 + 2*x^4])
```

3.341.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {1668, 27, 2244, 27, 2232, 27, 1509, 2226, 1416, 2220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (2x^2+3) \sqrt{2x^4+2x^2+1}} \, dx$$

$$\Big\downarrow \text{1668}$$

$$\frac{1}{9} \int -\frac{2(2x^4+7x^2+9)}{x^2 (2x^2+3) \sqrt{2x^4+2x^2+1}} dx - \frac{\sqrt{2x^4+2x^2+1}}{9x^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2}{9} \int \frac{2x^4+7x^2+9}{x^2 (2x^2+3) \sqrt{2x^4+2x^2+1}} dx - \frac{\sqrt{2x^4+2x^2+1}}{9x^3}$$

$$\Big\downarrow \text{2244}$$

$$-\frac{2}{9} \left(-\frac{1}{3} \int -\frac{3(12x^4+20x^2+1)}{(2x^2+3) \sqrt{2x^4+2x^2+1}} dx - \frac{3\sqrt{2x^4+2x^2+1}}{x} \right) - \frac{\sqrt{2x^4+2x^2+1}}{9x^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2}{9} \left(\int \frac{12x^4+20x^2+1}{(2x^2+3) \sqrt{2x^4+2x^2+1}} dx - \frac{3\sqrt{2x^4+2x^2+1}}{x} \right) - \frac{\sqrt{2x^4+2x^2+1}}{9x^3}$$

$$\Big\downarrow \text{2232}$$

$$-\frac{2}{9} \left(-3\sqrt{2} \int \frac{1-\sqrt{2}x^2}{\sqrt{2x^4+2x^2+1}} dx + \frac{1}{4} \int \frac{4\left(2\left(1+3\sqrt{2}\right) x^2+9\sqrt{2}+1\right)}{(2x^2+3) \sqrt{2x^4+2x^2+1}} dx - \frac{3\sqrt{2x^4+2x^2+1}}{x} \right) - \frac{\sqrt{2x^4+2x^2+1}}{9x^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2}{9} \left(-3\sqrt{2} \int \frac{1-\sqrt{2}x^2}{\sqrt{2x^4+2x^2+1}} dx + \int \frac{2\left(1+3\sqrt{2}\right) x^2+9\sqrt{2}+1}{(2x^2+3) \sqrt{2x^4+2x^2+1}} dx - \frac{3\sqrt{2x^4+2x^2+1}}{x} \right) - \frac{\sqrt{2x^4+2x^2+1}}{9x^3}$$

$$\Big\downarrow \text{1509}$$

$$-\frac{2}{9}\left(\int \frac{2\left(1+3\sqrt{2}\right)x^2+9\sqrt{2}+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx-3\sqrt{2}\left(\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-\frac{x\sqrt{2x^4}}{\sqrt{2}}\right.\right.$$

$$\frac{\sqrt{2x^4+2x^2+1}}{9x^3}$$

$$\Big\downarrow \; 2226$$

$$-\frac{2}{9}\left(\frac{1}{7}\left(1+19\sqrt{2}\right)\int \frac{1}{\sqrt{2x^4+2x^2+1}}dx+\frac{2}{7}\left(2+3\sqrt{2}\right)\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx-3\sqrt{2}\left(\frac{\left(\sqrt{2}x^2+1\right)\sqrt{}}{}\right.\right.$$

$$\frac{\sqrt{2x^4+2x^2+1}}{9x^3}$$

$$\Big\downarrow \; 1416$$

$$-\frac{2}{9}\left(\frac{2}{7}\left(2+3\sqrt{2}\right)\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx+\frac{\left(1+19\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{}\right.\right.}{14\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}\right.$$

$$\frac{\sqrt{2x^4+2x^2+1}}{9x^3}$$

$$\Big\downarrow \; 2220$$

$$-\frac{2}{9}\left(\frac{\left(1+19\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{14\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-3\sqrt{2}\left(\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x}{\left(\sqrt{2}x^2\right.}}}{\sqrt[4]{}}\right.\right.$$

$$\frac{\sqrt{2x^4+2x^2+1}}{9x^3}$$

input
```
Int[1/(x^4*(3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

3.341. $\int \frac{1}{x^4(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$

output
```
-1/9*Sqrt[1 + 2*x^2 + 2*x^4]/x^3 - (2*((-3*Sqrt[1 + 2*x^2 + 2*x^4])/x - 3*
Sqrt[2]*(-((x*Sqrt[1 + 2*x^2 + 2*x^4])/(1 + Sqrt[2]*x^2)) + ((1 + Sqrt[2]*
x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1
/4)*x], (2 - Sqrt[2])/4])/(2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])) + ((1 + 19*Sq
rt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Ell
ipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(14*2^(1/4)*Sqrt[1 + 2*x^2 +
 2*x^4]) + (2*(2 + 3*Sqrt[2])*(-1/2*((3 - Sqrt[2])*ArcTan[(Sqrt[5/3]*x)/Sq
rt[1 + 2*x^2 + 2*x^4]])/Sqrt[30] + ((3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(
1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2
*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(12*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4])
))/7))/9
```

3.341.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1668
```
Int[(x_)^(m_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4]), x_Symbol] :> Simp[x^(m + 1)*(Sqrt[a + b*x^2 + c*x^4]/(a*d*(m + 1))), x
] - Simp[1/(a*d*(m + 1))   Int[(x^(m + 2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x
^4]))*Simp[a*e*(m + 1) + b*d*(m + 2) + (b*e*(m + 2) + c*d*(m + 3))*x^2 + c*
e*(m + 3)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c,
0] && ILtQ[m/2, 0]
```

3.341. $\int \frac{1}{x^4(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$

rule 2220
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a
+ b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*El
lipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &
& EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[-b + c*(d/e) + a*(e/d)]
```

rule 2226
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[(A*(c*d + a*e*q)
- a*B*(e + d*q))/(c*d^2 - a*e^2)   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Simp[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2))   Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

rule 2232
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, Simp[-C/(e*q)   Int[(1 - q*x^2)/Sqrt[a + b*x^2
 + c*x^4], x], x] + Simp[1/(c*e)   Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d -
 a*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
&&  !GtQ[b^2 - 4*a*c, 0]
```

rule 2244
```
Int[((Px_)*(x_)^(m_))/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_
.)*(x_)^4]), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 2], C
 = Coeff[Px, x, 4]}, Simp[A*x^(m + 1)*(Sqrt[a + b*x^2 + c*x^4]/(a*d*(m + 1)
)), x] + Simp[1/(a*d*(m + 1))   Int[(x^(m + 2)/((d + e*x^2)*Sqrt[a + b*x^2
+ c*x^4]))*Simp[a*B*d*(m + 1) - A*(a*e*(m + 1) + b*d*(m + 2)) + (a*C*d*(m +
 1) - A*(b*e*(m + 2) + c*d*(m + 3)))*x^2 - A*c*e*(m + 3)*x^4, x], x], x]] /
; FreeQ[{a, b, c, d, e}, x] && PolyQ[Px, x^2, 2] && NeQ[b^2 - 4*a*c, 0] &&
ILtQ[m/2, 0]
```

3.341. $\int \frac{1}{x^4(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$

3.341.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.46 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.61

method	result
risch	$\frac{12x^6+10x^4+4x^2-1}{9x^3\sqrt{2x^4+2x^2+1}}-\frac{2\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{9\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(\frac{2}{3}-\frac{2i}{3}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1+i},\cdots\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x\cdots}}$
default	$\frac{2\sqrt{2x^4+2x^2+1}}{3x}+\frac{\left(\frac{2}{3}-\frac{2i}{3}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)-E\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\right)}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}-\frac{\sqrt{2x^4+2x^2+1}}{9x^3}-\cdots$
elliptic	$-\frac{\sqrt{2x^4+2x^2+1}}{9x^3}+\frac{2\sqrt{2x^4+2x^2+1}}{3x}+\frac{4\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{9\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}-\frac{2i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(\cdots\right.}{3\sqrt{-1+i}\,\sqrt{2x^4+2x\cdots}}$

input
```
int(1/x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/9*(12*x^6+10*x^4+4*x^2-1)/x^3/(2*x^4+2*x^2+1)^(1/2)-2/9/(-1+I)^(1/2)*(1+
(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1
+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+(2/3-2/3*I)/(-1+I)^(1/2)*(1+(1-I)*x^2
)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2
),1/2*2^(1/2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^
(1/2)))+4/27/(-1+I)^(1/2)*(1-I*x^2+x^2)^(1/2)*(1+I*x^2+x^2)^(1/2)/(2*x^4+2
*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2
))
```

3.341.5 Fricas [F]

$$\int \frac{1}{x^4\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx=\int \frac{1}{\sqrt{2\,x^4+2\,x^2+1}(2\,x^2+3)x^4}\,dx$$

input
```
integrate(1/x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(2*x^4 + 2*x^2 + 1)/(4*x^10 + 10*x^8 + 8*x^6 + 3*x^4), x)
```

3.341.6 Sympy [F]

$$\int \frac{1}{x^4 (3 + 2x^2) \sqrt{1 + 2x^2 + 2x^4}} \, dx = \int \frac{1}{x^4 \cdot (2x^2 + 3) \sqrt{2x^4 + 2x^2 + 1}} \, dx$$

input
```
integrate(1/x**4/(2*x**2+3)/(2*x**4+2*x**2+1)**(1/2),x)
```

output
```
Integral(1/(x**4*(2*x**2 + 3)*sqrt(2*x**4 + 2*x**2 + 1)), x)
```

3.341.7 Maxima [F]

$$\int \frac{1}{x^4 (3 + 2x^2) \sqrt{1 + 2x^2 + 2x^4}} \, dx = \int \frac{1}{\sqrt{2\,x^4 + 2\,x^2 + 1}(2\,x^2 + 3)x^4} \, dx$$

input
```
integrate(1/x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)*x^4), x)
```

3.341.8 Giac [F]

$$\int \frac{1}{x^4 (3 + 2x^2) \sqrt{1 + 2x^2 + 2x^4}} \, dx = \int \frac{1}{\sqrt{2\,x^4 + 2\,x^2 + 1}(2\,x^2 + 3)x^4} \, dx$$

input
```
integrate(1/x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)*x^4), x)
```

3.341.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (3+2x^2) \sqrt{1+2x^2+2x^4}} \, dx = \int \frac{1}{x^4 \, (2\,x^2+3) \, \sqrt{2\,x^4+2\,x^2+1}} \, dx$$

input
```
int(1/(x^4*(2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(1/2)),x)
```

output
```
int(1/(x^4*(2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(1/2)), x)
```

3.342 $\int \frac{x^7}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx$

3.342.1 Optimal result

Integrand size = 29, antiderivative size = 236

$$\int \frac{x^7}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx = \frac{a(b^2d-2acd-abe)+(b^3d-3abcd-ab^2e+2a^2ce)\,x^2}{c\,(b^2-4ac)\,(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}} + \frac{\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2c^{3/2}e} - \frac{d^3\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2e\,(cd^2-bde+ae^2)^{3/2}}$$

output
```
1/2*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(3/2)/e-1/2*d
^3*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x
^4+b*x^2+a)^(1/2))/e/(a*e^2-b*d*e+c*d^2)^(3/2)+(a*(-a*b*e-2*a*c*d+b^2*d)+(
2*a^2*c*e-a*b^2*e-3*a*b*c*d+b^3*d)*x^2)/c/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)
/(c*x^4+b*x^2+a)^(1/2)
```

3.342.2 Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.06

$$\int \frac{x^7}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx = \frac{-b^3dx^2+ab(-bd+3cdx^2+bex^2)+a^2(be+2c(d-ex^2))}{c\,(-b^2+4ac)\,(cd^2+e(-bd+ae))\sqrt{a+bx^2+cx^4}} - \frac{d^3\sqrt{-cd^2+bde-ae^2}\arctan\left(\frac{\sqrt{c}(d+ex^2)-e\sqrt{a+bx^2+cx^4}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{e\,(cd^2+e(-bd+ae))^2} - \frac{\log\left(ce\left(b+2cx^2-2\sqrt{c}\sqrt{a+bx^2+cx^4}\right)\right)}{2c^{3/2}e}$$

input
```
Integrate[x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

output
```
(-(b^3*d*x^2) + a*b*(-(b*d) + 3*c*d*x^2 + b*e*x^2) + a^2*(b*e + 2*c*(d - e
*x^2)))/(c*(-b^2 + 4*a*c)*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[a + b*x^2 + c*x^
4]) - (d^3*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x^2) - e*
Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(e*(c*d^2 + e*(-
(b*d) + a*e))^2) - Log[c*e*(b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4
])]/(2*c^(3/2)*e)
```

3.342.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {1578, 1264, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}dx$$

$$\Big\downarrow \text{ 1578}$$

$$\frac{1}{2}\int \frac{x^6}{(ex^2+d)(cx^4+bx^2+a)^{3/2}}dx^2$$

$$\Big\downarrow \text{ 1264}$$

$$\frac{1}{2}\left(\frac{2\left(x^2\left(2a^2ce-ab^2e-3abcd+b^3d\right)+a\left(-abe-2acd+b^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}-\frac{2\int \frac{\frac{\left(b^2-4ac\right)d(bd-ae)}{cd^2-bed+ae^2}-\left(b^2-4ac\right)x^2}{2c(ex^2+d)\sqrt{cx^4+bx^2+a}}dx^2}{b^2-4ac}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(\frac{2\left(x^2\left(2a^2ce-ab^2e-3abcd+b^3d\right)+a\left(-abe-2acd+b^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}-\frac{\int \frac{\frac{\left(b^2-4ac\right)d(bd-ae)}{cd^2-bed+ae^2}-\left(b^2-4ac\right)x^2}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx^2}{c\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \text{ 1269}$$

$$\frac{1}{2}\left(\frac{2\left(x^2\left(2a^2ce-ab^2e-3abcd+b^3d\right)+a\left(-abe-2acd+b^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}-\frac{\frac{cd^3\left(b^2-4ac\right)\int\frac{1}{\left(ex^2+d\right)\sqrt{cx^4+bx^2+a}}dx^2}{e(ae^2-bde+cd^2)}-\frac{\left(b^2-4ac\right)\int\frac{}{\sqrt{}}}{}}{c\left(b^2-4ac\right)}\right.$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}\left(\frac{2\left(x^2\left(2a^2ce-ab^2e-3abcd+b^3d\right)+a\left(-abe-2acd+b^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}-\frac{\frac{cd^3\left(b^2-4ac\right)\int\frac{1}{\left(ex^2+d\right)\sqrt{cx^4+bx^2+a}}dx^2}{e(ae^2-bde+cd^2)}-\frac{2\left(b^2-4ac\right)\int}{}}{c\left(b^2-4ac\right)}\right.$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{2\left(x^2\left(2a^2ce-ab^2e-3abcd+b^3d\right)+a\left(-abe-2acd+b^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}-\frac{\frac{cd^3\left(b^2-4ac\right)\int\frac{1}{\left(ex^2+d\right)\sqrt{cx^4+bx^2+a}}dx^2}{e(ae^2-bde+cd^2)}-\frac{\left(b^2-4ac\right)\text{arc}}{}}{c\left(b^2-4ac\right)}\right.$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{2}\left(\frac{2\left(x^2\left(2a^2ce-ab^2e-3abcd+b^3d\right)+a\left(-abe-2acd+b^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}-\frac{-\frac{2cd^3\left(b^2-4ac\right)\int\frac{1}{4\left(cd^2-bed+ae^2\right)-x^4}d\left(-\frac{(2cd-be)x^2+bd}{\sqrt{cx^4+bx^2+}}\right.}{e(ae^2-bde+cd^2)}}{c\left(b^2-4\right.}\right.$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{2\left(x^2\left(2a^2ce-ab^2e-3abcd+b^3d\right)+a\left(-abe-2acd+b^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}-\frac{\frac{cd^3\left(b^2-4ac\right)\text{arctanh}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{e(ae^2-bde+cd^2)^{3/2}}-}{c\left(b^2-4ac\right)}\right.$$

input
```
Int[x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

3.342. $\int\frac{x^7}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx$

output
```
((2*(a*(b^2*d - 2*a*c*d - a*b*e) + (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*
e)*x^2))/(c*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4])
 - (-(((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x
^4])])/(Sqrt[c]*e)) + (c*(b^2 - 4*a*c)*d^3*ArcTanh[(b*d - 2*a*e + (2*c*d -
 b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(e*(c
*d^2 - b*d*e + a*e^2)^(3/2)))/(c*(b^2 - 4*a*c)))/2
```

3.342.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1264
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)
^m*(f + g*x)^n, a + b*x + c*x^2, x], R = Coeff[PolynomialRemainder[(d + e*x
)^m*(f + g*x)^n, a + b*x + c*x^2, x], x, 0], S = Coeff[PolynomialRemainder[
(d + e*x)^m*(f + g*x)^n, a + b*x + c*x^2, x], x, 1]}, Simp[(b*R - 2*a*S + (
2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + S
imp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*E
xpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m - ((2*p + 3)*(2*c*R - b*S)
)/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[n, 1]
 && LtQ[p, -1] && ILtQ[m, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
```

3.342. $\int \frac{x^7}{(d+ex^2)(a+bx^2+cx^4)^{3/2}} \, dx$

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && !IGtQ[m, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.342.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.55

method	result
pseudoelliptic	$\frac{\sqrt{c\,x^4+b\,x^2+a}\left(ac-\frac{b^2}{4}\right)d^3c^{\frac{5}{2}}\ln\left(\frac{2\sqrt{c\,x^4+b\,x^2+a}\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,e+\left(b\,x^2+2a\right)e-d\left(2c\,x^2+b\right)}{e\,x^2+d}\right)+e\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\left(\sqrt{c\,x^4+b\cdots}\right.}{2\sqrt{\frac{a\,e^2\cdots}{}}}$
elliptic	$\frac{\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{2e\,c^{\frac{3}{2}}}-\frac{2c\,d^3\ln\left(\frac{\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\sqrt{c\left(x^2+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x^2\cdots\right)}{e}}}{x^2+\frac{d}{e}}\right.}{e^2\left(e\sqrt{-4ac+b^2}-be+2cd\right)\left(e\sqrt{-4ac+b^2}+be-2cd\right)\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\cdots}$
default	$-\frac{\frac{x^2}{2c\sqrt{c\,x^4+b\,x^2+a}}-\frac{b\left(-\frac{1}{c\sqrt{c\,x^4+b\,x^2+a}}-\frac{b\left(2c\,x^2+b\right)}{c\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+a}}\right)}{4c}+\frac{\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{2c^{\frac{3}{2}}}}{e}+\frac{d^2\left(2c\,x^2+b\right)}{e^3\sqrt{c\,x^4+b\,x^2+a}\,(4ac-\cdots}$

input
```
int(x^7/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*((c*x^4+b*x^2+a)
^(1/2)*(a*c-1/4*b^2)*d^3*c^(5/2)*ln((2*(c*x^4+b*x^2+a)^(1/2)*((a*e^2-b*d*e
+c*d^2)/e^2)^(1/2)*e+(b*x^2+2*a)*e-d*(2*c*x^2+b))/(e*x^2+d))+e*((a*e^2-b*d
*e+c*d^2)/e^2)^(1/2)*((c*x^4+b*x^2+a)^(1/2)*(a*e^2-b*d*e+c*d^2)*c*(a*c-1/4
*b^2)*ln((2*c*x^2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c^(1/2))-ln(2)*(a*e^2
-b*d*e+c*d^2)*c*(a*c-1/4*b^2)*(c*x^4+b*x^2+a)^(1/2)+e*((-a*e*x^2+d*(3/2*b*
x^2+a))*a*c+1/2*b*(b*x^2+a)*(a*e-b*d))*c^(3/2)))/e^2/(a*e^2-b*d*e+c*d^2)/(
a*c-1/4*b^2)/c^(5/2)
```

3.342.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1202 vs. $2(214) = 428$.

Time = 37.45 (sec) , antiderivative size = 4901, normalized size of antiderivative = 20.77

$$\int \frac{x^7}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^7/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*
b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (
a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3
)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b
*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^
4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^
2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x^2)*sqrt(c
)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 +
b)*sqrt(c) - 4*a*c) + ((b^2*c^3 - 4*a*c^4)*d^3*x^4 + (b^3*c^2 - 4*a*b*c^3)
*d^3*x^2 + (a*b^2*c^2 - 4*a^2*c^3)*d^3)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-(
(8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 +
(b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 -
4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 +
 b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)) - 4*(a^3*b*c*e^4 - (a*b^2*c^2
- 2*a^2*c^3)*d^3*e + (a*b^3*c - a^2*b*c^2)*d^2*e^2 - 2*(a^2*b^2*c - a^3*c^
2)*d*e^3 - ((b^3*c^2 - 3*a*b*c^3)*d^3*e - (b^4*c - 2*a*b^2*c^2 - 2*a^2*c^3
)*d^2*e^2 + (2*a*b^3*c - 5*a^2*b*c^2)*d*e^3 - (a^2*b^2*c - 2*a^3*c^2)*e^4)
*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a*b^2*c^4 - 4*a^2*c^5)*d^4*e - 2*(a*b^3*c
^3 - 4*a^2*b*c^4)*d^3*e^2 + (a*b^4*c^2 - 2*a^2*b^2*c^3 - 8*a^3*c^4)*d^2*e^
3 - 2*(a^2*b^3*c^2 - 4*a^3*b*c^3)*d*e^4 + (a^3*b^2*c^2 - 4*a^4*c^3)*e^5...
```

3.342.6 Sympy [F]

$$\int \frac{x^7}{(d+ex^2)\,(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^7}{(d+ex^2)\,(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**7/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral(x**7/((d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

3.342.7 Maxima [F]

$$\int \frac{x^7}{(d+ex^2)\,(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^7}{(cx^4+bx^2+a)^{\frac{3}{2}}(ex^2+d)}\,dx$$

input
```
integrate(x^7/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^7/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)), x)
```

3.342.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^7}{(d+ex^2)\,(a+bx^2+cx^4)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^7/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.342.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{(d+ex^2)\,(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^7}{(e\,x^2+d)\,\left(c\,x^4+b\,x^2+a\right)^{3/2}}\,dx$$

input
```
int(x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x)
```

output
```
int(x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

3.343 $\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx$

3.343.1 Optimal result

Integrand size = 29, antiderivative size = 167

$$\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx = -\frac{a(bd-2ae)+(b^2d-2acd-abe)x^2}{(b^2-4ac)(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}} + \frac{d^2\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2(cd^2-bde+ae^2)^{3/2}}$$

output
```
1/2*d^2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)
/(c*x^4+b*x^2+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(3/2)+(-a*(-2*a*e+b*d)-(-a*b*e
-2*a*c*d+b^2*d)*x^2)/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*x^4+b*x^2+a)^(1/2
)
```

3.343.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.08

$$\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx = \frac{-2a^2e+b^2dx^2-2acdx^2+ab(d-ex^2)}{(b^2-4ac)(-cd^2+e(bd-ae))\sqrt{a+bx^2+cx^4}} + \frac{d^2\sqrt{-cd^2+bde-ae^2}\arctan\left(\frac{\sqrt{c}(d+ex^2)-e\sqrt{a+bx^2+cx^4}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{(cd^2+e(-bd+ae))^2}$$

input
```
Integrate[x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

output
```
(-2*a^2*e + b^2*d*x^2 - 2*a*c*d*x^2 + a*b*(d - e*x^2))/((b^2 - 4*a*c)*(-(c
*d^2) + e*(b*d - a*e))*Sqrt[a + b*x^2 + c*x^4]) + (d^2*Sqrt[-(c*d^2) + b*d
*e - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x^2) - e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[
-(c*d^2) + e*(b*d - a*e)]])/(c*d^2 + e*(-(b*d) + a*e))^2
```

3.343.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1578, 1264, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}dx$$

$$\Big\downarrow \text{ 1578}$$

$$\frac{1}{2}\int \frac{x^4}{(ex^2+d)(cx^4+bx^2+a)^{3/2}}dx^2$$

$$\Big\downarrow \text{ 1264}$$

$$\frac{1}{2}\left(-\frac{2\int -\frac{(b^2-4ac)d^2}{2(cd^2-bed+ae^2)(ex^2+d)\sqrt{cx^4+bx^2+a}}dx^2}{b^2-4ac}-\frac{2(x^2(-abe-2acd+b^2d)+a(bd-2ae))}{(b^2-4ac)\sqrt{a+bx^2+cx^4}(ae^2-bde+cd^2)}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(\frac{d^2\int \frac{1}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx^2}{ae^2-bde+cd^2}-\frac{2(x^2(-abe-2acd+b^2d)+a(bd-2ae))}{(b^2-4ac)\sqrt{a+bx^2+cx^4}(ae^2-bde+cd^2)}\right)$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{1}{2}\left(-\frac{2d^2\int \frac{1}{4(cd^2-bed+ae^2)-x^4}d\left(-\frac{(2cd-be)x^2+bd-2ae}{\sqrt{cx^4+bx^2+a}}\right)}{ae^2-bde+cd^2}-\frac{2(x^2(-abe-2acd+b^2d)+a(bd-2ae))}{(b^2-4ac)\sqrt{a+bx^2+cx^4}(ae^2-bde+cd^2)}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}\left(\frac{d^2\text{arctanh}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{(ae^2-bde+cd^2)^{3/2}}-\frac{2(x^2(-abe-2acd+b^2d)+a(bd-2ae))}{(b^2-4ac)\sqrt{a+bx^2+cx^4}\,(ae^2-bde+cd^2)}\right)$$

input
```
Int[x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

output
```
((-2*(a*(b*d - 2*a*e) + (b^2*d - 2*a*c*d - a*b*e)*x^2))/((b^2 - 4*a*c)*(c*
d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4]) + (d^2*ArcTanh[(b*d - 2*a*e
+ (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4
])])/(c*d^2 - b*d*e + a*e^2)^(3/2))/2
```

3.343.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1264
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)
^m*(f + g*x)^n, a + b*x + c*x^2, x], R = Coeff[PolynomialRemainder[(d + e*x
)^m*(f + g*x)^n, a + b*x + c*x^2, x], x, 0], S = Coeff[PolynomialRemainder[
(d + e*x)^m*(f + g*x)^n, a + b*x + c*x^2, x], x, 1]}, Simp[(b*R - 2*a*S + (
2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + S
imp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*E
xpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m - ((2*p + 3)*(2*c*R - b*S)
)/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[n, 1]
 && LtQ[p, -1] && ILtQ[m, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.343.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.37

method	result
pseudoelliptic	$-\frac{\sqrt{c x^4+b x^2+a}\, d^2\left(ac-\frac{b^2}{4}\right)\ln\left(\frac{2\sqrt{c x^4+b x^2+a}\sqrt{\frac{a e^2-bde+c d^2}{e^2}}\, e+\left(b x^2+2a\right)e-d\left(2c x^2+b\right)}{e x^2+d}\right)+\left(a\left(\frac{b x^2}{2}+a\right)e-\frac{d\left(\left(-2c x^2+b\cdots\right.\right.}{2}\cdots\right.}{2\sqrt{c x^4+b x^2+a}\sqrt{\frac{a e^2-bde+c d^2}{e^2}}\, e\left(a e^2-bde+c d^2\right)\left(ac-\frac{b^2}{4}\right)}$
elliptic	$\frac{2c d^2\ln\left(\frac{\frac{2a e^2-2bde+2c d^2}{e^2}+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a e^2-bde+c d^2}{e^2}}\sqrt{c\left(x^2+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a e^2-bde+c d^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{\left(e\sqrt{-4ac+b^2}-be+2cd\right)\left(e\sqrt{-4ac+b^2}+be-2cd\right)e\sqrt{\frac{a e^2-bde+c d^2}{e^2}}}+\frac{\left(b+\sqrt{\cdots}\right.}{\cdots}$
default	$-\frac{b x^2+2a}{e\sqrt{c x^4+b x^2+a}\,(4ac-b^2)}-\frac{d(2c x^2+b)}{e^2\sqrt{c x^4+b x^2+a}\,(4ac-b^2)}+\frac{d^2\left(-\frac{2c\sqrt{c\left(x^2-\frac{-b+\sqrt{-4ac+b^2}}{2c}\right)^2+\sqrt{-4ac+b^2}\left(x^2-\frac{-b+\sqrt{\cdots}}{2\cdots}\right.}}{\left(e\sqrt{-4ac+b^2}-be+2cd\right)\left(-4ac+b^2\right)\left(x^2-\frac{-b+\sqrt{-4ac\cdots}}{2c}\right.}\cdots\right.}{\cdots}$

input
```
int(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*((c*x^4+b*x^2+a)^(1/2)*d^2*(a*c-1/4*b^2)*ln((2*(c*x^4+b*x^2+a)^(1/2)*
((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*e+(b*x^2+2*a)*e-d*(2*c*x^2+b))/(e*x^2+d))+
(a*(1/2*b*x^2+a)*e-1/2*d*((-2*c*x^2+b)*a+b^2*x^2))*e*((a*e^2-b*d*e+c*d^2)/
e^2)^(1/2))/(c*x^4+b*x^2+a)^(1/2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)/e/(a*e^2
-b*d*e+c*d^2)/(a*c-1/4*b^2)
```

3.343.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 669 vs. $2(155) = 310$.

Time = 0.54 (sec) , antiderivative size = 1381, normalized size of antiderivative = 8.27

$$\int \frac{x^5}{(d+ex^2)\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(((b^2*c - 4*a*c^2)*d^2*x^4 + (b^3 - 4*a*b*c)*d^2*x^2 + (a*b^2 - 4*a^
2*c)*d^2)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2
+ 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d
^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt
(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*
e*x^2 + d^2)) - 4*(a*b*c*d^3 + 3*a^2*b*d*e^2 - 2*a^3*e^3 - (a*b^2 + 2*a^2*
c)*d^2*e - (a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 + (b^3 - a*b*c)*d^2*e - 2*(a
*b^2 - a^2*c)*d*e^2)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a*b^2*c^2 - 4*a^2*c^3
)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2
)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^
2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^
2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*
a^3*c^2)*e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d
^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d
*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x^2), 1/2*(((b^2*c - 4*a*c^2)*d^2*x^4 +
(b^3 - 4*a*b*c)*d^2*x^2 + (a*b^2 - 4*a^2*c)*d^2)*sqrt(-c*d^2 + b*d*e - a*e
^2)*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c
*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2
- a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - 2*(a*b*c*d^3 +
 3*a^2*b*d*e^2 - 2*a^3*e^3 - (a*b^2 + 2*a^2*c)*d^2*e - (a^2*b*e^3 - (b^...
```

3.343.6 Sympy [F]

$$\int \frac{x^5}{(d+ex^2)\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \int \frac{x^5}{(d+ex^2)\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**5/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral(x**5/((d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

3.343.7 Maxima [F]

$$\int \frac{x^5}{(d+ex^2)\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \int \frac{x^5}{(cx^4+bx^2+a)^{\frac{3}{2}}(ex^2+d)}\,dx$$

input
```
integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^5/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)), x)
```

3.343.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 468 vs. $2(155) = 310$.

Time = 0.31 (sec) , antiderivative size = 468, normalized size of antiderivative = 2.80

$$\int \frac{x^5}{(d+ex^2)\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{d^2\arctan\left(-\frac{\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{(cd^2-bde+ae^2)\sqrt{-cd^2+bde-ae^2}} - \frac{\frac{\left(b^2cd^3-2\,ac^2d^3-b^3d^2e+abcd^2e+2\,ab^2de^2-2\,a^2cde^2-a^2be^3\right)x^2}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,abc^2d^3e+b^4d^2e^2-2\,ab^2cd^2e^2-8\,a^2c^2d^2e^2-2\,ab^3de^3+8\,a^2bcde^3+a^2b^2e^4-4\,a^3ce^4} + \frac{\cdots}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,a\cdots}}{\sqrt{cx^4+bx^2+a}}$$

input
```
integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
d^2*arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*e + sqrt(c)*d)/sqrt(-
c*d^2 + b*d*e - a*e^2))/((c*d^2 - b*d*e + a*e^2)*sqrt(-c*d^2 + b*d*e - a*e
^2)) - ((b^2*c*d^3 - 2*a*c^2*d^3 - b^3*d^2*e + a*b*c*d^2*e + 2*a*b^2*d*e^2
 - 2*a^2*c*d*e^2 - a^2*b*e^3)*x^2/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3
*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2
 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4) + (a*b*c*d
^3 - a*b^2*d^2*e - 2*a^2*c*d^2*e + 3*a^2*b*d*e^2 - 2*a^3*e^3)/(b^2*c^2*d^4
 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c
*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e
^4 - 4*a^3*c*e^4))/sqrt(c*x^4 + b*x^2 + a)
```

3.343.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^5}{(e\,x^2+d)\,(c\,x^4+b\,x^2+a)^{3/2}}\,dx$$

input
```
int(x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x)
```

output
```
int(x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

3.344 $\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx$

3.344.1 Optimal result

Integrand size = 29, antiderivative size = 159

$$\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx=\frac{a(2cd-be)+c(bd-2ae)x^2}{(b^2-4ac)(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}}-\frac{de\,\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2(cd^2-bde+ae^2)^{3/2}}$$

output
```
-1/2*d*e*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2
)/(c*x^4+b*x^2+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(3/2)+(a*(-b*e+2*c*d)+c*(-2*a
*e+b*d)*x^2)/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*x^4+b*x^2+a)^(1/2)
```

3.344.2 Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.12

$$\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx=\frac{\frac{(cd^2+e(-bd+ae))(-abe+bcdx^2+2ac(d-ex^2))}{\sqrt{a+bx^2+cx^4}}+(-b^2+4ac)\,de\sqrt{-cd^2+e(bd-}{(b^2-4ac)(cd^2+e(-bd+ae))^2}$$

input
```
Integrate[x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

output
```
(((c*d^2 + e*(-(b*d) + a*e))*(-(a*b*e) + b*c*d*x^2 + 2*a*c*(d - e*x^2)))/S
qrt[a + b*x^2 + c*x^4] + (-b^2 + 4*a*c)*d*e*Sqrt[-(c*d^2) + e*(b*d - a*e)]
*ArcTan[(Sqrt[c]*(d + e*x^2) - e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) +
e*(b*d - a*e)]])/((b^2 - 4*a*c)*(c*d^2 + e*(-(b*d) + a*e))^2)
```

3.344.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1578, 1235, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{x^2}{(ex^2+d)(cx^4+bx^2+a)^{3/2}}dx^2$$

$$\Big\downarrow \text{1235}$$

$$\frac{1}{2}\left(\frac{2(cx^2(bd-2ae)+a(2cd-be))}{(b^2-4ac)\sqrt{a+bx^2+cx^4}(ae^2-bde+cd^2)}-\frac{2\int \frac{(b^2-4ac)de}{2(ex^2+d)\sqrt{cx^4+bx^2+a}}dx^2}{(b^2-4ac)(ae^2-bde+cd^2)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{2(cx^2(bd-2ae)+a(2cd-be))}{(b^2-4ac)\sqrt{a+bx^2+cx^4}(ae^2-bde+cd^2)}-\frac{de\int \frac{1}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx^2}{ae^2-bde+cd^2}\right)$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{2}\left(\frac{2de\int \frac{1}{4(cd^2-bed+ae^2)-x^4}d\left(-\frac{(2cd-be)x^2+bd-2ae}{\sqrt{cx^4+bx^2+a}}\right)}{ae^2-bde+cd^2}+\frac{2(cx^2(bd-2ae)+a(2cd-be))}{(b^2-4ac)\sqrt{a+bx^2+cx^4}(ae^2-bde+cd^2)}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{2(cx^2(bd-2ae)+a(2cd-be))}{(b^2-4ac)\sqrt{a+bx^2+cx^4}(ae^2-bde+cd^2)}-\frac{de\,\text{arctanh}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{(ae^2-bde+cd^2)^{3/2}}\right)$$

input
```
Int[x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

output
```
((2*(a*(2*c*d - b*e) + c*(b*d - 2*a*e)*x^2))/((b^2 - 4*a*c)*(c*d^2 - b*d*e
 + a*e^2)*Sqrt[a + b*x^2 + c*x^4]) - (d*e*ArcTanh[(b*d - 2*a*e + (2*c*d -
b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(c*d^2
 - b*d*e + a*e^2)^(3/2))/2
```

3.344.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

3.344. $\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx$

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.344.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.36

method	result
pseudoelliptic	$\frac{\frac{d \ln\left(\frac{\frac{2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,e+\left(b\,x^2+2a\right)e-d\left(2c\,x^2+b\right)}{e\,x^2+d}\right)\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+a}}{2}+\left(a\left(2c\,x^2+b\right)e-2\left(\frac{b\,x^2}{2}+a\right)cd\right)\sqrt{\frac{a\,e}{}}}{4\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,\left(a\,e^2-bde+c\,d^2\right)\left(ac-\frac{b^2}{4}\right)}$
elliptic	$-\frac{2cd\ln\left(\frac{\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,\sqrt{c\left(x^2+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a\,e^2-bde+c\,d^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{\left(e\sqrt{-4ac+b^2}-be+2cd\right)\left(e\sqrt{-4ac+b^2}+be-2cd\right)\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}}-\frac{\left(-b+\right.}{}$
default	$\frac{2c\,x^2+b}{e\sqrt{c\,x^4+b\,x^2+a}\,(4ac-b^2)}-\frac{d\left(-\frac{2c\sqrt{c\left(x^2-\frac{-b+\sqrt{-4ac+b^2}}{2c}\right)^2+\sqrt{-4ac+b^2}\,\left(x^2-\frac{-b+\sqrt{-4ac+b^2}}{2c}\right)}}{\left(e\sqrt{-4ac+b^2}-be+2cd\right)\left(-4ac+b^2\right)\left(x^2-\frac{-b+\sqrt{-4ac+b^2}}{2c}\right)}+\frac{2c\sqrt{c\left(x^2+\frac{b+\sqrt{-4ac+b^2}}{2c}\right.}}{\left(e\sqrt{-4ac+b^2}+be-2cd\right.}\right.}{}$

input
```
int(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/(c*x^4+b*x^2+a)^(1/2)*(1/2*d*ln((2*(c*x^4+b*x^2+a)^(1/2)*((a*e^2-b*d*e
+c*d^2)/e^2)^(1/2)*e+(b*x^2+2*a)*e-d*(2*c*x^2+b))/(e*x^2+d))*(4*a*c-b^2)*(
c*x^4+b*x^2+a)^(1/2)+(a*(2*c*x^2+b)*e-2*(1/2*b*x^2+a)*c*d)*((a*e^2-b*d*e+c
*d^2)/e^2)^(1/2))/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)/(a*e^2-b*d*e+c*d^2)/(a*c
-1/4*b^2)
```

3.344.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 653 vs. $2(147) = 294$.

Time = 0.58 (sec) , antiderivative size = 1349, normalized size of antiderivative = 8.48

$$\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(((b^2*c - 4*a*c^2)*d*e*x^4 + (b^3 - 4*a*b*c)*d*e*x^2 + (a*b^2 - 4*a^
2*c)*d*e)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2
+ 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d
^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt
(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*
e*x^2 + d^2)) + 4*(2*a*c^2*d^3 - 3*a*b*c*d^2*e - a^2*b*e^3 + (a*b^2 + 2*a^
2*c)*d*e^2 + (b*c^2*d^3 + 3*a*b*c*d*e^2 - 2*a^2*c*e^3 - (b^2*c + 2*a*c^2)*
d^2*e)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b
^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*
(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^
4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)
*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*
x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 -
2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^
3 - 4*a^3*b*c)*e^4)*x^2), -1/2*(((b^2*c - 4*a*c^2)*d*e*x^4 + (b^3 - 4*a*b*
c)*d*e*x^2 + (a*b^2 - 4*a^2*c)*d*e)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1
/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2
 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a
^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - 2*(2*a*c^2*d^3 - 3*a*b*c*d^
2*e - a^2*b*e^3 + (a*b^2 + 2*a^2*c)*d*e^2 + (b*c^2*d^3 + 3*a*b*c*d*e^2 ...
```

3.344.6 Sympy [F]

$$\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral(x**3/((d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

3.344.7 Maxima [F]

$$\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^3}{(cx^4+bx^2+a)^{\frac{3}{2}}(ex^2+d)}\,dx$$

input
```
integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^3/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)), x)
```

3.344.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 451 vs. $2(147) = 294$.

Time = 0.29 (sec) , antiderivative size = 451, normalized size of antiderivative = 2.84

$$\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx = -\frac{de\arctan\left(-\frac{\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{(cd^2-bde+ae^2)\sqrt{-cd^2+bde-ae^2}} + \frac{\frac{(bc^2d^3-b^2cd^2e-2\,ac^2d^2e+3\,abcde^2-2\,a^2ce^3)x^2}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,abc^2d^3e+b^4d^2e^2-2\,ab^2cd^2e^2-8\,a^2c^2d^2e^2-2\,ab^3de^3+8\,a^2bcde^3+a^2b^2e^4-4\,a^3ce^4} + \frac{\cdots}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,a\cdots}}{\sqrt{cx^4+bx^2+a}}$$

input
```
integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-d*e*arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*e + sqrt(c)*d)/sqrt(
-c*d^2 + b*d*e - a*e^2))/((c*d^2 - b*d*e + a*e^2)*sqrt(-c*d^2 + b*d*e - a*
e^2)) + ((b*c^2*d^3 - b^2*c*d^2*e - 2*a*c^2*d^2*e + 3*a*b*c*d*e^2 - 2*a^2*
c*e^3)*x^2/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e +
b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^
2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4) + (2*a*c^2*d^3 - 3*a*b*c*d^2*e +
a*b^2*d*e^2 + 2*a^2*c*d*e^2 - a^2*b*e^3)/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^
3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*
d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4))/sq
rt(c*x^4 + b*x^2 + a)
```

3.344.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^3}{(e\,x^2+d)\,\left(c\,x^4+b\,x^2+a\right)^{3/2}}\,dx$$

input
```
int(x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x)
```

output
```
int(x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

3.345 $\int \frac{x}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx$

3.345.1 Optimal result

Integrand size = 27, antiderivative size = 166

$$\int \frac{x}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx = -\frac{bcd-b^2e+2ace+c(2cd-be)x^2}{(b^2-4ac)(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}} + \frac{e^2\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2(cd^2-bde+ae^2)^{3/2}}$$

output
```
1/2*e^2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)
/(c*x^4+b*x^2+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(3/2)+(-b*c*d+b^2*e-2*a*c*e-c*
(-b*e+2*c*d)*x^2)/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*x^4+b*x^2+a)^(1/2)
```

3.345.2 Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.10

$$\int \frac{x}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx = \frac{-b^2e+2c(ae+cdx^2)+bc(d-ex^2)}{(b^2-4ac)(-cd^2+e(bd-ae))\sqrt{a+bx^2+cx^4}} + \frac{e^2\sqrt{-cd^2+e(bd-ae)}\arctan\left(\frac{\sqrt{-cd^2+e(bd-ae)}x^2}{\sqrt{a}(d+ex^2)-d\sqrt{a+bx^2+cx^4}}\right)}{(cd^2+e(-bd+ae))^2}$$

input
```
Integrate[x/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

output
```
(-(b^2*e) + 2*c*(a*e + c*d*x^2) + b*c*(d - e*x^2))/((b^2 - 4*a*c)*(-(c*d^2
) + e*(b*d - a*e))*Sqrt[a + b*x^2 + c*x^4]) + (e^2*Sqrt[-(c*d^2) + e*(b*d
- a*e)]*ArcTan[(Sqrt[-(c*d^2) + e*(b*d - a*e)]*x^2)/(Sqrt[a]*(d + e*x^2) -
 d*Sqrt[a + b*x^2 + c*x^4])])/(c*d^2 + e*(-(b*d) + a*e))^2
```

3.345.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1576, 1165, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}dx$$

$$\Big\downarrow \text{ 1576}$$

$$\frac{1}{2}\int \frac{1}{(ex^2+d)(cx^4+bx^2+a)^{3/2}}dx^2$$

$$\Big\downarrow \text{ 1165}$$

$$\frac{1}{2}\left(-\frac{2\int -\frac{(b^2-4ac)e^2}{2(ex^2+d)\sqrt{cx^4+bx^2+a}}dx^2}{(b^2-4ac)(ae^2-bde+cd^2)}-\frac{2(2ace+b^2(-e)+cx^2(2cd-be)+bcd)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}(ae^2-bde+cd^2)}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(\frac{e^2\int \frac{1}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx^2}{ae^2-bde+cd^2}-\frac{2(2ace+b^2(-e)+cx^2(2cd-be)+bcd)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}(ae^2-bde+cd^2)}\right)$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{1}{2}\left(-\frac{2e^2\int \frac{1}{4(cd^2-bed+ae^2)-x^4}d\left(-\frac{(2cd-be)x^2+bd-2ae}{\sqrt{cx^4+bx^2+a}}\right)}{ae^2-bde+cd^2}-\frac{2(2ace+b^2(-e)+cx^2(2cd-be)+bcd)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}(ae^2-bde+cd^2)}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}\left(\frac{e^2\text{arctanh}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{(ae^2-bde+cd^2)^{3/2}}-\frac{2(2ace+b^2(-e)+cx^2(2cd-be)+bcd)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}(ae^2-bde+cd^2)}\right)$$

input
```
Int[x/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

output
```
((-2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x^2))/((b^2 - 4*a*c)*(c*d^
2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4]) + (e^2*ArcTanh[(b*d - 2*a*e +
(2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])
])/(c*d^2 - b*d*e + a*e^2)^(3/2))/2
```

3.345.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

3.345. $\int \frac{x}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx$

3.345.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.34

method	result
pseudoelliptic	$\frac{-e\sqrt{c\,x^4+b\,x^2+a}\left(ac-\frac{b^2}{4}\right)\ln\left(\frac{2\sqrt{c\,x^4+b\,x^2+a}\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\,e+\left(b\,x^2+2a\right)e-d\left(2c\,x^2+b\right)}{e\,x^2+d}\right)+\left(c^2d\,x^2+\left(\left(-\frac{b\,x^2}{2}+a\right)e+\frac{bd}{2}\right)c}{2\sqrt{c\,x^4+b\,x^2+a}\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\left(a\,e^2-bde+c\,d^2\right)\left(ac-\frac{b^2}{4}\right)}$
default	$-\frac{2c\sqrt{c\left(x^2-\frac{-b+\sqrt{-4ac+b^2}}{2c}\right)^2+\sqrt{-4ac+b^2}\left(x^2-\frac{-b+\sqrt{-4ac+b^2}}{2c}\right)}}{\left(e\sqrt{-4ac+b^2}-be+2cd\right)\left(-4ac+b^2\right)\left(x^2-\frac{-b+\sqrt{-4ac+b^2}}{2c}\right)}+\frac{2c\sqrt{c\left(x^2+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)^2-\sqrt{-4ac+b^2}\left(x^2+\frac{b+}{}\right.}}{\left(e\sqrt{-4ac+b^2}+be-2cd\right)\left(-4ac+b^2\right)\left(x^2+\frac{b+\sqrt{-}}{}\right.}$
elliptic	$-\frac{2c\sqrt{c\left(x^2-\frac{-b+\sqrt{-4ac+b^2}}{2c}\right)^2+\sqrt{-4ac+b^2}\left(x^2-\frac{-b+\sqrt{-4ac+b^2}}{2c}\right)}}{\left(e\sqrt{-4ac+b^2}-be+2cd\right)\left(-4ac+b^2\right)\left(x^2-\frac{-b+\sqrt{-4ac+b^2}}{2c}\right)}+\frac{2c\sqrt{c\left(x^2+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)^2-\sqrt{-4ac+b^2}\left(x^2+\frac{b+}{}\right.}}{\left(e\sqrt{-4ac+b^2}+be-2cd\right)\left(-4ac+b^2\right)\left(x^2+\frac{b+\sqrt{-}}{}\right.}$

input
```
int(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-e*(c*x^4+b*x^2+a)^(1/2)*(a*c-1/4*b^2)*ln((2*(c*x^4+b*x^2+a)^(1/2)*((
a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*e+(b*x^2+2*a)*e-d*(2*c*x^2+b))/(e*x^2+d))+(c
^2*d*x^2+((-1/2*b*x^2+a)*e+1/2*b*d)*c-1/2*b^2*e)*((a*e^2-b*d*e+c*d^2)/e^2)
^(1/2))/(c*x^4+b*x^2+a)^(1/2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)/(a*e^2-b*d*e
+c*d^2)/(a*c-1/4*b^2)
```

3.345.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 668 vs. 2(154) = 308.

Time = 0.56 (sec) , antiderivative size = 1379, normalized size of antiderivative = 8.31

$$\int \frac{x}{(d+ex^2)\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(((b^2*c - 4*a*c^2)*e^2*x^4 + (b^3 - 4*a*b*c)*e^2*x^2 + (a*b^2 - 4*a^
2*c)*e^2)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2
+ 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d
^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt
(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*
e*x^2 + d^2)) - 4*(b*c^2*d^3 - 2*(b^2*c - a*c^2)*d^2*e + (b^3 - a*b*c)*d*e
^2 - (a*b^2 - 2*a^2*c)*e^3 + (2*c^3*d^3 - 3*b*c^2*d^2*e - a*b*c*e^3 + (b^2
*c + 2*a*c^2)*d*e^2)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a*b^2*c^2 - 4*a^2*c^3
)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2
)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^
2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^
2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*
a^3*c^2)*e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d
^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d
*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x^2), 1/2*(((b^2*c - 4*a*c^2)*e^2*x^4 +
(b^3 - 4*a*b*c)*e^2*x^2 + (a*b^2 - 4*a^2*c)*e^2)*sqrt(-c*d^2 + b*d*e - a*e
^2)*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c
*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2
- a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - 2*(b*c^2*d^3 -
 2*(b^2*c - a*c^2)*d^2*e + (b^3 - a*b*c)*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 ...
```

3.345.6 Sympy [F]

$$\int \frac{x}{(d+ex^2)\,(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{x}{(d+ex^2)\,(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral(x/((d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

3.345.7 Maxima [F]

$$\int \frac{x}{(d+ex^2)\,(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{x}{(cx^4+bx^2+a)^{\frac{3}{2}}(ex^2+d)}\,dx$$

input
```
integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)), x)
```

3.345.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 468 vs. $2(154) = 308$.

Time = 0.28 (sec) , antiderivative size = 468, normalized size of antiderivative = 2.82

$$\int \frac{x}{(d+ex^2)\,(a+bx^2+cx^4)^{3/2}}\,dx = \frac{e^2\arctan\left(-\frac{\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{(cd^2-bde+ae^2)\sqrt{-cd^2+bde-ae^2}} - \frac{\frac{\left(2\,c^3d^3-3\,bc^2d^2e+b^2cde^2+2\,ac^2de^2-abce^3\right)x^2}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,abc^2d^3e+b^4d^2e^2-2\,ab^2cd^2e^2-8\,a^2c^2d^2e^2-2\,ab^3de^3+8\,a^2bcde^3+a^2b^2e^4-4\,a^3ce^4} + \frac{bc^2d\cdots}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,\cdots}}{\sqrt{cx^4+bx^2+a}}$$

input
```
integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
e^2*arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*e + sqrt(c)*d)/sqrt(-
c*d^2 + b*d*e - a*e^2))/((c*d^2 - b*d*e + a*e^2)*sqrt(-c*d^2 + b*d*e - a*e
^2)) - ((2*c^3*d^3 - 3*b*c^2*d^2*e + b^2*c*d*e^2 + 2*a*c^2*d*e^2 - a*b*c*e
^3)*x^2/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4
*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b
*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4) + (b*c^2*d^3 - 2*b^2*c*d^2*e + 2*a*c
^2*d^2*e + b^3*d*e^2 - a*b*c*d*e^2 - a*b^2*e^3 + 2*a^2*c*e^3)/(b^2*c^2*d^4
 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c
*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e
^4 - 4*a^3*c*e^4))/sqrt(c*x^4 + b*x^2 + a)
```

3.345.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{x}{(e\,x^2+d)\,\left(c\,x^4+b\,x^2+a\right)^{3/2}}\,dx$$

input
```
int(x/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x)
```

output
```
int(x/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

3.346 $\int \frac{1}{x(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx$

3.346.1 Optimal result

Integrand size = 29, antiderivative size = 266

$$\int \frac{1}{x(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx = \frac{b^2-2ac+bcx^2}{a(b^2-4ac)\,d\sqrt{a+bx^2+cx^4}} + \frac{e(bcd-b^2e+2ace+c(2cd-be)x^2)}{(b^2-4ac)\,d\,(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}} - \frac{\operatorname{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}d} - \frac{e^3\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2d\,(cd^2-bde+ae^2)^{3/2}}$$

output
```
-1/2*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(3/2)/d-1/2*
e^3*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*
x^4+b*x^2+a)^(1/2))/d/(a*e^2-b*d*e+c*d^2)^(3/2)+(b*c*x^2-2*a*c+b^2)/a/(-4*
a*c+b^2)/d/(c*x^4+b*x^2+a)^(1/2)+e*(b*c*d-b^2*e+2*a*c*e+c*(-b*e+2*c*d)*x^2
)/(-4*a*c+b^2)/d/(a*e^2-b*d*e+c*d^2)/(c*x^4+b*x^2+a)^(1/2)
```

3.346.2 Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.93

$$\int \frac{1}{x(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx = \frac{b^3e-bc(3ae+cdx^2)+2ac^2(d-ex^2)+b^2c(-d+ex^2)}{a(-b^2+4ac)(cd^2+e(-bd+ae))\sqrt{a+bx^2+cx^4}} - \frac{e^3\sqrt{-cd^2+e(bd-ae)}\arctan\left(\frac{\sqrt{c}(d+ex^2)-e\sqrt{a+bx^2+cx^4}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{d(cd^2+e(-bd+ae))^2} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{a^{3/2}d}$$

input
```
Integrate[1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

output
```
(b^3*e - b*c*(3*a*e + c*d*x^2) + 2*a*c^2*(d - e*x^2) + b^2*c*(-d + e*x^2))
/(a*(-b^2 + 4*a*c)*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[a + b*x^2 + c*x^4]) - (
e^3*Sqrt[-(c*d^2) + e*(b*d - a*e)]*ArcTan[(Sqrt[c]*(d + e*x^2) - e*Sqrt[a
+ b*x^2 + c*x^4])/Sqrt[-(c*d^2) + e*(b*d - a*e)]]])/(d*(c*d^2 + e*(-(b*d) +
 a*e))^2) + ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]]/(a^(3
/2)*d)
```

3.346.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1578, 1289, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{1}{x^2(ex^2+d)(cx^4+bx^2+a)^{3/2}}dx^2$$

$$\Big\downarrow \text{1289}$$

$$\frac{1}{2}\int \left(\frac{1}{dx^2(cx^4+bx^2+a)^{3/2}}-\frac{e}{d(ex^2+d)(cx^4+bx^2+a)^{3/2}}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{\operatorname{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{a^{3/2}d}-\frac{e^3\operatorname{arctanh}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{d\left(ae^2-bde+cd^2\right)^{3/2}}+\frac{2e\left(2ace+b^2(-e)+cx^2(2cd-be)+\right.}{d\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(ae^2-bde\right.}\right.$$

input
```
Int[1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

output
```
((2*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*d*Sqrt[a + b*x^2 + c*x^4]) +
 (2*e*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x^2))/((b^2 - 4*a*c)*d*(c
*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4]) - ArcTanh[(2*a + b*x^2)/(2*
Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/(a^(3/2)*d) - (e^3*ArcTanh[(b*d - 2*a*e
+ (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4
])])/(d*(c*d^2 - b*d*e + a*e^2)^(3/2)))/2
```

3.346.3.1 Defintions of rubi rules used

rule 1289
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0]))
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.346. $\int \frac{1}{x(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx$

3.346.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.07

method	result
pseudoelliptic	$\frac{\frac{1}{a\sqrt{c\,x^4+b\,x^2+a}}-\frac{b\left(2c\,x^2+b\right)}{a\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+a}}-\frac{\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{a^{\frac{3}{2}}}+\frac{-\frac{e^2}{\sqrt{c\,x^4+b\,x^2+a}}+\frac{(be-2cd)e\left(2c\,x^2+b\right)}{\sqrt{c\,x^4+b\,x^2+a}\left(4ac-b^2\right)}+\frac{e^2\ln\cdots}{\cdots}}{\cdots}}{2d}$
default	$\frac{\frac{1}{2a\sqrt{c\,x^4+b\,x^2+a}}-\frac{b\left(2c\,x^2+b\right)}{2a\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+a}}-\frac{\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{2a^{\frac{3}{2}}}}{d}-\frac{e\left(-\frac{2c\sqrt{c\left(x^2-\frac{-b+\sqrt{-4ac+b^2}}{2c}\right)^2+\sqrt{-4ac\cdots}}}{\left(e\sqrt{-4ac+b^2}-be+2cd\right)\left(-4ac+b^2\right)\cdots}\cdots\right)}{\cdots}$
elliptic	$\frac{2c\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{d\left(-b+\sqrt{-4ac+b^2}\right)\left(b+\sqrt{-4ac+b^2}\right)\sqrt{a}}-\frac{2c\,e^2\ln\left(\frac{\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\sqrt{c\left(x^2+\frac{d}{e}\right)^2+\cdots}}{x^2+\frac{d}{e}}\right)}{\left(e\sqrt{-4ac+b^2}-be+2cd\right)\left(e\sqrt{-4ac+b^2}+be-2cd\right)d\sqrt{\frac{a\,e^2\cdots}{\cdots}}}$

input
```
int(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*(1/a/(c*x^4+b*x^2+a)^(1/2)-b/a*(2*c*x^2+b)/(4*a*c-b^2)/(c*x^4+b*x^2+
a)^(1/2)-1/a^(3/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)+1/(
a*e^2-b*d*e+c*d^2)*(-e^2/(c*x^4+b*x^2+a)^(1/2)+(b*e-2*c*d)*e*(2*c*x^2+b)/(
c*x^4+b*x^2+a)^(1/2)/(4*a*c-b^2)+e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2
*(c*x^4+b*x^2+a)^(1/2)*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*e+(b*x^2+2*a)*e-d*(
2*c*x^2+b))/(e*x^2+d))))
```

3.346.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1202 vs. 2(242) = 484.

Time = 2.44 (sec) , antiderivative size = 4909, normalized size of antiderivative = 18.45

$$\int \frac{1}{x\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(((a^2*b^2*c - 4*a^3*c^2)*e^3*x^4 + (a^2*b^3 - 4*a^3*b*c)*e^3*x^2 + (
a^3*b^2 - 4*a^4*c)*e^3)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b
*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^
2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b
*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(
e^2*x^4 + 2*d*e*x^2 + d^2)) + ((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c -
4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^
3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4
- 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^
2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^4 + (
(b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3
*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a
^3*b*c)*e^4)*x^2)*sqrt(a)*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x
^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 4*((a*b^2*c^2 - 2*a^
2*c^3)*d^4 - (2*a*b^3*c - 5*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 2*a^
3*c^2)*d^2*e^2 - (a^2*b^3 - 3*a^3*b*c)*d*e^3 + (a*b*c^3*d^4 - 2*(a*b^2*c^2
 - a^2*c^3)*d^3*e + (a*b^3*c - a^2*b*c^2)*d^2*e^2 - (a^2*b^2*c - 2*a^3*c^2
)*d*e^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a^3*b^2*c^2 - 4*a^4*c^3)*d^5 - 2*
(a^3*b^3*c - 4*a^4*b*c^2)*d^4*e + (a^3*b^4 - 2*a^4*b^2*c - 8*a^5*c^2)*d^3*
e^2 - 2*(a^4*b^3 - 4*a^5*b*c)*d^2*e^3 + (a^5*b^2 - 4*a^6*c)*d*e^4 + ((a...
```

3.346.6 Sympy [F]

$$\int \frac{1}{x\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \int \frac{1}{x\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral(1/(x*(d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

3.346.7 Maxima [F]

$$\int \frac{1}{x\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \int \frac{1}{(cx^4+bx^2+a)^{\frac{3}{2}}(ex^2+d)x}\,dx$$

input
```
integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*x), x)
```

3.346.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.346.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \int \frac{1}{x\,\left(e\,x^2+d\right)\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}\,dx$$

input
```
int(1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x)
```

output
```
int(1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

3.347 $\int \frac{1}{x^3(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx$

3.347.1 Optimal result

Integrand size = 29, antiderivative size = 419

$$\int \frac{1}{x^3\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}}\,dx=-\frac{e(b^2-2ac+bcx^2)}{a\left(b^2-4ac\right)d^2\sqrt{a+bx^2+cx^4}}+\frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)dx^2\sqrt{a+bx^2+cx^4}}-\frac{e^2(bcd-b^2e+2ace+c(2cd-be)x^2)}{\left(b^2-4ac\right)d^2\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}}-\frac{\left(3b^2-8ac\right)\sqrt{a+bx^2+cx^4}}{2a^2\left(b^2-4ac\right)dx^2}+\frac{3b\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{5/2}d}+\frac{e\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}d^2}+\frac{e^4\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2d^2\left(cd^2-bde+ae^2\right)^{3/2}}$$

output
```
3/4*b*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(5/2)/d+1/2
*e*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(3/2)/d^2+1/2*
e^4*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*
x^4+b*x^2+a)^(1/2))/d^2/(a*e^2-b*d*e+c*d^2)^(3/2)-e*(b*c*x^2-2*a*c+b^2)/a/
(-4*a*c+b^2)/d^2/(c*x^4+b*x^2+a)^(1/2)+(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/
d/x^2/(c*x^4+b*x^2+a)^(1/2)-e^2*(b*c*d-b^2*e+2*a*c*e+c*(-b*e+2*c*d)*x^2)/(
-4*a*c+b^2)/d^2/(a*e^2-b*d*e+c*d^2)/(c*x^4+b*x^2+a)^(1/2)-1/2*(-8*a*c+3*b^
2)*(c*x^4+b*x^2+a)^(1/2)/a^2/(-4*a*c+b^2)/d/x^2
```

3.347.2 Mathematica [A] (verified)

Time = 1.68 (sec) , antiderivative size = 363, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^3 (d+ex^2) (a+bx^2+cx^4)^{3/2}} \, dx =$$

$$-\frac{\frac{d(4a^3ce^2+3b^2d(-cd+be)x^2(b+cx^2)+a^2(-b^2e^2+4bce(-d+ex^2)+4c^2(d^2+dex^2+e^2x^4))+a(8c^3d^2x^4+b^3e(d-ex^2)+10bc^2dx^2(d-ex^2)-b^2c(d^2+}{a^2(b^2-4ac)(-cd^2+e(bd-ae))x^2\sqrt{a+bx^2+cx^4}}}{2d^2}$$

input
```
Integrate[1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

output
```
-1/2*((d*(4*a^3*c*e^2 + 3*b^2*d*(-(c*d) + b*e)*x^2*(b + c*x^2) + a^2*(-(b^
2*e^2) + 4*b*c*e*(-d + e*x^2) + 4*c^2*(d^2 + d*e*x^2 + e^2*x^4)) + a*(8*c^
3*d^2*x^4 + b^3*e*(d - e*x^2) + 10*b*c^2*d*x^2*(d - e*x^2) - b^2*c*(d^2 +
12*d*e*x^2 + e^2*x^4))))/(a^2*(b^2 - 4*a*c)*(-(c*d^2) + e*(b*d - a*e))*x^2
*Sqrt[a + b*x^2 + c*x^4]) - (2*e^4*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(
Sqrt[c]*(d + e*x^2) - e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + e*(b*d -
a*e)]])/(c*d^2 + e*(-(b*d) + a*e))^2 + ((3*b*d + 2*a*e)*ArcTanh[(Sqrt[c]*x
^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/a^(5/2))/d^2
```

3.347.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 416, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1578, 1289, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (d+ex^2) (a+bx^2+cx^4)^{3/2}} \, dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2} \int \frac{1}{x^4 (ex^2+d) (cx^4+bx^2+a)^{3/2}} dx^2$$

$$\Big\downarrow \text{1289}$$

$$\frac{1}{2} \int \left(\frac{e^2}{d^2 (ex^2+d) (cx^4+bx^2+a)^{3/2}} - \frac{e}{d^2x^2 (cx^4+bx^2+a)^{3/2}} + \frac{1}{dx^4 (cx^4+bx^2+a)^{3/2}} \right) dx^2$$

$\downarrow$ 2009

$$\frac{1}{2}\left(\frac{e\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{a^{3/2}d^2}+\frac{3b\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{5/2}d}-\frac{(3b^2-8ac)\sqrt{a+bx^2+cx^4}}{a^2dx^2(b^2-4ac)}+\frac{e^4\text{arctanh}\left(\frac{-2a}{2\sqrt{a+bx}}\right.}{d^2(ae^2-b}\right.$$

input
```
Int[1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

output
```
((-2*e*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*d^2*Sqrt[a + b*x^2 + c*x^
4]) + (2*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*d*x^2*Sqrt[a + b*x^2 +
c*x^4]) - (2*e^2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x^2))/((b^2 -
4*a*c)*d^2*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4]) - ((3*b^2 - 8*
a*c)*Sqrt[a + b*x^2 + c*x^4])/(a^2*(b^2 - 4*a*c)*d*x^2) + (3*b*ArcTanh[(2*
a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*a^(5/2)*d) + (e*ArcTan
h[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(a^(3/2)*d^2) + (e^4
*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*
Sqrt[a + b*x^2 + c*x^4])])/(d^2*(c*d^2 - b*d*e + a*e^2)^(3/2)))/2
```

3.347.3.1 Defintions of rubi rules used

rule 1289
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0]))
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.347.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 413, normalized size of antiderivative = 0.99

method	result
risch	$-\frac{\sqrt{c x^4+b x^2+a}}{2a^2 d x^2}-\frac{-\frac{(2ae+3bd)\ln\left(\frac{2a+b x^2+2\sqrt{a}\,\sqrt{c x^4+b x^2+a}}{x^2}\right)}{2d\sqrt{a}}+\frac{a^2 e^3 \ln\left(\frac{\frac{2a e^2-2bde+2c d^2}{e^2}+\frac{(be-2cd)\left(x^2+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a e^2-bd}{e}}}{}\right.}{d\left(a e^2-bde+c d^2\right)}}{}$
pseudoelliptic	$\frac{\frac{\sqrt{c x^4+b x^2+a}\, e^3 x^2\left(a^{\frac{5}{2}} b^2-4a^{\frac{7}{2}} c\right)\ln\left(\frac{2\sqrt{c x^4+b x^2+a}\,\sqrt{\frac{a e^2-bde+c d^2}{e^2}}\, e+\left(b x^2+2a\right)e-d\left(2c x^2+b\right)}{e x^2+d}\right)}{4}+\left(\left(a e^2-bde+c d^2\right)\left(ae+\frac{3b}{2}\right.\right.}{}$
elliptic	$-\frac{2c\left(-\frac{\sqrt{c x^4+b x^2+a}}{a x^2}+\frac{b\ln\left(\frac{2a+b x^2+2\sqrt{a}\,\sqrt{c x^4+b x^2+a}}{x^2}\right)}{2a^{\frac{3}{2}}}\right)}{d\left(-b+\sqrt{-4ac+b^2}\right)\left(b+\sqrt{-4ac+b^2}\right)}+\frac{8c^2(ae+bd)\ln\left(\frac{2a+b x^2+2\sqrt{a}\,\sqrt{c x^4+b x^2+a}}{x^2}\right)}{d^2\left(-b+\sqrt{-4ac+b^2}\right)^2\left(b+\sqrt{-4ac+b^2}\right)^2\sqrt{a}}+\frac{2c e^3\ln\left(\right.}{}$
default	$\frac{-\frac{1}{2a x^2\sqrt{c x^4+b x^2+a}}-\frac{3b\left(\frac{1}{a\sqrt{c x^4+b x^2+a}}-\frac{b\left(2c x^2+b\right)}{a\left(4ac-b^2\right)\sqrt{c x^4+b x^2+a}}-\frac{\ln\left(\frac{2a+b x^2+2\sqrt{a}\,\sqrt{c x^4+b x^2+a}}{x^2}\right)}{a^{\frac{3}{2}}}\right)}{4a}-\frac{2c\left(2c x^2+b\right)}{a\left(4ac-b^2\right)\sqrt{c x^4+b}}}{d}$

input
```
int(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^2/d*(c*x^4+b*x^2+a)^(1/2)/x^2-1/2/a^2/d*(-1/2*(2*a*e+3*b*d)/d/a^(1/
2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)+a^2*e^3/d/(a*e^2-b*
d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(
b*e-2*c*d)/e*(x^2+1/e*d)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+1/e*d)^
2+(b*e-2*c*d)/e*(x^2+1/e*d)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+1/e*d))-2
*d/(a*e^2-b*d*e+c*d^2)*(-(a*c^2*e-b^2*c*e+b*c^2*d)/(c*x^4+b*x^2+a)^(1/2)*(
b*x^2+2*a)/(4*a*c-b^2)+(2*a*b*c*e-a*c^2*d-b^3*e+b^2*c*d)*(2*c*x^2+b)/(4*a*
c-b^2)/(c*x^4+b*x^2+a)^(1/2)))
```

3.347.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1597 vs. $2(379) = 758$.

Time = 5.17 (sec) , antiderivative size = 6486, normalized size of antiderivative = 15.48

$$\int \frac{1}{x^3 (d+ex^2) (a+bx^2+cx^4)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.347.6 Sympy [F]

$$\int \frac{1}{x^3 (d+ex^2) (a+bx^2+cx^4)^{3/2}} \, dx = \int \frac{1}{x^3 (d+ex^2) (a+bx^2+cx^4)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/x**3/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral(1/(x**3*(d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

3.347.7 Maxima [F]

$$\int \frac{1}{x^3 (d+ex^2) (a+bx^2+cx^4)^{3/2}} \, dx = \int \frac{1}{(cx^4+bx^2+a)^{\frac{3}{2}} (ex^2+d)x^3} \, dx$$

input
```
integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*x^3), x)
```

3.347.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 777 vs. $2(379) = 758$.

Time = 0.48 (sec) , antiderivative size = 777, normalized size of antiderivative = 1.85

$$\int \frac{1}{x^3 (d+ex^2) (a+bx^2+cx^4)^{3/2}}\,dx = \frac{e^4 \arctan\left(-\frac{\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{(cd^4-bd^3e+ad^2e^2)\sqrt{-cd^2+bde-ae^2}}$$
$$-\frac{\frac{(a^2b^2c^3d^3-2\,a^3c^4d^3-2\,a^2b^3c^2d^2e+5\,a^3bc^3d^2e+a^2b^4cde^2-2\,a^3b^2c^2de^2-2\,a^4c^3de^2-a^3b^3ce^3+3\,a^4bc^2e^3)x^2}{a^4b^2c^2d^4-4\,a^5c^3d^4-2\,a^4b^3cd^3e+8\,a^5bc^2d^3e+a^4b^4d^2e^2-2\,a^5b^2cd^2e^2-8\,a^6c^2d^2e^2-2\,a^5b^3de^3+8\,a^6bcde^3+a^6b^2e^4-4\,a^7ce^4}+\frac{a^2b^3c^2d^3-3\,a^3bc^3}{a^4b^2c^2d^4-4\,a^5c^3d}}{\sqrt{cx^4+bx^2+a}}$$
$$-\frac{(3\,bd+2\,ae)\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{2\,\sqrt{-a}a^2d^2}+\frac{(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a})b+2\,a\sqrt{c}}{2\left(\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^2-a\right)a^2d}$$

input
```
integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
e^4*arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*e + sqrt(c)*d)/sqrt(-
c*d^2 + b*d*e - a*e^2))/((c*d^4 - b*d^3*e + a*d^2*e^2)*sqrt(-c*d^2 + b*d*e
 - a*e^2)) - ((a^2*b^2*c^3*d^3 - 2*a^3*c^4*d^3 - 2*a^2*b^3*c^2*d^2*e + 5*a
^3*b*c^3*d^2*e + a^2*b^4*c*d*e^2 - 2*a^3*b^2*c^2*d*e^2 - 2*a^4*c^3*d*e^2 -
 a^3*b^3*c*e^3 + 3*a^4*b*c^2*e^3)*x^2/(a^4*b^2*c^2*d^4 - 4*a^5*c^3*d^4 - 2
*a^4*b^3*c*d^3*e + 8*a^5*b*c^2*d^3*e + a^4*b^4*d^2*e^2 - 2*a^5*b^2*c*d^2*e
^2 - 8*a^6*c^2*d^2*e^2 - 2*a^5*b^3*d*e^3 + 8*a^6*b*c*d*e^3 + a^6*b^2*e^4 -
 4*a^7*c*e^4) + (a^2*b^3*c^2*d^3 - 3*a^3*b*c^3*d^3 - 2*a^2*b^4*c*d^2*e + 7
*a^3*b^2*c^2*d^2*e - 2*a^4*c^3*d^2*e + a^2*b^5*d*e^2 - 3*a^3*b^3*c*d*e^2 -
 a^4*b*c^2*d*e^2 - a^3*b^4*e^3 + 4*a^4*b^2*c*e^3 - 2*a^5*c^2*e^3)/(a^4*b^2
*c^2*d^4 - 4*a^5*c^3*d^4 - 2*a^4*b^3*c*d^3*e + 8*a^5*b*c^2*d^3*e + a^4*b^4
*d^2*e^2 - 2*a^5*b^2*c*d^2*e^2 - 8*a^6*c^2*d^2*e^2 - 2*a^5*b^3*d*e^3 + 8*a
^6*b*c*d*e^3 + a^6*b^2*e^4 - 4*a^7*c*e^4))/sqrt(c*x^4 + b*x^2 + a) - 1/2*(
3*b*d + 2*a*e)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/(
sqrt(-a)*a^2*d^2) + 1/2*((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*b + 2*a*s
qrt(c))/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)*a^2*d)
```

3.347.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (d + ex^2) (a + bx^2 + cx^4)^{3/2}} dx = \int \frac{1}{x^3 (e x^2 + d) (c x^4 + b x^2 + a)^{3/2}} dx$$

input
```
int(1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x)
```

output
```
int(1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

3.348 $\int \frac{x^8}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

3.348.1 Optimal result

Integrand size = 29, antiderivative size = 449

$$\begin{aligned}
\int \frac{x^8}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx &= \frac{x^3(1-2x^2)}{20\sqrt{1+2x^2+2x^4}}+\frac{1}{20}x\sqrt{1+2x^2+2x^4} \\
&+\frac{x\sqrt{1+2x^2+2x^4}}{10\sqrt{2}\left(1+\sqrt{2}x^2\right)}+\frac{27}{80}\sqrt{\frac{3}{5}}\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) \\
&-\frac{\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{10\ 2^{3/4}\sqrt{1+2x^2+2x^4}} \\
&+\frac{\left(-2+7\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{8\ 2^{3/4}\left(-2+3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}} \\
&+\frac{27\left(3+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{1}{24}\left(12-11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{80\ 2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}
\end{aligned}$$

output
```
27/400*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)+1/20*x^3*(-2*
x^2+1)/(2*x^4+2*x^2+1)^(1/2)+1/20*x*(2*x^4+2*x^2+1)^(1/2)+1/20*x*(2*x^4+2*
x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))-1/20*(cos(2*arctan(2^(1/4)*x))^2)^(1/
2)/cos(2*arctan(2^(1/4)*x))*EllipticE(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1
/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1
/4)/(2*x^4+2*x^2+1)^(1/2)+27/160*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*
arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(2^(1/4)*x)),1/2-11/24*2^(1/2),1
/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*
2^(1/2))^2)^(1/2)*2^(1/4)/(2-3*2^(1/2))/(2*x^4+2*x^2+1)^(1/2)+1/16*(cos(2*
arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arcta
n(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(-2+7*2^(1/2))*(1+x^2*2^(1/2))*((2*x^
4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(-2+3*2^(1/2))/(2*x^4+2*x^2+1)
^(1/2)
```

3.348.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.27 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.44

$$\int \frac{x^8}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = \frac{4x+12x^3-4i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}E\big(i\operatorname{arcsinh}\big(\sqrt{1-}$$

input
```
Integrate[x^8/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

output
```
(4*x + 12*x^3 - (4*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x
^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] - (29 - 33*I)*Sqrt[1 - I]*Sqrt[
1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x],
 I] + 27*(1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*Ellipti
cPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(80*Sqrt[1 + 2*x^2 + 2*x^4])
```

3.348.3 Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {1638, 27, 2206, 27, 1511, 1416, 1509, 2220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.348. $\int \frac{x^8}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

$$\int \frac{x^8}{(2x^2+3)(2x^4+2x^2+1)^{3/2}}dx$$

$$\Big\downarrow \text{1638}$$

$$\frac{81\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{20\left(2-3\sqrt{2}\right)}-\frac{\int \frac{-10\left(2-3\sqrt{2}\right)x^6+6\left(5+6\sqrt{2}\right)x^4+9\left(4+3\sqrt{2}\right)x^2+27}{2(2x^4+2x^2+1)^{3/2}}dx}{10\left(2-3\sqrt{2}\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{81\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{20\left(2-3\sqrt{2}\right)}-\frac{\int \frac{-10\left(2-3\sqrt{2}\right)x^6+6\left(5+6\sqrt{2}\right)x^4+9\left(4+3\sqrt{2}\right)x^2+27}{(2x^4+2x^2+1)^{3/2}}dx}{20\left(2-3\sqrt{2}\right)}$$

$$\Big\downarrow \text{2206}$$

$$\frac{81\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{20\left(2-3\sqrt{2}\right)}-\frac{\frac{1}{4}\int \frac{4\left(-2\left(2-3\sqrt{2}\right)x^2-3\sqrt{2}+29\right)}{\sqrt{2x^4+2x^2+1}}dx-\frac{x\left(3\left(2-3\sqrt{2}\right)x^2-3\sqrt{2}+2\right)}{\sqrt{2x^4+2x^2+1}}}{20\left(2-3\sqrt{2}\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{81\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{20\left(2-3\sqrt{2}\right)}-\frac{\int \frac{-2\left(2-3\sqrt{2}\right)x^2-3\sqrt{2}+29}{\sqrt{2x^4+2x^2+1}}dx-\frac{x\left(3\left(2-3\sqrt{2}\right)x^2-3\sqrt{2}+2\right)}{\sqrt{2x^4+2x^2+1}}}{20\left(2-3\sqrt{2}\right)}$$

$$\Big\downarrow \text{1511}$$

$$\frac{81\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{20\left(2-3\sqrt{2}\right)}-$$
$$\frac{5\left(7-\sqrt{2}\right)\int \frac{1}{\sqrt{2x^4+2x^2+1}}dx-2\left(3-\sqrt{2}\right)\int \frac{1-\sqrt{2}x^2}{\sqrt{2x^4+2x^2+1}}dx-\frac{x\left(3\left(2-3\sqrt{2}\right)x^2-3\sqrt{2}+2\right)}{\sqrt{2x^4+2x^2+1}}}{20\left(2-3\sqrt{2}\right)}$$

$$\Big\downarrow \text{1416}$$

$$\frac{81\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{20\left(2-3\sqrt{2}\right)}-$$
$$\frac{-2\left(3-\sqrt{2}\right)\int \frac{1-\sqrt{2}x^2}{\sqrt{2x^4+2x^2+1}}dx+\frac{5\left(7-\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{2\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-\frac{x\left(3\left(2-3\sqrt{2}\right)x^2-3\sqrt{2}+\right.}{\sqrt{2x^4+2x^2+1}}}{20\left(2-3\sqrt{2}\right)}$$

$$\Big\downarrow \text{1509}$$

3.348. $\int \frac{x^8}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

$$\frac{81\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{20\left(2-3\sqrt{2}\right)}-\frac{\frac{5\left(7-\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{2\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-2(3-\sqrt{2})\left(\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\right.}{\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}\right.}{20\left(2-3\sqrt{2}\right)}$$

$\downarrow$ 2220

$$\frac{81\left(\frac{\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticPi}\left(\frac{1}{24}\left(12-11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{12\ 2^{3/4}\sqrt{2x^4+2x^2+1}}-\frac{\left(3-\sqrt{2}\right)\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{2\sqrt{30}}\right)}{20\left(2-3\sqrt{2}\right)}-\frac{\frac{5\left(7-\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{2\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-2(3-\sqrt{2})\left(\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\right.}{\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}\right.}{20\left(2-3\sqrt{2}\right)}$$

input
```
Int[x^8/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

output
```
-1/20*(-((x*(2 - 3*Sqrt[2] + 3*(2 - 3*Sqrt[2])*x^2))/Sqrt[1 + 2*x^2 + 2*x^
4]) - 2*(3 - Sqrt[2])*(-((x*Sqrt[1 + 2*x^2 + 2*x^4])/(1 + Sqrt[2]*x^2)) +
((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE
[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]))
 + (5*(7 - Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2
]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(2*2^(1/4)*Sqrt
[1 + 2*x^2 + 2*x^4]))/(2 - 3*Sqrt[2]) + (81*(-1/2*((3 - Sqrt[2])*ArcTan[(S
qrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/Sqrt[30] + ((3 + Sqrt[2])*(1 + Sqrt[
2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*
Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(12*2^(3/4)*Sqrt[1 + 2
*x^2 + 2*x^4])))/(20*(2 - 3*Sqrt[2]))
```

3.348. $\int \frac{x^8}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

3.348.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1638
```
Int[((x_)^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[(-(-d/e)^(m/2))*((c*d^2 - b*d*e + a*e^2)^(p + 1/2)
/(e^(2*p)*(Rt[c/a, 2]*d - e)))   Int[(1 + Rt[c/a, 2]*x^2)/((d + e*x^2)*Sqrt
[a + b*x^2 + c*x^4]), x], x] + Simp[(c*d^2 - b*d*e + a*e^2)^(p + 1/2)/(Rt[c
/a, 2]*d - e)   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[((Rt[c/a, 2]*d - e)*(
c*d^2 - b*d*e + a*e^2)^(-p - 1/2)*x^m + ((-d/e)^(m/2)*(1 + Rt[c/a, 2]*x^2)*
(a + b*x^2 + c*x^4)^(-p - 1/2))/e^(2*p))/(d + e*x^2), x], x], x] /; FreeQ[{
a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && ILtQ[p + 1/2, 0] && IGtQ[m/2,
0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

3.348. $\int \frac{x^8}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2220
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a
+ b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*El
lipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &
& EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[-b + c*(d/e) + a*(e/d)]
```

3.348.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.94 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.55

method	result
risch	$\frac{x(3x^2+1)}{20\sqrt{2x^4+2x^2+1}}-\frac{29\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{40\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{1}{20}+\frac{i}{20}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1+i}\cdots\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x\cdots}}$
elliptic	$-\frac{4\left(-\frac{3}{80}x^3-\frac{1}{80}x\right)}{\sqrt{2x^4+2x^2+1}}-\frac{31\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{40\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\cdots\right.}{20\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$
default	$\frac{27x^3}{16\sqrt{2x^4+2x^2+1}}-\frac{11\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{4\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(\frac{47}{32}-\frac{47i}{32}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1+i},\cdots\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2\cdots}}$

input
```
int(x^8/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

3.348. $\int \frac{x^8}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

output
```
1/20*x*(3*x^2+1)/(2*x^4+2*x^2+1)^(1/2)-29/40/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1
/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2
*2^(1/2)+1/2*I*2^(1/2))+(-1/20+1/20*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1
+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1
/2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))+27
/40/(-1+I)^(1/2)*(1-I*x^2+x^2)^(1/2)*(1+I*x^2+x^2)^(1/2)/(2*x^4+2*x^2+1)^(
1/2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

3.348.5 Fricas [F]

$$\int \frac{x^8}{(3+2x^2)\left(1+2x^2+2x^4\right)^{3/2}}\,dx = \int \frac{x^8}{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}\left(2\,x^2+3\right)}\,dx$$

input
```
integrate(x^8/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(2*x^4 + 2*x^2 + 1)*x^8/(8*x^10 + 28*x^8 + 40*x^6 + 32*x^4 +
14*x^2 + 3), x)
```

3.348.6 Sympy [F]

$$\int \frac{x^8}{(3+2x^2)\left(1+2x^2+2x^4\right)^{3/2}}\,dx = \int \frac{x^8}{(2x^2+3)\left(2x^4+2x^2+1\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**8/(2*x**2+3)/(2*x**4+2*x**2+1)**(3/2),x)
```

output
```
Integral(x**8/((2*x**2 + 3)*(2*x**4 + 2*x**2 + 1)**(3/2)), x)
```

3.348.7 Maxima [F]

$$\int \frac{x^8}{(3+2x^2)\left(1+2x^2+2x^4\right)^{3/2}}\,dx = \int \frac{x^8}{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}\left(2\,x^2+3\right)}\,dx$$

input
```
integrate(x^8/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^8/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

3.348.8 Giac [F]

$$\int \frac{x^8}{(3+2x^2)\left(1+2x^2+2x^4\right)^{3/2}}\,dx = \int \frac{x^8}{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}\left(2\,x^2+3\right)}\,dx$$

input
```
integrate(x^8/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^8/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

3.348.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{(3+2x^2)\left(1+2x^2+2x^4\right)^{3/2}}\,dx = \int \frac{x^8}{\left(2\,x^2+3\right)\,\left(2\,x^4+2\,x^2+1\right)^{3/2}}\,dx$$

input
```
int(x^8/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)),x)
```

output
```
int(x^8/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)), x)
```

3.349 $\int \frac{x^6}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

3.349.1 Optimal result

Integrand size = 29, antiderivative size = 423

$$\begin{aligned}\int \frac{x^6}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx &= \frac{x(1-2x^2)}{20\sqrt{1+2x^2+2x^4}} \\ &+ \frac{x\sqrt{1+2x^2+2x^4}}{10\sqrt{2}\,(1+\sqrt{2}x^2)} - \frac{9}{40}\sqrt{\frac{3}{5}}\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) \\ &- \frac{(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)\mid\frac{1}{4}(2-\sqrt{2})\right)}{10\ 2^{3/4}\sqrt{1+2x^2+2x^4}} \\ &- \frac{\left(\sqrt[4]{2}+2^{3/4}\right)(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{8\left(-2+3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}} \\ &- \frac{9(3+\sqrt{2})(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\,\text{EllipticPi}\left(\frac{1}{24}(12-11\sqrt{2}),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{40\ 2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}\end{aligned}$$

output
```
-9/200*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)+1/20*x*(-2*x^
2+1)/(2*x^4+2*x^2+1)^(1/2)+1/20*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(
1/2))-1/20*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*Ell
ipticE(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2
*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)-9/80*
(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin
(2*arctan(2^(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))
*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2-3*2^
(1/2))/(2*x^4+2*x^2+1)^(1/2)-1/8*(2^(1/4)+2^(3/4))*(cos(2*arctan(2^(1/4)*x
))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/
2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(
1/2)/(-2+3*2^(1/2))/(2*x^4+2*x^2+1)^(1/2)
```

3.349.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.68 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.47

$$\int \frac{x^6}{(3+2x^2)\left(1+2x^2+2x^4\right)^{3/2}}\,dx = \frac{2x-4x^3-2i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}E\left(i\operatorname{arcsinh}\left(\sqrt{1-\right.\right.}}{}$$

input
```
Integrate[x^6/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

output
```
(2*x - 4*x^3 - (2*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^
2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] + (8 - 6*I)*Sqrt[1 - I]*Sqrt[1 +
 (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I]
 - 9*(1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[
1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(40*Sqrt[1 + 2*x^2 + 2*x^4])
```

3.349.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {1638, 25, 2206, 27, 1511, 1416, 1509, 2220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(2x^2+3)\left(2x^4+2x^2+1\right)^{3/2}}\,dx$$

$\Big\downarrow$ 1638

$$-\frac{27\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{10\left(2-3\sqrt{2}\right)}-\frac{\int -\frac{2\left(5+6\sqrt{2}\right)x^4+3\left(4+3\sqrt{2}\right)x^2+9}{\left(2x^4+2x^2+1\right)^{3/2}}dx}{10\left(2-3\sqrt{2}\right)}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{2\left(5+6\sqrt{2}\right)x^4+3\left(4+3\sqrt{2}\right)x^2+9}{\left(2x^4+2x^2+1\right)^{3/2}}dx}{10\left(2-3\sqrt{2}\right)}-\frac{27\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{10\left(2-3\sqrt{2}\right)}$$

$\Big\downarrow$ 2206

$$\frac{\frac{1}{4}\int \frac{2\left(2\left(2-3\sqrt{2}\right)x^2+3\sqrt{2}+16\right)}{\sqrt{2x^4+2x^2+1}}dx+\frac{x\left(-2\left(2-3\sqrt{2}\right)x^2-3\sqrt{2}+2\right)}{2\sqrt{2x^4+2x^2+1}}}{10\left(2-3\sqrt{2}\right)}-\frac{27\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{10\left(2-3\sqrt{2}\right)}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{2}\int \frac{2\left(2-3\sqrt{2}\right)x^2+3\sqrt{2}+16}{\sqrt{2x^4+2x^2+1}}dx+\frac{x\left(-2\left(2-3\sqrt{2}\right)x^2-3\sqrt{2}+2\right)}{2\sqrt{2x^4+2x^2+1}}}{10\left(2-3\sqrt{2}\right)}-\frac{27\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{10\left(2-3\sqrt{2}\right)}$$

$\Big\downarrow$ 1511

$$\frac{\frac{1}{2}\left(5\left(2+\sqrt{2}\right)\int \frac{1}{\sqrt{2x^4+2x^2+1}}dx+2\left(3-\sqrt{2}\right)\int \frac{1-\sqrt{2}x^2}{\sqrt{2x^4+2x^2+1}}dx\right)+\frac{x\left(-2\left(2-3\sqrt{2}\right)x^2-3\sqrt{2}+2\right)}{2\sqrt{2x^4+2x^2+1}}}{10\left(2-3\sqrt{2}\right)}-\frac{27\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{10\left(2-3\sqrt{2}\right)}$$

$\Big\downarrow$ 1416

$$\frac{\frac{1}{2}\left(2\left(3-\sqrt{2}\right)\int \frac{1-\sqrt{2}x^2}{\sqrt{2x^4+2x^2+1}}dx+\frac{5\left(2+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{2\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}\right)+\frac{x\left(-2\left(2-3\sqrt{2}\right)x^2\cdots\right.}{2\sqrt{2x^4+2x^2\cdots}}}{10\left(2-3\sqrt{2}\right)}-\frac{27\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{10\left(2-3\sqrt{2}\right)}$$

$\Big\downarrow$ 1509

3.349. $\int \frac{x^6}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

$$\frac{\frac{1}{2}\left(\frac{5\left(2+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{2\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}+2(3-\sqrt{2})\left(\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}\right.\right.}{\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}\right.\right.}{10\left(2-3\sqrt{2}\right)}$$

$$\frac{27\int\frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{10\left(2-3\sqrt{2}\right)}$$

$$\Big\downarrow \; 2220$$

$$\frac{\frac{1}{2}\left(\frac{5\left(2+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{2\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}+2(3-\sqrt{2})\left(\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}\right.\right.}{\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}\right.\right.}{10\left(2-3\sqrt{2}\right)}$$

$$\frac{27\left(\frac{\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticPi}\left(\frac{1}{24}\left(12-11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{12\ 2^{3/4}\sqrt{2x^4+2x^2+1}}-\frac{\left(3-\sqrt{2}\right)\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{2\sqrt{30}}\right)}{10\left(2-3\sqrt{2}\right)}$$

input
```
Int[x^6/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

output
```
((x*(2 - 3*Sqrt[2] - 2*(2 - 3*Sqrt[2])*x^2))/(2*Sqrt[1 + 2*x^2 + 2*x^4]) +
 (2*(3 - Sqrt[2])*(-((x*Sqrt[1 + 2*x^2 + 2*x^4])/(1 + Sqrt[2]*x^2)) + ((1
+ Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*A
rcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])) + (
5*(2 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^
2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(2*2^(1/4)*Sqrt[1 +
 2*x^2 + 2*x^4]))/2)/(10*(2 - 3*Sqrt[2])) - (27*(-1/2*((3 - Sqrt[2])*ArcTa
n[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/Sqrt[30] + ((3 + Sqrt[2])*(1 + S
qrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 -
 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(12*2^(3/4)*Sqrt[1
 + 2*x^2 + 2*x^4])))/(10*(2 - 3*Sqrt[2]))
```

3.349. $\int \frac{x^6}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

3.349.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1638
```
Int[((x_)^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[(-(-d/e)^(m/2))*((c*d^2 - b*d*e + a*e^2)^(p + 1/2)
/(e^(2*p)*(Rt[c/a, 2]*d - e)))   Int[(1 + Rt[c/a, 2]*x^2)/((d + e*x^2)*Sqrt
[a + b*x^2 + c*x^4]), x], x] + Simp[(c*d^2 - b*d*e + a*e^2)^(p + 1/2)/(Rt[c
/a, 2]*d - e)   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[((Rt[c/a, 2]*d - e)*(
c*d^2 - b*d*e + a*e^2)^(-p - 1/2)*x^m + ((-d/e)^(m/2)*(1 + Rt[c/a, 2]*x^2)*
(a + b*x^2 + c*x^4)^(-p - 1/2))/e^(2*p))/(d + e*x^2), x], x], x] /; FreeQ[{
a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && ILtQ[p + 1/2, 0] && IGtQ[m/2,
0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

3.349. $\int \frac{x^6}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2220
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a
+ b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*El
lipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &
& EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[-b + c*(d/e) + a*(e/d)]
```

3.349.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.52 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.58

method	result
risch	$-\frac{x(2x^2-1)}{20\sqrt{2x^4+2x^2+1}}+\frac{2\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{5\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{1}{20}+\frac{i}{20}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1+\cdots\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2\cdots}}$
elliptic	$-\frac{4\left(\frac{1}{40}x^3-\frac{1}{80}x\right)}{\sqrt{2x^4+2x^2+1}}+\frac{7\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{20\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{20\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$
default	$-\frac{9x^3}{8\sqrt{2x^4+2x^2+1}}+\frac{7\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{4\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{17}{16}+\frac{17i}{16}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1+\cdots\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x\cdots}}$

input
```
int(x^6/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/20*x*(2*x^2-1)/(2*x^4+2*x^2+1)^(1/2)+2/5/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/
2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*
2^(1/2)+1/2*I*2^(1/2))+(-1/20+1/20*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+
(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1/
2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))-9/2
0/(-1+I)^(1/2)*(1-I*x^2+x^2)^(1/2)*(1+I*x^2+x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/
2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

3.349.5 Fricas [F]

$$\int \frac{x^6}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = \int \frac{x^6}{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}\left(2\,x^2+3\right)}\,dx$$

input
```
integrate(x^6/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(2*x^4 + 2*x^2 + 1)*x^6/(8*x^10 + 28*x^8 + 40*x^6 + 32*x^4 +
14*x^2 + 3), x)
```

3.349.6 Sympy [F]

$$\int \frac{x^6}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = \int \frac{x^6}{(2x^2+3)\,(2x^4+2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**6/(2*x**2+3)/(2*x**4+2*x**2+1)**(3/2),x)
```

output
```
Integral(x**6/((2*x**2 + 3)*(2*x**4 + 2*x**2 + 1)**(3/2)), x)
```

3.349.7 Maxima [F]

$$\int \frac{x^6}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = \int \frac{x^6}{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}(2\,x^2+3)}\,dx$$

input
```
integrate(x^6/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^6/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

3.349.8 Giac [F]

$$\int \frac{x^6}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = \int \frac{x^6}{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}(2\,x^2+3)}\,dx$$

input
```
integrate(x^6/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^6/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

3.349.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = \int \frac{x^6}{(2\,x^2+3)\,(2\,x^4+2\,x^2+1)^{3/2}}\,dx$$

input
```
int(x^6/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)),x)
```

output
```
int(x^6/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)), x)
```

3.350 $\int \frac{x^4}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

3.350.1 Optimal result

Integrand size = 29, antiderivative size = 422

$$\begin{aligned}
\int \frac{x^4}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx &= -\frac{x(2+x^2)}{10\sqrt{1+2x^2+2x^4}} \\
&+ \frac{x\sqrt{1+2x^2+2x^4}}{10\sqrt{2}\,(1+\sqrt{2}x^2)} + \frac{3}{20}\sqrt{\frac{3}{5}}\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) \\
&- \frac{(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)\left|\frac{1}{4}(2-\sqrt{2})\right.\right)}{10\ 2^{3/4}\sqrt{1+2x^2+2x^4}} \\
&+ \frac{(2+\sqrt{2})(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{4\ 2^{3/4}(-2+3\sqrt{2})\sqrt{1+2x^2+2x^4}} \\
&+ \frac{3(3+\sqrt{2})(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\,\text{EllipticPi}\left(\frac{1}{24}(12-11\sqrt{2}),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{20\ 2^{3/4}(2-3\sqrt{2})\sqrt{1+2x^2+2x^4}}
\end{aligned}$$

output
```
3/100*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)-1/10*x*(x^2+2)
/(2*x^4+2*x^2+1)^(1/2)+1/20*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2)
)-1/20*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*Ellipti
cE(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4
+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)+3/40*(cos
(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*a
rctan(2^(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*(1+
x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2-3*2^(1/2
))/(2*x^4+2*x^2+1)^(1/2)+1/8*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arct
an(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(
2+2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/
4)/(-2+3*2^(1/2))/(2*x^4+2*x^2+1)^(1/2)
```

3.350.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.10 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.47

$$\int \frac{x^4}{(3+2x^2)\,(1+2x^2+2x^4)^{3/2}}\,dx =$$
$$-\frac{4x+2x^3+i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}E\left(i\text{arcsinh}\left(\sqrt{1-i}x\right)\middle|i\right)+(1-2i)\sqrt{1-i}\sqrt{1+(1-}{}$$

input
```
Integrate[x^4/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

output
```
-1/20*(4*x + 2*x^3 + I*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*
x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] + (1 - 2*I)*Sqrt[1 - I]*Sqrt[1
 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x],
I] - 3*(1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticP
i[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/Sqrt[1 + 2*x^2 + 2*x^4]
```

3.350.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {1638, 27, 2206, 27, 1511, 1416, 1509, 2220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(2x^2+3)(2x^4+2x^2+1)^{3/2}}dx$$

$$\Big\downarrow \text{1638}$$

$$\frac{9\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{5\left(2-3\sqrt{2}\right)}-\frac{\int \frac{2\left(9\sqrt{2}x^4+\left(4+3\sqrt{2}\right)x^2+3\right)}{(2x^4+2x^2+1)^{3/2}}dx}{10\left(2-3\sqrt{2}\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{9\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{5\left(2-3\sqrt{2}\right)}-\frac{\int \frac{9\sqrt{2}x^4+\left(4+3\sqrt{2}\right)x^2+3}{(2x^4+2x^2+1)^{3/2}}dx}{5\left(2-3\sqrt{2}\right)}$$

$$\Big\downarrow \text{2206}$$

$$\frac{9\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{5\left(2-3\sqrt{2}\right)}-\frac{\frac{1}{4}\int \frac{2\left(2\left(1+3\sqrt{2}\right)-\left(2-3\sqrt{2}\right)x^2\right)}{\sqrt{2x^4+2x^2+1}}dx+\frac{\left(2-3\sqrt{2}\right)x(x^2+2)}{2\sqrt{2x^4+2x^2+1}}}{5\left(2-3\sqrt{2}\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{9\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{5\left(2-3\sqrt{2}\right)}-\frac{\frac{1}{2}\int \frac{2\left(1+3\sqrt{2}\right)-\left(2-3\sqrt{2}\right)x^2}{\sqrt{2x^4+2x^2+1}}dx+\frac{\left(2-3\sqrt{2}\right)x(x^2+2)}{2\sqrt{2x^4+2x^2+1}}}{5\left(2-3\sqrt{2}\right)}$$

$$\Big\downarrow \text{1511}$$

$$\frac{9\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{5\left(2-3\sqrt{2}\right)}-\frac{\frac{1}{2}\left(5\left(1+\sqrt{2}\right)\int \frac{1}{\sqrt{2x^4+2x^2+1}}dx-\left(3-\sqrt{2}\right)\int \frac{1-\sqrt{2}x^2}{\sqrt{2x^4+2x^2+1}}dx\right)+\frac{\left(2-3\sqrt{2}\right)x(x^2+2)}{2\sqrt{2x^4+2x^2+1}}}{5\left(2-3\sqrt{2}\right)}$$

$$\Big\downarrow \text{1416}$$

$$\frac{\frac{9\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{5\left(2-3\sqrt{2}\right)}-\frac{\frac{1}{2}\left(\frac{5\left(1+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{2\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-\left(3-\sqrt{2}\right)\int \frac{1-\sqrt{2}x^2}{\sqrt{2x^4+2x^2+1}}dx\right)+\frac{\left(2-3\sqrt{2}\right)x\left(x^2+2\right)}{2\sqrt{2x^4+2x^2+1}}}{5\left(2-3\sqrt{2}\right)}}{}$$

$$\Big\downarrow \text{1509}$$

$$\frac{9\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{5\left(2-3\sqrt{2}\right)}-\frac{\frac{1}{2}\left(\frac{5\left(1+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{2\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-\left(3-\sqrt{2}\right)\left(\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right.\right.}{\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}\right.\right.}{5\left(2-3\sqrt{2}\right)}$$

$$\Big\downarrow \text{2220}$$

$$\frac{9\left(\frac{\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticPi}\left(\frac{1}{24}\left(12-11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{12\ 2^{3/4}\sqrt{2x^4+2x^2+1}}-\frac{\left(3-\sqrt{2}\right)\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{2\sqrt{30}}\right)}{5\left(2-3\sqrt{2}\right)}-\frac{\frac{1}{2}\left(\frac{5\left(1+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{2\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-\left(3-\sqrt{2}\right)\left(\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right.\right.}{\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}\right.\right.}{5\left(2-3\sqrt{2}\right)}$$

input
```
Int[x^4/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

3.350. $\int \frac{x^4}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

output
```
-1/5*(((2 - 3*Sqrt[2])*x*(2 + x^2))/(2*Sqrt[1 + 2*x^2 + 2*x^4]) + (-((3 -
Sqrt[2])*(-((x*Sqrt[1 + 2*x^2 + 2*x^4])/(1 + Sqrt[2]*x^2)) + ((1 + Sqrt[2]
*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(
1/4)*x], (2 - Sqrt[2])/4])/(2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]))) + (5*(1 + S
qrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*El
lipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(2*2^(1/4)*Sqrt[1 + 2*x^2 +
 2*x^4]))/2)/(2 - 3*Sqrt[2]) + (9*(-1/2*((3 - Sqrt[2])*ArcTan[(Sqrt[5/3]*x
)/Sqrt[1 + 2*x^2 + 2*x^4]])/Sqrt[30] + ((3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sq
rt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/2
4, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(12*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x
^4])))/(5*(2 - 3*Sqrt[2]))
```

3.350.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

3.350. $\int \frac{x^4}{(3+2x^2)(1+2x^2+2x^4)^{3/2}} \, dx$

rule 1638
```
Int[((x_)^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[(-(-d/e)^(m/2))*((c*d^2 - b*d*e + a*e^2)^(p + 1/2)
/(e^(2*p)*(Rt[c/a, 2]*d - e)))   Int[(1 + Rt[c/a, 2]*x^2)/((d + e*x^2)*Sqrt
[a + b*x^2 + c*x^4]), x], x] + Simp[(c*d^2 - b*d*e + a*e^2)^(p + 1/2)/(Rt[c
/a, 2]*d - e)   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[((Rt[c/a, 2]*d - e)*(
c*d^2 - b*d*e + a*e^2)^(-p - 1/2)*x^m + ((-d/e)^(m/2)*(1 + Rt[c/a, 2]*x^2)*
(a + b*x^2 + c*x^4)^(-p - 1/2))/e^(2*p))/(d + e*x^2), x], x], x] /; FreeQ[{
a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && ILtQ[p + 1/2, 0] && IGtQ[m/2,
0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2220
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a
+ b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*El
lipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &
& EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[-b + c*(d/e) + a*(e/d)]
```

3.350. $\int \frac{x^4}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

3.350.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.65 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.58

method	result
risch	$-\frac{x(x^2+2)}{10\sqrt{2x^4+2x^2+1}}-\frac{\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{10\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{1}{20}+\frac{i}{20}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1+i}\cdots\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x\cdots}}$
elliptic	$-\frac{4\left(\frac{1}{40}x^3+\frac{1}{20}x\right)}{\sqrt{2x^4+2x^2+1}}-\frac{3\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{20\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{20\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}\cdots$
default	$\frac{3x^3}{4\sqrt{2x^4+2x^2+1}}-\frac{\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(\frac{5}{8}-\frac{5i}{8}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\cdots\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$

input
```
int(x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/10*x*(x^2+2)/(2*x^4+2*x^2+1)^(1/2)-1/10/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2
)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2
^(1/2)+1/2*I*2^(1/2))+(-1/20+1/20*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(
1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2
)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))+3/10
/(-1+I)^(1/2)*(1-I*x^2+x^2)^(1/2)*(1+I*x^2+x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2
)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

3.350.5 Fricas [F]

$$\int \frac{x^4}{(3+2x^2)\left(1+2x^2+2x^4\right)^{3/2}}\,dx=\int \frac{x^4}{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}\left(2\,x^2+3\right)}\,dx$$

input
```
integrate(x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(2*x^4 + 2*x^2 + 1)*x^4/(8*x^10 + 28*x^8 + 40*x^6 + 32*x^4 +
14*x^2 + 3), x)
```

3.350.6 Sympy [F]

$$\int \frac{x^4}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = \int \frac{x^4}{(2x^2+3)(2x^4+2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4/(2*x**2+3)/(2*x**4+2*x**2+1)**(3/2),x)
```

output
```
Integral(x**4/((2*x**2 + 3)*(2*x**4 + 2*x**2 + 1)**(3/2)), x)
```

3.350.7 Maxima [F]

$$\int \frac{x^4}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = \int \frac{x^4}{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}(2\,x^2+3)}\,dx$$

input
```
integrate(x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^4/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

3.350.8 Giac [F]

$$\int \frac{x^4}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = \int \frac{x^4}{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}(2\,x^2+3)}\,dx$$

input
```
integrate(x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^4/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

3.350.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = \int \frac{x^4}{(2\,x^2+3)\,\left(2\,x^4+2\,x^2+1\right)^{3/2}}\,dx$$

input
```
int(x^4/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)),x)
```

output
```
int(x^4/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)), x)
```

3.351 $\int \frac{x^2}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

3.351.1 Optimal result

Integrand size = 29, antiderivative size = 423

$$\int \frac{x^2}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = \frac{x(3+4x^2)}{10\sqrt{1+2x^2+2x^4}} - \frac{\sqrt{2}x\sqrt{1+2x^2+2x^4}}{5\left(1+\sqrt{2}x^2\right)} - \frac{1}{10}\sqrt{\frac{3}{5}}\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)$$
$$+\frac{\sqrt[4]{2}\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{5\sqrt{1+2x^2+2x^4}}$$
$$-\frac{\left(\sqrt[4]{2}+2^{3/4}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{4\left(-2+3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}$$
$$-\frac{\left(3+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{1}{24}\left(12-11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{10\,2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}$$

output
```
-1/50*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)+1/10*x*(4*x^2+
3)/(2*x^4+2*x^2+1)^(1/2)-1/5*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2
))+1/5*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*Ellipti
cE(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4
+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)-1/20*(cos
(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*a
rctan(2^(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*(1+
x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2-3*2^(1/2
))/(2*x^4+2*x^2+1)^(1/2)-1/4*(2^(1/4)+2^(3/4))*(cos(2*arctan(2^(1/4)*x))^2
)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2
-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)
/(-2+3*2^(1/2))/(2*x^4+2*x^2+1)^(1/2)
```

3.351.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.00 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.47

$$\int \frac{x^2}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = \frac{6x+8x^3+4i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}E\big(i\text{arcsinh}\big(\sqrt{1-}$$

input
```
Integrate[x^2/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

output
```
(6*x + 8*x^3 + (4*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^
2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] - (1 + 3*I)*Sqrt[1 - I]*Sqrt[1 +
 (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I]
 - 2*(1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[
1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(20*Sqrt[1 + 2*x^2 + 2*x^4])
```

3.351.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {1638, 27, 2206, 27, 1511, 1416, 1509, 2220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(2x^2+3)(2x^4+2x^2+1)^{3/2}}dx$$

$\Big\downarrow$ 1638

$$-\frac{6\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{5\left(2-3\sqrt{2}\right)}-\frac{\int -\frac{2\left(6\sqrt{2}x^4+3\left(2-\sqrt{2}\right)x^2+2\right)}{(2x^4+2x^2+1)^{3/2}}dx}{10\left(2-3\sqrt{2}\right)}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{6\sqrt{2}x^4+3\left(2-\sqrt{2}\right)x^2+2}{(2x^4+2x^2+1)^{3/2}}dx}{5\left(2-3\sqrt{2}\right)}-\frac{6\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{5\left(2-3\sqrt{2}\right)}$$

$\Big\downarrow$ 2206

$$\frac{\frac{1}{4}\int -\frac{2\left(4\left(2-3\sqrt{2}\right)x^2-9\sqrt{2}+2\right)}{\sqrt{2x^4+2x^2+1}}dx+\frac{\left(2-3\sqrt{2}\right)x(4x^2+3)}{2\sqrt{2x^4+2x^2+1}}}{5\left(2-3\sqrt{2}\right)}-\frac{6\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{5\left(2-3\sqrt{2}\right)}$$

$\Big\downarrow$ 27

$$\frac{\frac{\left(2-3\sqrt{2}\right)x(4x^2+3)}{2\sqrt{2x^4+2x^2+1}}-\frac{1}{2}\int \frac{4\left(2-3\sqrt{2}\right)x^2-9\sqrt{2}+2}{\sqrt{2x^4+2x^2+1}}dx}{5\left(2-3\sqrt{2}\right)}-\frac{6\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{5\left(2-3\sqrt{2}\right)}$$

$\Big\downarrow$ 1511

$$\frac{\frac{1}{2}\left(5\left(2+\sqrt{2}\right)\int \frac{1}{\sqrt{2x^4+2x^2+1}}dx-4\left(3-\sqrt{2}\right)\int \frac{1-\sqrt{2}x^2}{\sqrt{2x^4+2x^2+1}}dx\right)+\frac{\left(2-3\sqrt{2}\right)x(4x^2+3)}{2\sqrt{2x^4+2x^2+1}}}{5\left(2-3\sqrt{2}\right)}-\frac{6\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{5\left(2-3\sqrt{2}\right)}$$

$\Big\downarrow$ 1416

$$\frac{\frac{1}{2}\left(\frac{5\left(2+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{2\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-4\left(3-\sqrt{2}\right)\int \frac{1-\sqrt{2}x^2}{\sqrt{2x^4+2x^2+1}}dx\right)+\frac{\left(2-3\sqrt{2}\right)x(4x^2+3}{2\sqrt{2x^4+2x^2+1}}}{5\left(2-3\sqrt{2}\right)}$$
$$\frac{6\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{5\left(2-3\sqrt{2}\right)}$$

$\Big\downarrow$ 1509

3.351. $\int \frac{x^2}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}dx$

$$\frac{\frac{1}{2}\left(\frac{5\left(2+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{2\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-4(3-\sqrt{2})\left(\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}\right.\right.}{\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}\right.\right.}{5\left(2-3\sqrt{2}\right)}$$

$$\frac{6\int\frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{5\left(2-3\sqrt{2}\right)}$$

$$\Big\downarrow \text{2220}$$

$$\frac{\frac{1}{2}\left(\frac{5\left(2+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{2\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-4(3-\sqrt{2})\left(\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}\right.\right.}{\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}\right.\right.}{5\left(2-3\sqrt{2}\right)}$$

$$\frac{6\left(\frac{\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\operatorname{EllipticPi}\left(\frac{1}{24}\left(12-11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{12\ 2^{3/4}\sqrt{2x^4+2x^2+1}}-\frac{\left(3-\sqrt{2}\right)\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{2\sqrt{30}}\right)}{5\left(2-3\sqrt{2}\right)}$$

input

```
Int[x^2/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

output

```
(((2 - 3*Sqrt[2])*x*(3 + 4*x^2))/(2*Sqrt[1 + 2*x^2 + 2*x^4]) + (-4*(3 - Sq
rt[2])*(-((x*Sqrt[1 + 2*x^2 + 2*x^4])/(1 + Sqrt[2]*x^2)) + ((1 + Sqrt[2]*x
^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/
4)*x], (2 - Sqrt[2])/4])/(2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])) + (5*(2 + Sqrt
[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Ellip
ticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(2*2^(1/4)*Sqrt[1 + 2*x^2 + 2*
x^4]))/2)/(5*(2 - 3*Sqrt[2])) - (6*(-1/2*((3 - Sqrt[2])*ArcTan[(Sqrt[5/3]*
x)/Sqrt[1 + 2*x^2 + 2*x^4]])/Sqrt[30] + ((3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*S
qrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/
24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(12*2^(3/4)*Sqrt[1 + 2*x^2 + 2*
x^4])))/(5*(2 - 3*Sqrt[2]))
```

3.351. $\int \frac{x^2}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

3.351.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1638
```
Int[((x_)^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[(-(-d/e)^(m/2))*((c*d^2 - b*d*e + a*e^2)^(p + 1/2)
/(e^(2*p)*(Rt[c/a, 2]*d - e)))   Int[(1 + Rt[c/a, 2]*x^2)/((d + e*x^2)*Sqrt
[a + b*x^2 + c*x^4]), x], x] + Simp[(c*d^2 - b*d*e + a*e^2)^(p + 1/2)/(Rt[c
/a, 2]*d - e)   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[((Rt[c/a, 2]*d - e)*(
c*d^2 - b*d*e + a*e^2)^(-p - 1/2)*x^m + ((-d/e)^(m/2)*(1 + Rt[c/a, 2]*x^2)*
(a + b*x^2 + c*x^4)^(-p - 1/2))/e^(2*p))/(d + e*x^2), x], x], x] /; FreeQ[{
a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && ILtQ[p + 1/2, 0] && IGtQ[m/2,
0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

3.351. $\int \frac{x^2}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2220
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a
+ b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*El
lipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &
& EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[-b + c*(d/e) + a*(e/d)]
```

3.351.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.27 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.58

method	result
risch	$\frac{x(4x^2+3)}{10\sqrt{2x^4+2x^2+1}} - \frac{\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i}, \frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{10\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}} + \frac{\left(\frac{1}{5}-\frac{i}{5}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1+i}, \frac{\sqrt{2}}{2}+\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$
elliptic	$-\frac{4\left(-\frac{1}{10}x^3-\frac{3}{40}x\right)}{\sqrt{2x^4+2x^2+1}} + \frac{\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i}, \frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{10\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}} - \frac{i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i}, \frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{5\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$
default	$-\frac{x^3}{2\sqrt{2x^4+2x^2+1}} + \frac{\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i}, \frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{2\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}} + \frac{\left(-\frac{1}{4}+\frac{i}{4}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1+i}, \right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+}}$

input
```
int(x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/10*x*(4*x^2+3)/(2*x^4+2*x^2+1)^(1/2)-1/10/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/
2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*
2^(1/2)+1/2*I*2^(1/2))+(1/5-1/5*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+
I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+
1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))-1/5/(-
1+I)^(1/2)*(1-I*x^2+x^2)^(1/2)*(1+I*x^2+x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*E
llipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

3.351.5 Fricas [F]

$$\int \frac{x^2}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = \int \frac{x^2}{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}(2\,x^2+3)}\,dx$$

input
```
integrate(x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(2*x^4 + 2*x^2 + 1)*x^2/(8*x^10 + 28*x^8 + 40*x^6 + 32*x^4 +
14*x^2 + 3), x)
```

3.351.6 Sympy [F]

$$\int \frac{x^2}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = \int \frac{x^2}{(2x^2+3)\,(2x^4+2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(2*x**2+3)/(2*x**4+2*x**2+1)**(3/2),x)
```

output
```
Integral(x**2/((2*x**2 + 3)*(2*x**4 + 2*x**2 + 1)**(3/2)), x)
```

3.351.7 Maxima [F]

$$\int \frac{x^2}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = \int \frac{x^2}{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}(2\,x^2+3)}\,dx$$

input
```
integrate(x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

3.351.8 Giac [F]

$$\int \frac{x^2}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = \int \frac{x^2}{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}(2\,x^2+3)}\,dx$$

input
```
integrate(x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

3.351.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = \int \frac{x^2}{(2\,x^2+3)\,(2\,x^4+2\,x^2+1)^{3/2}}\,dx$$

input
```
int(x^2/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)),x)
```

output
```
int(x^2/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)), x)
```

3.352 $\int \frac{1}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

3.352.1 Optimal result

Integrand size = 26, antiderivative size = 422

$$\begin{aligned}\int \frac{1}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx &= -\frac{x(1+3x^2)}{5\sqrt{1+2x^2+2x^4}}+\frac{3x\sqrt{1+2x^2+2x^4}}{5\sqrt{2}\,(1+\sqrt{2}x^2)}\\
&+\frac{\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)}{5\sqrt{15}}-\frac{3(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{5\,2^{3/4}\sqrt{1+2x^2+2x^4}}\\
&+\frac{(2+\sqrt{2})\,(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{2\,2^{3/4}\,(-2+3\sqrt{2})\,\sqrt{1+2x^2+2x^4}}\\
&+\frac{(3+\sqrt{2})\,(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{(1+\sqrt{2}x^2)^2}}\,\text{EllipticPi}\left(\frac{1}{24}(12-11\sqrt{2}),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{15\,2^{3/4}\,(2-3\sqrt{2})\,\sqrt{1+2x^2+2x^4}}\end{aligned}$$

output
```
1/75*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)-1/5*x*(3*x^2+1)
/(2*x^4+2*x^2+1)^(1/2)+3/10*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2)
)-3/10*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*Ellipti
cE(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4
+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)+1/30*(cos
(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*a
rctan(2^(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*(1+
x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2-3*2^(1/2
))/(2*x^4+2*x^2+1)^(1/2)+1/4*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arct
an(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(
2+2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/
4)/(-2+3*2^(1/2))/(2*x^4+2*x^2+1)^(1/2)
```

3.352.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.15 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.47

$$\int \frac{1}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = \frac{-6x-18x^3-9i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}E\big(i\text{arcsinh}\big(\sqrt{1}$$

input
```
Integrate[1/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

output
```
(-6*x - 18*x^3 - (9*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*
x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] + (6 + 3*I)*Sqrt[1 - I]*Sqrt[1
 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x],
I] + 2*(1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticP
i[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(30*Sqrt[1 + 2*x^2 + 2*x^4])
```

3.352.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {1547, 27, 2206, 27, 1511, 1416, 1509, 2220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^2+3)\left(2x^4+2x^2+1\right)^{3/2}}\,dx$$

$\Big\downarrow$ 1547

$$\frac{4\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{5\left(2-3\sqrt{2}\right)}-\frac{\int -\frac{2\left(-4\sqrt{2}x^4-2\left(2-\sqrt{2}\right)x^2-5\sqrt{2}+2\right)}{\left(2x^4+2x^2+1\right)^{3/2}}dx}{10\left(2-3\sqrt{2}\right)}$$

$\Big\downarrow$ 27

$$\frac{4\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{5\left(2-3\sqrt{2}\right)}+\frac{\int \frac{-4\sqrt{2}x^4-2\left(2-\sqrt{2}\right)x^2-5\sqrt{2}+2}{\left(2x^4+2x^2+1\right)^{3/2}}dx}{5\left(2-3\sqrt{2}\right)}$$

$\Big\downarrow$ 2206

$$\frac{4\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{5\left(2-3\sqrt{2}\right)}+\frac{\frac{1}{4}\int \frac{4\left(3\left(2-3\sqrt{2}\right)x^2+4\left(1-2\sqrt{2}\right)\right)}{\sqrt{2x^4+2x^2+1}}dx-\frac{x\left(3\left(2-3\sqrt{2}\right)x^2-3\sqrt{2}+2\right)}{\sqrt{2x^4+2x^2+1}}}{5\left(2-3\sqrt{2}\right)}$$

$\Big\downarrow$ 27

$$\frac{4\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{5\left(2-3\sqrt{2}\right)}+\frac{\int \frac{3\left(2-3\sqrt{2}\right)x^2+4\left(1-2\sqrt{2}\right)}{\sqrt{2x^4+2x^2+1}}dx-\frac{x\left(3\left(2-3\sqrt{2}\right)x^2-3\sqrt{2}+2\right)}{\sqrt{2x^4+2x^2+1}}}{5\left(2-3\sqrt{2}\right)}$$

$\Big\downarrow$ 1511

$$\frac{-\frac{5}{2}\left(2+2\sqrt{2}\right)\int \frac{1}{\sqrt{2x^4+2x^2+1}}dx+3\left(3-\sqrt{2}\right)\int \frac{1-\sqrt{2}x^2}{\sqrt{2x^4+2x^2+1}}dx-\frac{x\left(3\left(2-3\sqrt{2}\right)x^2-3\sqrt{2}+2\right)}{\sqrt{2x^4+2x^2+1}}}{5\left(2-3\sqrt{2}\right)}+\frac{4\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{5\left(2-3\sqrt{2}\right)}$$

$\Big\downarrow$ 1416

$$\frac{3\left(3-\sqrt{2}\right)\int \frac{1-\sqrt{2}x^2}{\sqrt{2x^4+2x^2+1}}dx-\frac{5\left(2+2\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{4\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-\frac{x\left(3\left(2-3\sqrt{2}\right)x^2-3\sqrt{2}+\right.}{\sqrt{2x^4+2x^2+1}}}{5\left(2-3\sqrt{2}\right)}$$
$$\frac{4\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{5\left(2-3\sqrt{2}\right)}$$

$\Big\downarrow$ 1509

3.352. $\int \frac{1}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

$$\frac{4\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{5\left(2-3\sqrt{2}\right)}+$$

$$\frac{-\frac{5\left(2+2\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{4\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}+3(3-\sqrt{2})\left(\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right.\right.}{\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}\right.}{5\left(2-3\sqrt{2}\right)}$$

$$\Big\downarrow \text{2220}$$

$$\frac{4\left(\frac{\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticPi}\left(\frac{1}{24}\left(12-11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{12\ 2^{3/4}\sqrt{2x^4+2x^2+1}}-\frac{\left(3-\sqrt{2}\right)\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{2\sqrt{30}}\right)}{5\left(2-3\sqrt{2}\right)}+$$

$$\frac{-\frac{5\left(2+2\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{4\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}+3(3-\sqrt{2})\left(\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right.\right.}{\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}\right.}{5\left(2-3\sqrt{2}\right)}$$

input
```
Int[1/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

output
```
(-((x*(2 - 3*Sqrt[2] + 3*(2 - 3*Sqrt[2])*x^2))/Sqrt[1 + 2*x^2 + 2*x^4]) +
3*(3 - Sqrt[2])*(-((x*Sqrt[1 + 2*x^2 + 2*x^4])/(1 + Sqrt[2]*x^2)) + ((1 +
Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*Arc
Tan[2^(1/4)*x], (2 - Sqrt[2])/4])/(2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])) - (5*
(2 + 2*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^
2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(4*2^(1/4)*Sqrt[1 +
 2*x^2 + 2*x^4]))/(5*(2 - 3*Sqrt[2])) + (4*(-1/2*((3 - Sqrt[2])*ArcTan[(Sq
rt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/Sqrt[30] + ((3 + Sqrt[2])*(1 + Sqrt[2
]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*S
qrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(12*2^(3/4)*Sqrt[1 + 2*
x^2 + 2*x^4])))/(5*(2 - 3*Sqrt[2]))
```

3.352. $\int \frac{1}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

3.352.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1547
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_.) + (e_.)*(x_)^2), x_Sym
bol] :> Simp[-(c*d^2 - b*d*e + a*e^2)^(p + 1/2)/(e^(2*p)*(Rt[c/a, 2]*d - e)
)   Int[(1 + Rt[c/a, 2]*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]
+ Simp[(c*d^2 - b*d*e + a*e^2)^(p + 1/2)/(Rt[c/a, 2]*d - e)   Int[(a + b*x^
2 + c*x^4)^p*ExpandToSum[((Rt[c/a, 2]*d - e)*(c*d^2 - b*d*e + a*e^2)^(-p -
1/2) + ((1 + Rt[c/a, 2]*x^2)*(a + b*x^2 + c*x^4)^(-p - 1/2))/e^(2*p))/(d +
e*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && N
eQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[p + 1/2, 0] && NeQ[c*d^2 - a*e^2, 0] &
& PosQ[c/a]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2220
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a
+ b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*El
lipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &
& EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[-b + c*(d/e) + a*(e/d)]
```

3.352.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.32 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.58

method	result
risch	$-\frac{x\left(3x^2+1\right)}{5\sqrt{2x^4+2x^2+1}}+\frac{2\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{5\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{3}{10}+\frac{3i}{10}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1+i}\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x}}$
default	$-\frac{4\left(\frac{3}{20}x^3+\frac{1}{20}x\right)}{\sqrt{2x^4+2x^2+1}}+\frac{\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{10\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{3i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{10\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$
elliptic	$-\frac{4\left(\frac{3}{20}x^3+\frac{1}{20}x\right)}{\sqrt{2x^4+2x^2+1}}+\frac{\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{10\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{3i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{10\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$

input
```
int(1/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*x*(3*x^2+1)/(2*x^4+2*x^2+1)^(1/2)+2/5/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2
)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2
^(1/2)+1/2*I*2^(1/2))+(-3/10+3/10*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(
1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2
)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))+2/15
/(-1+I)^(1/2)*(1-I*x^2+x^2)^(1/2)*(1+I*x^2+x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2
)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

3.352.5 Fricas [F]

$$\int \frac{1}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}(2\,x^2+3)}\,dx$$

input
```
integrate(1/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(2*x^4 + 2*x^2 + 1)/(8*x^10 + 28*x^8 + 40*x^6 + 32*x^4 + 14*x
^2 + 3), x)
```

3.352.6 Sympy [F]

$$\int \frac{1}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2x^2+3)(2x^4+2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x**2+3)/(2*x**4+2*x**2+1)**(3/2),x)
```

output
```
Integral(1/((2*x**2 + 3)*(2*x**4 + 2*x**2 + 1)**(3/2)), x)
```

3.352.7 Maxima [F]

$$\int \frac{1}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}(2\,x^2+3)}\,dx$$

input
```
integrate(1/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

3.352.8 Giac [F]

$$\int \frac{1}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}(2\,x^2+3)}\,dx$$

input
```
integrate(1/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

3.352.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = \int \frac{1}{(2\,x^2+3)\,(2\,x^4+2\,x^2+1)^{3/2}}\,dx$$

input
```
int(1/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)),x)
```

output
```
int(1/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)), x)
```

3.353 $\int \frac{1}{x^2(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

3.353.1 Optimal result

Integrand size = 29, antiderivative size = 468

$$\begin{aligned}\int \frac{1}{x^2(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx &= -\frac{x}{3\sqrt{1+2x^2+2x^4}}+\frac{2x(1+3x^2)}{15\sqrt{1+2x^2+2x^4}} \\
&-\frac{\sqrt{1+2x^2+2x^4}}{3x}+\frac{2\sqrt{2}x\sqrt{1+2x^2+2x^4}}{15\left(1+\sqrt{2}x^2\right)}-\frac{2\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)}{15\sqrt{15}} \\
&-\frac{2\sqrt[4]{2}(1+\sqrt{2}x^2)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right)|\frac{1}{4}(2-\sqrt{2})\right)}{15\sqrt{1+2x^2+2x^4}} \\
&+\frac{\left(-7+3\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{3\ 2^{3/4}\left(-2+3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}} \\
&-\frac{\sqrt[4]{2}(3+\sqrt{2})\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{1}{24}(12-11\sqrt{2}),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{45\left(2-3\sqrt{2}\right)\sqrt{1+2x^2+2x^4}}\end{aligned}$$

output
```
-2/225*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)-1/3*x/(2*x^4+
2*x^2+1)^(1/2)+2/15*x*(3*x^2+1)/(2*x^4+2*x^2+1)^(1/2)-1/3*(2*x^4+2*x^2+1)^
(1/2)/x+2/15*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))-2/15*(cos(2*a
rctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticE(sin(2*arctan
(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^
2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)-1/45*(cos(2*arctan(2^(1/
4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(2^(1/4)*x
)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*(1+x^2*2^(1/2))*((
2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2-3*2^(1/2))/(2*x^4+2*x^2
+1)^(1/2)+1/6*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*
EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(-7+3*2^(1/2))*(
1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(-2+3*2^(
1/2))/(2*x^4+2*x^2+1)^(1/2)
```

3.353.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.21 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.45

$$\int \frac{1}{x^2 (3 + 2x^2) (1 + 2x^2 + 2x^4)^{3/2}} \, dx = \frac{-12i\sqrt{1-i}x\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}E\left(i\operatorname{arcsinh}\left(\sqrt{1-i}x\right)\right|}{}$$

input
```
Integrate[1/(x^2*(3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

output
```
((-12*I)*Sqrt[1 - I]*x*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*Ellipti
cE[I*ArcSinh[Sqrt[1 - I]*x], I] - (27 - 39*I)*Sqrt[1 - I]*x*Sqrt[1 + (1 -
I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] - 2*(
15 + 39*x^2 + 12*x^4 + 2*(1 - I)^(3/2)*x*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1
 + I)*x^2]*EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I]))/(90*x*Sqrt
[1 + 2*x^2 + 2*x^4])
```

3.353.3 Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {1642, 2198, 27, 2199, 1604, 1511, 1416, 1509, 2220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (2x^2+3) (2x^4+2x^2+1)^{3/2}} dx$$

$$\Big\downarrow \text{1642}$$

$$\frac{\int \frac{8\sqrt{2}x^6+4\left(2-\sqrt{2}\right)x^4-2\left(2-5\sqrt{2}\right)x^2+5\left(2-3\sqrt{2}\right)}{x^2(2x^4+2x^2+1)^{3/2}}dx}{15\left(2-3\sqrt{2}\right)}-\frac{8\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{15\left(2-3\sqrt{2}\right)}$$

$$\Big\downarrow \text{2198}$$

$$\frac{\frac{1}{4}\int \frac{4\left(-6\left(2-3\sqrt{2}\right)x^4-\left(18-31\sqrt{2}\right)x^2+5\left(2-3\sqrt{2}\right)\right)}{x^2\sqrt{2x^4+2x^2+1}}dx-\frac{3x\left(-2\left(2-3\sqrt{2}\right)x^2-3\sqrt{2}+2\right)}{\sqrt{2x^4+2x^2+1}}}{15\left(2-3\sqrt{2}\right)}-\frac{8\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{15\left(2-3\sqrt{2}\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{-6\left(2-3\sqrt{2}\right)x^4-\left(18-31\sqrt{2}\right)x^2+5\left(2-3\sqrt{2}\right)}{x^2\sqrt{2x^4+2x^2+1}}dx-\frac{3x\left(-2\left(2-3\sqrt{2}\right)x^2-3\sqrt{2}+2\right)}{\sqrt{2x^4+2x^2+1}}}{15\left(2-3\sqrt{2}\right)}-\frac{8\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{15\left(2-3\sqrt{2}\right)}$$

$$\Big\downarrow \text{2199}$$

$$\frac{\int \frac{2\left(2-3\sqrt{2}\right)-\left(18-31\sqrt{2}\right)x^2}{x^2\sqrt{2x^4+2x^2+1}}dx-\frac{3x\left(-2\left(2-3\sqrt{2}\right)x^2-3\sqrt{2}+2\right)}{\sqrt{2x^4+2x^2+1}}-\frac{3\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}{x}}{15\left(2-3\sqrt{2}\right)}-\frac{8\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{15\left(2-3\sqrt{2}\right)}$$

$$\Big\downarrow \text{1604}$$

$$\frac{-\int \frac{-4\left(2-3\sqrt{2}\right)x^2-31\sqrt{2}+18}{\sqrt{2x^4+2x^2+1}}dx-\frac{3x\left(-2\left(2-3\sqrt{2}\right)x^2-3\sqrt{2}+2\right)}{\sqrt{2x^4+2x^2+1}}-\frac{5\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}{x}}{15\left(2-3\sqrt{2}\right)}-\frac{8\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{15\left(2-3\sqrt{2}\right)}$$

$$\Big\downarrow \text{1511}$$

$$\frac{-5(6-7\sqrt{2})\int \frac{1}{\sqrt{2x^4+2x^2+1}}dx+4(3-\sqrt{2})\int \frac{1-\sqrt{2}x^2}{\sqrt{2x^4+2x^2+1}}dx-\frac{3x\left(-2\left(2-3\sqrt{2}\right)x^2-3\sqrt{2}+2\right)}{\sqrt{2x^4+2x^2+1}}-\frac{5\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}{x}}{15\left(2-3\sqrt{2}\right)}-\frac{8\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{15\left(2-3\sqrt{2}\right)}$$

$$\Big\downarrow \text{1416}$$

$$\frac{4(3-\sqrt{2})\int \frac{1-\sqrt{2}x^2}{\sqrt{2x^4+2x^2+1}}dx-\frac{5\left(6-7\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{2\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-\frac{3x\left(-2\left(2-3\sqrt{2}\right)x^2-3\sqrt{\right.}}{\sqrt{2x^4+2x^2+1}}}{15\left(2-3\sqrt{2}\right)}$$

$$\frac{8\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{15\left(2-3\sqrt{2}\right)}$$

$$\Big\downarrow \text{1509}$$

$$\frac{-\frac{5\left(6-7\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{2\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}+4(3-\sqrt{2})\left(\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right.\right.}{\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}\right.}{15\left(2-3\sqrt{2}\right)}$$

$$\frac{8\int \frac{\sqrt{2}x^2+1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}dx}{15\left(2-3\sqrt{2}\right)}$$

$$\Big\downarrow \text{2220}$$

$$\frac{-\frac{5\left(6-7\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{2\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}+4(3-\sqrt{2})\left(\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\arctan\left(\sqrt[4]{2}x\right.\right.}{\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}\right.}{15\left(2-3\sqrt{2}\right)}$$

$$\frac{8\left(\frac{\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticPi}\left(\frac{1}{24}\left(12-11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{12\ 2^{3/4}\sqrt{2x^4+2x^2+1}}-\frac{\left(3-\sqrt{2}\right)\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{2\sqrt{30}}\right)}{15\left(2-3\sqrt{2}\right)}$$

input
```
Int[1/(x^2*(3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

3.353. $\int \frac{1}{x^2(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

output
```
((-3*x*(2 - 3*Sqrt[2] - 2*(2 - 3*Sqrt[2])*x^2))/Sqrt[1 + 2*x^2 + 2*x^4] -
(5*(2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])/x + 4*(3 - Sqrt[2])*(-((x*Sqrt
[1 + 2*x^2 + 2*x^4])/(1 + Sqrt[2]*x^2)) + ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x
^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[
2])/4])/(2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])) - (5*(6 - 7*Sqrt[2])*(1 + Sqrt[
2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2
^(1/4)*x], (2 - Sqrt[2])/4])/(2*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]))/(15*(2 -
 3*Sqrt[2])) - (8*(-1/2*((3 - Sqrt[2])*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2
 + 2*x^4]])/Sqrt[30] + ((3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 +
2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1
/4)*x], (2 - Sqrt[2])/4])/(12*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4])))/(15*(2 -
3*Sqrt[2]))
```

3.353.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

3.353. $\int \frac{1}{x^2(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1642
```
Int[((x_)^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[(-(-d/e)^(m/2))*((c*d^2 - b*d*e + a*e^2)^(p + 1/2)
/(e^(2*p)*(Rt[c/a, 2]*d - e)))   Int[(1 + Rt[c/a, 2]*x^2)/((d + e*x^2)*Sqrt
[a + b*x^2 + c*x^4]), x], x] + Simp[(-d/e)^(m/2)*((c*d^2 - b*d*e + a*e^2)^(
p + 1/2)/(Rt[c/a, 2]*d - e))   Int[x^m*(a + b*x^2 + c*x^4)^p*ExpandToSum[((
(Rt[c/a, 2]*d - e)*(c*d^2 - b*d*e + a*e^2)^(-p - 1/2))/(-d/e)^(m/2) + ((1 +
 Rt[c/a, 2]*x^2)*(a + b*x^2 + c*x^4)^(-p - 1/2))/(e^(2*p)*x^m))/(d + e*x^2)
, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && ILtQ[p
+ 1/2, 0] && ILtQ[m/2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

rule 2199
```
Int[(Px_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> With[{q = Expon[Px, x^2]}, Simp[Coeff[Px, x^2, q]*(d*x)^(m + 2*q
- 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*d^(2*q - 3)*(m + 4*p + 2*q + 1))), x]
+ Int[(d*x)^m*(a + b*x^2 + c*x^4)^p*ExpandToSum[Px - Coeff[Px, x^2, q]*x^(2
*q) - Coeff[Px, x^2, q]*((a*(m + 2*q - 3)*x^(2*(q - 2)) + b*(m + 2*p + 2*q
- 1)*x^(2*(q - 1)))/(c*(m + 4*p + 2*q + 1))), x], x] /; GtQ[q, 1] && NeQ[m
+ 4*p + 2*q + 1, 0]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Px, x^2] && N
eQ[b^2 - 4*a*c, 0]
```

3.353. $\int \frac{1}{x^2(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

rule 2220
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a
+ b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*El
lipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &
& EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[-b + c*(d/e) + a*(e/d)]
```

3.353.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.98 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.54

method	result
risch	$-\frac{4x^4+13x^2+5}{15x\sqrt{2x^4+2x^2+1}}-\frac{3\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{5\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{2}{15}+\frac{2i}{15}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(F\left(x\sqrt{-1\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+}}$
elliptic	$-\frac{4\left(-\frac{1}{10}x^3+\frac{1}{20}x\right)}{\sqrt{2x^4+2x^2+1}}-\frac{\sqrt{2x^4+2x^2+1}}{3x}-\frac{11\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{15\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{2i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,F}{15\sqrt{-1+i}\,\sqrt{2x^4+}}$
default	$-\frac{x}{3\sqrt{2x^4+2x^2+1}}-\frac{\sqrt{2x^4+2x^2+1}}{3x}-\frac{\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{3\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{1}{3}+\frac{i}{3}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x}}{\sqrt{}}$

input
```
int(1/x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(4*x^4+13*x^2+5)/x/(2*x^4+2*x^2+1)^(1/2)-3/5/(-1+I)^(1/2)*(1+(1-I)*x
^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/
2),1/2*2^(1/2)+1/2*I*2^(1/2))+(-2/15+2/15*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1
/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/
2*2^(1/2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2
)))-4/45/(-1+I)^(1/2)*(1-I*x^2+x^2)^(1/2)*(1+I*x^2+x^2)^(1/2)/(2*x^4+2*x^2
+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

3.353.5 Fricas [F]

$$\int \frac{1}{x^2 (3+2x^2) (1+2x^2+2x^4)^{3/2}} \, dx = \int \frac{1}{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}(2\,x^2+3)x^2} \, dx$$

input
```
integrate(1/x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(2*x^4 + 2*x^2 + 1)/(8*x^12 + 28*x^10 + 40*x^8 + 32*x^6 + 14*
x^4 + 3*x^2), x)
```

3.353.6 Sympy [F]

$$\int \frac{1}{x^2 (3+2x^2) (1+2x^2+2x^4)^{3/2}} \, dx = \int \frac{1}{x^2 \cdot (2x^2+3) (2x^4+2x^2+1)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/x**2/(2*x**2+3)/(2*x**4+2*x**2+1)**(3/2),x)
```

output
```
Integral(1/(x**2*(2*x**2 + 3)*(2*x**4 + 2*x**2 + 1)**(3/2)), x)
```

3.353.7 Maxima [F]

$$\int \frac{1}{x^2 (3+2x^2) (1+2x^2+2x^4)^{3/2}} \, dx = \int \frac{1}{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}(2\,x^2+3)x^2} \, dx$$

input
```
integrate(1/x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)*x^2), x)
```

3.353.8 Giac [F]

$$\int \frac{1}{x^2 (3+2x^2) (1+2x^2+2x^4)^{3/2}} \, dx = \int \frac{1}{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}(2\,x^2+3)x^2} \, dx$$

input
```
integrate(1/x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)*x^2), x)
```

3.353.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (3+2x^2) (1+2x^2+2x^4)^{3/2}} \, dx = \int \frac{1}{x^2\,\left(2\,x^2+3\right)\,\left(2\,x^4+2\,x^2+1\right)^{3/2}} \, dx$$

input
```
int(1/(x^2*(2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)),x)
```

output
```
int(1/(x^2*(2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)), x)
```

3.354 $\int \frac{x^7\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

3.354.1 Optimal result

Integrand size = 29, antiderivative size = 406

$$\int \frac{x^7\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx=\frac{(b^2-ac)\sqrt{d+ex^2}}{c^3}-\frac{(cd+be)\left(d+ex^2\right)^{3/2}}{3c^2e^2}+\frac{\left(d+ex^2\right)^{5/2}}{5ce^2}-\frac{\left(b^2cd-ac^2d-b^3e+2abce-\frac{b^3cd-3abc^2d-b^4e+4ab^2ce-2a^2c^2e}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}c^{7/2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}-\frac{\left(b^2cd-ac^2d-b^3e+2abce+\frac{b^3cd-3abc^2d-b^4e+4ab^2ce-2a^2c^2e}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}c^{7/2}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

output
```
-1/3*(b*e+c*d)*(e*x^2+d)^(3/2)/c^2/e^2+1/5*(e*x^2+d)^(5/2)/c/e^2+(-a*c+b^2
)*(e*x^2+d)^(1/2)/c^3-1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e
*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*(b^2*c*d-a*c^2*d-b^3*e+2*a*b*c*e+(2*a^2*c^
2*e-4*a*b^2*c*e+3*a*b*c^2*d+b^4*e-b^3*c*d)/(-4*a*c+b^2)^(1/2))/c^(7/2)*2^(
1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)-1/2*arctanh(2^(1/2)*c^(1/2)*(e
*x^2+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*(b^2*c*d-a*c^2*d-b^3
*e+2*a*b*c*e+(-2*a^2*c^2*e+4*a*b^2*c*e-3*a*b*c^2*d-b^4*e+b^3*c*d)/(-4*a*c+
b^2)^(1/2))/c^(7/2)*2^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.354.2 Mathematica [A] (verified)

Time = 1.30 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.17

$$\int \frac{x^7\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

$$= \frac{\frac{2\sqrt{c}\sqrt{d+ex^2}\left(15b^2e^2+c^2\left(-2d^2+dex^2+3e^2x^4\right)-5ce\left(3ae+b\left(d+ex^2\right)\right)\right)}{e^2} - \frac{15\sqrt{2}\left(-b^4e+ac^2\left(\sqrt{b^2-4ac}d-2ae\right)+b^2c\left(-\sqrt{b^2-4ac}d+4ae\right)+b^3\left(cd\cdots\right.\right.}{\sqrt{b^2-4ac}\sqrt{-2c\cdots}}}{\cdots}$$

input
```
Integrate[(x^7*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

output
```
((2*Sqrt[c]*Sqrt[d + e*x^2]*(15*b^2*e^2 + c^2*(-2*d^2 + d*e*x^2 + 3*e^2*x^
4) - 5*c*e*(3*a*e + b*(d + e*x^2))))/e^2 - (15*Sqrt[2]*(-(b^4*e) + a*c^2*(
Sqrt[b^2 - 4*a*c]*d - 2*a*e) + b^2*c*(-(Sqrt[b^2 - 4*a*c]*d) + 4*a*e) + b^
3*(c*d + Sqrt[b^2 - 4*a*c]*e) - a*b*c*(3*c*d + 2*Sqrt[b^2 - 4*a*c]*e))*Arc
Tan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c
]*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e]) - (15*
Sqrt[2]*(b^4*e + a*c^2*(Sqrt[b^2 - 4*a*c]*d + 2*a*e) - b^2*c*(Sqrt[b^2 - 4
*a*c]*d + 4*a*e) + a*b*c*(3*c*d - 2*Sqrt[b^2 - 4*a*c]*e) + b^3*(-(c*d) + S
qrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d
+ (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + (b + Sqrt[
b^2 - 4*a*c])*e]))/(30*c^(7/2))
```

3.354.3 Rubi [A] (warning: unable to verify)

Time = 4.30 (sec) , antiderivative size = 397, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1578, 1199, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{x^6\sqrt{ex^2+d}}{cx^4+bx^2+a}dx^2$$

$$\downarrow 1199$$

$$\frac{\int \left(\frac{x^8}{ce} - \frac{(cd+be)x^4}{c^2e} + \frac{(b^2-ac)e}{c^3} - \frac{(b^2-ac)(cd^2-bed+ae^2)-(-eb^3+cdb^2+2aceb-ac^2d)x^4}{c^3e\left(\frac{cx^8}{e^2} - \frac{(2cd-be)x^4}{e^2} + a + \frac{d(cd-be)}{e^2}\right)}\right) d\sqrt{ex^2+d}}{e}$$

$$\downarrow 2009$$

$$\frac{-\frac{e\left(-\frac{-2a^2c^2e+4ab^2ce-3abc^2d+b^4(-e)+b^3cd}{\sqrt{b^2-4ac}}+2abce-ac^2d+b^3(-e)+b^2cd\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}c^{7/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}} - \frac{e\left(\frac{-2a^2c^2e+4ab^2ce-3abc^2d+b^4(\cdots}{\sqrt{b^2-4ac}}\cdots\right.}{\cdots}}{e}$$

input
```
Int[(x^7*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

output
```
(-1/3*((c*d + b*e)*x^6)/(c^2*e) + x^10/(5*c*e) + ((b^2 - a*c)*e*Sqrt[d + e
*x^2])/c^3 - (e*(b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*e - (b^3*c*d - 3*a*b*
c^2*d - b^4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqr
t[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(S
qrt[2]*c^(7/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (e*(b^2*c*d - a*
c^2*d - b^3*e + 2*a*b*c*e + (b^3*c*d - 3*a*b*c^2*d - b^4*e + 4*a*b^2*c*e -
 2*a^2*c^2*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])
/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(7/2)*Sqrt[2*c*d - (
b + Sqrt[b^2 - 4*a*c])*e]))/e
```

3.354.3.1 Defintions of rubi rules used

rule 1199
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Denominator[m]}, Simp[q/e   Subs
t[Int[ExpandIntegrand[x^(q*(m + 1) - 1)*(((e*f - d*g)/e + g*(x^q/e))^n/((c*
d^2 - b*d*e + a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))), x],
 x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Integer
Q[n] && FractionQ[m]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.354. $\int \frac{x^7\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.354.4 Maple [A] (verified)

Time = 2.39 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.17

method	result
risch	$-\frac{(-3e^2c^2x^4+5bce^2x^2-c^2dex^2+15e^2ac-15b^2e^2+5bcde+2c^2d^2)\sqrt{ex^2+d}}{15e^2c^3}-\frac{\sqrt{2}\left(\sqrt{\left(be-2cd+\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}\right)c}\left(\left(\cdots\right.\right.\right.}{\cdots}$
pseudoelliptic	$-\frac{\left(\left(\left(abc-\frac{1}{2}b^3\right)e-\frac{dc\left(ac-b^2\right)}{2}\right)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}+e\left(\left(-2ab^2c+\frac{1}{2}b^4+a^2c^2\right)e+\frac{3cdb\left(ac-\frac{b^2}{3}\right)}{2}\right)\right)e^2\sqrt{2}\sqrt{\left(be-2cd+\sqrt{-4e\cdots}\right.}}{\cdots}$
default	$\frac{\frac{x^2\left(ex^2+d\right)^{\frac{3}{2}}}{5e}-\frac{2d\left(ex^2+d\right)^{\frac{3}{2}}}{15e^2}}{c}-\frac{b\left(ex^2+d\right)^{\frac{3}{2}}}{3c^2e}+\frac{-\left(ac-b^2\right)\sqrt{ex^2+d}+\frac{\left(-2a^2c^2e^2+4ab^2ce^2-3abc^2de-b^4e^2+b^3cde+2\sqrt{-e^2\left(4\cdots\right.}\right.}{\cdots}}{\cdots}$

input
```
int(x^7*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(-3*c^2*e^2*x^4+5*b*c*e^2*x^2-c^2*d*e*x^2+15*a*c*e^2-15*b^2*e^2+5*b*
c*d*e+2*c^2*d^2)*(e*x^2+d)^(1/2)/e^2/c^3-1/c^3/((b*e-2*c*d+(-4*e^2*(a*c-1/
4*b^2))^(1/2))*c)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))
*c)^(1/2)/(-4*e^2*(a*c-1/4*b^2))^(1/2)*(((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))
^(1/2))*c)^(1/2)*((-1/2*a*c^2*d+b*(a*e+1/2*b*d)*c-1/2*b^3*e)*(-4*e^2*(a*c-
1/4*b^2))^(1/2)+e*(a*(a*e+3/2*b*d)*c^2+(-2*a*b^2*e-1/2*b^3*d)*c+1/2*b^4*e)
)*arctanh(c*(e*x^2+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1
/2))*c)^(1/2))+((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*((1/2*a
*c^2*d+(-a*b*e-1/2*b^2*d)*c+1/2*b^3*e)*(-4*e^2*(a*c-1/4*b^2))^(1/2)+e*(a*(
a*e+3/2*b*d)*c^2+(-2*a*b^2*e-1/2*b^3*d)*c+1/2*b^4*e))*arctan(c*(e*x^2+d)^(
1/2)*2^(1/2)/((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)))
```

3.354.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5829 vs. $2(356) = 712$.

Time = 257.63 (sec) , antiderivative size = 5829, normalized size of antiderivative = 14.36

$$\int \frac{x^7\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^7*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.354.6 Sympy [F]

$$\int \frac{x^7\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \int \frac{x^7\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

input
```
integrate(x**7*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
```

output
```
Integral(x**7*sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

3.354.7 Maxima [F]

$$\int \frac{x^7\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \int \frac{\sqrt{ex^2+d}x^7}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^7*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x^2 + d)*x^7/(c*x^4 + b*x^2 + a), x)
```

3.354.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 959 vs. $2(356) = 712$.

Time = 0.33 (sec) , antiderivative size = 959, normalized size of antiderivative = 2.36

$$\int \frac{x^7\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx =$$

$$-\frac{(((b^4c-5\,ab^2c^2+4\,a^2c^3)d-(b^5-6\,ab^3c+8\,a^2bc^2)e)c^2e^2+2\,(b^3c^4-3\,abc^5)d^2e-(3\,b^4c^3-11\,ab^2c^4+}{}$$

$$+\frac{(((b^4c-5\,ab^2c^2+4\,a^2c^3)d-(b^5-6\,ab^3c+8\,a^2bc^2)e)c^2e^2+2\,(b^3c^4-3\,abc^5)d^2e-(3\,b^4c^3-11\,ab^2c^4+}{}$$

$$+\frac{3\,(ex^2+d)^{\frac{5}{2}}c^4e^8-5\,(ex^2+d)^{\frac{3}{2}}c^4de^8-5\,(ex^2+d)^{\frac{3}{2}}bc^3e^9+15\,\sqrt{ex^2+d}b^2c^2e^{10}-15\,\sqrt{ex^2+d}ac^3e^{10}}{15\,c^5e^{10}}$$

input
```
integrate(x^7*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
-(((b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d - (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*e
)*c^2*e^2 + 2*(b^3*c^4 - 3*a*b*c^5)*d^2*e - (3*b^4*c^3 - 11*a*b^2*c^4 + 4*
a^2*c^5)*d*e^2 + (b^5*c^2 - 4*a*b^3*c^3 + 2*a^2*b*c^4)*e^3 - 2*((b^2*c^3 -
 a*c^4)*sqrt(b^2 - 4*a*c)*d^2 - (b^3*c^2 - a*b*c^3)*sqrt(b^2 - 4*a*c)*d*e
+ (a*b^2*c^2 - a^2*c^3)*sqrt(b^2 - 4*a*c)*e^2)*abs(c)*abs(e))*arctan(2*sqr
t(1/2)*sqrt(e*x^2 + d)/sqrt(-(2*c^6*d*e^12 - b*c^5*e^13 + sqrt(-4*(c^6*d^2
*e^12 - b*c^5*d*e^13 + a*c^5*e^14)*c^6*e^12 + (2*c^6*d*e^12 - b*c^5*e^13)^
2))/(c^6*e^12)))/((2*sqrt(b^2 - 4*a*c)*c^4*d - (b^2*c^3 - 4*a*c^4 + sqrt(b
^2 - 4*a*c)*b*c^3)*e)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*c^2
*abs(e)) + (((b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d - (b^5 - 6*a*b^3*c + 8*a^
2*b*c^2)*e)*c^2*e^2 + 2*(b^3*c^4 - 3*a*b*c^5)*d^2*e - (3*b^4*c^3 - 11*a*b^
2*c^4 + 4*a^2*c^5)*d*e^2 + (b^5*c^2 - 4*a*b^3*c^3 + 2*a^2*b*c^4)*e^3 + 2*(
(b^2*c^3 - a*c^4)*sqrt(b^2 - 4*a*c)*d^2 - (b^3*c^2 - a*b*c^3)*sqrt(b^2 - 4
*a*c)*d*e + (a*b^2*c^2 - a^2*c^3)*sqrt(b^2 - 4*a*c)*e^2)*abs(c)*abs(e))*ar
ctan(2*sqrt(1/2)*sqrt(e*x^2 + d)/sqrt(-(2*c^6*d*e^12 - b*c^5*e^13 - sqrt(-
4*(c^6*d^2*e^12 - b*c^5*d*e^13 + a*c^5*e^14)*c^6*e^12 + (2*c^6*d*e^12 - b*
c^5*e^13)^2))/(c^6*e^12)))/((2*sqrt(b^2 - 4*a*c)*c^4*d + (b^2*c^3 - 4*a*c^
4 - sqrt(b^2 - 4*a*c)*b*c^3)*e)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)
*c)*e)*c^2*abs(e)) + 1/15*(3*(e*x^2 + d)^(5/2)*c^4*e^8 - 5*(e*x^2 + d)^(3/
2)*c^4*d*e^8 - 5*(e*x^2 + d)^(3/2)*b*c^3*e^9 + 15*sqrt(e*x^2 + d)*b^2*c...
```

3.354.9 Mupad [B] (verification not implemented)

Time = 8.90 (sec) , antiderivative size = 11195, normalized size of antiderivative = 27.57

$$\int \frac{x^7\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((x^7*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

output
```
(d + e*x^2)^(1/2)*((3*d^2)/(c*e^2) - (a*e^4 + c*d^2*e^2 - b*d*e^3)/(c^2*e^
4) + (((3*d)/(c*e^2) + (b*e^3 - 2*c*d*e^2)/(c^2*e^4))*(b*e^3 - 2*c*d*e^2))
/(c*e^2)) - (d + e*x^2)^(3/2)*(d/(c*e^2) + (b*e^3 - 2*c*d*e^2)/(3*c^2*e^4)
) + atan(((((16*a^3*c^6*e^4 + 4*a*b^4*c^4*e^4 - 4*b^5*c^4*d*e^3 - 20*a^2*b
^2*c^5*e^4 + 16*a^2*c^7*d^2*e^2 + 4*b^4*c^5*d^2*e^2 + 20*a*b^3*c^5*d*e^3 -
 16*a^2*b*c^6*d*e^3 - 20*a*b^2*c^6*d^2*e^2)/c^5 - (2*(d + e*x^2)^(1/2)*(-(
b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^
4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3
*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4
*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/
2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2
)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^
4*c^7 - 8*a*b^2*c^8)))^(1/2)*(4*b^3*c^7*e^3 - 8*b^2*c^8*d*e^2 - 16*a*b*c^8
*e^3 + 32*a*c^9*d*e^2))/c^5)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2
)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^
2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e
 + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*
a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2
) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^
2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (2*(d + ...
```

3.354. $\int \frac{x^7\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

3.355 $\int \frac{x^5\sqrt{d+ex^2}}{a+bx^2+cx^4} dx$

3.355.1 Optimal result

Integrand size = 29, antiderivative size = 324

$$\int \frac{x^5\sqrt{d+ex^2}}{a+bx^2+cx^4} dx = -\frac{b\sqrt{d+ex^2}}{c^2}+\frac{(d+ex^2)^{3/2}}{3ce}+\frac{\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}c^{5/2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}+\frac{\left(bcd-b^2e+ace+\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}c^{5/2}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

output
```
1/3*(e*x^2+d)^(3/2)/c/e-b*(e*x^2+d)^(1/2)/c^2+1/2*arctanh(2^(1/2)*c^(1/2)*
(e*x^2+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*(b*c*d-b^2*e+a*c*e
+(-3*a*b*c*e+2*a*c^2*d+b^3*e-b^2*c*d)/(-4*a*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/
(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+
d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*(b*c*d-b^2*e+a*c*e+(3*a*b
*c*e-2*a*c^2*d-b^3*e+b^2*c*d)/(-4*a*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(2*c*d-e
*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.355.2 Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.18

$$\int \frac{x^5\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

$$= \frac{\frac{2\sqrt{c}\sqrt{d+ex^2}(-3be+c(d+ex^2))}{e} + \frac{3\sqrt{2}\left(-b^3e+bc\left(-\sqrt{b^2-4ac}d+3ae\right)+b^2\left(cd+\sqrt{b^2-4ac}e\right)-ac\left(2cd+\sqrt{b^2-4ac}e\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-2cd+be-\sqrt{b^2-4a}}}\right.}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}}}{6c^{5/2}}$$

input
```
Integrate[(x^5*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

output
```
((2*Sqrt[c]*Sqrt[d + e*x^2]*(-3*b*e + c*(d + e*x^2)))/e + (3*Sqrt[2]*(-(b^
3*e) + b*c*(-(Sqrt[b^2 - 4*a*c]*d) + 3*a*e) + b^2*(c*d + Sqrt[b^2 - 4*a*c]
*e) - a*c*(2*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d +
e*x^2])/Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt
[-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e]) + (3*Sqrt[2]*(b^3*e - b*c*(Sqrt[b^2
 - 4*a*c]*d + 3*a*e) + a*c*(2*c*d - Sqrt[b^2 - 4*a*c]*e) + b^2*(-(c*d) + Sq
rt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d +
 (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + (b + Sqrt[b
^2 - 4*a*c])*e]))/(6*c^(5/2))
```

3.355.3 Rubi [A] (warning: unable to verify)

Time = 2.10 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1578, 1199, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{x^4\sqrt{ex^2+d}}{cx^4+bx^2+a}dx^2$$

$$\Big\downarrow \text{1199}$$

$$\frac{\int \left(\frac{x^4}{c}-\frac{be}{c^2}+\frac{b(cd^2-bed+ae^2)-(-eb^2+cdb+ace)x^4}{c^2e\left(\frac{cx^8}{e^2}-\frac{(2cd-be)x^4}{e^2}+a+\frac{d(cd-be)}{e^2}\right)}\right)d\sqrt{ex^2+d}}{e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{e\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}c^{5/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{e\left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\text{arct}}{\sqrt{2}c^{5/2}\sqrt{2cd-e\left(\sqrt{b^2-4ac}\right.}}}{e}$$

input
```
Int[(x^5*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

output
```
(x^6/(3*c) - (b*e*Sqrt[d + e*x^2])/c^2 + (e*(b*c*d - b^2*e + a*c*e - (b^2*
c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*S
qrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]
*c^(5/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (e*(b*c*d - b^2*e + a*
c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh
[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]
])/(Sqrt[2]*c^(5/2)*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/e
```

3.355.3.1 Defintions of rubi rules used

rule 1199
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Denominator[m]}, Simp[q/e   Subs
t[Int[ExpandIntegrand[x^(q*(m + 1) - 1)*(((e*f - d*g)/e + g*(x^q/e))^n/((c*
d^2 - b*d*e + a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))), x],
 x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Integer
Q[n] && FractionQ[m]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.355.4 Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.22

method	result
risch	$-\frac{(-c x^2 e+3be-cd)\sqrt{e x^2+d}}{3e c^2} - \frac{\sqrt{2}\left(-\left(((ae+bd)c-b^2e)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}+2a c^2 de+(-3ab e^2-b^2de)c+b^3e^2\right)\sqrt{\left(be-2c\right.} \cdots}{\cdots}$
default	$\frac{(e x^2+d)^{\frac{3}{2}}}{3ce} - \frac{b\sqrt{e x^2+d}+\frac{\left(3ab e^2c-2a c^2de-b^3e^2+b^2cde+\sqrt{-e^2(4ac-b^2)}\,ace-\sqrt{-e^2(4ac-b^2)}\,b^2e+\sqrt{-e^2(4ac-b^2)}\,bcd\right)\sqrt{2}\,\mathrm{arct} \cdots}{2\sqrt{-e^2(4ac-b^2)}\sqrt{\left(be-2cd+\sqrt{-e^2(4ac-b^2)}\right)c} \cdots}}{\cdots}$
pseudoelliptic	$-\frac{-\left(((ac-b^2)e+bcd)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}+(-3abc+b^3)e^2+d(2a c^2-b^2c)e\right)e\sqrt{2}\sqrt{\left(be-2cd+\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}\right)c}\,\mathrm{arctanh}\left(\cdots\right.}{\cdots}$

input
```
int(x^5*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-c*e*x^2+3*b*e-c*d)*(e*x^2+d)^(1/2)/e/c^2-1/2/c^2/((b*e-2*c*d+(-4*e^
2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*2^(1/2)*(-(((a*e+b*d)*c-b^2*e)*(-4*e^2*(a
*c-1/4*b^2))^(1/2)+2*a*c^2*d*e+(-3*a*b*e^2-b^2*d*e)*c+b^3*e^2)*((b*e-2*c*d
+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x^2+d)^(1/2)*2^(1/2)/
((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))+arctan(c*(e*x^2+d)^(1
/2)*2^(1/2)/((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))*((-b*e+2*c
*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*(((a*e+b*d)*c-b^2*e)*(-4*e^2*(a*
c-1/4*b^2))^(1/2)-2*a*c^2*d*e+(3*a*b*e^2+b^2*d*e)*c-b^3*e^2))/((-b*e+2*c*d
+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)/(-4*e^2*(a*c-1/4*b^2))^(1/2)
```

3.355.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4182 vs. $2(280) = 560$.

Time = 102.89 (sec) , antiderivative size = 4182, normalized size of antiderivative = 12.91

$$\int \frac{x^5\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^5*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
-1/12*(3*sqrt(1/2)*c^2*e*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5
- 5*a*b^3*c + 5*a^2*b*c^2)*e + (b^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^
4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^
3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4
)*e^2)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(-(2*(a^2*b^4*c - 2
*a^3*b^2*c^2)*d^2 - 2*(a^2*b^5 - 2*a^3*b^3*c - a^4*b*c^2)*d*e + 2*(a^3*b^4
 - 3*a^4*b^2*c + a^5*c^2)*e^2 + ((a^2*b^4*c - 2*a^3*b^2*c^2)*d*e - (a^2*b^
5 - 3*a^3*b^3*c + a^4*b*c^2)*e^2)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^7*
c - 7*a*b^5*c^2 + 14*a^2*b^3*c^3 - 8*a^3*b*c^4)*d - (b^8 - 8*a*b^6*c + 20*
a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4)*e - (b^5*c^5 - 7*a*b^3*c^6 + 12*
a^2*b*c^7)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c -
5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2
*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))*sqrt(((b^
4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e + (b^
2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^
7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c +
11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))/(b^
2*c^5 - 4*a*c^6)) - ((a^2*b^2*c^5 - 4*a^3*c^6)*e*x^2 + 2*(a^2*b^2*c^5 - 4*
a^3*c^6)*d)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c -
 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11...
```

3.355.6 Sympy [F]

$$\int \frac{x^5\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \int \frac{x^5\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

input
```
integrate(x**5*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
```

output
```
Integral(x**5*sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

3.355.7 Maxima [F]

$$\int \frac{x^5\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \int \frac{\sqrt{ex^2+d}x^5}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^5*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x^2 + d)*x^5/(c*x^4 + b*x^2 + a), x)
```

3.355.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 776 vs. 2(280) = 560.

Time = 0.32 (sec) , antiderivative size = 776, normalized size of antiderivative = 2.40

$$\int \frac{x^5\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

$$= \frac{(((b^3c-4abc^2)d-(b^4-5ab^2c+4a^2c^2)e)c^2e^2+2(b^2c^4-2ac^5)d^2e-(3b^3c^3-8abc^4)de^2+(b^4c^2-3ab}{(2\sqrt{b^2-4ac}c^3d-(b^2c^2-4}$$

$$-\frac{(((b^3c-4abc^2)d-(b^4-5ab^2c+4a^2c^2)e)c^2e^2+2(b^2c^4-2ac^5)d^2e-(3b^3c^3-8abc^4)de^2+(b^4c^2-3}{(2\sqrt{b^2-4ac}c^3d+(b^2c^2-}$$

$$+\frac{(ex^2+d)^{\frac{3}{2}}c^2e^2-3\sqrt{ex^2+d}bce^3}{3c^3e^3}$$

input
```
integrate(x^5*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
(((b^3*c - 4*a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e)*c^2*e^2 + 2*(b^
2*c^4 - 2*a*c^5)*d^2*e - (3*b^3*c^3 - 8*a*b*c^4)*d*e^2 + (b^4*c^2 - 3*a*b^
2*c^3)*e^3 - 2*(sqrt(b^2 - 4*a*c)*b*c^3*d^2 - sqrt(b^2 - 4*a*c)*b^2*c^2*d*
e + sqrt(b^2 - 4*a*c)*a*b*c^2*e^2)*abs(c)*abs(e))*arctan(2*sqrt(1/2)*sqrt(
e*x^2 + d)/sqrt(-(2*c^4*d*e^4 - b*c^3*e^5 + sqrt(-4*(c^4*d^2*e^4 - b*c^3*d
*e^5 + a*c^3*e^6)*c^4*e^4 + (2*c^4*d*e^4 - b*c^3*e^5)^2))/(c^4*e^4)))/((2*
sqrt(b^2 - 4*a*c)*c^3*d - (b^2*c^2 - 4*a*c^3 + sqrt(b^2 - 4*a*c)*b*c^2)*e)
*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*c^2*abs(e)) - (((b^3*c -
 4*a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e)*c^2*e^2 + 2*(b^2*c^4 - 2*
a*c^5)*d^2*e - (3*b^3*c^3 - 8*a*b*c^4)*d*e^2 + (b^4*c^2 - 3*a*b^2*c^3)*e^3
 + 2*(sqrt(b^2 - 4*a*c)*b*c^3*d^2 - sqrt(b^2 - 4*a*c)*b^2*c^2*d*e + sqrt(b
^2 - 4*a*c)*a*b*c^2*e^2)*abs(c)*abs(e))*arctan(2*sqrt(1/2)*sqrt(e*x^2 + d)
/sqrt(-(2*c^4*d*e^4 - b*c^3*e^5 - sqrt(-4*(c^4*d^2*e^4 - b*c^3*d*e^5 + a*c
^3*e^6)*c^4*e^4 + (2*c^4*d*e^4 - b*c^3*e^5)^2))/(c^4*e^4)))/((2*sqrt(b^2 -
 4*a*c)*c^3*d + (b^2*c^2 - 4*a*c^3 - sqrt(b^2 - 4*a*c)*b*c^2)*e)*sqrt(-4*c
^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*c^2*abs(e)) + 1/3*((e*x^2 + d)^(3/
2)*c^2*e^2 - 3*sqrt(e*x^2 + d)*b*c*e^3)/(c^3*e^3)
```

3.355.9 Mupad [B] (verification not implemented)

Time = 8.68 (sec) , antiderivative size = 8222, normalized size of antiderivative = 25.38

$$\int \frac{x^5\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((x^5*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

output
```
(d + e*x^2)^(3/2)/(3*c*e) - atan(((((4*a*b^3*c^3*e^4 - 16*a^2*b*c^4*e^4 -
4*b^4*c^3*d*e^3 + 4*b^3*c^4*d^2*e^2 - 16*a*b*c^5*d^2*e^2 + 16*a*b^2*c^4*d*
e^3)/c^3 - (2*(d + e*x^2)^(1/2)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c -
b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*
(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e -
b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) -
3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c
^6)))^(1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6*e^3 + 32*a*c^7*d
*e^2))/c^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*
c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(
1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b
^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c
 - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*(d
+ e*x^2)^(1/2)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^
2*e^2 + b^4*c^2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*b^3*c^2*d*e
^3 - 10*a^2*b*c^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2))/c^3)*(-(b^7*e + 8*a^3*c^4*
d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b
^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*
d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a
*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c...
```

3.355. $\int \frac{x^5\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

3.356 $\int \frac{x^3\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

3.356.1 Optimal result

Integrand size = 29, antiderivative size = 292

$$\int \frac{x^3\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \frac{\sqrt{d+ex^2}}{c}+\frac{\left(bcd-b^2e+2ace-\sqrt{b^2-4ac}(cd-be)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{\left(bcd-b^2e+2ace+\sqrt{b^2-4ac}(cd-be)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

output
```
(e*x^2+d)^(1/2)/c+1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b-
(-4*a*c+b^2)^(1/2)))^(1/2))*(b*c*d-b^2*e+2*a*c*e-(-b*e+c*d)*(-4*a*c+b^2)^(
1/2))/c^(3/2)*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^
(1/2)-1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)
^(1/2)))^(1/2))*(b*c*d-b^2*e+2*a*c*e+(-b*e+c*d)*(-4*a*c+b^2)^(1/2))/c^(3/2
)*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.356.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.95 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.20

$$\int \frac{x^3\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$
$$=\frac{2\sqrt{c}\sqrt{d+ex^2}-\frac{\left(-ibcd-c\sqrt{-b^2+4ac}d+ib^2e-2iace+b\sqrt{-b^2+4ac}e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-2cd+be-i\sqrt{-b^2+4ac}e}}\right)}{\sqrt{-\frac{b^2}{2}+2ac}\sqrt{-2cd+\left(b-i\sqrt{-b^2+4ac}\right)e}}-\frac{\left(ibcd-c\sqrt{-b^2+4ac}d-ib^2e+2\cdots\right.}{\sqrt{-\frac{b^2}{2}\cdots}}}{2c^{3/2}}$$

input
```
Integrate[(x^3*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

output
```
(2*Sqrt[c]*Sqrt[d + e*x^2] - (((-I)*b*c*d - c*Sqrt[-b^2 + 4*a*c]*d + I*b^2
*e - (2*I)*a*c*e + b*Sqrt[-b^2 + 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d
+ e*x^2])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[-1/2*b^2 + 2
*a*c]*Sqrt[-2*c*d + (b - I*Sqrt[-b^2 + 4*a*c])*e]) - ((I*b*c*d - c*Sqrt[-b
^2 + 4*a*c]*d - I*b^2*e + (2*I)*a*c*e + b*Sqrt[-b^2 + 4*a*c]*e)*ArcTan[(Sq
rt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e]
])/(Sqrt[-1/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b + I*Sqrt[-b^2 + 4*a*c])*e]))/(
2*c^(3/2))
```

3.356.3 Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1578, 1196, 25, 1197, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$
$$\Big\downarrow \text{ 1578}$$
$$\frac{1}{2}\int \frac{x^2\sqrt{ex^2+d}}{cx^4+bx^2+a}dx^2$$
$$\Big\downarrow \text{ 1196}$$

$$\frac{1}{2}\left(\frac{\int -\frac{ae-(cd-be)x^2}{\sqrt{ex^2+d}(cx^4+bx^2+a)}dx^2}{c}+\frac{2\sqrt{d+ex^2}}{c}\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\left(\frac{2\sqrt{d+ex^2}}{c}-\frac{\int \frac{ae-(cd-be)x^2}{\sqrt{ex^2+d}(cx^4+bx^2+a)}dx^2}{c}\right)$$

$$\Big\downarrow \; 1197$$

$$\frac{1}{2}\left(\frac{2\sqrt{d+ex^2}}{c}-\frac{2\int \frac{-((cd-be)x^4)+cd^2+ae^2-bde}{cx^8-(2cd-be)x^4+cd^2+ae^2-bde}d\sqrt{ex^2+d}}{c}\right)$$

$$\Big\downarrow \; 1480$$

$$\frac{1}{2}\left(\frac{2\sqrt{d+ex^2}}{c}-\frac{2\left(\frac{\left(-\sqrt{b^2-4ac}(cd-be)+2ace+b^2(-e)+bcd\right)\int \frac{1}{cx^4+\frac{1}{2}\left(\left(b-\sqrt{b^2-4ac}\right)e-2cd\right)}d\sqrt{ex^2+d}}{2\sqrt{b^2-4ac}}-\frac{\left(\sqrt{b^2-4ac}(cd-be)+2ace+b^2(-\cdots}{\cdots}\cdots\right)}{c}\cdots\right.$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(\frac{2\sqrt{d+ex^2}}{c}-\frac{2\left(\frac{\left(\sqrt{b^2-4ac}(cd-be)+2ace+b^2(-e)+bcd\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\left(-\sqrt{b^2-4ac}(cd-be)+2ace+b^2(-e)+b\cdots}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{2\cdots}}\cdots\right)}{c}\cdots\right.$$

input
```
Int[(x^3*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

3.356. $\int \frac{x^3\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

output
```
((2*Sqrt[d + e*x^2])/c - (2*(-(((b*c*d - b^2*e + 2*a*c*e - Sqrt[b^2 - 4*a*
c]*(c*d - b*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b
- Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d -
(b - Sqrt[b^2 - 4*a*c])*e])) + ((b*c*d - b^2*e + 2*a*c*e + Sqrt[b^2 - 4*a*
c]*(c*d - b*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b
+ Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d -
(b + Sqrt[b^2 - 4*a*c])*e])))/c)/2
```

3.356.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1196
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Simp[1/c   Int
[(d + e*x)^(m - 1)*(Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x]/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && FractionQ[m] &
& GtQ[m, 0]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.356. $\int \frac{x^3\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

3.356.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.08

method	result
risch	$\frac{\sqrt{e x^2+d}}{c}-\frac{\sqrt{2}\left(-\frac{\left(2e^2ac-b^2e^2+bcde+\sqrt{-e^2\left(4ac-b^2\right)}\,be-\sqrt{-e^2\left(4ac-b^2\right)}\,cd\right)\operatorname{arctanh}\left(\frac{c\sqrt{e x^2+d}\,\sqrt{2}}{\sqrt{\left(-be+2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right)}{\sqrt{\left(-be+2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}+ \cdots\right.}{2c\sqrt{-e^2(4ac- \cdots}}$
default	$\frac{\sqrt{e x^2+d}-\frac{\left(-2e^2ac+b^2e^2-bcde+\sqrt{-e^2\left(4ac-b^2\right)}\,be-\sqrt{-e^2\left(4ac-b^2\right)}\,cd\right)\sqrt{2}\arctan\left(\frac{c\sqrt{e x^2+d}\,\sqrt{2}}{\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right)}{2\sqrt{-e^2\left(4ac-b^2\right)}\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}+\frac{\left(2e^2ac-b^2 \cdots\right.}{\cdots}}{c}$
pseudoelliptic	$\frac{\sqrt{e x^2+d}-\frac{\left(-2e^2ac+b^2e^2-bcde+\sqrt{-e^2\left(4ac-b^2\right)}\,be-\sqrt{-e^2\left(4ac-b^2\right)}\,cd\right)\sqrt{2}\arctan\left(\frac{c\sqrt{e x^2+d}\,\sqrt{2}}{\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right)}{2\sqrt{-e^2\left(4ac-b^2\right)}\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}+\frac{\left(2e^2ac-b^2 \cdots\right.}{\cdots}}{c}$

input
```
int(x^3*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
(e*x^2+d)^(1/2)/c-1/2*2^(1/2)/c/(-e^2*(4*a*c-b^2))^(1/2)*(-(2*e^2*a*c-b^2*
e^2+b*c*d*e+(-e^2*(4*a*c-b^2))^(1/2)*b*e-(-e^2*(4*a*c-b^2))^(1/2)*c*d)/((-
b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x^2+d)^(1/2)*2^(
1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))+(-2*e^2*a*c+b^2*e^2-
b*c*d*e+(-e^2*(4*a*c-b^2))^(1/2)*b*e-(-e^2*(4*a*c-b^2))^(1/2)*c*d)/((b*e-2
*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x^2+d)^(1/2)*2^(1/2)/(
(b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)))
```

3.356.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2435 vs. 2(247) = 494.

Time = 29.97 (sec) , antiderivative size = 2435, normalized size of antiderivative = 8.34

$$\int \frac{x^3\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
1/4*(sqrt(1/2)*c*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e + (b^2*c^3
- 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c
+ a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log((2*a*b^2*c*
d^2 - 2*a*b^3*d*e + 2*(a^2*b^2 - a^3*c)*e^2 + (a*b^2*c*d*e - (a*b^3 - a^2*
b*c)*e^2)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^4*c - 4*a*b^2*c^2)*d - (b^
5 - 5*a*b^3*c + 4*a^2*b*c^2)*e - (b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*sqrt(
(b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/
(b^2*c^6 - 4*a*c^7)))*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e + (b^2
*c^3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b
^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) - ((a*b^2*
c^3 - 4*a^2*c^4)*e*x^2 + 2*(a*b^2*c^3 - 4*a^2*c^4)*d)*sqrt((b^2*c^2*d^2 -
2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*
c^7)))/x^2) - sqrt(1/2)*c*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e +
(b^2*c^3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2
*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log((2
*a*b^2*c*d^2 - 2*a*b^3*d*e + 2*(a^2*b^2 - a^3*c)*e^2 + (a*b^2*c*d*e - (a*b
^3 - a^2*b*c)*e^2)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^4*c - 4*a*b^2*c^2
)*d - (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e - (b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c
^5)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c
^2)*e^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b...
```

3.356.6 Sympy [F]

$$\int \frac{x^3\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \int \frac{x^3\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

input
```
integrate(x**3*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
```

output
```
Integral(x**3*sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

3.356.7 Maxima [F]

$$\int \frac{x^3\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \int \frac{\sqrt{ex^2+d}x^3}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^3*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x^2 + d)*x^3/(c*x^4 + b*x^2 + a), x)
```

3.356.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 626 vs. 2(247) = 494.

Time = 0.33 (sec) , antiderivative size = 626, normalized size of antiderivative = 2.14

$$\int \frac{x^3\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \frac{\sqrt{ex^2+d}}{c}$$

$$-\frac{\left(2\,bc^4d^2e+\left((b^2c-4\,ac^2)d-(b^3-4\,abc)e\right)c^2e^2-(3\,b^2c^3-4\,ac^4)de^2+(b^3c^2-2\,abc^3)e^3-2\left(\sqrt{b^2-4\,a\right.}{\left(2\,\sqrt{b^2-4\,ac}c^2d-\left(b^2c-4\,ac^2+\sqrt{b^2-4\,ac\right.}}$$

$$+\frac{\left(2\,bc^4d^2e+\left((b^2c-4\,ac^2)d-(b^3-4\,abc)e\right)c^2e^2-(3\,b^2c^3-4\,ac^4)de^2+(b^3c^2-2\,abc^3)e^3+2\left(\sqrt{b^2-4\,a\right.}{\left(2\,\sqrt{b^2-4\,ac}c^2d+\left(b^2c-4\,ac^2-\sqrt{b^2-4\,ac\right.}}$$

input
```
integrate(x^3*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
sqrt(e*x^2 + d)/c - (2*b*c^4*d^2*e + ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c
)*e)*c^2*e^2 - (3*b^2*c^3 - 4*a*c^4)*d*e^2 + (b^3*c^2 - 2*a*b*c^3)*e^3 - 2
*(sqrt(b^2 - 4*a*c)*c^3*d^2 - sqrt(b^2 - 4*a*c)*b*c^2*d*e + sqrt(b^2 - 4*a
*c)*a*c^2*e^2)*abs(c)*abs(e))*arctan(2*sqrt(1/2)*sqrt(e*x^2 + d)/sqrt(-(2*
c^2*d - b*c*e + sqrt(-4*(c^2*d^2 - b*c*d*e + a*c*e^2)*c^2 + (2*c^2*d - b*c
*e)^2))/c^2))/((2*sqrt(b^2 - 4*a*c)*c^2*d - (b^2*c - 4*a*c^2 + sqrt(b^2 -
4*a*c)*b*c)*e)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*c^2*abs(e)
) + (2*b*c^4*d^2*e + ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*c^2*e^2 - (
3*b^2*c^3 - 4*a*c^4)*d*e^2 + (b^3*c^2 - 2*a*b*c^3)*e^3 + 2*(sqrt(b^2 - 4*a
*c)*c^3*d^2 - sqrt(b^2 - 4*a*c)*b*c^2*d*e + sqrt(b^2 - 4*a*c)*a*c^2*e^2)*a
bs(c)*abs(e))*arctan(2*sqrt(1/2)*sqrt(e*x^2 + d)/sqrt(-(2*c^2*d - b*c*e -
sqrt(-4*(c^2*d^2 - b*c*d*e + a*c*e^2)*c^2 + (2*c^2*d - b*c*e)^2))/c^2))/((
2*sqrt(b^2 - 4*a*c)*c^2*d + (b^2*c - 4*a*c^2 - sqrt(b^2 - 4*a*c)*b*c)*e)*s
qrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*c^2*abs(e))
```

3.356.9 Mupad [B] (verification not implemented)

Time = 9.19 (sec) , antiderivative size = 5705, normalized size of antiderivative = 19.54

$$\int \frac{x^3\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((x^3*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

output
```
(d + e*x^2)^(1/2)/c - atan(((((16*a^2*c^3*e^4 - 4*a*b^2*c^2*e^4 + 16*a*c^4
*d^2*e^2 + 4*b^3*c^2*d*e^3 - 4*b^2*c^3*d^2*e^2 - 16*a*b*c^3*d*e^3)/c - (2*
(d + e*x^2)^(1/2)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) +
 b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c -
 b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3
- 8*a*b^2*c^4)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3 +
 32*a*c^5*d*e^2))/c)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2
) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*
c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c
^3 - 8*a*b^2*c^4)))^(1/2) - (2*(d + e*x^2)^(1/2)*(b^4*e^4 + 2*a^2*c^2*e^4
- 2*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*
b*c^2*d*e^3))/c)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) +
b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c -
b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 -
 8*a*b^2*c^4)))^(1/2)*1i - (((16*a^2*c^3*e^4 - 4*a*b^2*c^2*e^4 + 16*a*c^4*
d^2*e^2 + 4*b^3*c^2*d*e^3 - 4*b^2*c^3*d^2*e^2 - 16*a*b*c^3*d*e^3)/c + (2*(
d + e*x^2)^(1/2)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) +
b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c -
b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 -
 8*a*b^2*c^4)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3...
```

3.356. $\int \frac{x^3\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

3.357 $\int \frac{x\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

3.357.1 Optimal result

Integrand size = 27, antiderivative size = 202

$$\int \frac{x\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = -\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} + \frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

output
```
-1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2
)))^(1/2))*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*2^(1/2)/c^(1/2)/(-4*a*c+
b^2)^(1/2)+1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b+(-4*a*c
+b^2)^(1/2)))^(1/2))*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)*2^(1/2)/c^(1/2
)/(-4*a*c+b^2)^(1/2)
```

3.357.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.27

$$\int \frac{x\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$
$$= \frac{\frac{\left(-2icd+\left(ib+\sqrt{-b^2+4ac}\right)e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-2cd+be-i\sqrt{-b^2+4ace}}}\right)}{\sqrt{-2cd+\left(b-i\sqrt{-b^2+4ac}\right)e}}+\frac{\left(2icd+\left(-ib+\sqrt{-b^2+4ac}\right)e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-2cd+be+i\sqrt{-b^2+4ace}}}\right)}{\sqrt{-2cd+\left(b+i\sqrt{-b^2+4ac}\right)e}}}{\sqrt{2}\sqrt{c}\sqrt{-b^2+4ac}}$$

input
```
Integrate[(x*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

output
```
((((-2*I)*c*d + (I*b + Sqrt[-b^2 + 4*a*c])*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt
[d + e*x^2])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/Sqrt[-2*c*d + (
b - I*Sqrt[-b^2 + 4*a*c])*e] + (((2*I)*c*d + ((-I)*b + Sqrt[-b^2 + 4*a*c])
*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e + I*Sqrt[-b
^2 + 4*a*c]*e]])/Sqrt[-2*c*d + (b + I*Sqrt[-b^2 + 4*a*c])*e])/(Sqrt[2]*Sqr
t[c]*Sqrt[-b^2 + 4*a*c])
```

3.357.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1576, 1148, 1450, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$
$$\Big\downarrow \text{1576}$$
$$\frac{1}{2}\int \frac{\sqrt{ex^2+d}}{cx^4+bx^2+a}dx^2$$
$$\Big\downarrow \text{1148}$$
$$e\int \frac{x^4}{cx^8-(2cd-be)x^4+cd^2+ae^2-bde}d\sqrt{ex^2+d}$$

$$\Big\downarrow \text{1450}$$

$$e\left(\frac{1}{2}\left(\frac{2cd-be}{e\sqrt{b^2-4ac}}+1\right)\int \frac{1}{cx^4+\frac{1}{2}\left(\left(b-\sqrt{b^2-4ac}\right)e-2cd\right)}d\sqrt{ex^2+d}+\frac{1}{2}\left(1-\frac{2cd-be}{e\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^4+\frac{1}{2}\left(\left(b\right.\right.}\right.$$

$$\Big\downarrow \text{221}$$

$$e\left(-\frac{\left(\frac{2cd-be}{e\sqrt{b^2-4ac}}+1\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(1-\frac{2cd-be}{e\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)$$

input
```
Int[(x*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

output
```
e*(-(((1 + (2*c*d - b*e)/(Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*S
qrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]
*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e])) - ((1 - (2*c*d - b*e)/(Sqrt[b^2
 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b +
Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e]))
```

3.357.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1148
```
Int[Sqrt[(d_.) + (e_.)*(x_)]/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[2*e   Subst[Int[x^2/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c
*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1450
```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^2/2)*(b/q + 1)   Int[(d*x)^(m - 2)/(b/
2 + q/2 + c*x^2), x], x] - Simp[(d^2/2)*(b/q - 1)   Int[(d*x)^(m - 2)/(b/2
- q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] &&
 GeQ[m, 2]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

3.357.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.12

method	result
default	$\frac{e\sqrt{2}\left(\frac{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)\arctan\left(\frac{c\sqrt{e\,x^2+d}\,\sqrt{2}}{\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right)}{\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}-\frac{\left(-be+2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)\operatorname{arctanh}\left(\frac{c\sqrt{e\,x^2+d}\,\sqrt{2}}{\sqrt{\left(-be+2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right)}{\sqrt{\left(-be+2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right)}{2\sqrt{-e^2(4ac-b^2)}}$
pseudoelliptic	$\frac{e\sqrt{2}\left(\frac{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)\arctan\left(\frac{c\sqrt{e\,x^2+d}\,\sqrt{2}}{\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right)}{\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}-\frac{\left(-be+2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)\operatorname{arctanh}\left(\frac{c\sqrt{e\,x^2+d}\,\sqrt{2}}{\sqrt{\left(-be+2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right)}{\sqrt{\left(-be+2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right)}{2\sqrt{-e^2(4ac-b^2)}}$

input
```
int(x*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*e*2^(1/2)/(-e^2*(4*a*c-b^2))^(1/2)*((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2
))/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x^2+d)^(1/2)
*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))-(-b*e+2*c*d+(-e^2
*(4*a*c-b^2))^(1/2))/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arcta
nh(c*(e*x^2+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/
2)))
```

3.357.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1085 vs. $2(163) = 326$.

Time = 6.08 (sec) , antiderivative size = 1085, normalized size of antiderivative = 5.37

$$\int \frac{x\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx =$$

$$-\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{\frac{2\,cd-be+(b^2c-4\,ac^2)\sqrt{\frac{e^2}{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(\frac{be^2x^2+2\,bde-2\,ae^2+2\,\sqrt{\frac{1}{2}}\sqrt{ex^2+d}\Big((b^2-4\,ac)e\right.$$

$$+\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{\frac{2\,cd-be+(b^2c-4\,ac^2)\sqrt{\frac{e^2}{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(\frac{be^2x^2+2\,bde-2\,ae^2-2\,\sqrt{\frac{1}{2}}\sqrt{ex^2+d}\Big((b^2-4\,ac)e\right.$$

$$-\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{\frac{2\,cd-be-(b^2c-4\,ac^2)\sqrt{\frac{e^2}{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(\frac{be^2x^2+2\,bde-2\,ae^2+2\,\sqrt{\frac{1}{2}}\sqrt{ex^2+d}\Big((b^2-4\,ac)e\right.$$

$$+\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{\frac{2\,cd-be-(b^2c-4\,ac^2)\sqrt{\frac{e^2}{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(\frac{be^2x^2+2\,bde-2\,ae^2-2\,\sqrt{\frac{1}{2}}\sqrt{ex^2+d}\Big((b^2-4\,ac)e\right.$$

input
```
integrate(x*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
-1/4*sqrt(1/2)*sqrt((2*c*d - b*e + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4
*a*c^3)))/(b^2*c - 4*a*c^2))*log((b*e^2*x^2 + 2*b*d*e - 2*a*e^2 + 2*sqrt(1
/2)*sqrt(e*x^2 + d)*((b^2 - 4*a*c)*e + (b^3*c - 4*a*b*c^2)*sqrt(e^2/(b^2*c
^2 - 4*a*c^3)))*sqrt((2*c*d - b*e + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 -
4*a*c^3)))/(b^2*c - 4*a*c^2)) + ((b^2*c - 4*a*c^2)*e*x^2 + 2*(b^2*c - 4*a*
c^2)*d)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/x^2) + 1/4*sqrt(1/2)*sqrt((2*c*d -
b*e + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2))*
log((b*e^2*x^2 + 2*b*d*e - 2*a*e^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^2 - 4
*a*c)*e + (b^3*c - 4*a*b*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))*sqrt((2*c*d -
 b*e + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2))
 + ((b^2*c - 4*a*c^2)*e*x^2 + 2*(b^2*c - 4*a*c^2)*d)*sqrt(e^2/(b^2*c^2 - 4
*a*c^3)))/x^2) - 1/4*sqrt(1/2)*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2)*sqrt(
e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2))*log((b*e^2*x^2 + 2*b*d*e - 2*
a*e^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^2 - 4*a*c)*e - (b^3*c - 4*a*b*c^2)
*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2)*sqrt
(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2)) - ((b^2*c - 4*a*c^2)*e*x^2 +
 2*(b^2*c - 4*a*c^2)*d)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/x^2) + 1/4*sqrt(1/2
)*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^
2*c - 4*a*c^2))*log((b*e^2*x^2 + 2*b*d*e - 2*a*e^2 - 2*sqrt(1/2)*sqrt(e*x^
2 + d)*((b^2 - 4*a*c)*e - (b^3*c - 4*a*b*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c...
```

3.357.6 Sympy [F]

$$\int \frac{x\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \int \frac{x\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

input
```
integrate(x*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
```

output
```
Integral(x*sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

3.357.7 Maxima [F]

$$\int \frac{x\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \int \frac{\sqrt{ex^2+dx}}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x^2 + d)*x/(c*x^4 + b*x^2 + a), x)
```

3.357.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 449 vs. $2(163) = 326$.

Time = 0.31 (sec) , antiderivative size = 449, normalized size of antiderivative = 2.22

$$\int \frac{x\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

$$= \frac{\left(\sqrt{-4\,c^2d+2\left(bc-\sqrt{b^2-4\,ac}c\right)e}(b^2-4\,ac)e^3-(4\,c^2d^2e-4\,bcde^2+b^2e^3)\sqrt{-4\,c^2d+2\left(bc-\sqrt{b^2-4\,a}\right.}\right.}{8\left(\sqrt{b^2-4\,ac}c^2d^2-\sqrt{b^2-4\,ac}bcde+\sqrt{b^2-4\,ac}ace^2\right)|c|}$$

$$-\frac{\left(\sqrt{-4\,c^2d+2\left(bc+\sqrt{b^2-4\,ac}c\right)e}(b^2-4\,ac)e^3-(4\,c^2d^2e-4\,bcde^2+b^2e^3)\sqrt{-4\,c^2d+2\left(bc+\sqrt{b^2-}\right.}\right.}{8\left(\sqrt{b^2-4\,ac}c^2d^2-\sqrt{b^2-4\,ac}bcde+\sqrt{b^2-4\,ac}ace^2\right)}$$

input
```
integrate(x*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/8*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*(b^2 - 4*a*c)*e^3 -
(4*c^2*d^2*e - 4*b*c*d*e^2 + b^2*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 -
4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x^2 + d)/sqrt(-(2*c*d - b*e + sqrt
((2*c*d - b*e)^2 - 4*(c*d^2 - b*d*e + a*e^2)*c))/c))/((sqrt(b^2 - 4*a*c)*c
^2*d^2 - sqrt(b^2 - 4*a*c)*b*c*d*e + sqrt(b^2 - 4*a*c)*a*c*e^2)*abs(c)*abs
(e)) - 1/8*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*(b^2 - 4*a*c)
*e^3 - (4*c^2*d^2*e - 4*b*c*d*e^2 + b^2*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt
(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x^2 + d)/sqrt(-(2*c*d - b*e
 - sqrt((2*c*d - b*e)^2 - 4*(c*d^2 - b*d*e + a*e^2)*c))/c))/((sqrt(b^2 - 4
*a*c)*c^2*d^2 - sqrt(b^2 - 4*a*c)*b*c*d*e + sqrt(b^2 - 4*a*c)*a*c*e^2)*abs
(c)*abs(e))
```

3.357.9 Mupad [B] (verification not implemented)

Time = 8.60 (sec) , antiderivative size = 717, normalized size of antiderivative = 3.55

$$\int \frac{x\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx =$$

$$-2\,\mathrm{atanh}\left(\frac{2\left(\sqrt{e\,x^2+d}\,(-2\,b^2\,c\,e^4+4\,b\,c^2\,d\,e^3-4\,c^3\,d^2\,e^2+4\,a\,c^2\,e^4)+\frac{\sqrt{e\,x^2+d}\,(8\,b^3\,c^2\,e^3-16\,d\,b^2\,c^3\,e^2-32\,a\,b\,c^3}{}\right.}{2\,c^2\,d^2\,e^3-2\,b}\right.$$

$$-2\,\mathrm{atanh}\left(\frac{2\left(\sqrt{e\,x^2+d}\,(-2\,b^2\,c\,e^4+4\,b\,c^2\,d\,e^3-4\,c^3\,d^2\,e^2+4\,a\,c^2\,e^4)-\frac{\sqrt{e\,x^2+d}\,(8\,b^3\,c^2\,e^3-16\,d\,b^2\,c^3\,e^2-32\,a\,b\,c^3}{}\right.}{2\,c^2\,d^2\,e^3-2\,b\,c}\right.$$

input
```
int((x*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

output
```
- 2*atanh((2*((d + e*x^2)^(1/2)*(4*a*c^2*e^4 - 2*b^2*c*e^4 - 4*c^3*d^2*e^2
 + 4*b*c^2*d*e^3) + ((d + e*x^2)^(1/2)*(8*b^3*c^2*e^3 - 16*b^2*c^3*d*e^2 -
 32*a*b*c^3*e^3 + 64*a*c^4*d*e^2)*(b^3*e + e*(-(4*a*c - b^2)^3)^(1/2) + 8*
a*c^2*d - 2*b^2*c*d - 4*a*b*c*e))/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*
(-(b^3*e + e*(-(4*a*c - b^2)^3)^(1/2) + 8*a*c^2*d - 2*b^2*c*d - 4*a*b*c*e)
/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2))/(2*c^2*d^2*e^3 + 2*a*c*e^5
 - 2*b*c*d*e^4))*(-(b^3*e + e*(-(4*a*c - b^2)^3)^(1/2) + 8*a*c^2*d - 2*b^2
*c*d - 4*a*b*c*e)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2) - 2*atanh(
(2*((d + e*x^2)^(1/2)*(4*a*c^2*e^4 - 2*b^2*c*e^4 - 4*c^3*d^2*e^2 + 4*b*c^2
*d*e^3) - ((d + e*x^2)^(1/2)*(8*b^3*c^2*e^3 - 16*b^2*c^3*d*e^2 - 32*a*b*c^
3*e^3 + 64*a*c^4*d*e^2)*(e*(-(4*a*c - b^2)^3)^(1/2) - b^3*e - 8*a*c^2*d +
2*b^2*c*d + 4*a*b*c*e))/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((e*(-(4*a
*c - b^2)^3)^(1/2) - b^3*e - 8*a*c^2*d + 2*b^2*c*d + 4*a*b*c*e)/(8*(b^4*c
+ 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2))/(2*c^2*d^2*e^3 + 2*a*c*e^5 - 2*b*c*d*
e^4))*((e*(-(4*a*c - b^2)^3)^(1/2) - b^3*e - 8*a*c^2*d + 2*b^2*c*d + 4*a*b
*c*e)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2)
```

3.358 $\int \frac{\sqrt{d+ex^2}}{x(a+bx^2+cx^4)}\,dx$

3.358.1 Optimal result

Integrand size = 29, antiderivative size = 281

$$\int \frac{\sqrt{d+ex^2}}{x(a+bx^2+cx^4)}\,dx = -\frac{\sqrt{d}\,\mathrm{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a} + \frac{\sqrt{c}\left(bd+\sqrt{b^2-4ac}\,d-2ae\right)\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{\sqrt{c}\left(bd-\sqrt{b^2-4ac}\,d-2ae\right)\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

output
```
-arctanh((e*x^2+d)^(1/2)/d^(1/2))*d^(1/2)/a+1/2*arctanh(2^(1/2)*c^(1/2)*(e
*x^2+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*c^(1/2)*(b*d-2*a*e+d
*(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)
^(1/2)))^(1/2)-1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b+(-4
*a*c+b^2)^(1/2)))^(1/2))*c^(1/2)*(b*d-2*a*e-d*(-4*a*c+b^2)^(1/2))/a*2^(1/2
)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.358.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{d+ex^2}}{x\left(a+bx^2+cx^4\right)}\,dx =$$

$$-\frac{\frac{\sqrt{2}\sqrt{c}\left(bd+\sqrt{b^2-4ac}d-2ae\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-2cd+be-\sqrt{b^2-4ac}e}}\right)}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}}+\frac{\sqrt{2}\sqrt{c}\left(-bd+\sqrt{b^2-4ac}d+2ae\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}}+2\sqrt{d}}{2a}$$

input
```
Integrate[Sqrt[d + e*x^2]/(x*(a + b*x^2 + c*x^4)),x]
```

output
```
-1/2*((Sqrt[2]*Sqrt[c]*(b*d + Sqrt[b^2 - 4*a*c]*d - 2*a*e)*ArcTan[(Sqrt[2]
*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]])/(Sqrt
[b^2 - 4*a*c]*Sqrt[-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*Sqrt[c]
*(-(b*d) + Sqrt[b^2 - 4*a*c]*d + 2*a*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e
*x^2])/Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[
-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]) + 2*Sqrt[d]*ArcTanh[Sqrt[d + e*x^2]/S
qrt[d]])/a
```

3.358.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1578, 1199, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}}{x\left(a+bx^2+cx^4\right)}\,dx$$

$$\Big\downarrow \; 1578$$

$$\frac{1}{2}\int \frac{\sqrt{ex^2+d}}{x^2\left(cx^4+bx^2+a\right)}dx^2$$

$$\Big\downarrow \; 1199$$

$$\frac{\int \left(\frac{e\left(-cdx^4+cd^2+ae^2-bde\right)}{a\left(cx^8-(2cd-be)x^4+cd^2+ae^2-bde\right)}-\frac{de}{a\left(d-x^4\right)}\right)d\sqrt{ex^2+d}}{e}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\sqrt{c}e\left(d\sqrt{b^2-4ac}-2ae+bd\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{c}e\left(-d\sqrt{b^2-4ac}-2ae+bd\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\sqrt{d}e\text{arcta}}{}}{e}$$

input
```
Int[Sqrt[d + e*x^2]/(x*(a + b*x^2 + c*x^4)),x]
```

output
```
(-((Sqrt[d]*e*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/a) + (Sqrt[c]*e*(b*d + Sqr
t[b^2 - 4*a*c]*d - 2*a*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2
*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*
d - (b - Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[c]*e*(b*d - Sqrt[b^2 - 4*a*c]*d -
2*a*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^
2 - 4*a*c])*e]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e]))/e
```

3.358.3.1 Defintions of rubi rules used

rule 1199
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Denominator[m]}, Simp[q/e   Subs
t[Int[ExpandIntegrand[x^(q*(m + 1) - 1)*(((e*f - d*g)/e + g*(x^q/e))^n/((c*
d^2 - b*d*e + a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))), x],
 x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Integer
Q[n] && FractionQ[m]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.358.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.27

method	result
pseudoelliptic	$-\frac{\sqrt{2}\,\sqrt{\left(be-2cd+\sqrt{-4e^{2}\left(ac-\frac{b^{2}}{4}\right)}\right)c}\,c\left(a\,e^{2}-\frac{bde}{2}-\frac{\sqrt{-4e^{2}\left(ac-\frac{b^{2}}{4}\right)}\,d}{2}\right)\operatorname{arctanh}\left(\frac{c\sqrt{e\,x^{2}+d}\,\sqrt{2}}{\sqrt{\left(-be+2cd+\sqrt{-4e^{2}\left(ac-\frac{b^{2}}{4}\right)}\right)c}}\right)+\sqrt{(}}{\sqrt{\left(-b\right.}}$
default	$\frac{\sqrt{e\,x^{2}+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^{2}+d}}{x}\right)}{a}-\frac{\sqrt{2}\,\sqrt{\left(be-2cd+\sqrt{-4e^{2}\left(ac-\frac{b^{2}}{4}\right)}\right)c}\,c\left(a\,e^{2}-\frac{bde}{2}-\frac{\sqrt{-4e^{2}\left(ac-\frac{b^{2}}{4}\right)}\,d}{2}\right)\operatorname{arctanh}\left(\right.}{}$

input
```
int((e*x^2+d)^(1/2)/x/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-(2^(1/2)*((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*c*(a*e^2-1/2*
b*d*e-1/2*(-4*e^2*(a*c-1/4*b^2))^(1/2)*d)*arctanh(c*(e*x^2+d)^(1/2)*2^(1/2
)/((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))+((-b*e+2*c*d+(-4*e^
2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*(2^(1/2)*c*(1/2*(-4*e^2*(a*c-1/4*b^2))^(1
/2)*d+e*(a*e-1/2*b*d))*arctan(c*(e*x^2+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-4*e^
2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))+d^(1/2)*arctanh((e*x^2+d)^(1/2)/d^(1/2))
*(-4*e^2*(a*c-1/4*b^2))^(1/2)*((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)
^(1/2)))/((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)/((b*e-2*c*d+(
-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)/(-4*e^2*(a*c-1/4*b^2))^(1/2)/a
```

3.358.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1557 vs. 2(232) = 464.

Time = 49.74 (sec) , antiderivative size = 3126, normalized size of antiderivative = 11.12

$$\int \frac{\sqrt{d+ex^2}}{x\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^(1/2)/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
[-1/4*(sqrt(1/2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sq
rt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*
c))*log(-(2*b^2*d^2 - 4*a*b*d*e + 2*a^2*e^2 + (b^2*d*e - a*b*e^2)*x^2 + 4*
sqrt(1/2)*(a^3*b^2 - 4*a^4*c)*sqrt(e*x^2 + d)*sqrt((b^2*d^2 - 2*a*b*d*e +
a^2*e^2)/(a^4*b^2 - 4*a^5*c))*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 -
4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b
^2 - 4*a^3*c)) - ((a^2*b^2 - 4*a^3*c)*e*x^2 + 2*(a^2*b^2 - 4*a^3*c)*d)*sqr
t((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/x^2) - sqrt(1/2)*a
*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*
b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log(-(2*b^2*d^
2 - 4*a*b*d*e + 2*a^2*e^2 + (b^2*d*e - a*b*e^2)*x^2 - 4*sqrt(1/2)*(a^3*b^2
 - 4*a^4*c)*sqrt(e*x^2 + d)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2
- 4*a^5*c))*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2
*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - (
(a^2*b^2 - 4*a^3*c)*e*x^2 + 2*(a^2*b^2 - 4*a^3*c)*d)*sqrt((b^2*d^2 - 2*a*b
*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/x^2) - sqrt(1/2)*a*sqrt(-(a*b*e - (b
^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(
a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log(-(2*b^2*d^2 - 4*a*b*d*e + 2*
a^2*e^2 + (b^2*d*e - a*b*e^2)*x^2 + 4*sqrt(1/2)*(a^3*b^2 - 4*a^4*c)*sqrt(e
*x^2 + d)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c))*sqr...
```

3.358.6 Sympy [F]

$$\int \frac{\sqrt{d+ex^2}}{x(a+bx^2+cx^4)}\,dx = \int \frac{\sqrt{d+ex^2}}{x(a+bx^2+cx^4)}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)/x/(c*x**4+b*x**2+a),x)
```

output
```
Integral(sqrt(d + e*x**2)/(x*(a + b*x**2 + c*x**4)), x)
```

3.358.7 Maxima [F]

$$\int \frac{\sqrt{d+ex^2}}{x\left(a+bx^2+cx^4\right)}\,dx = \int \frac{\sqrt{ex^2+d}}{(cx^4+bx^2+a)x}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x), x)
```

3.358.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 724 vs. $2(232) = 464$.

Time = 0.31 (sec) , antiderivative size = 724, normalized size of antiderivative = 2.58

$$\int \frac{\sqrt{d+ex^2}}{x\left(a+bx^2+cx^4\right)}\,dx = \frac{d\arctan\left(\frac{\sqrt{ex^2+d}}{\sqrt{-d}}\right)}{a\sqrt{-d}}$$

$$-\frac{\left(\sqrt{-4\,c^2d+2\left(bc-\sqrt{b^2-4\,ac}c\right)e}(b^2-4\,ac)a^2de^2-2\left(\sqrt{b^2-4\,ac}acd^2-\sqrt{b^2-4\,ac}abde+\sqrt{b^2-4\,a\right.\right.}{}$$

$$+\frac{\left(\sqrt{-4\,c^2d+2\left(bc+\sqrt{b^2-4\,ac}c\right)e}(b^2-4\,ac)a^2de^2+2\left(\sqrt{b^2-4\,ac}acd^2-\sqrt{b^2-4\,ac}abde+\sqrt{b^2-4\,a\right.\right.}{}$$

input
```
integrate((e*x^2+d)^(1/2)/x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
d*arctan(sqrt(e*x^2 + d)/sqrt(-d))/(a*sqrt(-d)) - 1/8*(sqrt(-4*c^2*d + 2*(
b*c - sqrt(b^2 - 4*a*c)*c)*e)*(b^2 - 4*a*c)*a^2*d*e^2 - 2*(sqrt(b^2 - 4*a*
c)*a*c*d^2 - sqrt(b^2 - 4*a*c)*a*b*d*e + sqrt(b^2 - 4*a*c)*a^2*e^2)*sqrt(-
4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(a)*abs(e) - (2*a^2*b*c*d^2*
e + 2*a^3*b*e^3 - (a^2*b^2 + 4*a^3*c)*d*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt
(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x^2 + d)/sqrt(-(2*a*c*d - a
*b*e + sqrt(-4*(a*c*d^2 - a*b*d*e + a^2*e^2)*a*c + (2*a*c*d - a*b*e)^2))/(
a*c)))/((sqrt(b^2 - 4*a*c)*a^2*c*d^2 - sqrt(b^2 - 4*a*c)*a^2*b*d*e + sqrt(
b^2 - 4*a*c)*a^3*e^2)*abs(a)*abs(c)*abs(e)) + 1/8*(sqrt(-4*c^2*d + 2*(b*c
+ sqrt(b^2 - 4*a*c)*c)*e)*(b^2 - 4*a*c)*a^2*d*e^2 + 2*(sqrt(b^2 - 4*a*c)*a
*c*d^2 - sqrt(b^2 - 4*a*c)*a*b*d*e + sqrt(b^2 - 4*a*c)*a^2*e^2)*sqrt(-4*c^
2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(a)*abs(e) - (2*a^2*b*c*d^2*e +
2*a^3*b*e^3 - (a^2*b^2 + 4*a^3*c)*d*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2
 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x^2 + d)/sqrt(-(2*a*c*d - a*b*e
 - sqrt(-4*(a*c*d^2 - a*b*d*e + a^2*e^2)*a*c + (2*a*c*d - a*b*e)^2))/(a*c)
))/((sqrt(b^2 - 4*a*c)*a^2*c*d^2 - sqrt(b^2 - 4*a*c)*a^2*b*d*e + sqrt(b^2
- 4*a*c)*a^3*e^2)*abs(a)*abs(c)*abs(e))
```

3.358.9 Mupad [B] (verification not implemented)

Time = 11.49 (sec) , antiderivative size = 10964, normalized size of antiderivative = 39.02

$$\int \frac{\sqrt{d+ex^2}}{x\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^2)^(1/2)/(x*(a + b*x^2 + c*x^4)),x)
```

output
```
atan((((d + e*x^2)^(1/2)*(2*a^2*c^3*e^12 + 6*c^5*d^4*e^8 - 8*b*c^4*d^3*e^9
 + 4*b^2*c^3*d^2*e^10 - 4*a*b*c^3*d*e^11) + ((b^4*d + 8*a^2*c^2*d - a*b^3*
e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*
c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((((b^4
*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c -
 b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a
^3*b^2*c)))^(1/2)*((d + e*x^2)^(1/2)*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e
*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4
*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e
^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 6
4*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64
*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) - 192*a^4*c^4*d*e^10 - 192*a^3
*c^5*d^3*e^8 + 48*a^2*b^2*c^4*d^3*e^8 - 48*a^2*b^3*c^3*d^2*e^9 + 192*a^3*b
*c^4*d^2*e^9 + 48*a^3*b^2*c^3*d*e^10) - (d + e*x^2)^(1/2)*(32*a^3*b*c^3*e^
11 + 48*a^3*c^4*d*e^10 - 8*a^2*b^3*c^2*e^11 + 144*a^2*c^5*d^3*e^8 + 16*b^4
*c^3*d^3*e^8 - 16*b^5*c^2*d^2*e^9 + 16*a*b^4*c^2*d*e^10 - 96*a*b^2*c^4*d^3
*e^8 + 96*a*b^3*c^3*d^2*e^9 - 144*a^2*b*c^4*d^2*e^9 - 72*a^2*b^2*c^3*d*e^1
0))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(
-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*
c^2 - 8*a^3*b^2*c)))^(1/2) + 12*a*c^5*d^4*e^8 + 12*a^2*c^4*d^2*e^10 - 4...
```

3.358. $\int \frac{\sqrt{d+ex^2}}{x(a+bx^2+cx^4)}\,dx$

3.359 $\int \frac{\sqrt{d+ex^2}}{x^3(a+bx^2+cx^4)}\,dx$

3.359.1 Optimal result

Integrand size = 29, antiderivative size = 382

$$\begin{aligned}
&\int \frac{\sqrt{d+ex^2}}{x^3(a+bx^2+cx^4)}\,dx \\
&= -\frac{\sqrt{d+ex^2}}{2ax^2}+\frac{e\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2a\sqrt{d}}+\frac{(bd-ae)\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^2\sqrt{d}} \\
&\quad-\frac{\sqrt{c}\left(b^2d-2acd-abe+\sqrt{b^2-4ac}(bd-ae)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} \\
&\quad+\frac{\sqrt{c}\left(b^2d-b\left(\sqrt{b^2-4ac}d+ae\right)-a\left(2cd-\sqrt{b^2-4ac}e\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}
\end{aligned}$$

output
```
1/2*e*arctanh((e*x^2+d)^(1/2)/d^(1/2))/a/d^(1/2)+(-a*e+b*d)*arctanh((e*x^2
+d)^(1/2)/d^(1/2))/a^2/d^(1/2)-1/2*(e*x^2+d)^(1/2)/a/x^2-1/2*arctanh(2^(1/
2)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*c^(1/2)
*(b^2*d-2*a*c*d-a*b*e+(-a*e+b*d)*(-4*a*c+b^2)^(1/2))/a^2*2^(1/2)/(-4*a*c+b
^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+1/2*arctanh(2^(1/2)*c^(1/
2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*c^(1/2)*(b^2*d-
2*a*c*d-a*b*e-(-a*e+b*d)*(-4*a*c+b^2)^(1/2))/a^2*2^(1/2)/(-4*a*c+b^2)^(1/2
)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.359.2 Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{d+ex^2}}{x^3\left(a+bx^2+cx^4\right)}\,dx$$
$$= \frac{-\frac{a\sqrt{d+ex^2}}{x^2} + \frac{\sqrt{2}\sqrt{c}\left(b^2d-2acd+b\sqrt{b^2-4ac}d-abe-a\sqrt{b^2-4ac}e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-2cd+be-\sqrt{b^2-4ac}e}}\right)}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}} + \frac{\sqrt{2}\sqrt{c}\left(-b^2d+2acd+b\sqrt{b^2-4ac}d+abe-\cdots\right.}{\sqrt{b^2-4ac}\sqrt{-\cdots}}}{2a^2}$$

input
```
Integrate[Sqrt[d + e*x^2]/(x^3*(a + b*x^2 + c*x^4)),x]
```

output
```
(-((a*Sqrt[d + e*x^2])/x^2) + (Sqrt[2]*Sqrt[c]*(b^2*d - 2*a*c*d + b*Sqrt[b
^2 - 4*a*c]*d - a*b*e - a*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqr
t[d + e*x^2])/Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 - 4*a*c
]*Sqrt[-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*Sqrt[c]*(-(b^2*d) +
 2*a*c*d + b*Sqrt[b^2 - 4*a*c]*d + a*b*e - a*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(
Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]]
)/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]) + ((2*b*d -
 a*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/Sqrt[d])/(2*a^2)
```

3.359.3 Rubi [A] (warning: unable to verify)

Time = 2.38 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1578, 1199, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}}{x^3\left(a+bx^2+cx^4\right)}\,dx$$
$$\Big\downarrow \; 1578$$
$$\frac{1}{2}\int \frac{\sqrt{ex^2+d}}{x^4\left(cx^4+bx^2+a\right)}dx^2$$
$$\Big\downarrow \; 1199$$

$$\frac{\int \left(\frac{de^2}{a(d-x^4)^2}+\frac{(bd-ae)e}{a^2(d-x^4)}-\frac{(b(cd^2-bed+ae^2)-c(bd-ae)x^4)e}{a^2(cx^8-(2cd-be)x^4+cd^2+ae^2-bde)}\right)d\sqrt{ex^2+d}}{e}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{\sqrt{c}e\left(\sqrt{b^2-4ac}(bd-ae)-abe-2acd+b^2d\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{c}e\left(-\sqrt{b^2-4ac}(bd-ae)-abe-2acd+b^2d\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\ldots}{\sqrt{2cd-\ldots}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}{e}$$

input
```
Int[Sqrt[d + e*x^2]/(x^3*(a + b*x^2 + c*x^4)),x]
```

output
```
((e^2*Sqrt[d + e*x^2])/(2*a*(d - x^4)) + (e^2*ArcTanh[Sqrt[d + e*x^2]/Sqrt
[d]])/(2*a*Sqrt[d]) + (e*(b*d - a*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(a^
2*Sqrt[d]) - (Sqrt[c]*e*(b^2*d - 2*a*c*d - a*b*e + Sqrt[b^2 - 4*a*c]*(b*d
- a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b
^2 - 4*a*c])*e]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^
2 - 4*a*c])*e]) + (Sqrt[c]*e*(b^2*d - 2*a*c*d - a*b*e - Sqrt[b^2 - 4*a*c]*
(b*d - a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + S
qrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sq
rt[b^2 - 4*a*c])*e]))/e
```

3.359.3.1 Defintions of rubi rules used

rule 1199
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Denominator[m]}, Simp[q/e   Subs
t[Int[ExpandIntegrand[x^(q*(m + 1) - 1)*(((e*f - d*g)/e + g*(x^q/e))^n/((c*
d^2 - b*d*e + a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))), x],
 x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Integer
Q[n] && FractionQ[m]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.359.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.95

method	result
risch	$-\frac{\sqrt{e\,x^2+d}}{2a\,x^2}-\frac{-\frac{(-ae+2bd)\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^2+d}}{x}\right)}{a\sqrt{d}}+\frac{c\sqrt{2}\left(-\frac{\left(ab\,e^2+2acde-b^2de+\sqrt{-e^2\left(4ac-b^2\right)}\,ae-\sqrt{-e^2\left(4ac-b^2\right)}\,bd\right)\operatorname{arctanh}}{\sqrt{\left(-be+2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right.}{}}{}$
pseudoelliptic	$\frac{\sqrt{2}\sqrt{\left(be-2cd+\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}\right)c}\,c\,x^2\left(\left(\frac{\left(ea\sqrt{d}-d^{\frac{3}{2}}b\right)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}}{2}+e\left(\left(ac-\frac{b^2}{2}\right)d^{\frac{3}{2}}+\frac{bea\sqrt{d}}{2}\right)\right)\operatorname{arctanh}\left(\frac{c}{\sqrt{\left(-be+2c\right.}}\right.\right.}{}$
default	$\frac{-\frac{\left(e\,x^2+d\right)^{\frac{3}{2}}}{2d\,x^2}+\frac{e\left(\sqrt{e\,x^2+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^2+d}}{x}\right)\right)}{2d}}{a}-\frac{b\left(\sqrt{e\,x^2+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^2+d}}{x}\right)\right)}{a^2}-\frac{-\sqrt{2}\sqrt{\left(be-2cd+\right.}}{}$

input
```
int((e*x^2+d)^(1/2)/x^3/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(e*x^2+d)^(1/2)/a/x^2-1/2/a*(-(-a*e+2*b*d)/a/d^(1/2)*ln((2*d+2*d^(1/2
)*(e*x^2+d)^(1/2))/x)+1/a*c*2^(1/2)/(-e^2*(4*a*c-b^2))^(1/2)*(-(a*b*e^2+2*
a*c*d*e-b^2*d*e+(-e^2*(4*a*c-b^2))^(1/2)*a*e-(-e^2*(4*a*c-b^2))^(1/2)*b*d)
/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x^2+d)^(1/2)
*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))+(-a*b*e^2-2*a*c*
d*e+b^2*d*e+(-e^2*(4*a*c-b^2))^(1/2)*a*e-(-e^2*(4*a*c-b^2))^(1/2)*b*d)/((b
*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x^2+d)^(1/2)*2^(1/
2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))))
```

3.359.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3290 vs. 2(309) = 618.

Time = 260.18 (sec) , antiderivative size = 6592, normalized size of antiderivative = 17.26

$$\int \frac{\sqrt{d+ex^2}}{x^3 (a+bx^2+cx^4)} \, dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^(1/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.359.6 Sympy [F]

$$\int \frac{\sqrt{d+ex^2}}{x^3 (a+bx^2+cx^4)} \, dx = \int \frac{\sqrt{d+ex^2}}{x^3 (a+bx^2+cx^4)} \, dx$$

input
```
integrate((e*x**2+d)**(1/2)/x**3/(c*x**4+b*x**2+a),x)
```

output
```
Integral(sqrt(d + e*x**2)/(x**3*(a + b*x**2 + c*x**4)), x)
```

3.359.7 Maxima [F]

$$\int \frac{\sqrt{d+ex^2}}{x^3 (a+bx^2+cx^4)} \, dx = \int \frac{\sqrt{ex^2+d}}{(cx^4+bx^2+a)x^3} \, dx$$

input
```
integrate((e*x^2+d)^(1/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^3), x)
```

3.359.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 784 vs. $2(309) = 618$.

Time = 0.33 (sec) , antiderivative size = 784, normalized size of antiderivative = 2.05

$$\int \frac{\sqrt{d+ex^2}}{x^3(a+bx^2+cx^4)}\,dx = -\frac{(2\,bd-ae)\arctan\left(\frac{\sqrt{ex^2+d}}{\sqrt{-d}}\right)}{2\,a^2\sqrt{-d}}$$

$$+\frac{\left(\sqrt{-4\,c^2d+2\,(bc-\sqrt{b^2-4\,ac}c)e}((b^3-4\,abc)d-(ab^2-4\,a^2c)e)e^2-2\,(\sqrt{b^2-4\,ac}bcd^2-\sqrt{b^2-4\,ac}\right.}{}$$

$$-\frac{\left(\sqrt{-4\,c^2d+2\,(bc+\sqrt{b^2-4\,ac}c)e}((b^3-4\,abc)d-(ab^2-4\,a^2c)e)e^2+2\,(\sqrt{b^2-4\,ac}bcd^2-\sqrt{b^2-4\,ac}\right.}{}$$

$$-\frac{\sqrt{ex^2+d}}{2\,ax^2}$$

input
```
integrate((e*x^2+d)^(1/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*(2*b*d - a*e)*arctan(sqrt(e*x^2 + d)/sqrt(-d))/(a^2*sqrt(-d)) + 1/8*(
sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^3 - 4*a*b*c)*d - (a*b
^2 - 4*a^2*c)*e)*e^2 - 2*(sqrt(b^2 - 4*a*c)*b*c*d^2 - sqrt(b^2 - 4*a*c)*b^
2*d*e + sqrt(b^2 - 4*a*c)*a*b*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a
*c)*c)*e)*abs(e) + (b^3*d*e^2 - a*b^2*e^3 - 2*(b^2*c - 2*a*c^2)*d^2*e)*sqr
t(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x
^2 + d)/sqrt(-(2*a^2*c*d - a^2*b*e + sqrt(-4*(a^2*c*d^2 - a^2*b*d*e + a^3*
e^2)*a^2*c + (2*a^2*c*d - a^2*b*e)^2))/(a^2*c)))/((sqrt(b^2 - 4*a*c)*a^2*c
*d^2 - sqrt(b^2 - 4*a*c)*a^2*b*d*e + sqrt(b^2 - 4*a*c)*a^3*e^2)*abs(c)*abs
(e)) - 1/8*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*((b^3 - 4*a*b
*c)*d - (a*b^2 - 4*a^2*c)*e)*e^2 + 2*(sqrt(b^2 - 4*a*c)*b*c*d^2 - sqrt(b^2
 - 4*a*c)*b^2*d*e + sqrt(b^2 - 4*a*c)*a*b*e^2)*sqrt(-4*c^2*d + 2*(b*c + sq
rt(b^2 - 4*a*c)*c)*e)*abs(e) + (b^3*d*e^2 - a*b^2*e^3 - 2*(b^2*c - 2*a*c^2
)*d^2*e)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1
/2)*sqrt(e*x^2 + d)/sqrt(-(2*a^2*c*d - a^2*b*e - sqrt(-4*(a^2*c*d^2 - a^2*
b*d*e + a^3*e^2)*a^2*c + (2*a^2*c*d - a^2*b*e)^2))/(a^2*c)))/((sqrt(b^2 -
4*a*c)*a^2*c*d^2 - sqrt(b^2 - 4*a*c)*a^2*b*d*e + sqrt(b^2 - 4*a*c)*a^3*e^2
)*abs(c)*abs(e)) - 1/2*sqrt(e*x^2 + d)/(a*x^2)
```

3.359.9 Mupad [B] (verification not implemented)

Time = 10.83 (sec) , antiderivative size = 19959, normalized size of antiderivative = 52.25

$$\int \frac{\sqrt{d+ex^2}}{x^3 (a+bx^2+cx^4)} \, dx = \text{Too large to display}$$

input
```
int((d + e*x^2)^(1/2)/(x^3*(a + b*x^2 + c*x^4)),x)
```

output
```
(atan((((a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*
e^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 1
8*a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/(2*a^4)
- (((16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4*b^3*
c^3*e^12 + 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^4*d^
3*e^9 - 27*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*
d^2*e^10 - 8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e^1
0 - 3*a^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 -
36*a^4*b*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11)/a^4 - ((a*e - 2*b*d)*(((d +
 e*x^2)^(1/2)*(240*a^6*b*c^4*e^11 + 64*a^6*c^5*d*e^10 + 20*a^4*b^5*c^2*e^1
1 - 140*a^5*b^3*c^3*e^11 + 160*a^5*c^6*d^3*e^8 - 32*a^2*b^6*c^3*d^3*e^8 +
32*a^2*b^7*c^2*d^2*e^9 + 224*a^3*b^4*c^4*d^3*e^8 - 208*a^3*b^5*c^3*d^2*e^9
 - 432*a^4*b^2*c^5*d^3*e^8 + 272*a^4*b^3*c^4*d^2*e^9 - 48*a^3*b^6*c^2*d*e^
10 + 348*a^4*b^4*c^3*d*e^10 + 224*a^5*b*c^5*d^2*e^9 - 648*a^5*b^2*c^4*d*e^
10))/(2*a^4) - ((a*e - 2*b*d)*((128*a^8*c^4*e^11 + 8*a^6*b^4*c^2*e^11 - 64
*a^7*b^2*c^3*e^11 + 128*a^7*c^5*d^2*e^9 + 32*a^5*b^3*c^4*d^3*e^8 - 24*a^5*
b^4*c^3*d^2*e^9 + 64*a^6*b^2*c^4*d^2*e^9 - 256*a^7*b*c^4*d*e^10 - 8*a^5*b^
5*c^2*d*e^10 - 128*a^6*b*c^5*d^3*e^8 + 96*a^6*b^3*c^3*d*e^10)/a^4 - ((d +
e*x^2)^(1/2)*(a*e - 2*b*d)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*
a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896...
```

3.360 $\int \frac{\sqrt{d+ex^2}}{x^5\left(a+bx^2+cx^4\right)}\,dx$

3.360.1 Optimal result

Integrand size = 29, antiderivative size = 552

$$\int \frac{\sqrt{d+ex^2}}{x^5\left(a+bx^2+cx^4\right)}\,dx$$

$$= -\frac{\sqrt{d+ex^2}}{4ax^4}+\frac{3e\sqrt{d+ex^2}}{8adx^2}+\frac{(bd-ae)\sqrt{d+ex^2}}{2a^2dx^2}-\frac{3e^2\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{8ad^{3/2}}$$

$$-\frac{e(bd-ae)\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2a^2d^{3/2}}-\frac{\left(b^2d-acd-abe\right)\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^3\sqrt{d}}$$

$$+\frac{\sqrt{c}\left(b^3d-ac\left(\sqrt{b^2-4ac}\,d-2ae\right)+b^2\left(\sqrt{b^2-4ac}\,d-ae\right)-ab\left(3cd+\sqrt{b^2-4ac}\,e\right)\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{\cdots}}{\sqrt{2cd-\left(b-\cdots\right.}}\right.}{\sqrt{2}a^3\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

$$-\frac{\sqrt{c}\left(b^3d-b^2\left(\sqrt{b^2-4ac}\,d+ae\right)+ac\left(\sqrt{b^2-4ac}\,d+2ae\right)-ab\left(3cd-\sqrt{b^2-4ac}\,e\right)\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{\cdots}}{\sqrt{2cd-\left(b+\cdots\right.}}\right.}{\sqrt{2}a^3\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

output
```
-3/8*e^2*arctanh((e*x^2+d)^(1/2)/d^(1/2))/a/d^(3/2)-1/2*e*(-a*e+b*d)*arcta
nh((e*x^2+d)^(1/2)/d^(1/2))/a^2/d^(3/2)-(-a*b*e-a*c*d+b^2*d)*arctanh((e*x^
2+d)^(1/2)/d^(1/2))/a^3/d^(1/2)-1/4*(e*x^2+d)^(1/2)/a/x^4+3/8*e*(e*x^2+d)^
(1/2)/a/d/x^2+1/2*(-a*e+b*d)*(e*x^2+d)^(1/2)/a^2/d/x^2+1/2*arctanh(2^(1/2)
*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*c^(1/2)*(
b^3*d-a*c*(-2*a*e+d*(-4*a*c+b^2)^(1/2))+b^2*(-a*e+d*(-4*a*c+b^2)^(1/2))-a*
b*(3*c*d+e*(-4*a*c+b^2)^(1/2)))/a^3*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b
-(-4*a*c+b^2)^(1/2)))^(1/2)-1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+d)^(1/2)/(2
*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*c^(1/2)*(b^3*d-b^2*(a*e+d*(-4*a*c+b^
2)^(1/2))+a*c*(2*a*e+d*(-4*a*c+b^2)^(1/2))-a*b*(3*c*d-e*(-4*a*c+b^2)^(1/2)
))/a^3*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.360.2 Mathematica [A] (verified)

Time = 1.70 (sec) , antiderivative size = 445, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{d+ex^2}}{x^5\left(a+bx^2+cx^4\right)}\,dx$$

$$= \frac{\frac{a\sqrt{d+ex^2}\left(4bdx^2-a\left(2d+ex^2\right)\right)}{dx^4} + \frac{4\sqrt{2}\sqrt{c}\left(-b^3d+ac\left(\sqrt{b^2-4ac}d-2ae\right)+b^2\left(-\sqrt{b^2-4ac}d+ae\right)+ab\left(3cd+\sqrt{b^2-4ac}e\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be-\sqrt{b^2}}}\right.}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}}}{}$$

input
```
Integrate[Sqrt[d + e*x^2]/(x^5*(a + b*x^2 + c*x^4)),x]
```

output
```
((a*Sqrt[d + e*x^2]*(4*b*d*x^2 - a*(2*d + e*x^2)))/(d*x^4) + (4*Sqrt[2]*Sq
rt[c]*(-(b^3*d) + a*c*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + b^2*(-(Sqrt[b^2 - 4*
a*c]*d) + a*e) + a*b*(3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c
]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 -
4*a*c]*Sqrt[-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e]) + (4*Sqrt[2]*Sqrt[c]*(b^3
*d - b^2*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*c*(Sqrt[b^2 - 4*a*c]*d + 2*a*e) +
 a*b*(-3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^
2])/Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*
c*d + (b + Sqrt[b^2 - 4*a*c])*e]) + ((-8*b^2*d^2 + 4*a*b*d*e + a*(8*c*d^2
+ a*e^2))*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/d^(3/2))/(8*a^3)
```

3.360.3 Rubi [A] (warning: unable to verify)

Time = 2.15 (sec) , antiderivative size = 587, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1578, 1199, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}}{x^5\left(a+bx^2+cx^4\right)}\,dx$$

$$\Big\downarrow \ \color{blue}{1578}$$

$$\frac{1}{2}\int \frac{\sqrt{ex^2+d}}{x^6\left(cx^4+bx^2+a\right)}dx^2$$

$$\Big\downarrow \ \color{blue}{1199}$$

$$\frac{\int \left(-\frac{de^3}{a(d-x^4)^3}-\frac{(bd-ae)e^2}{a^2(d-x^4)^2}-\frac{(db^2-aeb-acd)e}{a^3(d-x^4)}+\frac{\left((b^2-ac)\left(cd^2-bed+ae^2\right)-c(db^2-aeb-acd)x^4\right)e}{a^3\left(cx^8-(2cd-be)x^4+cd^2+ae^2-bde\right)}\right)d\sqrt{ex^2+d}}{e}$$

$$\Big\downarrow \ \color{blue}{2009}$$

$$-\frac{e\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\left(-abe-acd+b^2d\right)}{a^3\sqrt{d}}+\frac{\sqrt{c}e\left(b^2\left(d\sqrt{b^2-4ac}-ae\right)-ab\left(e\sqrt{b^2-4ac}+3cd\right)-ac\left(d\sqrt{b^2-4ac}-2ae\right)+b^3d\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{}}{\sqrt{2cd-e(}}\right.}{\sqrt{2}a^3\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

input
```
Int[Sqrt[d + e*x^2]/(x^5*(a + b*x^2 + c*x^4)),x]
```

output
```
(-1/4*(e^3*Sqrt[d + e*x^2])/(a*(d - x^4)^2) - (3*e^3*Sqrt[d + e*x^2])/(8*a
*d*(d - x^4)) - (e^2*(b*d - a*e)*Sqrt[d + e*x^2])/(2*a^2*d*(d - x^4)) - (3
*e^3*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(8*a*d^(3/2)) - (e^2*(b*d - a*e)*Ar
cTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(2*a^2*d^(3/2)) - (e*(b^2*d - a*c*d - a*b*
e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(a^3*Sqrt[d]) + (Sqrt[c]*e*(b^3*d - a
*c*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + b^2*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*b*(
3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sq
rt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a^3*Sqrt[b^2 - 4*a*c]*Sqr
t[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[c]*e*(b^3*d - b^2*(Sqrt[b^2
- 4*a*c]*d + a*e) + a*c*(Sqrt[b^2 - 4*a*c]*d + 2*a*e) - a*b*(3*c*d - Sqrt[
b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b
 + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a^3*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b
+ Sqrt[b^2 - 4*a*c])*e]))/e
```

3.360.3.1 Defintions of rubi rules used

rule 1199
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Denominator[m]}, Simp[q/e   Subs
t[Int[ExpandIntegrand[x^(q*(m + 1) - 1)*(((e*f - d*g)/e + g*(x^q/e))^n/((c*
d^2 - b*d*e + a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))), x],
 x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Integer
Q[n] && FractionQ[m]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.360.4 Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 463, normalized size of antiderivative = 0.84

method	result
risch	$-\frac{\sqrt{e x^2+d}\,(a e x^2-4 b d x^2+2 d a)}{8 a^2 x^4 d}-\frac{-\frac{(e^2 a^2+4abde+8d^2ac-8b^2d^2)\ln\left(\frac{2d+2\sqrt{d}\sqrt{e x^2+d}}{x}\right)}{a\sqrt{d}}-\frac{8d\sqrt{2}\,c\left(\left(\left(\frac{(-acd-b(ae-bd))\sqrt{-4e^2 \cdots}}{2}\cdots\right.\right.\right.}{\cdots}}{\cdots}$
pseudoelliptic	$-\frac{-8\sqrt{2}\,c\sqrt{\left(be-2cd+\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}\right)c}\,x^4\left(\frac{\left(-a d^{\frac{3}{2}} b e-d^{\frac{5}{2}}(ac-b^2)\right)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}}{2}+e\left(ae\left(ac-\frac{b^2}{2}\right)d^{\frac{3}{2}}-\frac{3d^{\frac{5}{2}}b\left(ac-\frac{b^2}{3}\cdots\right.}{2}\cdots\right.\right.}{\cdots}$
default	$\frac{-\frac{(e x^2+d)^{\frac{3}{2}}}{4 d x^4}-\frac{e\left(-\frac{(e x^2+d)^{\frac{3}{2}}}{2 d x^2}+\frac{e\left(\sqrt{e x^2+d}-\sqrt{d}\ln\left(\frac{2d+2\sqrt{d}\sqrt{e x^2+d}}{x}\right)\right)}{2d}\right)}{4d}}{a}-\frac{b\left(-\frac{(e x^2+d)^{\frac{3}{2}}}{2 d x^2}+\frac{e\left(\sqrt{e x^2+d}-\sqrt{d}\ln\left(\frac{2d+2\sqrt{d}\cdots}{x}\right)\cdots\right.}{2d}\cdots\right.}{a^2}\cdots$

input
```
int((e*x^2+d)^(1/2)/x^5/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(e*x^2+d)^(1/2)*(a*e*x^2-4*b*d*x^2+2*a*d)/a^2/x^4/d-1/8/a^2/d*(-(a^2*
e^2+4*a*b*d*e+8*a*c*d^2-8*b^2*d^2)/a/d^(1/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(
1/2))/x)-8*d/a/((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*2^(1/2)*
c/((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*((1/2*(-a*c*d-b*(a*e
-b*d))*(-4*e^2*(a*c-1/4*b^2))^(1/2)+e*(a*(a*e-3/2*b*d)*c-1/2*b^2*(a*e-b*d)
))*((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x^2+d)^
(1/2)*2^(1/2)/((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))+arctan(
c*(e*x^2+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/
2))*((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*(1/2*(a*c*d+b*(a*e
-b*d))*(-4*e^2*(a*c-1/4*b^2))^(1/2)+e*(a*(a*e-3/2*b*d)*c-1/2*b^2*(a*e-b*d)
)))/(-4*e^2*(a*c-1/4*b^2))^(1/2))
```

3.360.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}}{x^5(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(1/2)/x^5/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.360.6 Sympy [F]

$$\int \frac{\sqrt{d+ex^2}}{x^5(a+bx^2+cx^4)}\,dx = \int \frac{\sqrt{d+ex^2}}{x^5(a+bx^2+cx^4)}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)/x**5/(c*x**4+b*x**2+a),x)
```

output
```
Integral(sqrt(d + e*x**2)/(x**5*(a + b*x**2 + c*x**4)), x)
```

3.360.7 Maxima [F]

$$\int \frac{\sqrt{d+ex^2}}{x^5(a+bx^2+cx^4)}\,dx = \int \frac{\sqrt{ex^2+d}}{(cx^4+bx^2+a)x^5}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/x^5/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^5), x)
```

3.360.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1057 vs. $2(468) = 936$.

Time = 0.35 (sec) , antiderivative size = 1057, normalized size of antiderivative = 1.91

$$\int \frac{\sqrt{d+ex^2}}{x^5(a+bx^2+cx^4)}\,dx =$$

$$-\frac{\left(\sqrt{-4\,c^2d+2\,\left(bc-\sqrt{b^2-4\,ac}c\right)e}((b^4-5\,ab^2c+4\,a^2c^2)d-(ab^3-4\,a^2bc)e)a^2e^2-2\,\left((ab^2c-a^2c^2)\sqrt{\right.}\right.}{}$$

$$+\frac{\left(\sqrt{-4\,c^2d+2\,\left(bc+\sqrt{b^2-4\,ac}c\right)e}((b^4-5\,ab^2c+4\,a^2c^2)d-(ab^3-4\,a^2bc)e)a^2e^2+2\,\left((ab^2c-a^2c^2)\sqrt{\right.}\right.}{}$$

$$+\frac{(8\,b^2d^2-8\,acd^2-4\,abde-a^2e^2)\arctan\left(\frac{\sqrt{ex^2+d}}{\sqrt{-d}}\right)}{8\,a^3\sqrt{-d}d}$$

$$+\frac{4\,(ex^2+d)^{\frac{3}{2}}bde-4\,\sqrt{ex^2+d}bd^2e-(ex^2+d)^{\frac{3}{2}}ae^2-\sqrt{ex^2+d}ade^2}{8\,a^2de^2x^4}$$

input
```
integrate((e*x^2+d)^(1/2)/x^5/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
-1/8*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^4 - 5*a*b^2*c +
 4*a^2*c^2)*d - (a*b^3 - 4*a^2*b*c)*e)*a^2*e^2 - 2*((a*b^2*c - a^2*c^2)*sq
rt(b^2 - 4*a*c)*d^2 - (a*b^3 - a^2*b*c)*sqrt(b^2 - 4*a*c)*d*e + (a^2*b^2 -
 a^3*c)*sqrt(b^2 - 4*a*c)*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*
c)*e)*abs(a)*abs(e) - (2*(a^2*b^3*c - 3*a^3*b*c^2)*d^2*e - (a^2*b^4 - a^3*
b^2*c - 4*a^4*c^2)*d*e^2 + (a^3*b^3 - 2*a^4*b*c)*e^3)*sqrt(-4*c^2*d + 2*(b
*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x^2 + d)/sqrt(-(2*
a^3*c*d - a^3*b*e + sqrt(-4*(a^3*c*d^2 - a^3*b*d*e + a^4*e^2)*a^3*c + (2*a
^3*c*d - a^3*b*e)^2))/(a^3*c)))/((sqrt(b^2 - 4*a*c)*a^4*c*d^2 - sqrt(b^2 -
 4*a*c)*a^4*b*d*e + sqrt(b^2 - 4*a*c)*a^5*e^2)*abs(a)*abs(c)*abs(e)) + 1/8
*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*((b^4 - 5*a*b^2*c + 4*a
^2*c^2)*d - (a*b^3 - 4*a^2*b*c)*e)*a^2*e^2 + 2*((a*b^2*c - a^2*c^2)*sqrt(b
^2 - 4*a*c)*d^2 - (a*b^3 - a^2*b*c)*sqrt(b^2 - 4*a*c)*d*e + (a^2*b^2 - a^3
*c)*sqrt(b^2 - 4*a*c)*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e
)*abs(a)*abs(e) - (2*(a^2*b^3*c - 3*a^3*b*c^2)*d^2*e - (a^2*b^4 - a^3*b^2*
c - 4*a^4*c^2)*d*e^2 + (a^3*b^3 - 2*a^4*b*c)*e^3)*sqrt(-4*c^2*d + 2*(b*c +
 sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x^2 + d)/sqrt(-(2*a^3*
c*d - a^3*b*e - sqrt(-4*(a^3*c*d^2 - a^3*b*d*e + a^4*e^2)*a^3*c + (2*a^3*c
*d - a^3*b*e)^2))/(a^3*c)))/((sqrt(b^2 - 4*a*c)*a^4*c*d^2 - sqrt(b^2 - 4*a
*c)*a^4*b*d*e + sqrt(b^2 - 4*a*c)*a^5*e^2)*abs(a)*abs(c)*abs(e)) + 1/8*...
```

3.360.9 Mupad [B] (verification not implemented)

Time = 11.97 (sec) , antiderivative size = 33925, normalized size of antiderivative = 61.46

$$\int \frac{\sqrt{d+ex^2}}{x^5\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^2)^(1/2)/(x^5*(a + b*x^2 + c*x^4)),x)
```

output

```
atan((((((2048*a^12*c^4*d*e^12 + 12288*a^10*c^6*d^5*e^8 + 14336*a^11*c^5*
d^3*e^10 + 2048*a^8*b^4*c^4*d^5*e^8 - 1536*a^8*b^5*c^3*d^4*e^9 - 512*a^8*b
^6*c^2*d^3*e^10 - 11264*a^9*b^2*c^5*d^5*e^8 + 7168*a^9*b^3*c^4*d^4*e^9 + 6
272*a^9*b^4*c^3*d^3*e^10 + 384*a^9*b^5*c^2*d^2*e^11 - 20480*a^10*b^2*c^4*d
^3*e^10 - 3072*a^10*b^3*c^3*d^2*e^11 - 4096*a^10*b*c^5*d^4*e^9 + 128*a^10*
b^4*c^2*d*e^12 + 6144*a^11*b*c^4*d^2*e^11 - 1024*a^11*b^2*c^3*d*e^12)/(64*
a^8*d^2) - ((d + e*x^2)^(1/2)*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2
)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^
2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c
 - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c -
 b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-
(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(2
4576*a^12*c^5*d^4*e^8 + 16384*a^13*c^4*d^2*e^10 + 2048*a^10*b^4*c^3*d^4*e^
8 - 2048*a^10*b^5*c^2*d^3*e^9 - 14336*a^11*b^2*c^4*d^4*e^8 + 15360*a^11*b^
3*c^3*d^3*e^9 + 1024*a^11*b^4*c^2*d^2*e^10 - 8192*a^12*b^2*c^3*d^2*e^10 -
28672*a^12*b*c^4*d^3*e^9))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(
4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25
*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4
*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d
*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*...
```

3.360. $\int \frac{\sqrt{d+ex^2}}{x^5(a+bx^2+cx^4)}\,dx$

3.361 $\int \frac{x^4\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

3.361.1 Optimal result

Integrand size = 29, antiderivative size = 390

$$\int \frac{x^4\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \frac{x\sqrt{d+ex^2}}{2c} - \frac{\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{\left(bcd-b^2e+ace+\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^2\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}} + \frac{(cd-2be)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^2\sqrt{e}}$$

output
```
1/2*(-2*b*e+c*d)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/c^2/e^(1/2)+1/2*x*(e*x
^2+d)^(1/2)/c-arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1
/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b*c*d-b^2*e+a*c*e+(-3*a*b*c*e+2*a*c^2*d
+b^3*e-b^2*c*d)/(-4*a*c+b^2)^(1/2))/c^2/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(
1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))
)^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b*c*d-b^2*e+a*c*e+(
3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)/(-4*a*c+b^2)^(1/2))/c^2/(b+(-4*a*c+b^2)
^(1/2))^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.361.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.01 (sec) , antiderivative size = 786, normalized size of antiderivative = 2.02

$$\int \frac{x^4\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$
$$= \frac{2cx\sqrt{d+ex^2}+\frac{4(cd-2be)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{-\sqrt{d}+\sqrt{d+ex^2}}\right)}{\sqrt{e}}+\operatorname{RootSum}\Big[ae^4+4bde^2\#1^2-4ae^3\#1^2+16cd^2\#1^4-8bd\ldots}{}$$

input
```
Integrate[(x^4*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

output
```
(2*c*x*Sqrt[d + e*x^2] + (4*(c*d - 2*b*e)*ArcTanh[(Sqrt[e]*x)/(-Sqrt[d] +
Sqrt[d + e*x^2])])/Sqrt[e] + RootSum[a*e^4 + 4*b*d*e^2*#1^2 - 4*a*e^3*#1^2
 + 16*c*d^2*#1^4 - 8*b*d*e*#1^4 + 6*a*e^2*#1^4 + 4*b*d*#1^6 - 4*a*e*#1^6 +
 a*#1^8 & , (a*c*d*e^3*Log[x] - a*b*e^4*Log[x] - a*c*d*e^3*Log[-Sqrt[d] +
Sqrt[d + e*x^2] - x*#1] + a*b*e^4*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1] +
 4*b*c*d^2*e*Log[x]*#1^2 - 4*b^2*d*e^2*Log[x]*#1^2 + a*c*d*e^2*Log[x]*#1^2
 + 3*a*b*e^3*Log[x]*#1^2 - 4*b*c*d^2*e*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*
#1]*#1^2 + 4*b^2*d*e^2*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^2 - a*c*d
*e^2*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^2 - 3*a*b*e^3*Log[-Sqrt[d]
+ Sqrt[d + e*x^2] - x*#1]*#1^2 - 4*b*c*d^2*Log[x]*#1^4 + 4*b^2*d*e*Log[x]*
#1^4 - a*c*d*e*Log[x]*#1^4 - 3*a*b*e^2*Log[x]*#1^4 + 4*b*c*d^2*Log[-Sqrt[d
] + Sqrt[d + e*x^2] - x*#1]*#1^4 - 4*b^2*d*e*Log[-Sqrt[d] + Sqrt[d + e*x^2
] - x*#1]*#1^4 + a*c*d*e*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^4 + 3*a
*b*e^2*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^4 - a*c*d*Log[x]*#1^6 + a
*b*e*Log[x]*#1^6 + a*c*d*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^6 - a*b
*e*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^6)/(b*d*e^2*#1 - a*e^3*#1 + 8
*c*d^2*#1^3 - 4*b*d*e*#1^3 + 3*a*e^2*#1^3 + 3*b*d*#1^5 - 3*a*e*#1^5 + a*#1
^7) & ])/(4*c^2)
```

3.361.3 Rubi [A] (verified)

Time = 1.84 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1614, 299, 224, 219, 2256, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{ 1614}$$

$$\frac{\int \frac{cex^2+cd-be}{\sqrt{ex^2+d}}dx}{c^2}-\frac{\int \frac{(-eb^2+cdb+ace)x^2+a(cd-be)}{\sqrt{ex^2+d}(cx^4+bx^2+a)}dx}{c^2}$$

$$\Big\downarrow \text{ 299}$$

$$\frac{\frac{1}{2}(cd-2be)\int \frac{1}{\sqrt{ex^2+d}}dx+\frac{1}{2}cx\sqrt{d+ex^2}}{c^2}-\frac{\int \frac{(-eb^2+cdb+ace)x^2+a(cd-be)}{\sqrt{ex^2+d}(cx^4+bx^2+a)}dx}{c^2}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\frac{1}{2}(cd-2be)\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}+\frac{1}{2}cx\sqrt{d+ex^2}}{c^2}-\frac{\int \frac{(-eb^2+cdb+ace)x^2+a(cd-be)}{\sqrt{ex^2+d}(cx^4+bx^2+a)}dx}{c^2}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)(cd-2be)}{2\sqrt{e}}+\frac{1}{2}cx\sqrt{d+ex^2}}{c^2}-\frac{\int \frac{(-eb^2+cdb+ace)x^2+a(cd-be)}{\sqrt{ex^2+d}(cx^4+bx^2+a)}dx}{c^2}$$

$$\Big\downarrow \text{ 2256}$$

$$\frac{\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)(cd-2be)}{2\sqrt{e}}+\frac{1}{2}cx\sqrt{d+ex^2}}{c^2}-\frac{\int \left(\frac{-eb^2+cdb+ace-\frac{eb^3-cdb^2-3aceb+2ac^2d}{\sqrt{b^2-4ac}}}{\left(2cx^2+b+\sqrt{b^2-4ac}\right)\sqrt{ex^2+d}}+\frac{-eb^2+cdb+ace+\frac{eb^3-cdb^2-3aceb+2ac^2d}{\sqrt{b^2-4ac}}}{\left(2cx^2+b-\sqrt{b^2-4ac}\right)\sqrt{ex^2+d}}\right)dx}{c^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)(cd-2be)}{2\sqrt{e}}+\frac{1}{2}cx\sqrt{d+ex^2}}{c^2}-\frac{\frac{\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\arctan\left(\right.}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+\right.}}}{c^2}$$

input
```
Int[(x^4*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

output
```
-((((b*c*d - b^2*e + a*c*e - (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqr
t[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b
 - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt
[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + ((b*c*d - b^2*e + a*c*e + (b^2*c*d
- 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (
b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])
])/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/
c^2) + ((c*x*Sqrt[d + e*x^2])/2 + ((c*d - 2*b*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[
d + e*x^2]])/(2*Sqrt[e]))/c^2
```

3.361.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

3.361. $\int \frac{x^4\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

rule 1614
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Simp[f^4/c^2   Int[(f*x)^(m - 4)*(c*d - b*e + c
*e*x^2)*(d + e*x^2)^(q - 1), x], x] - Simp[f^4/c^2   Int[(f*x)^(m - 4)*(d +
 e*x^2)^(q - 1)*(Simp[a*(c*d - b*e) + (b*c*d - b^2*e + a*c*e)*x^2, x]/(a +
b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c,
 0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2256
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

3.361.4 Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 360, normalized size of antiderivative = 0.92

method	result
risch	$\frac{x\sqrt{e x^2+d}}{2c}-\frac{\frac{(2be-cd)\ln\left(x\sqrt{e}+\sqrt{e x^2+d}\right)}{c\sqrt{e}}+\frac{a\sqrt{2}\left(\frac{\left(2acde-b^2de+bc d^2+\sqrt{-d^2\left(4ac-b^2\right)}\,be-\sqrt{-d^2\left(4ac-b^2\right)}\,cd\right)\arctan\left(\frac{\cdots}{x\sqrt{\left(-2a\cdots\right.}}\right.}{\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}\cdots\right.}{\cdots}}{\cdots}$
default	$\frac{\frac{x\sqrt{e x^2+d}}{2}+\frac{d\ln\left(x\sqrt{e}+\sqrt{e x^2+d}\right)}{2\sqrt{e}}}{c}-\frac{a\sqrt{2}\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\left(\frac{(-be+cd)\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}}{2}+d\left(ace-\frac{b(be-cd)}{2}\right)\cdots\right.}{\cdots}$
pseudoelliptic	$-\frac{a\sqrt{2}\left(\frac{\left(cd\sqrt{e}-e^{\frac{3}{2}}b\right)\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}}{2}+\left(\left(ac-\frac{b^2}{2}\right)e^{\frac{3}{2}}+\frac{b\sqrt{e}\,cd}{2}\right)d\right)\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\,\operatorname{arctanh}\left(\frac{\cdots}{x\sqrt{\left(2ae-\cdots\right.}}\right.}{\cdots}$

input
```
int(x^4*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(e*x^2+d)^(1/2)/c-1/2/c*((2*b*e-c*d)/c*ln(x*e^(1/2)+(e*x^2+d)^(1/2))
/e^(1/2)+1/c*a*2^(1/2)/(-d^2*(4*a*c-b^2))^(1/2)*((2*a*c*d*e-b^2*d*e+b*c*d^
2+(-d^2*(4*a*c-b^2))^(1/2)*b*e-(-d^2*(4*a*c-b^2))^(1/2)*c*d)/((-2*a*e+b*d+
(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2)*arctan(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((-2
*a*e+b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2))-(-2*a*c*d*e+b^2*d*e-b*c*d^2+(
-d^2*(4*a*c-b^2))^(1/2)*b*e-(-d^2*(4*a*c-b^2))^(1/2)*c*d)/((2*a*e-b*d+(-d^
2*(4*a*c-b^2))^(1/2))*a)^(1/2)*arctanh(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((2*a*e
-b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2))))
```

3.361.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3263 vs. $2(338) = 676$.

Time = 14.87 (sec) , antiderivative size = 6534, normalized size of antiderivative = 16.75

$$\int \frac{x^4\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.361.6 Sympy [F]

$$\int \frac{x^4\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \int \frac{x^4\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

input
```
integrate(x**4*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
```

output
```
Integral(x**4*sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

3.361.7 Maxima [F]

$$\int \frac{x^4\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \int \frac{\sqrt{ex^2+d}x^4}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^4*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x^2 + d)*x^4/(c*x^4 + b*x^2 + a), x)
```

3.361.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^4\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.361.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \int \frac{x^4\,\sqrt{e\,x^2+d}}{c\,x^4+b\,x^2+a}\,dx$$

input
```
int((x^4*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

output
```
int((x^4*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4), x)
```

3.362 $\int \frac{x^2\sqrt{d+ex^2}}{a+bx^2+cx^4} dx$

3.362.1 Optimal result

Integrand size = 29, antiderivative size = 324

$$\int \frac{x^2\sqrt{d+ex^2}}{a+bx^2+cx^4} dx = \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} + \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}} + \frac{\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c}$$

output
```
arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*e^(1/2)/c+arctan(x*(2*c*d-e*(b-(-4*a*c+
b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(c*d-b*e+
(-2*a*c*e+b^2*e-b*c*d)/(-4*a*c+b^2)^(1/2))/c/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2
)))^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(
1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(c*d-b*e+(2*a*c
*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))/c/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-
e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.362.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 7468 vs. 2(324) = 648.

Time = 16.20 (sec) , antiderivative size = 7468, normalized size of antiderivative = 23.05

$$\int \frac{x^2\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \text{Result too large to show}$$

input
```
Integrate[(x^2*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

output
```
Result too large to show
```

3.362.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1616, 224, 219, 2256, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \ 1616$$

$$\frac{e\int \frac{1}{\sqrt{ex^2+d}}dx}{c}-\frac{\int \frac{ae-(cd-be)x^2}{\sqrt{ex^2+d}(cx^4+bx^2+a)}dx}{c}$$

$$\Big\downarrow \ 224$$

$$\frac{e\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{c}-\frac{\int \frac{ae-(cd-be)x^2}{\sqrt{ex^2+d}(cx^4+bx^2+a)}dx}{c}$$

$$\Big\downarrow \ 219$$

$$\frac{\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c}-\frac{\int \frac{ae-(cd-be)x^2}{\sqrt{ex^2+d}(cx^4+bx^2+a)}dx}{c}$$

$$\Big\downarrow \ 2256$$

$$\frac{\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c}-\frac{\int \left(\frac{-cd+be-\frac{-eb^2+cdb+2ace}{\sqrt{b^2-4ac}}}{\left(2cx^2+b+\sqrt{b^2-4ac}\right)\sqrt{ex^2+d}}+\frac{-cd+be+\frac{-eb^2+cdb+2ace}{\sqrt{b^2-4ac}}}{\left(2cx^2+b-\sqrt{b^2-4ac}\right)\sqrt{ex^2+d}}\right)dx}{c}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c}-\frac{-\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}{c}$$

input
```
Int[(x^2*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

output
```
-((-(((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt
[d + e*x^2])])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4
*a*c])*e])) - ((c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a
*c]]*Sqrt[d + e*x^2])])/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqr
t[b^2 - 4*a*c])*e]))/c) + (Sqrt[e]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/c
```

3.362.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 1616
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Simp[e*(f^2/c)   Int[(f*x)^(m - 2)*(d + e*x^2)^
(q - 1), x], x] - Simp[f^2/c   Int[(f*x)^(m - 2)*(d + e*x^2)^(q - 1)*(Simp[
a*e - (c*d - b*e)*x^2, x]/(a + b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && !IntegerQ[q] && GtQ[q, 0] && GtQ[m,
1] && LeQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.362. $\int \frac{x^2\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

rule 2256
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

3.362.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.14

method	result
default	$-\frac{a\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\,\sqrt{2}\left(bde-2c\,d^2+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\,e\right)\operatorname{arctanh}\left(\frac{a\sqrt{e\,x^2+d}\,\sqrt{2}}{x\sqrt{\left(2ae-bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}}\right)-}{2\sqrt{}}$
pseudoelliptic	$-\frac{a\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\,\sqrt{2}\left(bde-2c\,d^2+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\,e\right)\operatorname{arctanh}\left(\frac{a\sqrt{e\,x^2+d}\,\sqrt{2}}{x\sqrt{\left(2ae-bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}}\right)-}{2\sqrt{}}$

input
```
int(x^2*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*(a*((-2*a*e+b*d+(
-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*2^(1/2)*(b*d*e-2*c*d^2+(-4*d^2*(a*c-
1/4*b^2))^(1/2)*e)*arctanh(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((2*a*e-b*d+(-4*d^2
*(a*c-1/4*b^2))^(1/2))*a)^(1/2))-(a*2^(1/2)*(-b*d*e+2*c*d^2+(-4*d^2*(a*c-1
/4*b^2))^(1/2)*e)*arctan(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((-2*a*e+b*d+(-4*d^2*
(a*c-1/4*b^2))^(1/2))*a)^(1/2))+2*(-4*d^2*(a*c-1/4*b^2))^(1/2)*arctanh((e*
x^2+d)^(1/2)/x/e^(1/2))*((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2
)*e^(1/2))*((2*a*e-b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2))/((2*a*e-b*d
+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)/(-4*d^2*(a*c-1/4*b^2))^(1/2)/c
```

3.362.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1626 vs. $2(278) = 556$.

Time = 2.50 (sec) , antiderivative size = 3260, normalized size of antiderivative = 10.06

$$\int \frac{x^2\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
[-1/4*(sqrt(1/2)*c*sqrt(-(b*c*d - (b^2 - 2*a*c)*e + (b^2*c^2 - 4*a*c^3)*sq
rt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^
3))*log(-((b^2*c^2 - 4*a*c^3)*d*x^2*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(
b^2*c^4 - 4*a*c^5)) + 2*a*c*d^2 - 2*a*b*d*e - (b*c*d^2 + 4*a*b*e^2 - (b^2
+ 4*a*c)*d*e)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^3*c^2 - 4*a*b*c^3)*x*s
qrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) - ((b^2*c - 4*a*c
^2)*d - (b^3 - 4*a*b*c)*e)*x)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e + (b^2*c^2 -
4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c
^2 - 4*a*c^3)))/x^2) - sqrt(1/2)*c*sqrt(-(b*c*d - (b^2 - 2*a*c)*e + (b^2*c
^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))/(
b^2*c^2 - 4*a*c^3))*log(-((b^2*c^2 - 4*a*c^3)*d*x^2*sqrt((c^2*d^2 - 2*b*c*
d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) + 2*a*c*d^2 - 2*a*b*d*e - (b*c*d^2 + 4
*a*b*e^2 - (b^2 + 4*a*c)*d*e)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^3*c^2
- 4*a*b*c^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) -
 ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*x)*sqrt(-(b*c*d - (b^2 - 2*a*c)
*e + (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4
*a*c^5)))/(b^2*c^2 - 4*a*c^3)))/x^2) + sqrt(1/2)*c*sqrt(-(b*c*d - (b^2 - 2
*a*c)*e - (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^
4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3))*log(((b^2*c^2 - 4*a*c^3)*d*x^2*sqrt((c
^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) - 2*a*c*d^2 + 2*a*b*...
```

3.362.6 Sympy [F]

$$\int \frac{x^2\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \int \frac{x^2\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

input
```
integrate(x**2*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
```

output
```
Integral(x**2*sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

3.362.7 Maxima [F]

$$\int \frac{x^2\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \int \frac{\sqrt{ex^2+d}x^2}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^2*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x^2 + d)*x^2/(c*x^4 + b*x^2 + a), x)
```

3.362.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.362.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \int \frac{x^2\,\sqrt{e\,x^2+d}}{c\,x^4+b\,x^2+a}\,dx$$

input
```
int((x^2*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

output
```
int((x^2*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4), x)
```

3.363 $\int \frac{\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

3.363.1 Optimal result

Integrand size = 26, antiderivative size = 240

$$\int \frac{\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c
+b^2)^(1/2))^(1/2))*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(-4*a*c+b^2)^(1
/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))
^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(2*c*d-e*(b+(-4*a*c+b
^2)^(1/2)))^(1/2)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.363.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2607 vs. $2(240) = 480$.

Time = 13.42 (sec) , antiderivative size = 2607, normalized size of antiderivative = 10.86

$$\int \frac{\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[d + e*x^2]/(a + b*x^2 + c*x^4),x]
```

output
```
(Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*S
qrt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c]*Log[-(Sqrt[2]*Sqrt[(-b + Sqrt[b^2
 - 4*a*c])/c]) + 2*x] - Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*(2*c*d + (-b +
Sqrt[b^2 - 4*a*c])*e)*Sqrt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c]*Log[Sqrt[2
]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c] + 2*x] - 2*c*Sqrt[(-b + Sqrt[b^2 - 4*a*
c])/c]*d*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[-(Sqrt[2]*Sqrt[-(
(b + Sqrt[b^2 - 4*a*c])/c)]) + 2*x] + b*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e
*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[-(Sqrt[2]*Sqrt[-((b + Sqr
t[b^2 - 4*a*c])/c)]) + 2*x] + Sqrt[b^2 - 4*a*c]*Sqrt[(-b + Sqrt[b^2 - 4*a*
c])/c]*e*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[-(Sqrt[2]*Sqrt[-(
(b + Sqrt[b^2 - 4*a*c])/c)]) + 2*x] + 2*c*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]
*d*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[Sqrt[2]*Sqrt[-((b + Sqr
t[b^2 - 4*a*c])/c)] + 2*x] - b*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*Sqrt[(2*
c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[Sqrt[2]*Sqrt[-((b + Sqrt[b^2 - 4*a
*c])/c)] + 2*x] - Sqrt[b^2 - 4*a*c]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*Sqr
t[(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[Sqrt[2]*Sqrt[-((b + Sqrt[b^2
- 4*a*c])/c)] + 2*x] + 2*c*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*d*Sqrt[2*d -
 ((b + Sqrt[b^2 - 4*a*c])*e)/c]*Log[4*d - 2*Sqrt[2]*Sqrt[(-b + Sqrt[b^2 -
4*a*c])/c]*e*x + 2*Sqrt[4*d + (2*(-b + Sqrt[b^2 - 4*a*c])*e)/c]*Sqrt[d + e
*x^2]] - b*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*e*Sqrt[2*d - ((b + Sqrt[b...
```

3.363.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.34, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1488, 301, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \; 1488$$

$$\frac{2c\int \frac{\sqrt{ex^2+d}}{2cx^2+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{2c\int \frac{\sqrt{ex^2+d}}{2cx^2+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \; 301$$

$$\frac{2c\left(\frac{\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)\int \frac{1}{\left(2cx^2+b-\sqrt{b^2-4ac}\right)\sqrt{ex^2+d}}dx}{2c}+\frac{e\int \frac{1}{\sqrt{ex^2+d}}dx}{2c}\right)}{\sqrt{b^2-4ac}}-\frac{2c\left(\frac{\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)\int \frac{1}{\left(2cx^2+b+\sqrt{b^2-4ac}\right)\sqrt{ex^2+d}}dx}{2c}+\frac{e\int \frac{1}{\sqrt{ex^2+d}}dx}{2c}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \; 224$$

$$\frac{2c\left(\frac{\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)\int \frac{1}{\left(2cx^2+b-\sqrt{b^2-4ac}\right)\sqrt{ex^2+d}}dx}{2c}+\frac{e\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{2c}\right)}{\sqrt{b^2-4ac}}-\frac{2c\left(\frac{\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)\int \frac{1}{\left(2cx^2+b+\sqrt{b^2-4ac}\right)\sqrt{ex^2+d}}dx}{2c}+\frac{e\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{2c}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \; 219$$

$$\frac{2c\left(\frac{\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)\int \frac{1}{\left(2cx^2+b-\sqrt{b^2-4ac}\right)\sqrt{ex^2+d}}dx}{2c}+\frac{\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c}\right)}{\sqrt{b^2-4ac}}-\frac{2c\left(\frac{\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)\int \frac{1}{\left(2cx^2+b+\sqrt{b^2-4ac}\right)\sqrt{ex^2+d}}dx}{2c}+\frac{\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \; 291$$

3.363. $\int \frac{\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

$$\frac{2c\left(\frac{\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)\int \frac{1}{-\frac{\left(\left(b-\sqrt{b^2-4ac}\right)e-2cd\right)x^2}{ex^2+d}+b-\sqrt{b^2-4ac}}d\frac{x}{\sqrt{ex^2+d}}}{2c}+\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c}\right)}{\sqrt{b^2-4ac}}-\frac{2c\left(\frac{\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)\int \frac{1}{-\frac{\left(\left(b+\sqrt{b^2-4ac}\right)e-2cd\right)x^2}{ex^2+d}+b+\sqrt{b^2-4ac}}d\frac{x}{\sqrt{ex^2+d}}}{2c}+\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{218}$$

$$\frac{2c\left(\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2c\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c}\right)}{\sqrt{b^2-4ac}}-\frac{2c\left(\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{2c\sqrt{\sqrt{b^2-4ac}+b}}+\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c}\right)}{\sqrt{b^2-4ac}}$$

input
```
Int[Sqrt[d + e*x^2]/(a + b*x^2 + c*x^4),x]
```

output
```
(2*c*((Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*ArcTan[(Sqrt[2*c*d - (b - S
qrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(2
*c*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[e]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*
x^2]])/(2*c)))/Sqrt[b^2 - 4*a*c] - (2*c*((Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a
*c])*e]*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[
b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*c*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (Sqrt
[e]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c)))/Sqrt[b^2 - 4*a*c]
```

3.363. $\int \frac{\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

3.363.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 1488
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/r)   Int[(d + e*x^2)^q/(b -
 r + 2*c*x^2), x], x] - Simp[2*(c/r)   Int[(d + e*x^2)^q/(b + r + 2*c*x^2),
 x], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

3.363. $\int \frac{\sqrt{d+ex^2}}{a+bx^2+cx^4} dx$

3.363.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.97

method	result
default	$-\frac{d\sqrt{2}\left(\frac{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)\arctan\left(\frac{a\sqrt{e\,x^2+d}\,\sqrt{2}}{x\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}\right)}{\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}-\frac{\left(2ae-bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)\operatorname{arctanh}\left(\frac{\cdots}{x\sqrt{\left(2ae\cdots\right.}}\right.}{\sqrt{\left(2ae-bd+\sqrt{-d^2\left(4ac-\cdots\right.}\right.}}\cdots\right.}{2\sqrt{-d^2(4ac-b^2)}}$
pseudoelliptic	$-\frac{d\sqrt{2}\left(\frac{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)\arctan\left(\frac{a\sqrt{e\,x^2+d}\,\sqrt{2}}{x\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}\right)}{\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}-\frac{\left(2ae-bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)\operatorname{arctanh}\left(\frac{\cdots}{x\sqrt{\left(2ae\cdots\right.}}\right.}{\sqrt{\left(2ae-bd+\sqrt{-d^2\left(4ac-\cdots\right.}\right.}}\cdots\right.}{2\sqrt{-d^2(4ac-b^2)}}$

input
```
int((e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*d*2^(1/2)/(-d^2*(4*a*c-b^2))^(1/2)*((-2*a*e+b*d+(-d^2*(4*a*c-b^2))^(1
/2))/((-2*a*e+b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2)*arctan(a/x*(e*x^2+d)^
(1/2)*2^(1/2)/((-2*a*e+b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2))-(2*a*e-b*d+
(-d^2*(4*a*c-b^2))^(1/2))/((2*a*e-b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2)*a
rctanh(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((2*a*e-b*d+(-d^2*(4*a*c-b^2))^(1/2))*a
)^(1/2)))
```

3.363.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 985 vs. 2(200) = 400.

Time = 0.83 (sec) , antiderivative size = 985, normalized size of antiderivative = 4.10

$$\int \frac{\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

$$= \frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{bd-2\,ae+(ab^2-4\,a^2c)\sqrt{\frac{d^2}{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(-\frac{(ab^2-4\,a^2c)d\sqrt{\frac{d^2}{a^2b^2-4\,a^3c}}x^2+4\sqrt{\frac{1}{2}}(a^2b^2-4\,a^3c)\sqrt{}}{}\right.$$

$$-\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{bd-2\,ae+(ab^2-4\,a^2c)\sqrt{\frac{d^2}{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(-\frac{(ab^2-4\,a^2c)d\sqrt{\frac{d^2}{a^2b^2-4\,a^3c}}x^2-4\sqrt{\frac{1}{2}}(a^2b^2-4\,a^3}{}\right.$$

$$+\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{bd-2\,ae-(ab^2-4\,a^2c)\sqrt{\frac{d^2}{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(\frac{(ab^2-4\,a^2c)d\sqrt{\frac{d^2}{a^2b^2-4\,a^3c}}x^2+4\sqrt{\frac{1}{2}}(a^2b^2-4\,a^3c)}{}\right.$$

$$-\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{bd-2\,ae-(ab^2-4\,a^2c)\sqrt{\frac{d^2}{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(\frac{(ab^2-4\,a^2c)d\sqrt{\frac{d^2}{a^2b^2-4\,a^3c}}x^2-4\sqrt{\frac{1}{2}}(a^2b^2-4\,a^3c)}{}\right.$$

input
```
integrate((e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

3.363. $\int \frac{\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

output
```
1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e + (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b^2 - 4
*a^3*c)))/(a*b^2 - 4*a^2*c))*log(-((a*b^2 - 4*a^2*c)*d*sqrt(d^2/(a^2*b^2 -
 4*a^3*c))*x^2 + 4*sqrt(1/2)*(a^2*b^2 - 4*a^3*c)*sqrt(e*x^2 + d)*sqrt(d^2/
(a^2*b^2 - 4*a^3*c))*x*sqrt(-(b*d - 2*a*e + (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^
2*b^2 - 4*a^3*c)))/(a*b^2 - 4*a^2*c)) - 2*a*d^2 + (b*d^2 - 4*a*d*e)*x^2)/x
^2) - 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e + (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b
^2 - 4*a^3*c)))/(a*b^2 - 4*a^2*c))*log(-((a*b^2 - 4*a^2*c)*d*sqrt(d^2/(a^2
*b^2 - 4*a^3*c))*x^2 - 4*sqrt(1/2)*(a^2*b^2 - 4*a^3*c)*sqrt(e*x^2 + d)*sqr
t(d^2/(a^2*b^2 - 4*a^3*c))*x*sqrt(-(b*d - 2*a*e + (a*b^2 - 4*a^2*c)*sqrt(d
^2/(a^2*b^2 - 4*a^3*c)))/(a*b^2 - 4*a^2*c)) - 2*a*d^2 + (b*d^2 - 4*a*d*e)*
x^2)/x^2) + 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e - (a*b^2 - 4*a^2*c)*sqrt(d^2/
(a^2*b^2 - 4*a^3*c)))/(a*b^2 - 4*a^2*c))*log(((a*b^2 - 4*a^2*c)*d*sqrt(d^2
/(a^2*b^2 - 4*a^3*c))*x^2 + 4*sqrt(1/2)*(a^2*b^2 - 4*a^3*c)*sqrt(e*x^2 + d
)*sqrt(d^2/(a^2*b^2 - 4*a^3*c))*x*sqrt(-(b*d - 2*a*e - (a*b^2 - 4*a^2*c)*s
qrt(d^2/(a^2*b^2 - 4*a^3*c)))/(a*b^2 - 4*a^2*c)) + 2*a*d^2 - (b*d^2 - 4*a*
d*e)*x^2)/x^2) - 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e - (a*b^2 - 4*a^2*c)*sqrt
(d^2/(a^2*b^2 - 4*a^3*c)))/(a*b^2 - 4*a^2*c))*log(((a*b^2 - 4*a^2*c)*d*sqr
t(d^2/(a^2*b^2 - 4*a^3*c))*x^2 - 4*sqrt(1/2)*(a^2*b^2 - 4*a^3*c)*sqrt(e*x^
2 + d)*sqrt(d^2/(a^2*b^2 - 4*a^3*c))*x*sqrt(-(b*d - 2*a*e - (a*b^2 - 4*a^2
*c)*sqrt(d^2/(a^2*b^2 - 4*a^3*c)))/(a*b^2 - 4*a^2*c)) + 2*a*d^2 - (b*d^...
```

3.363.6 Sympy [F]

$$\int \frac{\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \int \frac{\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
```

output
```
Integral(sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

3.363.7 Maxima [F]

$$\int \frac{\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \int \frac{\sqrt{ex^2+d}}{cx^4+bx^2+a}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x^2 + d)/(c*x^4 + b*x^2 + a), x)
```

3.363.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
Timed out
```

3.363.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \int \frac{\sqrt{e\,x^2+d}}{c\,x^4+b\,x^2+a}\,dx$$

input
```
int((d + e*x^2)^(1/2)/(a + b*x^2 + c*x^4),x)
```

output
```
int((d + e*x^2)^(1/2)/(a + b*x^2 + c*x^4), x)
```

3.364 $\int \frac{\sqrt{d+ex^2}}{x^2(a+bx^2+cx^4)}\,dx$

3.364.1 Optimal result

Integrand size = 29, antiderivative size = 291

$$\int \frac{\sqrt{d+ex^2}}{x^2(a+bx^2+cx^4)}\,dx = -\frac{\sqrt{d+ex^2}}{ax} - \frac{c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

output
```
-(e*x^2+d)^(1/2)/a/x-c*arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*
x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(d+(-2*a*e+b*d)/(-4*a*c+b^2)^(1
/2))/a/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)
-c*arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*
a*c+b^2)^(1/2))^(1/2))*(d+(2*a*e-b*d)/(-4*a*c+b^2)^(1/2))/a/(b+(-4*a*c+b^2
)^(1/2))^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.364.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.73 (sec) , antiderivative size = 626, normalized size of antiderivative = 2.15

$$\int \frac{\sqrt{d+ex^2}}{x^2(a+bx^2+cx^4)}\,dx = -\frac{\sqrt{d+ex^2}}{ax}$$
$$+\frac{\text{RootSum}\left[ae^4+4bde^2\#1^2-4ae^3\#1^2+16cd^2\#1^4-8bde\#1^4+6ae^2\#1^4+4bd\#1^6-4ae\#1^6+a\#1^8\right.}{}$$

input
```
Integrate[Sqrt[d + e*x^2]/(x^2*(a + b*x^2 + c*x^4)),x]
```

output
```
-(Sqrt[d + e*x^2]/(a*x)) + RootSum[a*e^4 + 4*b*d*e^2*#1^2 - 4*a*e^3*#1^2 +
 16*c*d^2*#1^4 - 8*b*d*e*#1^4 + 6*a*e^2*#1^4 + 4*b*d*#1^6 - 4*a*e*#1^6 + a
*#1^8 & , (b*d*e^3*Log[x] - a*e^4*Log[x] - b*d*e^3*Log[-Sqrt[d] + Sqrt[d +
 e*x^2] - x*#1] + a*e^4*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1] + 4*c*d^2*e
*Log[x]*#1^2 - 3*b*d*e^2*Log[x]*#1^2 + 3*a*e^3*Log[x]*#1^2 - 4*c*d^2*e*Log
[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^2 + 3*b*d*e^2*Log[-Sqrt[d] + Sqrt[d
 + e*x^2] - x*#1]*#1^2 - 3*a*e^3*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1
^2 - 4*c*d^2*Log[x]*#1^4 + 3*b*d*e*Log[x]*#1^4 - 3*a*e^2*Log[x]*#1^4 + 4*c
*d^2*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^4 - 3*b*d*e*Log[-Sqrt[d] +
Sqrt[d + e*x^2] - x*#1]*#1^4 + 3*a*e^2*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*
#1]*#1^4 - b*d*Log[x]*#1^6 + a*e*Log[x]*#1^6 + b*d*Log[-Sqrt[d] + Sqrt[d +
 e*x^2] - x*#1]*#1^6 - a*e*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^6)/(b
*d*e^2*#1 - a*e^3*#1 + 8*c*d^2*#1^3 - 4*b*d*e*#1^3 + 3*a*e^2*#1^3 + 3*b*d*
#1^5 - 3*a*e*#1^5 + a*#1^7) & ]/(4*a)
```

3.364.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1618, 242, 2256, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}}{x^2(a+bx^2+cx^4)}\,dx$$
$$\Big\downarrow \text{1618}$$

$$\frac{d\int \frac{1}{x^2\sqrt{ex^2+d}}dx}{a} - \frac{\int \frac{cdx^2+bd-ae}{\sqrt{ex^2+d}(cx^4+bx^2+a)}dx}{a}$$

$\Big\downarrow$ 242

$$-\frac{\int \frac{cdx^2+bd-ae}{\sqrt{ex^2+d}(cx^4+bx^2+a)}dx}{a} - \frac{\sqrt{d+ex^2}}{ax}$$

$\Big\downarrow$ 2256

$$-\frac{\int \left(\frac{cd-\frac{c(bd-2ae)}{\sqrt{b^2-4ac}}}{\left(2cx^2+b+\sqrt{b^2-4ac}\right)\sqrt{ex^2+d}} + \frac{cd+\frac{c(bd-2ae)}{\sqrt{b^2-4ac}}}{\left(2cx^2+b-\sqrt{b^2-4ac}\right)\sqrt{ex^2+d}}\right)dx}{a} - \frac{\sqrt{d+ex^2}}{ax}$$

$\Big\downarrow$ 2009

$$-\frac{\frac{c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}} + \frac{c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}{a} - \frac{\sqrt{d+ex^2}}{ax}$$

input
```
Int[Sqrt[d + e*x^2]/(x^2*(a + b*x^2 + c*x^4)),x]
```

output
```
-(Sqrt[d + e*x^2]/(a*x)) - ((c*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]
*Sqrt[d + e*x^2])])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^
2 - 4*a*c])*e]) + (c*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*
c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d +
e*x^2])])/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e]))/a
```

3.364.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 1618
```
Int[(((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4), x_Symbol] :> Simp[d/a   Int[(f*x)^m*(d + e*x^2)^(q - 1), x],
x] - Simp[1/(a*f^2)   Int[(f*x)^(m + 2)*(d + e*x^2)^(q - 1)*(Simp[b*d - a*e
 + c*d*x^2, x]/(a + b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && LtQ[m, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2256
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

3.364.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.12

method	result
risch	$-\frac{\sqrt{e x^2+d}}{a x}-\frac{\sqrt{2}\left(\frac{\left(abde+2d^2ac-b^2d^2+\sqrt{-d^2\left(4ac-b^2\right)}\,ae-\sqrt{-d^2\left(4ac-b^2\right)}\,bd\right)\arctan\left(\frac{a\sqrt{e x^2+d}\,\sqrt{2}}{x\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}\right)}{\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}-\cdots\right)}{2a\sqrt{-d^2(4\cdots}}$
pseudoelliptic	$\frac{-\frac{\sqrt{e x^2+d}}{x}-\frac{\left(abde+2d^2ac-b^2d^2+\sqrt{-d^2\left(4ac-b^2\right)}\,ae-\sqrt{-d^2\left(4ac-b^2\right)}\,bd\right)\sqrt{2}\arctan\left(\frac{a\sqrt{e x^2+d}\,\sqrt{2}}{x\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}\right)}{2\sqrt{-d^2\left(4ac-b^2\right)}\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}+\frac{\left(-abd\cdots\right.}{\cdots}}{a}$
default	$\frac{-\frac{\left(e x^2+d\right)^{\frac{3}{2}}}{dx}+\frac{2e\left(\frac{x\sqrt{e x^2+d}}{2}+\frac{d\ln\left(x\sqrt{e}+\sqrt{e x^2+d}\right)}{2\sqrt{e}}\right)}{d}}{a}+\frac{\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\,\sqrt{2}\left((ae-bd)\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\cdots\right.}{\cdots}$

input
```
int((e*x^2+d)^(1/2)/x^2/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-(e*x^2+d)^(1/2)/a/x-1/2*2^(1/2)/a/(-d^2*(4*a*c-b^2))^(1/2)*((a*b*d*e+2*d^
2*a*c-b^2*d^2+(-d^2*(4*a*c-b^2))^(1/2)*a*e-(-d^2*(4*a*c-b^2))^(1/2)*b*d)/(
(-2*a*e+b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2)*arctan(a/x*(e*x^2+d)^(1/2)*
2^(1/2)/((-2*a*e+b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2))-(-a*b*d*e-2*d^2*a
*c+b^2*d^2+(-d^2*(4*a*c-b^2))^(1/2)*a*e-(-d^2*(4*a*c-b^2))^(1/2)*b*d)/((2*
a*e-b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2)*arctanh(a/x*(e*x^2+d)^(1/2)*2^(
1/2)/((2*a*e-b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2)))
```

3.364.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2402 vs. 2(249) = 498.

Time = 1.63 (sec) , antiderivative size = 2402, normalized size of antiderivative = 8.25

$$\int \frac{\sqrt{d+ex^2}}{x^2\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^(1/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(1/2)*a*x*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e + (a^3*
b^2 - 4*a^4*c)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*
b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((2*a^2*
b*c*d*e + (a^3*b^2*c - 4*a^4*c^2)*d*x^2*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2
*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)) - 2*(a*b
^2*c - a^2*c^2)*d^2 + (4*a^2*b*c*e^2 + (b^3*c - a*b*c^2)*d^2 - (5*a*b^2*c
- 4*a^2*c^2)*d*e)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((a^4*b^3 - 4*a^5*b*c)
*x*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*
c)*d*e)/(a^6*b^2 - 4*a^7*c)) - ((a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*d - (a^2
*b^3 - 4*a^3*b*c)*e)*x)*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e + (
a^3*b^2 - 4*a^4*c)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2
*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)))/x^2) -
 sqrt(1/2)*a*x*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e + (a^3*b^2 -
 4*a^4*c)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 -
 a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((2*a^2*b*c*d
*e + (a^3*b^2*c - 4*a^4*c^2)*d*x^2*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c +
a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)) - 2*(a*b^2*c
- a^2*c^2)*d^2 + (4*a^2*b*c*e^2 + (b^3*c - a*b*c^2)*d^2 - (5*a*b^2*c - 4*a
^2*c^2)*d*e)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((a^4*b^3 - 4*a^5*b*c)*x*sq
rt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)...
```

3.364.6 Sympy [F]

$$\int \frac{\sqrt{d+ex^2}}{x^2\left(a+bx^2+cx^4\right)}\,dx = \int \frac{\sqrt{d+ex^2}}{x^2\left(a+bx^2+cx^4\right)}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)/x**2/(c*x**4+b*x**2+a),x)
```

output
```
Integral(sqrt(d + e*x**2)/(x**2*(a + b*x**2 + c*x**4)), x)
```

3.364.7 Maxima [F]

$$\int \frac{\sqrt{d+ex^2}}{x^2\left(a+bx^2+cx^4\right)}\,dx = \int \frac{\sqrt{ex^2+d}}{(cx^4+bx^2+a)x^2}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^2), x)
```

3.364.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}}{x^2\left(a+bx^2+cx^4\right)}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(1/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
Timed out
```

3.364.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}}{x^2\left(a+bx^2+cx^4\right)}\,dx=\int \frac{\sqrt{e\,x^2+d}}{x^2\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

input
```
int((d + e*x^2)^(1/2)/(x^2*(a + b*x^2 + c*x^4)),x)
```

output
```
int((d + e*x^2)^(1/2)/(x^2*(a + b*x^2 + c*x^4)), x)
```

3.365 $\int \frac{\sqrt{d+ex^2}}{x^4(a+bx^2+cx^4)}\,dx$

3.365.1 Optimal result

Integrand size = 29, antiderivative size = 373

$$\int \frac{\sqrt{d+ex^2}}{x^4(a+bx^2+cx^4)}\,dx = -\frac{\sqrt{d+ex^2}}{3ax^3}+\frac{2e\sqrt{d+ex^2}}{3adx}+\frac{(bd-ae)\sqrt{d+ex^2}}{a^2dx}+\frac{c\left(bd-ae+\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}+\frac{c\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^2\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

output
```
-1/3*(e*x^2+d)^(1/2)/a/x^3+2/3*e*(e*x^2+d)^(1/2)/a/d/x+(-a*e+b*d)*(e*x^2+d
)^(1/2)/a^2/d/x+c*arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d
)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b*d-a*e+(-a*b*e-2*a*c*d+b^2*d)/(-4*
a*c+b^2)^(1/2))/a^2/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(b-(-4*a*c+b^2)
^(1/2))^(1/2)+c*arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^
(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b*d-a*e+(a*b*e+2*a*c*d-b^2*d)/(-4*a*c
+b^2)^(1/2))/a^2/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/
2)))^(1/2)
```

3.365.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 7477 vs. 2(373) = 746.

Time = 16.38 (sec) , antiderivative size = 7477, normalized size of antiderivative = 20.05

$$\int \frac{\sqrt{d+ex^2}}{x^4 (a+bx^2+cx^4)} \, dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[d + e*x^2]/(x^4*(a + b*x^2 + c*x^4)),x]
```

output
```
Result too large to show
```

3.365.3 Rubi [A] (verified)

Time = 1.59 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1618, 245, 242, 2246, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}}{x^4 (a+bx^2+cx^4)} \, dx$$

$$\Big\downarrow \text{1618}$$

$$\frac{d \int \frac{1}{x^4\sqrt{ex^2+d}}dx}{a} - \frac{\int \frac{cdx^2+bd-ae}{x^2\sqrt{ex^2+d}(cx^4+bx^2+a)}dx}{a}$$

$$\Big\downarrow \text{245}$$

$$\frac{d\left(-\frac{2e\int \frac{1}{x^2\sqrt{ex^2+d}}dx}{3d} - \frac{\sqrt{d+ex^2}}{3dx^3}\right)}{a} - \frac{\int \frac{cdx^2+bd-ae}{x^2\sqrt{ex^2+d}(cx^4+bx^2+a)}dx}{a}$$

$$\Big\downarrow \text{242}$$

$$\frac{d\left(\frac{2e\sqrt{d+ex^2}}{3d^2x} - \frac{\sqrt{d+ex^2}}{3dx^3}\right)}{a} - \frac{\int \frac{cdx^2+bd-ae}{x^2\sqrt{ex^2+d}(cx^4+bx^2+a)}dx}{a}$$

$$\Big\downarrow \text{2246}$$

$$\frac{d\left(\frac{2e\sqrt{d+ex^2}}{3d^2x} - \frac{\sqrt{d+ex^2}}{3dx^3}\right)}{a} - \frac{\int \left(\frac{bd-ae}{ax^2\sqrt{ex^2+d}} + \frac{-db^2+aeb-c(bd-ae)x^2+acd}{a\sqrt{ex^2+d}(cx^4+bx^2+a)}\right)dx}{a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\left(\frac{2e\sqrt{d+ex^2}}{3d^2x}-\frac{\sqrt{d+ex^2}}{3dx^3}\right)}{a}-\frac{-\frac{c\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{c\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\sqrt{d+ex^2}}{a\cdots}}{a}$$

input
```
Int[Sqrt[d + e*x^2]/(x^4*(a + b*x^2 + c*x^4)),x]
```

output
```
(d*(-1/3*Sqrt[d + e*x^2]/(d*x^3) + (2*e*Sqrt[d + e*x^2])/(3*d^2*x)))/a - (
-(((b*d - a*e)*Sqrt[d + e*x^2])/(a*d*x)) - (c*(b*d - a*e + (b^2*d - 2*a*c*
d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])
*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a*Sqrt[b - Sqrt[b^
2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (c*(b*d - a*e - (b^
2*d - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a*Sqrt
[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/a
```

3.365.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 1618
```
Int[(((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4), x_Symbol] :> Simp[d/a   Int[(f*x)^m*(d + e*x^2)^(q - 1), x],
x] - Simp[1/(a*f^2)   Int[(f*x)^(m + 2)*(d + e*x^2)^(q - 1)*(Simp[b*d - a*e
 + c*d*x^2, x]/(a + b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && LtQ[m, 0]
```

3.365. $\int \frac{\sqrt{d+ex^2}}{x^4(a+bx^2+cx^4)}\,dx$

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2246
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(f*x)^m*(d +
e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x
] && PolyQ[Px, x] && IntegerQ[p]
```

3.365.4 Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.07

method	result
risch	$-\frac{\sqrt{e x^2+d}\,(ae x^2-3bd x^2+da)}{3a^2x^3d}-\frac{\sqrt{2}\left(\left(\frac{((be+cd)a-b^2d)\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}}{2}+d\left(a^2ce-\frac{b(be+3cd)a}{2}+\frac{b^3d}{2}\right)\right)\sqrt{\left(-2ae+bd+\sqrt{\cdots}\right.}\right.}{\cdots}$
pseudoelliptic	$-\frac{\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\left(\frac{\left(d\left(ac-b^2\right)+abe\right)\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}}{2}+d\left(\frac{\left(-3abc+b^3\right)d}{2}+ae\left(ac-\frac{b^2}{2}\right)\right)\right)\sqrt{2}\,d\,x^3\arctan\cdots}{\cdots}$
default	$-\frac{\left(e x^2+d\right)^{\frac{3}{2}}}{3ad\,x^3}-\frac{b\left(-\frac{\left(e x^2+d\right)^{\frac{3}{2}}}{dx}+\frac{2e\left(\frac{x\sqrt{e x^2+d}}{2}+\frac{d\ln\left(x\sqrt{e}+\sqrt{e x^2+d}\right)}{2\sqrt{e}}\right)}{d}\right)}{a^2}-\frac{\sqrt{2}\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\left(\cdots\right.}{\cdots}$

input
```
int((e*x^2+d)^(1/2)/x^4/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(e*x^2+d)^(1/2)*(a*e*x^2-3*b*d*x^2+a*d)/a^2/x^3/d-1/a^2/((2*a*e-b*d+(
-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*2^(1/2)/((-2*a*e+b*d+(-4*d^2*(a*c-1/
4*b^2))^(1/2))*a)^(1/2)*((1/2*((b*e+c*d)*a-b^2*d)*(-4*d^2*(a*c-1/4*b^2))^(
1/2)+d*(a^2*c*e-1/2*b*(b*e+3*c*d)*a+1/2*b^3*d))*((-2*a*e+b*d+(-4*d^2*(a*c-
1/4*b^2))^(1/2))*a)^(1/2)*arctanh(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((2*a*e-b*d+
(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2))+((2*a*e-b*d+(-4*d^2*(a*c-1/4*b^2))
^(1/2))*a)^(1/2)*arctan(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((-2*a*e+b*d+(-4*d^2*(
a*c-1/4*b^2))^(1/2))*a)^(1/2))*(1/2*((-b*e-c*d)*a+b^2*d)*(-4*d^2*(a*c-1/4*
b^2))^(1/2)+d*(a^2*c*e-1/2*b*(b*e+3*c*d)*a+1/2*b^3*d)))/(-4*d^2*(a*c-1/4*b
^2))^(1/2)
```

3.365.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4095 vs. $2(323) = 646$.

Time = 6.51 (sec) , antiderivative size = 4095, normalized size of antiderivative = 10.98

$$\int \frac{\sqrt{d+ex^2}}{x^4\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^(1/2)/x^4/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
1/12*(3*sqrt(1/2)*a^2*d*x^3*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*
b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e - (a^5*b^2 - 4*a^6*c)*sqrt(((b^8 - 6*a*b^
6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5
*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2
*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(((a^5*b^2*c^2
- 4*a^6*c^3)*d*x^2*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3
 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*
e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)) +
2*(a*b^4*c^2 - 3*a^2*b^2*c^3 + a^3*c^4)*d^2 - 2*(a^2*b^3*c^2 - 2*a^3*b*c^3
)*d*e - ((b^5*c^2 - 3*a*b^3*c^3 + a^2*b*c^4)*d^2 - (5*a*b^4*c^2 - 14*a^2*b
^2*c^3 + 4*a^3*c^4)*d*e + 4*(a^2*b^3*c^2 - 2*a^3*b*c^3)*e^2)*x^2 + 2*sqrt(
1/2)*sqrt(e*x^2 + d)*((a^6*b^4 - 6*a^7*b^2*c + 8*a^8*c^2)*x*sqrt(((b^8 - 6
*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^
2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^
4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)) + ((a*b^7 - 7*a^2*b^5*c + 13*a^3*b^
3*c^2 - 4*a^4*b*c^3)*d - (a^2*b^6 - 6*a^3*b^4*c + 8*a^4*b^2*c^2)*e)*x)*sqr
t(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*
e - (a^5*b^2 - 4*a^6*c)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^
2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^
3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^1...
```

3.365.6 Sympy [F]

$$\int \frac{\sqrt{d+ex^2}}{x^4\left(a+bx^2+cx^4\right)}\,dx = \int \frac{\sqrt{d+ex^2}}{x^4\left(a+bx^2+cx^4\right)}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)/x**4/(c*x**4+b*x**2+a),x)
```

output
```
Integral(sqrt(d + e*x**2)/(x**4*(a + b*x**2 + c*x**4)), x)
```

3.365.7 Maxima [F]

$$\int \frac{\sqrt{d+ex^2}}{x^4\left(a+bx^2+cx^4\right)}\,dx=\int \frac{\sqrt{ex^2+d}}{(cx^4+bx^2+a)x^4}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/x^4/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^4), x)
```

3.365.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}}{x^4\left(a+bx^2+cx^4\right)}\,dx=\text{Timed out}$$

input
```
integrate((e*x^2+d)^(1/2)/x^4/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
Timed out
```

3.365.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}}{x^4\left(a+bx^2+cx^4\right)}\,dx=\int \frac{\sqrt{e\,x^2+d}}{x^4\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

input
```
int((d + e*x^2)^(1/2)/(x^4*(a + b*x^2 + c*x^4)),x)
```

output
```
int((d + e*x^2)^(1/2)/(x^4*(a + b*x^2 + c*x^4)), x)
```

3.366 $\int \frac{\sqrt{d+ex^2}}{x^6(a+bx^2+cx^4)}\,dx$

3.366.1 Optimal result

Integrand size = 29, antiderivative size = 512

$$\begin{aligned}
&\int \frac{\sqrt{d+ex^2}}{x^6\left(a+bx^2+cx^4\right)}\,dx \\
&= -\frac{\sqrt{d+ex^2}}{5ax^5}+\frac{4e\sqrt{d+ex^2}}{15adx^3}+\frac{(bd-ae)\sqrt{d+ex^2}}{3a^2dx^3}-\frac{8e^2\sqrt{d+ex^2}}{15ad^2x} \\
&\quad -\frac{2e(bd-ae)\sqrt{d+ex^2}}{3a^2d^2x}-\frac{\left(b^2d-acd-abe\right)\sqrt{d+ex^2}}{a^3dx} \\
&\quad -\frac{c\left(b^2d-acd-abe+\frac{b^3d-3abcd-ab^2e+2a^2ce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^3\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} \\
&\quad -\frac{c\left(b^2d-acd-abe-\frac{b^3d-3abcd-ab^2e+2a^2ce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^3\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}
\end{aligned}$$

output
```
-1/5*(e*x^2+d)^(1/2)/a/x^5+4/15*e*(e*x^2+d)^(1/2)/a/d/x^3+1/3*(-a*e+b*d)*(
e*x^2+d)^(1/2)/a^2/d/x^3-8/15*e^2*(e*x^2+d)^(1/2)/a/d^2/x-2/3*e*(-a*e+b*d)
*(e*x^2+d)^(1/2)/a^2/d^2/x-(-a*b*e-a*c*d+b^2*d)*(e*x^2+d)^(1/2)/a^3/d/x-c*
arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c
+b^2)^(1/2))^(1/2))*(b^2*d-a*c*d-a*b*e+(2*a^2*c*e-a*b^2*e-3*a*b*c*d+b^3*d)
/(-4*a*c+b^2)^(1/2))/a^3/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(b-(-4*a*c
+b^2)^(1/2))^(1/2)-c*arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^
2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2*d-a*c*d-a*b*e+(-2*a^2*c*e+a*
b^2*e+3*a*b*c*d-b^3*d)/(-4*a*c+b^2)^(1/2))/a^3/(b+(-4*a*c+b^2)^(1/2))^(1/2
)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.366.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 10511 vs. $2(512) = 1024$.

Time = 16.52 (sec) , antiderivative size = 10511, normalized size of antiderivative = 20.53

$$\int \frac{\sqrt{d+ex^2}}{x^6(a+bx^2+cx^4)}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[d + e*x^2]/(x^6*(a + b*x^2 + c*x^4)),x]
```

output
```
Result too large to show
```

3.366.3 Rubi [A] (verified)

Time = 2.51 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1618, 245, 245, 242, 2246, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}}{x^6(a+bx^2+cx^4)}\,dx$$

$$\Big\downarrow \text{1618}$$

$$\frac{d\int \frac{1}{x^6\sqrt{ex^2+d}}dx}{a}-\frac{\int \frac{cdx^2+bd-ae}{x^4\sqrt{ex^2+d}(cx^4+bx^2+a)}dx}{a}$$

$$\Big\downarrow \; 245$$

$$\frac{d\left(-\frac{4e\int \frac{1}{x^4\sqrt{ex^2+d}}dx}{5d}-\frac{\sqrt{d+ex^2}}{5dx^5}\right)}{a}-\frac{\int \frac{cdx^2+bd-ae}{x^4\sqrt{ex^2+d}(cx^4+bx^2+a)}dx}{a}$$

$$\Big\downarrow \; 245$$

$$\frac{d\left(-\frac{4e\left(-\frac{2e\int \frac{1}{x^2\sqrt{ex^2+d}}dx}{3d}-\frac{\sqrt{d+ex^2}}{3dx^3}\right)}{5d}-\frac{\sqrt{d+ex^2}}{5dx^5}\right)}{a}-\frac{\int \frac{cdx^2+bd-ae}{x^4\sqrt{ex^2+d}(cx^4+bx^2+a)}dx}{a}$$

$$\Big\downarrow \; 242$$

$$\frac{d\left(-\frac{4e\left(\frac{2e\sqrt{d+ex^2}}{3d^2x}-\frac{\sqrt{d+ex^2}}{3dx^3}\right)}{5d}-\frac{\sqrt{d+ex^2}}{5dx^5}\right)}{a}-\frac{\int \frac{cdx^2+bd-ae}{x^4\sqrt{ex^2+d}(cx^4+bx^2+a)}dx}{a}$$

$$\Big\downarrow \; 2246$$

$$\frac{d\left(-\frac{4e\left(\frac{2e\sqrt{d+ex^2}}{3d^2x}-\frac{\sqrt{d+ex^2}}{3dx^3}\right)}{5d}-\frac{\sqrt{d+ex^2}}{5dx^5}\right)}{a}-\frac{\int \left(\frac{bd-ae}{ax^4\sqrt{ex^2+d}}+\frac{-db^2+aeb+acd}{a^2x^2\sqrt{ex^2+d}}+\frac{db^3-aeb^2-2acdb+c(db^2-aeb-acd)x^2+a^2ce}{a^2\sqrt{ex^2+d}(cx^4+bx^2+a)}\right)dx}{a}$$

$$\Big\downarrow \; 2009$$

$$\frac{d\left(-\frac{4e\left(\frac{2e\sqrt{d+ex^2}}{3d^2x}-\frac{\sqrt{d+ex^2}}{3dx^3}\right)}{5d}-\frac{\sqrt{d+ex^2}}{5dx^5}\right)}{a}-\frac{\frac{c\left(\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{c\left(-\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)\arctan\left(\frac{x\sqrt{2cd-e(\cdots}}{\sqrt{\sqrt{b^2-4a\cdots}}}\right)}{a^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\cdots}{a}$$

input
```
Int[Sqrt[d + e*x^2]/(x^6*(a + b*x^2 + c*x^4)),x]
```

3.366. $\int \frac{\sqrt{d+ex^2}}{x^6(a+bx^2+cx^4)}\,dx$

output
```
(d*(-1/5*Sqrt[d + e*x^2]/(d*x^5) - (4*e*(-1/3*Sqrt[d + e*x^2]/(d*x^3) + (2
*e*Sqrt[d + e*x^2])/(3*d^2*x)))/(5*d)))/a - (-1/3*((b*d - a*e)*Sqrt[d + e*
x^2])/(a*d*x^3) + (2*e*(b*d - a*e)*Sqrt[d + e*x^2])/(3*a*d^2*x) + ((b^2*d
- a*c*d - a*b*e)*Sqrt[d + e*x^2])/(a^2*d*x) + (c*(b^2*d - a*c*d - a*b*e +
(b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d
+ e*x^2])])/(a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 -
4*a*c])*e]) + (c*(b^2*d - a*c*d - a*b*e - (b^3*d - 3*a*b*c*d - a*b^2*e + 2
*a^2*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*
e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a^2*Sqrt[b + Sqrt[b
^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/a
```

3.366.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 1618
```
Int[(((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4), x_Symbol] :> Simp[d/a   Int[(f*x)^m*(d + e*x^2)^(q - 1), x],
x] - Simp[1/(a*f^2)   Int[(f*x)^(m + 2)*(d + e*x^2)^(q - 1)*(Simp[b*d - a*e
 + c*d*x^2, x]/(a + b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && LtQ[m, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2246
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(f*x)^m*(d +
e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x
] && PolyQ[Px, x] && IntegerQ[p]
```

3.366. $\int \frac{\sqrt{d+ex^2}}{x^6(a+bx^2+cx^4)}\,dx$

3.366.4 Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 500, normalized size of antiderivative = 0.98

method	result
risch	$-\frac{\sqrt{e\,x^2+d}\,(-2a^2e^2x^4-5abde\,x^4-15ac\,d^2x^4+15b^2d^2x^4+a^2de\,x^2-5ab\,d^2x^2+3a^2d^2)}{15d^2a^3x^5}+\frac{\sqrt{2}\left(-\left((a^2ce+(-b^2e-2bcd)a+\right.\right.}{}$
pseudoelliptic	$\frac{5\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\left(\left(\left(\left(-2abc+b^3\right)d+ae\left(ac-b^2\right)\right)\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}+\left(-2a^2c^2+4a\,b^2c-b^4\right)d^2+e\left(-3cb\,a^2+a\,b^3\right)d\right)}{2}$ $-\frac{}{}$
default	$\frac{-\frac{\left(e\,x^2+d\right)^{\frac{3}{2}}}{5d\,x^5}+\frac{2e\left(e\,x^2+d\right)^{\frac{3}{2}}}{15d^2x^3}}{a}+\frac{b\left(e\,x^2+d\right)^{\frac{3}{2}}}{3a^2d\,x^3}+\frac{\left(-ac+b^2\right)\left(-\frac{\left(e\,x^2+d\right)^{\frac{3}{2}}}{dx}+\frac{2e\left(\frac{x\sqrt{e\,x^2+d}}{2}+\frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{2\sqrt{e}}\right)}{d}\right)}{a^3}-\frac{\left(\left(\right.\right.}{}$

input
```
int((e*x^2+d)^(1/2)/x^6/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(e*x^2+d)^(1/2)*(-2*a^2*e^2*x^4-5*a*b*d*e*x^4-15*a*c*d^2*x^4+15*b^2*
d^2*x^4+a^2*d*e*x^2-5*a*b*d^2*x^2+3*a^2*d^2)/d^2/a^3/x^5+1/2/a^3/((2*a*e-b
*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*2^(1/2)/((-2*a*e+b*d+(-4*d^2*(a*
c-1/4*b^2))^(1/2))*a)^(1/2)*(-((a^2*c*e+(-b^2*e-2*b*c*d)*a+b^3*d)*(-4*d^2*
(a*c-1/4*b^2))^(1/2)+(-3*b*c*d*e-2*c^2*d^2)*a^2+b^2*d*(b*e+4*c*d)*a-b^4*d^
2)*((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*arctanh(a/x*(e*x^2+
d)^(1/2)*2^(1/2)/((2*a*e-b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2))+((2*a
*e-b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*arctan(a/x*(e*x^2+d)^(1/2)*2
^(1/2)/((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2))*((a^2*c*e+(-b^
2*e-2*b*c*d)*a+b^3*d)*(-4*d^2*(a*c-1/4*b^2))^(1/2)+(3*b*c*d*e+2*c^2*d^2)*a
^2+(-b^3*d*e-4*b^2*c*d^2)*a+b^4*d^2))/(-4*d^2*(a*c-1/4*b^2))^(1/2)
```

3.366.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5773 vs. 2(450) = 900.

Time = 15.78 (sec) , antiderivative size = 5773, normalized size of antiderivative = 11.28

$$\int \frac{\sqrt{d+ex^2}}{x^6\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^(1/2)/x^6/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.366.6 Sympy [F]

$$\int \frac{\sqrt{d+ex^2}}{x^6\left(a+bx^2+cx^4\right)}\,dx = \int \frac{\sqrt{d+ex^2}}{x^6\left(a+bx^2+cx^4\right)}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)/x**6/(c*x**4+b*x**2+a),x)
```

output
```
Integral(sqrt(d + e*x**2)/(x**6*(a + b*x**2 + c*x**4)), x)
```

3.366.7 Maxima [F]

$$\int \frac{\sqrt{d+ex^2}}{x^6\left(a+bx^2+cx^4\right)}\,dx = \int \frac{\sqrt{ex^2+d}}{(cx^4+bx^2+a)x^6}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/x^6/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^6), x)
```

3.366.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}}{x^6(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(1/2)/x^6/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
Timed out
```

3.366.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}}{x^6(a+bx^2+cx^4)}\,dx = \int \frac{\sqrt{e\,x^2+d}}{x^6\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

input
```
int((d + e*x^2)^(1/2)/(x^6*(a + b*x^2 + c*x^4)),x)
```

output
```
int((d + e*x^2)^(1/2)/(x^6*(a + b*x^2 + c*x^4)), x)
```

3.367 $\int \frac{x^3(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx$

3.367.1 Optimal result

Integrand size = 29, antiderivative size = 460

$$\int \frac{x^3(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \frac{(cd-be)\sqrt{d+ex^2}}{c^2}+\frac{(d+ex^2)^{3/2}}{3c}$$

$$+\frac{\left(b^3e^2-b^2e\left(2cd+\sqrt{b^2-4ac}e\right)+c\left(a\sqrt{b^2-4ac}e^2-cd\left(\sqrt{b^2-4ac}d-4ae\right)\right)+bc\left(cd^2+e\left(2\sqrt{b^2-4ac}d-\right.\right.\right.}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

$$-\frac{\left(b^3e^2-b^2e\left(2cd-\sqrt{b^2-4ac}e\right)+bc\left(cd^2-e\left(2\sqrt{b^2-4ac}d+3ae\right)\right)-c\left(a\sqrt{b^2-4ac}e^2-cd\left(\sqrt{b^2-4ac}d+\right.\right.\right.}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

output
```
1/3*(e*x^2+d)^(3/2)/c+(-b*e+c*d)*(e*x^2+d)^(1/2)/c^2+1/2*arctanh(2^(1/2)*c
^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*(b^3*e^2-b^
2*e*(2*c*d+e*(-4*a*c+b^2)^(1/2))+c*(a*e^2*(-4*a*c+b^2)^(1/2)-c*d*(-4*a*e+d
*(-4*a*c+b^2)^(1/2)))+b*c*(c*d^2+e*(-3*a*e+2*d*(-4*a*c+b^2)^(1/2))))/c^(5/
2)*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)-1/2*a
rctanh(2^(1/2)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1
/2))*(b^3*e^2-b^2*e*(2*c*d-e*(-4*a*c+b^2)^(1/2))-c*(a*e^2*(-4*a*c+b^2)^(1/
2)-c*d*(4*a*e+d*(-4*a*c+b^2)^(1/2)))+b*c*(c*d^2-e*(3*a*e+2*d*(-4*a*c+b^2)^
(1/2))))/c^(5/2)*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)
))^(1/2)
```

3.367.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.76 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.09

$$\int \frac{x^3 (d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \frac{2\sqrt{c}\sqrt{d+ex^2}(4cd-3be+cex^2)+\frac{3\left(ib^3e^2+b^2e\left(-2icd+\sqrt{-b^2+4ac}e\right)+ibc\left(cd^2+e\left(2i\sqrt{-b^2+4ac}d-3\right.\right.\right.}{\sqrt{-\frac{b^2}{2}+}}}{}$$

input
```
Integrate[(x^3*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

output
```
(2*Sqrt[c]*Sqrt[d + e*x^2]*(4*c*d - 3*b*e + c*e*x^2) + (3*(I*b^3*e^2 + b^2
*e*((-2*I)*c*d + Sqrt[-b^2 + 4*a*c]*e) + I*b*c*(c*d^2 + e*((2*I)*Sqrt[-b^2
 + 4*a*c]*d - 3*a*e)) + c*(-(a*Sqrt[-b^2 + 4*a*c]*e^2) + c*d*(Sqrt[-b^2 +
4*a*c]*d + (4*I)*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c
*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[-1/2*b^2 + 2*a*c]*Sqrt[-2*c*d +
 (b - I*Sqrt[-b^2 + 4*a*c])*e]) + (3*((-I)*b^3*e^2 + b^2*e*((2*I)*c*d + Sq
rt[-b^2 + 4*a*c]*e) + b*c*((-I)*c*d^2 + e*(-2*Sqrt[-b^2 + 4*a*c]*d + (3*I)
*a*e)) + c*(-(a*Sqrt[-b^2 + 4*a*c]*e^2) + c*d*(Sqrt[-b^2 + 4*a*c]*d - (4*I
)*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e + I*Sq
rt[-b^2 + 4*a*c]*e]])/(Sqrt[-1/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b + I*Sqrt[-b
^2 + 4*a*c])*e]))/(6*c^(5/2))
```

3.367.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 398, normalized size of antiderivative = 0.87, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1578, 1196, 25, 1196, 1197, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \; 1578$$

$$\frac{1}{2}\int \frac{x^2 (ex^2+d)^{3/2}}{cx^4+bx^2+a}dx^2$$

$$\downarrow 1196$$

$$\frac{1}{2}\left(\frac{\int -\frac{\sqrt{ex^2+d}(ae-(cd-be)x^2)}{cx^4+bx^2+a}dx^2}{c}+\frac{2(d+ex^2)^{3/2}}{3c}\right)$$

$$\downarrow 25$$

$$\frac{1}{2}\left(\frac{2(d+ex^2)^{3/2}}{3c}-\frac{\int \frac{\sqrt{ex^2+d}(ae-(cd-be)x^2)}{cx^4+bx^2+a}dx^2}{c}\right)$$

$$\downarrow 1196$$

$$\frac{1}{2}\left(\frac{2(d+ex^2)^{3/2}}{3c}-\frac{\frac{\int \frac{ae(2cd-be)-\left(c^2d^2+b^2e^2-ce(2bd+ae)\right)x^2}{\sqrt{ex^2+d}\left(cx^4+bx^2+a\right)}dx^2}{c}-\frac{2\sqrt{d+ex^2}(cd-be)}{c}}{c}\right)$$

$$\downarrow 1197$$

$$\frac{1}{2}\left(\frac{2(d+ex^2)^{3/2}}{3c}-\frac{\frac{2\int \frac{(cd-be)\left(cd^2-bed+ae^2\right)-\left(c^2d^2+b^2e^2-ce(2bd+ae)\right)x^4}{cx^8-(2cd-be)x^4+cd^2+ae^2-bde}d\sqrt{ex^2+d}}{c}-\frac{2\sqrt{d+ex^2}(cd-be)}{c}}{c}\right)$$

$$\downarrow 1480$$

$$\frac{1}{2}\left(\frac{2(d+ex^2)^{3/2}}{3c}-\frac{2\left(-\frac{1}{2}\left(\frac{-bc\left(cd^2-3ae^2\right)-4ac^2de+b^3\left(-e^2\right)+2b^2cde}{\sqrt{b^2-4ac}}-ce(ae+2bd)+b^2e^2+c^2d^2\right)\int \frac{1}{cx^4+\frac{1}{2}\left(\left(b-\sqrt{b^2-4ac}\right)e-2cd\right)}d\sqrt{ex^2+d}-\right.}{c}\right.$$

$$\downarrow 221$$

$$\frac{1}{2}\left(\frac{2(d+ex^2)^{3/2}}{3c}-\frac{\frac{2\left(\frac{\left(\frac{-bc\left(cd^2-3ae^2\right)-4ac^2de+b^3\left(-e^2\right)+2b^2cde}{\sqrt{b^2-4ac}}-ce(ae+2bd)+b^2e^2+c^2d^2\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(-\frac{-bc\left(cd^2\right.}{}\right.}{}\right.}{c}}{c}\right.$$

3.367. $\int \frac{x^3(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx$

input
```
Int[(x^3*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

output
```
((2*(d + e*x^2)^(3/2))/(3*c) - ((-2*(c*d - b*e)*Sqrt[d + e*x^2])/c + (2*((
(c^2*d^2 + b^2*e^2 - c*e*(2*b*d + a*e) + (2*b^2*c*d*e - 4*a*c^2*d*e - b^3*
e^2 - b*c*(c*d^2 - 3*a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*S
qrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]
*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + ((c^2*d^2 + b^2*e^2 - c*e*(2*b
*d + a*e) - (2*b^2*c*d*e - 4*a*c^2*d*e - b^3*e^2 - b*c*(c*d^2 - 3*a*e^2))/
Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d -
(b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]*Sqrt[2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e])))/c)/c)/2
```

3.367.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1196
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Simp[1/c   Int
[(d + e*x)^(m - 1)*(Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x]/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && FractionQ[m] &
& GtQ[m, 0]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

3.367. $\int \frac{x^3(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx$

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.367.4 Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 447, normalized size of antiderivative = 0.97

method	result
risch	$-\frac{(-c x^2 e+3be-4cd)\sqrt{e x^2+d}}{3c^2} - \frac{\sqrt{2}\left(-\left(\left(-c^2d^2+(a e^2+2bde)c-b^2e^2\right)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}+(4d e^2 a+b d^2 e)c^2+(-3e^3ab-\right.\right.}{}$
default	$-\frac{-\left(\left((ac-b^2)e^2+2bcde-c^2d^2\right)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}+(-3abc+b^3)e^3+2d(2a c^2-b^2c)e^2+b c^2d^2e\right)\sqrt{2}\sqrt{\left(be-2cd+\sqrt{-4e^2\left(ac\right.}\right.}}{}$
pseudoelliptic	$-\frac{-\left(\left((ac-b^2)e^2+2bcde-c^2d^2\right)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}+(-3abc+b^3)e^3+2d(2a c^2-b^2c)e^2+b c^2d^2e\right)\sqrt{2}\sqrt{\left(be-2cd+\sqrt{-4e^2\left(ac\right.}\right.}}{}$

input
```
int(x^3*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-c*e*x^2+3*b*e-4*c*d)*(e*x^2+d)^(1/2)/c^2-1/2/c^2/((b*e-2*c*d+(-4*e^
2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*2^(1/2)*(-((-c^2*d^2+(a*e^2+2*b*d*e)*c-b^
2*e^2)*(-4*e^2*(a*c-1/4*b^2))^(1/2)+(4*a*d*e^2+b*d^2*e)*c^2+(-3*a*b*e^3-2*
b^2*d*e^2)*c+b^3*e^3)*((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*a
rctanh(c*(e*x^2+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2)
)*c)^(1/2))+((-c^2*d^2+(a*e^2+2*b*d*e)*c-b^2*e^2)*(-4*e^2*(a*c-1/4*b^2))^(
1/2)+(-4*a*d*e^2-b*d^2*e)*c^2+(3*a*b*e^3+2*b^2*d*e^2)*c-b^3*e^3)*arctan(c*
(e*x^2+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)
)*((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))/((-b*e+2*c*d+(-4*e^
2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)/(-4*e^2*(a*c-1/4*b^2))^(1/2)
```

3.367.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8200 vs. 2(400) = 800.

Time = 249.64 (sec) , antiderivative size = 8200, normalized size of antiderivative = 17.83

$$\int \frac{x^3(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.367.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.367.7 Maxima [F]

$$\int \frac{x^3(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}x^3}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^3*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^(3/2)*x^3/(c*x^4 + b*x^2 + a), x)
```

3.367.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 867 vs. $2(400) = 800$.

Time = 0.39 (sec) , antiderivative size = 867, normalized size of antiderivative = 1.88

$$\int \frac{x^3(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \frac{(ex^2+d)^{\frac{3}{2}}c^2 + 3\sqrt{ex^2+d}c^2d - 3\sqrt{ex^2+d}bce}{3c^3}$$

$$-\frac{(2bc^5d^3e + ((b^2c^2-4ac^3)d^2 - 2(b^3c-4abc^2)de + (b^4-5ab^2c+4a^2c^2)e^2)c^2e^2 - (5b^2c^4-8ac^5)d^2e^2 +}{(}$$

$$+\frac{(2bc^5d^3e + ((b^2c^2-4ac^3)d^2 - 2(b^3c-4abc^2)de + (b^4-5ab^2c+4a^2c^2)e^2)c^2e^2 - (5b^2c^4-8ac^5)d^2e^2 + 2}{(2}$$

input
```
integrate(x^3*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/3*((e*x^2 + d)^(3/2)*c^2 + 3*sqrt(e*x^2 + d)*c^2*d - 3*sqrt(e*x^2 + d)*b
*c*e)/c^3 - (2*b*c^5*d^3*e + ((b^2*c^2 - 4*a*c^3)*d^2 - 2*(b^3*c - 4*a*b*c
^2)*d*e + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^2)*c^2*e^2 - (5*b^2*c^4 - 8*a*c^
5)*d^2*e^2 + 2*(2*b^3*c^3 - 5*a*b*c^4)*d*e^3 - (b^4*c^2 - 3*a*b^2*c^3)*e^4
 - 2*(sqrt(b^2 - 4*a*c)*c^4*d^3 - 2*sqrt(b^2 - 4*a*c)*b*c^3*d^2*e - sqrt(b
^2 - 4*a*c)*a*b*c^2*e^3 + (b^2*c^2 + a*c^3)*sqrt(b^2 - 4*a*c)*d*e^2)*abs(c
)*abs(e))*arctan(2*sqrt(1/2)*sqrt(e*x^2 + d)/sqrt(-(2*c^4*d - b*c^3*e + sq
rt(-4*(c^4*d^2 - b*c^3*d*e + a*c^3*e^2)*c^4 + (2*c^4*d - b*c^3*e)^2))/c^4)
)/((2*sqrt(b^2 - 4*a*c)*c^3*d - (b^2*c^2 - 4*a*c^3 + sqrt(b^2 - 4*a*c)*b*c
^2)*e)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*c^2*abs(e)) + (2*b
*c^5*d^3*e + ((b^2*c^2 - 4*a*c^3)*d^2 - 2*(b^3*c - 4*a*b*c^2)*d*e + (b^4 -
 5*a*b^2*c + 4*a^2*c^2)*e^2)*c^2*e^2 - (5*b^2*c^4 - 8*a*c^5)*d^2*e^2 + 2*(
2*b^3*c^3 - 5*a*b*c^4)*d*e^3 - (b^4*c^2 - 3*a*b^2*c^3)*e^4 + 2*(sqrt(b^2 -
 4*a*c)*c^4*d^3 - 2*sqrt(b^2 - 4*a*c)*b*c^3*d^2*e - sqrt(b^2 - 4*a*c)*a*b*
c^2*e^3 + (b^2*c^2 + a*c^3)*sqrt(b^2 - 4*a*c)*d*e^2)*abs(c)*abs(e))*arctan
(2*sqrt(1/2)*sqrt(e*x^2 + d)/sqrt(-(2*c^4*d - b*c^3*e - sqrt(-4*(c^4*d^2 -
 b*c^3*d*e + a*c^3*e^2)*c^4 + (2*c^4*d - b*c^3*e)^2))/c^4))/((2*sqrt(b^2 -
 4*a*c)*c^3*d + (b^2*c^2 - 4*a*c^3 - sqrt(b^2 - 4*a*c)*b*c^2)*e)*sqrt(-4*c
^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*c^2*abs(e))
```

3.367.9 Mupad [B] (verification not implemented)

Time = 9.38 (sec) , antiderivative size = 16951, normalized size of antiderivative = 36.85

$$\int \frac{x^3(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((x^3*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x)
```

output
```
(d + e*x^2)^(3/2)/(3*c) - atan((((((4*a*b^3*c^3*e^5 - 16*a^2*b*c^4*e^5 + 16
*a*c^6*d^3*e^2 + 16*a^2*c^5*d*e^4 - 4*b^4*c^3*d*e^4 - 4*b^2*c^5*d^3*e^2 +
8*b^3*c^4*d^2*e^3 - 32*a*b*c^5*d^2*e^3 + 12*a*b^2*c^4*d*e^4)/c^3 - (2*(d +
 e*x^2)^(1/2)*(-(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c
^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*
b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*
b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^
2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e
^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e
^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) +
 2*b^7*e^3 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*c^4*d^3 - 40*a^3*b*
c^3*e^3 + 48*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^3*c^2*e^3 - 18*a*b
^5*c*e^3 - 6*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*c^2*d*e^2 + 72*a^
2*b*c^4*d^2*e - 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2
*c^6)))^(1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6*e^3 + 32*a*c^7
*d*e^2))/c^3)*(-(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c
^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*
b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*
b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^
2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^...
```

3.368 $\int \frac{x(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx$

3.368.1 Optimal result

Integrand size = 27, antiderivative size = 327

$$\int \frac{x(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \frac{e\sqrt{d+ex^2}}{c} - \frac{\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}d+ae\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} + \frac{\left(2c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-2ce\left(bd+\sqrt{b^2-4ac}d+ae\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

output
```
e*(e*x^2+d)^(1/2)/c-1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(
b-(-4*a*c+b^2)^(1/2)))^(1/2))*(2*c^2*d^2+b*e^2*(b-(-4*a*c+b^2)^(1/2))-2*c*
e*(b*d+a*e-d*(-4*a*c+b^2)^(1/2)))/c^(3/2)*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*
d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+d)^(1
/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*(2*c^2*d^2+b*e^2*(b+(-4*a*c+b^
2)^(1/2))-2*c*e*(b*d+a*e+d*(-4*a*c+b^2)^(1/2)))/c^(3/2)*2^(1/2)/(-4*a*c+b^
2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.368.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.20 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.14

$$\int \frac{x(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \frac{2\sqrt{c}e\sqrt{d+ex^2} + \frac{\left(-2ic^2d^2-b\left(ib+\sqrt{-b^2+4ac}\right)e^2+2ce\left(ibd+\sqrt{-b^2+4ac}d+iae\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-2cd+be-i\sqrt{-b^2+}}}\right.}{\sqrt{-\frac{b^2}{2}+2ac}\sqrt{-2cd+\left(b-i\sqrt{-b^2+4ac}\right)e}}}{2c^{3/}}$$

input
```
Integrate[(x*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

output
```
(2*Sqrt[c]*e*Sqrt[d + e*x^2] + (((-2*I)*c^2*d^2 - b*(I*b + Sqrt[-b^2 + 4*a
*c])*e^2 + 2*c*e*(I*b*d + Sqrt[-b^2 + 4*a*c]*d + I*a*e))*ArcTan[(Sqrt[2]*S
qrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqr
t[-1/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b - I*Sqrt[-b^2 + 4*a*c])*e]) + (((2*I)
*c^2*d^2 - b*((-I)*b + Sqrt[-b^2 + 4*a*c])*e^2 + 2*c*e*((-I)*b*d + Sqrt[-b
^2 + 4*a*c]*d - I*a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c
*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[-1/2*b^2 + 2*a*c]*Sqrt[-2*c*d +
 (b + I*Sqrt[-b^2 + 4*a*c])*e]))/(2*c^(3/2))
```

3.368.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1576, 1146, 1197, 25, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(d+ex^2)^{3/2}}{a+bx^2+cx^4}dx$$

$$\Big\downarrow \text{ 1576}$$

$$\frac{1}{2}\int \frac{(ex^2+d)^{3/2}}{cx^4+bx^2+a}dx^2$$

$$\Big\downarrow \text{ 1146}$$

$$\frac{1}{2}\left(\frac{\int \frac{cd^2-ae^2+e(2cd-be)x^2}{\sqrt{ex^2+d}(cx^4+bx^2+a)}dx^2}{c}+\frac{2e\sqrt{d+ex^2}}{c}\right)$$

$$\Big\downarrow \text{1197}$$

$$\frac{1}{2}\left(\frac{2\int -\frac{e(-((2cd-be)x^4)+cd^2+ae^2-bde)}{cx^8-(2cd-be)x^4+cd^2+ae^2-bde}d\sqrt{ex^2+d}}{c}+\frac{2e\sqrt{d+ex^2}}{c}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{2e\sqrt{d+ex^2}}{c}-\frac{2\int \frac{e(-((2cd-be)x^4)+cd^2+ae^2-bde)}{cx^8-(2cd-be)x^4+cd^2+ae^2-bde}d\sqrt{ex^2+d}}{c}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{2e\sqrt{d+ex^2}}{c}-\frac{2e\int \frac{-((2cd-be)x^4)+cd^2+ae^2-bde}{cx^8-(2cd-be)x^4+cd^2+ae^2-bde}d\sqrt{ex^2+d}}{c}\right)$$

$$\Big\downarrow \text{1480}$$

$$\frac{1}{2}\left(\frac{2e\sqrt{d+ex^2}}{c}-\frac{2e\left(-\frac{1}{2}\left(\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{e\sqrt{b^2-4ac}}-be+2cd\right)\int \frac{1}{cx^4+\frac{1}{2}\left(\left(b-\sqrt{b^2-4ac}\right)e-2cd\right)}d\sqrt{ex^2+d}-\frac{1}{2}\left(-\frac{-2ce(a\right.\right.}{c}\right.$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{2e\sqrt{d+ex^2}}{c}-\frac{2e\left(\frac{\left(\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{e\sqrt{b^2-4ac}}-be+2cd\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(-\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{e\sqrt{b^2-4ac}}-be+2cd\right.}{\sqrt{2}\sqrt{c}\sqrt{2cd-e(}}\right.}{c}\right.$$

input
```
Int[(x*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

3.368. $\int \frac{x\left(d+ex^2\right)^{3/2}}{a+bx^2+cx^4}\,dx$

output
```
((2*e*Sqrt[d + e*x^2])/c - (2*e*(((2*c*d - b*e + (2*c^2*d^2 + b^2*e^2 - 2*
c*e*(b*d + a*e))/(Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d +
e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]*Sqrt[2*
c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + ((2*c*d - b*e - (2*c^2*d^2 + b^2*e^2 -
 2*c*e*(b*d + a*e))/(Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d
 + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]*Sqrt
[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])))/c)/2
```

3.368.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1146
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Simp[1/c   Int[(d + e*x)^
(m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
 x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

3.368. $\int \frac{x(d+ex^2)^{3/2}}{a+bx^2+cx^4} \, dx$

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

3.368.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.02

method	result
risch	$\frac{e\sqrt{e x^2+d}}{c}-\frac{e\sqrt{2}\left(-\frac{\left(2e^2ac-b^2e^2+2bcde-2c^2d^2+\sqrt{-e^2\left(4ac-b^2\right)}\,be-2\sqrt{-e^2\left(4ac-b^2\right)}\,cd\right)\operatorname{arctanh}\left(\frac{c\sqrt{e x^2+d}\,\sqrt{2}}{\sqrt{\left(-be+2cd+\sqrt{-e^2\left(4ac-b\right.}\right.}}\right.}{\sqrt{\left(-be+2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right.}{2c\sqrt{}}$
default	$\frac{e\left(\sqrt{e x^2+d}-\frac{\left(-2e^2ac+b^2e^2-2bcde+2c^2d^2+\sqrt{-e^2\left(4ac-b^2\right)}\,be-2\sqrt{-e^2\left(4ac-b^2\right)}\,cd\right)\sqrt{2}\,\arctan\left(\frac{c\sqrt{e x^2+d}\,\sqrt{2}}{\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right)}{2\sqrt{-e^2\left(4ac-b^2\right)}\,\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}-\right.}{c}$
pseudoelliptic	$\frac{e\left(\sqrt{e x^2+d}-\frac{\left(-2e^2ac+b^2e^2-2bcde+2c^2d^2+\sqrt{-e^2\left(4ac-b^2\right)}\,be-2\sqrt{-e^2\left(4ac-b^2\right)}\,cd\right)\sqrt{2}\,\arctan\left(\frac{c\sqrt{e x^2+d}\,\sqrt{2}}{\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right)}{2\sqrt{-e^2\left(4ac-b^2\right)}\,\sqrt{\left(be-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}-\right.}{c}$

input
```
int(x*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
e*(e*x^2+d)^(1/2)/c-1/2/c*e*2^(1/2)/(-e^2*(4*a*c-b^2))^(1/2)*(-(2*e^2*a*c-
b^2*e^2+2*b*c*d*e-2*c^2*d^2+(-e^2*(4*a*c-b^2))^(1/2)*b*e-2*(-e^2*(4*a*c-b^
2))^(1/2)*c*d)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(
e*x^2+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))+(-
2*e^2*a*c+b^2*e^2-2*b*c*d*e+2*c^2*d^2+(-e^2*(4*a*c-b^2))^(1/2)*b*e-2*(-e^2
*(4*a*c-b^2))^(1/2)*c*d)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*ar
ctan(c*(e*x^2+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1
/2)))
```

3.368.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4444 vs. 2(278) = 556.

Time = 65.87 (sec) , antiderivative size = 4444, normalized size of antiderivative = 13.59

$$\int \frac{x(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(1/2)*c*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*
e^2 - (b^3 - 3*a*b*c)*e^3 + (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b
*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5
 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c
^4))*log(-(6*b*c^3*d^5*e - 6*(2*b^2*c^2 + a*c^3)*d^4*e^2 + 8*(b^3*c + 2*a*
b*c^2)*d^3*e^3 - 2*(b^4 + 6*a*b^2*c + 2*a^2*c^2)*d^2*e^4 + 2*(2*a*b^3 + a^
2*b*c)*d*e^5 - 2*(a^2*b^2 - a^3*c)*e^6 + (3*b*c^3*d^4*e^2 - 6*b^2*c^2*d^3*
e^3 + 2*(2*b^3*c + a*b*c^2)*d^2*e^4 - (b^4 + 2*a*b^2*c)*d*e^5 + (a*b^3 - a
^2*b*c)*e^6)*x^2 + 2*sqrt(1/2)*(3*(b^2*c^3 - 4*a*c^4)*d^3*e - 6*(b^3*c^2 -
 4*a*b*c^3)*d^2*e^2 + (4*b^4*c - 17*a*b^2*c^2 + 4*a^2*c^3)*d*e^3 - (b^5 -
5*a*b^3*c + 4*a^2*b*c^2)*e^4 + ((b^3*c^4 - 4*a*b*c^5)*d - (b^4*c^3 - 6*a*b
^2*c^4 + 8*a^2*c^5)*e)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c
^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2
*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))*sqrt(e*x^2 + d)*sqrt((2*c^3*d^3 - 3*b*c^2
*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 + (b^2*c^3 - 4*a*
c^4)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*
e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^
6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) + (2*(b^2*c^4 - 4*a*c^5)*d^3 - 2*(b^3*
c^3 - 4*a*b*c^4)*d^2*e + 2*(a*b^2*c^3 - 4*a^2*c^4)*d*e^2 + ((b^2*c^4 - 4*a
*c^5)*d^2*e - (b^3*c^3 - 4*a*b*c^4)*d*e^2 + (a*b^2*c^3 - 4*a^2*c^4)*e^3...
```

3.368.6 Sympy [F]

$$\int \frac{x(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \int \frac{x(d+ex^2)^{\frac{3}{2}}}{a+bx^2+cx^4}\,dx$$

input
```
integrate(x*(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

output
```
Integral(x*(d + e*x**2)**(3/2)/(a + b*x**2 + c*x**4), x)
```

3.368.7 Maxima [F]

$$\int \frac{x(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}x}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^(3/2)*x/(c*x^4 + b*x^2 + a), x)
```

3.368.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 802 vs. $2(278) = 556$.

Time = 0.34 (sec) , antiderivative size = 802, normalized size of antiderivative = 2.45

$$\int \frac{x(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \frac{\sqrt{ex^2+d}e}{c}$$

$$+ \frac{\left(\sqrt{-4\,c^2d+2\left(bc-\sqrt{b^2-4\,ac}c\right)e}(2\left(b^2c-4\,ac^2\right)de-(b^3-4\,abc)e^2)c^2e^2-2\left(\sqrt{b^2-4\,ac}c^3d^2e-\sqrt{b^2-\right.\right.}{}$$

$$- \frac{\left(\sqrt{-4\,c^2d+2\left(bc+\sqrt{b^2-4\,ac}c\right)e}(2\left(b^2c-4\,ac^2\right)de-(b^3-4\,abc)e^2)c^2e^2+2\left(\sqrt{b^2-4\,ac}c^3d^2e-\sqrt{b^2-\right.\right.}{}$$

input
```
integrate(x*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
sqrt(e*x^2 + d)*e/c + 1/8*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e
)*(2*(b^2*c - 4*a*c^2)*d*e - (b^3 - 4*a*b*c)*e^2)*c^2*e^2 - 2*(sqrt(b^2 -
4*a*c)*c^3*d^2*e - sqrt(b^2 - 4*a*c)*b*c^2*d*e^2 + sqrt(b^2 - 4*a*c)*a*c^2
*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c)*abs(e) - (4*
c^5*d^3*e - 6*b*c^4*d^2*e^2 + 4*(b^2*c^3 - a*c^4)*d*e^3 - (b^3*c^2 - 2*a*b
*c^3)*e^4)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt
(1/2)*sqrt(e*x^2 + d)/sqrt(-(2*c^2*d - b*c*e + sqrt(-4*(c^2*d^2 - b*c*d*e
+ a*c*e^2)*c^2 + (2*c^2*d - b*c*e)^2))/c^2))/((sqrt(b^2 - 4*a*c)*c^4*d^2 -
 sqrt(b^2 - 4*a*c)*b*c^3*d*e + sqrt(b^2 - 4*a*c)*a*c^3*e^2)*c^2*abs(e)) -
1/8*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*(2*(b^2*c - 4*a*c^2)
*d*e - (b^3 - 4*a*b*c)*e^2)*c^2*e^2 + 2*(sqrt(b^2 - 4*a*c)*c^3*d^2*e - sqr
t(b^2 - 4*a*c)*b*c^2*d*e^2 + sqrt(b^2 - 4*a*c)*a*c^2*e^3)*sqrt(-4*c^2*d +
2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(c)*abs(e) - (4*c^5*d^3*e - 6*b*c^4*d^
2*e^2 + 4*(b^2*c^3 - a*c^4)*d*e^3 - (b^3*c^2 - 2*a*b*c^3)*e^4)*sqrt(-4*c^2
*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x^2 + d)/
sqrt(-(2*c^2*d - b*c*e - sqrt(-4*(c^2*d^2 - b*c*d*e + a*c*e^2)*c^2 + (2*c^
2*d - b*c*e)^2))/c^2))/((sqrt(b^2 - 4*a*c)*c^4*d^2 - sqrt(b^2 - 4*a*c)*b*c
^3*d*e + sqrt(b^2 - 4*a*c)*a*c^3*e^2)*c^2*abs(e))
```

3.368.9 Mupad [B] (verification not implemented)

Time = 9.76 (sec) , antiderivative size = 12392, normalized size of antiderivative = 37.90

$$\int \frac{x(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((x*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x)
```

output
```
(e*(d + e*x^2)^(1/2))/c - atan(((((16*a^2*c^3*e^5 - 4*a*b^2*c^2*e^5 + 16*a
*c^4*d^2*e^3 + 4*b^3*c^2*d*e^4 - 4*b^2*c^3*d^2*e^3 - 16*a*b*c^3*d*e^4)/c -
 (2*(d + e*x^2)^(1/2)*(-(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a
^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b
^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 1
6*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*
e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5
- 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*b
^2*c^3*d^3 + 24*a^2*b*c^2*e^3 - 48*a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*
b^3*c*e^3 - 6*b^4*c*d*e^2 - 24*a*b*c^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16
*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2
 - 16*a*b*c^4*e^3 + 32*a*c^5*d*e^2))/c)*(-(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*
b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*
a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4
- (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*
e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^
2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^5*e^3 +
16*a*c^4*d^3 - 4*b^2*c^3*d^3 + 24*a^2*b*c^2*e^3 - 48*a^2*c^3*d*e^2 + 6*b^3
*c^2*d^2*e - 14*a*b^3*c*e^3 - 6*b^4*c*d*e^2 - 24*a*b*c^3*d^2*e + 36*a*b^2*
c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*(d + e...
```

3.368. $\int \frac{x(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx$

3.369 $\int \frac{(d+ex^2)^{3/2}}{x(a+bx^2+cx^4)}\,dx$

3.369.1 Optimal result

Integrand size = 29, antiderivative size = 346

$$\int \frac{(d+ex^2)^{3/2}}{x(a+bx^2+cx^4)}\,dx = -\frac{d^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a} - \frac{\left(a\sqrt{b^2-4ac}e^2-cd\left(\sqrt{b^2-4ac}d-4ae\right)-b(cd^2+ae^2)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{\left(a\sqrt{b^2-4ac}e^2-cd\left(\sqrt{b^2-4ac}d+4ae\right)+b(cd^2+ae^2)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

output
```
-d^(3/2)*arctanh((e*x^2+d)^(1/2)/d^(1/2))/a-1/2*arctanh(2^(1/2)*c^(1/2)*(e
*x^2+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*(-b*(a*e^2+c*d^2)+a*
e^2*(-4*a*c+b^2)^(1/2)-c*d*(-4*a*e+d*(-4*a*c+b^2)^(1/2)))/a*2^(1/2)/c^(1/2
)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)-1/2*arctanh(2^
(1/2)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*(b*(
a*e^2+c*d^2)+a*e^2*(-4*a*c+b^2)^(1/2)-c*d*(4*a*e+d*(-4*a*c+b^2)^(1/2)))/a*
2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.369.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.39 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.10

$$\int \frac{(d+ex^2)^{3/2}}{x(a+bx^2+cx^4)}\,dx =$$

$$-\frac{\frac{\sqrt{2}\left(-a\sqrt{-b^2+4ac}e^2+cd\left(\sqrt{-b^2+4ac}d+4iae\right)-ib(cd^2+ae^2)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-2cd+be-i\sqrt{-b^2+4ac}e}}\right)}{\sqrt{c}\sqrt{-b^2+4ac}\sqrt{-2cd+\left(b-i\sqrt{-b^2+4ac}\right)e}}+\frac{\sqrt{2}\left(-a\sqrt{-b^2+4ac}e^2+cd\left(\sqrt{-b^2+4ac}d-4i\right.\right.}{\sqrt{c}\sqrt{-b^2+4ac}\sqrt{}}}{2a}$$

input
```
Integrate[(d + e*x^2)^(3/2)/(x*(a + b*x^2 + c*x^4)),x]
```

output
```
-1/2*((Sqrt[2]*(-(a*Sqrt[-b^2 + 4*a*c]*e^2) + c*d*(Sqrt[-b^2 + 4*a*c]*d +
(4*I)*a*e) - I*b*(c*d^2 + a*e^2))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])
/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[c]*Sqrt[-b^2 + 4*a*c]
*Sqrt[-2*c*d + (b - I*Sqrt[-b^2 + 4*a*c])*e]) + (Sqrt[2]*(-(a*Sqrt[-b^2 +
4*a*c]*e^2) + c*d*(Sqrt[-b^2 + 4*a*c]*d - (4*I)*a*e) + I*b*(c*d^2 + a*e^2)
)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2
 + 4*a*c]*e]])/(Sqrt[c]*Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + (b + I*Sqrt[-b^2
+ 4*a*c])*e]) + 2*d^(3/2)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/a
```

3.369.3 Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1578, 1199, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{3/2}}{x(a+bx^2+cx^4)}dx$$

$$\Big\downarrow \ 1578$$

$$\frac{1}{2}\int \frac{(ex^2+d)^{3/2}}{x^2(cx^4+bx^2+a)}dx^2$$

$$\Big\downarrow \ 1199$$

$$\frac{\int \left(\frac{e\left(d\left(cd^2-bed+ae^2\right)-\left(cd^2-ae^2\right)x^4\right)}{a\left(cx^8-(2cd-be)x^4+cd^2+ae^2-bde\right)}-\frac{d^2e}{a(d-x^4)}\right)d\sqrt{ex^2+d}}{e}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{e\left(-cd\left(d\sqrt{b^2-4ac}-4ae\right)+ae^2\sqrt{b^2-4ac}-b(ae^2+cd^2)\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{e\left(-cd\left(d\sqrt{b^2-4ac}+4ae\right)+ae^2\sqrt{b^2-4ac}+b(ae^2\right.}{\sqrt{2}a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd}}}{e}$$

input
```
Int[(d + e*x^2)^(3/2)/(x*(a + b*x^2 + c*x^4)),x]
```

output
```
(-((d^(3/2)*e*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/a) - (e*(a*Sqrt[b^2 - 4*a*
c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e) - b*(c*d^2 + a*e^2))*ArcTanh[(S
qrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/
(Sqrt[2]*a*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*
e]) - (e*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e) + b*
(c*d^2 + a*e^2))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b
 + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e]))/e
```

3.369.3.1 Defintions of rubi rules used

rule 1199
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Denominator[m]}, Simp[q/e   Subs
t[Int[ExpandIntegrand[x^(q*(m + 1) - 1)*(((e*f - d*g)/e + g*(x^q/e))^n/((c*
d^2 - b*d*e + a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))), x],
 x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Integer
Q[n] && FractionQ[m]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.369. $\int \frac{\left(d+ex^2\right)^{3/2}}{x(a+bx^2+cx^4)}\,dx$

3.369.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.16

method	result
pseudoelliptic	$\frac{-\sqrt{2}\sqrt{\left(be-2cd+\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}\right)c}\left(\left(a e^2-c d^2\right)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}-e^3ab+4acd e^2-d^2ebc\right)\operatorname{arctanh}\left(\frac{c\sqrt{e x^2+d}\cdots}{\sqrt{\left(-be+2cd+\sqrt{-4e\cdots}\right.}}\right.}{\cdots}$
default	$\frac{\frac{\left(e x^2+d\right)^{\frac{3}{2}}}{3}+d\left(\sqrt{e x^2+d}-\sqrt{d}\ln\left(\frac{2d+2\sqrt{d}\sqrt{e x^2+d}}{x}\right)\right)}{a}+\frac{-\sqrt{2}\sqrt{\left(be-2cd+\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}\right)c}\left(\left(a e^2-c d^2\right)\sqrt{-4e^2\left(ac\cdots\right.}\right.}{\cdots}$

input
```
int((e*x^2+d)^(3/2)/x/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*(-2^(1/2)*((b*e-2*
c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*((a*e^2-c*d^2)*(-4*e^2*(a*c-1/4
*b^2))^(1/2)-e^3*a*b+4*a*c*d*e^2-d^2*e*b*c)*arctanh(c*(e*x^2+d)^(1/2)*2^(1
/2)/((-b*e+2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))+((-b*e+2*c*d+(-4*
e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2)*(2^(1/2)*((a*e^2-c*d^2)*(-4*e^2*(a*c-1/
4*b^2))^(1/2)+e*(a*b*e^2-4*a*c*d*e+b*c*d^2))*arctan(c*(e*x^2+d)^(1/2)*2^(1
/2)/((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))*c)^(1/2))-2*d^(3/2)*arctanh(
(e*x^2+d)^(1/2)/d^(1/2))*(-4*e^2*(a*c-1/4*b^2))^(1/2)*((b*e-2*c*d+(-4*e^2*
(a*c-1/4*b^2))^(1/2))*c)^(1/2)))/((b*e-2*c*d+(-4*e^2*(a*c-1/4*b^2))^(1/2))
*c)^(1/2)/(-4*e^2*(a*c-1/4*b^2))^(1/2)/a
```

3.369.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\left(d+ex^2\right)^{3/2}}{x\left(a+bx^2+cx^4\right)}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(3/2)/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.369.6 Sympy [F]

$$\int \frac{(d+ex^2)^{3/2}}{x(a+bx^2+cx^4)}\,dx=\int \frac{(d+ex^2)^{\frac{3}{2}}}{x(a+bx^2+cx^4)}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)/x/(c*x**4+b*x**2+a),x)
```

output
```
Integral((d + e*x**2)**(3/2)/(x*(a + b*x**2 + c*x**4)), x)
```

3.369.7 Maxima [F]

$$\int \frac{(d+ex^2)^{3/2}}{x(a+bx^2+cx^4)}\,dx=\int \frac{(ex^2+d)^{\frac{3}{2}}}{(cx^4+bx^2+a)x}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^(3/2)/((c*x^4 + b*x^2 + a)*x), x)
```

3.369.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 838 vs. 2(292) = 584.

Time = 0.34 (sec) , antiderivative size = 838, normalized size of antiderivative = 2.42

$$\int \frac{(d+ex^2)^{3/2}}{x(a+bx^2+cx^4)}\,dx=\frac{d^2\arctan\left(\frac{\sqrt{ex^2+d}}{\sqrt{-d}}\right)}{a\sqrt{-d}}$$

$$-\frac{\left(\sqrt{-4\,c^2d+2\left(bc-\sqrt{b^2-4\,ac}c\right)e}((b^2c-4\,ac^2)d^2-(ab^2-4\,a^2c)e^2)a^2e^2-2\left(\sqrt{b^2-4\,ac}ac^2d^3-\sqrt{b^2-\right.}{}$$

$$+\frac{\left(\sqrt{-4\,c^2d+2\left(bc+\sqrt{b^2-4\,ac}c\right)e}((b^2c-4\,ac^2)d^2-(ab^2-4\,a^2c)e^2)a^2e^2+2\left(\sqrt{b^2-4\,ac}ac^2d^3-\sqrt{b^2-\right.}{}$$

input
```
integrate((e*x^2+d)^(3/2)/x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
d^2*arctan(sqrt(e*x^2 + d)/sqrt(-d))/(a*sqrt(-d)) - 1/8*(sqrt(-4*c^2*d + 2
*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^2*c - 4*a*c^2)*d^2 - (a*b^2 - 4*a^2*c)
*e^2)*a^2*e^2 - 2*(sqrt(b^2 - 4*a*c)*a*c^2*d^3 - sqrt(b^2 - 4*a*c)*a*b*c*d
^2*e + sqrt(b^2 - 4*a*c)*a^2*c*d*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 -
4*a*c)*c)*e)*abs(a)*abs(e) - (2*a^2*b*c^2*d^3*e + 6*a^3*b*c*d*e^3 - a^3*b^
2*e^4 - (a^2*b^2*c + 8*a^3*c^2)*d^2*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2
 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x^2 + d)/sqrt(-(2*a*c*d - a*b*e
 + sqrt(-4*(a*c*d^2 - a*b*d*e + a^2*e^2)*a*c + (2*a*c*d - a*b*e)^2))/(a*c)
))/((sqrt(b^2 - 4*a*c)*a^2*c^2*d^2 - sqrt(b^2 - 4*a*c)*a^2*b*c*d*e + sqrt(
b^2 - 4*a*c)*a^3*c*e^2)*abs(a)*abs(c)*abs(e)) + 1/8*(sqrt(-4*c^2*d + 2*(b*
c + sqrt(b^2 - 4*a*c)*c)*e)*((b^2*c - 4*a*c^2)*d^2 - (a*b^2 - 4*a^2*c)*e^2
)*a^2*e^2 + 2*(sqrt(b^2 - 4*a*c)*a*c^2*d^3 - sqrt(b^2 - 4*a*c)*a*b*c*d^2*e
 + sqrt(b^2 - 4*a*c)*a^2*c*d*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*
c)*c)*e)*abs(a)*abs(e) - (2*a^2*b*c^2*d^3*e + 6*a^3*b*c*d*e^3 - a^3*b^2*e^
4 - (a^2*b^2*c + 8*a^3*c^2)*d^2*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4
*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x^2 + d)/sqrt(-(2*a*c*d - a*b*e - s
qrt(-4*(a*c*d^2 - a*b*d*e + a^2*e^2)*a*c + (2*a*c*d - a*b*e)^2))/(a*c)))/(
(sqrt(b^2 - 4*a*c)*a^2*c^2*d^2 - sqrt(b^2 - 4*a*c)*a^2*b*c*d*e + sqrt(b^2
- 4*a*c)*a^3*c*e^2)*abs(a)*abs(c)*abs(e))
```

3.369.9 Mupad [B] (verification not implemented)

Time = 11.99 (sec) , antiderivative size = 28434, normalized size of antiderivative = 82.18

$$\int \frac{(d+ex^2)^{3/2}}{x(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^2)^(3/2)/(x*(a + b*x^2 + c*x^4)),x)
```

output
```
atan((((d + e*x^2)^(1/2)*(2*a^4*c*e^16 + 6*c^5*d^8*e^8 - 16*a*c^4*d^6*e^10
 - 16*b*c^4*d^7*e^9 + 4*b^4*c*d^4*e^12 + 16*a^2*c^3*d^4*e^12 + 8*a^3*c^2*d
^2*e^14 + 24*b^2*c^3*d^6*e^10 - 16*b^3*c^2*d^5*e^11 - 8*a^3*b*c*d*e^15 - 8
*a*b^3*c*d^3*e^13 + 16*a*b^2*c^2*d^4*e^12 - 24*a^2*b*c^2*d^3*e^13 + 12*a^2
*b^2*c*d^2*e^14) + (-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*
a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*
a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c -
128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*
c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*
d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3
*d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2
*e - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c
- 8*a^3*b^2*c^2)))^(1/2)*(((-(((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d
^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2
*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*
b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^
4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 -
3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16
*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b
^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 +...
```

3.369. $\int \frac{(d+ex^2)^{3/2}}{x(a+bx^2+cx^4)}\,dx$

3.370 $\int \frac{(d+ex^2)^{3/2}}{x^3(a+bx^2+cx^4)}\,dx$

3.370.1 Optimal result

Integrand size = 29, antiderivative size = 417

$$\int \frac{(d+ex^2)^{3/2}}{x^3(a+bx^2+cx^4)}\,dx = -\frac{d\sqrt{d+ex^2}}{2ax^2} + \frac{\sqrt{d}\,e\,\mathrm{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2a} + \frac{\sqrt{d}(bd-2ae)\mathrm{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^2}$$
$$-\frac{\sqrt{c}\left(b^2d^2+bd\left(\sqrt{b^2-4ac}\,d-2ae\right)-2a\left(cd^2+e\left(\sqrt{b^2-4ac}\,d-ae\right)\right)\right)\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$
$$+\frac{\sqrt{c}\left(b^2d^2-bd\left(\sqrt{b^2-4ac}\,d+2ae\right)-2a\left(cd^2-e\left(\sqrt{b^2-4ac}\,d+ae\right)\right)\right)\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

output
```
1/2*e*arctanh((e*x^2+d)^(1/2)/d^(1/2))*d^(1/2)/a+(-2*a*e+b*d)*arctanh((e*x
^2+d)^(1/2)/d^(1/2))*d^(1/2)/a^2-1/2*d*(e*x^2+d)^(1/2)/a/x^2-1/2*arctanh(2
^(1/2)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*c^(
1/2)*(b^2*d^2+b*d*(-2*a*e+d*(-4*a*c+b^2)^(1/2))-2*a*(c*d^2+e*(-a*e+d*(-4*a
*c+b^2)^(1/2))))/a^2*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(
1/2)))^(1/2)+1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b+(-4*a
*c+b^2)^(1/2)))^(1/2))*c^(1/2)*(b^2*d^2-b*d*(2*a*e+d*(-4*a*c+b^2)^(1/2))-2
*a*(c*d^2-e*(a*e+d*(-4*a*c+b^2)^(1/2))))/a^2*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2
*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.370.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.74 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.02

$$\int \frac{\left(d+ex^2\right)^{3/2}}{x^3\left(a+bx^2+cx^4\right)}\,dx = \frac{-\frac{ad\sqrt{d+ex^2}}{x^2}+\frac{\sqrt{2}\sqrt{c}\left(-ib^2d^2+bd\left(\sqrt{-b^2+4ac}d+2iae\right)-2ia\left(-cd^2+e\left(-i\sqrt{-b^2+4ac}d+ae\right)\right)\right)\arctan\left(\frac{\quad}{\sqrt{-\cdots}}\right.}{\sqrt{-b^2+4ac}\sqrt{-2cd+\left(b-i\sqrt{-b^2+4ac}\right)e}}}{\quad}$$

input
```
Integrate[(d + e*x^2)^(3/2)/(x^3*(a + b*x^2 + c*x^4)),x]
```

output
```
(-((a*d*Sqrt[d + e*x^2])/x^2) + (Sqrt[2]*Sqrt[c]*((-I)*b^2*d^2 + b*d*(Sqrt
[-b^2 + 4*a*c]*d + (2*I)*a*e) - (2*I)*a*(-(c*d^2) + e*((-I)*Sqrt[-b^2 + 4*
a*c]*d + a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e
 - I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + (b - I*Sqrt
[-b^2 + 4*a*c])*e]) + (Sqrt[2]*Sqrt[c]*(I*b^2*d^2 + b*d*(Sqrt[-b^2 + 4*a*c
]*d - (2*I)*a*e) + (2*I)*a*(-(c*d^2) + e*(I*Sqrt[-b^2 + 4*a*c]*d + a*e)))*
ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 +
 4*a*c]*e]])/(Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + (b + I*Sqrt[-b^2 + 4*a*c])*
e]) + Sqrt[d]*(2*b*d - 3*a*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(2*a^2)
```

3.370.3 Rubi [A] (warning: unable to verify)

Time = 1.80 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1578, 1199, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{3/2}}{x^3(a+bx^2+cx^4)}\,dx$$

$$\Big\downarrow \ 1578$$

$$\frac{1}{2}\int \frac{(ex^2+d)^{3/2}}{x^4(cx^4+bx^2+a)}dx^2$$

$$\Big\downarrow \ 1199$$

$$\frac{\int \left(\frac{d^2e^2}{a(d-x^4)^2}+\frac{d(bd-2ae)e}{a^2(d-x^4)}-\frac{((bd-ae)(cd^2-bed+ae^2)-cd(bd-2ae)x^4)e}{a^2(cx^8-(2cd-be)x^4+cd^2+ae^2-bde)}\right)d\sqrt{ex^2+d}}{e}$$

$$\Big\downarrow \ 2009$$

$$-\frac{\sqrt{c}e\left(-2a\left(e\left(d\sqrt{b^2-4ac}-ae\right)+cd^2\right)+bd\left(d\sqrt{b^2-4ac}-2ae\right)+b^2d^2\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{c}e\left(-bd\left(d\sqrt{b^2-4ac}+2ae\right)+2ae\left(d\right.\right.}{\sqrt{2}a}$$

input
```
Int[(d + e*x^2)^(3/2)/(x^3*(a + b*x^2 + c*x^4)),x]
```

output
```
((d*e^2*Sqrt[d + e*x^2])/(2*a*(d - x^4)) + (Sqrt[d]*e^2*ArcTanh[Sqrt[d + e
*x^2]/Sqrt[d]])/(2*a) + (Sqrt[d]*e*(b*d - 2*a*e)*ArcTanh[Sqrt[d + e*x^2]/S
qrt[d]])/a^2 - (Sqrt[c]*e*(b^2*d^2 + b*d*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) - 2
*a*(c*d^2 + e*(Sqrt[b^2 - 4*a*c]*d - a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[
d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a^2*Sqrt[b^
2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[c]*e*(b^2*d^2
- 2*a*c*d^2 + 2*a*e*(Sqrt[b^2 - 4*a*c]*d + a*e) - b*d*(Sqrt[b^2 - 4*a*c]*d
 + 2*a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqr
t[b^2 - 4*a*c])*e]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt
[b^2 - 4*a*c])*e]))/e
```

3.370.3.1 Defintions of rubi rules used

rule 1199
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Denominator[m]}, Simp[q/e   Subs
t[Int[ExpandIntegrand[x^(q*(m + 1) - 1)*(((e*f - d*g)/e + g*(x^q/e))^n/((c*
d^2 - b*d*e + a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))), x],
 x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Integer
Q[n] && FractionQ[m]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.370.4 Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 399, normalized size of antiderivative = 0.96

method	result
risch	$-\frac{d\sqrt{e\,x^2+d}}{2a\,x^2}-\frac{\frac{\sqrt{d}\,(3ae-2bd)\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^2+d}}{x}\right)}{a}+\frac{c\sqrt{2}\left(-\frac{\left(-2e^3a^2+2abd\,e^2+2ac\,d^2e-b^2d^2e+2\sqrt{-e^2\left(4ac-b^2\right)}\,ade-\sqrt{-e^2\left(4\cdots\right.}\right.}{\sqrt{\left(-be+2cd+\sqrt{-e^2\left(\cdots\right.}\right.}}\cdots\right.}{\cdots}}{\cdots}$
pseudoelliptic	$-\frac{\sqrt{\left(be-2cd+\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}\right)c}\,\sqrt{2}\,c\left(\left(-d^{\frac{3}{2}}ae+\frac{b\,d^{\frac{5}{2}}}{2}\right)\sqrt{-4e^2\left(ac-\frac{b^2}{4}\right)}+e\left(\left(-ac+\frac{b^2}{2}\right)d^{\frac{5}{2}}+ae\left(ea\sqrt{d}-d^{\frac{3}{2}}b\right)\right)\right)x^2\,\mathrm{ar}\cdots}{\cdots}$
default	$\frac{-\frac{\left(e\,x^2+d\right)^{\frac{5}{2}}}{2d\,x^2}+\frac{3e\left(\frac{\left(e\,x^2+d\right)^{\frac{3}{2}}}{3}+d\left(\sqrt{e\,x^2+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^2+d}}{x}\right)\right)\right)}{2d}}{a}-\frac{b\left(\frac{\left(e\,x^2+d\right)^{\frac{3}{2}}}{3}+d\left(\sqrt{e\,x^2+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{\cdots}}{x}\right.\right.\right.\cdots}{a^2}$

input
```
int((e*x^2+d)^(3/2)/x^3/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*d*(e*x^2+d)^(1/2)/a/x^2-1/2/a*(d^(1/2)*(3*a*e-2*b*d)/a*ln((2*d+2*d^(1
/2)*(e*x^2+d)^(1/2))/x)+1/a*c*2^(1/2)/(-e^2*(4*a*c-b^2))^(1/2)*(-(-2*e^3*a
^2+2*a*b*d*e^2+2*a*c*d^2*e-b^2*d^2*e+2*(-e^2*(4*a*c-b^2))^(1/2)*a*d*e-(-e^
2*(4*a*c-b^2))^(1/2)*b*d^2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2
)*arctanh(c*(e*x^2+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))
*c)^(1/2))+(2*e^3*a^2-2*a*b*d*e^2-2*a*c*d^2*e+b^2*d^2*e+2*(-e^2*(4*a*c-b^2
))^(1/2)*a*d*e-(-e^2*(4*a*c-b^2))^(1/2)*b*d^2)/((b*e-2*c*d+(-e^2*(4*a*c-b^
2))^(1/2))*c)^(1/2)*arctan(c*(e*x^2+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*
a*c-b^2))^(1/2))*c)^(1/2))))
```

3.370.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}}{x^3(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(3/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.370.6 Sympy [F]

$$\int \frac{(d+ex^2)^{3/2}}{x^3(a+bx^2+cx^4)}\,dx = \int \frac{(d+ex^2)^{\frac{3}{2}}}{x^3(a+bx^2+cx^4)}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)/x**3/(c*x**4+b*x**2+a),x)
```

output
```
Integral((d + e*x**2)**(3/2)/(x**3*(a + b*x**2 + c*x**4)), x)
```

3.370.7 Maxima [F]

$$\int \frac{(d+ex^2)^{3/2}}{x^3(a+bx^2+cx^4)}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}}{(cx^4+bx^2+a)x^3}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^(3/2)/((c*x^4 + b*x^2 + a)*x^3), x)
```

3.370.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 898 vs. $2(355) = 710$.

Time = 0.33 (sec) , antiderivative size = 898, normalized size of antiderivative = 2.15

$$\int \frac{(d+ex^2)^{3/2}}{x^3(a+bx^2+cx^4)}\,dx = -\frac{(2\,bd^2-3\,ade)\arctan\left(\frac{\sqrt{ex^2+d}}{\sqrt{-d}}\right)}{2\,a^2\sqrt{-d}}$$

$$+\frac{\left(\sqrt{-4\,c^2d+2\left(bc-\sqrt{b^2-4\,ac}c\right)e}((b^3-4\,abc)d^2-2\,(ab^2-4\,a^2c)de)e^2-2\left(\sqrt{b^2-4\,ac}bcd^3+2\sqrt{b^2-4\right.}{}$$

$$-\frac{\left(\sqrt{-4\,c^2d+2\left(bc+\sqrt{b^2-4\,ac}c\right)e}((b^3-4\,abc)d^2-2\,(ab^2-4\,a^2c)de)e^2+2\left(\sqrt{b^2-4\,ac}bcd^3+2\sqrt{b^2-4\right.}{}$$

$$-\frac{\sqrt{ex^2+d}d}{2\,ax^2}$$

input
```
integrate((e*x^2+d)^(3/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*(2*b*d^2 - 3*a*d*e)*arctan(sqrt(e*x^2 + d)/sqrt(-d))/(a^2*sqrt(-d)) +
 1/8*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^3 - 4*a*b*c)*d^
2 - 2*(a*b^2 - 4*a^2*c)*d*e)*e^2 - 2*(sqrt(b^2 - 4*a*c)*b*c*d^3 + 2*sqrt(b
^2 - 4*a*c)*a*b*d*e^2 - sqrt(b^2 - 4*a*c)*a^2*e^3 - (b^2 + a*c)*sqrt(b^2 -
 4*a*c)*d^2*e)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(e) + (
2*a^2*b*e^4 - 2*(b^2*c - 2*a*c^2)*d^3*e + (b^3 + 2*a*b*c)*d^2*e^2 - 2*(a*b
^2 + 2*a^2*c)*d*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*arc
tan(2*sqrt(1/2)*sqrt(e*x^2 + d)/sqrt(-(2*a^2*c*d - a^2*b*e + sqrt(-4*(a^2*
c*d^2 - a^2*b*d*e + a^3*e^2)*a^2*c + (2*a^2*c*d - a^2*b*e)^2))/(a^2*c)))/(
(sqrt(b^2 - 4*a*c)*a^2*c*d^2 - sqrt(b^2 - 4*a*c)*a^2*b*d*e + sqrt(b^2 - 4*
a*c)*a^3*e^2)*abs(c)*abs(e)) - 1/8*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*
a*c)*c)*e)*((b^3 - 4*a*b*c)*d^2 - 2*(a*b^2 - 4*a^2*c)*d*e)*e^2 + 2*(sqrt(b
^2 - 4*a*c)*b*c*d^3 + 2*sqrt(b^2 - 4*a*c)*a*b*d*e^2 - sqrt(b^2 - 4*a*c)*a^
2*e^3 - (b^2 + a*c)*sqrt(b^2 - 4*a*c)*d^2*e)*sqrt(-4*c^2*d + 2*(b*c + sqrt
(b^2 - 4*a*c)*c)*e)*abs(e) + (2*a^2*b*e^4 - 2*(b^2*c - 2*a*c^2)*d^3*e + (b
^3 + 2*a*b*c)*d^2*e^2 - 2*(a*b^2 + 2*a^2*c)*d*e^3)*sqrt(-4*c^2*d + 2*(b*c
+ sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(e*x^2 + d)/sqrt(-(2*a^2
*c*d - a^2*b*e - sqrt(-4*(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*a^2*c + (2*a^2*
c*d - a^2*b*e)^2))/(a^2*c)))/((sqrt(b^2 - 4*a*c)*a^2*c*d^2 - sqrt(b^2 - 4*
a*c)*a^2*b*d*e + sqrt(b^2 - 4*a*c)*a^3*e^2)*abs(c)*abs(e)) - 1/2*sqrt(e...
```

3.370.9 Mupad [B] (verification not implemented)

Time = 11.06 (sec) , antiderivative size = 35855, normalized size of antiderivative = 85.98

$$\int \frac{(d+ex^2)^{3/2}}{x^3(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^2)^(3/2)/(x^3*(a + b*x^2 + c*x^4)),x)
```

output
```
(d^(1/2)*atan(((d^(1/2)*(3*a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(4*a^6*c^3*e^1
6 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*
c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c
^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^
3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c
^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^
5*e^11 - 60*a^4*b*c^4*d^3*e^13))/(2*a^4) - (d^(1/2)*((56*a^4*c^6*d^6*e^9 -
 44*a^5*c^5*d^4*e^11 - 100*a^6*c^4*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e^8 - 39*
a^2*b^5*c^3*d^5*e^10 - 11*a^2*b^6*c^2*d^4*e^11 - 108*a^3*b^2*c^5*d^6*e^9 +
 96*a^3*b^3*c^4*d^5*e^10 + 111*a^3*b^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2*d^3*e
^12 - 237*a^4*b^2*c^4*d^4*e^11 - 161*a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^4*c^2
*d^2*e^13 + 111*a^5*b^2*c^3*d^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c^4*d
^7*e^8 + 6*a*b^6*c^3*d^6*e^9 + 2*a*b^7*c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*e^8
 + 92*a^4*b*c^5*d^5*e^10 + 252*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14)/
a^4 + (d^(1/2)*(3*a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(64*a^7*b*c^3*e^13 + 35
2*a^7*c^4*d*e^12 - 16*a^6*b^3*c^2*e^13 - 160*a^5*c^6*d^5*e^8 + 736*a^6*c^5
*d^3*e^10 + 32*a^2*b^6*c^3*d^5*e^8 - 32*a^2*b^7*c^2*d^4*e^9 - 224*a^3*b^4*
c^4*d^5*e^8 + 144*a^3*b^5*c^3*d^4*e^9 + 112*a^3*b^6*c^2*d^3*e^10 + 432*a^4
*b^2*c^5*d^5*e^8 + 144*a^4*b^3*c^4*d^4*e^9 - 716*a^4*b^4*c^3*d^3*e^10 - 13
2*a^4*b^5*c^2*d^2*e^11 + 936*a^5*b^2*c^4*d^3*e^10 + 860*a^5*b^3*c^3*d^2...
```

3.370. $\int \frac{(d+ex^2)^{3/2}}{x^3(a+bx^2+cx^4)}\,dx$

3.371 $\int \frac{x^4(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx$

3.371.1 Optimal result

Integrand size = 29, antiderivative size = 595

$$\begin{aligned}
\int \frac{x^4(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx &= \frac{(3cd-4be)x\sqrt{d+ex^2}}{8c^2}+\frac{x(d+ex^2)^{3/2}}{4c} \\
&- \frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2c^3\sqrt{b-\sqrt{b^2-4ac}}} \\
&- \frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\left(bcd-b^2e+ace+\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2c^3\sqrt{b+\sqrt{b^2-4ac}}} \\
&+ \frac{d(3cd-4be)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8c^2\sqrt{e}} \\
&- \frac{\sqrt{e}\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^3} \\
&- \frac{\sqrt{e}\left(bcd-b^2e+ace+\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^3}
\end{aligned}$$

output
```
1/4*x*(e*x^2+d)^(3/2)/c+1/8*d*(-4*b*e+3*c*d)*arctanh(x*e^(1/2)/(e*x^2+d)^(
1/2))/c^2/e^(1/2)-1/2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*(b*c*d-b^2*e+a*c*
e+(-3*a*b*c*e+2*a*c^2*d+b^3*e-b^2*c*d)/(-4*a*c+b^2)^(1/2))*e^(1/2)/c^3-1/2
*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*(b*c*d-b^2*e+a*c*e+(3*a*b*c*e-2*a*c^2*
d-b^3*e+b^2*c*d)/(-4*a*c+b^2)^(1/2))*e^(1/2)/c^3+1/8*(-4*b*e+3*c*d)*x*(e*x
^2+d)^(1/2)/c^2-1/2*arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2
+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b*c*d-b^2*e+a*c*e+(-3*a*b*c*e+2*a
*c^2*d+b^3*e-b^2*c*d)/(-4*a*c+b^2)^(1/2))*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))
^(1/2)/c^3/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x*(2*c*d-e*(b+(-4*a*c+b
^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b*c*d-b^2
*e+a*c*e+(3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)/(-4*a*c+b^2)^(1/2))*(2*c*d-e*
(b+(-4*a*c+b^2)^(1/2)))^(1/2)/c^3/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.371.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 2.30 (sec) , antiderivative size = 1407, normalized size of antiderivative = 2.36

$$\int \frac{x^4(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^4*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

output
```
(c*x*Sqrt[d + e*x^2]*(5*c*d - 4*b*e + 2*c*e*x^2) + (2*(3*c^2*d^2 + 8*b^2*e
^2 - 4*c*e*(3*b*d + 2*a*e))*ArcTanh[(Sqrt[e]*x)/(-Sqrt[d] + Sqrt[d + e*x^2
])])/Sqrt[e] - 2*RootSum[a*e^4 + 4*b*d*e^2*#1^2 - 4*a*e^3*#1^2 + 16*c*d^2*
#1^4 - 8*b*d*e*#1^4 + 6*a*e^2*#1^4 + 4*b*d*#1^6 - 4*a*e*#1^6 + a*#1^8 & ,
(-(a*c^2*d^2*e^3*Log[x]) + 2*a*b*c*d*e^4*Log[x] - a*b^2*e^5*Log[x] + a^2*c
*e^5*Log[x] + a*c^2*d^2*e^3*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1] - 2*a*b
*c*d*e^4*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1] + a*b^2*e^5*Log[-Sqrt[d] +
 Sqrt[d + e*x^2] - x*#1] - a^2*c*e^5*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1
] - 4*b*c^2*d^3*e*Log[x]*#1^2 + 8*b^2*c*d^2*e^2*Log[x]*#1^2 - 5*a*c^2*d^2*
e^2*Log[x]*#1^2 - 4*b^3*d*e^3*Log[x]*#1^2 + 2*a*b*c*d*e^3*Log[x]*#1^2 + 3*
a*b^2*e^4*Log[x]*#1^2 - 3*a^2*c*e^4*Log[x]*#1^2 + 4*b*c^2*d^3*e*Log[-Sqrt[
d] + Sqrt[d + e*x^2] - x*#1]*#1^2 - 8*b^2*c*d^2*e^2*Log[-Sqrt[d] + Sqrt[d
+ e*x^2] - x*#1]*#1^2 + 5*a*c^2*d^2*e^2*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x
*#1]*#1^2 + 4*b^3*d*e^3*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^2 - 2*a*
b*c*d*e^3*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^2 - 3*a*b^2*e^4*Log[-S
qrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^2 + 3*a^2*c*e^4*Log[-Sqrt[d] + Sqrt[d
+ e*x^2] - x*#1]*#1^2 + 4*b*c^2*d^3*Log[x]*#1^4 - 8*b^2*c*d^2*e*Log[x]*#1^
4 + 5*a*c^2*d^2*e*Log[x]*#1^4 + 4*b^3*d*e^2*Log[x]*#1^4 - 2*a*b*c*d*e^2*Lo
g[x]*#1^4 - 3*a*b^2*e^3*Log[x]*#1^4 + 3*a^2*c*e^3*Log[x]*#1^4 - 4*b*c^2*d^
3*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^4 + 8*b^2*c*d^2*e*Log[-Sqrt...
```

3.371.3 Rubi [A] (verified)

Time = 2.09 (sec) , antiderivative size = 594, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1614, 299, 211, 224, 219, 2256, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \left(d+ex^2\right)^{3/2}}{a+bx^2+cx^4} \, dx$$

$$\Big\downarrow \text{1614}$$

$$\frac{\int \sqrt{ex^2+d}\left(cex^2+cd-be\right)dx}{c^2}-\frac{\int \frac{\sqrt{ex^2+d}\left(\left(-eb^2+cdb+ace\right)x^2+a(cd-be)\right)}{cx^4+bx^2+a}dx}{c^2}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{1}{4}(3cd-4be)\int \sqrt{ex^2+d}dx+\frac{1}{4}cx\left(d+ex^2\right)^{3/2}}{c^2}-\frac{\int \frac{\sqrt{ex^2+d}\left(\left(-eb^2+cdb+ace\right)x^2+a(cd-be)\right)}{cx^4+bx^2+a}dx}{c^2}$$

$$\Big\downarrow \ 211$$

$$\frac{\frac{1}{4}(3cd-4be)\left(\frac{1}{2}d\int \frac{1}{\sqrt{ex^2+d}}dx+\frac{1}{2}x\sqrt{d+ex^2}\right)+\frac{1}{4}cx\left(d+ex^2\right)^{3/2}}{c^2}-\frac{\int \frac{\sqrt{ex^2+d}\left(\left(-eb^2+cdb+ace\right)x^2+a(cd-be)\right)}{cx^4+bx^2+a}dx}{c^2}$$

$$\Big\downarrow \ 224$$

$$\frac{\frac{1}{4}(3cd-4be)\left(\frac{1}{2}d\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}+\frac{1}{2}x\sqrt{d+ex^2}\right)+\frac{1}{4}cx\left(d+ex^2\right)^{3/2}}{c^2}-\frac{\int \frac{\sqrt{ex^2+d}\left(\left(-eb^2+cdb+ace\right)x^2+a(cd-be)\right)}{cx^4+bx^2+a}dx}{c^2}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{1}{4}\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{e}}+\frac{1}{2}x\sqrt{d+ex^2}\right)(3cd-4be)+\frac{1}{4}cx\left(d+ex^2\right)^{3/2}}{c^2}-\frac{\int \frac{\sqrt{ex^2+d}\left(\left(-eb^2+cdb+ace\right)x^2+a(cd-be)\right)}{cx^4+bx^2+a}dx}{c^2}$$

$$\Big\downarrow \ 2256$$

$$\frac{\frac{1}{4}\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{e}}+\frac{1}{2}x\sqrt{d+ex^2}\right)(3cd-4be)+\frac{1}{4}cx\left(d+ex^2\right)^{3/2}}{c^2}-\frac{\int \left(\frac{\sqrt{ex^2+d}\left(-eb^2+cdb+ace-\frac{eb^3-cdb^2-3aceb+2ac^2d}{\sqrt{b^2-4ac}}\right)}{2cx^2+b+\sqrt{b^2-4ac}}+\frac{\left(-eb^2+cdb+ace+\frac{eb^3-cdb^2-3aceb+2ac^2d}{\sqrt{b^2-4ac}}\right)\sqrt{ex^2+d}}{2cx^2+b-\sqrt{b^2-4ac}}\right)dx}{c^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{1}{4}\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{e}}+\frac{1}{2}x\sqrt{d+ex^2}\right)(3cd-4be)+\frac{1}{4}cx\left(d+ex^2\right)^{3/2}}{c^2}-\frac{\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2c\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(\frac{3abce-2c\cdots}{\cdots}\right.}{\cdots}}{\cdots}$$

input
```
Int[(x^4*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

3.371. $\int \frac{x^4\left(d+ex^2\right)^{3/2}}{a+bx^2+cx^4}\,dx$

output
```
-(((Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(b*c*d - b^2*e + a*c*e - (b^2*
c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d
 - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x
^2])])/(2*c*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e]*(b*c*d - b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*
e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(
Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*c*Sqrt[b + Sqrt[b^2 - 4*
a*c]]) + (Sqrt[e]*(b*c*d - b^2*e + a*c*e - (b^2*c*d - 2*a*c^2*d - b^3*e +
3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c)
+ (Sqrt[e]*(b*c*d - b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c
*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c))/c^2) +
 ((c*x*(d + e*x^2)^(3/2))/4 + ((3*c*d - 4*b*e)*((x*Sqrt[d + e*x^2])/2 + (d
*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*Sqrt[e])))/4)/c^2
```

3.371.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

3.371. $\int \frac{x^4(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx$

rule 1614
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Simp[f^4/c^2   Int[(f*x)^(m - 4)*(c*d - b*e + c
*e*x^2)*(d + e*x^2)^(q - 1), x], x] - Simp[f^4/c^2   Int[(f*x)^(m - 4)*(d +
 e*x^2)^(q - 1)*(Simp[a*(c*d - b*e) + (b*c*d - b^2*e + a*c*e)*x^2, x]/(a +
b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c,
 0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2256
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

3.371.4 Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 460, normalized size of antiderivative = 0.77

method	result
risch	$-\frac{x\left(-2c\,x^{2}e+4be-5cd\right)\sqrt{e\,x^{2}+d}}{8c^{2}}-\frac{\frac{\left(8e^{2}ac-8b^{2}e^{2}+12bcde-3c^{2}d^{2}\right)\ln\left(x\sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{c\sqrt{e}}-\frac{4a\sqrt{2}\left(\left(\left(e^{2}ac-(be-cd)^{2}\right)\sqrt{-4d^{2}\left(ac-\right.}\right.\right.}{}}{}$
pseudoelliptic	$-\frac{2\left(a\sqrt{2}\sqrt{\left(-2ae+bd+\sqrt{-4d^{2}\left(ac-\frac{b^{2}}{4}\right)}\right)a}\left(\frac{\left(\frac{\left(-ac+b^{2}\right)e^{\frac{5}{2}}}{2}+\frac{c^{2}d^{2}\sqrt{e}}{2}-b\,e^{\frac{3}{2}}cd\right)\sqrt{-4d^{2}\left(ac-\frac{b^{2}}{4}\right)}}{2}+\left(dc\left(ac-\frac{b^{2}}{2}\right)e^{\frac{3}{2}}-\frac{3b\left(\right.}{}\right.\right.\right.}{}$
default	$\frac{\frac{x\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{4}+\frac{3d\left(\frac{x\sqrt{e\,x^{2}+d}}{2}+\frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{2\sqrt{e}}\right)}{4}}{c}-\frac{2\left(a\sqrt{2}\sqrt{\left(-2ae+bd+\sqrt{-4d^{2}\left(ac-\frac{b^{2}}{4}\right)}\right)a}\left(\frac{\left(\frac{\left(-ac+b^{2}\right)e^{\frac{5}{2}}}{2}+\frac{c^{2}d^{2}}{2}\right.}{}\right.\right.}{}$

input
```
int(x^4*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/8*x*(-2*c*e*x^2+4*b*e-5*c*d)*(e*x^2+d)^(1/2)/c^2-1/8/c^2*((8*a*c*e^2-8*
b^2*e^2+12*b*c*d*e-3*c^2*d^2)/c*ln(x*e^(1/2)+(e*x^2+d)^(1/2))/e^(1/2)-4/c/
((2*a*e-b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*a*2^(1/2)/((-2*a*e+b*d+
(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*(((e^2*a*c-(b*e-c*d)^2)*(-4*d^2*(a*
c-1/4*b^2))^(1/2)+3*(e*c*(b*e-4/3*c*d)*a-1/3*b*(b*e-c*d)^2)*d)*((-2*a*e+b*
d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*arctanh(a/x*(e*x^2+d)^(1/2)*2^(1/
2)/((2*a*e-b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2))-((2*a*e-b*d+(-4*d^2
*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*arctan(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((-2*a*
e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2))*((e^2*a*c-(b*e-c*d)^2)*(-4*d
^2*(a*c-1/4*b^2))^(1/2)-3*(e*c*(b*e-4/3*c*d)*a-1/3*b*(b*e-c*d)^2)*d))/(-4*
d^2*(a*c-1/4*b^2))^(1/2))
```

3.371.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9908 vs. $2(515) = 1030$.

Time = 290.82 (sec) , antiderivative size = 19825, normalized size of antiderivative = 33.32

$$\int \frac{x^4(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.371.6 Sympy [F]

$$\int \frac{x^4(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \int \frac{x^4(d+ex^2)^{\frac{3}{2}}}{a+bx^2+cx^4}\,dx$$

input
```
integrate(x**4*(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

output
```
Integral(x**4*(d + e*x**2)**(3/2)/(a + b*x**2 + c*x**4), x)
```

3.371.7 Maxima [F]

$$\int \frac{x^4(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}x^4}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^4*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^(3/2)*x^4/(c*x^4 + b*x^2 + a), x)
```

3.371.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^4(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.371.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \int \frac{x^4\,(e\,x^2+d)^{3/2}}{c\,x^4+b\,x^2+a}\,dx$$

input
```
int((x^4*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x)
```

output
```
int((x^4*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4), x)
```

3.372 $\int \frac{x^2(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx$

3.372.1 Optimal result

Integrand size = 29, antiderivative size = 491

$$
\begin{aligned}
\int \frac{x^2(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx &= \frac{ex\sqrt{d+ex^2}}{2c} \\
&+ \frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2c^2\sqrt{b-\sqrt{b^2-4ac}}} \\
&+ \frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2c^2\sqrt{b+\sqrt{b^2-4ac}}} \\
&+ \frac{d\sqrt{e}\,\mathrm{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c} + \frac{\sqrt{e}\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\mathrm{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^2} \\
&+ \frac{\sqrt{e}\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\mathrm{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^2}
\end{aligned}
$$

output
```
1/2*d*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*e^(1/2)/c+1/2*arctanh(x*e^(1/2)/(
e*x^2+d)^(1/2))*(c*d-b*e+(-2*a*c*e+b^2*e-b*c*d)/(-4*a*c+b^2)^(1/2))*e^(1/2
)/c^2+1/2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*(c*d-b*e+(2*a*c*e-b^2*e+b*c*d
)/(-4*a*c+b^2)^(1/2))*e^(1/2)/c^2+1/2*e*x*(e*x^2+d)^(1/2)/c+1/2*arctan(x*(
2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2
))^(1/2))*(c*d-b*e+(-2*a*c*e+b^2*e-b*c*d)/(-4*a*c+b^2)^(1/2))*(2*c*d-e*(b-
(-4*a*c+b^2)^(1/2)))^(1/2)/c^2/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan(x*(
2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2
))^(1/2))*(c*d-b*e+(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*(2*c*d-e*(b+(
-4*a*c+b^2)^(1/2)))^(1/2)/c^2/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.372.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.38 (sec) , antiderivative size = 916, normalized size of antiderivative = 1.87

$$\int \frac{x^2(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \frac{2cex\sqrt{d+ex^2}+4\sqrt{e}(3cd-2be)\text{arctanh}\left(\frac{\sqrt{e}x}{-\sqrt{d}+\sqrt{d+ex^2}}\right)+\text{RootSum}\left[ae^4+4bde^2\#1\right.}{}$$

input
```
Integrate[(x^2*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

output
```
(2*c*e*x*Sqrt[d + e*x^2] + 4*Sqrt[e]*(3*c*d - 2*b*e)*ArcTanh[(Sqrt[e]*x)/(
-Sqrt[d] + Sqrt[d + e*x^2])] + RootSum[a*e^4 + 4*b*d*e^2*#1^2 - 4*a*e^3*#1
^2 + 16*c*d^2*#1^4 - 8*b*d*e*#1^4 + 6*a*e^2*#1^4 + 4*b*d*#1^6 - 4*a*e*#1^6
 + a*#1^8 & , (2*a*c*d*e^4*Log[x] - a*b*e^5*Log[x] - 2*a*c*d*e^4*Log[-Sqrt
[d] + Sqrt[d + e*x^2] - x*#1] + a*b*e^5*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x
*#1] - 4*c^2*d^3*e*Log[x]*#1^2 + 8*b*c*d^2*e^2*Log[x]*#1^2 - 4*b^2*d*e^3*L
og[x]*#1^2 - 2*a*c*d*e^3*Log[x]*#1^2 + 3*a*b*e^4*Log[x]*#1^2 + 4*c^2*d^3*e
*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^2 - 8*b*c*d^2*e^2*Log[-Sqrt[d]
+ Sqrt[d + e*x^2] - x*#1]*#1^2 + 4*b^2*d*e^3*Log[-Sqrt[d] + Sqrt[d + e*x^2
] - x*#1]*#1^2 + 2*a*c*d*e^3*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^2 -
 3*a*b*e^4*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^2 + 4*c^2*d^3*Log[x]*
#1^4 - 8*b*c*d^2*e*Log[x]*#1^4 + 4*b^2*d*e^2*Log[x]*#1^4 + 2*a*c*d*e^2*Log
[x]*#1^4 - 3*a*b*e^3*Log[x]*#1^4 - 4*c^2*d^3*Log[-Sqrt[d] + Sqrt[d + e*x^2
] - x*#1]*#1^4 + 8*b*c*d^2*e*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^4 -
 4*b^2*d*e^2*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^4 - 2*a*c*d*e^2*Log
[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^4 + 3*a*b*e^3*Log[-Sqrt[d] + Sqrt[d
 + e*x^2] - x*#1]*#1^4 - 2*a*c*d*e*Log[x]*#1^6 + a*b*e^2*Log[x]*#1^6 + 2*a
*c*d*e*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^6 - a*b*e^2*Log[-Sqrt[d]
+ Sqrt[d + e*x^2] - x*#1]*#1^6)/(b*d*e^2*#1 - a*e^3*#1 + 8*c*d^2*#1^3 - 4*
b*d*e*#1^3 + 3*a*e^2*#1^3 + 3*b*d*#1^5 - 3*a*e*#1^5 + a*#1^7) & ])/(4*c...
```

3.372.3 Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1616, 211, 224, 219, 2256, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (d+ex^2)^{3/2}}{a+bx^2+cx^4} \, dx$$

$$\Big\downarrow \text{1616}$$

$$\frac{e\int \sqrt{ex^2+d}dx}{c}-\frac{\int \frac{\sqrt{ex^2+d}(ae-(cd-be)x^2)}{cx^4+bx^2+a}dx}{c}$$

$$\Big\downarrow \text{211}$$

$$\frac{e\left(\frac{1}{2}d\int \frac{1}{\sqrt{ex^2+d}}dx+\frac{1}{2}x\sqrt{d+ex^2}\right)}{c}-\frac{\int \frac{\sqrt{ex^2+d}(ae-(cd-be)x^2)}{cx^4+bx^2+a}dx}{c}$$

$$\Big\downarrow \; 224$$

$$\frac{e\left(\frac{1}{2}d\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}+\frac{1}{2}x\sqrt{d+ex^2}\right)}{c}-\frac{\int \frac{\sqrt{ex^2+d}\left(ae-(cd-be)x^2\right)}{cx^4+bx^2+a}dx}{c}$$

$$\Big\downarrow \; 219$$

$$\frac{e\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{e}}+\frac{1}{2}x\sqrt{d+ex^2}\right)}{c}-\frac{\int \frac{\sqrt{ex^2+d}\left(ae-(cd-be)x^2\right)}{cx^4+bx^2+a}dx}{c}$$

$$\Big\downarrow \; 2256$$

$$\frac{e\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{e}}+\frac{1}{2}x\sqrt{d+ex^2}\right)}{c}-\frac{\int \left(\frac{\sqrt{ex^2+d}\left(-cd+be-\frac{-eb^2+cdb+2ace}{\sqrt{b^2-4ac}}\right)}{2cx^2+b+\sqrt{b^2-4ac}}+\frac{\left(-cd+be+\frac{-eb^2+cdb+2ace}{\sqrt{b^2-4ac}}\right)\sqrt{ex^2+d}}{2cx^2+b-\sqrt{b^2-4ac}}\right)dx}{c}$$

$$\Big\downarrow \; 2009$$

$$\frac{e\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{e}}+\frac{1}{2}x\sqrt{d+ex^2}\right)}{c}-\frac{-\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2c\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right.}{2c\sqrt{\sqrt{b^2-4ac}+b}}}{}$$

input
```
Int[(x^2*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

3.372. $\int \frac{x^2\left(d+ex^2\right)^{3/2}}{a+bx^2+cx^4}\,dx$

output
```
(e*((x*Sqrt[d + e*x^2])/2 + (d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*Sq
rt[e])))/c - (-1/2*(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*d - b*e - (
b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt
[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c*Sq
rt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c*d
 - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d -
 (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2
])])/(2*c*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (Sqrt[e]*(c*d - b*e - (b*c*d - b^
2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2
*c) - (Sqrt[e]*(c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*A
rcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c))/c
```

3.372.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 1616
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Simp[e*(f^2/c)   Int[(f*x)^(m - 2)*(d + e*x^2)^
(q - 1), x], x] - Simp[f^2/c   Int[(f*x)^(m - 2)*(d + e*x^2)^(q - 1)*(Simp[
a*e - (c*d - b*e)*x^2, x]/(a + b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m,
1] && LeQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2256
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

3.372.4 Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 394, normalized size of antiderivative = 0.80

method	result
risch	$\frac{ex\sqrt{e\,x^2+d}}{2c}-\frac{\frac{\sqrt{e}\,(2be-3cd)\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{c}+\frac{a\sqrt{2}\left(\frac{\left(2acd\,e^2-b^2d\,e^2+2d^2ebc-2c^2d^3+\sqrt{-d^2\left(4ac-b^2\right)}\,b\,e^2-2\sqrt{-d^2\left(4ac-b^2\right)}\,c \ldots}{\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2 \ldots}}\right.}{\ldots}}{\ldots}$
default	$-\frac{a\left(\left(e^{\frac{3}{2}}cd-\frac{b\,e^{\frac{5}{2}}}{2}\right)\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}+d\left(\left(ac-\frac{b^2}{2}\right)e^{\frac{5}{2}}+dc\left(-cd\sqrt{e}+e^{\frac{3}{2}}b\right)\right)\right)\sqrt{2}\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\,\operatorname{arcta}\ldots}{\ldots}$
pseudoelliptic	$-\frac{a\left(\left(e^{\frac{3}{2}}cd-\frac{b\,e^{\frac{5}{2}}}{2}\right)\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}+d\left(\left(ac-\frac{b^2}{2}\right)e^{\frac{5}{2}}+dc\left(-cd\sqrt{e}+e^{\frac{3}{2}}b\right)\right)\right)\sqrt{2}\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\,\operatorname{arcta}\ldots}{\ldots}$

input
```
int(x^2*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*e*x*(e*x^2+d)^(1/2)/c-1/2/c*(e^(1/2)*(2*b*e-3*c*d)/c*ln(x*e^(1/2)+(e*x
^2+d)^(1/2))+1/c*a*2^(1/2)/(-d^2*(4*a*c-b^2))^(1/2)*((2*a*c*d*e^2-b^2*d*e^
2+2*d^2*e*b*c-2*c^2*d^3+(-d^2*(4*a*c-b^2))^(1/2)*b*e^2-2*(-d^2*(4*a*c-b^2)
)^(1/2)*c*d*e)/((-2*a*e+b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2)*arctan(a/x*
(e*x^2+d)^(1/2)*2^(1/2)/((-2*a*e+b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2))-(
-2*a*c*d*e^2+b^2*d*e^2-2*d^2*e*b*c+2*c^2*d^3+(-d^2*(4*a*c-b^2))^(1/2)*b*e^
2-2*(-d^2*(4*a*c-b^2))^(1/2)*c*d*e)/((2*a*e-b*d+(-d^2*(4*a*c-b^2))^(1/2))*
a)^(1/2)*arctanh(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((2*a*e-b*d+(-d^2*(4*a*c-b^2)
)^(1/2))*a)^(1/2))))
```

3.372.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5955 vs. 2(415) = 830.

Time = 54.07 (sec) , antiderivative size = 11917, normalized size of antiderivative = 24.27

$$\int \frac{x^2(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.372.6 Sympy [F]

$$\int \frac{x^2(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \int \frac{x^2(d+ex^2)^{\frac{3}{2}}}{a+bx^2+cx^4}\,dx$$

input
```
integrate(x**2*(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

output
```
Integral(x**2*(d + e*x**2)**(3/2)/(a + b*x**2 + c*x**4), x)
```

3.372.7 Maxima [F]

$$\int \frac{x^2(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}x^2}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^2*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^(3/2)*x^2/(c*x^4 + b*x^2 + a), x)
```

3.372.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^2(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.372.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \int \frac{x^2\,(e\,x^2+d)^{3/2}}{c\,x^4+b\,x^2+a}\,dx$$

input
```
int((x^2*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x)
```

output
```
int((x^2*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4), x)
```

3.373 $\int \frac{(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx$

3.373.1 Optimal result

Integrand size = 26, antiderivative size = 487

$$\int \frac{(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \frac{\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}\,d+ae\right)\right)\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)ex}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$
$$-\frac{\left(2c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-2ce\left(bd+\sqrt{b^2-4ac}\,d+ae\right)\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)ex}}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$
$$+\frac{\sqrt{e}\left(3cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c\sqrt{b^2-4ac}}$$
$$-\frac{\sqrt{e}\left(3cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c\sqrt{b^2-4ac}}$$

output
```
1/2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*(3*c*d-e*(b-(-4*a*c+b^2)^(1/2)))*e^
(1/2)/c/(-4*a*c+b^2)^(1/2)-1/2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*(3*c*d-e
*(b+(-4*a*c+b^2)^(1/2)))*e^(1/2)/c/(-4*a*c+b^2)^(1/2)+arctan(x*(2*c*d-e*(b
-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*
(2*c^2*d^2+b*e^2*(b-(-4*a*c+b^2)^(1/2))-2*c*e*(b*d+a*e-d*(-4*a*c+b^2)^(1/2
)))/c/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(b-(-4*a*c
+b^2)^(1/2))^(1/2)-arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+
d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(2*c^2*d^2+b*e^2*(b+(-4*a*c+b^2)^(1
/2))-2*c*e*(b*d+a*e+d*(-4*a*c+b^2)^(1/2)))/c/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c
+b^2)^(1/2))^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.373.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 8958 vs. $2(487) = 974$.

Time = 16.20 (sec) , antiderivative size = 8958, normalized size of antiderivative = 18.39

$$\int \frac{(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \text{Result too large to show}$$

input
```
Integrate[(d + e*x^2)^(3/2)/(a + b*x^2 + c*x^4),x]
```

output
```
Result too large to show
```

3.373.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1488, 318, 398, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{1488}$$

$$\frac{2c\int \frac{(ex^2+d)^{3/2}}{2cx^2+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{2c\int \frac{(ex^2+d)^{3/2}}{2cx^2+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \; 318$$

$$\frac{2c\left(\frac{\int \frac{2e\left(3cd-\left(b-\sqrt{b^2-4ac}\right)e\right)x^2+d\left(4cd-\left(b-\sqrt{b^2-4ac}\right)e\right)}{\left(2cx^2+b-\sqrt{b^2-4ac}\right)\sqrt{ex^2+d}}dx}{4c}+\frac{ex\sqrt{d+ex^2}}{4c}\right)}{\sqrt{b^2-4ac}}-\frac{2c\left(\frac{\int \frac{2e\left(3cd-\left(b+\sqrt{b^2-4ac}\right)e\right)x^2+d\left(4cd-\left(b+\sqrt{b^2-4ac}\right)e\right)}{\left(2cx^2+b+\sqrt{b^2-4ac}\right)\sqrt{ex^2+d}}dx}{4c}+\frac{ex\sqrt{d+ex^2}}{4c}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \; 398$$

$$\frac{2c\left(\frac{\frac{2\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\int \frac{1}{\left(2cx^2+b-\sqrt{b^2-4ac}\right)\sqrt{ex^2+d}}dx}{c}+\frac{e\left(3cd-e\left(b-\sqrt{b^2-4ac}\right)\right)\int \frac{1}{\sqrt{ex^2+d}}dx}{c}}{4c}+\frac{ex\sqrt{d+ex^2}}{4c}\right)}{\sqrt{b^2-4ac}}-\frac{2c\left(\frac{\frac{2\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\int \frac{1}{\left(2cx^2+b+\sqrt{b^2-4ac}\right)\sqrt{ex^2+d}}dx}{c}+\frac{e\left(3cd-e\left(\sqrt{b^2-4ac}+b\right)\right)\int \frac{1}{\sqrt{ex^2+d}}dx}{c}}{4c}+\frac{ex\sqrt{d+ex^2}}{4c}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \; 224$$

$$\frac{2c\left(\frac{\frac{2\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\int \frac{1}{\left(2cx^2+b-\sqrt{b^2-4ac}\right)\sqrt{ex^2+d}}dx}{c}+\frac{e\left(3cd-e\left(b-\sqrt{b^2-4ac}\right)\right)\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{c}}{4c}+\frac{ex\sqrt{d+ex^2}}{4c}\right)}{\sqrt{b^2-4ac}}-\frac{2c\left(\frac{\frac{2\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\int \frac{1}{\left(2cx^2+b+\sqrt{b^2-4ac}\right)\sqrt{ex^2+d}}dx}{c}+\frac{e\left(3cd-e\left(\sqrt{b^2-4ac}+b\right)\right)\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{c}}{4c}+\frac{ex\sqrt{d+ex^2}}{4c}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \; 219$$

3.373. $\int \frac{\left(d+ex^2\right)^{3/2}}{a+bx^2+cx^4}\,dx$

$$\frac{2c\left(\frac{\frac{2\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\int\frac{1}{\left(2cx^2+b-\sqrt{b^2-4ac}\right)\sqrt{ex^2+d}}dx}{c}+\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(3cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}{c}}{4c}+ex\ldots\right)}{\sqrt{b^2-4ac}}$$

$$\frac{2c\left(\frac{\frac{2\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\int\frac{1}{\left(2cx^2+b+\sqrt{b^2-4ac}\right)\sqrt{ex^2+d}}dx}{c}+\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(3cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}{c}}{4c}+ex\sqrt{\ldots}\right)}{\sqrt{b^2-4ac}}$$

$\downarrow$ 291

$$\frac{2c\left(\frac{\frac{2\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\int\frac{1}{-\frac{\left(\left(b-\sqrt{b^2-4ac}\right)e-2cd\right)x^2}{ex^2+d}+b-\sqrt{b^2-4ac}}d\frac{x}{\sqrt{ex^2+d}}}{c}+\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(3cd-e\left(b-\ldots\right)\right)}{c}}{4c}\ldots\right)}{\sqrt{b^2-4ac}}$$

$$\frac{2c\left(\frac{\frac{2\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\int\frac{1}{-\frac{\left(\left(b+\sqrt{b^2-4ac}\right)e-2cd\right)x^2}{ex^2+d}+b+\sqrt{b^2-4ac}}d\frac{x}{\sqrt{ex^2+d}}}{c}+\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(3cd-e\left(\sqrt{b^2\ldots}\right)\right)}{c}}{4c}\ldots\right)}{\sqrt{b^2-4ac}}$$

$\downarrow$ 218

$$\frac{2c\left(\frac{\frac{2\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(3cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}{c}}{4c}+ex\ldots\right)}{\sqrt{b^2-4ac}}$$

$$\frac{2c\left(\frac{\frac{2\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(3cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}{c}}{4c}+ex\sqrt{\ldots}\right)}{\sqrt{b^2-4ac}}$$

input
```
Int[(d + e*x^2)^(3/2)/(a + b*x^2 + c*x^4),x]
```

3.373. $\int \frac{(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx$

output
```
(2*c*((e*x*Sqrt[d + e*x^2])/(4*c) + ((2*(2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a
*c])*e^2 - 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(Sqrt[2*c*d - (
b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])
])/(c*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e])
 + (Sqrt[e]*(3*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d
 + e*x^2]])/c)/(4*c)))/Sqrt[b^2 - 4*a*c] - (2*c*((e*x*Sqrt[d + e*x^2])/(4*
c) + ((2*(2*c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^
2 - 4*a*c]*d + a*e))*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(S
qrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c*Sqrt[b + Sqrt[b^2 - 4*a*c
]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[e]*(3*c*d - (b + Sqrt[
b^2 - 4*a*c])*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/c)/(4*c)))/Sqrt[b^2
 - 4*a*c]
```

3.373.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

3.373. $\int \frac{(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx$

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 1488
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/r)   Int[(d + e*x^2)^q/(b -
 r + 2*c*x^2), x], x] - Simp[2*(c/r)   Int[(d + e*x^2)^q/(b + r + 2*c*x^2),
 x], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

3.373.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 411, normalized size of antiderivative = 0.84

method	result
default	$-\frac{\sqrt{\left(-2ae+bd+\sqrt{-4d^{2}\left(ac-\frac{b^{2}}{4}\right)}\right)a}\,\sqrt{2}\,\left(\left(a\,e^{2}-c\,d^{2}\right)\sqrt{-4d^{2}\left(ac-\frac{b^{2}}{4}\right)}+d\left(ab\,e^{2}-4acde+bc\,d^{2}\right)\right)\operatorname{arctanh}\left(\frac{a\sqrt{e\ldots}}{x\sqrt{\left(2ae-bd+\ldots\right.}}\right.}{\ldots}$
pseudoelliptic	$-\frac{\sqrt{\left(-2ae+bd+\sqrt{-4d^{2}\left(ac-\frac{b^{2}}{4}\right)}\right)a}\,\sqrt{2}\,\left(\left(a\,e^{2}-c\,d^{2}\right)\sqrt{-4d^{2}\left(ac-\frac{b^{2}}{4}\right)}+d\left(ab\,e^{2}-4acde+bc\,d^{2}\right)\right)\operatorname{arctanh}\left(\frac{a\sqrt{e\ldots}}{x\sqrt{\left(2ae-bd+\ldots\right.}}\right.}{\ldots}$

input
```
int((e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*(((-2*a*e+b*d+(-4
*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*2^(1/2)*((a*e^2-c*d^2)*(-4*d^2*(a*c-1/
4*b^2))^(1/2)+d*(a*b*e^2-4*a*c*d*e+b*c*d^2))*arctanh(a/x*(e*x^2+d)^(1/2)*2
^(1/2)/((2*a*e-b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2))-(2^(1/2)*((a*e^
2-c*d^2)*(-4*d^2*(a*c-1/4*b^2))^(1/2)-a*b*d*e^2+4*a*c*d^2*e-b*c*d^3)*arcta
n(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a
)^(1/2))+2*(-4*d^2*(a*c-1/4*b^2))^(1/2)*arctanh((e*x^2+d)^(1/2)/x/e^(1/2))
*((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*e^(3/2))*((2*a*e-b*d+
(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2))/((2*a*e-b*d+(-4*d^2*(a*c-1/4*b^2))
^(1/2))*a)^(1/2)/(-4*d^2*(a*c-1/4*b^2))^(1/2)/c
```

3.373.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3857 vs. $2(415) = 830$.

Time = 15.60 (sec) , antiderivative size = 7721, normalized size of antiderivative = 15.85

$$\int \frac{(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.373.6 Sympy [F]

$$\int \frac{(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \int \frac{(d+ex^2)^{\frac{3}{2}}}{a+bx^2+cx^4}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

output
```
Integral((d + e*x**2)**(3/2)/(a + b*x**2 + c*x**4), x)
```

3.373.7 Maxima [F]

$$\int \frac{(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}}{cx^4+bx^2+a}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^(3/2)/(c*x^4 + b*x^2 + a), x)
```

3.373.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.373.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \int \frac{(e\,x^2+d)^{3/2}}{c\,x^4+b\,x^2+a}\,dx$$

input
```
int((d + e*x^2)^(3/2)/(a + b*x^2 + c*x^4),x)
```

output
```
int((d + e*x^2)^(3/2)/(a + b*x^2 + c*x^4), x)
```

3.374 $\int \frac{(d+ex^2)^{3/2}}{x^2(a+bx^2+cx^4)}\,dx$

3.374.1 Optimal result

Integrand size = 29, antiderivative size = 260

$$\int \frac{(d+ex^2)^{3/2}}{x^2(a+bx^2+cx^4)}\,dx = -\frac{d\sqrt{d+ex^2}}{ax} - \frac{\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)^{3/2}\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{3/2}} + \frac{\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)^{3/2}\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{3/2}}$$

output
```
-arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*
c+b^2)^(1/2))^(1/2))*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(3/2)/(b-(-4*a*c+b^2
)^(1/2))^(3/2)/(-4*a*c+b^2)^(1/2)+arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))
)^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(2*c*d-e*(b+(-4*a*c+
b^2)^(1/2)))^(3/2)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(3/2)-d*(e*x^
2+d)^(1/2)/a/x
```

3.374.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 7491 vs. 2(260) = 520.

Time = 16.33 (sec) , antiderivative size = 7491, normalized size of antiderivative = 28.81

$$\int \frac{(d+ex^2)^{3/2}}{x^2(a+bx^2+cx^4)}\,dx = \text{Result too large to show}$$

input
```
Integrate[(d + e*x^2)^(3/2)/(x^2*(a + b*x^2 + c*x^4)),x]
```

output
```
Result too large to show
```

3.374.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.63, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1618, 247, 224, 219, 2256, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{3/2}}{x^2(a+bx^2+cx^4)}dx$$

$$\Big\downarrow \text{1618}$$

$$\frac{d\int \frac{\sqrt{ex^2+d}}{x^2}dx}{a}-\frac{\int \frac{(cdx^2+bd-ae)\sqrt{ex^2+d}}{cx^4+bx^2+a}dx}{a}$$

$$\Big\downarrow \text{247}$$

$$\frac{d\left(e\int \frac{1}{\sqrt{ex^2+d}}dx-\frac{\sqrt{d+ex^2}}{x}\right)}{a}-\frac{\int \frac{(cdx^2+bd-ae)\sqrt{ex^2+d}}{cx^4+bx^2+a}dx}{a}$$

$$\Big\downarrow \text{224}$$

$$\frac{d\left(e\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}-\frac{\sqrt{d+ex^2}}{x}\right)}{a}-\frac{\int \frac{(cdx^2+bd-ae)\sqrt{ex^2+d}}{cx^4+bx^2+a}dx}{a}$$

$$\Big\downarrow \text{219}$$

$$\frac{d\left(\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{\sqrt{d+ex^2}}{x}\right)}{a}-\frac{\int \frac{(cdx^2+bd-ae)\sqrt{ex^2+d}}{cx^4+bx^2+a}dx}{a}$$

$$\Big\downarrow \text{2256}$$

$$\frac{d\left(\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{\sqrt{d+ex^2}}{x}\right)}{a}-\frac{\int\left(\frac{\sqrt{ex^2+d}\left(cd-\frac{c(bd-2ae)}{\sqrt{b^2-4ac}}\right)}{2cx^2+b+\sqrt{b^2-4ac}}+\frac{\left(cd+\frac{c(bd-2ae)}{\sqrt{b^2-4ac}}\right)\sqrt{ex^2+d}}{2cx^2+b-\sqrt{b^2-4ac}}\right)dx}{a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\left(\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{\sqrt{d+ex^2}}{x}\right)}{a}-$$

$$\frac{\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+\right.}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+e}}\right.}{2\sqrt{\sqrt{b^2-4ac}+b}}}{a}$$

input
```
Int[(d + e*x^2)^(3/2)/(x^2*(a + b*x^2 + c*x^4)),x]
```

output
```
(d*(-(Sqrt[d + e*x^2]/x) + Sqrt[e]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]))/
a - ((Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(d + (b*d - 2*a*e)/Sqrt[b^2
- 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqr
t[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt
[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*
ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*
a*c]]*Sqrt[d + e*x^2])])/(2*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (Sqrt[e]*(d - (
b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/2 +
(Sqrt[e]*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d
+ e*x^2]])/2)/a
```

3.374.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

3.374. $\int \frac{(d+ex^2)^{3/2}}{x^2(a+bx^2+cx^4)}\,dx$

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 1618
```
Int[(((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4), x_Symbol] :> Simp[d/a   Int[(f*x)^m*(d + e*x^2)^(q - 1), x],
x] - Simp[1/(a*f^2)   Int[(f*x)^(m + 2)*(d + e*x^2)^(q - 1)*(Simp[b*d - a*e
 + c*d*x^2, x]/(a + b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && LtQ[m, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2256
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

3.374.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.33

method	result
risch	$-\frac{d\sqrt{e\,x^2+d}}{ax}-\frac{d\sqrt{2}\left(\frac{\left(-2e^2a^2+2abde+2d^2ac-b^2d^2+2\sqrt{-d^2\left(4ac-b^2\right)}\,ae-\sqrt{-d^2\left(4ac-b^2\right)}\,bd\right)\arctan\left(\frac{a\sqrt{e\,x^2+d}\,\sqrt{2}}{x\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4a \cdots\right)}\right)}}\right)}{\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}\cdots\right)}{2\cdots}$
pseudoelliptic	$-\frac{d\left(\frac{2\sqrt{e\,x^2+d}}{x}+\frac{\left(-2e^2a^2+2abde+2d^2ac-b^2d^2+2\sqrt{-d^2\left(4ac-b^2\right)}\,ae-\sqrt{-d^2\left(4ac-b^2\right)}\,bd\right)\sqrt{2}\,\arctan\left(\frac{a\sqrt{e\,x^2+d}\,\sqrt{2}}{x\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b \cdots\right)}\right)}}\right)}{\sqrt{-d^2\left(4ac-b^2\right)}\,\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}\cdots\right)}{2\cdots}$
default	$\frac{-\frac{\left(e\,x^2+d\right)^{\frac{5}{2}}}{dx}+\frac{4e\left(\frac{x\left(e\,x^2+d\right)^{\frac{3}{2}}}{4}+\frac{3d\left(\frac{x\sqrt{e\,x^2+d}}{2}+\frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{2\sqrt{e}}\right)}{4}\right)}{d}}{a}+\frac{\left(\left(-\frac{b\sqrt{e}\,d}{2}+e^{\frac{3}{2}}a\right)\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}-b\,e^{\frac{3}{2}}ad+a\cdots\right)}{\cdots}$

input
```
int((e*x^2+d)^(3/2)/x^2/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-d*(e*x^2+d)^(1/2)/a/x-1/2/a*d*2^(1/2)/(-d^2*(4*a*c-b^2))^(1/2)*((-2*e^2*a
^2+2*a*b*d*e+2*d^2*a*c-b^2*d^2+2*(-d^2*(4*a*c-b^2))^(1/2)*a*e-(-d^2*(4*a*c
-b^2))^(1/2)*b*d)/((-2*a*e+b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2)*arctan(a
/x*(e*x^2+d)^(1/2)*2^(1/2)/((-2*a*e+b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2)
)-(2*e^2*a^2-2*a*b*d*e-2*d^2*a*c+b^2*d^2+2*(-d^2*(4*a*c-b^2))^(1/2)*a*e-(-
d^2*(4*a*c-b^2))^(1/2)*b*d)/((2*a*e-b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2)
*arctanh(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((2*a*e-b*d+(-d^2*(4*a*c-b^2))^(1/2))
*a)^(1/2)))
```

3.374.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4059 vs. $2(218) = 436$.

Time = 7.92 (sec) , antiderivative size = 4059, normalized size of antiderivative = 15.61

$$\int \frac{(d+ex^2)^{3/2}}{x^2(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^(3/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(1/2)*a*x*sqrt(-(3*a^2*b*d*e^2 - 2*a^3*e^3 + (b^3 - 3*a*b*c)*d^3
 - 3*(a*b^2 - 2*a^2*c)*d^2*e + (a^3*b^2 - 4*a^4*c)*sqrt(-(18*a^3*b*d^3*e^3
 - 9*a^4*d^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d
^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4
*a^4*c))*log(-(12*a^3*b*d^3*e^3 - 6*a^4*d^2*e^4 - 2*(a*b^2*c - a^2*c^2)*d^
6 + 2*(a*b^3 + 2*a^2*b*c)*d^5*e - 4*(2*a^2*b^2 + a^3*c)*d^4*e^2 + ((a^3*b^
2*c - 4*a^4*c^2)*d^3 - (a^3*b^3 - 4*a^4*b*c)*d^2*e + (a^4*b^2 - 4*a^5*c)*d
*e^2)*x^2*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^4 - 2*a*b^2*c + a^2
*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(
a^6*b^2 - 4*a^7*c)) + (27*a^3*b*d^2*e^4 - 12*a^4*d*e^5 + (b^3*c - a*b*c^2)
*d^6 - (b^4 + 6*a*b^2*c - 4*a^2*c^2)*d^5*e + 2*(4*a*b^3 + 5*a^2*b*c)*d^4*e
^2 - 2*(11*a^2*b^2 + 4*a^3*c)*d^3*e^3)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*(
((a^4*b^3 - 4*a^5*b*c)*d - 2*(a^5*b^2 - 4*a^6*c)*e)*x*sqrt(-(18*a^3*b*d^3*
e^3 - 9*a^4*d^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c
)*d^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)) - ((a*b^4
- 5*a^2*b^2*c + 4*a^3*c^2)*d^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d^3*e + 3*(a^3*b^
2 - 4*a^4*c)*d^2*e^2)*x)*sqrt(-(3*a^2*b*d*e^2 - 2*a^3*e^3 + (b^3 - 3*a*b*c
)*d^3 - 3*(a*b^2 - 2*a^2*c)*d^2*e + (a^3*b^2 - 4*a^4*c)*sqrt(-(18*a^3*b*d^
3*e^3 - 9*a^4*d^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b
*c)*d^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3...
```

3.374.6 Sympy [F]

$$\int \frac{(d+ex^2)^{3/2}}{x^2(a+bx^2+cx^4)}\,dx = \int \frac{(d+ex^2)^{\frac{3}{2}}}{x^2(a+bx^2+cx^4)}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)/x**2/(c*x**4+b*x**2+a),x)
```

output
```
Integral((d + e*x**2)**(3/2)/(x**2*(a + b*x**2 + c*x**4)), x)
```

3.374.7 Maxima [F]

$$\int \frac{(d+ex^2)^{3/2}}{x^2(a+bx^2+cx^4)}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}}{(cx^4+bx^2+a)x^2}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^(3/2)/((c*x^4 + b*x^2 + a)*x^2), x)
```

3.374.8 Giac [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}}{x^2(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(3/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
Timed out
```

3.374.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}}{x^2(a+bx^2+cx^4)}\,dx = \int \frac{(e\,x^2+d)^{3/2}}{x^2\,(c\,x^4+b\,x^2+a)}\,dx$$

input
```
int((d + e*x^2)^(3/2)/(x^2*(a + b*x^2 + c*x^4)),x)
```

output
```
int((d + e*x^2)^(3/2)/(x^2*(a + b*x^2 + c*x^4)), x)
```

3.375 $\int \frac{(d+ex^2)^{3/2}}{x^4(a+bx^2+cx^4)}\,dx$

3.375.1 Optimal result

Integrand size = 29, antiderivative size = 523

$$\int \frac{(d+ex^2)^{3/2}}{x^4(a+bx^2+cx^4)}\,dx = \frac{(bd-ae)\sqrt{d+ex^2}}{a^2x} - \frac{(d+ex^2)^{3/2}}{3ax^3} + \frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\left(bd-ae+\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2a^2\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2a^2\sqrt{b+\sqrt{b^2-4ac}}} - \frac{\sqrt{e}(bd-ae)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{a^2} + \frac{\sqrt{e}\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2a^2} + \frac{\sqrt{e}\left(bd-ae+\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2a^2}$$

output
```
-1/3*(e*x^2+d)^(3/2)/a/x^3-(-a*e+b*d)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*e
^(1/2)/a^2+1/2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*(b*d-a*e+(a*b*e+2*a*c*d-
b^2*d)/(-4*a*c+b^2)^(1/2))*e^(1/2)/a^2+1/2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/
2))*(b*d-a*e+(-a*b*e-2*a*c*d+b^2*d)/(-4*a*c+b^2)^(1/2))*e^(1/2)/a^2+(-a*e+
b*d)*(e*x^2+d)^(1/2)/a^2/x+1/2*arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(
1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b*d-a*e+(-a*b*e-2*a*c*
d+b^2*d)/(-4*a*c+b^2)^(1/2))*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/a^2/(b
-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(
1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b*d-a*e+(a*b*e+2*a*c*d
-b^2*d)/(-4*a*c+b^2)^(1/2))*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a^2/(b+
(-4*a*c+b^2)^(1/2))^(1/2)
```

3.375.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.49 (sec) , antiderivative size = 1095, normalized size of antiderivative = 2.09

$$\int \frac{(d+ex^2)^{3/2}}{x^4(a+bx^2+cx^4)}\,dx = \frac{\sqrt{d+ex^2}(-ad+3bdx^2-4aex^2)}{3a^2x^3}$$

$$-\frac{\text{RootSum}\Big[ae^4+4bde^2\#1^2-4ae^3\#1^2+16cd^2\#1^4-8bde\#1^4+6ae^2\#1^4+4bd\#1^6-4ae\#1^6+a\#1^8\&,$$

input
```
Integrate[(d + e*x^2)^(3/2)/(x^4*(a + b*x^2 + c*x^4)),x]
```

output
```
(Sqrt[d + e*x^2]*(-(a*d) + 3*b*d*x^2 - 4*a*e*x^2))/(3*a^2*x^3) - RootSum[a
*e^4 + 4*b*d*e^2*#1^2 - 4*a*e^3*#1^2 + 16*c*d^2*#1^4 - 8*b*d*e*#1^4 + 6*a*
e^2*#1^4 + 4*b*d*#1^6 - 4*a*e*#1^6 + a*#1^8 & , (b^2*d^2*e^3*Log[x] - a*c*
d^2*e^3*Log[x] - 2*a*b*d*e^4*Log[x] + a^2*e^5*Log[x] - b^2*d^2*e^3*Log[-Sq
rt[d] + Sqrt[d + e*x^2] - x*#1] + a*c*d^2*e^3*Log[-Sqrt[d] + Sqrt[d + e*x^
2] - x*#1] + 2*a*b*d*e^4*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1] - a^2*e^5*
Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1] + 4*b*c*d^3*e*Log[x]*#1^2 - 3*b^2*d
^2*e^2*Log[x]*#1^2 - 5*a*c*d^2*e^2*Log[x]*#1^2 + 6*a*b*d*e^3*Log[x]*#1^2 -
 3*a^2*e^4*Log[x]*#1^2 - 4*b*c*d^3*e*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1
]*#1^2 + 3*b^2*d^2*e^2*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^2 + 5*a*c
*d^2*e^2*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^2 - 6*a*b*d*e^3*Log[-Sq
rt[d] + Sqrt[d + e*x^2] - x*#1]*#1^2 + 3*a^2*e^4*Log[-Sqrt[d] + Sqrt[d + e
*x^2] - x*#1]*#1^2 - 4*b*c*d^3*Log[x]*#1^4 + 3*b^2*d^2*e*Log[x]*#1^4 + 5*a
*c*d^2*e*Log[x]*#1^4 - 6*a*b*d*e^2*Log[x]*#1^4 + 3*a^2*e^3*Log[x]*#1^4 + 4
*b*c*d^3*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^4 - 3*b^2*d^2*e*Log[-Sq
rt[d] + Sqrt[d + e*x^2] - x*#1]*#1^4 - 5*a*c*d^2*e*Log[-Sqrt[d] + Sqrt[d +
 e*x^2] - x*#1]*#1^4 + 6*a*b*d*e^2*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*
#1^4 - 3*a^2*e^3*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^4 - b^2*d^2*Log
[x]*#1^6 + a*c*d^2*Log[x]*#1^6 + 2*a*b*d*e*Log[x]*#1^6 - a^2*e^2*Log[x]*#1
^6 + b^2*d^2*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^6 - a*c*d^2*Log[...
```

3.375.3 Rubi [A] (verified)

Time = 1.74 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1618, 242, 2246, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)^{3/2}}{x^4\left(a+bx^2+cx^4\right)}dx$$

$$\Big\downarrow \text{ 1618}$$

$$\frac{d\int \frac{\sqrt{ex^2+d}}{x^4}dx}{a}-\frac{\int \frac{\left(cdx^2+bd-ae\right)\sqrt{ex^2+d}}{x^2\left(cx^4+bx^2+a\right)}dx}{a}$$

$$\Big\downarrow \text{ 242}$$

$$-\frac{\int \frac{\left(cdx^2+bd-ae\right)\sqrt{ex^2+d}}{x^2\left(cx^4+bx^2+a\right)}dx}{a}-\frac{\left(d+ex^2\right)^{3/2}}{3ax^3}$$

$$\Big\downarrow \text{2246}$$

$$-\frac{\int \left(\frac{\sqrt{ex^2+d}(bd-ae)}{ax^2}+\frac{\sqrt{ex^2+d}(-db^2+aeb-c(bd-ae)x^2+acd)}{a(cx^4+bx^2+a)}\right)dx}{a}-\frac{(d+ex^2)^{3/2}}{3ax^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right.}{2a\sqrt{\sqrt{b^2-4ac}+b}}}{}$$

$$\frac{(d+ex^2)^{3/2}}{3ax^3}$$

input
```
Int[(d + e*x^2)^(3/2)/(x^4*(a + b*x^2 + c*x^4)),x]
```

output
```
-1/3*(d + e*x^2)^(3/2)/(a*x^3) - (-(((b*d - a*e)*Sqrt[d + e*x^2])/(a*x)) -
 (Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(b*d - a*e + (b^2*d - 2*a*c*d -
a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*
x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]])/(2*a*Sqrt[b - Sqrt[b^2
- 4*a*c]]) - (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(b*d - a*e - (b^2*d
- 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]])/(2*a*Sqrt[b
 + Sqrt[b^2 - 4*a*c]]) + (Sqrt[e]*(b*d - a*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d +
 e*x^2]])/a - (Sqrt[e]*(b*d - a*e - (b^2*d - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4
*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*a) - (Sqrt[e]*(b*d - a*e +
 (b^2*d - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d +
 e*x^2]])/(2*a))/a
```

3.375.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 1618
```
Int[(((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4), x_Symbol] :> Simp[d/a   Int[(f*x)^m*(d + e*x^2)^(q - 1), x],
x] - Simp[1/(a*f^2)   Int[(f*x)^(m + 2)*(d + e*x^2)^(q - 1)*(Simp[b*d - a*e
 + c*d*x^2, x]/(a + b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && LtQ[m, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2246
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(f*x)^m*(d +
e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x
] && PolyQ[Px, x] && IntegerQ[p]
```

3.375.4 Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 451, normalized size of antiderivative = 0.86

method	result
risch	$-\frac{\sqrt{e x^2+d}\,(4ae x^2-3bd x^2+da)}{3a^2x^3}-\frac{\sqrt{2}\left(-\left(\left(e^2a^2+(-2bde-c d^2)a+b^2d^2\right)\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}-\left((b e^2+4dce)a^2+(-2b^2de-\cdots\right.\right.\right.}{\cdots}$
pseudoelliptic	$\frac{\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\left(\left((-ac+b^2)d^2-2abde+e^2a^2\right)\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}-\left((-3abc+b^3)d^2+2e(2c a^2-b^2a)d+a^2\cdots\right.\right.}{\cdots}$
default	$\frac{-\frac{(e x^2+d)^{\frac{5}{2}}}{3d x^3}+\frac{2e\left(-\frac{(e x^2+d)^{\frac{5}{2}}}{dx}+\frac{4e\left(\frac{x(e x^2+d)^{\frac{3}{2}}}{4}+\frac{3d\left(\frac{x\sqrt{e x^2+d}}{2}+\frac{d\ln\left(x\sqrt{e}+\sqrt{e x^2+d}\right)}{2\sqrt{e}}\right)}{4}\right)}{d}\right)}{3d}}{a}-\frac{b\left(-\frac{(e x^2+d)^{\frac{5}{2}}}{dx}+\frac{4e\left(\frac{x(e x\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}$

input
```
int((e*x^2+d)^(3/2)/x^4/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

3.375. $\int \frac{(d+ex^2)^{3/2}}{x^4(a+bx^2+cx^4)}\,dx$

output
```
-1/3*(e*x^2+d)^(1/2)*(4*a*e*x^2-3*b*d*x^2+a*d)/a^2/x^3-1/2/a^2/((2*a*e-b*d
+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*2^(1/2)*(-((e^2*a^2+(-2*b*d*e-c*d^
2)*a+b^2*d^2)*(-4*d^2*(a*c-1/4*b^2))^(1/2)-((b*e^2+4*c*d*e)*a^2+(-2*b^2*d*
e-3*b*c*d^2)*a+b^3*d^2)*d)*((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(
1/2)*arctanh(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((2*a*e-b*d+(-4*d^2*(a*c-1/4*b^2)
)^(1/2))*a)^(1/2))+((e^2*a^2+(-2*b*d*e-c*d^2)*a+b^2*d^2)*(-4*d^2*(a*c-1/4*
b^2))^(1/2)+((b*e^2+4*c*d*e)*a^2+(-2*b^2*d*e-3*b*c*d^2)*a+b^3*d^2)*d)*((2*
a*e-b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*arctan(a/x*(e*x^2+d)^(1/2)*
2^(1/2)/((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)))/((-2*a*e+b*d
+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)/(-4*d^2*(a*c-1/4*b^2))^(1/2)
```

3.375.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7830 vs. $2(447) = 894$.

Time = 32.46 (sec) , antiderivative size = 7830, normalized size of antiderivative = 14.97

$$\int \frac{(d+ex^2)^{3/2}}{x^4(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^(3/2)/x^4/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.375.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}}{x^4(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d)**(3/2)/x**4/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.375.7 Maxima [F]

$$\int \frac{(d+ex^2)^{3/2}}{x^4(a+bx^2+cx^4)}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}}{(cx^4+bx^2+a)x^4}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)/x^4/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^(3/2)/((c*x^4 + b*x^2 + a)*x^4), x)
```

3.375.8 Giac [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}}{x^4(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(3/2)/x^4/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
Timed out
```

3.375.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}}{x^4(a+bx^2+cx^4)}\,dx = \int \frac{(e\,x^2+d)^{3/2}}{x^4\,(c\,x^4+b\,x^2+a)}\,dx$$

input
```
int((d + e*x^2)^(3/2)/(x^4*(a + b*x^2 + c*x^4)),x)
```

output
```
int((d + e*x^2)^(3/2)/(x^4*(a + b*x^2 + c*x^4)), x)
```

3.376 $\int \frac{x^5\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

3.376.1 Optimal result

Integrand size = 29, antiderivative size = 281

$$\int \frac{x^5\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = -\frac{b\sqrt{1-x^2}}{c^2}-\frac{(1-x^2)^{3/2}}{3c}+\frac{\left(b^2-ac+bc-\frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b+2c-\sqrt{b^2-4ac}}}+\frac{\left(b^2-ac+bc+\frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b+2c+\sqrt{b^2-4ac}}}$$

output
```
-1/3*(-x^2+1)^(3/2)/c-b*(-x^2+1)^(1/2)/c^2+1/2*arctanh(2^(1/2)*c^(1/2)*(-x
^2+1)^(1/2)/(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2))*(b^2-a*c+b*c+(3*a*b*c+2*a*c^
2-b^3-b^2*c)/(-4*a*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(b+2*c-(-4*a*c+b^2)^(1/2)
)^(1/2)+1/2*arctanh(2^(1/2)*c^(1/2)*(-x^2+1)^(1/2)/(b+2*c+(-4*a*c+b^2)^(1/
2))^(1/2))*(b^2-a*c+b*c+(-3*a*b*c-2*a*c^2+b^3+b^2*c)/(-4*a*c+b^2)^(1/2))/c
^(5/2)*2^(1/2)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.376.2 Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.23

$$\int \frac{x^5\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$$
$$= \frac{2\sqrt{c}\sqrt{1-x^2}(-3b+c(-1+x^2)) - \frac{3\sqrt{2}\left(b^3+bc\left(-3a+\sqrt{b^2-4ac}\right)+b^2\left(c+\sqrt{b^2-4ac}\right)-ac\left(2c+\sqrt{b^2-4ac}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-b-2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{-b-2c-\sqrt{b^2-4ac}}}}{6c^{5/2}}$$

input
```
Integrate[(x^5*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

output
```
(2*Sqrt[c]*Sqrt[1 - x^2]*(-3*b + c*(-1 + x^2)) - (3*Sqrt[2]*(b^3 + b*c*(-3
*a + Sqrt[b^2 - 4*a*c]) + b^2*(c + Sqrt[b^2 - 4*a*c]) - a*c*(2*c + Sqrt[b^
2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[-b - 2*c - Sqrt[b
^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*c]]) - (3
*Sqrt[2]*(-b^3 + a*c*(2*c - Sqrt[b^2 - 4*a*c]) + b*c*(3*a + Sqrt[b^2 - 4*a
*c]) + b^2*(-c + Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2]
)/Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b - 2*c +
Sqrt[b^2 - 4*a*c]]))/(6*c^(5/2))
```

3.376.3 Rubi [A] (warning: unable to verify)

Time = 3.18 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1578, 1199, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$$
$$\Big\downarrow \text{ 1578}$$
$$\frac{1}{2}\int \frac{x^4\sqrt{1-x^2}}{cx^4+bx^2+a}dx^2$$
$$\Big\downarrow \text{ 1199}$$
$$-\int \left(\frac{x^4}{c}-\frac{b(a+b+c)-(b^2+cb-ac)\,x^4}{c^2\,(cx^8-(b+2c)x^4+a+b+c)}+\frac{b}{c^2}\right)d\sqrt{1-x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{-3abc-2ac^2+b^3+b^2c}{\sqrt{b^2-4ac}}-ac+b^2+bc\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}c^{5/2}\sqrt{-\sqrt{b^2-4ac}+b+2c}}+\frac{\left(\frac{-3abc-2ac^2+b^3+b^2c}{\sqrt{b^2-4ac}}-ac+b^2+bc\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b+2c}}-\frac{b\sqrt{1-x^2}}{c^2}-\frac{x^6}{3c}$$

input
```
Int[(x^5*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

output
```
-1/3*x^6/c - (b*Sqrt[1 - x^2])/c^2 + ((b^2 - a*c + b*c - (b^3 - 3*a*b*c +
b^2*c - 2*a*c^2)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2]
)/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b + 2*c - Sqrt
[b^2 - 4*a*c]]) + ((b^2 - a*c + b*c + (b^3 - 3*a*b*c + b^2*c - 2*a*c^2)/Sq
rt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c + Sq
rt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

3.376.3.1 Defintions of rubi rules used

rule 1199
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Denominator[m]}, Simp[q/e   Subs
t[Int[ExpandIntegrand[x^(q*(m + 1) - 1)*(((e*f - d*g)/e + g*(x^q/e))^n/((c*
d^2 - b*d*e + a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))), x],
 x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Integer
Q[n] && FractionQ[m]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.376. $\int \frac{x^5\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

3.376.4 Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.12

method	result
pseudoelliptic	$-\frac{3\left(\left(\left(\frac{\left(\left(-a+b\right)c+b^{2}\right)\sqrt{-4ac+b^{2}}}{3}+\frac{2a\,c^{2}}{3}+b\left(a-\frac{b}{3}\right)c-\frac{b^{3}}{3}\right)\sqrt{2}\,\sqrt{\left(b+2c+\sqrt{-4ac+b^{2}}\right)c}\,\arctan\left(\frac{c\sqrt{-x^{2}+1}\,\sqrt{2}}{\sqrt{\left(\sqrt{-4ac+b^{2}}-b-2c\right)c}}\right)+\sqrt{(\cdots}\right.\right.}{2\sqrt{-4\cdots}}$
risch	$\frac{\left(-c\,x^{2}+3b+c\right)\left(x^{2}-1\right)}{3c^{2}\sqrt{-x^{2}+1}}-\frac{2a\left(\frac{\left(-2ac\sqrt{-4ac+b^{2}}+b^{2}\sqrt{-4ac+b^{2}}+\sqrt{-4ac+b^{2}}\,bc+4abc+4a\,c^{2}-b^{3}-b^{2}c\right)\arctan\left(\frac{\frac{2a\left(\sqrt{-x^{2}+1}-1\right)\cdots}{x^{2}}}{2\sqrt{4ac-2b^{2}-2\sqrt{-4a\cdots}}}\right.}{\left(8ac-2b^{2}\right)\sqrt{4ac-2b^{2}-2\sqrt{-4ac+b^{2}}\,a-2b\sqrt{-4ac+b^{2}}-2ab}}\right.}{2a\left(\frac{\left(-2ac\sqrt{-4ac+b^{2}}+b^{2}\sqrt{-4ac+b^{2}}+\sqrt{-4ac+b^{2}}\,bc+4abc+4a\,c^{2}-b^{3}-b^{2}c\right)\arctan\left(\frac{\frac{2a\left(\sqrt{-x^{2}+1}-1\right)^{2}}{x^{2}}+2\sqrt{-4\cdots}}{2\sqrt{4ac-2b^{2}-2\sqrt{-4ac+b^{2}}\,a-2\cdots}}\right.}{\left(8ac-2b^{2}\right)\sqrt{4ac-2b^{2}-2\sqrt{-4ac+b^{2}}\,a-2b\sqrt{-4ac+b^{2}}-2ab}}\right.}$
default	$-\frac{\left(-x^{2}+1\right)^{\frac{3}{2}}}{3c}-\frac{\cdots}{\cdots}$

input
```
int(x^5*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-3/2/(-4*a*c+b^2)^(1/2)/(((-4*a*c+b^2)^(1/2)-b-2*c)*c)^(1/2)/((b+2*c+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*((1/3*((-a+b)*c+b^2)*(-4*a*c+b^2)^(1/2)+2/3*a*c^2+
b*(a-1/3*b)*c-1/3*b^3)*2^(1/2)*((b+2*c+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
(c*(-x^2+1)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b-2*c)*c)^(1/2))+(((-4*a*c+
b^2)^(1/2)-b-2*c)*c)^(1/2)*(2^(1/2)*(1/3*((a-b)*c-b^2)*(-4*a*c+b^2)^(1/2)+
2/3*a*c^2+b*(a-1/3*b)*c-1/3*b^3)*arctanh(c*(-x^2+1)^(1/2)*2^(1/2)/((b+2*c+
(-4*a*c+b^2)^(1/2))*c)^(1/2))+2/3*(b+1/3*c-1/3*c*x^2)*(-4*a*c+b^2)^(1/2)*(
(b+2*c+(-4*a*c+b^2)^(1/2))*c)^(1/2)*(-x^2+1)^(1/2)))/c^2
```

3.376.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3615 vs. $2(237) = 474$.

Time = 3.73 (sec) , antiderivative size = 3615, normalized size of antiderivative = 12.86

$$\int \frac{x^5\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^5*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
-1/6*(3*sqrt(1/2)*c^2*sqrt((b^5 + 2*a^2*c^3 + (5*a^2*b - 4*a*b^2)*c^2 - (5
*a*b^3 - b^4)*c - (b^2*c^5 - 4*a*c^6)*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b
^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5
 + b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*
c^6))*log(-(2*a^3*b^4 + (a^2*b^2*c^5 - 4*a^3*c^6)*x^2*sqrt((b^8 + (a^4 - 4
*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^
2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11))
+ 2*(a^5 - 2*a^4*b)*c^2 + (a^2*b^5 + (a^4*b - 2*a^3*b^2)*c^2 - (3*a^3*b^3
- a^2*b^4)*c)*x^2 - 2*(3*a^4*b^2 - a^3*b^3)*c + sqrt(1/2)*((b^5*c^5 - 7*a*
b^3*c^6 + 12*a^2*b*c^7)*x^2*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 -
2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^
2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)) + (b^8 + 4*(a^4 - 2*a^3*b)
*c^4 - (17*a^3*b^2 - 14*a^2*b^3)*c^3 + (20*a^2*b^4 - 7*a*b^5)*c^2 - (8*a*b
^6 - b^7)*c)*x^2)*sqrt((b^5 + 2*a^2*c^3 + (5*a^2*b - 4*a*b^2)*c^2 - (5*a*b
^3 - b^4)*c - (b^2*c^5 - 4*a*c^6)*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*
c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b
^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)
) - 2*(a^3*b^4 + (a^5 - 2*a^4*b)*c^2 - (3*a^4*b^2 - a^3*b^3)*c)*sqrt(-x^2
+ 1))/x^2) - 3*sqrt(1/2)*c^2*sqrt((b^5 + 2*a^2*c^3 + (5*a^2*b - 4*a*b^2)*c
^2 - (5*a*b^3 - b^4)*c - (b^2*c^5 - 4*a*c^6)*sqrt((b^8 + (a^4 - 4*a^3*b...
```

3.376.6 Sympy [F]

$$\int \frac{x^5\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \int \frac{x^5\sqrt{-(x-1)(x+1)}}{a+bx^2+cx^4}\,dx$$

input
```
integrate(x**5*(-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
```

output
```
Integral(x**5*sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)
```

3.376.7 Maxima [F]

$$\int \frac{x^5\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \int \frac{\sqrt{-x^2+1}x^5}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^5*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^2 + 1)*x^5/(c*x^4 + b*x^2 + a), x)
```

3.376.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4637 vs. $2(237) = 474$.

Time = 1.15 (sec) , antiderivative size = 4637, normalized size of antiderivative = 16.50

$$\int \frac{x^5\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^5*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/8*(2*b^6*c^4 - 14*a*b^4*c^5 + 6*b^5*c^5 + 24*a^2*b^2*c^6 - 40*a*b^3*c^6
+ 4*b^4*c^6 + 64*a^2*b*c^7 - 24*a*b^2*c^7 + 32*a^2*c^8 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 7*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 5*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 12
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b^
2*c^4 + 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)
*c)*a*b^3*c^4 - 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2
- 4*a*c)*c)*b^4*c^4 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqr
t(b^2 - 4*a*c)*c)*a^2*b*c^5 + 43*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c
^2 - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b
*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^3*c^5 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*c^6 + 48*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b*c^6 - 10*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^2*c^6 + 20*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*c^7 - 2*(b^2 -
 4*a*c)*b^4*c^4 + 6*(b^2 - 4*a*c)*a*b^2*c^5 - 6*(b^2 - 4*a*c)*b^3*c^5 + 16
*(b^2 - 4*a*c)*a*b*c^6 - 4*(b^2 - 4*a*c)*b^2*c^6 + 8*(b^2 - 4*a*c)*a*c^7 -
 (2*b^6*c^2 - 18*a*b^4*c^3 + 2*b^5*c^3 + 48*a^2*b^2*c^4 - 16*a*b^3*c^4 - 3
2*a^3*c^5 + 32*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 ...
```

3.376.9 Mupad [B] (verification not implemented)

Time = 8.34 (sec) , antiderivative size = 917, normalized size of antiderivative = 3.26

$$\int \frac{x^5\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \sqrt{1-x^2}\left(\frac{2}{3\,c}-\frac{\frac{b}{c}+1}{c}+\frac{x^2}{3\,c}\right)$$

$$-\frac{\ln\left(\frac{\frac{\left(x\,\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}-1\right)\,1\mathrm{i}}{\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\left(b^3\,c+b^4-b^3\,\sqrt{b^2-4\,a\,c}+4\,a^2\,c^2+2\,a\,c^2\,\sqrt{b^2-4\,a\,c}-b^2\,c\,\sqrt{b^2-\right.}{4\,c^3\,\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}\,\left(4\,a\,c-b^2\right)}$$

$$-\frac{\ln\left(\frac{\frac{\left(x\,\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}+1\right)\,1\mathrm{i}}{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\left(b^3\,c+b^4+b^3\,\sqrt{b^2-4\,a\,c}+4\,a^2\,c^2-2\,a\,c^2\,\sqrt{b^2-4\,a\,c}+b^2\,c\,\sqrt{b^2-\right.}{4\,c^3\,\left(4\,a\,c-b^2\right)\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}$$

$$-\frac{\ln\left(\frac{\frac{\left(x\,\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}-1\right)\,1\mathrm{i}}{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\left(b^3\,c+b^4+b^3\,\sqrt{b^2-4\,a\,c}+4\,a^2\,c^2-2\,a\,c^2\,\sqrt{b^2-4\,a\,c}+b^2\,c\,\sqrt{b^2-\right.}{4\,c^3\,\left(4\,a\,c-b^2\right)\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}$$

$$-\frac{\ln\left(\frac{\frac{\left(x\,\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}+1\right)\,1\mathrm{i}}{\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\left(b^3\,c+b^4-b^3\,\sqrt{b^2-4\,a\,c}+4\,a^2\,c^2+2\,a\,c^2\,\sqrt{b^2-4\,a\,c}-b^2\,c\,\sqrt{b^2-\right.}{4\,c^3\,\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}\,\left(4\,a\,c-b^2\right)}$$

input
```
int((x^5*(1 - x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

output
```
(1 - x^2)^(1/2)*(2/(3*c) - (b/c + 1)/c + x^2/(3*c)) - (log((((x*(-(b - (b^
2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c) +
 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1
/2)))*(b^3*c + b^4 - b^3*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2 + 2*a*c^2*(b^2 -
4*a*c)^(1/2) - b^2*c*(b^2 - 4*a*c)^(1/2) - 4*a*b*c^2 - 5*a*b^2*c + 3*a*b*c
*(b^2 - 4*a*c)^(1/2)))/(4*c^3*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*
(4*a*c - b^2)) - (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1
i)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x +
(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b^3*c + b^4 + b^3*(b^2 - 4*a*c
)^(1/2) + 4*a^2*c^2 - 2*a*c^2*(b^2 - 4*a*c)^(1/2) + b^2*c*(b^2 - 4*a*c)^(1
/2) - 4*a*b*c^2 - 5*a*b^2*c - 3*a*b*c*(b^2 - 4*a*c)^(1/2)))/(4*c^3*(4*a*c
- b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) - (log((((x*(-(b + (b^
2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) +
 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1
/2)))*(b^3*c + b^4 + b^3*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2 - 2*a*c^2*(b^2 -
4*a*c)^(1/2) + b^2*c*(b^2 - 4*a*c)^(1/2) - 4*a*b*c^2 - 5*a*b^2*c - 3*a*b*c
*(b^2 - 4*a*c)^(1/2)))/(4*c^3*(4*a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*
c) + 1)^(1/2)) - (log((((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1
i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x +
(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b^3*c + b^4 - b^3*(b^2 - 4*...
```

3.376. $\int \frac{x^5\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

3.377 $\int \frac{x^3\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

3.377.1 Optimal result

Integrand size = 29, antiderivative size = 229

$$\int \frac{x^3\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \frac{\sqrt{1-x^2}}{c} - \frac{\left(b+c-\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+2c-\sqrt{b^2-4ac}}} - \frac{\left(b+c+\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+2c+\sqrt{b^2-4ac}}}$$

output
```
(-x^2+1)^(1/2)/c-1/2*arctanh(2^(1/2)*c^(1/2)*(-x^2+1)^(1/2)/(b+2*c-(-4*a*c
+b^2)^(1/2))^(1/2))*(b+c+(2*a*c-b^2-b*c)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/
2)/(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctanh(2^(1/2)*c^(1/2)*(-x^2+1)^(
1/2)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2))*(b+c+(-2*a*c+b^2+b*c)/(-4*a*c+b^2)^
(1/2))/c^(3/2)*2^(1/2)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.377.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.24

$$\int \frac{x^3\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \frac{\sqrt{1-x^2}}{c} + \frac{\left(b^2-2ac+bc+b\sqrt{b^2-4ac}+c\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-b-2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{-b-2c-\sqrt{b^2-4ac}}} + \frac{\left(-b^2+2ac-bc+b\sqrt{b^2-4ac}+c\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-b-2c+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{-b-2c+\sqrt{b^2-4ac}}}$$

input
```
Integrate[(x^3*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

output
```
Sqrt[1 - x^2]/c + ((b^2 - 2*a*c + b*c + b*Sqrt[b^2 - 4*a*c] + c*Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[-b - 2*c - Sqrt[b^2 -
 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[-b - 2*c - Sqrt[b^2 - 4
*a*c]]) + ((-b^2 + 2*a*c - b*c + b*Sqrt[b^2 - 4*a*c] + c*Sqrt[b^2 - 4*a*c]
)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]
]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]])
```

3.377.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1578, 1196, 1197, 25, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{1-x^2}}{a+bx^2+cx^4}dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{x^2\sqrt{1-x^2}}{cx^4+bx^2+a}dx^2$$

$$\Big\downarrow \text{1196}$$

$$\frac{1}{2}\left(\frac{\int \frac{(b+c)x^2+a}{\sqrt{1-x^2}(cx^4+bx^2+a)}dx^2}{c}+\frac{2\sqrt{1-x^2}}{c}\right)$$

$$\Big\downarrow \text{1197}$$

$$\frac{1}{2}\left(\frac{2\int -\frac{-((b+c)x^4)+a+b+c}{cx^8-(b+2c)x^4+a+b+c}d\sqrt{1-x^2}}{c}+\frac{2\sqrt{1-x^2}}{c}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{2\sqrt{1-x^2}}{c}-\frac{2\int \frac{-((b+c)x^4)+a+b+c}{cx^8-(b+2c)x^4+a+b+c}d\sqrt{1-x^2}}{c}\right)$$

$$\Big\downarrow \text{1480}$$

$$\frac{1}{2}\left(\frac{2\left(\frac{1}{2}\left(\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\int\frac{1}{cx^4+\frac{1}{2}\left(-b-2c-\sqrt{b^2-4ac}\right)}d\sqrt{1-x^2}+\frac{1}{2}\left(-\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\int\frac{1}{cx^4+\frac{1}{2}\left(-b-2c+\sqrt{b^2-4a\right.}}\right.}{c}\right.$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{2\left(-\frac{\left(-\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\sqrt{c}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{\left(\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{c}+\frac{2\sqrt{1-x^2}}{c}\right)$$

input
```
Int[(x^3*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

output
```
((2*Sqrt[1 - x^2])/c + (2*(-(((b + c - (b^2 - 2*a*c + b*c)/Sqrt[b^2 - 4*a*
c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*
c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]])) - ((b + c + (b^
2 - 2*a*c + b*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2]
)/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + 2*c + Sqrt
[b^2 - 4*a*c]])))/c)/2
```

3.377.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1196
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Simp[1/c   Int
[(d + e*x)^(m - 1)*(Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x]/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && FractionQ[m] &
& GtQ[m, 0]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

3.377.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$\frac{\sqrt{-x^{2}+1}+\frac{\left(b\sqrt{-4ac+b^{2}}+\sqrt{-4ac+b^{2}}\,c+2ac-b^{2}-bc\right)\sqrt{2}\,\arctan\left(\frac{c\sqrt{-x^{2}+1}\,\sqrt{2}}{\sqrt{\left(\sqrt{-4ac+b^{2}}-b-2c\right)c}}\right)}{2\sqrt{-4ac+b^{2}}\,\sqrt{\left(\sqrt{-4ac+b^{2}}-b-2c\right)c}}-\frac{\left(b\sqrt{-4ac+b^{2}}+\sqrt{-4ac+b^{2}}\,c-2ac+b^{2}+bc\cdots\right.}{2\sqrt{-4ac+b^{2}}\,\sqrt{\left(b+\cdots\right.}}}{c}$
risch	$-\frac{x^{2}-1}{c\sqrt{-x^{2}+1}}+\frac{2a\left(\frac{\left(b\sqrt{-4ac+b^{2}}+2\sqrt{-4ac+b^{2}}\,c+4ac-b^{2}\right)\arctan\left(\frac{\frac{2a\left(\sqrt{-x^{2}+1}-1\right)^{2}}{x^{2}}+2\sqrt{-4ac+b^{2}}+2a+2b}{2\sqrt{4ac-2b^{2}-2\sqrt{-4ac+b^{2}}\,a-2b\sqrt{-4ac+b^{2}}-2ab}}\right)}{\left(8ac-2b^{2}\right)\sqrt{4ac-2b^{2}-2\sqrt{-4ac+b^{2}}\,a-2b\sqrt{-4ac+b^{2}}-2ab}}-\frac{\left(-b\sqrt{-4a\cdots\right.}{\cdots}\right.}{c}$
default	$\frac{2a\left(\frac{\left(b\sqrt{-4ac+b^{2}}+2\sqrt{-4ac+b^{2}}\,c+4ac-b^{2}\right)\arctan\left(\frac{\frac{2a\left(\sqrt{-x^{2}+1}-1\right)^{2}}{x^{2}}+2\sqrt{-4ac+b^{2}}+2a+2b}{2\sqrt{4ac-2b^{2}-2\sqrt{-4ac+b^{2}}\,a-2b\sqrt{-4ac+b^{2}}-2ab}}\right)}{\left(8ac-2b^{2}\right)\sqrt{4ac-2b^{2}-2\sqrt{-4ac+b^{2}}\,a-2b\sqrt{-4ac+b^{2}}-2ab}}-\frac{\left(-b\sqrt{-4ac+b^{2}}-2\sqrt{-4ac+b^{2}}\cdots\right.}{\left(8ac-2b\cdots\right.}\right.}{c}$

input
```
int(x^3*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/c*((-x^2+1)^(1/2)+1/2*(b*(-4*a*c+b^2)^(1/2)+(-4*a*c+b^2)^(1/2)*c+2*a*c-b
^2-b*c)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b-2*c)*c)^(1/2)*ar
ctan(c*(-x^2+1)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b-2*c)*c)^(1/2))-1/2*(b
*(-4*a*c+b^2)^(1/2)+(-4*a*c+b^2)^(1/2)*c-2*a*c+b^2+b*c)/(-4*a*c+b^2)^(1/2)
*2^(1/2)/((b+2*c+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*(-x^2+1)^(1/2)*2^(
1/2)/((b+2*c+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

3.377.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2053 vs. $2(187) = 374$.

Time = 1.47 (sec) , antiderivative size = 2053, normalized size of antiderivative = 8.97

$$\int \frac{x^3\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
1/2*(sqrt(1/2)*c*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c - (b^2*c^3 - 4*a*c^
4)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a
*c^7)))/(b^2*c^3 - 4*a*c^4))*log((2*a^2*b^2 + (a*b^2*c^3 - 4*a^2*c^4)*x^2*
sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^
7)) + (a*b^3 - (a^2*b - a*b^2)*c)*x^2 - 2*(a^3 - a^2*b)*c + sqrt(1/2)*((b^
4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*x^2*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 -
 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)) + (b^5 + 4*(a^2*b - a*b^2)*c^2 -
(5*a*b^3 - b^4)*c)*x^2)*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c - (b^2*c^3 -
 4*a*c^4)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^
6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) - 2*(a^2*b^2 - (a^3 - a^2*b)*c)*sqrt(-
x^2 + 1))/x^2) - sqrt(1/2)*c*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c - (b^2*
c^3 - 4*a*c^4)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b
^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log((2*a^2*b^2 + (a*b^2*c^3 - 4*a
^2*c^4)*x^2*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*
c^6 - 4*a*c^7)) + (a*b^3 - (a^2*b - a*b^2)*c)*x^2 - 2*(a^3 - a^2*b)*c - sq
rt(1/2)*((b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*x^2*sqrt((b^4 + (a^2 - 2*a*b
+ b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)) + (b^5 + 4*(a^2*b - a
*b^2)*c^2 - (5*a*b^3 - b^4)*c)*x^2)*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c
- (b^2*c^3 - 4*a*c^4)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3
)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) - 2*(a^2*b^2 - (a^3 - a...
```

3.377.6 Sympy [F]

$$\int \frac{x^3\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \int \frac{x^3\sqrt{-(x-1)(x+1)}}{a+bx^2+cx^4}\,dx$$

input
```
integrate(x**3*(-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
```

output
```
Integral(x**3*sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)
```

3.377.7 Maxima [F]

$$\int \frac{x^3\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \int \frac{\sqrt{-x^2+1}x^3}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^3*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^2 + 1)*x^3/(c*x^4 + b*x^2 + a), x)
```

3.377.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4060 vs. $2(187) = 374$.

Time = 1.07 (sec) , antiderivative size = 4060, normalized size of antiderivative = 17.73

$$\int \frac{x^3\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
sqrt(-x^2 + 1)/c - 1/8*(2*b^5*c^4 - 12*a*b^3*c^5 + 6*b^4*c^5 + 16*a^2*b*c^
6 - 32*a*b^2*c^6 + 4*b^3*c^6 + 32*a^2*c^7 - 16*a*b*c^7 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 5*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b*c
^4 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)
*a*b^2*c^4 - 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4
*a*c)*c)*b^3*c^4 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b
^2 - 4*a*c)*c)*a^2*c^5 + 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 -
sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*
c^2 - sqrt(b^2 - 4*a*c)*c)*b^2*c^5 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*
c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*c^6 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b*c^6 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(
b^2 - 4*a*c)*a*b*c^5 - 6*(b^2 - 4*a*c)*b^2*c^5 + 8*(b^2 - 4*a*c)*a*c^6 - 4
*(b^2 - 4*a*c)*b*c^6 - (2*b^5*c^2 - 16*a*b^3*c^3 + 2*b^4*c^3 + 32*a^2*b*c^
4 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^
2 - sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c
^2 - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c
- 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr...
```

3.377. $\int \frac{x^3\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

3.377.9 Mupad [B] (verification not implemented)

Time = 8.35 (sec) , antiderivative size = 776, normalized size of antiderivative = 3.39

$$\int \frac{x^3\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \frac{\sqrt{1-x^2}}{c} - \frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}-1\right)1\mathrm{i}}{\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}}\right)\left(4\,a\,c^2-b^2\,c-b^3+b^2\,\sqrt{b^2-4\,a\,c}+4\,a\,b\,c-2\,a\,c\,\sqrt{b^2-4\,a\,c}+b\,c\,\surd\right.}{4\,c^2\,\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}\,\left(4\,a\,c-b^2\right)} + \frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}-1\right)1\mathrm{i}}{\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}}\right)\left(b^2\,c-4\,a\,c^2+b^3+b^2\,\sqrt{b^2-4\,a\,c}-4\,a\,b\,c-2\,a\,c\,\sqrt{b^2-4\,a\,c}+b\,c\,\surd\right.}{4\,c^2\,\left(4\,a\,c-b^2\right)\,\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}} - \frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}+1\right)1\mathrm{i}}{\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}}\right)\left(4\,a\,c^2-b^2\,c-b^3+b^2\,\sqrt{b^2-4\,a\,c}+4\,a\,b\,c-2\,a\,c\,\sqrt{b^2-4\,a\,c}+b\,c\,\surd\right.}{4\,c^2\,\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}\,\left(4\,a\,c-b^2\right)} + \frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}+1\right)1\mathrm{i}}{\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}}\right)\left(b^2\,c-4\,a\,c^2+b^3+b^2\,\sqrt{b^2-4\,a\,c}-4\,a\,b\,c-2\,a\,c\,\sqrt{b^2-4\,a\,c}+b\,c\,\surd\right.}{4\,c^2\,\left(4\,a\,c-b^2\right)\,\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}$$

input
```
int((x^3*(1 - x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

output
```
(1 - x^2)^(1/2)/c - (log((((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1
)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x
 - (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(4*a*c^2 - b^2*c - b^3 + b^2
*(b^2 - 4*a*c)^(1/2) + 4*a*b*c - 2*a*c*(b^2 - 4*a*c)^(1/2) + b*c*(b^2 - 4*
a*c)^(1/2)))/(4*c^2*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b
^2)) + (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b + (
b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b + (b^
2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b^2*c - 4*a*c^2 + b^3 + b^2*(b^2 - 4*a*c
)^(1/2) - 4*a*b*c - 2*a*c*(b^2 - 4*a*c)^(1/2) + b*c*(b^2 - 4*a*c)^(1/2)))/
(4*c^2*(4*a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) - (log((
((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b - (b^2 - 4*a*c)^
(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b - (b^2 - 4*a*c)^(1
/2))/(2*c))^(1/2)))*(4*a*c^2 - b^2*c - b^3 + b^2*(b^2 - 4*a*c)^(1/2) + 4*a
*b*c - 2*a*c*(b^2 - 4*a*c)^(1/2) + b*c*(b^2 - 4*a*c)^(1/2)))/(4*c^2*((b -
(b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2)) + (log((((x*(-(b + (b
^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b + (b^2 - 4*a*c)^(1/2))/(2*c)
+ 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(
1/2)))*(b^2*c - 4*a*c^2 + b^3 + b^2*(b^2 - 4*a*c)^(1/2) - 4*a*b*c - 2*a*c*
(b^2 - 4*a*c)^(1/2) + b*c*(b^2 - 4*a*c)^(1/2)))/(4*c^2*(4*a*c - b^2)*((b +
 (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2))
```

3.377. $\int \frac{x^3\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

3.378 $\int \frac{x\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

3.378.1 Optimal result

Integrand size = 27, antiderivative size = 182

$$\int \frac{x\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = -\frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} + \frac{\sqrt{b+2c+\sqrt{b^2-4ac}}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

output
```
-1/2*arctanh(2^(1/2)*c^(1/2)*(-x^2+1)^(1/2)/(b+2*c-(-4*a*c+b^2)^(1/2))^(1/
2))*(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)+1/
2*arctanh(2^(1/2)*c^(1/2)*(-x^2+1)^(1/2)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2))
*(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)
```

3.378.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.93

$$\int \frac{x\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \frac{\sqrt{-b-2c-\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-b-2c-\sqrt{b^2-4ac}}}\right) - \sqrt{-b-2c+\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-b-2c+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

input
```
Integrate[(x*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

output
```
(Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])
/Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*c]]] - Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]]*
ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

3.378.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1576, 1148, 1450, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{1-x^2}}{a+bx^2+cx^4} \, dx$$

$$\Big\downarrow \text{1576}$$

$$\frac{1}{2}\int \frac{\sqrt{1-x^2}}{cx^4+bx^2+a}dx^2$$

$$\Big\downarrow \text{1148}$$

$$-\int \frac{x^4}{cx^8-(b+2c)x^4+a+b+c}d\sqrt{1-x^2}$$

$$\Big\downarrow \text{1450}$$

$$-\frac{1}{2}\left(\frac{b+2c}{\sqrt{b^2-4ac}}+1\right)\int \frac{1}{cx^4+\frac{1}{2}\left(-b-2c-\sqrt{b^2-4ac}\right)}d\sqrt{1-x^2}-\frac{1}{2}\left(1-\frac{b+2c}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^4+\frac{1}{2}\left(-b-2c+\sqrt{b^2-4ac}\right)}d\sqrt{1-x^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{\left(1-\frac{b+2c}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\sqrt{c}\sqrt{-\sqrt{b^2-4ac}+b+2c}}+\frac{\left(\frac{b+2c}{\sqrt{b^2-4ac}}+1\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b+2c}}$$

input
```
Int[(x*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

output
```
((1 - (b + 2*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])
/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + 2*c - Sqrt[
b^2 - 4*a*c]]) + ((1 + (b + 2*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[
c]*Sqrt[1 - x^2])/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqr
t[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

3.378.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1148
```
Int[Sqrt[(d_.) + (e_.)*(x_)]/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[2*e   Subst[Int[x^2/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c
*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1450
```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^2/2)*(b/q + 1)   Int[(d*x)^(m - 2)/(b/
2 + q/2 + c*x^2), x], x] - Simp[(d^2/2)*(b/q - 1)   Int[(d*x)^(m - 2)/(b/2
- q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] &&
 GeQ[m, 2]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

3.378.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.89

method	result
pseudoelliptic	$-\frac{\sqrt{2}\left(\frac{\left(\sqrt{-4ac+b^2}-b-2c\right)\arctan\left(\frac{c\sqrt{-x^2+1}\,\sqrt{2}}{\sqrt{\left(\sqrt{-4ac+b^2}-b-2c\right)c}}\right)}{\sqrt{\left(\sqrt{-4ac+b^2}-b-2c\right)c}}-\frac{\left(b+2c+\sqrt{-4ac+b^2}\right)\operatorname{arctanh}\left(\frac{c\sqrt{-x^2+1}\,\sqrt{2}}{\sqrt{\left(b+2c+\sqrt{-4ac+b^2}\right)c}}\right)}{\sqrt{\left(b+2c+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}}$
default	$2a\left(\frac{\left(-2\sqrt{-4ac+b^2}\,a-b\sqrt{-4ac+b^2}+4ac-b^2\right)\arctan\left(\frac{\frac{2a\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+2\sqrt{-4ac+b^2}+2a+2b}{2\sqrt{4ac-2b^2-2\sqrt{-4ac+b^2}\,a-2b\sqrt{-4ac+b^2}-2ab}}\right)}{2a(4ac-b^2)\sqrt{4ac-2b^2-2\sqrt{-4ac+b^2}\,a-2b\sqrt{-4ac+b^2}-2ab}}-\frac{\left(2\sqrt{-4ac+b^2}\,a\right.}{}\right.$

input
```
int(x*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*2^(1/2)/(-4*a*c+b^2)^(1/2)*(((-4*a*c+b^2)^(1/2)-b-2*c)/(((-4*a*c+b^2)
^(1/2)-b-2*c)*c)^(1/2)*arctan(c*(-x^2+1)^(1/2)*2^(1/2)/(((-4*a*c+b^2)^(1/2
)-b-2*c)*c)^(1/2))-(b+2*c+(-4*a*c+b^2)^(1/2))/((b+2*c+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctanh(c*(-x^2+1)^(1/2)*2^(1/2)/((b+2*c+(-4*a*c+b^2)^(1/2))*c)^(
1/2)))
```

3.378.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 871 vs. 2(143) = 286.

Time = 0.75 (sec) , antiderivative size = 871, normalized size of antiderivative = 4.79

$$\int \frac{x\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx =$$

$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b+2\,c-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(\frac{bx^2+\frac{(b^2c-4\,ac^2)x^2}{\sqrt{b^2c^2-4\,ac^3}}+\sqrt{\frac{1}{2}}\left((b^2-4\,ac)x^2+\frac{(b^3c-4\,abc^2)x^2}{\sqrt{b^2c^2-4\,ac^3}}\right)\sqrt{\frac{b+2\,c-\frac{b^2c-4}{\sqrt{b^2c^2-}}}{b^2c-4\,ac^2}}}{x^2}\right.$$

$$+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b+2\,c-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(\frac{bx^2+\frac{(b^2c-4\,ac^2)x^2}{\sqrt{b^2c^2-4\,ac^3}}-\sqrt{\frac{1}{2}}\left((b^2-4\,ac)x^2+\frac{(b^3c-4\,abc^2)x^2}{\sqrt{b^2c^2-4\,ac^3}}\right)\sqrt{\frac{b+2\,c-\frac{b^2c-4}{\sqrt{b^2c^2-}}}{b^2c-4\,ac^2}}}{x^2}\right.$$

$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b+2\,c+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(\frac{bx^2-\frac{(b^2c-4\,ac^2)x^2}{\sqrt{b^2c^2-4\,ac^3}}+\sqrt{\frac{1}{2}}\left((b^2-4\,ac)x^2-\frac{(b^3c-4\,abc^2)x^2}{\sqrt{b^2c^2-4\,ac^3}}\right)\sqrt{\frac{b+2\,c+\frac{b^2c-4}{\sqrt{b^2c^2-}}}{b^2c-4\,ac^2}}}{x^2}\right.$$

$$+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b+2\,c+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(\frac{bx^2-\frac{(b^2c-4\,ac^2)x^2}{\sqrt{b^2c^2-4\,ac^3}}-\sqrt{\frac{1}{2}}\left((b^2-4\,ac)x^2-\frac{(b^3c-4\,abc^2)x^2}{\sqrt{b^2c^2-4\,ac^3}}\right)\sqrt{\frac{b+2\,c+\frac{b^2c-4}{\sqrt{b^2c^2-}}}{b^2c-4\,ac^2}}}{x^2}\right.$$

input
```
integrate(x*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

3.378. $\int \frac{x\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

output
```
-1/2*sqrt(1/2)*sqrt((b + 2*c - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/
(b^2*c - 4*a*c^2))*log((b*x^2 + (b^2*c - 4*a*c^2)*x^2/sqrt(b^2*c^2 - 4*a*c
^3) + sqrt(1/2)*((b^2 - 4*a*c)*x^2 + (b^3*c - 4*a*b*c^2)*x^2/sqrt(b^2*c^2
- 4*a*c^3))*sqrt((b + 2*c - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^
2*c - 4*a*c^2)) - 2*sqrt(-x^2 + 1)*a + 2*a)/x^2) + 1/2*sqrt(1/2)*sqrt((b +
 2*c - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))*log((
b*x^2 + (b^2*c - 4*a*c^2)*x^2/sqrt(b^2*c^2 - 4*a*c^3) - sqrt(1/2)*((b^2 -
4*a*c)*x^2 + (b^3*c - 4*a*b*c^2)*x^2/sqrt(b^2*c^2 - 4*a*c^3))*sqrt((b + 2*
c - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2)) - 2*sqrt
(-x^2 + 1)*a + 2*a)/x^2) - 1/2*sqrt(1/2)*sqrt((b + 2*c + (b^2*c - 4*a*c^2)
/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))*log((b*x^2 - (b^2*c - 4*a*c^2
)*x^2/sqrt(b^2*c^2 - 4*a*c^3) + sqrt(1/2)*((b^2 - 4*a*c)*x^2 - (b^3*c - 4*
a*b*c^2)*x^2/sqrt(b^2*c^2 - 4*a*c^3))*sqrt((b + 2*c + (b^2*c - 4*a*c^2)/sq
rt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2)) - 2*sqrt(-x^2 + 1)*a + 2*a)/x^2)
 + 1/2*sqrt(1/2)*sqrt((b + 2*c + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3)
)/(b^2*c - 4*a*c^2))*log((b*x^2 - (b^2*c - 4*a*c^2)*x^2/sqrt(b^2*c^2 - 4*a
*c^3) - sqrt(1/2)*((b^2 - 4*a*c)*x^2 - (b^3*c - 4*a*b*c^2)*x^2/sqrt(b^2*c^
2 - 4*a*c^3))*sqrt((b + 2*c + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(
b^2*c - 4*a*c^2)) - 2*sqrt(-x^2 + 1)*a + 2*a)/x^2)
```

3.378.6 Sympy [F]

$$\int \frac{x\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \int \frac{x\sqrt{-(x-1)(x+1)}}{a+bx^2+cx^4}\,dx$$

input
```
integrate(x*(-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
```

output
```
Integral(x*sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)
```

3.378.7 Maxima [F]

$$\int \frac{x\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \int \frac{\sqrt{-x^2+1}x}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^2 + 1)*x/(c*x^4 + b*x^2 + a), x)
```

3.378.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 591 vs. $2(143) = 286$.

Time = 1.57 (sec) , antiderivative size = 591, normalized size of antiderivative = 3.25

$$\int \frac{x\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$$

$$= \frac{\left(2\,b^2c^2 - 8\,ac^3 - \sqrt{2}\sqrt{b^2-4\,ac}\sqrt{-bc-2\,c^2-\sqrt{b^2-4\,ac}c}b^2 + 4\,\sqrt{2}\sqrt{b^2-4\,ac}\sqrt{-bc-2\,c^2-\sqrt{b^2-4\,a}}\right.}{2\,(b^4}$$

$$+ \frac{\left(2\,b^2c^2 - 8\,ac^3 - \sqrt{2}\sqrt{b^2-4\,ac}\sqrt{-bc-2\,c^2+\sqrt{b^2-4\,ac}c}b^2 + 4\,\sqrt{2}\sqrt{b^2-4\,ac}\sqrt{-bc-2\,c^2+\sqrt{b^2-4}}\right.}{2\,(b}$$

input
```
integrate(x*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/2*(2*b^2*c^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - s
qrt(b^2 - 4*a*c)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 -
sqrt(b^2 - 4*a*c)*c)*a*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 -
 sqrt(b^2 - 4*a*c)*c)*b*c - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2
- sqrt(b^2 - 4*a*c)*c)*c^2 - 2*(b^2 - 4*a*c)*c^2)*arctan(2*sqrt(1/2)*sqrt(
-x^2 + 1)/sqrt(-(b + 2*c + sqrt((b + 2*c)^2 - 4*(a + b + c)*c))/c))/((b^4
- 8*a*b^2*c + 2*b^3*c + 16*a^2*c^2 - 8*a*b*c^2 + 5*b^2*c^2 - 20*a*c^3)*abs
(c)) + 1/2*(2*b^2*c^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*
c^2 + sqrt(b^2 - 4*a*c)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2
*c^2 + sqrt(b^2 - 4*a*c)*c)*a*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c -
2*c^2 + sqrt(b^2 - 4*a*c)*c)*b*c - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c -
 2*c^2 + sqrt(b^2 - 4*a*c)*c)*c^2 - 2*(b^2 - 4*a*c)*c^2)*arctan(2*sqrt(1/2
)*sqrt(-x^2 + 1)/sqrt(-(b + 2*c - sqrt((b + 2*c)^2 - 4*(a + b + c)*c))/c))
/((b^4 - 8*a*b^2*c + 2*b^3*c + 16*a^2*c^2 - 8*a*b*c^2 + 5*b^2*c^2 - 20*a*c
^3)*abs(c))
```

3.378.9 Mupad [B] (verification not implemented)

Time = 8.30 (sec) , antiderivative size = 649, normalized size of antiderivative = 3.57

$$\int \frac{x\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}-1\right)1\mathrm{i}}{\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}}\right)\left(4ac+b\sqrt{b^2-4ac}+2c\sqrt{b^2-4ac}-b^2\right)}{4c\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}\,(4ac-b^2)} - \frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}-1\right)1\mathrm{i}}{\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}}\right)\left(b\sqrt{b^2-4ac}-4ac+2c\sqrt{b^2-4ac}+b^2\right)}{4c\,(4ac-b^2)\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}} + \frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}+1\right)1\mathrm{i}}{\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}}\right)\left(4ac+b\sqrt{b^2-4ac}+2c\sqrt{b^2-4ac}-b^2\right)}{4c\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}\,(4ac-b^2)} - \frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}+1\right)1\mathrm{i}}{\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}}\right)\left(b\sqrt{b^2-4ac}-4ac+2c\sqrt{b^2-4ac}+b^2\right)}{4c\,(4ac-b^2)\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}$$

input
```
int((x*(1 - x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

output
```
(log((((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b - (b^2 - 4
*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b - (b^2 - 4*a
*c)^(1/2))/(2*c))^(1/2)))*(4*a*c + b*(b^2 - 4*a*c)^(1/2) + 2*c*(b^2 - 4*a*
c)^(1/2) - b^2))/(4*c*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c -
 b^2)) - (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b +
 (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b + (
b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b*(b^2 - 4*a*c)^(1/2) - 4*a*c + 2*c*(b
^2 - 4*a*c)^(1/2) + b^2))/(4*c*(4*a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2
*c) + 1)^(1/2)) + (log((((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*
1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x +
 (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(4*a*c + b*(b^2 - 4*a*c)^(1/2)
 + 2*c*(b^2 - 4*a*c)^(1/2) - b^2))/(4*c*((b - (b^2 - 4*a*c)^(1/2))/(2*c) +
 1)^(1/2)*(4*a*c - b^2)) - (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1
/2) + 1)*1i)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)
*1i)/(x + (-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b*(b^2 - 4*a*c)^(1/2
) - 4*a*c + 2*c*(b^2 - 4*a*c)^(1/2) + b^2))/(4*c*(4*a*c - b^2)*((b + (b^2
- 4*a*c)^(1/2))/(2*c) + 1)^(1/2))
```

3.378. $\int \frac{x\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

3.379 $\int \frac{\sqrt{1-x^2}}{x(a+bx^2+cx^4)}\,dx$

3.379.1 Optimal result

Integrand size = 29, antiderivative size = 241

$$\int \frac{\sqrt{1-x^2}}{x\left(a+bx^2+cx^4\right)}\,dx = -\frac{\operatorname{arctanh}\left(\sqrt{1-x^2}\right)}{a} + \frac{\sqrt{c}\left(2a+b+\sqrt{b^2-4ac}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{b+2c-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(2a+b-\sqrt{b^2-4ac}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{b+2c+\sqrt{b^2-4ac}}}$$

output
```
-arctanh((-x^2+1)^(1/2))/a+1/2*arctanh(2^(1/2)*c^(1/2)*(-x^2+1)^(1/2)/(b+2
*c-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(2*a+b+(-4*a*c+b^2)^(1/2))/a*2^(1/2)
/(-4*a*c+b^2)^(1/2)/(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctanh(2^(1/2)*c
^(1/2)*(-x^2+1)^(1/2)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(2*a+b-(-4
*a*c+b^2)^(1/2))/a*2^(1/2)/(-4*a*c+b^2)^(1/2)/(b+2*c+(-4*a*c+b^2)^(1/2))^(
1/2)
```

3.379.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{1-x^2}}{x(a+bx^2+cx^4)}\,dx = -\frac{\frac{\sqrt{2}\sqrt{c}\left(-2a-b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-b-2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{-b-2c-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(2a+b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-b-2c+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{-b-2c+\sqrt{b^2-4ac}}}-\log\left(-1+\sqrt{1-\right.}{2a}$$

input
```
Integrate[Sqrt[1 - x^2]/(x*(a + b*x^2 + c*x^4)),x]
```

output
```
-1/2*((Sqrt[2]*Sqrt[c]*(-2*a - b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt
[c]*Sqrt[1 - x^2])/Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]
*Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(2*a + b + Sqrt[b^
2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[-b - 2*c + Sqrt[b^
2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]]) - Log
[-1 + Sqrt[1 - x^2]] + Log[a*(1 + Sqrt[1 - x^2])])/a
```

3.379.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1578, 1199, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x^2}}{x(a+bx^2+cx^4)}\,dx$$

$$\Big\downarrow \ 1578$$

$$\frac{1}{2}\int \frac{\sqrt{1-x^2}}{x^2(cx^4+bx^2+a)}dx^2$$

$$\Big\downarrow \ 1199$$

$$-\int \left(\frac{1}{a(1-x^4)}-\frac{-cx^4+a+b+c}{a(cx^8-(b+2c)x^4+a+b+c)}\right)d\sqrt{1-x^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2a+b\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}+b+2c}}+\frac{\sqrt{c}\left(1-\frac{2a+b}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b+2c}}-\frac{\operatorname{arctanh}\left(\sqrt{1-x^2}\right)}{a}$$

input
```
Int[Sqrt[1 - x^2]/(x*(a + b*x^2 + c*x^4)),x]
```

output
```
-(ArcTanh[Sqrt[1 - x^2]]/a) + (Sqrt[c]*(2*a + b + Sqrt[b^2 - 4*a*c])*ArcTa
nh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sq
rt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(1
 - (2*a + b)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sq
rt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + 2*c + Sqrt[b^2 - 4*a
*c]])
```

3.379.3.1 Defintions of rubi rules used

rule 1199
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Denominator[m]}, Simp[q/e   Subs
t[Int[ExpandIntegrand[x^(q*(m + 1) - 1)*(((e*f - d*g)/e + g*(x^q/e))^n/((c*
d^2 - b*d*e + a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))), x],
 x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Integer
Q[n] && FractionQ[m]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.379.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.12

method	result
pseudoelliptic	$-\frac{\sqrt{2}\sqrt{\left(b+2c+\sqrt{-4ac+b^2}\right)c}\,c\left(a+\frac{b}{2}+\frac{\sqrt{-4ac+b^2}}{2}\right)\arctan\left(\frac{c\sqrt{-x^2+1}\,\sqrt{2}}{\sqrt{\left(\sqrt{-4ac+b^2}-b-2c\right)c}}\right)+\sqrt{\left(\sqrt{-4ac+b^2}-b-2c\right)c}\left(c\sqrt{2}\left(a+\right.\right.}{\sqrt{-4ac+b^2}\sqrt{\left(\sqrt{-4ac+b^2}-b-\right.}}$
default	$\frac{\sqrt{-x^2+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)}{a}-\frac{2a\left(\frac{\left(-\sqrt{-4ac+b^2}\,ab+2ac\sqrt{-4ac+b^2}-b^2\sqrt{-4ac+b^2}+4c\,a^2-b^2a+4abc-b^3\right)\arctan\left(\frac{2a\left(\sqrt{-}\right.}{2\sqrt{4ac-2b}}\right.}{2a\left(4ac-b^2\right)\sqrt{4ac-2b^2-2\sqrt{-4ac+b^2}\,a-2b\sqrt{-4ac+b^2}-}}\right.}{}$

input
```
int((-x^2+1)^(1/2)/x/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/(-4*a*c+b^2)^(1/2)*(2^(1/2)*((b+2*c+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*(a+1
/2*b+1/2*(-4*a*c+b^2)^(1/2))*arctan(c*(-x^2+1)^(1/2)*2^(1/2)/(((-4*a*c+b^2
)^(1/2)-b-2*c)*c)^(1/2))+(((-4*a*c+b^2)^(1/2)-b-2*c)*c)^(1/2)*(c*2^(1/2)*(
a+1/2*b-1/2*(-4*a*c+b^2)^(1/2))*arctanh(c*(-x^2+1)^(1/2)*2^(1/2)/((b+2*c+(
-4*a*c+b^2)^(1/2))*c)^(1/2))-1/2*(-4*a*c+b^2)^(1/2)*((b+2*c+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*(ln((-x^2+1)^(1/2)-1)-ln(1+(-x^2+1)^(1/2)))))/(((-4*a*c+b^2
)^(1/2)-b-2*c)*c)^(1/2)/((b+2*c+(-4*a*c+b^2)^(1/2))*c)^(1/2)/a
```

3.379.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1232 vs. 2(196) = 392.

Time = 2.72 (sec) , antiderivative size = 1232, normalized size of antiderivative = 5.11

$$\int \frac{\sqrt{1-x^2}}{x\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

input
```
integrate((-x^2+1)^(1/2)/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
1/2*(sqrt(1/2)*a*sqrt((a*b + b^2 - 2*a*c + (a^2*b^2 - 4*a^3*c)*sqrt((a^2 +
 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log((2*sqrt(1/2)*
(a^3*b^2 - 4*a^4*c)*x^2*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c))*sqrt
((a*b + b^2 - 2*a*c + (a^2*b^2 - 4*a^3*c)*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^
2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) + (a^2*b^2 - 4*a^3*c)*x^2*sqrt((a^2 +
2*a*b + b^2)/(a^4*b^2 - 4*a^5*c)) + (a*b + b^2)*x^2 + 2*a^2 + 2*a*b - 2*(a
^2 + a*b)*sqrt(-x^2 + 1))/x^2) - sqrt(1/2)*a*sqrt((a*b + b^2 - 2*a*c + (a^
2*b^2 - 4*a^3*c)*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 -
 4*a^3*c))*log(-(2*sqrt(1/2)*(a^3*b^2 - 4*a^4*c)*x^2*sqrt((a^2 + 2*a*b + b
^2)/(a^4*b^2 - 4*a^5*c))*sqrt((a*b + b^2 - 2*a*c + (a^2*b^2 - 4*a^3*c)*sqr
t((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - (a^2*b^
2 - 4*a^3*c)*x^2*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c)) - (a*b + b^
2)*x^2 - 2*a^2 - 2*a*b + 2*(a^2 + a*b)*sqrt(-x^2 + 1))/x^2) + sqrt(1/2)*a*
sqrt((a*b + b^2 - 2*a*c - (a^2*b^2 - 4*a^3*c)*sqrt((a^2 + 2*a*b + b^2)/(a^
4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log(-(2*sqrt(1/2)*(a^3*b^2 - 4*a^4
*c)*x^2*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c))*sqrt((a*b + b^2 - 2*
a*c - (a^2*b^2 - 4*a^3*c)*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c)))/(
a^2*b^2 - 4*a^3*c)) + (a^2*b^2 - 4*a^3*c)*x^2*sqrt((a^2 + 2*a*b + b^2)/(a^
4*b^2 - 4*a^5*c)) - (a*b + b^2)*x^2 - 2*a^2 - 2*a*b + 2*(a^2 + a*b)*sqrt(-
x^2 + 1))/x^2) - sqrt(1/2)*a*sqrt((a*b + b^2 - 2*a*c - (a^2*b^2 - 4*a^3...
```

3.379.6 Sympy [F]

$$\int \frac{\sqrt{1-x^2}}{x\left(a+bx^2+cx^4\right)}\,dx = \int \frac{\sqrt{-\left(x-1\right)\left(x+1\right)}}{x\left(a+bx^2+cx^4\right)}\,dx$$

input
```
integrate((-x**2+1)**(1/2)/x/(c*x**4+b*x**2+a),x)
```

output
```
Integral(sqrt(-(x - 1)*(x + 1))/(x*(a + b*x**2 + c*x**4)), x)
```

3.379.7 Maxima [F]

$$\int \frac{\sqrt{1-x^2}}{x\left(a+bx^2+cx^4\right)}\,dx=\int \frac{\sqrt{-x^2+1}}{(cx^4+bx^2+a)x}\,dx$$

input
```
integrate((-x^2+1)^(1/2)/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^2 + 1)/((c*x^4 + b*x^2 + a)*x), x)
```

3.379.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3639 vs. $2(196) = 392$.

Time = 0.97 (sec) , antiderivative size = 3639, normalized size of antiderivative = 15.10

$$\int \frac{\sqrt{1-x^2}}{x\left(a+bx^2+cx^4\right)}\,dx=\text{Too large to display}$$

input
```
integrate((-x^2+1)^(1/2)/x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*log(sqrt(-x^2 + 1) + 1)/a + 1/2*log(-sqrt(-x^2 + 1) + 1)/a + 1/8*(4*a
^3*b^3*c^2 + 2*a^2*b^4*c^2 - 16*a^4*b*c^3 + 4*a^2*b^3*c^3 - 32*a^4*c^4 - 1
6*a^3*b*c^4 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4
*a*c)*c)*a^3*b^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2
- 4*a*c)*c)*a^2*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt
(b^2 - 4*a*c)*c)*a^4*b*c - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 -
 sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c -
2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^4*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 9*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 20*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 10*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^
3 - 4*(b^2 - 4*a*c)*a^3*b*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2 - 8*(b^2 - 4*a
*c)*a^3*c^3 - 4*(b^2 - 4*a*c)*a^2*b*c^3 + (2*b^4*c^2 - 16*a*b^2*c^3 + 32*a
^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c
)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*
c)*a*b^2*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*
a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2
- 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - s...
```

3.379.9 Mupad [B] (verification not implemented)

Time = 8.40 (sec) , antiderivative size = 669, normalized size of antiderivative = 2.78

$$\int \frac{\sqrt{1-x^2}}{x\left(a+bx^2+cx^4\right)}\,dx$$
$$= \frac{\ln\left(\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}}\right)}{a} + \frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}+1\right)\,1\mathrm{i}}{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\left(4\,a\,c+2\,a\,\sqrt{b^2-4\,a\,c}+b\,\sqrt{b^2-4\,a\,c}-b^2\right)}{4\,a\,\left(4\,a\,c-b^2\right)\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}} - \frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}+1\right)\,1\mathrm{i}}{\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\left(2\,a\,\sqrt{b^2-4\,a\,c}-4\,a\,c+b\,\sqrt{b^2-4\,a\,c}+b^2\right)}{4\,a\,\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}\,\left(4\,a\,c-b^2\right)} + \frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}-1\right)\,1\mathrm{i}}{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\left(4\,a\,c+2\,a\,\sqrt{b^2-4\,a\,c}+b\,\sqrt{b^2-4\,a\,c}-b^2\right)}{4\,a\,\left(4\,a\,c-b^2\right)\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}} - \frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}-1\right)\,1\mathrm{i}}{\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\left(2\,a\,\sqrt{b^2-4\,a\,c}-4\,a\,c+b\,\sqrt{b^2-4\,a\,c}+b^2\right)}{4\,a\,\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}\,\left(4\,a\,c-b^2\right)}$$

input
```
int((1 - x^2)^(1/2)/(x*(a + b*x^2 + c*x^4)),x)
```

output
```
log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2))/a + (log((((x*(-(b + (b^2 - 4*a*c)^
(1/2))/(2*c))^(1/2) + 1)*1i)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) +
 (1 - x^2)^(1/2)*1i)/(x + (-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(4*a*
c + 2*a*(b^2 - 4*a*c)^(1/2) + b*(b^2 - 4*a*c)^(1/2) - b^2))/(4*a*(4*a*c -
b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) - (log((((x*(-(b - (b^2
- 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1
)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2
)))*(2*a*(b^2 - 4*a*c)^(1/2) - 4*a*c + b*(b^2 - 4*a*c)^(1/2) + b^2))/(4*a*
((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2)) + (log((((x*(-(
b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b + (b^2 - 4*a*c)^(1/2))/
(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b + (b^2 - 4*a*c)^(1/2))/(2
*c))^(1/2)))*(4*a*c + 2*a*(b^2 - 4*a*c)^(1/2) + b*(b^2 - 4*a*c)^(1/2) - b^
2))/(4*a*(4*a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) - (log
((((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b - (b^2 - 4*a*c
)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b - (b^2 - 4*a*c)^
(1/2))/(2*c))^(1/2)))*(2*a*(b^2 - 4*a*c)^(1/2) - 4*a*c + b*(b^2 - 4*a*c)^(
1/2) + b^2))/(4*a*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2
))
```

3.379. $\int \frac{\sqrt{1-x^2}}{x(a+bx^2+cx^4)}\,dx$

3.380 $\int \frac{\sqrt{1-x^2}}{x^3(a+bx^2+cx^4)}\,dx$

3.380.1 Optimal result

Integrand size = 29, antiderivative size = 290

$$\int \frac{\sqrt{1-x^2}}{x^3(a+bx^2+cx^4)}\,dx = -\frac{1}{4a\left(1-\sqrt{1-x^2}\right)}+\frac{1}{4a\left(1+\sqrt{1-x^2}\right)} + \frac{(a+2b)\text{arctanh}\left(\sqrt{1-x^2}\right)}{2a^2} - \frac{\sqrt{c}\left(a+b+\frac{b^2+a(b-2c)}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b+2c-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(a+b-\frac{b^2+a(b-2c)}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b+2c+\sqrt{b^2-4ac}}}$$

output
```
1/2*(a+2*b)*arctanh((-x^2+1)^(1/2))/a^2-1/4/a/(1-(-x^2+1)^(1/2))+1/4/a/(1+
(-x^2+1)^(1/2))-1/2*arctanh(2^(1/2)*c^(1/2)*(-x^2+1)^(1/2)/(b+2*c-(-4*a*c+
b^2)^(1/2))^(1/2))*c^(1/2)*(a+b+(b^2+a*(b-2*c))/(-4*a*c+b^2)^(1/2))/a^2*2^
(1/2)/(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctanh(2^(1/2)*c^(1/2)*(-x^2+1
)^(1/2)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(a+b+(-b^2-a*(b-2*c))/(-
4*a*c+b^2)^(1/2))/a^2*2^(1/2)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.380.2 Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{1-x^2}}{x^3 (a+bx^2+cx^4)}\,dx$$
$$= \frac{-\frac{a\sqrt{1-x^2}}{x^2} + \frac{\sqrt{2}\sqrt{c}\left(b\left(-b+\sqrt{b^2-4ac}\right)+a\left(-b+2c+\sqrt{b^2-4ac}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-b-2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{-b-2c-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(b\left(b+\sqrt{b^2-4ac}\right)+a\left(b-2c+\sqrt{b^2-4a\cdots\right.\right.}{\sqrt{b^2-4ac}\sqrt{-b-2c+\cdots}}}{2a^2}$$

input
```
Integrate[Sqrt[1 - x^2]/(x^3*(a + b*x^2 + c*x^4)),x]
```

output
```
(-((a*Sqrt[1 - x^2])/x^2) + (Sqrt[2]*Sqrt[c]*(b*(-b + Sqrt[b^2 - 4*a*c]) +
 a*(-b + 2*c + Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/
Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b - 2*c - Sq
rt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(b*(b + Sqrt[b^2 - 4*a*c]) + a*(b - 2
*c + Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[-b -
2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b - 2*c + Sqrt[b^2 - 4
*a*c]]) + (a + 2*b)*ArcTanh[Sqrt[1 - x^2]])/(2*a^2)
```

3.380.3 Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1578, 1199, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x^2}}{x^3 (a+bx^2+cx^4)}\,dx$$
$$\Big\downarrow \text{ 1578}$$
$$\frac{1}{2}\int \frac{\sqrt{1-x^2}}{x^4 (cx^4+bx^2+a)}dx^2$$
$$\Big\downarrow \text{ 1199}$$

$$-\int \left(-\frac{a+2b}{2a^2(1-x^4)}+\frac{b(a+b+c)-(a+b)cx^4}{a^2(cx^8-(b+2c)x^4+a+b+c)}+\frac{1}{4a\left(1-\sqrt{1-x^2}\right)^2}+\frac{1}{4a\left(\sqrt{1-x^2}+1\right)^2}\right)d\sqrt{1-x^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c}\left(\frac{a(b-2c)+b^2}{\sqrt{b^2-4ac}}+a+b\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}a^2\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{\sqrt{c}\left(-\frac{a(b-2c)+b^2}{\sqrt{b^2-4ac}}+a+b\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}a^2\sqrt{\sqrt{b^2-4ac}+b+2c}}+\frac{(a+2b)\text{arctanh}\left(\sqrt{1-x^2}\right)}{2a^2}-\frac{1}{4a\left(1-\sqrt{1-x^2}\right)}+\frac{1}{4a\left(\sqrt{1-x^2}+1\right)}$$

input
```
Int[Sqrt[1 - x^2]/(x^3*(a + b*x^2 + c*x^4)),x]
```

output
```
-1/4*1/(a*(1 - Sqrt[1 - x^2])) + 1/(4*a*(1 + Sqrt[1 - x^2])) + ((a + 2*b)*
ArcTanh[Sqrt[1 - x^2]])/(2*a^2) - (Sqrt[c]*(a + b + (b^2 + a*(b - 2*c))/Sq
rt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c - Sq
rt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - (Sqrt
[c]*(a + b - (b^2 + a*(b - 2*c))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[
c]*Sqrt[1 - x^2])/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b
+ 2*c + Sqrt[b^2 - 4*a*c]])
```

3.380.3.1 Defintions of rubi rules used

rule 1199
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Denominator[m]}, Simp[q/e   Subs
t[Int[ExpandIntegrand[x^(q*(m + 1) - 1)*(((e*f - d*g)/e + g*(x^q/e))^n/((c*
d^2 - b*d*e + a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))), x],
 x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Integer
Q[n] && FractionQ[m]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.380.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.14

method	result
pseudoelliptic	$-\frac{-2\sqrt{\left(b+2c+\sqrt{-4ac+b^2}\right)c}\,\sqrt{2}\,c\,x^2\left((a+b)\sqrt{-4ac+b^2}+a(b-2c)+b^2\right)\arctan\left(\frac{c\sqrt{-x^2+1}\,\sqrt{2}}{\sqrt{\left(\sqrt{-4ac+b^2}-b-2c\right)c}}\right)+\left(-2\left((-a-b)\sqrt{-\cdots}\right.\right.}{\cdots}$
risch	$\frac{x^2-1}{2a\,x^2\sqrt{-x^2+1}}+\frac{-\frac{(-a-2b)\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)}{a}+\frac{2\left(2\sqrt{-4ac+b^2}\,a^2c-\sqrt{-4ac+b^2}\,a\,b^2+3\sqrt{-4ac+b^2}\,abc-\sqrt{-4ac+b^2}\,b^3+4cb\,a^2-4\cdots\right.}{a\left(4ac-b^2\right)\sqrt{4ac-2b^2-2\sqrt{-\cdots}}}}{\cdots}$
default	$\frac{-\frac{\left(-x^2+1\right)^{\frac{3}{2}}}{2x^2}-\frac{\sqrt{-x^2+1}}{2}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)}{2}}{a}-\frac{b\left(\sqrt{-x^2+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)\right)}{a^2}-\frac{-2a\left(\frac{\left(2\sqrt{-4ac+b^2}\,a^2c-\sqrt{-4ac+b\cdots}\right.}{\cdots}\right.}{\cdots}$

input
```
int((-x^2+1)^(1/2)/x^3/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4/(-4*a*c+b^2)^(1/2)*(-2*((b+2*c+(-4*a*c+b^2)^(1/2))*c)^(1/2)*2^(1/2)*c
*x^2*((a+b)*(-4*a*c+b^2)^(1/2)+a*(b-2*c)+b^2)*arctan(c*(-x^2+1)^(1/2)*2^(1
/2)/(((-4*a*c+b^2)^(1/2)-b-2*c)*c)^(1/2))+(-2*((-a-b)*(-4*a*c+b^2)^(1/2)+a
*(b-2*c)+b^2)*2^(1/2)*c*x^2*arctanh(c*(-x^2+1)^(1/2)*2^(1/2)/((b+2*c+(-4*a
*c+b^2)^(1/2))*c)^(1/2))+((b+2*c+(-4*a*c+b^2)^(1/2))*c)^(1/2)*(-4*a*c+b^2)
^(1/2)*(-x^2*(a+2*b)*ln(1+(-x^2+1)^(1/2))+x^2*(a+2*b)*ln((-x^2+1)^(1/2)-1)
+2*a*(-x^2+1)^(1/2)))*(((-4*a*c+b^2)^(1/2)-b-2*c)*c)^(1/2))/(((-4*a*c+b^2)
^(1/2)-b-2*c)*c)^(1/2)/((b+2*c+(-4*a*c+b^2)^(1/2))*c)^(1/2)/a^2/x^2
```

3.380.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2799 vs. $2(236) = 472$.

Time = 7.50 (sec) , antiderivative size = 2799, normalized size of antiderivative = 9.65

$$\int \frac{\sqrt{1-x^2}}{x^3(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate((-x^2+1)^(1/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(1/2)*a^2*x^2*sqrt((a*b^3 + b^4 + 2*a^2*c^2 - (3*a^2*b + 4*a*b^2
)*c - (a^4*b^2 - 4*a^5*c)*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b +
 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)
))/(a^4*b^2 - 4*a^5*c))*log(((a^4*b^2*c - 4*a^5*c^2)*x^2*sqrt((a^2*b^4 + 2
*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 +
2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)) + 2*(a^3 + 2*a^2*b)*c^2 + ((a^2*b + 2*a*b
^2)*c^2 - (a*b^3 + b^4)*c)*x^2 - 2*(a^2*b^2 + a*b^3)*c + sqrt(1/2)*((a^5*b
^3 - 4*a^6*b*c)*x^2*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2
*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)) + (a
^2*b^4 + a*b^5 + 4*(a^4 + 2*a^3*b)*c^2 - (5*a^3*b^2 + 6*a^2*b^3)*c)*x^2)*s
qrt((a*b^3 + b^4 + 2*a^2*c^2 - (3*a^2*b + 4*a*b^2)*c - (a^4*b^2 - 4*a^5*c)
*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*
b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) -
 2*((a^3 + 2*a^2*b)*c^2 - (a^2*b^2 + a*b^3)*c)*sqrt(-x^2 + 1))/x^2) - sqrt
(1/2)*a^2*x^2*sqrt((a*b^3 + b^4 + 2*a^2*c^2 - (3*a^2*b + 4*a*b^2)*c - (a^4
*b^2 - 4*a^5*c)*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2
)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^
2 - 4*a^5*c))*log(((a^4*b^2*c - 4*a^5*c^2)*x^2*sqrt((a^2*b^4 + 2*a*b^5 + b
^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c
)/(a^8*b^2 - 4*a^9*c)) + 2*(a^3 + 2*a^2*b)*c^2 + ((a^2*b + 2*a*b^2)*c^2...
```

3.380.6 Sympy [F]

$$\int \frac{\sqrt{1-x^2}}{x^3(a+bx^2+cx^4)}\,dx = \int \frac{\sqrt{-(x-1)(x+1)}}{x^3(a+bx^2+cx^4)}\,dx$$

input
```
integrate((-x**2+1)**(1/2)/x**3/(c*x**4+b*x**2+a),x)
```

output
```
Integral(sqrt(-(x - 1)*(x + 1))/(x**3*(a + b*x**2 + c*x**4)), x)
```

3.380.7 Maxima [F]

$$\int \frac{\sqrt{1-x^2}}{x^3 (a+bx^2+cx^4)}\,dx = \int \frac{\sqrt{-x^2+1}}{(cx^4+bx^2+a)x^3}\,dx$$

input
```
integrate((-x^2+1)^(1/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^2 + 1)/((c*x^4 + b*x^2 + a)*x^3), x)
```

3.380.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1675 vs. $2(236) = 472$.

Time = 1.57 (sec) , antiderivative size = 1675, normalized size of antiderivative = 5.78

$$\int \frac{\sqrt{1-x^2}}{x^3 (a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate((-x^2+1)^(1/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
-1/4*(sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqr
t(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(-b*c - 2*c^
2 - sqrt(b^2 - 4*a*c)*c)*b^4*c + 2*b^5*c + 16*sqrt(2)*sqrt(-b*c - 2*c^2 -
sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 -
4*a*c)*c)*a*b^2*c^2 + 5*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b
^3*c^2 - 16*a*b^3*c^2 + 2*b^4*c^2 - 20*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^
2 - 4*a*c)*c)*a*b*c^3 + 32*a^2*b*c^3 - 12*a*b^2*c^3 + 16*a^2*c^4 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^4 + 6*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^3*c -
8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*c
^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*
a*b*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c
)*c)*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 -
 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^3*c + 8*(b^2 - 4*a*c)*a*b*c^2 - 2*(b^
2 - 4*a*c)*b^2*c^2 + 4*(b^2 - 4*a*c)*a*c^3)*arctan(2*sqrt(1/2)*sqrt(-x^2 +
 1)/sqrt(-(a^2*b + 2*a^2*c + sqrt(-4*(a^3 + a^2*b + a^2*c)*a^2*c + (a^2*b
+ 2*a^2*c)^2))/(a^2*c)))/((a^2*b^4 - 8*a^3*b^2*c + 2*a^2*b^3*c + 16*a^4*c^
2 - 8*a^3*b*c^2 + 5*a^2*b^2*c^2 - 20*a^3*c^3)*abs(c)) - 1/4*(sqrt(2)*sqrt(
-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(-b*c - 2*c^2 +...
```

3.380.9 Mupad [B] (verification not implemented)

Time = 8.24 (sec) , antiderivative size = 825, normalized size of antiderivative = 2.84

$$\int \frac{\sqrt{1-x^2}}{x^3(a+bx^2+cx^4)}\,dx = \frac{\ln\left(\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}}\right)}{2\,a} - \frac{\ln\left(\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}}\right)\,(a+b)}{a^2} - \frac{\sqrt{1-x^2}}{2\,a\,x^2}$$

$$-\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}+1\right)1\mathrm{i}}{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\left(4\,a^2\,c-a\,b^2-b^3+b^2\sqrt{b^2-4\,a\,c}+4\,a\,b\,c+a\,b\sqrt{b^2-4\,a\,c}-2\,a\,c\right.}{4\,a^2\,(4\,a\,c-b^2)\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}$$

$$+\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}+1\right)1\mathrm{i}}{\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\left(a\,b^2-4\,a^2\,c+b^3+b^2\sqrt{b^2-4\,a\,c}-4\,a\,b\,c+a\,b\sqrt{b^2-4\,a\,c}-2\,a\,c\right.}{4\,a^2\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}\,(4\,a\,c-b^2)}$$

$$-\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}-1\right)1\mathrm{i}}{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\left(4\,a^2\,c-a\,b^2-b^3+b^2\sqrt{b^2-4\,a\,c}+4\,a\,b\,c+a\,b\sqrt{b^2-4\,a\,c}-2\,a\,c\right.}{4\,a^2\,(4\,a\,c-b^2)\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}$$

$$+\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}-1\right)1\mathrm{i}}{\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\left(a\,b^2-4\,a^2\,c+b^3+b^2\sqrt{b^2-4\,a\,c}-4\,a\,b\,c+a\,b\sqrt{b^2-4\,a\,c}-2\,a\,c\right.}{4\,a^2\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}\,(4\,a\,c-b^2)}$$

input
```
int((1 - x^2)^(1/2)/(x^3*(a + b*x^2 + c*x^4)),x)
```

output
```
log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2))/(2*a) - (log((1/x^2 - 1)^(1/2) - (1
/x^2)^(1/2))*(a + b))/a^2 - (1 - x^2)^(1/2)/(2*a*x^2) - (log((((x*(-(b + (
b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b + (b^2 - 4*a*c)^(1/2))/(2*c)
 + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^
(1/2)))*(4*a^2*c - a*b^2 - b^3 + b^2*(b^2 - 4*a*c)^(1/2) + 4*a*b*c + a*b*(
b^2 - 4*a*c)^(1/2) - 2*a*c*(b^2 - 4*a*c)^(1/2)))/(4*a^2*(4*a*c - b^2)*((b
+ (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) + (log((((x*(-(b - (b^2 - 4*a*c)^
(1/2))/(2*c))^(1/2) + 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) +
 (1 - x^2)^(1/2)*1i)/(x + (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(a*b^
2 - 4*a^2*c + b^3 + b^2*(b^2 - 4*a*c)^(1/2) - 4*a*b*c + a*b*(b^2 - 4*a*c)^
(1/2) - 2*a*c*(b^2 - 4*a*c)^(1/2)))/(4*a^2*((b - (b^2 - 4*a*c)^(1/2))/(2*c
) + 1)^(1/2)*(4*a*c - b^2)) - (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))
^(1/2) - 1)*1i)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1
/2)*1i)/(x - (-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(4*a^2*c - a*b^2 -
 b^3 + b^2*(b^2 - 4*a*c)^(1/2) + 4*a*b*c + a*b*(b^2 - 4*a*c)^(1/2) - 2*a*c
*(b^2 - 4*a*c)^(1/2)))/(4*a^2*(4*a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*
c) + 1)^(1/2)) + (log((((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1
i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x -
(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(a*b^2 - 4*a^2*c + b^3 + b^2*(b
^2 - 4*a*c)^(1/2) - 4*a*b*c + a*b*(b^2 - 4*a*c)^(1/2) - 2*a*c*(b^2 - 4*...
```

3.380. $\int \frac{\sqrt{1-x^2}}{x^3(a+bx^2+cx^4)}\,dx$

3.381 $\int \frac{x^4\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

3.381.1 Optimal result

Integrand size = 29, antiderivative size = 325

$$\int \frac{x^4\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \frac{x\sqrt{1-x^2}}{2c}+\frac{(2b+c)\arcsin(x)}{2c^2} - \frac{\left(b^2-ac+bc-\frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{b+2c-\sqrt{b^2-4ac}}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{1-x^2}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{b+2c-\sqrt{b^2-4ac}}} - \frac{\left(b^2-ac+bc+\frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{b+2c+\sqrt{b^2-4ac}}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{1-x^2}}\right)}{c^2\sqrt{b+\sqrt{b^2-4ac}}\sqrt{b+2c+\sqrt{b^2-4ac}}}$$

output
```
1/2*(2*b+c)*arcsin(x)/c^2+1/2*x*(-x^2+1)^(1/2)/c-arctan(x*(b+2*c-(-4*a*c+b
^2)^(1/2))^(1/2)/(-x^2+1)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b^2-a*c+b*c
+(3*a*b*c+2*a*c^2-b^3-b^2*c)/(-4*a*c+b^2)^(1/2))/c^2/(b-(-4*a*c+b^2)^(1/2)
)^(1/2)/(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)-arctan(x*(b+2*c+(-4*a*c+b^2)^(1/2
))^(1/2)/(-x^2+1)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2-a*c+b*c+(-3*a*b
*c-2*a*c^2+b^3+b^2*c)/(-4*a*c+b^2)^(1/2))/c^2/(b+(-4*a*c+b^2)^(1/2))^(1/2)
/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.381.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.64 (sec) , antiderivative size = 588, normalized size of antiderivative = 1.81

$$\int \frac{x^4\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$$
$$= \frac{2cx\sqrt{1-x^2}+4(2b+c)\arctan\left(\frac{x}{-1+\sqrt{1-x^2}}\right)+\text{RootSum}\left[a+4a\#1^2+4b\#1^2+6a\#1^4+8b\#1^4+16c\#1\right.}{}$$

input
```
Integrate[(x^4*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

output
```
(2*c*x*Sqrt[1 - x^2] + 4*(2*b + c)*ArcTan[x/(-1 + Sqrt[1 - x^2])] + RootSu
m[a + 4*a*#1^2 + 4*b*#1^2 + 6*a*#1^4 + 8*b*#1^4 + 16*c*#1^4 + 4*a*#1^6 + 4
*b*#1^6 + a*#1^8 & , (-(a*b*Log[x]) - a*c*Log[x] + a*b*Log[-1 + Sqrt[1 - x
^2] - x*#1] + a*c*Log[-1 + Sqrt[1 - x^2] - x*#1] - 3*a*b*Log[x]*#1^2 - 4*b
^2*Log[x]*#1^2 + a*c*Log[x]*#1^2 - 4*b*c*Log[x]*#1^2 + 3*a*b*Log[-1 + Sqrt
[1 - x^2] - x*#1]*#1^2 + 4*b^2*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^2 - a*c*L
og[-1 + Sqrt[1 - x^2] - x*#1]*#1^2 + 4*b*c*Log[-1 + Sqrt[1 - x^2] - x*#1]*
#1^2 - 3*a*b*Log[x]*#1^4 - 4*b^2*Log[x]*#1^4 + a*c*Log[x]*#1^4 - 4*b*c*Log
[x]*#1^4 + 3*a*b*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^4 + 4*b^2*Log[-1 + Sqrt
[1 - x^2] - x*#1]*#1^4 - a*c*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^4 + 4*b*c*L
og[-1 + Sqrt[1 - x^2] - x*#1]*#1^4 - a*b*Log[x]*#1^6 - a*c*Log[x]*#1^6 + a
*b*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^6 + a*c*Log[-1 + Sqrt[1 - x^2] - x*#1
]*#1^6)/(a*#1 + b*#1 + 3*a*#1^3 + 4*b*#1^3 + 8*c*#1^3 + 3*a*#1^5 + 3*b*#1^
5 + a*#1^7) & ])/(4*c^2)
```

3.381.3 Rubi [A] (verified)

Time = 2.03 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1614, 299, 223, 2256, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$$
$$\Big\downarrow \; 1614$$

$$\frac{\int \frac{-cx^2+b+c}{\sqrt{1-x^2}}dx}{c^2} - \frac{\int \frac{(b^2+cb-ac)x^2+a(b+c)}{\sqrt{1-x^2}(cx^4+bx^2+a)}dx}{c^2}$$

$$\Big\downarrow \; 299$$

$$\frac{\frac{1}{2}(2b+c)\int \frac{1}{\sqrt{1-x^2}}dx + \frac{1}{2}c\sqrt{1-x^2}x}{c^2} - \frac{\int \frac{(b^2+cb-ac)x^2+a(b+c)}{\sqrt{1-x^2}(cx^4+bx^2+a)}dx}{c^2}$$

$$\Big\downarrow \; 223$$

$$\frac{\frac{1}{2}\arcsin(x)(2b+c) + \frac{1}{2}c\sqrt{1-x^2}x}{c^2} - \frac{\int \frac{(b^2+cb-ac)x^2+a(b+c)}{\sqrt{1-x^2}(cx^4+bx^2+a)}dx}{c^2}$$

$$\Big\downarrow \; 2256$$

$$\frac{\frac{1}{2}\arcsin(x)(2b+c) + \frac{1}{2}c\sqrt{1-x^2}x}{c^2} - \frac{\int \left(\frac{b^2+cb-ac-\frac{-b^3-cb^2+3acb+2ac^2}{\sqrt{b^2-4ac}}}{\sqrt{1-x^2}\left(2cx^2+b+\sqrt{b^2-4ac}\right)} + \frac{b^2+cb-ac+\frac{-b^3-cb^2+3acb+2ac^2}{\sqrt{b^2-4ac}}}{\sqrt{1-x^2}\left(2cx^2+b-\sqrt{b^2-4ac}\right)}\right)dx}{c^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{1}{2}\arcsin(x)(2b+c) + \frac{1}{2}c\sqrt{1-x^2}x}{c^2} - \frac{\frac{\left(-\frac{-3abc-2ac^2+b^3+b^2c}{\sqrt{b^2-4ac}}-ac+b^2+bc\right)\arctan\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{-\sqrt{b^2-4ac}+b+2c}} + \frac{\left(\frac{-3abc-2ac^2+b^3+b^2c}{\sqrt{b^2-4ac}}-ac+b^2+bc\right)\arctan\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\sqrt{b^2-4ac}+b+2c}}}{c^2}$$

input
```
Int[(x^4*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

output
```
((c*x*Sqrt[1 - x^2])/2 + ((2*b + c)*ArcSin[x])/2)/c^2 - (((b^2 - a*c + b*c
 - (b^3 - 3*a*b*c + b^2*c - 2*a*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[b + 2
*c - Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])])/(
Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) + ((b^2 - a
*c + b*c + (b^3 - 3*a*b*c + b^2*c - 2*a*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[b + 2*c + Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x
^2])])/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]))/c^
2
```

3.381. $\int \frac{x^4\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

3.381.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1614
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Simp[f^4/c^2   Int[(f*x)^(m - 4)*(c*d - b*e + c
*e*x^2)*(d + e*x^2)^(q - 1), x], x] - Simp[f^4/c^2   Int[(f*x)^(m - 4)*(d +
 e*x^2)^(q - 1)*(Simp[a*(c*d - b*e) + (b*c*d - b^2*e + a*c*e)*x^2, x]/(a +
b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c,
 0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2256
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

3.381.4 Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.77

method	result
risch	$-\frac{x(x^2-1)}{2c\sqrt{-x^2+1}}+\frac{\frac{(2b+c)\arcsin(x)}{c}+\frac{a\sqrt{2}\left(\frac{\left(b\sqrt{-4ac+b^2}+\sqrt{-4ac+b^2}\,c+2ac-b^2-bc\right)\arctan\left(\frac{a\sqrt{-x^2+1}\sqrt{2}}{x\sqrt{\left(2a+b+\sqrt{-4ac+b^2}\right)a}}\right)}{\sqrt{\left(2a+b+\sqrt{-4ac+b^2}\right)a}}-\frac{\left(b\sqrt{-4a\cdots\right.}{\cdots}\right.}{c\sqrt{-4ac+b^2}\cdots}}{2c}$
pseudoelliptic	$\frac{a\sqrt{2}\sqrt{\left(2a+b+\sqrt{-4ac+b^2}\right)a}\left(\frac{(-b-c)\sqrt{-4ac+b^2}}{2}+ac-\frac{b(b+c)}{2}\right)\operatorname{arctanh}\left(\frac{a\sqrt{-x^2+1}\sqrt{2}}{x\sqrt{\left(-b+\sqrt{-4ac+b^2}-2a\right)a}}\right)+\left(a\sqrt{2}\left(\frac{(b+c)\sqrt{-4ac}\cdots}{2}\cdots\right.\right.}{\sqrt{\left(-b+\sqrt{-4\cdots}\right.}}$
default	$\frac{\frac{x\sqrt{-x^2+1}}{2}+\frac{\arcsin(x)}{2}}{c}+\frac{a\sqrt{2}\sqrt{\left(2a+b+\sqrt{-4ac+b^2}\right)a}\left(\frac{(-b-c)\sqrt{-4ac+b^2}}{2}+ac-\frac{b(b+c)}{2}\right)\operatorname{arctanh}\left(\frac{a\sqrt{-x^2+1}\sqrt{2}}{x\sqrt{\left(-b+\sqrt{-4ac+b^2}-2a\right)a\cdots}}\right.}{\cdots}$

input
```
int(x^4*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x/c*(x^2-1)/(-x^2+1)^(1/2)+1/2/c*((2*b+c)/c*arcsin(x)+1/c*a*2^(1/2)/(
-4*a*c+b^2)^(1/2)*((b*(-4*a*c+b^2)^(1/2)+(-4*a*c+b^2)^(1/2)*c+2*a*c-b^2-b*
c)/((2*a+b+(-4*a*c+b^2)^(1/2))*a)^(1/2)*arctan(a/x*(-x^2+1)^(1/2)*2^(1/2)/
((2*a+b+(-4*a*c+b^2)^(1/2))*a)^(1/2))-(b*(-4*a*c+b^2)^(1/2)+(-4*a*c+b^2)^(
1/2)*c-2*a*c+b^2+b*c)/((-b+(-4*a*c+b^2)^(1/2)-2*a)*a)^(1/2)*arctanh(a/x*(-
x^2+1)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)-2*a)*a)^(1/2))))
```

3.381.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2860 vs. $2(279) = 558$.

Time = 1.30 (sec) , antiderivative size = 2860, normalized size of antiderivative = 8.80

$$\int \frac{x^4\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(1/2)*c^2*sqrt(-(b^4 + (2*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c +
 (b^2*c^4 - 4*a*c^5)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^
2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))/(b
^2*c^4 - 4*a*c^5))*log(-(2*a^2*b^3 - 2*a^3*c^2 - 2*(a^2*b^3 - a^3*c^2 - (2
*a^3*b - a^2*b^2)*c)*x^2 - 2*(2*a^3*b - a^2*b^2)*c + sqrt(1/2)*((b^6 + 4*a
^2*b*c^3 + (8*a^2*b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)*c)*sqrt(-x^2 + 1)*x
 - (b^6 + 4*a^2*b*c^3 + (8*a^2*b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)*c)*x -
 ((b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*sqrt(-x^2 + 1)*x - (b^4*c^4 - 6*a*b^
2*c^5 + 8*a^2*c^6)*x)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a
^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))*s
qrt(-(b^4 + (2*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c + (b^2*c^4 - 4*a*c^5)*
sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4
)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)) -
2*(a^2*b^3 - a^3*c^2 - (2*a^3*b - a^2*b^2)*c)*sqrt(-x^2 + 1))/x^2) - sqrt(
1/2)*c^2*sqrt(-(b^4 + (2*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c + (b^2*c^4 -
 4*a*c^5)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 - 6*a
*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*
a*c^5))*log(-(2*a^2*b^3 - 2*a^3*c^2 - 2*(a^2*b^3 - a^3*c^2 - (2*a^3*b - a^
2*b^2)*c)*x^2 - 2*(2*a^3*b - a^2*b^2)*c - sqrt(1/2)*((b^6 + 4*a^2*b*c^3 +
(8*a^2*b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)*c)*sqrt(-x^2 + 1)*x - (b^6 ...
```

3.381.6 Sympy [F]

$$\int \frac{x^4\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \int \frac{x^4\sqrt{-(x-1)(x+1)}}{a+bx^2+cx^4}\,dx$$

input
```
integrate(x**4*(-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
```

output
```
Integral(x**4*sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)
```

3.381.7 Maxima [F]

$$\int \frac{x^4\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \int \frac{\sqrt{-x^2+1}x^4}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^4*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^2 + 1)*x^4/(c*x^4 + b*x^2 + a), x)
```

3.381.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1710 vs. $2(279) = 558$.

Time = 1.42 (sec) , antiderivative size = 1710, normalized size of antiderivative = 5.26

$$\int \frac{x^4\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/4*(3*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^2*b^3 + 2*sqrt(2)
*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a*b^4 - 2*a^2*b^4 - sqrt(2)*sqrt(
2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*b^5 + 2*a*b^5 - 12*sqrt(2)*sqrt(2*a^2 +
 a*b + sqrt(b^2 - 4*a*c)*a)*a^3*b*c - 8*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^
2 - 4*a*c)*a)*a^2*b^2*c + 12*a^3*b^2*c + 8*sqrt(2)*sqrt(2*a^2 + a*b + sqrt
(b^2 - 4*a*c)*a)*a*b^3*c - 16*a^2*b^3*c - 16*a^4*c^2 - 16*sqrt(2)*sqrt(2*a
^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^2*b*c^2 + 32*a^3*b*c^2 - 3*sqrt(2)*sqrt(
2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b^2 - 2*sqrt(2)*s
qrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^3 + sqrt(2)*s
qrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*b^4 + 6*sqrt(2)*s
qrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*c + 4*sqrt(2)
*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b*c - 6*sqr
t(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^2*c + 8
*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*c^2
 + 2*(b^2 - 4*a*c)*a^2*b^2 - 2*(b^2 - 4*a*c)*a*b^3 - 4*(b^2 - 4*a*c)*a^3*c
 + 8*(b^2 - 4*a*c)*a^2*b*c)*abs(a)*arctan(-1/2*sqrt(2)*(x/(sqrt(-x^2 + 1)
- 1) - (sqrt(-x^2 + 1) - 1)/x)/sqrt((2*a*c^2 + b*c^2 + sqrt(-4*(a*c^2 + b*
c^2 + c^3)*a*c^2 + (2*a*c^2 + b*c^2)^2))/(a*c^2)))/(3*a^4*b^2*c^2 + 2*a^3*
b^3*c^2 - a^2*b^4*c^2 - 12*a^5*c^3 - 8*a^4*b*c^3 + 8*a^3*b^2*c^3 - 16*a^4*
c^4) + 1/4*(3*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^2*b^3 +...
```

3.381.9 Mupad [B] (verification not implemented)

Time = 8.24 (sec) , antiderivative size = 1024, normalized size of antiderivative = 3.15

$$\int \frac{x^4\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \operatorname{asin}(x)\left(\frac{\frac{b}{c}+1}{c}-\frac{1}{2\,c}\right)+\frac{x\sqrt{1-x^2}}{2\,c} - \frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}-1\right)1\mathrm{i}}{\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\left(b^2\left(-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}\right)^{3/2}+a\,b\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}+2\,a\,c\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}-2}{2\,c\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}\,(4\,a\,c-b^2)} + \frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}+1\right)1\mathrm{i}}{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\left(b^2\left(-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}\right)^{3/2}+a\,b\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}+2\,a\,c\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}-2}{2\,c\,(4\,a\,c-b^2)\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}} + \frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}+1\right)1\mathrm{i}}{\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\left(b^2\left(-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}\right)^{3/2}+a\,b\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}+2\,a\,c\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}-2}{2\,c\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}\,(4\,a\,c-b^2)} - \frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}-1\right)1\mathrm{i}}{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\left(b^2\left(-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}\right)^{3/2}+a\,b\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}+2\,a\,c\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}-2}{2\,c\,(4\,a\,c-b^2)\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}$$

input
```
int((x^4*(1 - x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

output
```
asin(x)*((b/c + 1)/c - 1/(2*c)) + (x*(1 - x^2)^(1/2))/(2*c) - (log((((x*(-
(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))
/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b - (b^2 - 4*a*c)^(1/2))/(
2*c))^(1/2)))*(b^2*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + a*b*(-(b - (
b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 2*a*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c)
)^(1/2) - 2*a*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + b*c*(-(b - (b^2
 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(2*c*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)
^(1/2)*(4*a*c - b^2)) + (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)
 + 1)*1i)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i
)/(x + (-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b^2*(-(b + (b^2 - 4*a*c
)^(1/2))/(2*c))^(3/2) + a*b*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 2*a
*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a*c*(-(b + (b^2 - 4*a*c)^(
1/2))/(2*c))^(3/2) + b*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(2*c*(
4*a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) + (log((((x*(-(b
 - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(
2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b - (b^2 - 4*a*c)^(1/2))/(2*
c))^(1/2)))*(b^2*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + a*b*(-(b - (b^
2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 2*a*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^
(1/2) - 2*a*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + b*c*(-(b - (b^2 -
 4*a*c)^(1/2))/(2*c))^(3/2)))/(2*c*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1...
```

3.381. $\int \frac{x^4\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

3.382 $\int \frac{x^2\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

3.382.1 Optimal result

Integrand size = 29, antiderivative size = 263

$$\int \frac{x^2\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = -\frac{\arcsin(x)}{c}+\frac{\left(b+c-\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{1-x^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{b+2c-\sqrt{b^2-4ac}}}+\frac{\left(b+c+\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{b+2c+\sqrt{b^2-4ac}}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{1-x^2}}\right)}{c\sqrt{b+\sqrt{b^2-4ac}}\sqrt{b+2c+\sqrt{b^2-4ac}}}$$

output
```
-arcsin(x)/c+arctan(x*(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)/(-x^2+1)^(1/2)/(b-(
-4*a*c+b^2)^(1/2))^(1/2))*(b+c+(2*a*c-b^2-b*c)/(-4*a*c+b^2)^(1/2))/c/(b-(-
4*a*c+b^2)^(1/2))^(1/2)/(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)+arctan(x*(b+2*c+(
-4*a*c+b^2)^(1/2))^(1/2)/(-x^2+1)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+c
+(-2*a*c+b^2+b*c)/(-4*a*c+b^2)^(1/2))/c/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(b+2*
c+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.382.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.41 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.57

$$\int \frac{x^2\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx =$$

$$-\frac{8\arctan\left(\frac{x}{-1+\sqrt{1-x^2}}\right)+\text{RootSum}\left[a+4a\#1^2+4b\#1^2+6a\#1^4+8b\#1^4+16c\#1^4+4a\#1^6+4b\#1^6\right.}{}$$

input
```
Integrate[(x^2*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

output
```
-1/4*(8*ArcTan[x/(-1 + Sqrt[1 - x^2])] + RootSum[a + 4*a*#1^2 + 4*b*#1^2 +
 6*a*#1^4 + 8*b*#1^4 + 16*c*#1^4 + 4*a*#1^6 + 4*b*#1^6 + a*#1^8 & , (-(a*L
og[x]) + a*Log[-1 + Sqrt[1 - x^2] - x*#1] - 3*a*Log[x]*#1^2 - 4*b*Log[x]*#
1^2 - 4*c*Log[x]*#1^2 + 3*a*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^2 + 4*b*Log[
-1 + Sqrt[1 - x^2] - x*#1]*#1^2 + 4*c*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^2
- 3*a*Log[x]*#1^4 - 4*b*Log[x]*#1^4 - 4*c*Log[x]*#1^4 + 3*a*Log[-1 + Sqrt[
1 - x^2] - x*#1]*#1^4 + 4*b*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^4 + 4*c*Log[
-1 + Sqrt[1 - x^2] - x*#1]*#1^4 - a*Log[x]*#1^6 + a*Log[-1 + Sqrt[1 - x^2]
 - x*#1]*#1^6)/(a*#1 + b*#1 + 3*a*#1^3 + 4*b*#1^3 + 8*c*#1^3 + 3*a*#1^5 +
3*b*#1^5 + a*#1^7) & ])/c
```

3.382.3 Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1616, 25, 223, 2256, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{1-x^2}}{a+bx^2+cx^4}dx$$

$$\Big\downarrow \text{1616}$$

$$-\frac{\int -\frac{(b+c)x^2+a}{\sqrt{1-x^2}(cx^4+bx^2+a)}dx}{c}-\frac{\int \frac{1}{\sqrt{1-x^2}}dx}{c}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(b+c)x^2+a}{\sqrt{1-x^2}(cx^4+bx^2+a)}dx}{c}-\frac{\int \frac{1}{\sqrt{1-x^2}}dx}{c}$$

$$\Big\downarrow \text{223}$$

$$\frac{\int \frac{(b+c)x^2+a}{\sqrt{1-x^2}(cx^4+bx^2+a)}dx}{c}-\frac{\arcsin(x)}{c}$$

$$\Big\downarrow \text{2256}$$

$$\frac{\int \left(\frac{b+c-\frac{-b^2-cb+2ac}{\sqrt{b^2-4ac}}}{\sqrt{1-x^2}\left(2cx^2+b+\sqrt{b^2-4ac}\right)}+\frac{b+c+\frac{-b^2-cb+2ac}{\sqrt{b^2-4ac}}}{\sqrt{1-x^2}\left(2cx^2+b-\sqrt{b^2-4ac}\right)}\right)dx}{c}-\frac{\arcsin(x)}{c}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\left(-\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\arctan\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{-\sqrt{b^2-4ac}+b+2c}}+\frac{\left(\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\arctan\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\sqrt{b^2-4ac}+b+2c}}}{c}-\frac{\arcsin(x)}{c}$$

input
```
Int[(x^2*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

output
```
-(ArcSin[x]/c) + (((b + c - (b^2 - 2*a*c + b*c)/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[1
- x^2])])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]])
+ ((b + c + (b^2 - 2*a*c + b*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[b + 2*c +
Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])])/(Sqrt[
b + Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]))/c
```

3.382.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1616
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Simp[e*(f^2/c)   Int[(f*x)^(m - 2)*(d + e*x^2)^
(q - 1), x], x] - Simp[f^2/c   Int[(f*x)^(m - 2)*(d + e*x^2)^(q - 1)*(Simp[
a*e - (c*d - b*e)*x^2, x]/(a + b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m,
1] && LeQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.382. $\int \frac{x^2\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

rule 2256
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

3.382.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.00

method	result
default	$\frac{a\sqrt{\left(2a+b+\sqrt{-4ac+b^2}\right)a}\,\sqrt{2}\left(b+2c+\sqrt{-4ac+b^2}\right)\operatorname{arctanh}\left(\frac{a\sqrt{-x^2+1}\,\sqrt{2}}{x\sqrt{\left(-b+\sqrt{-4ac+b^2}-2a\right)a}}\right)+\sqrt{\left(-b+\sqrt{-4ac+b^2}-2a\right)a}\left(a\sqrt{2}\right.}{2\sqrt{\left(-b+\sqrt{-4ac+b^2}-2a\right)a}\,\sqrt{-4ac}}$
pseudoelliptic	$\frac{a\sqrt{\left(2a+b+\sqrt{-4ac+b^2}\right)a}\,\sqrt{2}\left(b+2c+\sqrt{-4ac+b^2}\right)\operatorname{arctanh}\left(\frac{a\sqrt{-x^2+1}\,\sqrt{2}}{x\sqrt{\left(-b+\sqrt{-4ac+b^2}-2a\right)a}}\right)+\sqrt{\left(-b+\sqrt{-4ac+b^2}-2a\right)a}\left(a\sqrt{2}\right.}{2\sqrt{\left(-b+\sqrt{-4ac+b^2}-2a\right)a}\,\sqrt{-4ac}}$

input
```
int(x^2*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/((-b+(-4*a*c+b^2)^(1/2)-2*a)*a)^(1/2)/(-4*a*c+b^2)^(1/2)/((2*a+b+(-4*a
*c+b^2)^(1/2))*a)^(1/2)*(a*((2*a+b+(-4*a*c+b^2)^(1/2))*a)^(1/2)*2^(1/2)*(b
+2*c+(-4*a*c+b^2)^(1/2))*arctanh(a/x*(-x^2+1)^(1/2)*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2)-2*a)*a)^(1/2))+((-b+(-4*a*c+b^2)^(1/2)-2*a)*a)^(1/2)*(a*2^(1/2)*
(b+2*c-(-4*a*c+b^2)^(1/2))*arctan(a/x*(-x^2+1)^(1/2)*2^(1/2)/((2*a+b+(-4*a
*c+b^2)^(1/2))*a)^(1/2))+2*arctan(1/x*(-x^2+1)^(1/2))*(-4*a*c+b^2)^(1/2)*(
(2*a+b+(-4*a*c+b^2)^(1/2))*a)^(1/2)))/c
```

3.382.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1430 vs. 2(223) = 446.

Time = 0.67 (sec) , antiderivative size = 1430, normalized size of antiderivative = 5.44

$$\int \frac{x^2\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(1/2)*c*sqrt(-(b^2 - (2*a - b)*c + (b^2*c^2 - 4*a*c^3)*sqrt((b^2
 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3))*log(-(2*(a*b +
a*c)*x^2 - 2*a*b - 2*a*c + sqrt(1/2)*((b^3 - 4*a*c^2 - (4*a*b - b^2)*c)*sq
rt(-x^2 + 1)*x - (b^3 - 4*a*c^2 - (4*a*b - b^2)*c)*x - ((b^3*c^2 - 4*a*b*c
^3)*sqrt(-x^2 + 1)*x - (b^3*c^2 - 4*a*b*c^3)*x)*sqrt((b^2 + 2*b*c + c^2)/(
b^2*c^4 - 4*a*c^5)))*sqrt(-(b^2 - (2*a - b)*c + (b^2*c^2 - 4*a*c^3)*sqrt((
b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3)) + 2*(a*b + a
*c)*sqrt(-x^2 + 1))/x^2) - sqrt(1/2)*c*sqrt(-(b^2 - (2*a - b)*c + (b^2*c^2
 - 4*a*c^3)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*
c^3))*log(-(2*(a*b + a*c)*x^2 - 2*a*b - 2*a*c - sqrt(1/2)*((b^3 - 4*a*c^2
- (4*a*b - b^2)*c)*sqrt(-x^2 + 1)*x - (b^3 - 4*a*c^2 - (4*a*b - b^2)*c)*x
- ((b^3*c^2 - 4*a*b*c^3)*sqrt(-x^2 + 1)*x - (b^3*c^2 - 4*a*b*c^3)*x)*sqrt(
(b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))*sqrt(-(b^2 - (2*a - b)*c + (b^2*
c^2 - 4*a*c^3)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4
*a*c^3)) + 2*(a*b + a*c)*sqrt(-x^2 + 1))/x^2) + sqrt(1/2)*c*sqrt(-(b^2 - (
2*a - b)*c - (b^2*c^2 - 4*a*c^3)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c
^5)))/(b^2*c^2 - 4*a*c^3))*log(-(2*(a*b + a*c)*x^2 - 2*a*b - 2*a*c + sqrt(
1/2)*((b^3 - 4*a*c^2 - (4*a*b - b^2)*c)*sqrt(-x^2 + 1)*x - (b^3 - 4*a*c^2
- (4*a*b - b^2)*c)*x + ((b^3*c^2 - 4*a*b*c^3)*sqrt(-x^2 + 1)*x - (b^3*c^2
- 4*a*b*c^3)*x)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))*sqrt(-(b...
```

3.382.6 Sympy [F]

$$\int \frac{x^2\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \int \frac{x^2\sqrt{-(x-1)(x+1)}}{a+bx^2+cx^4}\,dx$$

input
```
integrate(x**2*(-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
```

output
```
Integral(x**2*sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)
```

3.382.7 Maxima [F]

$$\int \frac{x^2\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \int \frac{\sqrt{-x^2+1}x^2}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^2*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^2 + 1)*x^2/(c*x^4 + b*x^2 + a), x)
```

3.382.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3580 vs. 2(223) = 446.

Time = 1.02 (sec) , antiderivative size = 3580, normalized size of antiderivative = 13.61

$$\int \frac{x^2\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*(pi*sgn(x) + 2*arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-
x^2 + 1) - 1)))/c + 1/8*((2*a^2*b^4 - 16*a^3*b^2*c + 32*a^4*c^2 + 3*sqrt(2
)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b^2 + 2*sq
rt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^3 - sq
rt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*b^4 - 12*s
qrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*c - 8
*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b*c
 + 8*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b
^2*c - 16*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c
)*a^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2 + 8*(b^2 - 4*a*c)*a^3*c)*c^2*abs(a) -
2*(3*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^3*b^2*c + 5*sqrt(2)
*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^2*b^3*c + sqrt(2)*sqrt(2*a^2 +
a*b + sqrt(b^2 - 4*a*c)*a)*a*b^4*c + 2*a^2*b^4*c - sqrt(2)*sqrt(2*a^2 + a*
b + sqrt(b^2 - 4*a*c)*a)*b^5*c + 2*a*b^5*c - 12*sqrt(2)*sqrt(2*a^2 + a*b +
 sqrt(b^2 - 4*a*c)*a)*a^4*c^2 - 20*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4
*a*c)*a)*a^3*b*c^2 + 3*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^2
*b^2*c^2 - 16*a^3*b^2*c^2 + 10*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c
)*a)*a*b^3*c^2 - 16*a^2*b^3*c^2 - sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*
a*c)*a)*b^4*c^2 + 2*a*b^4*c^2 - 28*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4
*a*c)*a)*a^3*c^3 + 32*a^4*c^3 - 24*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 ...
```

3.382.9 Mupad [B] (verification not implemented)

Time = 8.34 (sec) , antiderivative size = 870, normalized size of antiderivative = 3.31

$$\int \frac{x^2\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = -\frac{\operatorname{asin}(x)}{c}$$

$$-\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}+1\right)1\mathrm{i}}{\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\left(2\,a\,\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}+b\,\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}+b\,\left(-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}\right)^{3/2}+2\,c\,\left(-\right.\right.}{\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}\,\left(8\,a\,c-2\,b^2\right)}$$

$$-\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}+1\right)1\mathrm{i}}{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\left(2\,a\,\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}+b\,\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}+b\,\left(-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}\right)^{3/2}+2\,c\,\left(-\right.\right.}{\left(8\,a\,c-2\,b^2\right)\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}$$

$$+\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}-1\right)1\mathrm{i}}{\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\left(2\,a\,\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}+b\,\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}+b\,\left(-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}\right)^{3/2}+2\,c\,\left(-\right.\right.}{\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}\,\left(8\,a\,c-2\,b^2\right)}$$

$$+\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}-1\right)1\mathrm{i}}{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\left(2\,a\,\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}+b\,\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}+b\,\left(-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}\right)^{3/2}+2\,c\,\left(-\right.\right.}{\left(8\,a\,c-2\,b^2\right)\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}$$

input
```
int((x^2*(1 - x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

output
```
(log((((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b - (b^2 - 4
*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b - (b^2 - 4*a
*c)^(1/2))/(2*c))^(1/2)))*(2*a*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) +
b*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + b*(-(b - (b^2 - 4*a*c)^(1/2))
/(2*c))^(3/2) + 2*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(((b - (b^2
 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(8*a*c - 2*b^2)) - (log((((x*(-(b - (b^2
 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c) +
1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/
2)))*(2*a*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + b*(-(b - (b^2 - 4*a*c
)^(1/2))/(2*c))^(1/2) + b*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + 2*c*(
-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(((b - (b^2 - 4*a*c)^(1/2))/(2*c
) + 1)^(1/2)*(8*a*c - 2*b^2)) - (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c
))^(1/2) + 1)*1i)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^
(1/2)*1i)/(x + (-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(2*a*(-(b + (b^2
 - 4*a*c)^(1/2))/(2*c))^(1/2) + b*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)
 + b*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + 2*c*(-(b + (b^2 - 4*a*c)^(
1/2))/(2*c))^(3/2)))/((8*a*c - 2*b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1
)^(1/2)) - asin(x)/c + (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)
- 1)*1i)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)
/(x - (-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(2*a*(-(b + (b^2 - 4*a...
```

3.382. $\int \frac{x^2\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

3.383 $\int \frac{\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

3.383.1 Optimal result

Integrand size = 26, antiderivative size = 220

$$\int \frac{\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{b+2c-\sqrt{b^2-4ac}}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{1-x^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{b+2c+\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{b+2c+\sqrt{b^2-4ac}}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{1-x^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
arctan(x*(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)/(-x^2+1)^(1/2)/(b-(-4*a*c+b^2)^(
1/2))^(1/2))*(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)/(-4*a*c+b^2)^(1/2)/(b-(-4*a*
c+b^2)^(1/2))^(1/2)-arctan(x*(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2)/(-x^2+1)^(1/
2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2)/(-4*a*c+
b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.383.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.30 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$$
$$= -\frac{1}{4}\text{RootSum}\Big[a+4a\#1^2+4b\#1^2+6a\#1^4+8b\#1^4+16c\#1^4+4a\#1^6+4b\#1^6$$
$$+a\#1^8\&, \frac{-\log(x)+\log\left(-1+\sqrt{1-x^2}-x\#1\right)+\log(x)\#1^2-\log\left(-1+\sqrt{1-x^2}-x\#1\right)\#1^2+\log(x}{a\#1+b\#1+3a\#1^3+4b\#1^3+8c}$$

input
```
Integrate[Sqrt[1 - x^2]/(a + b*x^2 + c*x^4),x]
```

output
```
-1/4*RootSum[a + 4*a*#1^2 + 4*b*#1^2 + 6*a*#1^4 + 8*b*#1^4 + 16*c*#1^4 + 4
*a*#1^6 + 4*b*#1^6 + a*#1^8 & , (-Log[x] + Log[-1 + Sqrt[1 - x^2] - x*#1]
+ Log[x]*#1^2 - Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^2 + Log[x]*#1^4 - Log[-1
 + Sqrt[1 - x^2] - x*#1]*#1^4 - Log[x]*#1^6 + Log[-1 + Sqrt[1 - x^2] - x*#
1]*#1^6)/(a*#1 + b*#1 + 3*a*#1^3 + 4*b*#1^3 + 8*c*#1^3 + 3*a*#1^5 + 3*b*#1
^5 + a*#1^7) & ]
```

3.383.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1488, 301, 223, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$$
$$\Big\downarrow \text{1488}$$
$$\frac{2c\int \frac{\sqrt{1-x^2}}{2cx^2+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{2c\int \frac{\sqrt{1-x^2}}{2cx^2+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$
$$\Big\downarrow \text{301}$$

$$\frac{2c\left(\frac{\left(-\sqrt{b^2-4ac}+b+2c\right)\int \frac{1}{\sqrt{1-x^2}\left(2cx^2+b-\sqrt{b^2-4ac}\right)}dx}{2c}-\frac{\int \frac{1}{\sqrt{1-x^2}}dx}{2c}\right)}{\sqrt{b^2-4ac}}-\frac{2c\left(\frac{\left(\sqrt{b^2-4ac}+b+2c\right)\int \frac{1}{\sqrt{1-x^2}\left(2cx^2+b+\sqrt{b^2-4ac}\right)}dx}{2c}-\frac{\int \frac{1}{\sqrt{1-x^2}}dx}{2c}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{223}$$

$$\frac{2c\left(\frac{\left(-\sqrt{b^2-4ac}+b+2c\right)\int \frac{1}{\sqrt{1-x^2}\left(2cx^2+b-\sqrt{b^2-4ac}\right)}dx}{2c}-\frac{\arcsin(x)}{2c}\right)}{\sqrt{b^2-4ac}}-\frac{2c\left(\frac{\left(\sqrt{b^2-4ac}+b+2c\right)\int \frac{1}{\sqrt{1-x^2}\left(2cx^2+b+\sqrt{b^2-4ac}\right)}dx}{2c}-\frac{\arcsin(x)}{2c}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{291}$$

$$\frac{2c\left(\frac{\left(-\sqrt{b^2-4ac}+b+2c\right)\int \frac{1}{-\frac{\left(-b-2c+\sqrt{b^2-4ac}\right)x^2}{1-x^2}+b-\sqrt{b^2-4ac}}d\frac{x}{\sqrt{1-x^2}}}{2c}-\frac{\arcsin(x)}{2c}\right)}{\sqrt{b^2-4ac}}-\frac{2c\left(\frac{\left(\sqrt{b^2-4ac}+b+2c\right)\int \frac{1}{-\frac{\left(-b-2c-\sqrt{b^2-4ac}\right)x^2}{1-x^2}+b+\sqrt{b^2-4ac}}d\frac{x}{\sqrt{1-x^2}}}{2c}-\frac{\arcsin(x)}{2c}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{218}$$

$$\frac{2c\left(\frac{\sqrt{-\sqrt{b^2-4ac}+b+2c}\arctan\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2c\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\arcsin(x)}{2c}\right)}{\sqrt{b^2-4ac}}-\frac{2c\left(\frac{\sqrt{\sqrt{b^2-4ac}+b+2c}\arctan\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2c\sqrt{\sqrt{b^2-4ac}+b}}-\frac{\arcsin(x)}{2c}\right)}{\sqrt{b^2-4ac}}$$

input
```
Int[Sqrt[1 - x^2]/(a + b*x^2 + c*x^4),x]
```

3.383. $\int \frac{\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

output
```
(2*c*(-1/2*ArcSin[x]/c + (Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[b
 + 2*c - Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])
])/(2*c*Sqrt[b - Sqrt[b^2 - 4*a*c]])))/Sqrt[b^2 - 4*a*c] - (2*c*(-1/2*ArcS
in[x]/c + (Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[b + 2*c + Sqrt[b
^2 - 4*a*c]]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])])/(2*c*Sqrt[b
+ Sqrt[b^2 - 4*a*c]])))/Sqrt[b^2 - 4*a*c]
```

3.383.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 1488
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/r)   Int[(d + e*x^2)^q/(b -
 r + 2*c*x^2), x], x] - Simp[2*(c/r)   Int[(d + e*x^2)^q/(b + r + 2*c*x^2),
 x], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

3.383. $\int \frac{\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

3.383.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.76

method	result
default	$-\frac{\sqrt{2}\left(-\frac{\left(-b+\sqrt{-4ac+b^2}-2a\right)\operatorname{arctanh}\left(\frac{a\sqrt{-x^2+1}\,\sqrt{2}}{x\sqrt{\left(-b+\sqrt{-4ac+b^2}-2a\right)a}}\right)}{\sqrt{\left(-b+\sqrt{-4ac+b^2}-2a\right)a}}+\frac{\left(2a+b+\sqrt{-4ac+b^2}\right)\arctan\left(\frac{a\sqrt{-x^2+1}\,\sqrt{2}}{x\sqrt{\left(2a+b+\sqrt{-4ac+b^2}\right)a}}\right)}{\sqrt{\left(2a+b+\sqrt{-4ac+b^2}\right)a}}\right)}{2\sqrt{-4ac+b^2}}$
pseudoelliptic	$-\frac{\sqrt{2}\left(-\frac{\left(-b+\sqrt{-4ac+b^2}-2a\right)\operatorname{arctanh}\left(\frac{a\sqrt{-x^2+1}\,\sqrt{2}}{x\sqrt{\left(-b+\sqrt{-4ac+b^2}-2a\right)a}}\right)}{\sqrt{\left(-b+\sqrt{-4ac+b^2}-2a\right)a}}+\frac{\left(2a+b+\sqrt{-4ac+b^2}\right)\arctan\left(\frac{a\sqrt{-x^2+1}\,\sqrt{2}}{x\sqrt{\left(2a+b+\sqrt{-4ac+b^2}\right)a}}\right)}{\sqrt{\left(2a+b+\sqrt{-4ac+b^2}\right)a}}\right)}{2\sqrt{-4ac+b^2}}$

input
```
int((-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*2^(1/2)/(-4*a*c+b^2)^(1/2)*(-(-b+(-4*a*c+b^2)^(1/2)-2*a)/((-b+(-4*a*c
+b^2)^(1/2)-2*a)*a)^(1/2)*arctanh(a/x*(-x^2+1)^(1/2)*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2)-2*a)*a)^(1/2))+(2*a+b+(-4*a*c+b^2)^(1/2))/((2*a+b+(-4*a*c+b^2)^
(1/2))*a)^(1/2)*arctan(a/x*(-x^2+1)^(1/2)*2^(1/2)/((2*a+b+(-4*a*c+b^2)^(1/
2))*a)^(1/2)))
```

3.383.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 759 vs. $2(180) = 360$.

Time = 0.35 (sec) , antiderivative size = 759, normalized size of antiderivative = 3.45

$$\int \frac{\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$$

$$= \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{2\,a+b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(-\frac{x^2+\frac{\sqrt{\frac{1}{2}}\left(\left(ab^2-4\,a^2c\right)\sqrt{-x^2+1}x-\left(ab^2-4\,a^2c\right)x\right)\sqrt{-\frac{2\,a+b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}}{\sqrt{a^2b^2-4\,a^3c}}+\sqrt{-x^2}}{x^2}\right.$$

$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{2\,a+b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(-\frac{x^2-\frac{\sqrt{\frac{1}{2}}\left(\left(ab^2-4\,a^2c\right)\sqrt{-x^2+1}x-\left(ab^2-4\,a^2c\right)x\right)\sqrt{-\frac{2\,a+b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}}{\sqrt{a^2b^2-4\,a^3c}}+\sqrt{-x}}{x^2}\right.$$

$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{2\,a+b-\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(-\frac{x^2+\frac{\sqrt{\frac{1}{2}}\left(\left(ab^2-4\,a^2c\right)\sqrt{-x^2+1}x-\left(ab^2-4\,a^2c\right)x\right)\sqrt{-\frac{2\,a+b-\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}}{\sqrt{a^2b^2-4\,a^3c}}+\sqrt{-x}}{x^2}\right.$$

$$+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{2\,a+b-\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(-\frac{x^2-\frac{\sqrt{\frac{1}{2}}\left(\left(ab^2-4\,a^2c\right)\sqrt{-x^2+1}x-\left(ab^2-4\,a^2c\right)x\right)\sqrt{-\frac{2\,a+b-\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}}{\sqrt{a^2b^2-4\,a^3c}}+\sqrt{-x}}{x^2}\right.$$

input
```
integrate((-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

3.383. $\int \frac{\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

output
```
1/2*sqrt(1/2)*sqrt(-(2*a + b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/
(a*b^2 - 4*a^2*c))*log(-(x^2 + sqrt(1/2)*((a*b^2 - 4*a^2*c)*sqrt(-x^2 + 1)
*x - (a*b^2 - 4*a^2*c)*x)*sqrt(-(2*a + b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2
- 4*a^3*c))/(a*b^2 - 4*a^2*c))/sqrt(a^2*b^2 - 4*a^3*c) + sqrt(-x^2 + 1) -
1)/x^2) - 1/2*sqrt(1/2)*sqrt(-(2*a + b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 -
4*a^3*c))/(a*b^2 - 4*a^2*c))*log(-(x^2 - sqrt(1/2)*((a*b^2 - 4*a^2*c)*sqrt
(-x^2 + 1)*x - (a*b^2 - 4*a^2*c)*x)*sqrt(-(2*a + b + (a*b^2 - 4*a^2*c)/sqr
t(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))/sqrt(a^2*b^2 - 4*a^3*c) + sqrt(-x
^2 + 1) - 1)/x^2) - 1/2*sqrt(1/2)*sqrt(-(2*a + b - (a*b^2 - 4*a^2*c)/sqrt(
a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(-(x^2 + sqrt(1/2)*((a*b^2 - 4*a
^2*c)*sqrt(-x^2 + 1)*x - (a*b^2 - 4*a^2*c)*x)*sqrt(-(2*a + b - (a*b^2 - 4*
a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))/sqrt(a^2*b^2 - 4*a^3*c)
 + sqrt(-x^2 + 1) - 1)/x^2) + 1/2*sqrt(1/2)*sqrt(-(2*a + b - (a*b^2 - 4*a^
2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(-(x^2 - sqrt(1/2)*((a
*b^2 - 4*a^2*c)*sqrt(-x^2 + 1)*x - (a*b^2 - 4*a^2*c)*x)*sqrt(-(2*a + b - (
a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))/sqrt(a^2*b^2
- 4*a^3*c) + sqrt(-x^2 + 1) - 1)/x^2)
```

3.383.6 Sympy [F]

$$\int \frac{\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \int \frac{\sqrt{-(x-1)(x+1)}}{a+bx^2+cx^4}\,dx$$

input
```
integrate((-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
```

output
```
Integral(sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)
```

3.383.7 Maxima [F]

$$\int \frac{\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \int \frac{\sqrt{-x^2+1}}{cx^4+bx^2+a}\,dx$$

input
```
integrate((-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^2 + 1)/(c*x^4 + b*x^2 + a), x)
```

3.383.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 641 vs. $2(180) = 360$.

Time = 1.08 (sec) , antiderivative size = 641, normalized size of antiderivative = 2.91

$$\int \frac{\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx =$$

$$-\frac{\left(2\,a^2b^2-8\,a^3c+3\,\sqrt{2}\sqrt{2\,a^2+ab+\sqrt{b^2-4\,ac}a}\sqrt{b^2-4\,ac}a^2+2\,\sqrt{2}\sqrt{2\,a^2+ab+\sqrt{b^2-4\,ac}a}\sqrt{b^2-\right.}{2\,(3}$$

$$+\frac{\left(2\,a^2b^2-8\,a^3c+3\,\sqrt{2}\sqrt{2\,a^2+ab-\sqrt{b^2-4\,ac}a}\sqrt{b^2-4\,ac}a^2+2\,\sqrt{2}\sqrt{2\,a^2+ab-\sqrt{b^2-4\,ac}a}\sqrt{b^2-\right.}{2\,(3}$$

input
```
integrate((-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*(2*a^2*b^2 - 8*a^3*c + 3*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)
*a)*sqrt(b^2 - 4*a*c)*a^2 + 2*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)
*a)*sqrt(b^2 - 4*a*c)*a*b - sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a
)*sqrt(b^2 - 4*a*c)*b^2 + 4*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a
)*sqrt(b^2 - 4*a*c)*a*c - 2*(b^2 - 4*a*c)*a^2)*abs(a)*arctan(-1/2*sqrt(2)*
(x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1)/x)/sqrt((2*a + b + sqrt((2*
a + b)^2 - 4*(a + b + c)*a))/a))/(3*a^4*b^2 + 2*a^3*b^3 - a^2*b^4 - 12*a^5
*c - 8*a^4*b*c + 8*a^3*b^2*c - 16*a^4*c^2) + 1/2*(2*a^2*b^2 - 8*a^3*c + 3*
sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2 + 2*
sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b - sq
rt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*b^2 + 4*sq
rt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*c - 2*(b
^2 - 4*a*c)*a^2)*abs(a)*arctan(-1/2*sqrt(2)*(x/(sqrt(-x^2 + 1) - 1) - (sqr
t(-x^2 + 1) - 1)/x)/sqrt((2*a + b - sqrt((2*a + b)^2 - 4*(a + b + c)*a))/a
))/(3*a^4*b^2 + 2*a^3*b^3 - a^2*b^4 - 12*a^5*c - 8*a^4*b*c + 8*a^3*b^2*c -
 16*a^4*c^2)
```

3.383.9 Mupad [B] (verification not implemented)

Time = 8.27 (sec) , antiderivative size = 989, normalized size of antiderivative = 4.50

$$\int \frac{\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx =$$

$$-\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}-1\right)1\mathrm{i}}{\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\left(b^2\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}+a\,b\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}-2\,a\,c\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}+2\,a\,c}{2\,a\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}\,\left(4\,a\,c-b^2\right)}$$

$$+\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}+1\right)1\mathrm{i}}{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\left(b^2\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}+a\,b\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}-2\,a\,c\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}+2\,a\,c}{2\,a\,\left(4\,a\,c-b^2\right)\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}$$

$$+\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}+1\right)1\mathrm{i}}{\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\left(b^2\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}+a\,b\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}-2\,a\,c\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}+2\,a\,c}{2\,a\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}\,\left(4\,a\,c-b^2\right)}$$

$$-\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}-1\right)1\mathrm{i}}{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\left(b^2\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}+a\,b\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}-2\,a\,c\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}+2\,a\,c}{2\,a\,\left(4\,a\,c-b^2\right)\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}$$

input
```
int((1 - x^2)^(1/2)/(a + b*x^2 + c*x^4),x)
```

output
```
(log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b + (b^2 - 4
*a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b + (b^2 - 4*a
*c)^(1/2))/(2*c))^(1/2)))*(b^2*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) +
a*b*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a*c*(-(b + (b^2 - 4*a*c)^
(1/2))/(2*c))^(1/2) + 2*a*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + b*c
*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(2*a*(4*a*c - b^2)*((b + (b^2
- 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) - (log((((x*(-(b - (b^2 - 4*a*c)^(1/2))/
(2*c))^(1/2) - 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x
^2)^(1/2)*1i)/(x - (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b^2*(-(b -
(b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + a*b*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))
^(1/2) - 2*a*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 2*a*c*(-(b - (b^
2 - 4*a*c)^(1/2))/(2*c))^(3/2) + b*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3
/2)))/(2*a*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2)) + (l
og((((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b - (b^2 - 4*a
*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b - (b^2 - 4*a*c
)^(1/2))/(2*c))^(1/2)))*(b^2*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + a*
b*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a*c*(-(b - (b^2 - 4*a*c)^(1
/2))/(2*c))^(1/2) + 2*a*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + b*c*(
-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(2*a*((b - (b^2 - 4*a*c)^(1/2))/
(2*c) + 1)^(1/2)*(4*a*c - b^2)) - (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))...
```

3.383. $\int \frac{\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

3.384 $\int \frac{\sqrt{1-x^2}}{x^2\left(a+bx^2+cx^4\right)}\,dx$

3.384.1 Optimal result

Integrand size = 29, antiderivative size = 265

$$\int \frac{\sqrt{1-x^2}}{x^2\left(a+bx^2+cx^4\right)}\,dx = -\frac{\sqrt{1-x^2}}{ax} - \frac{c\left(1+\frac{2a+b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{1-x^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{b+2c-\sqrt{b^2-4ac}}} - \frac{c\left(1-\frac{2a+b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{b+2c+\sqrt{b^2-4ac}}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{1-x^2}}\right)}{a\sqrt{b+\sqrt{b^2-4ac}}\sqrt{b+2c+\sqrt{b^2-4ac}}}$$

output
```
-(-x^2+1)^(1/2)/a/x-c*arctan(x*(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)/(-x^2+1)^(
1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(1+(2*a+b)/(-4*a*c+b^2)^(1/2))/a/(b-(-4
*a*c+b^2)^(1/2))^(1/2)/(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)-c*arctan(x*(b+2*c+
(-4*a*c+b^2)^(1/2))^(1/2)/(-x^2+1)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(1+
(-2*a-b)/(-4*a*c+b^2)^(1/2))/a/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(b+2*c+(-4*a*c
+b^2)^(1/2))^(1/2)
```

3.384.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.45 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.78

$$\int \frac{\sqrt{1-x^2}}{x^2\left(a+bx^2+cx^4\right)}\,dx = -\frac{\sqrt{1-x^2}}{ax} + \frac{\text{RootSum}\left[a+4a\#1^2+4b\#1^2+6a\#1^4+8b\#1^4+16c\#1^4+4a\#1^6+4b\#1^6+a\#1^8\&, \frac{-a\log(x)-b\log(a}{}\right.}{}$$

input
```
Integrate[Sqrt[1 - x^2]/(x^2*(a + b*x^2 + c*x^4)),x]
```

output
```
-(Sqrt[1 - x^2]/(a*x)) + RootSum[a + 4*a*#1^2 + 4*b*#1^2 + 6*a*#1^4 + 8*b*
#1^4 + 16*c*#1^4 + 4*a*#1^6 + 4*b*#1^6 + a*#1^8 & , (-(a*Log[x]) - b*Log[x
] + a*Log[-1 + Sqrt[1 - x^2] - x*#1] + b*Log[-1 + Sqrt[1 - x^2] - x*#1] -
3*a*Log[x]*#1^2 - 3*b*Log[x]*#1^2 - 4*c*Log[x]*#1^2 + 3*a*Log[-1 + Sqrt[1
- x^2] - x*#1]*#1^2 + 3*b*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^2 + 4*c*Log[-1
 + Sqrt[1 - x^2] - x*#1]*#1^2 - 3*a*Log[x]*#1^4 - 3*b*Log[x]*#1^4 - 4*c*Lo
g[x]*#1^4 + 3*a*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^4 + 3*b*Log[-1 + Sqrt[1
- x^2] - x*#1]*#1^4 + 4*c*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^4 - a*Log[x]*#
1^6 - b*Log[x]*#1^6 + a*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^6 + b*Log[-1 + S
qrt[1 - x^2] - x*#1]*#1^6)/(a*#1 + b*#1 + 3*a*#1^3 + 4*b*#1^3 + 8*c*#1^3 +
 3*a*#1^5 + 3*b*#1^5 + a*#1^7) & ]/(4*a)
```

3.384.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1618, 242, 2256, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x^2}}{x^2 \left(a+b x^2+c x^4\right)} \, dx$$

$$\Big\downarrow \ 1618$$

$$\frac{\int \frac{1}{x^2\sqrt{1-x^2}}dx}{a}-\frac{\int \frac{cx^2+a+b}{\sqrt{1-x^2}(cx^4+bx^2+a)}dx}{a}$$

$$\Big\downarrow \ 242$$

$$-\frac{\int \frac{cx^2+a+b}{\sqrt{1-x^2}(cx^4+bx^2+a)}dx}{a}-\frac{\sqrt{1-x^2}}{ax}$$

$$\Big\downarrow \ 2256$$

$$-\frac{\int \left(\frac{c-\frac{(2a+b)c}{\sqrt{b^2-4ac}}}{\sqrt{1-x^2}\left(2cx^2+b+\sqrt{b^2-4ac}\right)}+\frac{\frac{(2a+b)c}{\sqrt{b^2-4ac}}+c}{\sqrt{1-x^2}\left(2cx^2+b-\sqrt{b^2-4ac}\right)}\right)dx}{a}-\frac{\sqrt{1-x^2}}{ax}$$

$$\Big\downarrow \ 2009$$

$$-\frac{\frac{c\left(\frac{2a+b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{-\sqrt{b^2-4ac}+b+2c}}+\frac{c\left(1-\frac{2a+b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\sqrt{b^2-4ac}+b+2c}}}{a}-\frac{\sqrt{1-x^2}}{ax}$$

input
```
Int[Sqrt[1 - x^2]/(x^2*(a + b*x^2 + c*x^4)),x]
```

output
```
-(Sqrt[1 - x^2]/(a*x)) - ((c*(1 + (2*a + b)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[b + 2*c - Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^
2])])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) + (c
*(1 - (2*a + b)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c
]]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])])/(Sqrt[b + Sqrt[b^2 - 4
*a*c]]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]))/a
```

3.384.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 1618
```
Int[(((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4), x_Symbol] :> Simp[d/a   Int[(f*x)^m*(d + e*x^2)^(q - 1), x],
x] - Simp[1/(a*f^2)   Int[(f*x)^(m + 2)*(d + e*x^2)^(q - 1)*(Simp[b*d - a*e
 + c*d*x^2, x]/(a + b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && LtQ[m, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2256
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

3.384.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.90

method	result
pseudoelliptic	$\frac{-\frac{\sqrt{-x^2+1}}{x}+\frac{\left(-2ac+b^2+\sqrt{-4ac+b^2}\,a+b\sqrt{-4ac+b^2}+ab\right)\sqrt{2}\,\arctan\left(\frac{a\sqrt{-x^2+1}\,\sqrt{2}}{x\sqrt{\left(2a+b+\sqrt{-4ac+b^2}\right)a}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(2a+b+\sqrt{-4ac+b^2}\right)a}}-\frac{\left(2ac-b^2+\sqrt{-4ac+b^2}\,a+b\sqrt{-4ac+\cdots}\right.}{2\sqrt{-4ac+b^2}\cdots}}{a}$
risch	$\frac{x^2-1}{ax\sqrt{-x^2+1}}+\frac{\sqrt{2}\left(\left(\left(-a-b\right)\sqrt{-4ac+b^2}+a(b-2c)+b^2\right)\sqrt{\left(2a+b+\sqrt{-4ac+b^2}\right)a}\,\operatorname{arctanh}\left(\frac{a\sqrt{-x^2+1}\,\sqrt{2}}{x\sqrt{\left(-b+\sqrt{-4ac+b^2}-2a\right)a}}\right)+\cdots\right.}{2a\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}-2a\cdots\right.}}$
default	$\frac{-\frac{\left(-x^2+1\right)^{\frac{3}{2}}}{x}-x\sqrt{-x^2+1}-\arcsin(x)}{a}+\frac{\left(\left(-a-b\right)\sqrt{-4ac+b^2}+a(b-2c)+b^2\right)\sqrt{2}\,\sqrt{\left(2a+b+\sqrt{-4ac+b^2}\right)a}\,\operatorname{arctanh}\left(\frac{a\cdots}{x\sqrt{\left(-b\cdots\right.}}\right.}{\cdots}$

input
```
int((-x^2+1)^(1/2)/x^2/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/a*(-1/x*(-x^2+1)^(1/2)+1/2*(-2*a*c+b^2+(-4*a*c+b^2)^(1/2)*a+b*(-4*a*c+b^
2)^(1/2)+a*b)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((2*a+b+(-4*a*c+b^2)^(1/2))*a)^(1
/2)*arctan(a/x*(-x^2+1)^(1/2)*2^(1/2)/((2*a+b+(-4*a*c+b^2)^(1/2))*a)^(1/2)
)-1/2*(2*a*c-b^2+(-4*a*c+b^2)^(1/2)*a+b*(-4*a*c+b^2)^(1/2)-a*b)/(-4*a*c+b^
2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)-2*a)*a)^(1/2)*arctanh(a/x*(-x^2+1
)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)-2*a)*a)^(1/2)))
```

3.384.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1998 vs. 2(221) = 442.

Time = 0.49 (sec) , antiderivative size = 1998, normalized size of antiderivative = 7.54

$$\int \frac{\sqrt{1-x^2}}{x^2\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

input
```
integrate((-x^2+1)^(1/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
1/2*(sqrt(1/2)*a*x*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c + (a^3*b^2 - 4*a
^4*c)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*
b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((2*a*c^2 - 2*(a*c^2 - (a*b + b^2
)*c)*x^2 - 2*(a*b + b^2)*c + sqrt(1/2)*((a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*
b + 5*a*b^2)*c)*sqrt(-x^2 + 1)*x - (a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b + 5
*a*b^2)*c)*x - ((a^3*b^3 - 4*a^4*b*c)*sqrt(-x^2 + 1)*x - (a^3*b^3 - 4*a^4*
b*c)*x)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^
6*b^2 - 4*a^7*c)))*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c + (a^3*b^2 - 4*a
^4*c)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*
b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)) - 2*(a*c^2 - (a*b + b^2)*c)*sqrt(-x^
2 + 1))/x^2) - sqrt(1/2)*a*x*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c + (a^3
*b^2 - 4*a^4*c)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2
)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((2*a*c^2 - 2*(a*c^2 -
(a*b + b^2)*c)*x^2 - 2*(a*b + b^2)*c - sqrt(1/2)*((a*b^3 + b^4 + 4*a^2*c^2
 - (4*a^2*b + 5*a*b^2)*c)*sqrt(-x^2 + 1)*x - (a*b^3 + b^4 + 4*a^2*c^2 - (4
*a^2*b + 5*a*b^2)*c)*x - ((a^3*b^3 - 4*a^4*b*c)*sqrt(-x^2 + 1)*x - (a^3*b^
3 - 4*a^4*b*c)*x)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b
^2)*c)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c + (a^3
*b^2 - 4*a^4*c)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2
)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)) - 2*(a*c^2 - (a*b + b^2...
```

3.384.6 Sympy [F]

$$\int \frac{\sqrt{1-x^2}}{x^2\left(a+bx^2+cx^4\right)}\,dx=\int \frac{\sqrt{-\left(x-1\right)\left(x+1\right)}}{x^2\left(a+bx^2+cx^4\right)}\,dx$$

input
```
integrate((-x**2+1)**(1/2)/x**2/(c*x**4+b*x**2+a),x)
```

output
```
Integral(sqrt(-(x - 1)*(x + 1))/(x**2*(a + b*x**2 + c*x**4)), x)
```

3.384.7 Maxima [F]

$$\int \frac{\sqrt{1-x^2}}{x^2(a+bx^2+cx^4)}\,dx = \int \frac{\sqrt{-x^2+1}}{(cx^4+bx^2+a)x^2}\,dx$$

input
```
integrate((-x^2+1)^(1/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^2 + 1)/((c*x^4 + b*x^2 + a)*x^2), x)
```

3.384.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3965 vs. $2(221) = 442$.

Time = 0.85 (sec) , antiderivative size = 3965, normalized size of antiderivative = 14.96

$$\int \frac{\sqrt{1-x^2}}{x^2(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate((-x^2+1)^(1/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/8*(4*a^6*b^3 + 6*a^5*b^4 + 2*a^4*b^5 - 16*a^7*b*c - 32*a^6*b^2*c - 12*a^
5*b^3*c + 32*a^7*c^2 + 16*a^6*b*c^2 + 6*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^
2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^6*b + 13*sqrt(2)*sqrt(2*a^2 + a*b + sqrt
(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^5*b^2 + 7*sqrt(2)*sqrt(2*a^2 + a*b +
sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^4*b^3 - sqrt(2)*sqrt(2*a^2 + a*b
+ sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*b^4 - sqrt(2)*sqrt(2*a^2 + a*
b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b^5 - 12*sqrt(2)*sqrt(2*a^2
 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^6*c - 6*sqrt(2)*sqrt(2*a
^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^5*b*c + 12*sqrt(2)*sqr
t(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^4*b^2*c + 6*sqrt(
2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*b^3*c - 1
6*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^5*c^
2 - 8*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^
4*b*c^2 - 4*(b^2 - 4*a*c)*a^6*b - 6*(b^2 - 4*a*c)*a^5*b^2 - 2*(b^2 - 4*a*c
)*a^4*b^3 + 8*(b^2 - 4*a*c)*a^6*c + 4*(b^2 - 4*a*c)*a^5*b*c - (2*a^3*b^4 +
 2*a^2*b^5 - 16*a^4*b^2*c - 16*a^3*b^3*c + 32*a^5*c^2 + 32*a^4*b*c^2 + 3*s
qrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*b^2 +
 5*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b
^3 + sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b
^4 - sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*...
```

3.384.9 Mupad [B] (verification not implemented)

Time = 8.17 (sec) , antiderivative size = 1234, normalized size of antiderivative = 4.66

$$\int \frac{\sqrt{1-x^2}}{x^2\,(a+bx^2+cx^4)}\,dx = -\frac{\sqrt{1-x^2}}{a\,x}$$

$$+\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}-1\right)\,1{}\mathrm{i}}{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}-\sqrt{1-x^2}\,1{}\mathrm{i}}{x-\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\left(b^3\,\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}+a\,b^2\,\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}-2\,a^2\,c\,\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}-2\,a}{2\,a^2\,\left(4\,a\,c-b^2\right)\,\sqrt{\frac{b+}{}}}$$

$$+\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}-1\right)\,1{}\mathrm{i}}{\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}}-\sqrt{1-x^2}\,1{}\mathrm{i}}{x-\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\left(b^3\,\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}+a\,b^2\,\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}-2\,a^2\,c\,\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}-2\,a}{2\,a^2\,\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}}$$

$$-\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}+1\right)\,1{}\mathrm{i}}{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}+\sqrt{1-x^2}\,1{}\mathrm{i}}{x+\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\left(b^3\,\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}+a\,b^2\,\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}-2\,a^2\,c\,\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}-2\,a}{2\,a^2\,\left(4\,a\,c-b^2\right)\,\sqrt{\frac{b+}{}}}$$

$$-\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}+1\right)\,1{}\mathrm{i}}{\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}}+\sqrt{1-x^2}\,1{}\mathrm{i}}{x+\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\left(b^3\,\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}+a\,b^2\,\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}-2\,a^2\,c\,\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}-2\,a}{2\,a^2\,\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}}$$

input
```
int((1 - x^2)^(1/2)/(x^2*(a + b*x^2 + c*x^4)),x)
```

output
```
(log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b + (b^2 - 4
*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b + (b^2 - 4*a
*c)^(1/2))/(2*c))^(1/2)))*(b^3*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) +
a*b^2*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a^2*c*(-(b + (b^2 - 4*a
*c)^(1/2))/(2*c))^(1/2) - 2*a*c^2*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)
 + b^2*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) - 3*a*b*c*(-(b + (b^2 -
4*a*c)^(1/2))/(2*c))^(1/2) + a*b*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2
)))/(2*a^2*(4*a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) - (1
 - x^2)^(1/2)/(a*x) + (log((((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) -
 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/
(x - (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b^3*(-(b - (b^2 - 4*a*c)^
(1/2))/(2*c))^(1/2) + a*b^2*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a
^2*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a*c^2*(-(b - (b^2 - 4*a*
c)^(1/2))/(2*c))^(3/2) + b^2*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) -
3*a*b*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + a*b*c*(-(b - (b^2 - 4*a
*c)^(1/2))/(2*c))^(3/2)))/(2*a^2*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/
2)*(4*a*c - b^2)) - (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1
)*1i)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x
 + (-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b^3*(-(b + (b^2 - 4*a*c)^(1
/2))/(2*c))^(1/2) + a*b^2*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*...
```

3.384. $\int \frac{\sqrt{1-x^2}}{x^2(a+bx^2+cx^4)}\,dx$

3.385 $\int \frac{x^2\sqrt{1-x^2}}{-1+x^2+x^4}\,dx$

3.385.1 Optimal result

Integrand size = 25, antiderivative size = 96

$$\int \frac{x^2\sqrt{1-x^2}}{-1+x^2+x^4}\,dx = -\arcsin(x)+\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\arctan\left(\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}x}{\sqrt{1-x^2}}\right) - \sqrt{\frac{1}{5}\left(-2+\sqrt{5}\right)}\operatorname{arctanh}\left(\frac{\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}x}{\sqrt{1-x^2}}\right)$$

output
```
-arcsin(x)-1/5*arctanh(1/2*x*(-2+2*5^(1/2))^(1/2)/(-x^2+1)^(1/2))*(-10+5*5
^(1/2))^(1/2)+1/5*arctan(1/2*x*(2+2*5^(1/2))^(1/2)/(-x^2+1)^(1/2))*(10+5*5
^(1/2))^(1/2)
```

3.385.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 199 vs. $2(96) = 192$.

Time = 0.44 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.07

$$\int \frac{x^2\sqrt{1-x^2}}{-1+x^2+x^4}\,dx = \frac{1}{5}\left(-10\arctan\left(\frac{x}{-1+\sqrt{1-x^2}}\right) + \sqrt{5\left(2+\sqrt{5}\right)}\arctan\left(\frac{\sqrt{-2+\sqrt{5}}x}{-1+\sqrt{1-x^2}}\right) + \sqrt{5\left(2+\sqrt{5}\right)}\arctan\left(\frac{\sqrt{2+\sqrt{5}}x}{-1+\sqrt{1-x^2}}\right) + \sqrt{5\left(-2+\sqrt{5}\right)}\operatorname{arctanh}\left(\frac{\sqrt{2+\sqrt{5}}x}{1-\sqrt{1-x^2}}\right) + \sqrt{5\left(-2+\sqrt{5}\right)}\operatorname{arctanh}\left(\frac{\sqrt{-2+\sqrt{5}}x}{-1+\sqrt{1-x^2}}\right)\right)$$

input
```
Integrate[(x^2*Sqrt[1 - x^2])/(-1 + x^2 + x^4),x]
```

output
```
(-10*ArcTan[x/(-1 + Sqrt[1 - x^2])] + Sqrt[5*(2 + Sqrt[5])]*ArcTan[(Sqrt[-
2 + Sqrt[5]]*x)/(-1 + Sqrt[1 - x^2])] + Sqrt[5*(2 + Sqrt[5])]*ArcTan[(Sqrt
[2 + Sqrt[5]]*x)/(-1 + Sqrt[1 - x^2])] + Sqrt[5*(-2 + Sqrt[5])]*ArcTanh[(S
qrt[2 + Sqrt[5]]*x)/(1 - Sqrt[1 - x^2])] + Sqrt[5*(-2 + Sqrt[5])]*ArcTanh[
(Sqrt[-2 + Sqrt[5]]*x)/(-1 + Sqrt[1 - x^2])])/5
```

3.385.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1616, 25, 223, 2256, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{1-x^2}}{x^4+x^2-1}\,dx$$

$$\Big\downarrow \text{ 1616}$$

$$-\int \frac{1}{\sqrt{1-x^2}}dx-\int -\frac{1-2x^2}{\sqrt{1-x^2}\left(-x^4-x^2+1\right)}dx$$

$$\Big\downarrow \text{ 25}$$

$$\int \frac{1-2x^2}{\sqrt{1-x^2}\left(-x^4-x^2+1\right)}dx-\int \frac{1}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{ 223}$$

$$\int \frac{1-2x^2}{\sqrt{1-x^2}\left(-x^4-x^2+1\right)}dx-\arcsin(x)$$

$$\Big\downarrow \text{ 2256}$$

$$\int \left(\frac{-2-\frac{4}{\sqrt{5}}}{\left(-2x^2-\sqrt{5}-1\right)\sqrt{1-x^2}}+\frac{-2+\frac{4}{\sqrt{5}}}{\left(-2x^2+\sqrt{5}-1\right)\sqrt{1-x^2}}\right)dx-\arcsin(x)$$

$$\Big\downarrow \text{ 2009}$$

$$-\arcsin(x)+\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\arctan\left(\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}x}{\sqrt{1-x^2}}\right)-\sqrt{\frac{1}{5}\left(\sqrt{5}-2\right)}\text{arctanh}\left(\frac{\sqrt{\frac{1}{2}\left(\sqrt{5}-1\right)}x}{\sqrt{1-x^2}}\right)$$

input
```
Int[(x^2*Sqrt[1 - x^2])/(-1 + x^2 + x^4),x]
```

output
```
-ArcSin[x] + Sqrt[(2 + Sqrt[5])/5]*ArcTan[(Sqrt[(1 + Sqrt[5])/2]*x)/Sqrt[1
 - x^2]] - Sqrt[(-2 + Sqrt[5])/5]*ArcTanh[(Sqrt[(-1 + Sqrt[5])/2]*x)/Sqrt[
1 - x^2]]
```

3.385.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1616
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Simp[e*(f^2/c)   Int[(f*x)^(m - 2)*(d + e*x^2)^
(q - 1), x], x] - Simp[f^2/c   Int[(f*x)^(m - 2)*(d + e*x^2)^(q - 1)*(Simp[
a*e - (c*d - b*e)*x^2, x]/(a + b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m,
1] && LeQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2256
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

3.385.4 Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.04

method	result
pseudoelliptic	$\frac{\left(5-3\sqrt{5}\right)\sqrt{2\sqrt{5}+2}\,\operatorname{arctanh}\left(\frac{2\sqrt{-x^{2}+1}}{x\sqrt{2\sqrt{5}-2}}\right)}{20}+\frac{\left(-3\sqrt{5}-5\right)\sqrt{2\sqrt{5}-2}\,\arctan\left(\frac{2\sqrt{-x^{2}+1}}{x\sqrt{2\sqrt{5}+2}}\right)}{20}+\arctan\left(\frac{\sqrt{-x^{2}+1}}{x}\right)$
default	$-\frac{\sqrt{2+\sqrt{5}}\,\sqrt{5}\,\arctan\left(\frac{\sqrt{-x^{2}+1}-1}{x\sqrt{2+\sqrt{5}}}\right)}{5}+\frac{\sqrt{\sqrt{5}-2}\,\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{-x^{2}+1}-1}{x\sqrt{\sqrt{5}-2}}\right)}{5}+2\arctan\left(\frac{\sqrt{-x^{2}+1}-1}{x}\right)-\frac{\sqrt{5}\,\text{arcta}}{}$
trager	$\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^{2}+1\right)\sqrt{-x^{2}+1}+x\right)-\operatorname{RootOf}\left(400\_Z^{4}+80\_Z^{2}\right.$

input
```
int(x^2*(-x^2+1)^(1/2)/(x^4+x^2-1),x,method=_RETURNVERBOSE)
```

output
```
1/20*(5-3*5^(1/2))*(2*5^(1/2)+2)^(1/2)*arctanh(2/x*(-x^2+1)^(1/2)/(2*5^(1/
2)-2)^(1/2))+1/20*(-3*5^(1/2)-5)*(2*5^(1/2)-2)^(1/2)*arctan(2/x*(-x^2+1)^(
1/2)/(2*5^(1/2)+2)^(1/2))+arctan(1/x*(-x^2+1)^(1/2))
```

3.385.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(71) = 142$.

Time = 0.29 (sec) , antiderivative size = 318, normalized size of antiderivative = 3.31

$$\int \frac{x^2\sqrt{1-x^2}}{-1+x^2+x^4}\,dx$$
$$= \frac{1}{10}\sqrt{5}\sqrt{-\sqrt{5}-2}\log\left(-\frac{2x^2+\sqrt{-x^2+1}\left(\left(\sqrt{5}x-x\right)\sqrt{-\sqrt{5}-2}+2\right)-\left(\sqrt{5}x-x\right)\sqrt{-\sqrt{5}-2}-2}{x^2}\right)$$
$$-\frac{1}{10}\sqrt{5}\sqrt{-\sqrt{5}-2}\log\left(-\frac{2x^2-\sqrt{-x^2+1}\left(\left(\sqrt{5}x-x\right)\sqrt{-\sqrt{5}-2}-2\right)+\left(\sqrt{5}x-x\right)\sqrt{-\sqrt{5}-2}-2}{x^2}\right)$$
$$+\frac{1}{10}\sqrt{5}\sqrt{\sqrt{5}-2}\log\left(-\frac{2x^2+\left(\sqrt{-x^2+1}\left(\sqrt{5}x+x\right)-\sqrt{5}x-x\right)\sqrt{\sqrt{5}-2}+2\sqrt{-x^2+1}-2}{x^2}\right)$$
$$-\frac{1}{10}\sqrt{5}\sqrt{\sqrt{5}-2}\log\left(-\frac{2x^2-\left(\sqrt{-x^2+1}\left(\sqrt{5}x+x\right)-\sqrt{5}x-x\right)\sqrt{\sqrt{5}-2}+2\sqrt{-x^2+1}-2}{x^2}\right)$$
$$+2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

input
```
integrate(x^2*(-x^2+1)^(1/2)/(x^4+x^2-1),x, algorithm="fricas")
```

output
```
1/10*sqrt(5)*sqrt(-sqrt(5) - 2)*log(-(2*x^2 + sqrt(-x^2 + 1)*((sqrt(5)*x -
 x)*sqrt(-sqrt(5) - 2) + 2) - (sqrt(5)*x - x)*sqrt(-sqrt(5) - 2) - 2)/x^2)
 - 1/10*sqrt(5)*sqrt(-sqrt(5) - 2)*log(-(2*x^2 - sqrt(-x^2 + 1)*((sqrt(5)*
x - x)*sqrt(-sqrt(5) - 2) - 2) + (sqrt(5)*x - x)*sqrt(-sqrt(5) - 2) - 2)/x
^2) + 1/10*sqrt(5)*sqrt(sqrt(5) - 2)*log(-(2*x^2 + (sqrt(-x^2 + 1)*(sqrt(5
)*x + x) - sqrt(5)*x - x)*sqrt(sqrt(5) - 2) + 2*sqrt(-x^2 + 1) - 2)/x^2) -
 1/10*sqrt(5)*sqrt(sqrt(5) - 2)*log(-(2*x^2 - (sqrt(-x^2 + 1)*(sqrt(5)*x +
 x) - sqrt(5)*x - x)*sqrt(sqrt(5) - 2) + 2*sqrt(-x^2 + 1) - 2)/x^2) + 2*ar
ctan((sqrt(-x^2 + 1) - 1)/x)
```

3.385.6 Sympy [F]

$$\int \frac{x^2\sqrt{1-x^2}}{-1+x^2+x^4}\,dx = \int \frac{x^2\sqrt{-(x-1)(x+1)}}{x^4+x^2-1}\,dx$$

input
```
integrate(x**2*(-x**2+1)**(1/2)/(x**4+x**2-1),x)
```

output
```
Integral(x**2*sqrt(-(x - 1)*(x + 1))/(x**4 + x**2 - 1), x)
```

3.385.7 Maxima [F]

$$\int \frac{x^2\sqrt{1-x^2}}{-1+x^2+x^4}\,dx = \int \frac{\sqrt{-x^2+1}x^2}{x^4+x^2-1}\,dx$$

input
```
integrate(x^2*(-x^2+1)^(1/2)/(x^4+x^2-1),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^2 + 1)*x^2/(x^4 + x^2 - 1), x)
```

3.385.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(71) = 142$.

Time = 0.33 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.18

$$\begin{aligned}
&\int \frac{x^2\sqrt{1-x^2}}{-1+x^2+x^4}\,dx \\
&= -\frac{1}{2}\pi\operatorname{sgn}(x) - \frac{1}{5}\sqrt{5\sqrt{5}+10}\arctan\left(-\frac{\frac{x}{\sqrt{-x^2+1}-1}-\frac{\sqrt{-x^2+1}-1}{x}}{\sqrt{2\sqrt{5}+2}}\right) \\
&\quad - \frac{1}{10}\sqrt{5\sqrt{5}-10}\log\left(\left|\sqrt{2\sqrt{5}-2}-\frac{x}{\sqrt{-x^2+1}-1}+\frac{\sqrt{-x^2+1}-1}{x}\right|\right) \\
&\quad + \frac{1}{10}\sqrt{5\sqrt{5}-10}\log\left(\left|-\sqrt{2\sqrt{5}-2}-\frac{x}{\sqrt{-x^2+1}-1}+\frac{\sqrt{-x^2+1}-1}{x}\right|\right) \\
&\quad - \arctan\left(-\frac{x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{2\left(\sqrt{-x^2+1}-1\right)}\right)
\end{aligned}$$

input
```
integrate(x^2*(-x^2+1)^(1/2)/(x^4+x^2-1),x, algorithm="giac")
```

output
```
-1/2*pi*sgn(x) - 1/5*sqrt(5*sqrt(5) + 10)*arctan(-(x/(sqrt(-x^2 + 1) - 1)
- (sqrt(-x^2 + 1) - 1)/x)/sqrt(2*sqrt(5) + 2)) - 1/10*sqrt(5*sqrt(5) - 10)
*log(abs(sqrt(2*sqrt(5) - 2) - x/(sqrt(-x^2 + 1) - 1) + (sqrt(-x^2 + 1) -
1)/x)) + 1/10*sqrt(5*sqrt(5) - 10)*log(abs(-sqrt(2*sqrt(5) - 2) - x/(sqrt(
-x^2 + 1) - 1) + (sqrt(-x^2 + 1) - 1)/x)) - arctan(-1/2*x*((sqrt(-x^2 + 1)
 - 1)^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1))
```

3.385.9 Mupad [B] (verification not implemented)

Time = 8.39 (sec) , antiderivative size = 383, normalized size of antiderivative = 3.99

$$\int \frac{x^2\sqrt{1-x^2}}{-1+x^2+x^4}\,dx = -\mathrm{asin}(x) - \frac{\ln\left(\frac{\frac{\left(x\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}-1\right)1\mathrm{i}}{\sqrt{\frac{3}{2}-\frac{\sqrt{5}}{2}}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\right)\left(\sqrt{5}-2\right)}{\left(2\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)\sqrt{\frac{3}{2}-\frac{\sqrt{5}}{2}}} + \frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}-1\right)1\mathrm{i}}{\sqrt{\frac{\sqrt{5}}{2}+\frac{3}{2}}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}}\right)\left(\sqrt{5}+2\right)}{\left(2\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)\sqrt{\frac{\sqrt{5}}{2}+\frac{3}{2}}} + \frac{\ln\left(\frac{\frac{\left(x\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}+1\right)1\mathrm{i}}{\sqrt{\frac{3}{2}-\frac{\sqrt{5}}{2}}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\right)\left(\sqrt{5}-2\right)}{\left(2\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)\sqrt{\frac{3}{2}-\frac{\sqrt{5}}{2}}} - \frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}+1\right)1\mathrm{i}}{\sqrt{\frac{\sqrt{5}}{2}+\frac{3}{2}}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}}\right)\left(\sqrt{5}+2\right)}{\left(2\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)\sqrt{\frac{\sqrt{5}}{2}+\frac{3}{2}}}$$

input
```
int((x^2*(1 - x^2)^(1/2))/(x^2 + x^4 - 1),x)
```

output
```
(log((((x*(- 5^(1/2)/2 - 1/2)^(1/2) - 1)*1i)/(5^(1/2)/2 + 3/2)^(1/2) - (1
- x^2)^(1/2)*1i)/(x - (- 5^(1/2)/2 - 1/2)^(1/2)))*(5^(1/2) + 2))/((2*(- 5^
(1/2)/2 - 1/2)^(1/2) + 4*(- 5^(1/2)/2 - 1/2)^(3/2))*(5^(1/2)/2 + 3/2)^(1/2
)) - (log((((x*(5^(1/2)/2 - 1/2)^(1/2) - 1)*1i)/(3/2 - 5^(1/2)/2)^(1/2) -
(1 - x^2)^(1/2)*1i)/(x - (5^(1/2)/2 - 1/2)^(1/2)))*(5^(1/2) - 2))/((2*(5^(
1/2)/2 - 1/2)^(1/2) + 4*(5^(1/2)/2 - 1/2)^(3/2))*(3/2 - 5^(1/2)/2)^(1/2))
- asin(x) + (log((((x*(5^(1/2)/2 - 1/2)^(1/2) + 1)*1i)/(3/2 - 5^(1/2)/2)^(
1/2) + (1 - x^2)^(1/2)*1i)/(x + (5^(1/2)/2 - 1/2)^(1/2)))*(5^(1/2) - 2))/(
(2*(5^(1/2)/2 - 1/2)^(1/2) + 4*(5^(1/2)/2 - 1/2)^(3/2))*(3/2 - 5^(1/2)/2)^
(1/2)) - (log((((x*(- 5^(1/2)/2 - 1/2)^(1/2) + 1)*1i)/(5^(1/2)/2 + 3/2)^(1
/2) + (1 - x^2)^(1/2)*1i)/(x + (- 5^(1/2)/2 - 1/2)^(1/2)))*(5^(1/2) + 2))/
((2*(- 5^(1/2)/2 - 1/2)^(1/2) + 4*(- 5^(1/2)/2 - 1/2)^(3/2))*(5^(1/2)/2 +
3/2)^(1/2))
```

3.385. $\int \frac{x^2\sqrt{1-x^2}}{-1+x^2+x^4}\,dx$

3.386 $\int \frac{x^8}{\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx$

3.386.1 Optimal result

Integrand size = 29, antiderivative size = 479

$$\begin{aligned}\int \frac{x^8}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx &= -\frac{3dx\sqrt{d+ex^2}}{8ce^2}-\frac{bx\sqrt{d+ex^2}}{2c^2e}+\frac{x^3\sqrt{d+ex^2}}{4ce}\\
&-\frac{\left(b^3-2abc-\frac{b^4-4ab^2c+2a^2c^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^3\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\\
&-\frac{\left(b^3-2abc+\frac{b^4-4ab^2c+2a^2c^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^3\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\\
&+\frac{3d^2\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8ce^{5/2}}+\frac{bd\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^2e^{3/2}}+\frac{(b^2-ac)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^3\sqrt{e}}\end{aligned}$$

output
```
3/8*d^2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/c/e^(5/2)+1/2*b*d*arctanh(x*e^(
1/2)/(e*x^2+d)^(1/2))/c^2/e^(3/2)+(-a*c+b^2)*arctanh(x*e^(1/2)/(e*x^2+d)^(
1/2))/c^3/e^(1/2)-3/8*d*x*(e*x^2+d)^(1/2)/c/e^2-1/2*b*x*(e*x^2+d)^(1/2)/c^
2/e+1/4*x^3*(e*x^2+d)^(1/2)/c/e-arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^
(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b^3-2*a*b*c+(-2*a^2*c
^2+4*a*b^2*c-b^4)/(-4*a*c+b^2)^(1/2))/c^3/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))
^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2
)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^3-2*a*b*c+(2*a^
2*c^2-4*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2))/c^3/(b+(-4*a*c+b^2)^(1/2))^(1/2)/
(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.386.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.37 (sec) , antiderivative size = 838, normalized size of antiderivative = 1.75

$$\int \frac{x^8}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

$$= \frac{\frac{c\sqrt{d+ex^2}(-3cdx-4bex+2cex^3)}{e^2} + \frac{2(3c^2d^2+8b^2e^2+4ce(bd-2ae))\operatorname{arctanh}\left(\frac{\sqrt{e}x}{-\sqrt{d}+\sqrt{d+ex^2}}\right)}{e^{5/2}} - 2\text{RootSum}\Big[ae^4+4bde^2\#1^2-4}{}$$

input
```
Integrate[x^8/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

output
```
((c*Sqrt[d + e*x^2]*(-3*c*d*x - 4*b*e*x + 2*c*e*x^3))/e^2 + (2*(3*c^2*d^2
+ 8*b^2*e^2 + 4*c*e*(b*d - 2*a*e))*ArcTanh[(Sqrt[e]*x)/(-Sqrt[d] + Sqrt[d
+ e*x^2])])/e^(5/2) - 2*RootSum[a*e^4 + 4*b*d*e^2*#1^2 - 4*a*e^3*#1^2 + 16
*c*d^2*#1^4 - 8*b*d*e*#1^4 + 6*a*e^2*#1^4 + 4*b*d*#1^6 - 4*a*e*#1^6 + a*#1
^8 & , (-(a*b^2*e^3*Log[x]) + a^2*c*e^3*Log[x] + a*b^2*e^3*Log[-Sqrt[d] +
Sqrt[d + e*x^2] - x*#1] - a^2*c*e^3*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]
 - 4*b^3*d*e*Log[x]*#1^2 + 8*a*b*c*d*e*Log[x]*#1^2 + 3*a*b^2*e^2*Log[x]*#1
^2 - 3*a^2*c*e^2*Log[x]*#1^2 + 4*b^3*d*e*Log[-Sqrt[d] + Sqrt[d + e*x^2] -
x*#1]*#1^2 - 8*a*b*c*d*e*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^2 - 3*a
*b^2*e^2*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^2 + 3*a^2*c*e^2*Log[-Sq
rt[d] + Sqrt[d + e*x^2] - x*#1]*#1^2 + 4*b^3*d*Log[x]*#1^4 - 8*a*b*c*d*Log
[x]*#1^4 - 3*a*b^2*e*Log[x]*#1^4 + 3*a^2*c*e*Log[x]*#1^4 - 4*b^3*d*Log[-Sq
rt[d] + Sqrt[d + e*x^2] - x*#1]*#1^4 + 8*a*b*c*d*Log[-Sqrt[d] + Sqrt[d + e
*x^2] - x*#1]*#1^4 + 3*a*b^2*e*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^4
 - 3*a^2*c*e*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^4 + a*b^2*Log[x]*#1
^6 - a^2*c*Log[x]*#1^6 - a*b^2*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^6
 + a^2*c*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^6)/(b*d*e^2*#1 - a*e^3*
#1 + 8*c*d^2*#1^3 - 4*b*d*e*#1^3 + 3*a*e^2*#1^3 + 3*b*d*#1^5 - 3*a*e*#1^5
+ a*#1^7) & ])/(8*c^3)
```

3.386.3 Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1626, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

$$\Big\downarrow \text{ 1626}$$

$$\int \left(\frac{b^2-ac}{c^3\sqrt{d+ex^2}}-\frac{bx^2(b^2-2ac)+a(b^2-ac)}{c^3\sqrt{d+ex^2}\,(a+bx^2+cx^4)}-\frac{bx^2}{c^2\sqrt{d+ex^2}}+\frac{x^4}{c\sqrt{d+ex^2}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\left(-\frac{2a^2c^2-4ab^2c+b^4}{\sqrt{b^2-4ac}}-2abc+b^3\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^3\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\frac{2a^2c^2-4ab^2c+b^4}{\sqrt{b^2-4ac}}-2abc+b^3\right)\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c^3\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{(b^2-ac)\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^3\sqrt{e}}+\frac{bd\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^2e^{3/2}}+\frac{3d^2\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8ce^{5/2}}-\frac{bx\sqrt{d+ex^2}}{2c^2e}-\frac{3dx\sqrt{d+ex^2}}{8ce^2}+\frac{x^3\sqrt{d+ex^2}}{4ce}$$

input
```
Int[x^8/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

output
```
(-3*d*x*Sqrt[d + e*x^2])/(8*c*e^2) - (b*x*Sqrt[d + e*x^2])/(2*c^2*e) + (x^
3*Sqrt[d + e*x^2])/(4*c*e) - ((b^3 - 2*a*b*c - (b^4 - 4*a*b^2*c + 2*a^2*c^
2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(
Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c^3*Sqrt[b - Sqrt[b^2 - 4*
a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - ((b^3 - 2*a*b*c + (b^4 -
4*a*b^2*c + 2*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c^3*Sq
rt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]) + (3*d^
2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(8*c*e^(5/2)) + (b*d*ArcTanh[(Sqrt
[e]*x)/Sqrt[d + e*x^2]])/(2*c^2*e^(3/2)) + ((b^2 - a*c)*ArcTanh[(Sqrt[e]*x
)/Sqrt[d + e*x^2]])/(c^3*Sqrt[e])
```

3.386.3.1 Defintions of rubi rules used

rule 1626
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] && !IntegerQ[q] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.386. $\int \frac{x^8}{\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx$

3.386.4 Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 394, normalized size of antiderivative = 0.82

method	result
risch	$-\frac{x(-2c\,x^2e+4be+3cd)\sqrt{e\,x^2+d}}{8e^2c^2}-\frac{\frac{\left(8e^2ac-8b^2e^2-4bcde-3c^2d^2\right)\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{c\sqrt{e}}+\frac{4e^2a\sqrt{2}\left(\frac{\left(-3abcd+b^3d+\sqrt{-d^2\left(4ac-b^2\right.}\right.}{}\right.}{}}{}$
default	$-\frac{\frac{ac\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{\sqrt{e}}-\frac{b^2\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{\sqrt{e}}-c^2\left(\frac{x^3\sqrt{e\,x^2+d}}{4e}-\frac{3d\left(\frac{x\sqrt{e\,x^2+d}}{2e}-\frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{2e^{\frac{3}{2}}}\right)}{4e}\right)+bc\left(\frac{x\sqrt{e\,x^2+d}}{2e}-\frac{d}{}\right.}{c^3}$
pseudoelliptic	$-\frac{-a\sqrt{2}\,e^{\frac{9}{2}}\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\left(\left(ac-b^2\right)\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}+3abcd-b^3d\right)\operatorname{arctanh}\left(\frac{a\sqrt{e\,x^2+d}\,\sqrt{2}}{x\sqrt{\left(2ae-bd+\sqrt{-4d^2\left(a\right.}\right.}}\right.}{}$

input
```
int(x^8/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*x*(-2*c*e*x^2+4*b*e+3*c*d)*(e*x^2+d)^(1/2)/e^2/c^2-1/8/e^2/c^2*((8*a*
c*e^2-8*b^2*e^2-4*b*c*d*e-3*c^2*d^2)/c*ln(x*e^(1/2)+(e*x^2+d)^(1/2))/e^(1/
2)+4*e^2/c*a*2^(1/2)/(-d^2*(4*a*c-b^2))^(1/2)*((-3*a*b*c*d+b^3*d+(-d^2*(4*
a*c-b^2))^(1/2)*a*c-(-d^2*(4*a*c-b^2))^(1/2)*b^2)/((-2*a*e+b*d+(-d^2*(4*a*
c-b^2))^(1/2))*a)^(1/2)*arctan(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((-2*a*e+b*d+(-
d^2*(4*a*c-b^2))^(1/2))*a)^(1/2))-(3*a*b*c*d-b^3*d+(-d^2*(4*a*c-b^2))^(1/2
)*a*c-(-d^2*(4*a*c-b^2))^(1/2)*b^2)/((2*a*e-b*d+(-d^2*(4*a*c-b^2))^(1/2))*
a)^(1/2)*arctanh(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((2*a*e-b*d+(-d^2*(4*a*c-b^2)
)^(1/2))*a)^(1/2))))
```

3.386.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9131 vs. $2(405) = 810$.

Time = 142.83 (sec) , antiderivative size = 18271, normalized size of antiderivative = 38.14

$$\int \frac{x^8}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(x^8/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.386.6 Sympy [F]

$$\int \frac{x^8}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{x^8}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

input
```
integrate(x**8/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

output
```
Integral(x**8/(sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

3.386.7 Maxima [F]

$$\int \frac{x^8}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{x^8}{(cx^4+bx^2+a)\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^8/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^8/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)), x)
```

3.386.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^8}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^8/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.386.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{x^8}{\sqrt{e\,x^2+d}\,(c\,x^4+b\,x^2+a)}\,dx$$

input
```
int(x^8/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)
```

output
```
int(x^8/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)
```

3.387 $\int \frac{x^6}{\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx$

3.387.1 Optimal result

Integrand size = 29, antiderivative size = 366

$$\int \frac{x^6}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \frac{x\sqrt{d+ex^2}}{2ce} + \frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} + \frac{\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^2\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}} - \frac{d\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2ce^{3/2}} - \frac{b\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2\sqrt{e}}$$

output
```
-1/2*d*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/c/e^(3/2)-b*arctanh(x*e^(1/2)/(e
*x^2+d)^(1/2))/c^2/e^(1/2)+1/2*x*(e*x^2+d)^(1/2)/c/e+arctan(x*(2*c*d-e*(b-
(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(
b^2-a*c-b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c^2/(2*c*d-e*(b-(-4*a*c+b^2)^(1
/2)))^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)
^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2-a*c+b*(-
3*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c^2/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-e*(
b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.387.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.78 (sec) , antiderivative size = 594, normalized size of antiderivative = 1.62

$$\int \frac{x^6}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \frac{x\sqrt{d+ex^2}}{2ce} + \frac{(-cd-2be)\text{arctanh}\left(\frac{\sqrt{e}x}{-\sqrt{d}+\sqrt{d+ex^2}}\right)}{c^2e^{3/2}} + \frac{\text{RootSum}\left[ae^4+4bde^2\#1^2-4ae^3\#1^2+16cd^2\#1^4-8bde\#1^4+6ae^2\#1^4+4bd\#1^6-4ae\#1^6+a\#1^8\right.}{}$$

input
```
Integrate[x^6/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

output
```
(x*Sqrt[d + e*x^2])/(2*c*e) + ((-(c*d) - 2*b*e)*ArcTanh[(Sqrt[e]*x)/(-Sqrt
[d] + Sqrt[d + e*x^2])])/(c^2*e^(3/2)) + RootSum[a*e^4 + 4*b*d*e^2*#1^2 -
4*a*e^3*#1^2 + 16*c*d^2*#1^4 - 8*b*d*e*#1^4 + 6*a*e^2*#1^4 + 4*b*d*#1^6 -
4*a*e*#1^6 + a*#1^8 & , (-(a*b*e^3*Log[x]) + a*b*e^3*Log[-Sqrt[d] + Sqrt[d
 + e*x^2] - x*#1] - 4*b^2*d*e*Log[x]*#1^2 + 4*a*c*d*e*Log[x]*#1^2 + 3*a*b*
e^2*Log[x]*#1^2 + 4*b^2*d*e*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^2 -
4*a*c*d*e*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^2 - 3*a*b*e^2*Log[-Sqr
t[d] + Sqrt[d + e*x^2] - x*#1]*#1^2 + 4*b^2*d*Log[x]*#1^4 - 4*a*c*d*Log[x]
*#1^4 - 3*a*b*e*Log[x]*#1^4 - 4*b^2*d*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#
1]*#1^4 + 4*a*c*d*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^4 + 3*a*b*e*Lo
g[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^4 + a*b*Log[x]*#1^6 - a*b*Log[-Sqr
t[d] + Sqrt[d + e*x^2] - x*#1]*#1^6)/(b*d*e^2*#1 - a*e^3*#1 + 8*c*d^2*#1^3
 - 4*b*d*e*#1^3 + 3*a*e^2*#1^3 + 3*b*d*#1^5 - 3*a*e*#1^5 + a*#1^7) & ]/(4*
c^2)
```

3.387.3 Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1626, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

$$\Big\downarrow \; 1626$$

$$\int \left(\frac{x^2(b^2-ac)+ab}{c^2\sqrt{d+ex^2}\,(a+bx^2+cx^4)}-\frac{b}{c^2\sqrt{d+ex^2}}+\frac{x^2}{c\sqrt{d+ex^2}}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{b\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2\sqrt{e}}-\frac{d\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2ce^{3/2}}+\frac{x\sqrt{d+ex^2}}{2ce}$$

input
```
Int[x^6/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

output
```
(x*Sqrt[d + e*x^2])/(2*c*e) + ((b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b
^2 - 4*a*c]]*Sqrt[d + e*x^2])]])/(c^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*
d - (b - Sqrt[b^2 - 4*a*c])*e]) + ((b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2
 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sq
rt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]])/(c^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[
2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]) - (d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^
2]])/(2*c*e^(3/2)) - (b*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(c^2*Sqrt[e]
)
```

3.387.3.1 Defintions of rubi rules used

rule 1626
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.387.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.83

method	result
risch	$\frac{x\sqrt{e\,x^2+d}}{2ce}-\frac{\frac{(2be+cd)\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{c\sqrt{e}}+\frac{ea\sqrt{2}\left(\frac{\left(2acd-b^2d+\sqrt{-d^2\left(4ac-b^2\right)}\,b\right)\arctan\left(\frac{a\sqrt{e\,x^2+d}\,\sqrt{2}}{x\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}\right)}{\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}-\right.}{c\sqrt{-d^2\left(4ac\right.}}}{2ec}$
default	$\frac{\frac{x\sqrt{e\,x^2+d}}{2e}-\frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{2e^{\frac{3}{2}}}}{c}-\frac{b\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{c^2\sqrt{e}}-\frac{a\sqrt{2}\left(\frac{\left(2acd-b^2d+\sqrt{-d^2\left(4ac-b^2\right)}\,b\right)\arctan\left(\frac{a\sqrt{e\,x^2}}{x\sqrt{\left(-2ae+bd+\sqrt{\right.}\right.}}\right.}{\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}\right.}{}$
pseudoelliptic	$-\frac{a\sqrt{2}\,\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\,e^{\frac{3}{2}}\left(-\frac{\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\,b}{2}+d\left(ac-\frac{b^2}{2}\right)\right)\operatorname{arctanh}\left(\frac{a\sqrt{e\,x^2+d}\,\sqrt{2}}{x\sqrt{\left(2ae-bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right.}}\right.}{}$

input
```
int(x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(e*x^2+d)^(1/2)/c/e-1/2/e/c*((2*b*e+c*d)/c*ln(x*e^(1/2)+(e*x^2+d)^(1
/2))/e^(1/2)+e/c*a*2^(1/2)/(-d^2*(4*a*c-b^2))^(1/2)*((2*a*c*d-b^2*d+(-d^2*
(4*a*c-b^2))^(1/2)*b)/((-2*a*e+b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2)*arct
an(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((-2*a*e+b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(
1/2))-(-2*a*c*d+b^2*d+(-d^2*(4*a*c-b^2))^(1/2)*b)/((2*a*e-b*d+(-d^2*(4*a*c
-b^2))^(1/2))*a)^(1/2)*arctanh(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((2*a*e-b*d+(-d
^2*(4*a*c-b^2))^(1/2))*a)^(1/2))))
```

3.387.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7323 vs. $2(308) = 616$.

Time = 52.68 (sec) , antiderivative size = 14654, normalized size of antiderivative = 40.04

$$\int \frac{x^6}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.387.6 Sympy [F]

$$\int \frac{x^6}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{x^6}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

input
```
integrate(x**6/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

output
```
Integral(x**6/(sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

3.387.7 Maxima [F]

$$\int \frac{x^6}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{x^6}{(cx^4+bx^2+a)\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^6/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)), x)
```

3.387.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^6}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.387.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{x^6}{\sqrt{e\,x^2+d}\,(c\,x^4+b\,x^2+a)}\,dx$$

input
```
int(x^6/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)
```

output
```
int(x^6/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)
```

3.388 $\int \frac{x^4}{\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx$

3.388.1 Optimal result

Integrand size = 29, antiderivative size = 298

$$\int \frac{x^4}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = -\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}}$$

output
```
arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/c/e^(1/2)-arctan(x*(2*c*d-e*(b-(-4*a*c+
b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(2*a*c
-b^2)/(-4*a*c+b^2)^(1/2))/c/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(b-(-4*
a*c+b^2)^(1/2))^(1/2)-arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x
^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/
2))/c/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.388.2 Mathematica [A] (verified)

Time = 10.67 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.98

$$\int \frac{x^4}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \frac{-\frac{\left(b+\frac{-b^2+2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-be+\sqrt{b^2-4ac}e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}}{c}$$

input
```
Integrate[x^4/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

output
```
(-(((b + (-b^2 + 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - b*e + Sqrt
[b^2 - 4*a*c]*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[
b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e])) - ((b +
(b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b + Sqrt[b
^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]) + ArcTanh[(Sqrt[e]*x
)/Sqrt[d + e*x^2]]/Sqrt[e])/c
```

3.388.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1626, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

$$\Big\downarrow \ 1626$$

$$\int \left(\frac{1}{c\sqrt{d+ex^2}}-\frac{a+bx^2}{c\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}}$$

input
```
Int[x^4/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

output
```
-(((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^
2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c*Sqrt[
b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e])) - ((b + (
b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c*Sqrt[b + Sqrt[
b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]) + ArcTanh[(Sqrt[e]*
x)/Sqrt[d + e*x^2]]/(c*Sqrt[e])
```

3.388.3.1 Defintions of rubi rules used

rule 1626
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.388.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.85

method	result
default	$\frac{\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{c\sqrt{e}}+\frac{a\sqrt{2}\left(\frac{\left(-bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)\arctan\left(\frac{a\sqrt{e\,x^2+d}\,\sqrt{2}}{x\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}\right)}{\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}-\frac{\left(bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)\operatorname{arct}\cdots}{\sqrt{\left(2ae-\cdots\right.}}\right.}{2c\sqrt{-d^2(4ac-b^2)}\cdots}$
pseudoelliptic	$-\frac{a\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\,\sqrt{2}\,\sqrt{e}\left(bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)\operatorname{arctanh}\left(\frac{a\sqrt{e\,x^2+d}\,\sqrt{2}}{x\sqrt{\left(2ae-bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}}\right)-\sqrt{\left(2\cdots\right.}}{2\sqrt{e}\sqrt{\left(-2\cdots\right.}}$

input
```
int(x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/c*ln(x*e^(1/2)+(e*x^2+d)^(1/2))/e^(1/2)+1/2/c*a*2^(1/2)/(-d^2*(4*a*c-b^2
))^(1/2)*((-b*d+(-d^2*(4*a*c-b^2))^(1/2))/((-2*a*e+b*d+(-d^2*(4*a*c-b^2))^
(1/2))*a)^(1/2)*arctan(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((-2*a*e+b*d+(-d^2*(4*a
*c-b^2))^(1/2))*a)^(1/2))-(b*d+(-d^2*(4*a*c-b^2))^(1/2))/((2*a*e-b*d+(-d^2
*(4*a*c-b^2))^(1/2))*a)^(1/2)*arctanh(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((2*a*e-
b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2)))
```

3.388.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5543 vs. 2(252) = 504.

Time = 9.05 (sec) , antiderivative size = 11094, normalized size of antiderivative = 37.23

$$\int \frac{x^4}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx=\text{Too large to display}$$

input
```
integrate(x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.388.6 Sympy [F]

$$\int \frac{x^4}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{x^4}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

input
```
integrate(x**4/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

output
```
Integral(x**4/(sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

3.388.7 Maxima [F]

$$\int \frac{x^4}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{x^4}{(cx^4+bx^2+a)\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)), x)
```

3.388.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^4}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.388.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{x^4}{\sqrt{e\,x^2+d}\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

input
```
int(x^4/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)
```

output
```
int(x^4/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)
```

3.389 $\int \frac{x^2}{\sqrt{d+ex^2}(a+bx^2+cx^4)} dx$

3.389.1 Optimal result

Integrand size = 29, antiderivative size = 240

$$\int \frac{x^2}{\sqrt{d+ex^2}(a+bx^2+cx^4)} dx = -\frac{\sqrt{b-\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} + \frac{\sqrt{b+\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

output
```
-arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*
c+b^2)^(1/2))^(1/2))*(b-(-4*a*c+b^2)^(1/2))^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*
d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))
)^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-4*a*c+b^2)^(1/2
))^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.389.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 8.66 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = -d\text{RootSum}\Big[ae^4+4bde^2\#1^2-4ae^3\#1^2+16cd^2\#1^4 -8bde\#1^4+6ae^2\#1^4+4bd\#1^6-4ae\#1^6 +a\#1^8\&, \frac{-e\log(x)\#1+e\log\left(-\sqrt{d}+\sqrt{d+ex^2}-x\#1\right)\#1+\log(x)\#1^3-\log\left(-\sqrt{d}+\sqrt{d+ex^2}-x\right.}{-bde^2+ae^3-8cd^2\#1^2+4bde\#1^2-3ae^2\#1^2-3bd\#1^4+3ae\#1^4-a\#1^6}$$

input
```
Integrate[x^2/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

output
```
-(d*RootSum[a*e^4 + 4*b*d*e^2*#1^2 - 4*a*e^3*#1^2 + 16*c*d^2*#1^4 - 8*b*d*
e*#1^4 + 6*a*e^2*#1^4 + 4*b*d*#1^6 - 4*a*e*#1^6 + a*#1^8 & , (-(e*Log[x]*#
1) + e*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1 + Log[x]*#1^3 - Log[-Sqrt
[d] + Sqrt[d + e*x^2] - x*#1]*#1^3)/(-(b*d*e^2) + a*e^3 - 8*c*d^2*#1^2 + 4
*b*d*e*#1^2 - 3*a*e^2*#1^2 - 3*b*d*#1^4 + 3*a*e*#1^4 - a*#1^6) & ])
```

3.389.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1626, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

$$\Big\downarrow \text{1626}$$

$$\int \left(\frac{1-\frac{b}{\sqrt{b^2-4ac}}}{\sqrt{d+ex^2}\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}+\frac{\frac{b}{\sqrt{b^2-4ac}}+1}{\sqrt{d+ex^2}\left(\sqrt{b^2-4ac}+b+2cx^2\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

input
```
Int[x^2/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

output
```
-((Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c]
)*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b^2 - 4*a*c]
*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e])) + (Sqrt[b + Sqrt[b^2 - 4*a*c]]*
ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*
a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e])
```

3.389.3.1 Defintions of rubi rules used

rule 1626
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.389.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.97

method	result
default	$\frac{\sqrt{2}\,ad\left(\operatorname{arctanh}\left(\frac{a\sqrt{e x^2+d}\,\sqrt{2}}{x\sqrt{\left(2ae-bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}}\right)\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}+\arctan\left(\frac{a\sqrt{e x^2+d}\,\sqrt{2}}{x\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac\cdots\right.}}\right.\right.}{\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\sqrt{\left(2ae-bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\cdots\right.}\right.}}$
pseudoelliptic	$\frac{\sqrt{2}\,ad\left(\operatorname{arctanh}\left(\frac{a\sqrt{e x^2+d}\,\sqrt{2}}{x\sqrt{\left(2ae-bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}}\right)\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}+\arctan\left(\frac{a\sqrt{e x^2+d}\,\sqrt{2}}{x\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac\cdots\right.}}\right.\right.}{\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\sqrt{\left(2ae-bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\cdots\right.}\right.}}$

input
```
int(x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2^(1/2)*a*d*(arctanh(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((2*a*e-b*d+(-4*d^2*(a*c-
1/4*b^2))^(1/2))*a)^(1/2))*((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(
1/2)+arctan(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2)
)^(1/2))*a)^(1/2))*((2*a*e-b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2))/(-4
*d^2*(a*c-1/4*b^2))^(1/2)/((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1
/2)/((2*a*e-b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)
```

3.389.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3395 vs. 2(200) = 400.

Time = 2.46 (sec) , antiderivative size = 3395, normalized size of antiderivative = 14.15

$$\int \frac{x^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e + ((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c
)*d*e + (a*b^2 - 4*a^2*c)*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*
c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 -
4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2 - (b^
3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2))*log((((b^2*c - 4*a*c^2)*d^3 - (
b^3 - 4*a*b*c)*d^2*e + (a*b^2 - 4*a^2*c)*d*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c
^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*
e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*x^2 + 2*a*d^
2 - (b*d^2 - 4*a*d*e)*x^2 + 2*sqrt(1/2)*((b^2 - 4*a*c)*d^2*x - ((b^3*c - 4
*a*b*c^2)*d^3 - (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e + 3*(a*b^3 - 4*a^2*b*c
)*d*e^2 - 2*(a^2*b^2 - 4*a^3*c)*e^3)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2
*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*
b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*x)*sqrt(e*x^2 + d)*sqrt
(-(b*d - 2*a*e + ((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4
*a^2*c)*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3
*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 +
 (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e +
 (a*b^2 - 4*a^2*c)*e^2)))/x^2) - 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e + ((b^2*
c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)*sqrt(d^2/(
(b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*...
```

3.389.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{x^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

input
```
integrate(x**2/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

3.389.7 Maxima [F]

$$\int \frac{x^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{x^2}{(cx^4+bx^2+a)\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)), x)
```

3.389.8 Giac [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.389.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{x^2}{\sqrt{e\,x^2+d}\,(c\,x^4+b\,x^2+a)}\,dx$$

input
```
int(x^2/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)
```

output
```
int(x^2/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)
```

3.390 $\int \frac{1}{\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx$

3.390.1 Optimal result

Integrand size = 26, antiderivative size = 243

$$\int \frac{1}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \frac{2c\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{2c\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

output
```
2*c*arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4
*a*c+b^2)^(1/2))^(1/2))/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))
)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-2*c*arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)
^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))/(-4*a*c+b^2)^
(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.390.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.14 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.74

$$\int \frac{1}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$
$$= -2e^{3/2}\text{RootSum}\Bigg[cd^4-4cd^3\#1+4bd^2e\#1+6cd^2\#1^2-8bde\#1^2+16ae^2\#1^2 -4cd\#1^3+4be\#1^3 +c\#1^4\&,\frac{\log\left(d+2ex^2-2\sqrt{e}x\sqrt{d+ex^2}-\#1\right)\#1}{-cd^3+bd^2e+3cd^2\#1-4bde\#1+8ae^2\#1-3cd\#1^2+3be\#1^2+c\#1^3}\&\Bigg]$$

input
```
Integrate[1/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

output
```
-2*e^(3/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 4*b*d^2*e*#1 + 6*c*d^2*#1^2 - 8*b*
d*e*#1^2 + 16*a*e^2*#1^2 - 4*c*d*#1^3 + 4*b*e*#1^3 + c*#1^4 & , (Log[d + 2
*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1)/(-(c*d^3) + b*d^2*e + 3*c*d
^2*#1 - 4*b*d*e*#1 + 8*a*e^2*#1 - 3*c*d*#1^2 + 3*b*e*#1^2 + c*#1^3) & ]
```

3.390.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1488, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

$$\Big\downarrow \text{1488}$$

$$\frac{2c\int \frac{1}{\left(2cx^2+b-\sqrt{b^2-4ac}\right)\sqrt{ex^2+d}}dx}{\sqrt{b^2-4ac}}-\frac{2c\int \frac{1}{\left(2cx^2+b+\sqrt{b^2-4ac}\right)\sqrt{ex^2+d}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{291}$$

$$\frac{2c\int \frac{1}{-\frac{\left(\left(b-\sqrt{b^2-4ac}\right)e-2cd\right)x^2}{ex^2+d}+b-\sqrt{b^2-4ac}}d\frac{x}{\sqrt{ex^2+d}}}{\sqrt{b^2-4ac}}-\frac{2c\int \frac{1}{-\frac{\left(\left(b+\sqrt{b^2-4ac}\right)e-2cd\right)x^2}{ex^2+d}+b+\sqrt{b^2-4ac}}d\frac{x}{\sqrt{ex^2+d}}}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{2c\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{2c\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$

input
```
Int[1/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

output
```
(2*c*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2
 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c
]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (2*c*ArcTan[(Sqrt[2*c*d - (b
 + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]
)/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^
2 - 4*a*c])*e])
```

3.390.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

3.390. $\int \frac{1}{\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx$

rule 1488
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/r)   Int[(d + e*x^2)^q/(b -
 r + 2*c*x^2), x], x] - Simp[2*(c/r)   Int[(d + e*x^2)^q/(b + r + 2*c*x^2),
 x], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

3.390.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.92

method	result
default	$-\frac{\sqrt{2}\left(\frac{\left(bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)\arctan\left(\frac{a\sqrt{e x^2+d}\,\sqrt{2}}{x\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}\right)}{\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}-\frac{\left(-bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)\operatorname{arctanh}\left(\frac{a\sqrt{e x^2+d}\,\sqrt{2}}{x\sqrt{\left(2ae-bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}\right)}{\sqrt{\left(2ae-bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}\right)}{2\sqrt{-d^2(4ac-b^2)}}$
pseudoelliptic	$-\frac{\sqrt{2}\left(\frac{\left(bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)\arctan\left(\frac{a\sqrt{e x^2+d}\,\sqrt{2}}{x\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}\right)}{\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}-\frac{\left(-bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)\operatorname{arctanh}\left(\frac{a\sqrt{e x^2+d}\,\sqrt{2}}{x\sqrt{\left(2ae-bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}\right)}{\sqrt{\left(2ae-bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}\right)}{2\sqrt{-d^2(4ac-b^2)}}$

input
```
int(1/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*2^(1/2)/(-d^2*(4*a*c-b^2))^(1/2)*((b*d+(-d^2*(4*a*c-b^2))^(1/2))/((-2
*a*e+b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2)*arctan(a/x*(e*x^2+d)^(1/2)*2^(
1/2)/((-2*a*e+b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2))-(-b*d+(-d^2*(4*a*c-b
^2))^(1/2))/((2*a*e-b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2)*arctanh(a/x*(e*
x^2+d)^(1/2)*2^(1/2)/((2*a*e-b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2)))
```

3.390.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4557 vs. 2(203) = 406.

Time = 6.17 (sec) , antiderivative size = 4557, normalized size of antiderivative = 18.75

$$\int \frac{1}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(1/2)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e - ((a*b^2*c - 4*a^2*c^2)*d^2
- (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)*sqrt((c^2*d^2 - 2*b*c
*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^
2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^
4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))/((a*b^2*c - 4*a^2*c^2)*d^2 - (a*
b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2))*log(-(2*a*c^2*d^2 - 2*a*b
*c*d*e + ((a*b^2*c^2 - 4*a^2*c^3)*d^3 - (a*b^3*c - 4*a^2*b*c^2)*d^2*e + (a
^2*b^2*c - 4*a^3*c^2)*d*e^2)*x^2*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^
2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4
- 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*
b^2 - 4*a^5*c)*e^4)) - (b*c^2*d^2 + 4*a*b*c*e^2 - (b^2*c + 4*a*c^2)*d*e)*x
^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((2*(a^2*b^2*c^2 - 4*a^3*c^3)*d^3 - 3*(a^
2*b^3*c - 4*a^3*b*c^2)*d^2*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d*e^2 -
 (a^3*b^3 - 4*a^4*b*c)*e^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b
^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2
*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2
 - 4*a^5*c)*e^4)) - ((a*b^2*c - 4*a^2*c^2)*d*e - (a*b^3 - 4*a^2*b*c)*e^2)*
x)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e - ((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 -
4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*
e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e...
```

3.390.6 Sympy [F]

$$\int \frac{1}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{1}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

input
```
integrate(1/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

output
```
Integral(1/(sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

3.390.7 Maxima [F]

$$\int \frac{1}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{1}{(cx^4+bx^2+a)\sqrt{ex^2+d}}\,dx$$

input
```
integrate(1/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)), x)
```

3.390.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.390.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{1}{\sqrt{e\,x^2+d}\,(c\,x^4+b\,x^2+a)}\,dx$$

input
```
int(1/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)
```

output
```
int(1/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)
```

3.391 $\int \frac{1}{x^2\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx$

3.391.1 Optimal result

Integrand size = 29, antiderivative size = 280

$$\int \frac{1}{x^2\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = -\frac{\sqrt{d+ex^2}}{adx} - \frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

output
```
-(e*x^2+d)^(1/2)/a/d/x-c*arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(
e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(1+b/(-4*a*c+b^2)^(1/2))/a/(2
*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-c*arctan
(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^
(1/2))^(1/2))*(1-b/(-4*a*c+b^2)^(1/2))/a/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c
*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.391.2 Mathematica [A] (verified)

Time = 10.80 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^2\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = -\frac{\frac{\sqrt{d+ex^2}}{dx}+\frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-be+\sqrt{b^2-4ac}e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}}+\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}}{a}$$

input
```
Integrate[1/(x^2*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

output
```
-((Sqrt[d + e*x^2]/(d*x) + (c*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d
 - b*e + Sqrt[b^2 - 4*a*c]*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x
^2])])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*
e]) + (c*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*
a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b + Sqrt
[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/a)
```

3.391.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1626, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

$$\Big\downarrow \; 1626$$

$$\int \left(\frac{-b-cx^2}{a\sqrt{d+ex^2}\,(a+bx^2+cx^4)}+\frac{1}{ax^2\sqrt{d+ex^2}}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\sqrt{d+ex^2}}{adx}$$

input
```
Int[1/(x^2*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

output
```
-(Sqrt[d + e*x^2]/(a*d*x)) - (c*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c
*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e
*x^2])])/(a*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c
])*e]) - (c*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a*Sqrt[b +
 Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

3.391.3.1 Defintions of rubi rules used

rule 1626
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.391.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.95

method	result
default	$-\frac{\sqrt{e x^2+d}}{a d x}+\frac{\sqrt{2}\left(\frac{\left(-2acd+b^2d+\sqrt{-d^2\left(4ac-b^2\right)}\,b\right)\arctan\left(\frac{a\sqrt{e x^2+d}\,\sqrt{2}}{x\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}\right)}{\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}-\frac{\left(2acd-b^2d+\sqrt{-d^2\left(4ac-b^2\right)}\cdots\right.}{\sqrt{\left(2a\cdots\right.}}\right.}{2a\sqrt{-d^2(4ac-b^2)}}$
risch	$-\frac{\sqrt{e x^2+d}}{a d x}+\frac{\sqrt{2}\left(\frac{\left(-2acd+b^2d+\sqrt{-d^2\left(4ac-b^2\right)}\,b\right)\arctan\left(\frac{a\sqrt{e x^2+d}\,\sqrt{2}}{x\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}\right)}{\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}-\frac{\left(2acd-b^2d+\sqrt{-d^2\left(4ac-b^2\right)}\cdots\right.}{\sqrt{\left(2a\cdots\right.}}\right.}{2a\sqrt{-d^2(4ac-b^2)}}$
pseudoelliptic	$-\frac{\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\,\sqrt{2}\,dx\left(\frac{\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\,b}{2}+d\left(ac-\frac{b^2}{2}\right)\right)\operatorname{arctanh}\left(\frac{a\sqrt{e x^2+d}\,\sqrt{2}}{x\sqrt{\left(2ae-bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}}\right)\cdots}{\sqrt{\left(2ae-\cdots\right.}}$

input
```
int(1/x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(e*x^2+d)^(1/2)/a/d/x+1/2*2^(1/2)/a/(-d^2*(4*a*c-b^2))^(1/2)*((-2*a*c*d+b
^2*d+(-d^2*(4*a*c-b^2))^(1/2)*b)/((-2*a*e+b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)
^(1/2)*arctan(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((-2*a*e+b*d+(-d^2*(4*a*c-b^2))^
(1/2))*a)^(1/2))-(2*a*c*d-b^2*d+(-d^2*(4*a*c-b^2))^(1/2)*b)/((2*a*e-b*d+(-
d^2*(4*a*c-b^2))^(1/2))*a)^(1/2)*arctanh(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((2*a
*e-b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2)))
```

3.391.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6431 vs. 2(236) = 472.

Time = 12.66 (sec) , antiderivative size = 6431, normalized size of antiderivative = 22.97

$$\int \frac{1}{x^2\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.391.6 Sympy [F]

$$\int \frac{1}{x^2\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{1}{x^2\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

input
```
integrate(1/x**2/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

3.391.7 Maxima [F]

$$\int \frac{1}{x^2\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{1}{(cx^4+bx^2+a)\sqrt{ex^2+d}\,x^2}\,dx$$

input
```
integrate(1/x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)*x^2), x)
```

3.391.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.391.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{1}{x^2\,\sqrt{e\,x^2+d}\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

input
```
int(1/(x^2*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)
```

output
```
int(1/(x^2*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)
```

3.392 $\int \frac{1}{x^4\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx$

3.392.1 Optimal result

Integrand size = 29, antiderivative size = 341

$$\int \frac{1}{x^4\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = -\frac{\sqrt{d+ex^2}}{3adx^3}+\frac{b\sqrt{d+ex^2}}{a^2dx}+\frac{2e\sqrt{d+ex^2}}{3ad^2x} + \frac{c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} + \frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^2\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

output
```
-1/3*(e*x^2+d)^(1/2)/a/d/x^3+b*(e*x^2+d)^(1/2)/a^2/d/x+2/3*e*(e*x^2+d)^(1/
2)/a/d^2/x+c*arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/
2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a^2/(
2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+c*arcta
n(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)
^(1/2))^(1/2))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/a^2/(b+(-4*a*c+b^2)^(1/2
))^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.392.2 Mathematica [A] (verified)

Time = 10.59 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^4\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$
$$= \frac{\frac{3b\sqrt{d+ex^2}}{dx}-\frac{a(d-2ex^2)\sqrt{d+ex^2}}{d^2x^3}+\frac{3c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-be+\sqrt{b^2-4ac}e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}}+\frac{3c\left(b+\frac{-b^2+2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)ex}}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right.}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}}{3a^2}$$

input
```
Integrate[1/(x^4*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

output
```
((3*b*Sqrt[d + e*x^2])/(d*x) - (a*(d - 2*e*x^2)*Sqrt[d + e*x^2])/(d^2*x^3)
 + (3*c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - b*e + S
qrt[b^2 - 4*a*c]*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]])/(Sq
rt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) + (3*c
*(b + (-b^2 + 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2
 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]])/(Sqrt[b +
 Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/(3*a^2)
```

3.392.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1626, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$
$$\Big\downarrow \text{1626}$$
$$\int \left(\frac{-ac+b^2+bcx^2}{a^2\sqrt{d+ex^2}\,(a+bx^2+cx^4)}-\frac{b}{a^2x^2\sqrt{d+ex^2}}+\frac{1}{ax^4\sqrt{d+ex^2}}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{c\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{b\sqrt{d+ex^2}}{a^2dx}+\frac{2e\sqrt{d+ex^2}}{3ad^2x}-\frac{\sqrt{d+ex^2}}{3adx^3}$$

input
```
Int[1/(x^4*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

output
```
-1/3*Sqrt[d + e*x^2]/(a*d*x^3) + (b*Sqrt[d + e*x^2])/(a^2*d*x) + (2*e*Sqrt
[d + e*x^2])/(3*a*d^2*x) + (c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*
Sqrt[d + e*x^2])])/(a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt
[b^2 - 4*a*c])*e]) + (c*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d
 + e*x^2])])/(a^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e])
```

3.392.3.1 Defintions of rubi rules used

rule 1626
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.392.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.97

method	result
risch	$-\frac{\sqrt{e\,x^2+d}\,(-2ae\,x^2-3bd\,x^2+da)}{3d^2a^2x^3}+\frac{\sqrt{2}\left(\frac{\left(3abcd-b^3d+\sqrt{-d^2\left(4ac-b^2\right)}\,ac-\sqrt{-d^2\left(4ac-b^2\right)}\,b^2\right)\arctan\left(\frac{a\sqrt{e\,x^2+d}\,\cdots}{x\sqrt{\left(-2ae+bd+\sqrt{-d^2\cdots}\right.}}\right.}{\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}\cdots\right.}{\cdots}$
default	$\frac{-\frac{\sqrt{e\,x^2+d}}{3d\,x^3}+\frac{2e\sqrt{e\,x^2+d}}{3d^2x}}{a}+\frac{b\sqrt{e\,x^2+d}}{a^2dx}+\frac{\sqrt{2}\left(\frac{\left(3abcd-b^3d+\sqrt{-d^2\left(4ac-b^2\right)}\,ac-\sqrt{-d^2\left(4ac-b^2\right)}\,b^2\right)\arctan\left(\frac{a\sqrt{e\,x^2\cdots}}{x\sqrt{\left(-2ae+bd+\sqrt{\cdots}\right.}}\right.}{\sqrt{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}\cdots\right.}{\cdots}$
pseudoelliptic	$\frac{3\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\,\sqrt{2}\,d^2\left(\left(ac-b^2\right)\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}+\left(-3abc+b^3\right)d\right)x^3\,\mathrm{arctanh}\left(\frac{a\sqrt{e\,x^2+d}\,\sqrt{2}}{x\sqrt{\left(2ae-bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\cdots\right.}}\right.\cdots}{2}$ $-\frac{\cdots}{\cdots}$

input

```
int(1/x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-1/3*(e*x^2+d)^(1/2)*(-2*a*e*x^2-3*b*d*x^2+a*d)/d^2/a^2/x^3+1/2/a^2*2^(1/2
)/(-d^2*(4*a*c-b^2))^(1/2)*((3*a*b*c*d-b^3*d+(-d^2*(4*a*c-b^2))^(1/2)*a*c-
(-d^2*(4*a*c-b^2))^(1/2)*b^2)/((-2*a*e+b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1
/2)*arctan(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((-2*a*e+b*d+(-d^2*(4*a*c-b^2))^(1/
2))*a)^(1/2))-(-3*a*b*c*d+b^3*d+(-d^2*(4*a*c-b^2))^(1/2)*a*c-(-d^2*(4*a*c-
b^2))^(1/2)*b^2)/((2*a*e-b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2)*arctanh(a/
x*(e*x^2+d)^(1/2)*2^(1/2)/((2*a*e-b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2)))
```

3.392.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8187 vs. $2(291) = 582$.

Time = 62.57 (sec) , antiderivative size = 8187, normalized size of antiderivative = 24.01

$$\int \frac{1}{x^4\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.392.6 Sympy [F]

$$\int \frac{1}{x^4\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{1}{x^4\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

input
```
integrate(1/x**4/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

output
```
Integral(1/(x**4*sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

3.392.7 Maxima [F]

$$\int \frac{1}{x^4\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{1}{(cx^4+bx^2+a)\sqrt{ex^2+d}x^4}\,dx$$

input
```
integrate(1/x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)*x^4), x)
```

3.392.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.392.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx = \int \frac{1}{x^4\,\sqrt{e\,x^2+d}\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

input
```
int(1/(x^4*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)
```

output
```
int(1/(x^4*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)
```

3.393 $\int \frac{1}{x^6\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx$

3.393.1 Optimal result

Integrand size = 29, antiderivative size = 443

$$\int \frac{1}{x^6\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = -\frac{\sqrt{d+ex^2}}{5adx^5}+\frac{b\sqrt{d+ex^2}}{3a^2dx^3}+\frac{4e\sqrt{d+ex^2}}{15ad^2x^3} - \frac{(b^2-ac)\sqrt{d+ex^2}}{a^3dx}-\frac{2be\sqrt{d+ex^2}}{3a^2d^2x}-\frac{8e^2\sqrt{d+ex^2}}{15ad^3x} - \frac{c\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^3\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{c\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^3\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

output
```
-1/5*(e*x^2+d)^(1/2)/a/d/x^5+1/3*b*(e*x^2+d)^(1/2)/a^2/d/x^3+4/15*e*(e*x^2
+d)^(1/2)/a/d^2/x^3-(-a*c+b^2)*(e*x^2+d)^(1/2)/a^3/d/x-2/3*b*e*(e*x^2+d)^(
1/2)/a^2/d^2/x-8/15*e^2*(e*x^2+d)^(1/2)/a/d^3/x-c*arctan(x*(2*c*d-e*(b-(-4
*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b^2
-a*c+b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a^3/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)
))^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-c*arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^
(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2-a*c-b*(-3
*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a^3/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-e*(b
+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.393.2 Mathematica [A] (verified)

Time = 11.23 (sec) , antiderivative size = 383, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^6\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx =$$

$$-\frac{\frac{15(b^2-ac)\sqrt{d+ex^2}}{dx}-\frac{5ab(d-2ex^2)\sqrt{d+ex^2}}{d^2x^3}+\frac{a^2\sqrt{d+ex^2}(3d^2-4dex^2+8e^2x^4)}{d^3x^5}+\frac{15c\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-be+\sqrt{b^2-4ac}}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+e}}\right.}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}}}{15a^3}$$

input
```
Integrate[1/(x^6*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

output
```
-1/15*((15*(b^2 - a*c)*Sqrt[d + e*x^2])/(d*x) - (5*a*b*(d - 2*e*x^2)*Sqrt[
d + e*x^2])/(d^2*x^3) + (a^2*Sqrt[d + e*x^2]*(3*d^2 - 4*d*e*x^2 + 8*e^2*x^
4))/(d^3*x^5) + (15*c*(b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*Ar
cTan[(Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*
c]]*Sqrt[d + e*x^2])])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d + (-b + Sqr
t[b^2 - 4*a*c])*e]) + (15*c*(b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2
- 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b
+ Sqrt[b^2 - 4*a*c])*e]))/a^3
```

3.393.3 Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1626, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

$$\Big\downarrow \text{1626}$$

$$\int \left(\frac{b^2-ac}{a^3x^2\sqrt{d+ex^2}}+\frac{-cx^2(b^2-ac)-b(b^2-2ac)}{a^3\sqrt{d+ex^2}\,(a+bx^2+cx^4)}-\frac{b}{a^2x^4\sqrt{d+ex^2}}+\frac{1}{ax^6\sqrt{d+ex^2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^3\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{c\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a^3\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{(b^2-ac)\sqrt{d+ex^2}}{a^3dx}-\frac{2be\sqrt{d+ex^2}}{3a^2d^2x}+\frac{b\sqrt{d+ex^2}}{3a^2dx^3}-\frac{8e^2\sqrt{d+ex^2}}{15ad^3x}+\frac{4e\sqrt{d+ex^2}}{15ad^2x^3}-\frac{\sqrt{d+ex^2}}{5adx^5}$$

input
```
Int[1/(x^6*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

output
```
-1/5*Sqrt[d + e*x^2]/(a*d*x^5) + (b*Sqrt[d + e*x^2])/(3*a^2*d*x^3) + (4*e*
Sqrt[d + e*x^2])/(15*a*d^2*x^3) - ((b^2 - a*c)*Sqrt[d + e*x^2])/(a^3*d*x)
- (2*b*e*Sqrt[d + e*x^2])/(3*a^2*d^2*x) - (8*e^2*Sqrt[d + e*x^2])/(15*a*d^
3*x) - (c*(b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d +
 e*x^2])])/(a^3*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4
*a*c])*e]) - (c*(b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sq
rt[d + e*x^2])])/(a^3*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e])
```

3.393.3.1 Defintions of rubi rules used

rule 1626
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.393. $\int \frac{1}{x^6\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx$

3.393.4 Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 410, normalized size of antiderivative = 0.93

method	result
risch	$-\frac{\sqrt{e\,x^2+d}\,(8a^2e^2x^4+10abde\,x^4-15ac\,d^2x^4+15b^2d^2x^4-4a^2de\,x^2-5ab\,d^2x^2+3a^2d^2)}{15d^3a^3x^5}-\frac{\sqrt{2}\left(\frac{\left(-2a^2c^2d+4a\,b^2cd-d\,b^4+2\sqrt{\cdots}\right.}{\cdots}\right.}{\cdots}$
default	$\frac{-\frac{\sqrt{e\,x^2+d}}{5d\,x^5}-\frac{4e\left(-\frac{\sqrt{e\,x^2+d}}{3d\,x^3}+\frac{2e\sqrt{e\,x^2+d}}{3d^2x}\right)}{5d}}{a}-\frac{b\left(-\frac{\sqrt{e\,x^2+d}}{3d\,x^3}+\frac{2e\sqrt{e\,x^2+d}}{3d^2x}\right)}{a^2}-\frac{(-ac+b^2)\sqrt{e\,x^2+d}}{a^3dx}-\frac{\sqrt{2}\left(\frac{\left(-2a^2c^2d+4a\,b^2\cdots\right.}{\cdots}\right.}{\cdots}$
pseudoelliptic	$-\frac{-5\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\left(\left(abc-\frac{1}{2}b^3\right)\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}+d\left(-2a\,b^2c+\frac{1}{2}b^4+a^2c^2\right)\right)\sqrt{2}\,d^3x^5\,\mathrm{arctanh}\left(\frac{\cdots}{x\sqrt{(2\cdots}}\right.}{\cdots}$

input
```
int(1/x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(e*x^2+d)^(1/2)*(8*a^2*e^2*x^4+10*a*b*d*e*x^4-15*a*c*d^2*x^4+15*b^2*
d^2*x^4-4*a^2*d*e*x^2-5*a*b*d^2*x^2+3*a^2*d^2)/d^3/a^3/x^5-1/2/a^3*2^(1/2)
/(-d^2*(4*a*c-b^2))^(1/2)*((-2*a^2*c^2*d+4*a*b^2*c*d-d*b^4+2*(-d^2*(4*a*c-
b^2))^(1/2)*a*b*c-(-d^2*(4*a*c-b^2))^(1/2)*b^3)/((-2*a*e+b*d+(-d^2*(4*a*c-
b^2))^(1/2))*a)^(1/2)*arctan(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((-2*a*e+b*d+(-d^
2*(4*a*c-b^2))^(1/2))*a)^(1/2))-(2*a^2*c^2*d-4*a*b^2*c*d+d*b^4+2*(-d^2*(4*
a*c-b^2))^(1/2)*a*b*c-(-d^2*(4*a*c-b^2))^(1/2)*b^3)/((2*a*e-b*d+(-d^2*(4*a
*c-b^2))^(1/2))*a)^(1/2)*arctanh(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((2*a*e-b*d+(
-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2)))
```

3.393.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9998 vs. $2(381) = 762$.

Time = 165.00 (sec) , antiderivative size = 9998, normalized size of antiderivative = 22.57

$$\int \frac{1}{x^6\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.393.6 Sympy [F]

$$\int \frac{1}{x^6\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{1}{x^6\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

input
```
integrate(1/x**6/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

output
```
Integral(1/(x**6*sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

3.393.7 Maxima [F]

$$\int \frac{1}{x^6\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{1}{(cx^4+bx^2+a)\sqrt{ex^2+d}\,x^6}\,dx$$

input
```
integrate(1/x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)*x^6), x)
```

3.393.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{x^6\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.393.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{1}{x^6\,\sqrt{e\,x^2+d}\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

input
```
int(1/(x^6*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)
```

output
```
int(1/(x^6*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)
```

3.394 $\int \frac{x^6}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx$

3.394.1 Optimal result

Integrand size = 29, antiderivative size = 350

$$\int \frac{x^6}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = -\frac{d^2x}{e(cd^2-bde+ae^2)\sqrt{d+ex^2}} + \frac{2\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)^{3/2}} + \frac{2\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b+\sqrt{b^2-4ac}}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{ce^{3/2}}$$

output
```
arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/c/e^(3/2)-d^2*x/e/(a*e^2-b*d*e+c*d^2)/(
e*x^2+d)^(1/2)+2*arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)
^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b^2-a*c-b*(-3*a*c+b^2)/(-4*a*c+b^2)^
(1/2))/c/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(3/2)/(b-(-4*a*c+b^2)^(1/2))^(1/
2)+2*arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-
4*a*c+b^2)^(1/2))^(1/2))*(b^2-a*c+b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c/(2*
c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(3/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.394.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 10546 vs. 2(350) = 700.

Time = 21.27 (sec) , antiderivative size = 10546, normalized size of antiderivative = 30.13

$$\int \frac{x^6}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \text{Result too large to show}$$

input
```
Integrate[x^6/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

output
```
Result too large to show
```

3.394.3 Rubi [A] (verified)

Time = 2.33 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.30, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1620, 252, 224, 219, 2246, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx$$

$$\Big\downarrow \; 1620$$

$$\frac{d^2\int \frac{x^2}{(ex^2+d)^{3/2}}dx}{ae^2-bde+cd^2}-\frac{\int \frac{x^2((bd-ae)x^2+ad)}{\sqrt{ex^2+d}(cx^4+bx^2+a)}dx}{ae^2-bde+cd^2}$$

$$\Big\downarrow \; 252$$

$$\frac{d^2\left(\frac{\int \frac{1}{\sqrt{ex^2+d}}dx}{e}-\frac{x}{e\sqrt{d+ex^2}}\right)}{ae^2-bde+cd^2}-\frac{\int \frac{x^2((bd-ae)x^2+ad)}{\sqrt{ex^2+d}(cx^4+bx^2+a)}dx}{ae^2-bde+cd^2}$$

$$\Big\downarrow \; 224$$

$$\frac{d^2\left(\frac{\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{e}-\frac{x}{e\sqrt{d+ex^2}}\right)}{ae^2-bde+cd^2}-\frac{\int \frac{x^2((bd-ae)x^2+ad)}{\sqrt{ex^2+d}(cx^4+bx^2+a)}dx}{ae^2-bde+cd^2}$$

$$\Big\downarrow \; 219$$

$$\frac{d^2\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{3/2}}-\frac{x}{e\sqrt{d+ex^2}}\right)}{ae^2-bde+cd^2}-\frac{\int\frac{x^2((bd-ae)x^2+ad)}{\sqrt{ex^2+d}(cx^4+bx^2+a)}dx}{ae^2-bde+cd^2}$$

$$\Big\downarrow \text{2246}$$

$$\frac{d^2\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{3/2}}-\frac{x}{e\sqrt{d+ex^2}}\right)}{ae^2-bde+cd^2}-\frac{\int\left(\frac{bd-ae}{c\sqrt{ex^2+d}}-\frac{(db^2-aeb-acd)x^2+a(bd-ae)}{c\sqrt{ex^2+d}(cx^4+bx^2+a)}\right)dx}{ae^2-bde+cd^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^2\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{3/2}}-\frac{x}{e\sqrt{d+ex^2}}\right)}{ae^2-bde+cd^2}-$$

$$\frac{-\frac{\left(-\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)\arctan\left(\frac{x\sqrt{2cd-e(\cdots}}{\sqrt{\sqrt{b^2-4a\cdots}}}\right.}{c\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}{ae^2-bde+cd^2}$$

input
```
Int[x^6/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

output
```
(d^2*(-(x/(e*Sqrt[d + e*x^2])) + ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/e^(3
/2)))/(c*d^2 - b*d*e + a*e^2) - (-(((b^2*d - a*c*d - a*b*e - (b^3*d - 3*a*
b*c*d - a*b^2*e + 2*a^2*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b -
Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(
c*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e])) -
((b^2*d - a*c*d - a*b*e + (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)/Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b +
Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[
2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]) + ((b*d - a*e)*ArcTanh[(Sqrt[e]*x)/Sqr
t[d + e*x^2]])/(c*Sqrt[e]))/(c*d^2 - b*d*e + a*e^2)
```

3.394. $\int \frac{x^6}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx$

3.394.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 1620
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Simp[d^2*(f^4/(c*d^2 - b*d*e + a*e^2))   Int[(f
*x)^(m - 4)*(d + e*x^2)^q, x], x] - Simp[f^4/(c*d^2 - b*d*e + a*e^2)   Int[
(f*x)^(m - 4)*(d + e*x^2)^(q + 1)*(Simp[a*d + (b*d - a*e)*x^2, x]/(a + b*x^
2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0]
&&  !IntegerQ[q] && LtQ[q, -1] && GtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2246
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(f*x)^m*(d +
e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x
] && PolyQ[Px, x] && IntegerQ[p]
```

3.394. $\int \frac{x^6}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx$

3.394.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 502, normalized size of antiderivative = 1.43

method	result
pseudoelliptic	$-\frac{\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\,a\sqrt{2}\,\sqrt{e\,x^2+d}\left(\left(\frac{\left(-e^{\frac{3}{2}}bd+a\,e^{\frac{5}{2}}\right)\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}}{2}+d\left(d\left(ac-\frac{b^2}{2}\right)e^{\frac{3}{2}}+\frac{ab\,e^{\frac{5}{2}}}{2}\right)\right)\operatorname{arctan}\cdots}{\cdots}$
default	$\frac{-\frac{x}{e\sqrt{e\,x^2+d}}+\frac{\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{e^{\frac{3}{2}}}}{c}-\frac{bx}{c^2d\sqrt{e\,x^2+d}}-\frac{\left((ae-bd)\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}+\left(\left(2ac-b^2\right)d+abe\right)d\right)a\sqrt{2}\,cd\sqrt{\left(-2ae+b\cdots\right.}}{\cdots}$

input
```
int(x^6/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/(-4*d^2*(a*c-1/4*b^2))^(1/2)/e^(3/2)/((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2)
)^(1/2))*a)^(1/2)/((2*a*e-b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*(((-2
*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*a*2^(1/2)*(e*x^2+d)^(1/2)*
(1/2*(-e^(3/2)*b*d+a*e^(5/2))*(-4*d^2*(a*c-1/4*b^2))^(1/2)+d*(d*(a*c-1/2*b
^2)*e^(3/2)+1/2*a*b*e^(5/2)))*arctanh(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((2*a*e-
b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2))+((2*a*e-b*d+(-4*d^2*(a*c-1/4*b
^2))^(1/2))*a)^(1/2)*(a*2^(1/2)*(1/2*(e^(3/2)*b*d-a*e^(5/2))*(-4*d^2*(a*c-
1/4*b^2))^(1/2)+d*(d*(a*c-1/2*b^2)*e^(3/2)+1/2*a*b*e^(5/2)))*(e*x^2+d)^(1/
2)*arctan(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^
(1/2))*a)^(1/2))-((e*x^2+d)^(1/2)*(a*e^2-b*d*e+c*d^2)*arctanh((e*x^2+d)^(1
/2)/x/e^(1/2))-e^(1/2)*c*d^2*x)*((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))
*a)^(1/2)*(-4*d^2*(a*c-1/4*b^2))^(1/2)))/(e*x^2+d)^(1/2)/(a*e^2-b*d*e+c*d^
2)/c
```

3.394.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^6}{(d+ex^2)^{3/2}\,(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x^6/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.394.6 Sympy [F]

$$\int \frac{x^6}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \int \frac{x^6}{(d+ex^2)^{\frac{3}{2}}(a+bx^2+cx^4)}\,dx$$

input
```
integrate(x**6/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

output
```
Integral(x**6/((d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)
```

3.394.7 Maxima [F]

$$\int \frac{x^6}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \int \frac{x^6}{(cx^4+bx^2+a)(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^6/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(x^6/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)), x)
```

3.394.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^6}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^6/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.394.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \int \frac{x^6}{(e\,x^2+d)^{3/2}\,(c\,x^4+b\,x^2+a)}\,dx$$

input
```
int(x^6/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x)
```

output
```
int(x^6/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)), x)
```

3.395 $\int \frac{x^4}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx$

3.395.1 Optimal result

Integrand size = 29, antiderivative size = 360

$$\int \frac{x^4}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \frac{dx}{(cd^2-bde+ae^2)\sqrt{d+ex^2}} - \frac{\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,(cd^2-bde+ae^2)} - \frac{\left(bd-ae+\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,(cd^2-bde+ae^2)}$$

output
```
d*x/(a*e^2-b*d*e+c*d^2)/(e*x^2+d)^(1/2)-arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^
(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b*d-a*e+(a*b*
e+2*a*c*d-b^2*d)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)/(2*c*d-e*(b-(-4*a
*c+b^2)^(1/2)))^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-arctan(x*(2*c*d-e*(b+(-
4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b*
d-a*e+(-a*b*e-2*a*c*d+b^2*d)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)/(b+(-
4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.395.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.69 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.25

$$\int \frac{x^4}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \frac{\frac{4dx}{\sqrt{d+ex^2}} - d\text{RootSum}\Big[ae^4 + 4bde^2\#1^2 - 4ae^3\#1^2 + 16cd^2\#1^4 - 8bde\#$$

input
```
Integrate[x^4/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

output
```
((4*d*x)/Sqrt[d + e*x^2] - d*RootSum[a*e^4 + 4*b*d*e^2*#1^2 - 4*a*e^3*#1^2
 + 16*c*d^2*#1^4 - 8*b*d*e*#1^4 + 6*a*e^2*#1^4 + 4*b*d*#1^6 - 4*a*e*#1^6 +
 a*#1^8 & , (-(a*e^3*Log[x]) + a*e^3*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1
] - 4*b*d*e*Log[x]*#1^2 + 7*a*e^2*Log[x]*#1^2 + 4*b*d*e*Log[-Sqrt[d] + Sqr
t[d + e*x^2] - x*#1]*#1^2 - 7*a*e^2*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]
*#1^2 + 4*b*d*Log[x]*#1^4 - 7*a*e*Log[x]*#1^4 - 4*b*d*Log[-Sqrt[d] + Sqrt[
d + e*x^2] - x*#1]*#1^4 + 7*a*e*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^
4 + a*Log[x]*#1^6 - a*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1]*#1^6)/(b*d*e^
2*#1 - a*e^3*#1 + 8*c*d^2*#1^3 - 4*b*d*e*#1^3 + 3*a*e^2*#1^3 + 3*b*d*#1^5
- 3*a*e*#1^5 + a*#1^7) & ])/(4*c*d^2 - 4*b*d*e + 4*a*e^2)
```

3.395.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1620, 208, 2256, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}dx$$

$$\Big\downarrow \text{1620}$$

$$\frac{d^2\int \frac{1}{(ex^2+d)^{3/2}}dx}{ae^2-bde+cd^2} - \frac{\int \frac{(bd-ae)x^2+ad}{\sqrt{ex^2+d}(cx^4+bx^2+a)}dx}{ae^2-bde+cd^2}$$

$$\Big\downarrow \text{208}$$

$$\frac{dx}{\sqrt{d+ex^2}\,(ae^2-bde+cd^2)}-\frac{\int\frac{(bd-ae)x^2+ad}{\sqrt{ex^2+d}(cx^4+bx^2+a)}dx}{ae^2-bde+cd^2}$$

$\downarrow$ 2256

$$\frac{dx}{\sqrt{d+ex^2}\,(ae^2-bde+cd^2)}-\frac{\int\left(\frac{bd-ae-\frac{-db^2+aeb+2acd}{\sqrt{b^2-4ac}}}{\left(2cx^2+b+\sqrt{b^2-4ac}\right)\sqrt{ex^2+d}}+\frac{bd-ae+\frac{-db^2+aeb+2acd}{\sqrt{b^2-4ac}}}{\left(2cx^2+b-\sqrt{b^2-4ac}\right)\sqrt{ex^2+d}}\right)dx}{ae^2-bde+cd^2}$$

$\downarrow$ 2009

$$\frac{dx}{\sqrt{d+ex^2}\,(ae^2-bde+cd^2)}-\frac{\frac{\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}{ae^2-bde+cd^2}$$

input
```
Int[x^4/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

output
```
(d*x)/((c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x^2]) - (((b*d - a*e - (b^2*d -
2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4
*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b - Sqr
t[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + ((b*d - a*e + (
b^2*d - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt
[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt
[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/(c*d^2 -
 b*d*e + a*e^2)
```

3.395.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 1620
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Simp[d^2*(f^4/(c*d^2 - b*d*e + a*e^2))   Int[(f
*x)^(m - 4)*(d + e*x^2)^q, x], x] - Simp[f^4/(c*d^2 - b*d*e + a*e^2)   Int[
(f*x)^(m - 4)*(d + e*x^2)^(q + 1)*(Simp[a*d + (b*d - a*e)*x^2, x]/(a + b*x^
2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0]
&&  !IntegerQ[q] && LtQ[q, -1] && GtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2256
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

3.395.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.06

method	result
pseudoelliptic	$\frac{d\left(\sqrt{2}\,a\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\,\sqrt{e\,x^2+d}\left(ae-\frac{bd}{2}-\frac{\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}}{2}\right)\operatorname{arctanh}\left(\frac{a\sqrt{e\,x^2+d}\,\sqrt{2}}{x\sqrt{\left(2ae-bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)}}\right.\right.}{\sqrt{\left(-2ae+bd+\sqrt{-4d^2}\right.}}$
default	$\frac{x}{cd\sqrt{e\,x^2+d}}+\frac{\sqrt{2}\,ac\,d^2\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\,\sqrt{e\,x^2+d}\left(ae-\frac{bd}{2}-\frac{\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}}{2}\right)\operatorname{arctanh}\left(\frac{a\sqrt{e}}{x\sqrt{\left(2ae-bd+\right.}}\right.}{c\sqrt{}}$

input
```
int(x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)/(e*x^2+d)^(1/2)/((2*
a*e-b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*d/(-4*d^2*(a*c-1/4*b^2))^(1
/2)*(2^(1/2)*a*((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*(e*x^2+
d)^(1/2)*(a*e-1/2*b*d-1/2*(-4*d^2*(a*c-1/4*b^2))^(1/2))*arctanh(a/x*(e*x^2
+d)^(1/2)*2^(1/2)/((2*a*e-b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2))+((2*
a*e-b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*(a*2^(1/2)*(e*x^2+d)^(1/2)*
(a*e-1/2*b*d+1/2*(-4*d^2*(a*c-1/4*b^2))^(1/2))*arctan(a/x*(e*x^2+d)^(1/2)*
2^(1/2)/((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2))+x*(-4*d^2*(a*
c-1/4*b^2))^(1/2)*((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)))/(a
*e^2-b*d*e+c*d^2)
```

3.395.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14462 vs. $2(318) = 636$.

Time = 156.12 (sec) , antiderivative size = 14462, normalized size of antiderivative = 40.17

$$\int \frac{x^4}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.395.6 Sympy [F]

$$\int \frac{x^4}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \int \frac{x^4}{(d+ex^2)^{\frac{3}{2}}(a+bx^2+cx^4)}\,dx$$

input
```
integrate(x**4/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

output
```
Integral(x**4/((d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)
```

3.395.7 Maxima [F]

$$\int \frac{x^4}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \int \frac{x^4}{(cx^4+bx^2+a)(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(x^4/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)), x)
```

3.395.8 Giac [F(-1)]

Timed out.

$$\int \frac{x^4}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
Timed out
```

3.395.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \int \frac{x^4}{(e\,x^2+d)^{3/2}\,(c\,x^4+b\,x^2+a)}\,dx$$

input
```
int(x^4/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x)
```

output
```
int(x^4/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)), x)
```

3.396 $\int \frac{x^2}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx$

3.396.1 Optimal result

Integrand size = 29, antiderivative size = 333

$$\int \frac{x^2}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = -\frac{ex}{(cd^2-bde+ae^2)\sqrt{d+ex^2}} + \frac{c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,(cd^2-bde+ae^2)} + \frac{c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,(cd^2-bde+ae^2)}$$

output

```
-e*x/(a*e^2-b*d*e+c*d^2)/(e*x^2+d)^(1/2)+c*arctan(x*(2*c*d-e*(b-(-4*a*c+b^
2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(d+(2*a*e-b
*d)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)
))^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+c*arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^
(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(d+(-2*a*e+b*d
)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*
c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.396.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.64 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.39

$$\int \frac{x^2}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx =$$

$$-\frac{\frac{4ex}{\sqrt{d+ex^2}} - \text{RootSum}\left[ae^4 + 4bde^2\#1^2 - 4ae^3\#1^2 + 16cd^2\#1^4 - 8bde\#1^4 + 6ae^2\#1^4 + 4bd\#1^6 - 4ae\#1^6 \right.}{}$$

input
```
Integrate[x^2/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

output
```
-(((4*e*x)/Sqrt[d + e*x^2] - RootSum[a*e^4 + 4*b*d*e^2*#1^2 - 4*a*e^3*#1^2
 + 16*c*d^2*#1^4 - 8*b*d*e*#1^4 + 6*a*e^2*#1^4 + 4*b*d*#1^6 - 4*a*e*#1^6 +
 a*#1^8 & , (-(a*e^4*Log[x]) + a*e^4*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#1
] - 4*c*d^2*e*Log[x]*#1^2 + 3*a*e^3*Log[x]*#1^2 + 4*c*d^2*e*Log[-Sqrt[d] +
 Sqrt[d + e*x^2] - x*#1]*#1^2 - 3*a*e^3*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x
*#1]*#1^2 + 4*c*d^2*Log[x]*#1^4 - 3*a*e^2*Log[x]*#1^4 - 4*c*d^2*Log[-Sqrt[
d] + Sqrt[d + e*x^2] - x*#1]*#1^4 + 3*a*e^2*Log[-Sqrt[d] + Sqrt[d + e*x^2]
 - x*#1]*#1^4 + a*e*Log[x]*#1^6 - a*e*Log[-Sqrt[d] + Sqrt[d + e*x^2] - x*#
1]*#1^6)/(b*d*e^2*#1 - a*e^3*#1 + 8*c*d^2*#1^3 - 4*b*d*e*#1^3 + 3*a*e^2*#1
^3 + 3*b*d*#1^5 - 3*a*e*#1^5 + a*#1^7) & ])/(4*c*d^2 - 4*b*d*e + 4*a*e^2))
```

3.396.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1622, 208, 2256, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}dx$$

$$\Big\downarrow \text{1622}$$

$$\frac{\int \frac{cdx^2+ae}{\sqrt{ex^2+d}(cx^4+bx^2+a)}dx}{ae^2-bde+cd^2} - \frac{de\int \frac{1}{(ex^2+d)^{3/2}}dx}{ae^2-bde+cd^2}$$

$$\Big\downarrow \text{208}$$

$$\frac{\int \frac{cdx^2+ae}{\sqrt{ex^2+d}(cx^4+bx^2+a)}dx}{ae^2-bde+cd^2}-\frac{ex}{\sqrt{d+ex^2}\,(ae^2-bde+cd^2)}$$

$$\Big\downarrow \text{2256}$$

$$\frac{\int \left(\frac{cd-\frac{c(2ae-bd)}{\sqrt{b^2-4ac}}}{\left(2cx^2+b+\sqrt{b^2-4ac}\right)\sqrt{ex^2+d}}+\frac{cd+\frac{c(2ae-bd)}{\sqrt{b^2-4ac}}}{\left(2cx^2+b-\sqrt{b^2-4ac}\right)\sqrt{ex^2+d}}\right)dx}{ae^2-bde+cd^2}-\frac{ex}{\sqrt{d+ex^2}\,(ae^2-bde+cd^2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}{ae^2-bde+cd^2}-\frac{ex}{\sqrt{d+ex^2}\,(ae^2-bde+cd^2)}$$

input
```
Int[x^2/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

output
```
-((e*x)/((c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x^2])) + ((c*(d - (b*d - 2*a*e
)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(S
qrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]
*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (c*(d + (b*d - 2*a*e)/Sqrt[b^2
 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sq
rt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*
d - (b + Sqrt[b^2 - 4*a*c])*e]))/(c*d^2 - b*d*e + a*e^2)
```

3.396.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

3.396. $\int \frac{x^2}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx$

rule 1622
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Simp[(-d)*e*(f^2/(c*d^2 - b*d*e + a*e^2))   Int
[(f*x)^(m - 2)*(d + e*x^2)^q, x], x] + Simp[f^2/(c*d^2 - b*d*e + a*e^2)   I
nt[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(Simp[a*e + c*d*x^2, x]/(a + b*x^2 + c
*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] &&  !
IntegerQ[q] && LtQ[q, -1] && GtQ[m, 1] && LeQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2256
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

3.396.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.18

method	result
default	$\frac{a\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\,\sqrt{2}\,\sqrt{e\,x^2+d}\left(-bde+2c\,d^2+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\,e\right)\operatorname{arctanh}\left(\frac{a\sqrt{e\,x^2+d}\,\sqrt{2}}{x\sqrt{\left(2ae-bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right.}\right.}}\right.}{\sqrt{\left(-2ae+bd+\sqrt{-4\right.}}}$
pseudoelliptic	$\frac{a\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\,\sqrt{2}\,\sqrt{e\,x^2+d}\left(-bde+2c\,d^2+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\,e\right)\operatorname{arctanh}\left(\frac{a\sqrt{e\,x^2+d}\,\sqrt{2}}{x\sqrt{\left(2ae-bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right.}\right.}}\right.}{\sqrt{\left(-2ae+bd+\sqrt{-4\right.}}}$

input
```
int(x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*(a*((-2*a*e+b*d+(-4*
d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*2^(1/2)*(e*x^2+d)^(1/2)*(-b*d*e+2*c*d^2
+(-4*d^2*(a*c-1/4*b^2))^(1/2)*e)*arctanh(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((2*a
*e-b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2))-((2*a*e-b*d+(-4*d^2*(a*c-1/
4*b^2))^(1/2))*a)^(1/2)*(a*2^(1/2)*(e*x^2+d)^(1/2)*(b*d*e-2*c*d^2+(-4*d^2*
(a*c-1/4*b^2))^(1/2)*e)*arctan(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((-2*a*e+b*d+(-
4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2))+2*(-4*d^2*(a*c-1/4*b^2))^(1/2)*((-2*
a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*e*x))/(e*x^2+d)^(1/2)/((2*a
*e-b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)/(-4*d^2*(a*c-1/4*b^2))^(1/2)
/(2*a*e^2-2*b*d*e+2*c*d^2)
```

3.396.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14146 vs. $2(291) = 582$.

Time = 111.97 (sec) , antiderivative size = 14146, normalized size of antiderivative = 42.48

$$\int \frac{x^2}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.396.6 Sympy [F]

$$\int \frac{x^2}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \int \frac{x^2}{(d+ex^2)^{\frac{3}{2}}(a+bx^2+cx^4)}\,dx$$

input
```
integrate(x**2/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

output
```
Integral(x**2/((d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)
```

3.396.7 Maxima [F]

$$\int \frac{x^2}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \int \frac{x^2}{(cx^4+bx^2+a)(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(x^2/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)), x)
```

3.396.8 Giac [F(-1)]

Timed out.

$$\int \frac{x^2}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
Timed out
```

3.396.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \int \frac{x^2}{(e\,x^2+d)^{3/2}\,(c\,x^4+b\,x^2+a)}\,dx$$

input
```
int(x^2/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x)
```

output
```
int(x^2/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)), x)
```

3.397 $\int \frac{1}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx$

3.397.1 Optimal result

Integrand size = 26, antiderivative size = 341

$$\int \frac{1}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \frac{e^2x}{d(cd^2-bde+ae^2)\sqrt{d+ex^2}} - \frac{c\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,(cd^2-bde+ae^2)} - \frac{c\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,(cd^2-bde+ae^2)}$$

output
```
e^2*x/d/(a*e^2-b*d*e+c*d^2)/(e*x^2+d)^(1/2)-c*arctan(x*(2*c*d-e*(b-(-4*a*c
+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(e+(b*e-
2*c*d)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1
/2)))^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-c*arctan(x*(2*c*d-e*(b+(-4*a*c+b^
2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(e+(-b*e+2*
c*d)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)/
(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.397.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 18.25 (sec) , antiderivative size = 2061, normalized size of antiderivative = 6.04

$$\int \frac{1}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

output
```
(2*c*x*(45*Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*
c])*e)*x^2*(d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))] + (30*
e*x^2*Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e
)*x^2*(d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))])/d - 45*Arc
Sin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 -
4*a*c] - 2*c*x^2)))]] - (30*e*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 -
 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/d - (45*(2*c*d
 + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2
- 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/(d*(-b + Sqrt
[b^2 - 4*a*c] - 2*c*x^2)) - (30*e*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^4
*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^
2 - 4*a*c] - 2*c*x^2)))]])/(d^2*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)) + 4*(-
(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2
*c*x^2))))^(5/2)*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt
[b^2 - 4*a*c] - 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*d + (-b +
Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))] + (4*e
*x^2*(-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a
*c] - 2*c*x^2))))^(5/2)*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b
 + Sqrt[b^2 - 4*a*c] - 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*d +
 (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)...
```

3.397.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1486, 208, 2256, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}dx$$
$$\Big\downarrow \text{1486}$$
$$\frac{e^2\int \frac{1}{(ex^2+d)^{3/2}}dx}{ae^2-bde+cd^2}+\frac{\int \frac{-cex^2+cd-be}{\sqrt{ex^2+d}(cx^4+bx^2+a)}dx}{ae^2-bde+cd^2}$$
$$\Big\downarrow \text{208}$$
$$\frac{\int \frac{-cex^2+cd-be}{\sqrt{ex^2+d}(cx^4+bx^2+a)}dx}{ae^2-bde+cd^2}+\frac{e^2x}{d\sqrt{d+ex^2}\,(ae^2-bde+cd^2)}$$
$$\Big\downarrow \text{2256}$$
$$\frac{\int \left(\frac{-ce-\frac{c(be-2cd)}{\sqrt{b^2-4ac}}}{\left(2cx^2+b-\sqrt{b^2-4ac}\right)\sqrt{ex^2+d}}+\frac{\frac{c(be-2cd)}{\sqrt{b^2-4ac}}-ce}{\left(2cx^2+b+\sqrt{b^2-4ac}\right)\sqrt{ex^2+d}}\right)dx}{ae^2-bde+cd^2}+\frac{e^2x}{d\sqrt{d+ex^2}\,(ae^2-bde+cd^2)}$$
$$\Big\downarrow \text{2009}$$
$$\frac{-\frac{c\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{c\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}{ae^2-bde+cd^2}+\frac{e^2x}{d\sqrt{d+ex^2}\,(ae^2-bde+cd^2)}$$

input
```
Int[1/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

output
```
(e^2*x)/(d*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x^2]) + (-((c*(e - (2*c*d -
b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)
/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b - Sqrt[b^2 - 4*a*
c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e])) - (c*(e + (2*c*d - b*e)/Sqrt
[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b
+ Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[
2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/(c*d^2 - b*d*e + a*e^2)
```

3.397. $\int \frac{1}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx$

3.397.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 1486
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Simp[e^2/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x^2)^q, x], x] + Simp[
1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x^2)^(q + 1)*((c*d - b*e - c*e*x^2)/
(a + b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[q] && LtQ[q, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2256
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

3.397.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.26

method	result
default	$-\frac{\left(\frac{(be-cd)\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}}{2}+\left(\frac{bcd}{2}+e\left(ac-\frac{b^2}{2}\right)\right)d\right)\sqrt{2}\,d\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\,\sqrt{e\,x^2+d}\,\operatorname{arctanh}\left(\frac{\cdots}{x\sqrt{\left(2ae-b\cdots\right.}}\right.\cdots}{\cdots}$
pseudoelliptic	$-\frac{\left(\frac{(be-cd)\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}}{2}+\left(\frac{bcd}{2}+e\left(ac-\frac{b^2}{2}\right)\right)d\right)\sqrt{2}\,d\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\,\sqrt{e\,x^2+d}\,\operatorname{arctanh}\left(\frac{\cdots}{x\sqrt{\left(2ae-b\cdots\right.}}\right.\cdots}{\cdots}$

input
```
int(1/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)/(e*x^2+d)^(1/2)/((2
*a*e-b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)/(-4*d^2*(a*c-1/4*b^2))^(1/
2)*((1/2*(b*e-c*d)*(-4*d^2*(a*c-1/4*b^2))^(1/2)+(1/2*b*c*d+e*(a*c-1/2*b^2)
)*d)*2^(1/2)*d*((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*(e*x^2+
d)^(1/2)*arctanh(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((2*a*e-b*d+(-4*d^2*(a*c-1/4*
b^2))^(1/2))*a)^(1/2))+(2^(1/2)*(1/2*(-b*e+c*d)*(-4*d^2*(a*c-1/4*b^2))^(1/
2)+(1/2*b*c*d+e*(a*c-1/2*b^2))*d)*d*(e*x^2+d)^(1/2)*arctan(a/x*(e*x^2+d)^(
1/2)*2^(1/2)/((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2))-e^2*x*(-
4*d^2*(a*c-1/4*b^2))^(1/2)*((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(
1/2))*((2*a*e-b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2))/(a*e^2-b*d*e+c*d
^2)/d
```

3.397.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17249 vs. 2(299) = 598.

Time = 217.47 (sec) , antiderivative size = 17249, normalized size of antiderivative = 50.58

$$\int \frac{1}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.397.6 Sympy [F]

$$\int \frac{1}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \int \frac{1}{(d+ex^2)^{\frac{3}{2}}(a+bx^2+cx^4)}\,dx$$

input
```
integrate(1/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

output
```
Integral(1/((d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)
```

3.397.7 Maxima [F]

$$\int \frac{1}{(d+ex^2)^{3/2}\,(a+bx^2+cx^4)}\,dx = \int \frac{1}{(cx^4+bx^2+a)(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)), x)
```

3.397.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^{3/2}\,(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
Timed out
```

3.397.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^{3/2}\,(a+bx^2+cx^4)}\,dx = \int \frac{1}{(e\,x^2+d)^{3/2}\,(c\,x^4+b\,x^2+a)}\,dx$$

input
```
int(1/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x)
```

output
```
int(1/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)), x)
```

3.398 $\int \frac{1}{x^2(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx$

3.398.1 Optimal result

Integrand size = 29, antiderivative size = 339

$$\int \frac{1}{x^2\left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)}\,dx = \frac{e(cd-be)x}{ad\left(cd^2+e(-bd+ae)\right)\sqrt{d+ex^2}} + \frac{-d-2ex^2}{ad^2x\sqrt{d+ex^2}} - \frac{2c^2\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)^{3/2}} - \frac{2c^2\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b+\sqrt{b^2-4ac}}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)^{3/2}}$$

output
```
e*(-b*e+c*d)*x/a/d/(c*d^2+e*(a*e-b*d))/(e*x^2+d)^(1/2)+(-2*e*x^2-d)/a/d^2/
x/(e*x^2+d)^(1/2)-2*c^2*arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e
*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(1+b/(-4*a*c+b^2)^(1/2))/a/(2*
c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(3/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-2*c^2*arc
tan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^
2)^(1/2))^(1/2))*(1-b/(-4*a*c+b^2)^(1/2))/a/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)
))^(3/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.398.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 17.54 (sec) , antiderivative size = 2158, normalized size of antiderivative = 6.37

$$\int \frac{1}{x^2 (d+ex^2)^{3/2} (a+bx^2+cx^4)} \, dx = \text{Result too large to show}$$

input

```
Integrate[1/(x^2*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

output

```
-((d + 2*e*x^2)/(a*d^2*x*Sqrt[d + e*x^2])) - ((c + (b*c)/Sqrt[b^2 - 4*a*c]
)*x*(45*Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])
*e)*x^2*(d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))] + (30*e*x
^2*Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x
^2*(d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))])/d - 45*ArcSin
[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a
*c] - 2*c*x^2)))]] - (30*e*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*
a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/d - (45*(2*c*d +
(-b + Sqrt[b^2 - 4*a*c])*e)*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4
*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/(d*(-b + Sqrt[b^
2 - 4*a*c] - 2*c*x^2)) - (30*e*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^4*Ar
cSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 -
 4*a*c] - 2*c*x^2)))]])/(d^2*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)) + 4*(-(((
2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*
x^2))))^(5/2)*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^
2 - 4*a*c] - 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*d + (-b + Sqr
t[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))] + (4*e*x^
2*(-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c]
 - 2*c*x^2))))^(5/2)*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b +
Sqrt[b^2 - 4*a*c] - 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*d +...
```

3.398.3 Rubi [A] (verified)

Time = 1.60 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.21, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1624, 245, 208, 2246, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (d+ex^2)^{3/2} (a+bx^2+cx^4)}dx$$

$$\Big\downarrow \ 1624$$

$$\frac{e^2 \int \frac{1}{x^2(ex^2+d)^{3/2}}dx}{ae^2-bde+cd^2}+\frac{\int \frac{-cex^2+cd-be}{x^2\sqrt{ex^2+d}(cx^4+bx^2+a)}dx}{ae^2-bde+cd^2}$$

$$\Big\downarrow \ 245$$

$$\frac{e^2\left(-\frac{2e\int \frac{1}{\left(ex^2+d\right)^{3/2}}dx}{d}-\frac{1}{dx\sqrt{d+ex^2}}\right)}{ae^2-bde+cd^2}+\frac{\int \frac{-cex^2+cd-be}{x^2\sqrt{ex^2+d}(cx^4+bx^2+a)}dx}{ae^2-bde+cd^2}$$

$$\Big\downarrow \ 208$$

$$\frac{\int \frac{-cex^2+cd-be}{x^2\sqrt{ex^2+d}(cx^4+bx^2+a)}dx}{ae^2-bde+cd^2}+\frac{e^2\left(-\frac{2ex}{d^2\sqrt{d+ex^2}}-\frac{1}{dx\sqrt{d+ex^2}}\right)}{ae^2-bde+cd^2}$$

$$\Big\downarrow \ 2246$$

$$\frac{\int \left(\frac{cd-be}{ax^2\sqrt{ex^2+d}}+\frac{eb^2-cdb-c(cd-be)x^2-ace}{a\sqrt{ex^2+d}(cx^4+bx^2+a)}\right)dx}{ae^2-bde+cd^2}+\frac{e^2\left(-\frac{2ex}{d^2\sqrt{d+ex^2}}-\frac{1}{dx\sqrt{d+ex^2}}\right)}{ae^2-bde+cd^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{c\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{c\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\sqrt{d+e\cdots}}{\cdots}}{ae^2-bde+cd^2}$$
$$\frac{e^2\left(-\frac{2ex}{d^2\sqrt{d+ex^2}}-\frac{1}{dx\sqrt{d+ex^2}}\right)}{ae^2-bde+cd^2}$$

input
```
Int[1/(x^2*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

3.398. $\int \frac{1}{x^2(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx$

output
```
(e^2*(-(1/(d*x*Sqrt[d + e*x^2])) - (2*e*x)/(d^2*Sqrt[d + e*x^2])))/(c*d^2
- b*d*e + a*e^2) + (-(((c*d - b*e)*Sqrt[d + e*x^2])/(a*d*x)) - (c*(c*d - b
*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b
- Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])
/(a*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) -
 (c*(c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d
 + e*x^2])])/(a*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e]))/(c*d^2 - b*d*e + a*e^2)
```

3.398.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 1624
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Simp[e^2/(c*d^2 - b*d*e + a*e^2)   Int[(f*x)^m*
(d + e*x^2)^q, x], x] + Simp[1/(c*d^2 - b*d*e + a*e^2)   Int[(f*x)^m*(d + e
*x^2)^(q + 1)*(Simp[c*d - b*e - c*e*x^2, x]/(a + b*x^2 + c*x^4)), x], x] /;
 FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] &&
 LtQ[q, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2246
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(f*x)^m*(d +
e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x
] && PolyQ[Px, x] && IntegerQ[p]
```

3.398. $\int \frac{1}{x^2(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx$

3.398.4 Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.51

method	result
default	$\frac{-\frac{1}{dx\sqrt{e\,x^2+d}}-\frac{2ex}{d^2\sqrt{e\,x^2+d}}}{a}-\frac{\sqrt{2}\,d\left(\left((ac-b^2)e+bcd\right)\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}+(2a\,c^2-b^2c)d^2+e(-3abc+b^3)d\right)\sqrt{\left(-2ae+bd+\sqrt{\cdots}\right.}}{\cdots}$
pseudoelliptic	$-\frac{\frac{\sqrt{2}\,d^2\left(\left(\left(ac-b^2\right)e+bcd\right)\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}+\left(2a\,c^2-b^2c\right)d^2+e\left(-3abc+b^3\right)d\right)\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\,x\sqrt{e\,x^2+d}\,\operatorname{arctanh}\left(\cdots\right.}{2}}{\cdots}$
risch	$-\frac{\sqrt{e\,x^2+d}}{d^2ax}-\frac{\frac{e^2a\sqrt{\left(x+\frac{\sqrt{-ed}}{e}\right)^2e-2\sqrt{-ed}\left(x+\frac{\sqrt{-ed}}{e}\right)}}{2d\left(a\,e^2-bde+c\,d^2\right)\left(x+\frac{\sqrt{-ed}}{e}\right)}+\frac{e^2a\sqrt{\left(x-\frac{\sqrt{-ed}}{e}\right)^2e+2\sqrt{-ed}\left(x-\frac{\sqrt{-ed}}{e}\right)}}{2d\left(a\,e^2-bde+c\,d^2\right)\left(x-\frac{\sqrt{-ed}}{e}\right)}-\frac{d\sqrt{2}\left(-\left(\left(ace-b^2e+bcd\right)\sqrt{\cdots}\right.\right.}{\cdots}}{\cdots}$

input
```
int(1/x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/a*(-1/d/x/(e*x^2+d)^(1/2)-2*e/d^2*x/(e*x^2+d)^(1/2))-1/2/a/((-2*a*e+b*d+
(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)/(e*x^2+d)^(1/2)/((2*a*e-b*d+(-4*d^2
*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*(2^(1/2)*d*(((a*c-b^2)*e+b*c*d)*(-4*d^2*(a
*c-1/4*b^2))^(1/2)+(2*a*c^2-b^2*c)*d^2+e*(-3*a*b*c+b^3)*d)*((-2*a*e+b*d+(-
4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*(e*x^2+d)^(1/2)*arctanh(a/x*(e*x^2+d)
^(1/2)*2^(1/2)/((2*a*e-b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2))-((2*a*e
-b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*(2^(1/2)*d*(e*x^2+d)^(1/2)*(((
a*c-b^2)*e+b*c*d)*(-4*d^2*(a*c-1/4*b^2))^(1/2)+(-2*a*c^2+b^2*c)*d^2+e*(3*a
*b*c-b^3)*d)*arctan(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((-2*a*e+b*d+(-4*d^2*(a*c-
1/4*b^2))^(1/2))*a)^(1/2))-2*(-4*d^2*(a*c-1/4*b^2))^(1/2)*((-2*a*e+b*d+(-4
*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*e*x*(b*e-c*d)))/(-4*d^2*(a*c-1/4*b^2))
^(1/2)/(a*e^2-b*d*e+c*d^2)/d
```

3.398.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(d+e x^2\right)^{3/2} \left(a+b x^2+c x^4\right)} \, dx = \text{Timed out}$$

input
```
integrate(1/x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.398.6 Sympy [F]

$$\int \frac{1}{x^2 \left(d+e x^2\right)^{3/2} \left(a+b x^2+c x^4\right)} \, dx = \int \frac{1}{x^2 \left(d+e x^2\right)^{\frac{3}{2}} \left(a+b x^2+c x^4\right)} \, dx$$

input
```
integrate(1/x**2/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

output
```
Integral(1/(x**2*(d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)
```

3.398.7 Maxima [F]

$$\int \frac{1}{x^2 \left(d+e x^2\right)^{3/2} \left(a+b x^2+c x^4\right)} \, dx = \int \frac{1}{(c x^4+b x^2+a)(e x^2+d)^{\frac{3}{2}} x^2} \, dx$$

input
```
integrate(1/x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)*x^2), x)
```

3.398.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(d+ex^2\right)^{3/2} \left(a+bx^2+cx^4\right)} \, dx = \text{Timed out}$$

input
```
integrate(1/x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
Timed out
```

3.398.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(d+ex^2\right)^{3/2} \left(a+bx^2+cx^4\right)} \, dx = \int \frac{1}{x^2 \left(e\,x^2+d\right)^{3/2} \left(c\,x^4+b\,x^2+a\right)} \,dx$$

input
```
int(1/(x^2*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x)
```

output
```
int(1/(x^2*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)), x)
```

3.399 $\int \frac{1}{x^4(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx$

3.399.1 Optimal result

Integrand size = 29, antiderivative size = 419

$$\int \frac{1}{x^4(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = -\frac{1}{3adx^3\sqrt{d+ex^2}}+\frac{3bd+4ae}{3a^2d^2x\sqrt{d+ex^2}}+\frac{2e(3bd+4ae)x}{3a^2d^3\sqrt{d+ex^2}}-\frac{e(bcd-b^2e+ace)\,x}{a^2d\,(cd^2+e(-bd+ae))\sqrt{d+ex^2}}+\frac{2c^2\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^2\sqrt{b-\sqrt{b^2-4ac}}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)^{3/2}}+\frac{2c^2\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^2\sqrt{b+\sqrt{b^2-4ac}}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)^{3/2}}$$

output
```
-1/3/a/d/x^3/(e*x^2+d)^(1/2)+1/3*(4*a*e+3*b*d)/a^2/d^2/x/(e*x^2+d)^(1/2)+2
/3*e*(4*a*e+3*b*d)*x/a^2/d^3/(e*x^2+d)^(1/2)-e*(a*c*e-b^2*e+b*c*d)*x/a^2/d
/(c*d^2+e*(a*e-b*d))/(e*x^2+d)^(1/2)+2*c^2*arctan(x*(2*c*d-e*(b-(-4*a*c+b^
2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-2*a*c+
b^2)/(-4*a*c+b^2)^(1/2))/a^2/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(3/2)/(b-(-4
*a*c+b^2)^(1/2))^(1/2)+2*c^2*arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/
2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(2*a*c-b^2)/(-4*a*c+b^
2)^(1/2))/a^2/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(3/2)/(b+(-4*a*c+b^2)^(1/2)
)^(1/2)
```

3.399.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 17.82 (sec) , antiderivative size = 2218, normalized size of antiderivative = 5.29

$$\int \frac{1}{x^4 (d+ex^2)^{3/2} (a+bx^2+cx^4)} \, dx = \text{Result too large to show}$$

input
```
Integrate[1/(x^4*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

output
```
(b*(d + 2*e*x^2))/(a^2*d^2*x*Sqrt[d + e*x^2]) - (d^2 - 4*d*e*x^2 - 8*e^2*x
^4)/(3*a*d^3*x^3*Sqrt[d + e*x^2]) + ((b*c + (c*(b^2 - 2*a*c))/Sqrt[b^2 - 4
*a*c])*x*(45*Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*
a*c])*e)*x^2*(d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))] + (3
0*e*x^2*Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])
*e)*x^2*(d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))])/d - 45*A
rcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2
- 4*a*c] - 2*c*x^2)))]] - (30*e*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2
 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/d - (45*(2*c
*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^
2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/(d*(-b + Sq
rt[b^2 - 4*a*c] - 2*c*x^2)) - (30*e*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x
^4*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[
b^2 - 4*a*c] - 2*c*x^2)))]])/(d^2*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)) + 4*
(-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] -
 2*c*x^2))))^(5/2)*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sq
rt[b^2 - 4*a*c] - 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*d + (-b
+ Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))] + (4
*e*x^2*(-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4
*a*c] - 2*c*x^2))))^(5/2)*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(...
```

3.399.3 Rubi [A] (verified)

Time = 2.45 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.30, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1624, 245, 245, 208, 2246, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(d+ex^2\right)^{3/2} \left(a+bx^2+cx^4\right)}dx$$

$$\Big\downarrow \; 1624$$

$$\frac{e^2 \int \frac{1}{x^4 \left(ex^2+d\right)^{3/2}}dx}{ae^2-bde+cd^2}+\frac{\int \frac{-cex^2+cd-be}{x^4 \sqrt{ex^2+d} \left(cx^4+bx^2+a\right)}dx}{ae^2-bde+cd^2}$$

$$\Big\downarrow \; 245$$

$$\frac{\int \frac{-cex^2+cd-be}{x^4 \sqrt{ex^2+d} \left(cx^4+bx^2+a\right)}dx}{ae^2-bde+cd^2}+\frac{e^2 \left(-\frac{4e \int \frac{1}{x^2 \left(ex^2+d\right)^{3/2}}dx}{3d}-\frac{1}{3dx^3 \sqrt{d+ex^2}}\right)}{ae^2-bde+cd^2}$$

$$\Big\downarrow \; 245$$

$$\frac{\int \frac{-cex^2+cd-be}{x^4 \sqrt{ex^2+d} \left(cx^4+bx^2+a\right)}dx}{ae^2-bde+cd^2}+\frac{e^2 \left(-\frac{4e \left(-\frac{2e \int \frac{1}{\left(ex^2+d\right)^{3/2}}dx}{d}-\frac{1}{dx \sqrt{d+ex^2}}\right)}{3d}-\frac{1}{3dx^3 \sqrt{d+ex^2}}\right)}{ae^2-bde+cd^2}$$

$$\Big\downarrow \; 208$$

$$\frac{\int \frac{-cex^2+cd-be}{x^4 \sqrt{ex^2+d} \left(cx^4+bx^2+a\right)}dx}{ae^2-bde+cd^2}+\frac{e^2 \left(-\frac{4e \left(-\frac{2ex}{d^2 \sqrt{d+ex^2}}-\frac{1}{dx \sqrt{d+ex^2}}\right)}{3d}-\frac{1}{3dx^3 \sqrt{d+ex^2}}\right)}{ae^2-bde+cd^2}$$

$$\Big\downarrow \; 2246$$

$$\frac{\int \left(\frac{cd-be}{ax^4 \sqrt{ex^2+d}}+\frac{eb^2-cdb-ace}{a^2 x^2 \sqrt{ex^2+d}}+\frac{-eb^3+cdb^2+2aceb+c\left(-eb^2+cdb+ace\right)x^2-ac^2 d}{a^2 \sqrt{ex^2+d} \left(cx^4+bx^2+a\right)}\right)dx}{ae^2-bde+cd^2}+\frac{e^2 \left(-\frac{4e \left(-\frac{2ex}{d^2 \sqrt{d+ex^2}}-\frac{1}{dx \sqrt{d+ex^2}}\right)}{3d}-\frac{1}{3dx^3 \sqrt{d+ex^2}}\right)}{ae^2-bde+cd^2}$$

$$\Big\downarrow \; 2009$$

3.399. $\int \frac{1}{x^4 \left(d+ex^2\right)^{3/2} \left(a+bx^2+cx^4\right)}dx$

$$\frac{\frac{c\left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{c\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\arctan}{a^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}\right.}}}{ae^2-bde+cd^2}$$

$$\frac{e^2\left(-\frac{4e\left(-\frac{2ex}{d^2\sqrt{d+ex^2}}-\frac{1}{dx\sqrt{d+ex^2}}\right)}{3d}-\frac{1}{3dx^3\sqrt{d+ex^2}}\right)}{ae^2-bde+cd^2}$$

input
```
Int[1/(x^4*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

output
```
(e^2*(-1/3*1/(d*x^3*Sqrt[d + e*x^2]) - (4*e*(-(1/(d*x*Sqrt[d + e*x^2])) -
(2*e*x)/(d^2*Sqrt[d + e*x^2])))/(3*d)))/(c*d^2 - b*d*e + a*e^2) + (-1/3*((
c*d - b*e)*Sqrt[d + e*x^2])/(a*d*x^3) + (2*e*(c*d - b*e)*Sqrt[d + e*x^2])/
(3*a*d^2*x) + ((b*c*d - b^2*e + a*c*e)*Sqrt[d + e*x^2])/(a^2*d*x) + (c*(b*
c*d - b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt
[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*
c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (c*(b*c*d - b^2*e + a*c*e - (b^2*c*d -
 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b
 + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]
)/(a^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]
))/(c*d^2 - b*d*e + a*e^2)
```

3.399.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 1624
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Simp[e^2/(c*d^2 - b*d*e + a*e^2)   Int[(f*x)^m*
(d + e*x^2)^q, x], x] + Simp[1/(c*d^2 - b*d*e + a*e^2)   Int[(f*x)^m*(d + e
*x^2)^(q + 1)*(Simp[c*d - b*e - c*e*x^2, x]/(a + b*x^2 + c*x^4)), x], x] /;
 FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] &&
 LtQ[q, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2246
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(f*x)^m*(d +
e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x
] && PolyQ[Px, x] && IntegerQ[p]
```

3.399.4 Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 610, normalized size of antiderivative = 1.46

method	result
pseudoelliptic	$-\frac{-3\sqrt{\left(-2ae+bd+\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}\right)a}\,\sqrt{2}\,d^3\left(\left(c\left(be-\frac{cd}{2}\right)a-\frac{b^2(be-cd)}{2}\right)\sqrt{-4d^2\left(ac-\frac{b^2}{4}\right)}+d\left(a^2c^2e+(-2b^2ce+\frac{3}{2}bc^2d)\right.\right.}{}$
default	$\frac{-\frac{1}{3d\,x^3\sqrt{e\,x^2+d}}-\frac{4e\left(-\frac{1}{dx\sqrt{e\,x^2+d}}-\frac{2ex}{d^2\sqrt{e\,x^2+d}}\right)}{3d}}{a}-\frac{b\left(-\frac{1}{dx\sqrt{e\,x^2+d}}-\frac{2ex}{d^2\sqrt{e\,x^2+d}}\right)}{a^2}+\frac{\sqrt{2}\,d\left(\left(-\frac{dc\left(ac-b^2\right)}{2}+be\left(ac-\frac{b^2}{2}\right)\right.\right.}{}$
risch	$-\frac{\sqrt{e\,x^2+d}\,\left(-5ae\,x^2-3bd\,x^2+da\right)}{3d^3a^2x^3}+\frac{\frac{e^3a^2\sqrt{\left(x-\frac{\sqrt{-ed}}{e}\right)^2e+2\sqrt{-ed}\left(x-\frac{\sqrt{-ed}}{e}\right)}}{2d\left(a\,e^2-bde+c\,d^2\right)\left(x-\frac{\sqrt{-ed}}{e}\right)}+\frac{e^3a^2\sqrt{\left(x+\frac{\sqrt{-ed}}{e}\right)^2e-2\sqrt{-ed}\left(x+\frac{\sqrt{-ed}}{e}\right)}}{2d\left(a\,e^2-bde+c\,d^2\right)\left(x+\frac{\sqrt{-ed}}{e}\right)}+}{}$

input
```
int(1/x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-3*((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*2^(1/2)*d^3*(
(c*(b*e-1/2*c*d)*a-1/2*b^2*(b*e-c*d))*(-4*d^2*(a*c-1/4*b^2))^(1/2)+d*(a^2*
c^2*e+(-2*b^2*c*e+3/2*b*c^2*d)*a+1/2*b^3*(b*e-c*d)))*x^3*(e*x^2+d)^(1/2)*a
rctanh(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((2*a*e-b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2
))*a)^(1/2))+((2*a*e-b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)*(-3*2^(1/2
)*d^3*(((-e*b*c+1/2*c^2*d)*a+1/2*b^2*(b*e-c*d))*(-4*d^2*(a*c-1/4*b^2))^(1/
2)+d*(a^2*c^2*e+(-2*b^2*c*e+3/2*b*c^2*d)*a+1/2*b^3*(b*e-c*d)))*x^3*(e*x^2+
d)^(1/2)*arctan(a/x*(e*x^2+d)^(1/2)*2^(1/2)/((-2*a*e+b*d+(-4*d^2*(a*c-1/4*
b^2))^(1/2))*a)^(1/2))+((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)
*(e^2*(-8*e^2*x^4-4*d*e*x^2+d^2)*a^2-(-c*d^2+e*(5*c*x^2+b)*d-2*b*e^2*x^2)*
(e*x^2+d)*d*a+3*b*d^2*x^2*(e*x^2+d)*(b*e-c*d))*(-4*d^2*(a*c-1/4*b^2))^(1/2
)))/((-2*a*e+b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1/2)/(e*x^2+d)^(1/2)/(-
4*d^2*(a*c-1/4*b^2))^(1/2)/((2*a*e-b*d+(-4*d^2*(a*c-1/4*b^2))^(1/2))*a)^(1
/2)/a^2/x^3/(a*e^2-b*d*e+c*d^2)/d^3
```

3.399.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (d+ex^2)^{3/2} (a+bx^2+cx^4)} \, dx = \text{Timed out}$$

input
```
integrate(1/x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.399.6 Sympy [F]

$$\int \frac{1}{x^4 (d+ex^2)^{3/2} (a+bx^2+cx^4)} \, dx = \int \frac{1}{x^4 (d+ex^2)^{\frac{3}{2}} (a+bx^2+cx^4)} \, dx$$

input
```
integrate(1/x**4/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

output
```
Integral(1/(x**4*(d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)
```

3.399.7 Maxima [F]

$$\int \frac{1}{x^4 (d+ex^2)^{3/2} (a+bx^2+cx^4)} \, dx = \int \frac{1}{(cx^4+bx^2+a)(ex^2+d)^{\frac{3}{2}}x^4} \, dx$$

input
```
integrate(1/x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)*x^4), x)
```

3.399.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (d+ex^2)^{3/2} (a+bx^2+cx^4)} \, dx = \text{Timed out}$$

input
```
integrate(1/x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
Timed out
```

3.399.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (d+ex^2)^{3/2} (a+bx^2+cx^4)} \, dx = \int \frac{1}{x^4\,(e\,x^2+d)^{3/2}\,(c\,x^4+b\,x^2+a)} \, dx$$

input
```
int(1/(x^4*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x)
```

output
```
int(1/(x^4*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)), x)
```

3.400 $\int \frac{(fx)^m(d+ex^2)^q}{a+bx^2+cx^4}\,dx$

3.400.1 Optimal result

Integrand size = 29, antiderivative size = 243

$$\int \frac{(fx)^m (d+ex^2)^q}{a+bx^2+cx^4}\,dx = \frac{2c(fx)^{1+m}(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\text{AppellF1}\left(\frac{1+m}{2},1,-q,\frac{3+m}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)f(1+m)} - \frac{2c(fx)^{1+m}(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\text{AppellF1}\left(\frac{1+m}{2},1,-q,\frac{3+m}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)f(1+m)}$$

output
```
2*c*(f*x)^(1+m)*(e*x^2+d)^q*AppellF1(1/2+1/2*m,1,-q,3/2+1/2*m,-2*c*x^2/(b-
(-4*a*c+b^2)^(1/2)),-e*x^2/d)/f/(1+m)/((1+e*x^2/d)^q)/(b-(-4*a*c+b^2)^(1/2
))/(-4*a*c+b^2)^(1/2)-2*c*(f*x)^(1+m)*(e*x^2+d)^q*AppellF1(1/2+1/2*m,1,-q,
3/2+1/2*m,-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)),-e*x^2/d)/f/(1+m)/((1+e*x^2/d)^q
)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

3.400.2 Mathematica [F]

$$\int \frac{(fx)^m (d+ex^2)^q}{a+bx^2+cx^4} \, dx = \int \frac{(fx)^m (d+ex^2)^q}{a+bx^2+cx^4} \, dx$$

input
```
Integrate[((f*x)^m*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

output
```
Integrate[((f*x)^m*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x]
```

3.400.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1628, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m (d+ex^2)^q}{a+bx^2+cx^4} \, dx$$

$$\Big\downarrow \text{1628}$$

$$\frac{2c \int \frac{(fx)^m (ex^2+d)^q}{2cx^2+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}} - \frac{2c \int \frac{(fx)^m (ex^2+d)^q}{2cx^2+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{395}$$

$$\frac{2c(d+ex^2)^q \left(\frac{ex^2}{d}+1\right)^{-q} \int \frac{(fx)^m \left(\frac{ex^2}{d}+1\right)^q}{2cx^2+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}} - \frac{2c(d+ex^2)^q \left(\frac{ex^2}{d}+1\right)^{-q} \int \frac{(fx)^m \left(\frac{ex^2}{d}+1\right)^q}{2cx^2+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{394}$$

$$\frac{2c(fx)^{m+1} (d+ex^2)^q \left(\frac{ex^2}{d}+1\right)^{-q} \text{AppellF1}\left(\frac{m+1}{2},1,-q,\frac{m+3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{f(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)} - \frac{2c(fx)^{m+1} (d+ex^2)^q \left(\frac{ex^2}{d}+1\right)^{-q} \text{AppellF1}\left(\frac{m+1}{2},1,-q,\frac{m+3}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{f(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

input
```
Int[((f*x)^m*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

output
```
(2*c*(f*x)^(1 + m)*(d + e*x^2)^q*AppellF1[(1 + m)/2, 1, -q, (3 + m)/2, (-2
*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(Sqrt[b^2 - 4*a*c]*(b - Sq
rt[b^2 - 4*a*c])*f*(1 + m)*(1 + (e*x^2)/d)^q) - (2*c*(f*x)^(1 + m)*(d + e*
x^2)^q*AppellF1[(1 + m)/2, 1, -q, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c]), -((e*x^2)/d)])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*f*(1 + m)
*(1 + (e*x^2)/d)^q)
```

3.400.3.1 Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 1628
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/r)   I
nt[(f*x)^m*((d + e*x^2)^q/(b - r + 2*c*x^2)), x], x] - Simp[2*(c/r)   Int[(
f*x)^m*((d + e*x^2)^q/(b + r + 2*c*x^2)), x], x]] /; FreeQ[{a, b, c, d, e,
f, m, q}, x] && NeQ[b^2 - 4*a*c, 0]
```

3.400.4 Maple [F]

$$\int \frac{(fx)^m \left(e\,x^2+d\right)^q}{c\,x^4+b\,x^2+a}dx$$

input
```
int((f*x)^m*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

output
```
int((f*x)^m*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

3.400.5 Fricas [F]

$$\int \frac{(fx)^m (d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q (fx)^m}{cx^4+bx^2+a}\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
integral((e*x^2 + d)^q*(f*x)^m/(c*x^4 + b*x^2 + a), x)
```

3.400.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(fx)^m (d+ex^2)^q}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.400.7 Maxima [F]

$$\int \frac{(fx)^m (d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q (fx)^m}{cx^4+bx^2+a}\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^q*(f*x)^m/(c*x^4 + b*x^2 + a), x)
```

3.400.8 Giac [F]

$$\int \frac{(fx)^m (d + ex^2)^q}{a + bx^2 + cx^4} dx = \int \frac{(ex^2 + d)^q (fx)^m}{cx^4 + bx^2 + a} dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^q*(f*x)^m/(c*x^4 + b*x^2 + a), x)
```

3.400.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^m (d + ex^2)^q}{a + bx^2 + cx^4} dx = \int \frac{(f\,x)^m\,(e\,x^2 + d)^q}{c\,x^4 + b\,x^2 + a} dx$$

input
```
int(((f*x)^m*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x)
```

output
```
int(((f*x)^m*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x)
```

3.401 $\int \frac{x^7(d+ex^2)^q}{a+bx^2+cx^4}\,dx$

3.401.1 Optimal result

Integrand size = 27, antiderivative size = 313

$$\int \frac{x^7(d+ex^2)^q}{a+bx^2+cx^4}\,dx = -\frac{(cd+be)(d+ex^2)^{1+q}}{2c^2e^2(1+q)}+\frac{(d+ex^2)^{2+q}}{2ce^2(2+q)}$$
$$+\frac{\left(a-\frac{b^2}{c}+\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}\right)(d+ex^2)^{1+q}\,\text{Hypergeometric2F1}\left(1,1+q,2+q,\frac{2c(d+ex^2)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2c\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+q)}$$
$$+\frac{\left(a-\frac{b^2}{c}-\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}\right)(d+ex^2)^{1+q}\,\text{Hypergeometric2F1}\left(1,1+q,2+q,\frac{2c(d+ex^2)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2c\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)(1+q)}$$

output
```
-1/2*(b*e+c*d)*(e*x^2+d)^(1+q)/c^2/e^2/(1+q)+1/2*(e*x^2+d)^(2+q)/c/e^2/(2+
q)+1/2*(e*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],2*c*(e*x^2+d)/(2*c*d-e*(b-
(-4*a*c+b^2)^(1/2))))*(a-b^2/c+b*(-3*a*c+b^2)/c/(-4*a*c+b^2)^(1/2))/c/(1+q
)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))+1/2*(e*x^2+d)^(1+q)*hypergeom([1, 1+q],
[2+q],2*c*(e*x^2+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))*(a-b^2/c-b*(-3*a*c+b
^2)/c/(-4*a*c+b^2)^(1/2))/c/(1+q)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))
```

3.401.2 Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.87

$$\int \frac{x^7(d+ex^2)^q}{a+bx^2+cx^4}\,dx$$
$$= \frac{(d+ex^2)^{1+q}\left(-\frac{cd+be}{e^2(1+q)}+\frac{c(d+ex^2)}{e^2(2+q)}+\frac{c\left(a-\frac{b^2}{c}+\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,1+q,2+q,\frac{2c(d+ex^2)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}\right)}{\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)e\right)(1+q)}+\frac{c\left(a-\frac{b^2}{c}\right.}{}}{2c^2}$$

input
```
Integrate[(x^7*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

output
```
((d + e*x^2)^(1 + q)*(-((c*d + b*e)/(e^2*(1 + q))) + (c*(d + e*x^2))/(e^2*
(2 + q)) + (c*(a - b^2/c + (b*(b^2 - 3*a*c))/(c*Sqrt[b^2 - 4*a*c]))*Hyperg
eometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d + (-b + Sqrt[b^2 - 4
*a*c])*e)])/((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*(1 + q)) + (c*(a - b^2/c
 - (b*(b^2 - 3*a*c))/(c*Sqrt[b^2 - 4*a*c]))*Hypergeometric2F1[1, 1 + q, 2
+ q, (2*c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/((2*c*d - (b
+ Sqrt[b^2 - 4*a*c])*e)*(1 + q))))/(2*c^2)
```

3.401.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1578, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7\left(d+ex^2\right)^q}{a+bx^2+cx^4}\,dx$$
$$\Big\downarrow \text{ 1578}$$
$$\frac{1}{2}\int \frac{x^6\left(ex^2+d\right)^q}{cx^4+bx^2+a}dx^2$$
$$\Big\downarrow \text{ 1200}$$

$$\frac{1}{2}\int \left(\frac{(-cd-be)\left(ex^2+d\right)^q}{c^2e}+\frac{\left(\frac{b^2}{c^2}-\frac{(b^2-3ac)b}{c^2\sqrt{b^2-4ac}}-\frac{a}{c}\right)\left(ex^2+d\right)^q}{2cx^2+b-\sqrt{b^2-4ac}}+\frac{\left(\frac{b^2}{c^2}+\frac{(b^2-3ac)b}{c^2\sqrt{b^2-4ac}}-\frac{a}{c}\right)\left(ex^2+d\right)^q}{2cx^2+b+\sqrt{b^2-4ac}}+\frac{\left(ex^2+d\right.}{ce}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{\left(\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}+a-\frac{b^2}{c}\right)\left(d+ex^2\right)^{q+1}\text{Hypergeometric2F1}\left(1,q+1,q+2,\frac{2c(ex^2+d)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{c(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{\left(-\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}+a\right.}{}\right.$$

input
```
Int[(x^7*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

output
```
(-(((c*d + b*e)*(d + e*x^2)^(1 + q))/(c^2*e^2*(1 + q))) + (d + e*x^2)^(2 +
 q)/(c*e^2*(2 + q)) + ((a - b^2/c + (b*(b^2 - 3*a*c))/(c*Sqrt[b^2 - 4*a*c]
))*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2)
)/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])/(c*(2*c*d - (b - Sqrt[b^2 - 4*a*c]
)*e)*(1 + q)) + ((a - b^2/c - (b*(b^2 - 3*a*c))/(c*Sqrt[b^2 - 4*a*c]))*(d
+ e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c
*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(
1 + q)))/2
```

3.401.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.401. $\int \frac{x^7(d+ex^2)^q}{a+bx^2+cx^4}\,dx$

3.401.4 Maple [F]

$$\int \frac{x^7 (e\,x^2+d)^q}{c\,x^4+b\,x^2+a}dx$$

input
```
int(x^7*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

output
```
int(x^7*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

3.401.5 Fricas [F]

$$\int \frac{x^7 (d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q x^7}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^7*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
integral((e*x^2 + d)^q*x^7/(c*x^4 + b*x^2 + a), x)
```

3.401.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^7 (d+ex^2)^q}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**7*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.401.7 Maxima [F]

$$\int \frac{x^7(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q x^7}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^7*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^q*x^7/(c*x^4 + b*x^2 + a), x)
```

3.401.8 Giac [F]

$$\int \frac{x^7(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q x^7}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^7*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^q*x^7/(c*x^4 + b*x^2 + a), x)
```

3.401.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^7(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{x^7\,(e\,x^2+d)^q}{c\,x^4+b\,x^2+a}\,dx$$

input
```
int((x^7*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x)
```

output
```
int((x^7*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x)
```

3.402 $\int \frac{x^5(d+ex^2)^q}{a+bx^2+cx^4}\,dx$

3.402.1 Optimal result

Integrand size = 27, antiderivative size = 256

$$\int \frac{x^5(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \frac{(d+ex^2)^{1+q}}{2ce(1+q)} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^{1+q}\text{Hypergeometric2F1}\left(1,1+q,2+q,\frac{2c(d+ex^2)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2c\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+q)} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^{1+q}\text{Hypergeometric2F1}\left(1,1+q,2+q,\frac{2c(d+ex^2)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2c\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)(1+q)}$$

output
```
1/2*(e*x^2+d)^(1+q)/c/e/(1+q)+1/2*(e*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q]
,2*c*(e*x^2+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))*(b+(2*a*c-b^2)/(-4*a*c+b^
2)^(1/2))/c/(1+q)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))+1/2*(e*x^2+d)^(1+q)*hyp
ergeom([1, 1+q],[2+q],2*c*(e*x^2+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))*(b+(
-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c/(1+q)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))
```

3.402.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.82

$$\int \frac{x^5(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \frac{(d+ex^2)^{1+q}\left(\frac{1}{e}+\frac{\left(b+\frac{-b^2+2ac}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,1+q,2+q,\frac{2c(d+ex^2)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}\right)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}+\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,1 \ldots\right.}{2cd-\left(b+\sqrt{b^2-4 \ldots\right.}\right.}{2c(1+q)}$$

input
```
Integrate[(x^5*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

output
```
((d + e*x^2)^(1 + q)*(e^(-1) + ((b + (-b^2 + 2*a*c)/Sqrt[b^2 - 4*a*c])*Hyp
ergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d + (-b + Sqrt[b^2
- 4*a*c])*e)])/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e) + ((b + (b^2 - 2*a*c)/
Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2
*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)))/
(2*c*(1 + q))
```

3.402.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1578, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(d+ex^2)^q}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \; 1578$$

$$\frac{1}{2}\int \frac{x^4(ex^2+d)^q}{cx^4+bx^2+a}dx^2$$

$$\Big\downarrow \; 1200$$

$$\frac{1}{2}\int \left(\frac{(ex^2+d)^q}{c}+\frac{\left(\frac{b^2-2ac}{c\sqrt{b^2-4ac}}-\frac{b}{c}\right)(ex^2+d)^q}{2cx^2+b-\sqrt{b^2-4ac}}+\frac{\left(-\frac{b}{c}-\frac{b^2-2ac}{c\sqrt{b^2-4ac}}\right)(ex^2+d)^q}{2cx^2+b+\sqrt{b^2-4ac}}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^{q+1}\text{Hypergeometric2F1}\left(1,q+1,q+2,\frac{2c(ex^2+d)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{c(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(d+ex\right.}{}\right.$$

input
```
Int[(x^5*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

output
```
((d + e*x^2)^(1 + q)/(c*e*(1 + q)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c]
)*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))
/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])/(c*(2*c*d - (b - Sqrt[b^2 - 4*a*c])
*e)*(1 + q)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(d + e*x^2)^(1 + q)*
Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^
2 - 4*a*c])*e)])/(c*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(1 + q)))/2
```

3.402.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.402.4 Maple [F]

$$\int \frac{x^5 (e\,x^2+d)^q}{c\,x^4+b\,x^2+a}dx$$

input
```
int(x^5*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

output
```
int(x^5*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

3.402.5 Fricas [F]

$$\int \frac{x^5(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q x^5}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^5*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
integral((e*x^2 + d)^q*x^5/(c*x^4 + b*x^2 + a), x)
```

3.402.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**5*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.402.7 Maxima [F]

$$\int \frac{x^5(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q x^5}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^5*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^q*x^5/(c*x^4 + b*x^2 + a), x)
```

3.402.8 Giac [F]

$$\int \frac{x^5(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q x^5}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^5*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^q*x^5/(c*x^4 + b*x^2 + a), x)
```

3.402.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{x^5\,(e\,x^2+d)^q}{c\,x^4+b\,x^2+a}\,dx$$

input
```
int((x^5*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x)
```

output
```
int((x^5*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x)
```

3.403 $\int \frac{x^3(d+ex^2)^q}{a+bx^2+cx^4}\,dx$

3.403.1 Optimal result

Integrand size = 27, antiderivative size = 210

$$\int \frac{x^3(d+ex^2)^q}{a+bx^2+cx^4}\,dx = -\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^2)^{1+q}\,\text{Hypergeometric2F1}\left(1,1+q,2+q,\frac{2c(d+ex^2)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+q)} - \frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^2)^{1+q}\,\text{Hypergeometric2F1}\left(1,1+q,2+q,\frac{2c(d+ex^2)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)(1+q)}$$

output
```
-1/2*(e*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],2*c*(e*x^2+d)/(2*c*d-e*(b-(-
4*a*c+b^2)^(1/2))))*(1-b/(-4*a*c+b^2)^(1/2))/(1+q)/(2*c*d-e*(b-(-4*a*c+b^2
)^(1/2)))-1/2*(e*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],2*c*(e*x^2+d)/(2*c*
d-e*(b+(-4*a*c+b^2)^(1/2))))*(1+b/(-4*a*c+b^2)^(1/2))/(1+q)/(2*c*d-e*(b+(-
4*a*c+b^2)^(1/2)))
```

3.403.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.87

$$\int \frac{x^3(d+ex^2)^q}{a+bx^2+cx^4}\,dx = -\frac{(d+ex^2)^{1+q}\left(\left(-bd+\sqrt{b^2-4ac}d+2ae\right)\text{Hypergeometric2F1}\left(1,1+q,2+q,\frac{2c(d+ex^2)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}\right)+(\right.}{4\sqrt{b^2-4ac}\left(cd^2+e(-bd+ae\right.}$$

input
```
Integrate[(x^3*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

output
```
-1/4*((d + e*x^2)^(1 + q)*((-(b*d) + Sqrt[b^2 - 4*a*c]*d + 2*a*e)*Hypergeo
metric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d + (-b + Sqrt[b^2 - 4*a
*c])*e)] + (b*d + Sqrt[b^2 - 4*a*c]*d - 2*a*e)*Hypergeometric2F1[1, 1 + q,
 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]))/(Sqrt[b^2
 - 4*a*c]*(c*d^2 + e*(-(b*d) + a*e))*(1 + q))
```

3.403.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1578, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(d+ex^2)^q}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{x^2(ex^2+d)^q}{cx^4+bx^2+a}dx^2$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{2}\int \left(\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(ex^2+d)^q}{2cx^2+b-\sqrt{b^2-4ac}}+\frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)(ex^2+d)^q}{2cx^2+b+\sqrt{b^2-4ac}}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^{q+1}\text{Hypergeometric2F1}\left(1,q+1,q+2,\frac{2c(ex^2+d)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}-\frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(d+\right.}{}\right.$$

input
```
Int[(x^3*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

output
```
(-(((1 - b/Sqrt[b^2 - 4*a*c])*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 +
 q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])/((2*c*d
 - (b - Sqrt[b^2 - 4*a*c])*e)*(1 + q))) - ((1 + b/Sqrt[b^2 - 4*a*c])*(d +
e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e)])/((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(1 +
q)))/2
```

3.403.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.403.4 Maple [F]

$$\int \frac{x^3(e\,x^2+d)^q}{c\,x^4+b\,x^2+a}dx$$

input
```
int(x^3*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

output
```
int(x^3*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

3.403.5 Fricas [F]

$$\int \frac{x^3(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q x^3}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^3*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
integral((e*x^2 + d)^q*x^3/(c*x^4 + b*x^2 + a), x)
```

3.403.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.403.7 Maxima [F]

$$\int \frac{x^3(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q x^3}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^3*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^q*x^3/(c*x^4 + b*x^2 + a), x)
```

3.403.8 Giac [F]

$$\int \frac{x^3(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q x^3}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^3*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^q*x^3/(c*x^4 + b*x^2 + a), x)
```

3.403.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{x^3\,(e\,x^2+d)^q}{c\,x^4+b\,x^2+a}\,dx$$

input
```
int((x^3*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x)
```

output
```
int((x^3*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x)
```

3.404 $\int \frac{x(d+ex^2)^q}{a+bx^2+cx^4}\,dx$

3.404.1 Optimal result

Integrand size = 25, antiderivative size = 198

$$\int \frac{x(d+ex^2)^q}{a+bx^2+cx^4}\,dx = -\frac{c(d+ex^2)^{1+q}\,\text{Hypergeometric2F1}\left(1,1+q,2+q,\frac{2c(d+ex^2)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{b^2-4ac}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+q)} + \frac{c(d+ex^2)^{1+q}\,\text{Hypergeometric2F1}\left(1,1+q,2+q,\frac{2c(d+ex^2)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{b^2-4ac}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)(1+q)}$$

output
```
-c*(e*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],2*c*(e*x^2+d)/(2*c*d-e*(b-(-4*
a*c+b^2)^(1/2))))/(1+q)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(1/2
)+c*(e*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],2*c*(e*x^2+d)/(2*c*d-e*(b+(-4
*a*c+b^2)^(1/2))))/(1+q)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)
))
```

3.404.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.85

$$\int \frac{x(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \frac{c(d+ex^2)^{1+q}\left(-\frac{\text{Hypergeometric2F1}\left(1,1+q,2+q,\frac{2c(d+ex^2)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}\right)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}+\frac{\text{Hypergeometric2F1}\left(1,1+q,2+q,\frac{2c(d+ex^2)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{b^2-4ac}(1+q)}$$

input
```
Integrate[(x*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

output
```
(c*(d + e*x^2)^(1 + q)*(-(Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x
^2))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)]/(2*c*d + (-b + Sqrt[b^2 - 4*a*c
])*e)) + Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b
+ Sqrt[b^2 - 4*a*c])*e)]/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)))/(Sqrt[b^2 -
 4*a*c]*(1 + q))
```

3.404.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1576, 1150, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(d+ex^2\right)^q}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{1576}$$

$$\frac{1}{2}\int \frac{\left(ex^2+d\right)^q}{cx^4+bx^2+a}dx^2$$

$$\Big\downarrow \text{1150}$$

$$\frac{1}{2}\int \left(\frac{2c\left(ex^2+d\right)^q}{\sqrt{b^2-4ac}\left(2cx^2+b-\sqrt{b^2-4ac}\right)}-\frac{2c\left(ex^2+d\right)^q}{\sqrt{b^2-4ac}\left(2cx^2+b+\sqrt{b^2-4ac}\right)}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2c(d+ex^2)^{q+1}\,\text{Hypergeometric2F1}\left(1,q+1,q+2,\frac{2c(ex^2+d)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{(q+1)\sqrt{b^2-4ac}\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}-\frac{2c(d+ex^2)^{q+1}\,\text{Hypergeometric2F}}{(q+1)\sqrt{b^2-4ac}\left(2c\right.}\right.$$

input
```
Int[(x*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

output
```
((-2*c*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*
x^2))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])/(Sqrt[b^2 - 4*a*c]*(2*c*d - (b
 - Sqrt[b^2 - 4*a*c])*e)*(1 + q)) + (2*c*(d + e*x^2)^(1 + q)*Hypergeometri
c2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e
)])/(Sqrt[b^2 - 4*a*c]*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(1 + q)))/2
```

3.404.3.1 Defintions of rubi rules used

rule 1150
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[ExpandIntegrand[(d + e*x)^m, 1/(a + b*x + c*x^2), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] &&  !IntegerQ[2*m]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.404.4 Maple [F]

$$\int \frac{x(e\,x^2+d)^q}{c\,x^4+b\,x^2+a}dx$$

input
```
int(x*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

output
```
int(x*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

3.404.5 Fricas [F]

$$\int \frac{x(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q x}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
integral((e*x^2 + d)^q*x/(c*x^4 + b*x^2 + a), x)
```

3.404.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate(x*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.404.7 Maxima [F]

$$\int \frac{x(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q x}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^q*x/(c*x^4 + b*x^2 + a), x)
```

3.404.8 Giac [F]

$$\int \frac{x(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q x}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^q*x/(c*x^4 + b*x^2 + a), x)
```

3.404.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{x\,(e\,x^2+d)^q}{c\,x^4+b\,x^2+a}\,dx$$

input
```
int((x*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x)
```

output
```
int((x*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x)
```

3.405 $\int \frac{(d+ex^2)^q}{x(a+bx^2+cx^4)}\,dx$

3.405.1 Optimal result

Integrand size = 27, antiderivative size = 262

$$\int \frac{(d+ex^2)^q}{x(a+bx^2+cx^4)}\,dx = \frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^2)^{1+q}\,\text{Hypergeometric2F1}\left(1,1+q,2+q,\frac{2c(d+ex^2)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2a\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+q)} + \frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^2)^{1+q}\,\text{Hypergeometric2F1}\left(1,1+q,2+q,\frac{2c(d+ex^2)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2a\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)(1+q)} - \frac{(d+ex^2)^{1+q}\,\text{Hypergeometric2F1}\left(1,1+q,2+q,1+\frac{ex^2}{d}\right)}{2ad(1+q)}$$

output
```
-1/2*(e*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],1+e*x^2/d)/a/d/(1+q)+1/2*c*(
e*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],2*c*(e*x^2+d)/(2*c*d-e*(b-(-4*a*c+
b^2)^(1/2))))*(1+b/(-4*a*c+b^2)^(1/2))/a/(1+q)/(2*c*d-e*(b-(-4*a*c+b^2)^(1
/2)))+1/2*c*(e*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],2*c*(e*x^2+d)/(2*c*d-
e*(b+(-4*a*c+b^2)^(1/2))))*(1-b/(-4*a*c+b^2)^(1/2))/a/(1+q)/(2*c*d-e*(b+(-
4*a*c+b^2)^(1/2)))
```

3.405.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex^2)^q}{x(a+bx^2+cx^4)}\,dx$$
$$= \frac{(d+ex^2)^{1+q}\left(\frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,1+q,2+q,\frac{2c(d+ex^2)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}\right)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}+\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,1+q,\ldots\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)\ldots}\right.}{2a(1+q)}$$

input
```
Integrate[(d + e*x^2)^q/(x*(a + b*x^2 + c*x^4)),x]
```

output
```
((d + e*x^2)^(1 + q)*((c*(1 + b/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, 1
+ q, 2 + q, (2*c*(d + e*x^2))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)])/(2*c*
d + (-b + Sqrt[b^2 - 4*a*c])*e) + (c*(1 - b/Sqrt[b^2 - 4*a*c])*Hypergeomet
ric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e)])/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e) - Hypergeometric2F1[1, 1 + q, 2
+ q, 1 + (e*x^2)/d]/d))/(2*a*(1 + q))
```

3.405.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1578, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^q}{x(a+bx^2+cx^4)}\,dx$$
$$\Big\downarrow \; 1578$$
$$\frac{1}{2}\int \frac{(ex^2+d)^q}{x^2(cx^4+bx^2+a)}dx^2$$
$$\Big\downarrow \; 1200$$
$$\frac{1}{2}\int \left(\frac{(-cx^2-b)(ex^2+d)^q}{a(cx^4+bx^2+a)}+\frac{(ex^2+d)^q}{ax^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(d+ex^2\right)^{q+1}\text{Hypergeometric2F1}\left(1,q+1,q+2,\frac{2c(ex^2+d)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{a(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(d+\right.}{}\right.$$

input
```
Int[(d + e*x^2)^q/(x*(a + b*x^2 + c*x^4)),x]
```

output
```
((c*(1 + b/Sqrt[b^2 - 4*a*c])*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 +
 q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])/(a*(2*c
*d - (b - Sqrt[b^2 - 4*a*c])*e)*(1 + q)) + (c*(1 - b/Sqrt[b^2 - 4*a*c])*(d
 + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*
c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(a*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*
(1 + q)) - ((d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, 1 + (e*
x^2)/d])/(a*d*(1 + q)))/2
```

3.405.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.405.4 Maple [F]

$$\int \frac{(e\,x^2+d)^q}{x\,(c\,x^4+b\,x^2+a)}dx$$

input
```
int((e*x^2+d)^q/x/(c*x^4+b*x^2+a),x)
```

output
```
int((e*x^2+d)^q/x/(c*x^4+b*x^2+a),x)
```

3.405.5 Fricas [F]

$$\int \frac{(d+ex^2)^q}{x\,(a+bx^2+cx^4)}\,dx = \int \frac{(ex^2+d)^q}{(cx^4+bx^2+a)x}\,dx$$

input
```
integrate((e*x^2+d)^q/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
integral((e*x^2 + d)^q/(c*x^5 + b*x^3 + a*x), x)
```

3.405.6 Sympy [F]

$$\int \frac{(d+ex^2)^q}{x\,(a+bx^2+cx^4)}\,dx = \int \frac{(d+ex^2)^q}{x\,(a+bx^2+cx^4)}\,dx$$

input
```
integrate((e*x**2+d)**q/x/(c*x**4+b*x**2+a),x)
```

output
```
Integral((d + e*x**2)**q/(x*(a + b*x**2 + c*x**4)), x)
```

3.405.7 Maxima [F]

$$\int \frac{(d+ex^2)^q}{x(a+bx^2+cx^4)}\,dx = \int \frac{(ex^2+d)^q}{(cx^4+bx^2+a)x}\,dx$$

input
```
integrate((e*x^2+d)^q/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x), x)
```

3.405.8 Giac [F]

$$\int \frac{(d+ex^2)^q}{x(a+bx^2+cx^4)}\,dx = \int \frac{(ex^2+d)^q}{(cx^4+bx^2+a)x}\,dx$$

input
```
integrate((e*x^2+d)^q/x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x), x)
```

3.405.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^q}{x(a+bx^2+cx^4)}\,dx = \int \frac{(e\,x^2+d)^q}{x\,(c\,x^4+b\,x^2+a)}\,dx$$

input
```
int((d + e*x^2)^q/(x*(a + b*x^2 + c*x^4)),x)
```

output
```
int((d + e*x^2)^q/(x*(a + b*x^2 + c*x^4)), x)
```

3.406 $\int \frac{(d+ex^2)^q}{x^3(a+bx^2+cx^4)}\,dx$

3.406.1 Optimal result

Integrand size = 27, antiderivative size = 322

$$\begin{aligned}
\int \frac{(d+ex^2)^q}{x^3(a+bx^2+cx^4)}\,dx &= -\frac{c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^{1+q}\,\text{Hypergeometric2F1}\left(1,1+q,2+q,\frac{2c(d+ex^2)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2a^2\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+q)} \\
&\quad -\frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^{1+q}\,\text{Hypergeometric2F1}\left(1,1+q,2+q,\frac{2c(d+ex^2)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2a^2\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)(1+q)} \\
&\quad +\frac{b(d+ex^2)^{1+q}\,\text{Hypergeometric2F1}\left(1,1+q,2+q,1+\frac{ex^2}{d}\right)}{2a^2d(1+q)} \\
&\quad +\frac{e(d+ex^2)^{1+q}\,\text{Hypergeometric2F1}\left(2,1+q,2+q,1+\frac{ex^2}{d}\right)}{2ad^2(1+q)}
\end{aligned}$$

output
```
1/2*b*(e*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],1+e*x^2/d)/a^2/d/(1+q)+1/2*
e*(e*x^2+d)^(1+q)*hypergeom([2, 1+q],[2+q],1+e*x^2/d)/a/d^2/(1+q)-1/2*c*(e
*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],2*c*(e*x^2+d)/(2*c*d-e*(b-(-4*a*c+b
^2)^(1/2))))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a^2/(1+q)/(2*c*d-e*(b-(-4
*a*c+b^2)^(1/2)))-1/2*c*(e*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],2*c*(e*x^
2+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/
a^2/(1+q)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))
```

3.406.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.80

$$\int \frac{(d+ex^2)^q}{x^3(a+bx^2+cx^4)}\,dx$$
$$= \frac{(d+ex^2)^{1+q}\left(-\frac{c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,1+q,2+q,\frac{2c(d+ex^2)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}\right)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}-\frac{c\left(b+\frac{-b^2+2ac}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,1+\right.}{2cd-\left(b+\sqrt{b^2-4a\right.}}\right.}{2a^2(1+q)}$$

input
```
Integrate[(d + e*x^2)^q/(x^3*(a + b*x^2 + c*x^4)),x]
```

output
```
((d + e*x^2)^(1 + q)*(-((c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Hypergeom
etric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d + (-b + Sqrt[b^2 - 4*a*
c])*e)])/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)) - (c*(b + (-b^2 + 2*a*c)/Sq
rt[b^2 - 4*a*c])*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c
*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e) + (b
*Hypergeometric2F1[1, 1 + q, 2 + q, 1 + (e*x^2)/d])/d + (a*e*Hypergeometri
c2F1[2, 1 + q, 2 + q, 1 + (e*x^2)/d])/d^2))/(2*a^2*(1 + q))
```

3.406.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1578, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^q}{x^3(a+bx^2+cx^4)}\,dx$$
$$\Big\downarrow \text{ 1578}$$
$$\frac{1}{2}\int \frac{(ex^2+d)^q}{x^4(cx^4+bx^2+a)}dx^2$$
$$\Big\downarrow \text{ 1200}$$
$$\frac{1}{2}\int \left(\frac{(b^2+cx^2b-ac)(ex^2+d)^q}{a^2(cx^4+bx^2+a)}-\frac{b(ex^2+d)^q}{a^2x^2}+\frac{(ex^2+d)^q}{ax^4}\right)dx^2$$

$$\downarrow 2009$$

$$\frac{1}{2}\left(-\frac{c\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)(d+ex^2)^{q+1}\text{Hypergeometric2F1}\left(1,q+1,q+2,\frac{2c(ex^2+d)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{a^2(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}-\frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(d\cdots}{\cdots}\right.$$

input
```
Int[(d + e*x^2)^q/(x^3*(a + b*x^2 + c*x^4)),x]
```

output
```
(-((c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(d + e*x^2)^(1 + q)*Hypergeome
tric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b - Sqrt[b^2 - 4*a*c]
)*e)])/(a^2*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*(1 + q))) - (c*(b - (b^2 -
 2*a*c)/Sqrt[b^2 - 4*a*c])*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q,
 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(a^2*(2*c*
d - (b + Sqrt[b^2 - 4*a*c])*e)*(1 + q)) + (b*(d + e*x^2)^(1 + q)*Hypergeom
etric2F1[1, 1 + q, 2 + q, 1 + (e*x^2)/d])/(a^2*d*(1 + q)) + (e*(d + e*x^2)
^(1 + q)*Hypergeometric2F1[2, 1 + q, 2 + q, 1 + (e*x^2)/d])/(a*d^2*(1 + q)
))/2
```

3.406.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.406. $\int \frac{(d+ex^2)^q}{x^3(a+bx^2+cx^4)}\,dx$

3.406.4 Maple [F]

$$\int \frac{(e\,x^2+d)^q}{x^3\,(c\,x^4+b\,x^2+a)}dx$$

input
```
int((e*x^2+d)^q/x^3/(c*x^4+b*x^2+a),x)
```

output
```
int((e*x^2+d)^q/x^3/(c*x^4+b*x^2+a),x)
```

3.406.5 Fricas [F]

$$\int \frac{(d+ex^2)^q}{x^3\,(a+bx^2+cx^4)}\,dx = \int \frac{(ex^2+d)^q}{(cx^4+bx^2+a)x^3}\,dx$$

input
```
integrate((e*x^2+d)^q/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
integral((e*x^2 + d)^q/(c*x^7 + b*x^5 + a*x^3), x)
```

3.406.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^q}{x^3\,(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d)**q/x**3/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.406.7 Maxima [F]

$$\int \frac{(d+ex^2)^q}{x^3(a+bx^2+cx^4)}\,dx = \int \frac{(ex^2+d)^q}{(cx^4+bx^2+a)x^3}\,dx$$

input
```
integrate((e*x^2+d)^q/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x^3), x)
```

3.406.8 Giac [F]

$$\int \frac{(d+ex^2)^q}{x^3(a+bx^2+cx^4)}\,dx = \int \frac{(ex^2+d)^q}{(cx^4+bx^2+a)x^3}\,dx$$

input
```
integrate((e*x^2+d)^q/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x^3), x)
```

3.406.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^q}{x^3(a+bx^2+cx^4)}\,dx = \int \frac{(e\,x^2+d)^q}{x^3\,(c\,x^4+b\,x^2+a)}\,dx$$

input
```
int((d + e*x^2)^q/(x^3*(a + b*x^2 + c*x^4)),x)
```

output
```
int((d + e*x^2)^q/(x^3*(a + b*x^2 + c*x^4)), x)
```

3.407 $\int \frac{x^6(d+ex^2)^q}{a+bx^2+cx^4}\,dx$

3.407.1 Optimal result

Integrand size = 27, antiderivative size = 339

$$\begin{aligned}\int \frac{x^6(d+ex^2)^q}{a+bx^2+cx^4}\,dx &= \frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\operatorname{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c^2\left(b-\sqrt{b^2-4ac}\right)}\\
&\quad+\frac{\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\operatorname{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c^2\left(b+\sqrt{b^2-4ac}\right)}\\
&\quad-\frac{bx(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\operatorname{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},-\frac{ex^2}{d}\right)}{c^2}\\
&\quad+\frac{x^3(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\operatorname{Hypergeometric2F1}\left(\frac{3}{2},-q,\frac{5}{2},-\frac{ex^2}{d}\right)}{3c}\end{aligned}$$

output
```
-b*x*(e*x^2+d)^q*hypergeom([1/2, -q],[3/2],-e*x^2/d)/c^2/((1+e*x^2/d)^q)+1
/3*x^3*(e*x^2+d)^q*hypergeom([3/2, -q],[5/2],-e*x^2/d)/c/((1+e*x^2/d)^q)+x
*(e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-e*x^2/
d)*(b^2-a*c-b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c^2/((1+e*x^2/d)^q)/(b-(-4*
a*c+b^2)^(1/2))+x*(e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b+(-4*a*c+b^
2)^(1/2)),-e*x^2/d)*(b^2-a*c+b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c^2/((1+e*
x^2/d)^q)/(b+(-4*a*c+b^2)^(1/2))
```

3.407.2 Mathematica [F]

$$\int \frac{x^6(d+ex^2)^q}{a+bx^2+cx^4}\,dx=\int \frac{x^6(d+ex^2)^q}{a+bx^2+cx^4}\,dx$$

input
```
Integrate[(x^6*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

output
```
Integrate[(x^6*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x]
```

3.407.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1626, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(d+ex^2)^q}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{1626}$$

$$\int \left(\frac{(x^2(b^2-ac)+ab)(d+ex^2)^q}{c^2(a+bx^2+cx^4)}-\frac{b(d+ex^2)^q}{c^2}+\frac{x^2(d+ex^2)^q}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c^2\left(b-\sqrt{b^2-4ac}\right)}+\frac{x\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c^2\left(\sqrt{b^2-4ac}+b\right)}-\frac{bx(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},-\frac{ex^2}{d}\right)}{c^2}+\frac{x^3(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{Hypergeometric2F1}\left(\frac{3}{2},-q,\frac{5}{2},-\frac{ex^2}{d}\right)}{3c}$$

input
```
Int[(x^6*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

output
```
((b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF
1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(c^2
*(b - Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q) + ((b^2 - a*c + (b*(b^2 - 3*a*
c))/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(c^2*(b + Sqrt[b^2 - 4*a*c])*(1
+ (e*x^2)/d)^q) - (b*x*(d + e*x^2)^q*Hypergeometric2F1[1/2, -q, 3/2, -((e*
x^2)/d)])/(c^2*(1 + (e*x^2)/d)^q) + (x^3*(d + e*x^2)^q*Hypergeometric2F1[3
/2, -q, 5/2, -((e*x^2)/d)])/(3*c*(1 + (e*x^2)/d)^q)
```

3.407.3.1 Defintions of rubi rules used

rule 1626
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.407.4 Maple [F]

$$\int \frac{x^6(e\,x^2+d)^q}{c\,x^4+b\,x^2+a}dx$$

input
```
int(x^6*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

output
```
int(x^6*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

3.407.5 Fricas [F]

$$\int \frac{x^6(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q x^6}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^6*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
integral((e*x^2 + d)^q*x^6/(c*x^4 + b*x^2 + a), x)
```

3.407.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^6(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**6*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.407.7 Maxima [F]

$$\int \frac{x^6(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q x^6}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^6*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^q*x^6/(c*x^4 + b*x^2 + a), x)
```

3.407.8 Giac [F]

$$\int \frac{x^6(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q x^6}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^6*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^q*x^6/(c*x^4 + b*x^2 + a), x)
```

3.407.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{x^6\,(e\,x^2+d)^q}{c\,x^4+b\,x^2+a}\,dx$$

input
```
int((x^6*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x)
```

output
```
int((x^6*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x)
```

3.408 $\int \frac{x^4(d+ex^2)^q}{a+bx^2+cx^4}\,dx$

3.408.1 Optimal result

Integrand size = 27, antiderivative size = 273

$$\int \frac{x^4(d+ex^2)^q}{a+bx^2+cx^4}\,dx = -\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\operatorname{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c\left(b-\sqrt{b^2-4ac}\right)} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\operatorname{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c\left(b+\sqrt{b^2-4ac}\right)} + \frac{x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\operatorname{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},-\frac{ex^2}{d}\right)}{c}$$

output
```
x*(e*x^2+d)^q*hypergeom([1/2, -q],[3/2],-e*x^2/d)/c/((1+e*x^2/d)^q)-x*(e*x
^2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-e*x^2/d)*(b
+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/c/((1+e*x^2/d)^q)/(b-(-4*a*c+b^2)^(1/2))-
x*(e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)),-e*x^2
/d)*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c/((1+e*x^2/d)^q)/(b+(-4*a*c+b^2)^
(1/2))
```

3.408.2 Mathematica [F]

$$\int \frac{x^4(d+ex^2)^q}{a+bx^2+cx^4}\,dx=\int \frac{x^4(d+ex^2)^q}{a+bx^2+cx^4}\,dx$$

input
```
Integrate[(x^4*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

output
```
Integrate[(x^4*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x]
```

3.408.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1626, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(d+ex^2)^q}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \; 1626$$

$$\int \left(\frac{(d+ex^2)^q}{c}-\frac{(a+bx^2)\,(d+ex^2)^q}{c\,(a+bx^2+cx^4)}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{x\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c\left(b-\sqrt{b^2-4ac}\right)}-\frac{x\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c\left(\sqrt{b^2-4ac}+b\right)}+\frac{x(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},-\frac{ex^2}{d}\right)}{c}$$

input
```
Int[(x^4*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

output
```
-(((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF1[1/2, 1,
-q, 3/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(c*(b - Sqrt[b
^2 - 4*a*c])*(1 + (e*x^2)/d)^q)) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*
x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c
]), -((e*x^2)/d)])/(c*(b + Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q) + (x*(d +
 e*x^2)^q*Hypergeometric2F1[1/2, -q, 3/2, -((e*x^2)/d)])/(c*(1 + (e*x^2)/d
)^q)
```

3.408.3.1 Defintions of rubi rules used

rule 1626
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.408.4 Maple [F]

$$\int \frac{x^4(e\,x^2+d)^q}{c\,x^4+b\,x^2+a}dx$$

input
```
int(x^4*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

output
```
int(x^4*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

3.408.5 Fricas [F]

$$\int \frac{x^4(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q x^4}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^4*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
integral((e*x^2 + d)^q*x^4/(c*x^4 + b*x^2 + a), x)
```

3.408.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.408.7 Maxima [F]

$$\int \frac{x^4(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q x^4}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^4*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^q*x^4/(c*x^4 + b*x^2 + a), x)
```

3.408.8 Giac [F]

$$\int \frac{x^4(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q x^4}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^4*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^q*x^4/(c*x^4 + b*x^2 + a), x)
```

3.408.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex^2)^q}{a+bx^2+cx^4}\,dx=\int \frac{x^4\,{\left(e\,x^2+d\right)}^q}{c\,x^4+b\,x^2+a}\,dx$$

input
```
int((x^4*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x)
```

output
```
int((x^4*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x)
```

3.409 $\int \frac{x^2(d+ex^2)^q}{a+bx^2+cx^4}\,dx$

3.409.1 Optimal result

Integrand size = 27, antiderivative size = 162

$$\int \frac{x^2(d+ex^2)^q}{a+bx^2+cx^4}\,dx = -\frac{x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\operatorname{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{\sqrt{b^2-4ac}} + \frac{x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\operatorname{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{\sqrt{b^2-4ac}}$$

output
```
-x*(e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-e*x^
2/d)/((1+e*x^2/d)^q)/(-4*a*c+b^2)^(1/2)+x*(e*x^2+d)^q*AppellF1(1/2,1,-q,3/
2,-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)),-e*x^2/d)/((1+e*x^2/d)^q)/(-4*a*c+b^2)^(
1/2)
```

3.409.2 Mathematica [F]

$$\int \frac{x^2(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{x^2(d+ex^2)^q}{a+bx^2+cx^4}\,dx$$

input
```
Integrate[(x^2*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

output
```
Integrate[(x^2*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x]
```

3.409.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1626, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (d+ex^2)^q}{a+bx^2+cx^4} \, dx$$

$$\Big\downarrow \text{1626}$$

$$\int \left(\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q}{-\sqrt{b^2-4ac}+b+2cx^2}+\frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)(d+ex^2)^q}{\sqrt{b^2-4ac}+b+2cx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{\sqrt{b^2-4ac}}-\frac{x(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{\sqrt{b^2-4ac}}$$

input
```
Int[(x^2*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

output
```
-((x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*
a*c]), -((e*x^2)/d)])/(Sqrt[b^2 - 4*a*c]*(1 + (e*x^2)/d)^q)) + (x*(d + e*x
^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), -((e*x
^2)/d)])/(Sqrt[b^2 - 4*a*c]*(1 + (e*x^2)/d)^q)
```

3.409.3.1 Defintions of rubi rules used

rule 1626
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.409.4 Maple [F]

$$\int \frac{x^2(e\,x^2+d)^q}{c\,x^4+b\,x^2+a}dx$$

input
```
int(x^2*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

output
```
int(x^2*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

3.409.5 Fricas [F]

$$\int \frac{x^2(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q x^2}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^2*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
integral((e*x^2 + d)^q*x^2/(c*x^4 + b*x^2 + a), x)
```

3.409.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.409.7 Maxima [F]

$$\int \frac{x^2(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q x^2}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^2*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^q*x^2/(c*x^4 + b*x^2 + a), x)
```

3.409.8 Giac [F]

$$\int \frac{x^2(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q x^2}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^2*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^q*x^2/(c*x^4 + b*x^2 + a), x)
```

3.409.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{x^2\,(e\,x^2+d)^q}{c\,x^4+b\,x^2+a}\,dx$$

input
```
int((x^2*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x)
```

output
```
int((x^2*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x)
```

3.410 $\int \frac{(d+ex^2)^q}{a+bx^2+cx^4} dx$

3.410.1 Optimal result

Integrand size = 24, antiderivative size = 190

$$\int \frac{(d+ex^2)^q}{a+bx^2+cx^4} dx = -\frac{2cx(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{b^2-4ac-b\sqrt{b^2-4ac}} - \frac{2cx(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{b^2-4ac+b\sqrt{b^2-4ac}}$$

output
```
-2*c*x*(e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-
e*x^2/d)/((1+e*x^2/d)^q)/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))-2*c*x*(e*x^2+d)^
q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)),-e*x^2/d)/((1+e*x^
2/d)^q)/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

3.410.2 Mathematica [F]

$$\int \frac{(d+ex^2)^q}{a+bx^2+cx^4} dx = \int \frac{(d+ex^2)^q}{a+bx^2+cx^4} dx$$

input
```
Integrate[(d + e*x^2)^q/(a + b*x^2 + c*x^4),x]
```

output
```
Integrate[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x]
```

3.410.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1488, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^q}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \; 1488$$

$$\frac{2c\int \frac{(ex^2+d)^q}{2cx^2+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{2c\int \frac{(ex^2+d)^q}{2cx^2+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \; 334$$

$$\frac{2c(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\int \frac{\left(\frac{ex^2}{d}+1\right)^q}{2cx^2+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{2c(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\int \frac{\left(\frac{ex^2}{d}+1\right)^q}{2cx^2+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \; 333$$

$$\frac{2cx(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2cx(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

input
```
Int[(d + e*x^2)^q/(a + b*x^2 + c*x^4),x]
```

output
```
(2*c*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b - Sqrt[b^2 -
4*a*c]), -((e*x^2)/d)])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*(1 + (e
*x^2)/d)^q) - (2*c*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4
*a*c])*(1 + (e*x^2)/d)^q)
```

3.410.3.1 Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 1488
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/r)   Int[(d + e*x^2)^q/(b -
 r + 2*c*x^2), x], x] - Simp[2*(c/r)   Int[(d + e*x^2)^q/(b + r + 2*c*x^2),
 x], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

3.410.4 Maple [F]

$$\int \frac{(e\,x^{2}+d)^{q}}{c\,x^{4}+b\,x^{2}+a}dx$$

input
```
int((e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

output
```
int((e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

3.410.5 Fricas [F]

$$\int \frac{(d+ex^2)^q}{a+bx^2+cx^4}\,dx=\int \frac{(ex^2+d)^q}{cx^4+bx^2+a}\,dx$$

input
```
integrate((e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
integral((e*x^2 + d)^q/(c*x^4 + b*x^2 + a), x)
```

3.410.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d)**q/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.410.7 Maxima [F]

$$\int \frac{(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q}{cx^4+bx^2+a}\,dx$$

input
```
integrate((e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^q/(c*x^4 + b*x^2 + a), x)
```

3.410.8 Giac [F]

$$\int \frac{(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^q}{cx^4+bx^2+a}\,dx$$

input
```
integrate((e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^q/(c*x^4 + b*x^2 + a), x)
```

3.410.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^q}{a+bx^2+cx^4}\,dx=\int \frac{{\left(e\,x^2+d\right)}^q}{c\,x^4+b\,x^2+a}\,dx$$

input
```
int((d + e*x^2)^q/(a + b*x^2 + c*x^4),x)
```

output
```
int((d + e*x^2)^q/(a + b*x^2 + c*x^4), x)
```

3.411 $\int \frac{(d+ex^2)^q}{x^2(a+bx^2+cx^4)}\,dx$

3.411.1 Optimal result

Integrand size = 27, antiderivative size = 264

$$\begin{aligned}\int \frac{(d+ex^2)^q}{x^2(a+bx^2+cx^4)}\,dx &= -\frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\operatorname{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a\left(b-\sqrt{b^2-4ac}\right)}\\&\quad-\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\operatorname{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a\left(b+\sqrt{b^2-4ac}\right)}\\&\quad-\frac{(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\operatorname{Hypergeometric2F1}\left(-\frac{1}{2},-q,\frac{1}{2},-\frac{ex^2}{d}\right)}{ax}\end{aligned}$$

output
```
-(e*x^2+d)^q*hypergeom([-1/2, -q],[1/2],-e*x^2/d)/a/x/((1+e*x^2/d)^q)-c*x*
(e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-e*x^2/d
)*(1+b/(-4*a*c+b^2)^(1/2))/a/((1+e*x^2/d)^q)/(b-(-4*a*c+b^2)^(1/2))-c*x*(e
*x^2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)),-e*x^2/d)*
(1-b/(-4*a*c+b^2)^(1/2))/a/((1+e*x^2/d)^q)/(b+(-4*a*c+b^2)^(1/2))
```

3.411.2 Mathematica [F]

$$\int \frac{(d+ex^2)^q}{x^2(a+bx^2+cx^4)}\,dx = \int \frac{(d+ex^2)^q}{x^2(a+bx^2+cx^4)}\,dx$$

input
```
Integrate[(d + e*x^2)^q/(x^2*(a + b*x^2 + c*x^4)),x]
```

output
```
Integrate[(d + e*x^2)^q/(x^2*(a + b*x^2 + c*x^4)), x]
```

3.411.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1626, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^q}{x^2(a+bx^2+cx^4)}\,dx$$

$$\Big\downarrow \text{1626}$$

$$\int \left(\frac{(-b-cx^2)(d+ex^2)^q}{a(a+bx^2+cx^4)}+\frac{(d+ex^2)^q}{ax^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{cx\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a\left(b-\sqrt{b^2-4ac}\right)}-\frac{cx\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a\left(\sqrt{b^2-4ac}+b\right)}-\frac{(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{Hypergeometric2F1}\left(-\frac{1}{2},-q,\frac{1}{2},-\frac{ex^2}{d}\right)}{ax}$$

input
```
Int[(d + e*x^2)^q/(x^2*(a + b*x^2 + c*x^4)),x]
```

output
```
-((c*(1 + b/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (
-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(a*(b - Sqrt[b^2 - 4*a*c
])*(1 + (e*x^2)/d)^q)) - (c*(1 - b/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*Appe
llF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(
a*(b + Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q) - ((d + e*x^2)^q*Hypergeometr
ic2F1[-1/2, -q, 1/2, -((e*x^2)/d)])/(a*x*(1 + (e*x^2)/d)^q)
```

3.411.3.1 Defintions of rubi rules used

rule 1626
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.411.4 Maple [F]

$$\int \frac{(e\,x^2+d)^q}{x^2\,(c\,x^4+b\,x^2+a)}dx$$

input
```
int((e*x^2+d)^q/x^2/(c*x^4+b*x^2+a),x)
```

output
```
int((e*x^2+d)^q/x^2/(c*x^4+b*x^2+a),x)
```

3.411.5 Fricas [F]

$$\int \frac{(d+ex^2)^q}{x^2\,(a+bx^2+cx^4)}\,dx=\int \frac{(ex^2+d)^q}{(cx^4+bx^2+a)x^2}\,dx$$

input
```
integrate((e*x^2+d)^q/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
integral((e*x^2 + d)^q/(c*x^6 + b*x^4 + a*x^2), x)
```

3.411.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^q}{x^2(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d)**q/x**2/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.411.7 Maxima [F]

$$\int \frac{(d+ex^2)^q}{x^2(a+bx^2+cx^4)}\,dx = \int \frac{(ex^2+d)^q}{(cx^4+bx^2+a)x^2}\,dx$$

input
```
integrate((e*x^2+d)^q/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x^2), x)
```

3.411.8 Giac [F]

$$\int \frac{(d+ex^2)^q}{x^2(a+bx^2+cx^4)}\,dx = \int \frac{(ex^2+d)^q}{(cx^4+bx^2+a)x^2}\,dx$$

input
```
integrate((e*x^2+d)^q/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x^2), x)
```

3.411.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^q}{x^2(a+bx^2+cx^4)}\,dx = \int \frac{(e\,x^2+d)^q}{x^2\,(c\,x^4+b\,x^2+a)}\,dx$$

input
```
int((d + e*x^2)^q/(x^2*(a + b*x^2 + c*x^4)),x)
```

output
```
int((d + e*x^2)^q/(x^2*(a + b*x^2 + c*x^4)), x)
```

3.412 $\int \frac{(d+ex^2)^q}{x^4(a+bx^2+cx^4)}\,dx$

3.412.1 Optimal result

Integrand size = 27, antiderivative size = 328

$$
\begin{aligned}
&\int \frac{(d+ex^2)^q}{x^4(a+bx^2+cx^4)}\,dx \\
&= \frac{c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\operatorname{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a^2\left(b-\sqrt{b^2-4ac}\right)} \\
&\quad+\frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\operatorname{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a^2\left(b+\sqrt{b^2-4ac}\right)} \\
&\quad-\frac{(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\operatorname{Hypergeometric2F1}\left(-\frac{3}{2},-q,-\frac{1}{2},-\frac{ex^2}{d}\right)}{3ax^3} \\
&\quad+\frac{b(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\operatorname{Hypergeometric2F1}\left(-\frac{1}{2},-q,\frac{1}{2},-\frac{ex^2}{d}\right)}{a^2x}
\end{aligned}
$$

output
```
-1/3*(e*x^2+d)^q*hypergeom([-3/2, -q],[-1/2],-e*x^2/d)/a/x^3/((1+e*x^2/d)^
q)+b*(e*x^2+d)^q*hypergeom([-1/2, -q],[1/2],-e*x^2/d)/a^2/x/((1+e*x^2/d)^q
)+c*x*(e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-e
*x^2/d)*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a^2/((1+e*x^2/d)^q)/(b-(-4*a*c
+b^2)^(1/2))+c*x*(e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b+(-4*a*c+b^2
)^(1/2)),-e*x^2/d)*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/a^2/((1+e*x^2/d)^q)/
(b+(-4*a*c+b^2)^(1/2))
```

3.412.2 Mathematica [F]

$$\int \frac{(d+ex^2)^q}{x^4(a+bx^2+cx^4)}\,dx = \int \frac{(d+ex^2)^q}{x^4(a+bx^2+cx^4)}\,dx$$

input
```
Integrate[(d + e*x^2)^q/(x^4*(a + b*x^2 + c*x^4)),x]
```

output
```
Integrate[(d + e*x^2)^q/(x^4*(a + b*x^2 + c*x^4)), x]
```

3.412.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1626, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^q}{x^4(a+bx^2+cx^4)}\,dx$$

$$\Big\downarrow \ 1626$$

$$\int \left(\frac{(-ac+b^2+bcx^2)(d+ex^2)^q}{a^2(a+bx^2+cx^4)} - \frac{b(d+ex^2)^q}{a^2x^2} + \frac{(d+ex^2)^q}{ax^4}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{cx\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a^2\left(b-\sqrt{b^2-4ac}\right)}+\frac{cx\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a^2\left(\sqrt{b^2-4ac}+b\right)}+\frac{b(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{Hypergeometric2F1}\left(-\frac{1}{2},-q,\frac{1}{2},-\frac{ex^2}{d}\right)}{a^2x}-\frac{(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{Hypergeometric2F1}\left(-\frac{3}{2},-q,-\frac{1}{2},-\frac{ex^2}{d}\right)}{3ax^3}$$

input
```
Int[(d + e*x^2)^q/(x^4*(a + b*x^2 + c*x^4)),x]
```

output
```
(c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF1[1/2, 1,
-q, 3/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(a^2*(b - Sqrt
[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q) + (c*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c
])*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c]), -((e*x^2)/d)])/(a^2*(b + Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q) - ((
d + e*x^2)^q*Hypergeometric2F1[-3/2, -q, -1/2, -((e*x^2)/d)])/(3*a*x^3*(1
+ (e*x^2)/d)^q) + (b*(d + e*x^2)^q*Hypergeometric2F1[-1/2, -q, 1/2, -((e*x
^2)/d)])/(a^2*x*(1 + (e*x^2)/d)^q)
```

3.412.3.1 Defintions of rubi rules used

rule 1626
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.412.4 Maple [F]

$$\int \frac{(e\,x^2+d)^q}{x^4\,(c\,x^4+b\,x^2+a)}dx$$

input
```
int((e*x^2+d)^q/x^4/(c*x^4+b*x^2+a),x)
```

output
```
int((e*x^2+d)^q/x^4/(c*x^4+b*x^2+a),x)
```

3.412.5 Fricas [F]

$$\int \frac{(d+ex^2)^q}{x^4\,(a+bx^2+cx^4)}\,dx = \int \frac{(ex^2+d)^q}{(cx^4+bx^2+a)x^4}\,dx$$

input
```
integrate((e*x^2+d)^q/x^4/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
integral((e*x^2 + d)^q/(c*x^8 + b*x^6 + a*x^4), x)
```

3.412.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^q}{x^4(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d)**q/x**4/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.412.7 Maxima [F]

$$\int \frac{(d+ex^2)^q}{x^4(a+bx^2+cx^4)}\,dx = \int \frac{(ex^2+d)^q}{(cx^4+bx^2+a)x^4}\,dx$$

input
```
integrate((e*x^2+d)^q/x^4/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x^4), x)
```

3.412.8 Giac [F]

$$\int \frac{(d+ex^2)^q}{x^4(a+bx^2+cx^4)}\,dx = \int \frac{(ex^2+d)^q}{(cx^4+bx^2+a)x^4}\,dx$$

input
```
integrate((e*x^2+d)^q/x^4/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x^4), x)
```

3.412.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^q}{x^4\,(a+bx^2+cx^4)}\,dx = \int \frac{(e\,x^2+d)^q}{x^4\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

input
```
int((d + e*x^2)^q/(x^4*(a + b*x^2 + c*x^4)),x)
```

output
```
int((d + e*x^2)^q/(x^4*(a + b*x^2 + c*x^4)), x)
```

3.413 $\int \frac{\sqrt{1+\frac{1}{c^2x^2}}}{\sqrt{1-c^4x^4}}\,dx$

3.413.1 Optimal result

Integrand size = 28, antiderivative size = 40

$$\int \frac{\sqrt{1+\frac{1}{c^2x^2}}}{\sqrt{1-c^4x^4}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{1-c^4x^4}}{c\sqrt{1+\frac{1}{c^2x^2}}\,x}\right)}{c}$$

output
```
-arctanh((-c^4*x^4+1)^(1/2)/c/x/(1+1/c^2/x^2)^(1/2))/c
```

3.413.2 Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt{1+\frac{1}{c^2x^2}}}{\sqrt{1-c^4x^4}}\,dx = -\frac{\sqrt{1+\frac{1}{c^2x^2}}\,x\operatorname{arctanh}\left(\frac{\sqrt{1-c^4x^4}}{\sqrt{1+c^2x^2}}\right)}{\sqrt{1+c^2x^2}}$$

input
```
Integrate[Sqrt[1 + 1/(c^2*x^2)]/Sqrt[1 - c^4*x^4],x]
```

output
```
-((Sqrt[1 + 1/(c^2*x^2)]*x*ArcTanh[Sqrt[1 - c^4*x^4]/Sqrt[1 + c^2*x^2]])/S
qrt[1 + c^2*x^2])
```

3.413.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1778, 1388, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{1}{c^2x^2}+1}}{\sqrt{1-c^4x^4}}dx$$

$$\Big\downarrow \text{1778}$$

$$\frac{x\sqrt{\frac{1}{c^2x^2}+1}\int \frac{\sqrt{c^2x^2+1}}{x\sqrt{1-c^4x^4}}dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{1388}$$

$$\frac{x\sqrt{\frac{1}{c^2x^2}+1}\int \frac{1}{x\sqrt{1-c^2x^2}}dx}{\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{243}$$

$$\frac{x\sqrt{\frac{1}{c^2x^2}+1}\int \frac{1}{x^2\sqrt{1-c^2x^2}}dx^2}{2\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{x\sqrt{\frac{1}{c^2x^2}+1}\int \frac{1}{\frac{1}{c^2}-\frac{x^4}{c^2}}d\sqrt{1-c^2x^2}}{c^2\sqrt{c^2x^2+1}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{x\sqrt{\frac{1}{c^2x^2}+1}\text{arctanh}\left(\sqrt{1-c^2x^2}\right)}{\sqrt{c^2x^2+1}}$$

input
```
Int[Sqrt[1 + 1/(c^2*x^2)]/Sqrt[1 - c^4*x^4],x]
```

output
```
-((Sqrt[1 + 1/(c^2*x^2)]*x*ArcTanh[Sqrt[1 - c^2*x^2]])/Sqrt[1 + c^2*x^2])
```

3.413.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

rule 1778
```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sym
bol] :> Simp[(e^IntPart[q]*((d + e*x^mn)^FracPart[q]/(1 + d*(1/(x^mn*e)))^F
racPart[q]))/x^(mn*FracPart[q])   Int[x^(mn*q)*(1 + d*(1/(x^mn*e)))^q*(a +
c*x^n2)^p, x], x] /; FreeQ[{a, c, d, e, mn, p, q}, x] && EqQ[n2, -2*mn] &&
 !IntegerQ[p] &&  !IntegerQ[q] && PosQ[n2]
```

3.413.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.38 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.52

method	result	size
default	$-\frac{\sqrt{\frac{c^{2} x^{2}+1}{c^{2} x^{2}}}\, x \sqrt{-c^{4} x^{4}+1}\, \operatorname{csgn}\left(\frac{1}{c}\right) \ln\left(\frac{2 \operatorname{csgn}\left(\frac{1}{c}\right) c \sqrt{-\frac{c^{2} x^{2}-1}{c^{2}}}+2}{c^{2} x}\right)}{\left(c^{2} x^{2}+1\right) \sqrt{-\frac{c^{2} x^{2}-1}{c^{2}}}\, c}$	101

input
```
int((1+1/c^2/x^2)^(1/2)/(-c^4*x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-((c^2*x^2+1)/c^2/x^2)^(1/2)*x*(-c^4*x^4+1)^(1/2)*csgn(1/c)*ln(2*(csgn(1/c
)*c*(-1/c^2*(c^2*x^2-1))^(1/2)+1)/c^2/x)/(c^2*x^2+1)/(-1/c^2*(c^2*x^2-1))^
(1/2)/c
```

3.413.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(36) = 72$.

Time = 0.25 (sec) , antiderivative size = 120, normalized size of antiderivative = 3.00

$$\int \frac{\sqrt{1+\frac{1}{c^2x^2}}}{\sqrt{1-c^4x^4}}\,dx = -\frac{\log\left(\frac{c^2x^2+\sqrt{-c^4x^4+1}cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{c^2x^2+1}\right)-\log\left(-\frac{c^2x^2-\sqrt{-c^4x^4+1}cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{c^2x^2+1}\right)}{2\,c}$$

input
```
integrate((1+1/c^2/x^2)^(1/2)/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(log((c^2*x^2 + sqrt(-c^4*x^4 + 1)*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2))
+ 1)/(c^2*x^2 + 1)) - log(-(c^2*x^2 - sqrt(-c^4*x^4 + 1)*c*x*sqrt((c^2*x^2
 + 1)/(c^2*x^2)) + 1)/(c^2*x^2 + 1)))/c
```

3.413.6 Sympy [F]

$$\int \frac{\sqrt{1+\frac{1}{c^2x^2}}}{\sqrt{1-c^4x^4}}\,dx = \int \frac{\sqrt{1+\frac{1}{c^2x^2}}}{\sqrt{-(cx-1)(cx+1)(c^2x^2+1)}}\,dx$$

input
```
integrate((1+1/c**2/x**2)**(1/2)/(-c**4*x**4+1)**(1/2),x)
```

output
```
Integral(sqrt(1 + 1/(c**2*x**2))/sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1)
), x)
```

3.413.7 Maxima [F]

$$\int \frac{\sqrt{1+\frac{1}{c^2x^2}}}{\sqrt{1-c^4x^4}}\,dx = \int \frac{\sqrt{\frac{1}{c^2x^2}+1}}{\sqrt{-c^4x^4+1}}\,dx$$

input
```
integrate((1+1/c^2/x^2)^(1/2)/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(1/(c^2*x^2) + 1)/sqrt(-c^4*x^4 + 1), x)
```

3.413.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{1+\frac{1}{c^2x^2}}}{\sqrt{1-c^4x^4}}\,dx = -\frac{\left(\log\left(\sqrt{-c^2x^2+1}+1\right)-\log\left(-\sqrt{-c^2x^2+1}+1\right)\right)|c|}{2\,c^2}$$

input
```
integrate((1+1/c^2/x^2)^(1/2)/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(log(sqrt(-c^2*x^2 + 1) + 1) - log(-sqrt(-c^2*x^2 + 1) + 1))*abs(c)/c
^2
```

3.413.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+\frac{1}{c^2x^2}}}{\sqrt{1-c^4x^4}}\,dx = \int \frac{\sqrt{\frac{1}{c^2\,x^2}+1}}{\sqrt{1-c^4\,x^4}}\,dx$$

input
```
int((1/(c^2*x^2) + 1)^(1/2)/(1 - c^4*x^4)^(1/2),x)
```

output
```
int((1/(c^2*x^2) + 1)^(1/2)/(1 - c^4*x^4)^(1/2), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
    return 1
  elif isinstance(expn,list):
     return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
  elif  is_sqrt(expn):
     if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
        return 1
     else:
        return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
  elif isinstance(expn,Pow):   #type(expn,'`^`')
     if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
        return expnType(expn.args[0])   #ExpnType(op(1,expn))
     elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
           return 1
        else:
           return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
     else:
        return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
  elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
     m1 = expnType(expn.args[0])
     m2 = expnType(list(expn.args[1:]))
     return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
     return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
  elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
     m1 = max(map(expnType, list(expn.args)))
     return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
  elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
     m1 = max(map(expnType, list(expn.args)))
     return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  elif is_appell_function(expn.func):
     m1 = max(map(expnType, list(expn.args)))
     return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  elif isinstance(expn,RootSum):
     m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
     return max(7,m1)
  elif str(expn).find("Integral") != -1:
     m1 = max(map(expnType, list(expn.args)))
     return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
     return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```